



“All things being equal, the simplest solution tends to be the best one.”

William of Ockham

Dimensionality Reduction: Overview

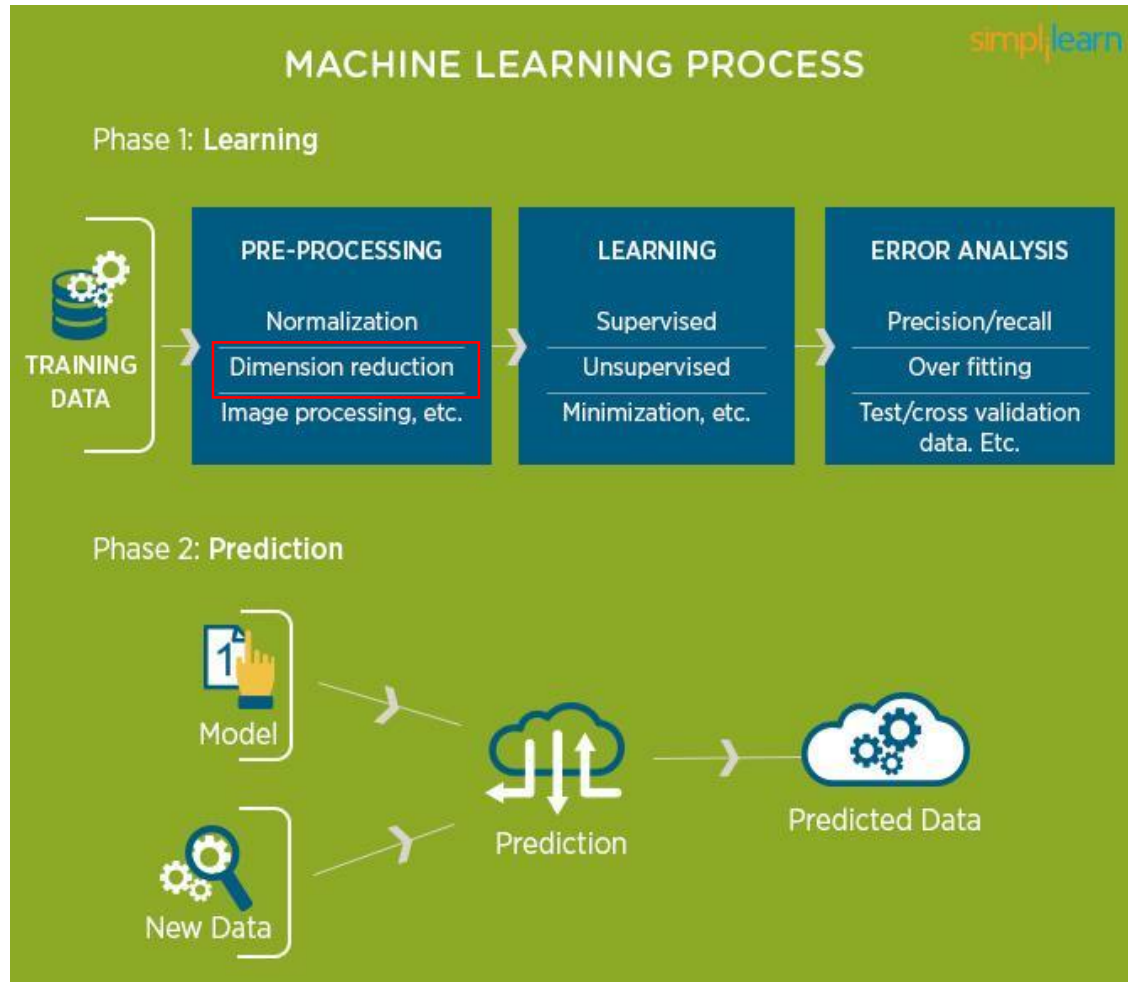
Pilsung Kang

School of Industrial Management Engineering

Korea University

Data Analytics Process

- Process of Business Analytics with Machine Learning



High-dimensional Data

- Examples of high dimensional data

Document classification:

Billions of documents x Thousands/
Millions of words/bigrams matrix



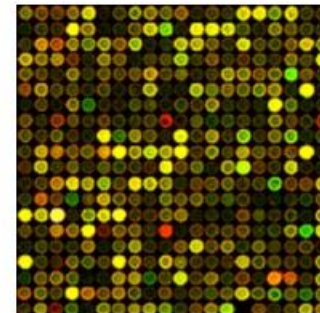
Recommendation systems:

