Assignment 1

Due Date: 2019/03/20, 1:30 PM

Remark. In this assignment, all the sequences are sequence of real numbers, and \lim is a shorthand of $\lim_{n\to\infty}$.

- **1.** Suppose that a subset S of \mathbb{R} contains a minimum $m \in \mathbb{R}$, namely $\min(S) = m$. Prove that $\inf(S) = m$.
- 2. Find infimum and supremum of the following sets, and prove your answer.
- (1) $(1, 2) = \{x \in \mathbb{R} : 1 < x < 2\}$
- $(2) \left\{ \frac{1}{1+n^2} : n \in \mathbb{N} \right\}$
- (3) $\{(-1)^n + (-1/2)^m : n, m \in \mathbb{N}\}$
- **3.** Define $X Y = \{x y : x \in X, y \in Y\}$. Let A and B be bounded and non-empty subsets of \mathbb{R} .
- (1) Prove that A B is also bounded.
- (2) Express $\sup(A B)$ in terms of $\sup A$, $\sup B$, $\inf A$, and $\inf B$.
- **4.** Suppose that a sequence $\langle a_n \rangle$ satisfies $\lim a_n = 0$, and that another sequence $\langle s_n \rangle$ satisfies $|s_n s| < a_n$ for all $n \in \mathbb{N}$. Prove that $\lim s_n = s$.

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- ${f 5.}$ Find the following limits and verify your answer rigorously.
- (1) $\lim \frac{\sqrt{n}}{2\sqrt{n}+7}$
- (2) $\lim \frac{2n^5 + \cos(n^8 + 1)}{n^5 + 1}$
- (3) $\lim \frac{3n^2+n(-1)^n}{n^2+2}$

- **6.** Prove that the sequence $\langle a_n \rangle$ defined by $a_n = \frac{(-2)^n + n}{2^n}$ is divergent.
- 7. Let $\langle s_n \rangle$ be a convergent sequence and let $\lim s_n = s$. Prove that $\lim f(s_n) = f(s)$ where f is given by:

(1)
$$f(x) = x^2 + 4x + 5$$

(2)
$$f(x) = \sqrt{\frac{1}{1+x^2}}$$

(3)
$$f(x) = x^{2019}$$

8. Let $\langle a_n \rangle$ be a divergent sequence. Define another sequence $\langle b_n \rangle$ as

$$b_n = \frac{1}{n} \sum_{k=1}^n a_k \ ; \ n \in \mathbb{N} \ .$$

Prove, or disprove by a counter example that the following statement: the sequence $\langle b_n \rangle$ is divergent.