

Ensemble Learning: CatBoost

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Prokhorenkova et al. (2018)

Background in brief

- \checkmark Assume we observe a dataset of examples $\mathcal{D} = \{(\mathbf{x}_k, y_k)\}_{k=1,...,n}$
 - where $\mathbf{x}_k = (x_k^1,...,x_k^m)$ is a random vector of m features and $y_k \in \mathbb{R}$ is a target
- \checkmark Gradient boosting builds iteratively a sequence of approximations $F^t:\mathbb{R}^m \to \mathbb{R}$ in a greedy fashion
 - $F^t: F^{t-1} + \alpha h^t$ (alpha is a step size and h is a base predictor)
 - lacksquare is chosen from a family of functions H in order to minimize the expected loss

$$h^t = \arg\min_{h \in H} \mathcal{L}(F^{t-1} + h) = \arg\min_{h \in H} \mathbb{E}L(y, F^{t-1}(\mathbf{x}) + h(\mathbf{x}))$$

 \blacksquare The gradient step h^t is chosen in such a way that $h^t(\mathbf{x})$ approximates $-g^t(\mathbf{x},y)$, where

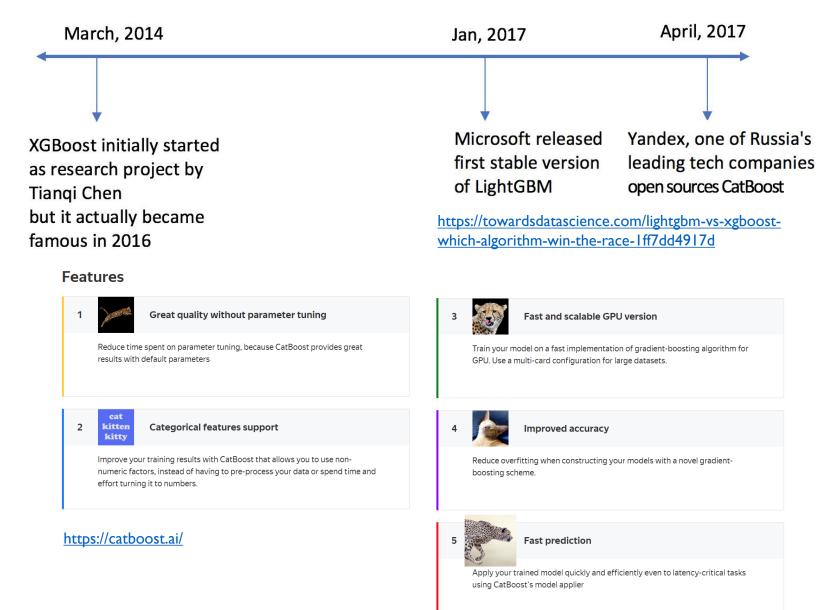
$$g^{t}(\mathbf{x}, y) := \left. \frac{\partial L(y, s)}{\partial s} \right|_{s = F^{t-1}(\mathbf{x})}$$

Usually the least squares approximation is used

$$h^t = \arg\min_{h \in H} \mathbb{E}(-g^t(\mathbf{x}, y) - h(\mathbf{x}))^2$$











Gradient Boosting

✓ Statistical Issue 1: Prediction Shift

$$\arg\min_{h\in H} \mathbb{E}(-g^t(\mathbf{x},y) - h(\mathbf{x}))^2 \approx \arg\min_{h\in H} \frac{1}{n} \sum_{k=1}^n (-g^t(\mathbf{x}_k,y_k) - h(\mathbf{x}_k))^2$$

Training Dataset: $\mathcal{D} = (\mathbf{x}_k, y_k)_{k=1,...,n}$

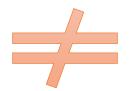
where
$$(\mathbf{x}_k = (x_k^1, ..., x_k^m), y_k \in \mathbb{R}$$

Conditional distribution

$$F^{t-1}(\mathbf{x}_k|\mathbf{x}_k)$$

for training example \mathbf{x}_k

is SHFTED from



Conditional distribution

$$F^{t-1}(\mathbf{x}|\mathbf{x})$$

for test example \mathbf{x}





- Gradient Boosting
 - ✓ Statistical Issue 2: Target Leakage
 - Target Statistic (TS): Replace the categorical feature with it mean target value in the training dataset
 - (Problem) Target value of an instance is used to compute its feature value (target information is leaked)

	•••	x ⁱ	•••	у
I ₁	•••	Α	•••	I
l ₂	•••	В	•••	I
l ₃	•••	С	•••	I
l ₄	•••	Α	•••	0
I ₅	•••	В	•••	I
I ₆	•••	С	•••	I
I ₇	•••	В	•••	0
I ₈	•••	С	•••	I
l ₉	•••	С	•••	I
I ₁₀	•••	С	•••	0