480,189 users x 17,770 movies matrix



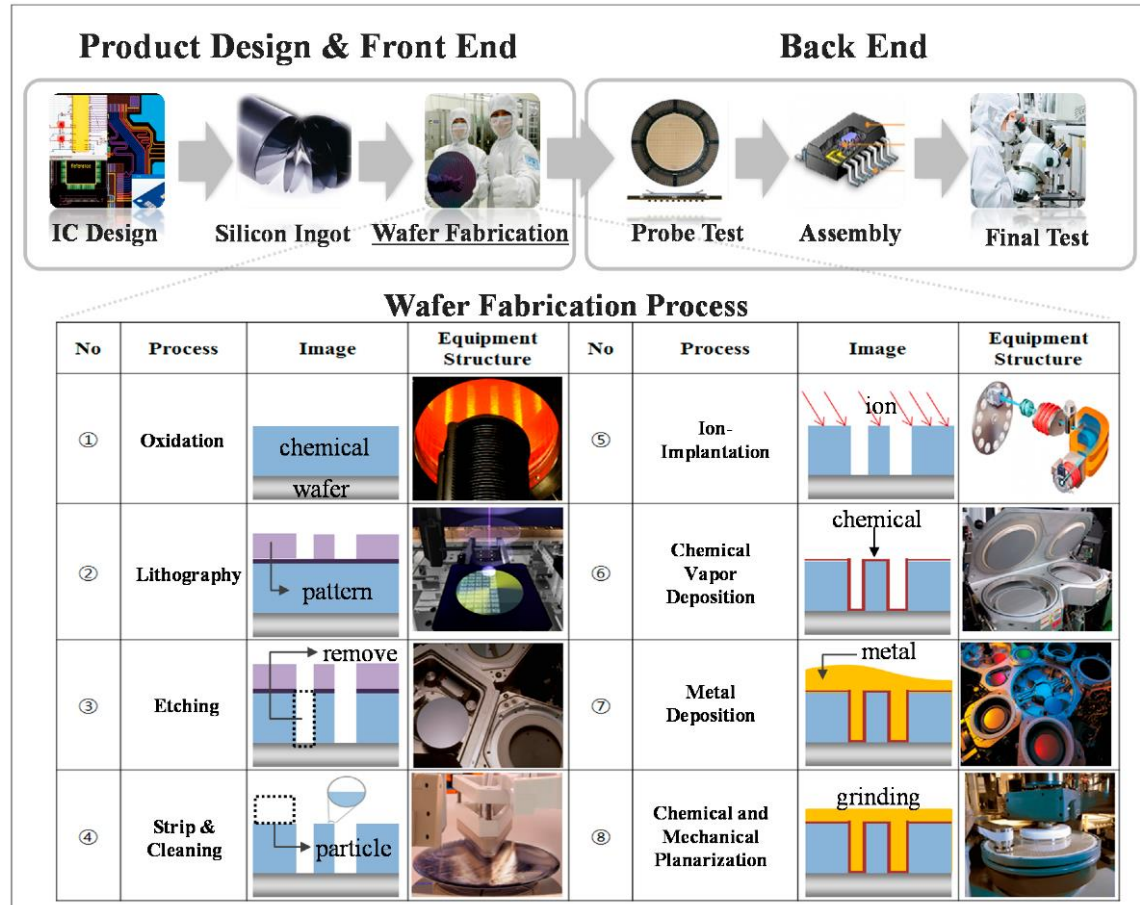
Clustering gene expression profiles:

10,000 genes x 1,000 conditions



High-dimensional Data

- Examples of high dimensional data in Manufacturing



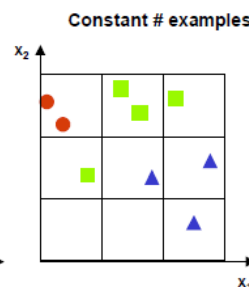
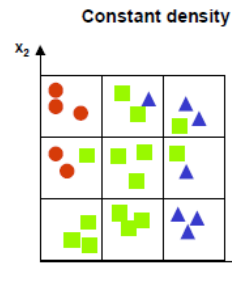
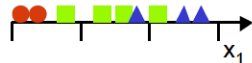
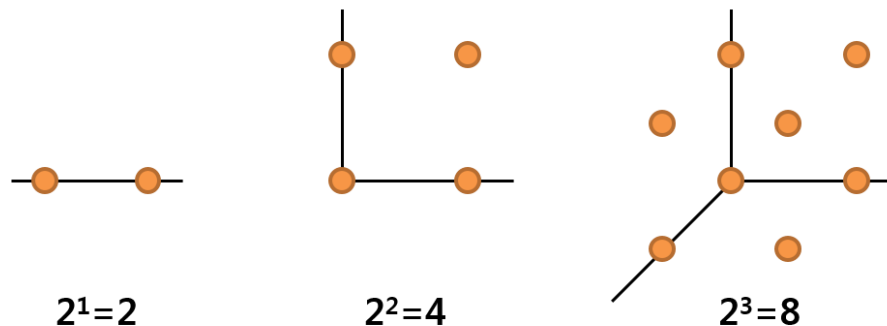
Park, S. H., Kim, S., & Baek, J. G. (2018). Kernel-Density-Based Particle Defect Management for Semiconductor Manufacturing Facilities. *Applied Sciences*, 8(2), 224.

Dimensionality Reduction: Overview

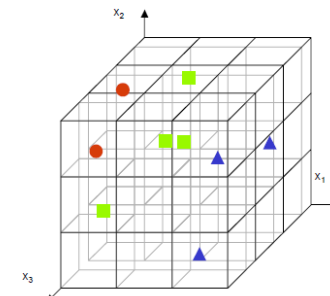
- Curse of dimensionality

- ✓ The number of instances increases exponentially to achieve the same explanation ability when the number of variables increases

“If there are various logical ways to explain a certain phenomenon, the simplest is the best” - Occam’s Razor



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Dimensionality Reduction: Overview

- Curse of dimensionality

- ✓ Sometimes, an intrinsic dimension is relatively low compared to the original dimension.

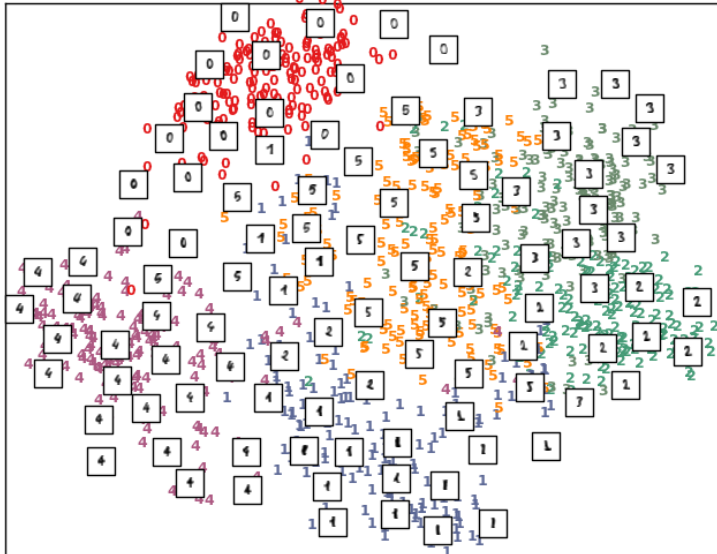
- Ex: handwritten digits in a 16 by 16 pixel (256 dimensions)
 - Reduced to two dimensions by PCA and ISOMAP



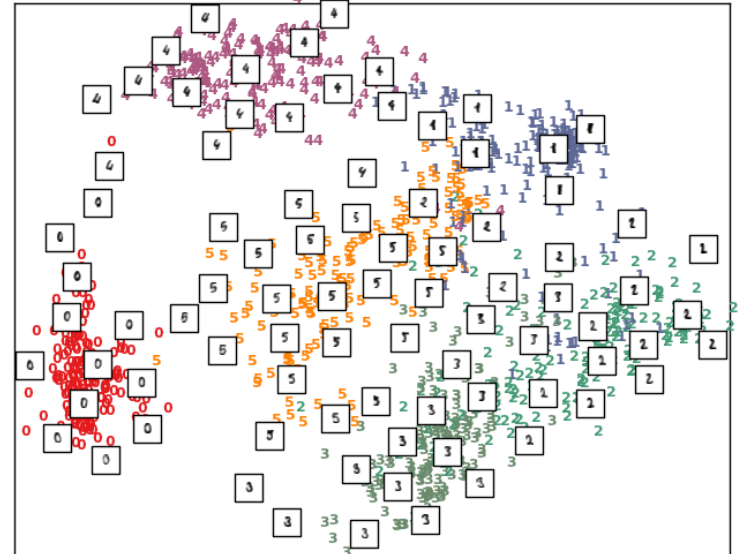
Dimensionality Reduction: Overview

- Curse of dimensionality
 - ✓ Sometimes, an intrinsic dimension is relatively low compared to the original dimension.
 - Ex: handwritten digits in a 16 by 16 pixel (256 dimensions)
 - Reduced to two dimensions by PCA and ISOMAP

Principal Components projection of the digits (time 0.01s)

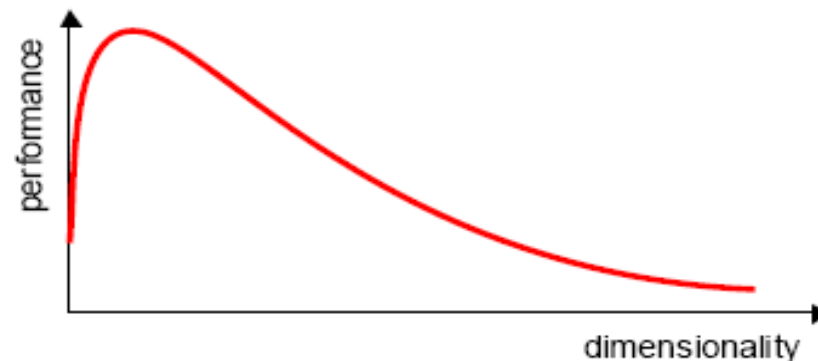


Isomap projection of the digits (time 1.51s)



Dimensionality Reduction: Overview

- Curse of dimensionality
 - ✓ Problems caused by high-dimensionality
 - Increase the probability of having noise in data → degenerate the prediction performance
 - Increase computational burden for training/applying prediction models
 - Require more number of examples to secure generalization ability of prediction model
 - ✓ To resolve the curse of dimensionality
 - Utilize domain knowledge
 - Use a regularization term in objective function
 - Employ a quantitative reduction technique



Dimensionality Reduction: Overview

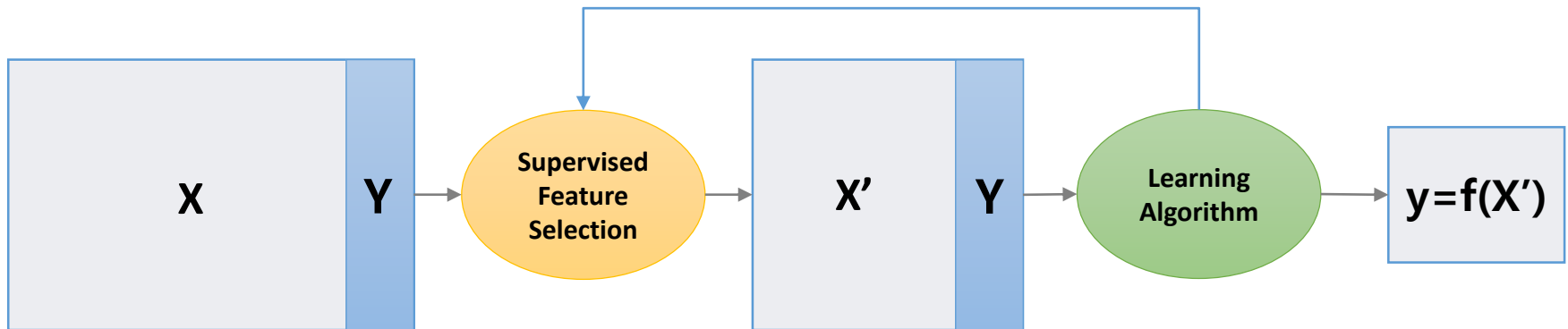
- Backgrounds
 - ✓ Theoretically, model performance improves when the number of variables increases
(Under variable independence condition)
 - ✓ In reality, model performance degenerates due to variable dependence, existence of noise, etc.
- Purpose
 - ✓ Identify a subset of variables that best fit the model
- Effect
 - ✓ Remove correlations between variables
 - ✓ Simplified post-processing
 - ✓ Remove redundant or unnecessary variables while keeping relevant information
 - ✓ Visualization can be possible

Dimensionality Reduction: Overview

- Supervised vs. Unsupervised Dimensionality Reduction

- ✓ Supervised dimensionality reduction

- Use data mining models to verify the reduced dimensions
- Dimensionality reduction results can be different according to the data mining algorithms employed

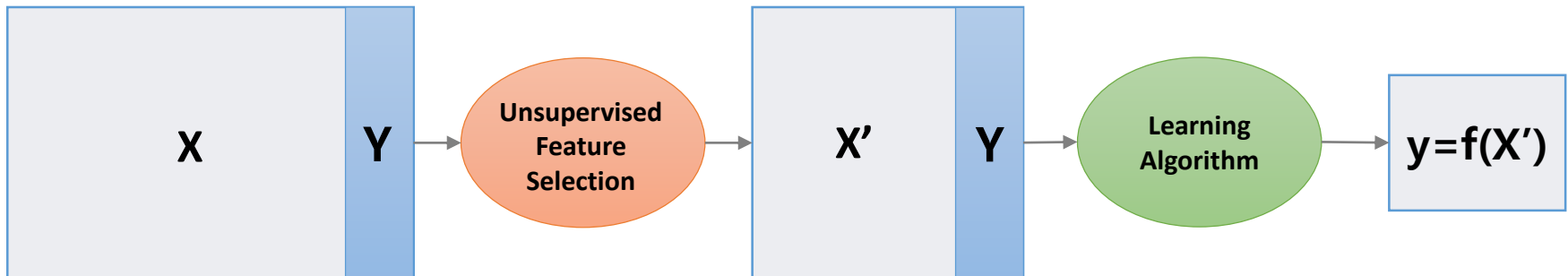


Dimensionality Reduction: Overview

- Supervised vs. Unsupervised Dimensionality Reduction

- ✓ Unsupervised dimensionality reduction

- Find a set of coordinate systems in a lower dimension that preserve the information (e.g., variance, distance, etc.) in the original input space as much as possible
- Do not use data mining models during the process
- Dimensionality reduction results are identical if the data and method is same



Dimensionality Reduction: Overview

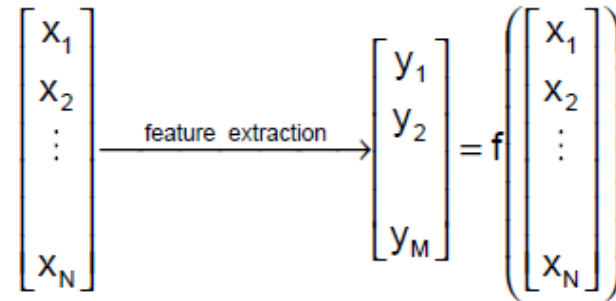
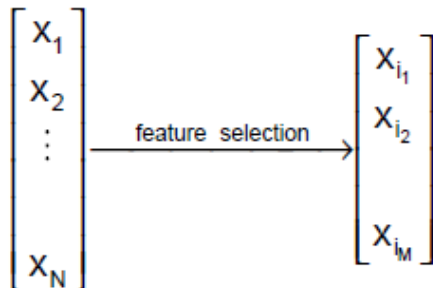
- Dimensionality reduction techniques

- ✓ Variable/feature selection

- Select a subset of variables from the original variable set
 - Filter – Variable selection and model training are independent
 - Wrapper – Variable selection is done to optimize the result of the considered data mining model

- ✓ Variable/feature extraction

- Extract a new smaller set of variables that preserve the characteristics of the original data
 - Performance metric that is independent from data mining models is used



Dimensionality Reduction: Overview

- Selection vs. Extraction

✓ Conceptual difference between variable selection and variable extraction

X_1	X_2	X_3	...	X_n
...
...
...
...
...

Variable selection

X_1	X_5	X_8
...
...
...
...
...

Variable extraction

Z_1	Z_2	Z_3
...
...
...
...
...

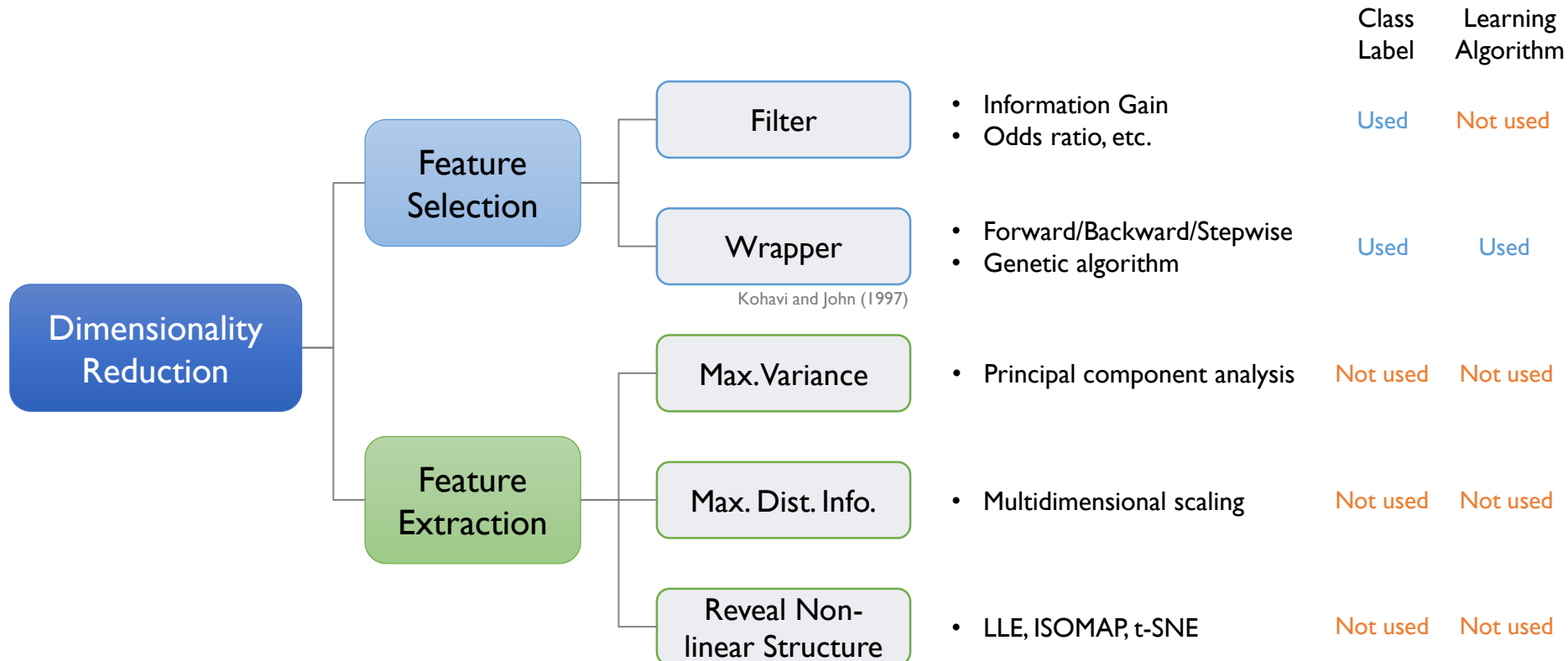
$$Z_1 = X_1 + 0.2 * X_2$$

$$Z_2 = X_3 - 2 * X_5$$

$$Z_3 = X_4 + X_6 - X_9$$

Dimensionality Reduction: Overview

- A simplified taxonomy of dimensionality reduction techniques



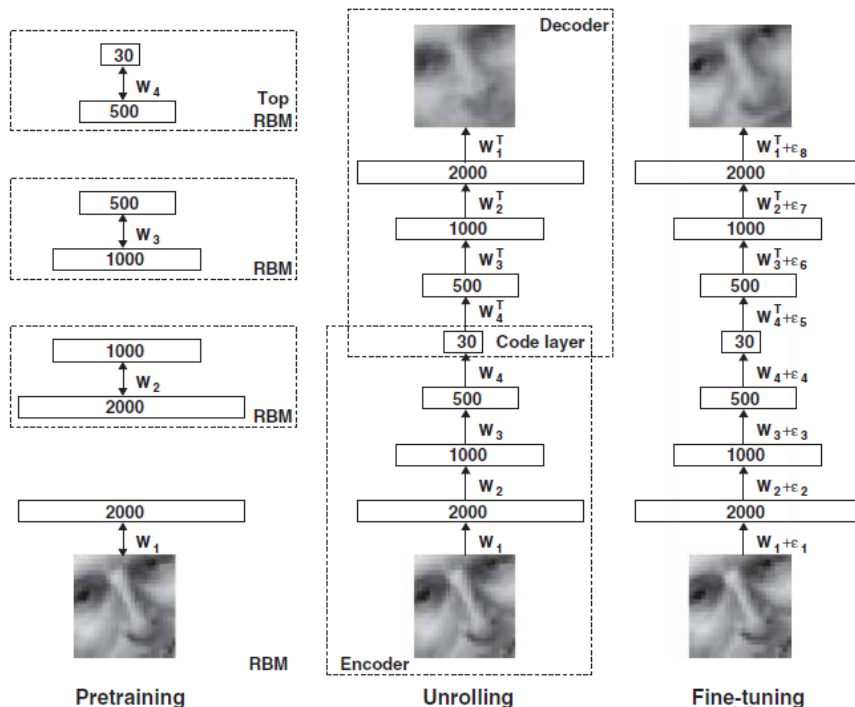
Dimensionality Reduction: Recent Trends

- Representation learning: Deep auto-encoder

- ✓ Try to extract (learn) features from very low-level components (e.g., image pixels, text words, etc.)

- Article recommendation

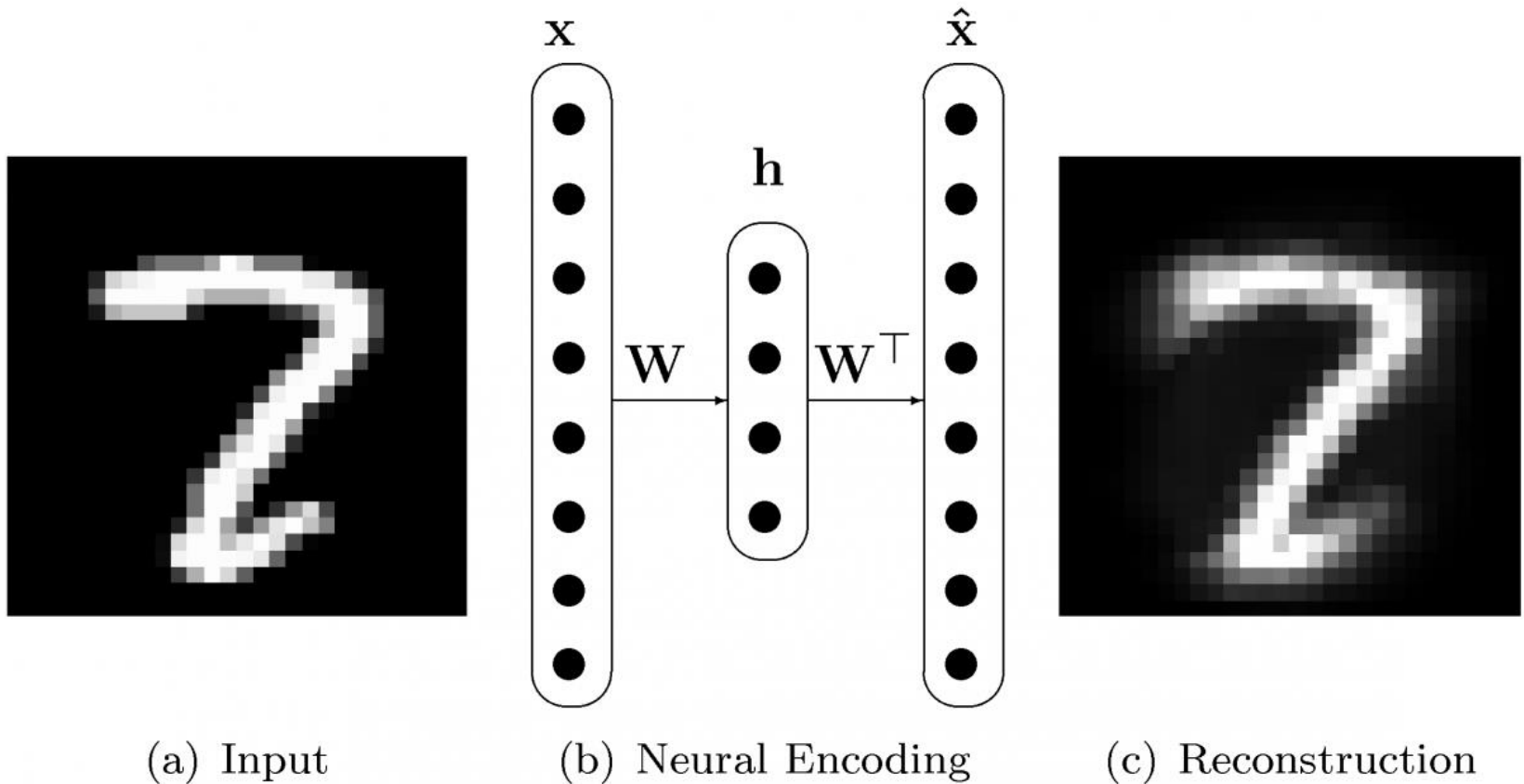
- Bengio, Y., Courville, A., & Vincent, P. (2013). Representation learning: A review and new perspectives. *IEEE transactions on pattern analysis and machine intelligence*, 35(8), 1798-1828.



Hinton and Salakhutdinov (2006)

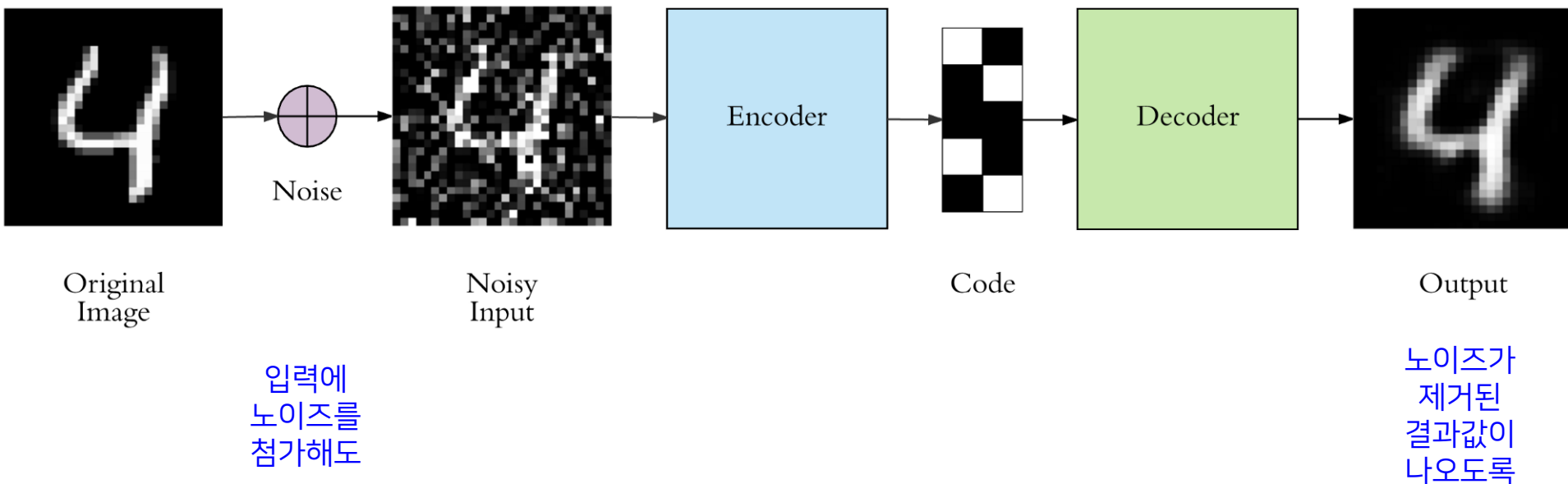
Dimensionality Reduction: Recent Trends

- Representation learning: Deep auto-encoder
 - ✓ Try to extract (learn) features from very low-level components (e.g., image pixels, text words, etc.)



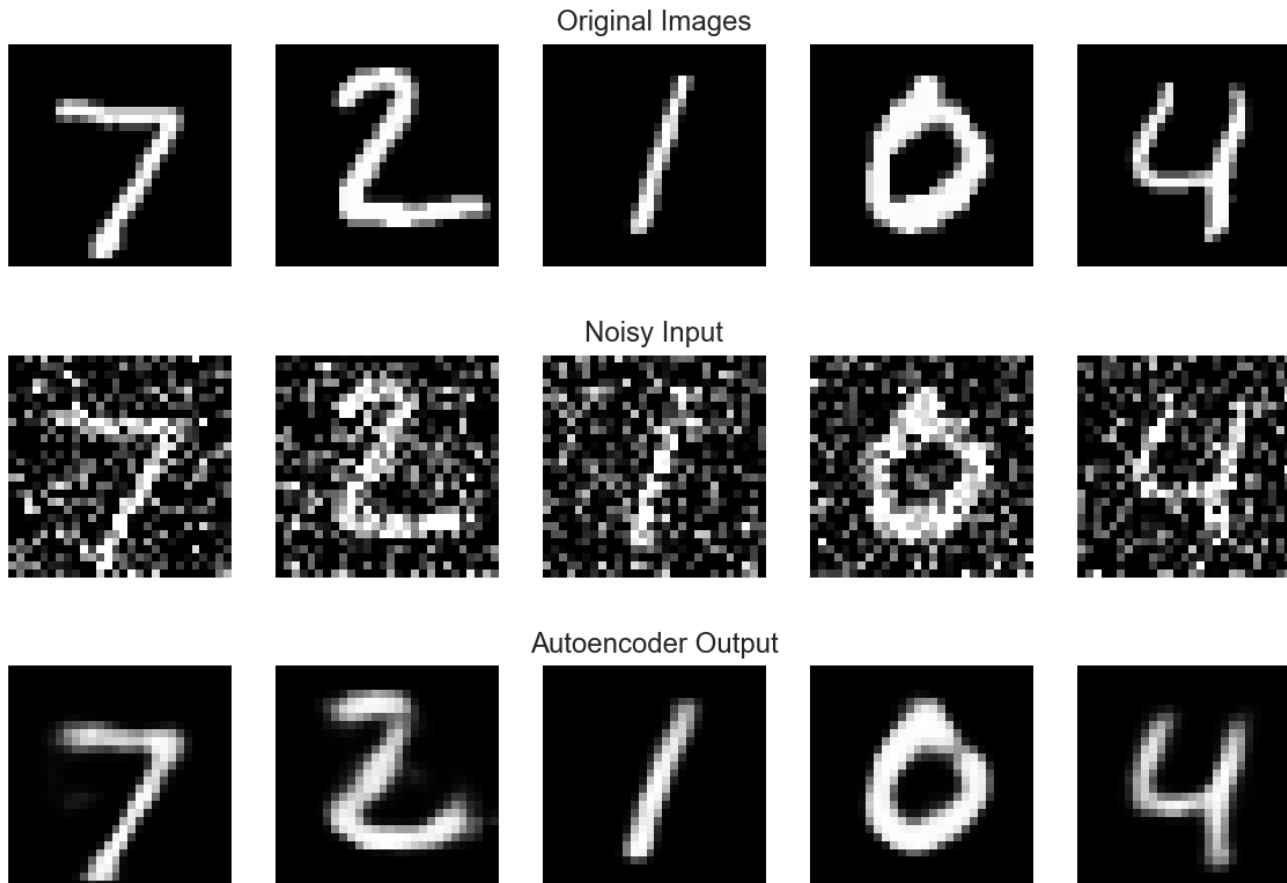
Dimensionality Reduction: Recent Trends

- Representation learning: De-noising Auto-Encoder
 - ✓ Auto-Encoder models are very sensitive to small perturbations of input data
 - ✓ Add noise to the input data but require to produce the original input without the noise



Dimensionality Reduction: Recent Trends

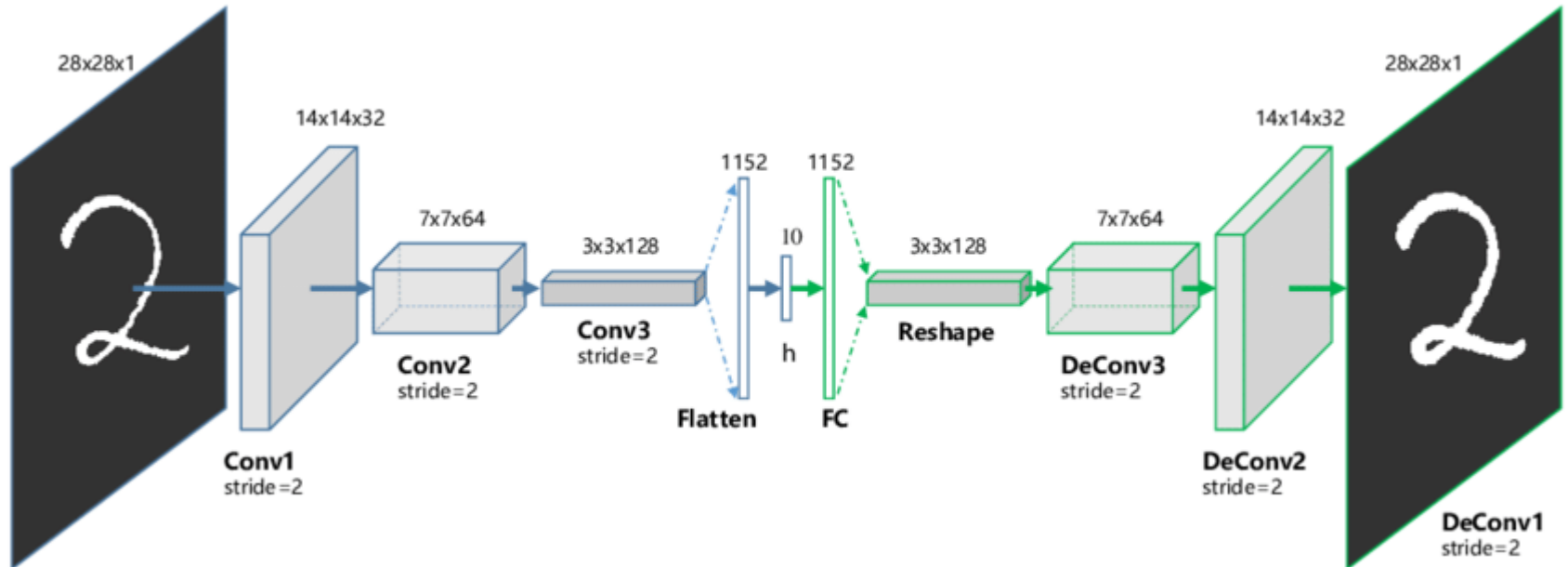
- How to generate noise?
 - ✓ Random Gaussian noise is generally used



<https://towardsdatascience.com/applied-deep-learning-part-3-autoencoders-lc083af4d798>

Dimensionality Reduction: Recent Trends

- Representation learning: Convolutional neural network
 - ✓ Try to extract (learn) features from very low-level components (e.g., image pixels, text words, etc.)

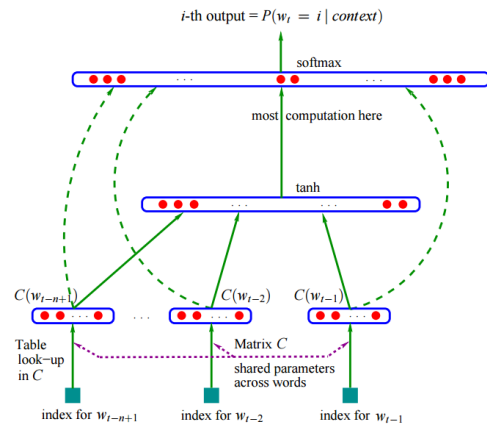


https://www.researchgate.net/figure/The-structure-of-proposed-Convolutional-AutoEncoders-CAE-for-MNIST-In-the-middle-there_fig1_320658590

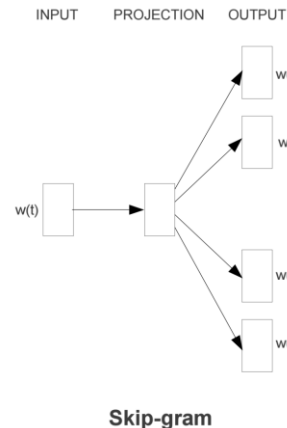
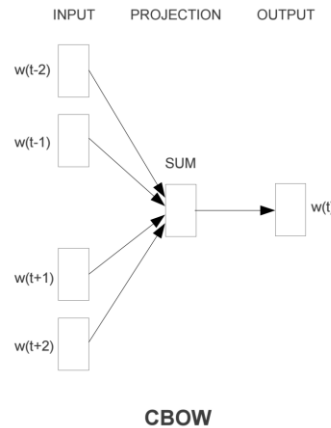
Dimensionality Reduction: Recent Trends

Representation learning: Word/Document Embedding

NNLM



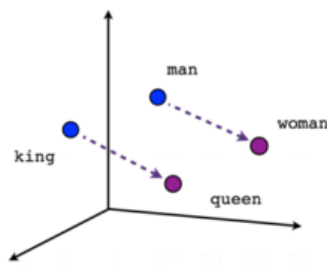
Word2Vec



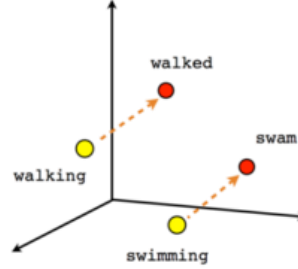
GloVe

frog nearest neighbors	Litoria	Leptodactylidae	Rana	Eleutherodactylus
<ul style="list-style-type: none"> frogs toad litoria leptodactylidae rana lizard eleutherodactylus 				

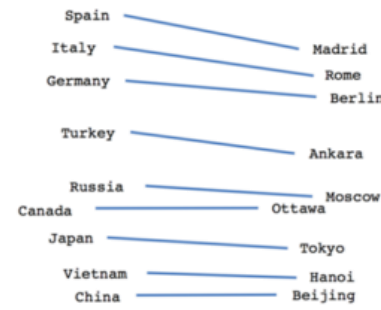
man -> woman	city -> zip	comparative -> superlative
<ul style="list-style-type: none"> uncle -> woman king -> queen man -> sir 	<ul style="list-style-type: none"> 9681Z -> Honolulu 97211 -> Nashville 95829 -> Sacramento 92804 -> Anaheim 	<ul style="list-style-type: none"> strong -> stronger clear -> clearer soft -> softer dark -> darker



Male-Female



Verb tense



Country-Capital

Dimensionality Reduction: Recent Trends

- Representation learning: Pre-trained models



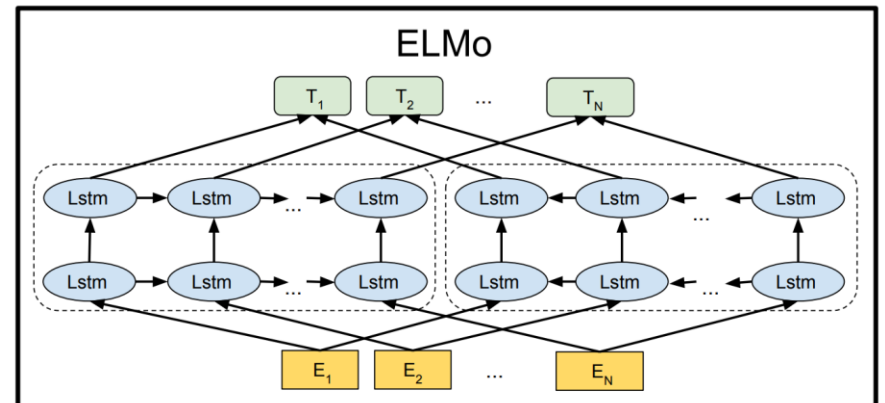
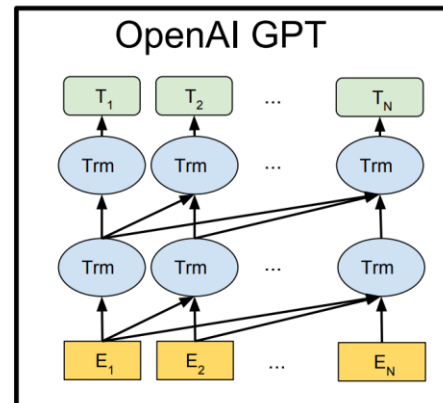
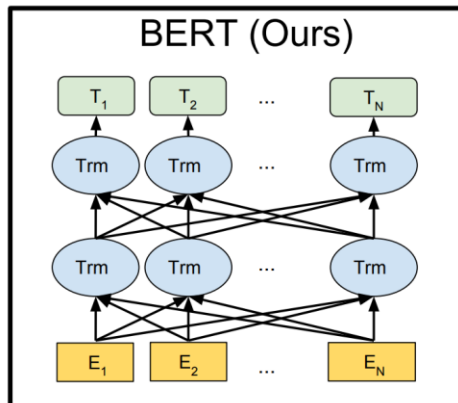
ELMo



BERT



GPT-2





References

Research Papers

- Bengio, Y., Courville, A., Vincent, P. (2013). Representation learning: A review and new perspectives, IEEE Transactions on Pattern Analysis and Machine Intelligence 35(8): 1798-1828.

Other materials

- Figure in the title page: <https://wattsupwiththat.com/2015/12/12/is-climate-forecasting-immune-from-occams-razor/>