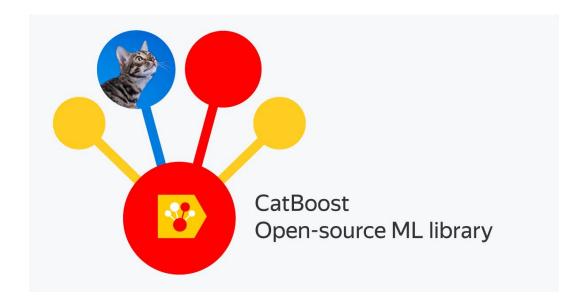


	•••	x ⁱ (TS)	• • •	у
I_1	•••	0.50	•••	I
l ₂	•••	0.67	•••	I
l ₃	•••	0.80	•••	I
I ₄	•••	0.50	•••	0
I ₅	•••	0.67	•••	I
I ₆	•••	0.80	•••	I
I ₇	•••	0.67	•••	0
I ₈	•••	0.80	•••	I
l ₉	•••	0.80	•••	I
I ₁₀	•••	0.80	•••	0





Solution



Target Leakage — Ordered Target Statistics

Prediction Shift — Ordered Boosting





- Categorical features in boosted tree
 - ✓ Basic approach: one-hot encoding

	•••	x ⁱ	•••
I ₁	•••	Α	•••
l ₂	•••	В	•••
l ₃	•••	С	•••
I ₄	•••	Α	•••
I ₅	•••	В	•••
I ₆	•••	С	•••
I ₇	•••	В	•••
I ₈	•••	С	•••
l ₉	•••	С	•••
I ₁₀	•••	С	•••



	•••	xi(A)	xi(B)	x ⁱ (C)	•••
I ₁	•••	- 1	0	0	•••
l ₂	•••	0	I	0	•••
l ₃	•••	0	0	I	•••
I ₄	•••	1	0	0	•••
I ₅	•••	0	I	0	•••
I ₆	•••	0	0	I	•••
l ₇	•••	0	I	0	•••
I ₈	•••	0	0	I	•••
l ₉	•••	0	0	I	•••
I ₁₀	•••	0	0	I	•••





- Categorical features in boosted tree
 - ✓ Another popular method: to group categories by target statistics (TS)
 - an estimate expected target value in each category
 - \bullet Greedy TS without smoothing $\, \hat{x}_k^i \, \approx \mathbb{E}(y \mid x^i \, = \, x_k^i) \,$
 - i is a categorical feature while k is training example index

	•••	x ⁱ	•••	у
I_1	•••	Α	•••	I
l ₂	•••	В	•••	I
l ₃	•••	С	•••	I
l ₄	•••	Α	•••	0
I ₅	•••	В	•••	I
I ₆	•••	С	•••	I
I ₇	•••	В	•••	0
I ₈		С	•••	I
l ₉	•••	С	•••	I
I ₁₀	•••	С	•••	0



1
)
1
)
)





- Categorical features in boosted tree
 - ✓ Another popular method: to group categories by target statistics (TS)
 - Greedy TS with smoothing

$$\hat{x}_k^i = \frac{\sum_{j=1}^n \mathbb{1}_{\{x_j^i = x_k^i\}} \cdot y_j + ap}{\sum_{j=1}^n \mathbb{1}_{\{x_j^i = x_k^i\}} + a}$$

- a > 0 is a parameter
- A common setting for p is the average target value in the dataset
- Used to remove the negative effect of low-frequency noisy categories





Greedy TS with smoothing example

 \checkmark a = 0.1 (parameter), p = 0.7 (computed from the training dataset)

√ For category A

	•••	x i	•••	у
I ₁	•••	Α	• • •	I
l ₂	•••	В	•••	I
l ₃	•••	С	•••	I
I ₄	•••	Α	•••	0
I ₅	•••	В	•••	I
I ₆	•••	С	•••	I
I ₇	•••	В	•••	0
I ₈	•••	С	•••	I
l ₉	•••	С	•••	0
I ₁₀	•••	С	•••	I

$$\hat{x}_k^A = \frac{\sum_{j=1}^n \mathbb{1}_{\{x_j^A = x_k^A\}} \cdot y_j + ap}{\sum_{j=1}^n \mathbb{1}_{\{x_j^A = x_k^A\}} + a}$$

$$= \frac{1 + 0.1 \times 0.7}{2 + 0.1} = 0.5095$$





Greedy TS with smoothing example

 \checkmark a = 0.1 (parameter), p = 0.7 (computed from the training dataset)

√ For category B

	•••	x ⁱ	•••	у
I ₁	•••	Α	•••	I
l ₂	•••	В	•••	1
l ₃	•••	С	•••	I
I ₄	•••	Α	•••	0
I ₅	•••	В	•••	I
I ₆	•••	С	•••	I
I ₇	•••	В	•••	0
I ₈	•••	С	•••	I
l ₉	•••	С	•••	0
I ₁₀	•••	С	•••	I

$$\hat{x}_k^B = \frac{\sum_{j=1}^n \mathbb{1}_{\{x_j^B = x_k^B\}} \cdot y_j + ap}{\sum_{j=1}^n \mathbb{1}_{\{x_j^B = x_k^B\}} + a}$$

$$= \frac{2 + 0.1 \times 0.7}{3 + 0.1} = 0.6677$$





Greedy TS with smoothing example

 \checkmark a = 0.1 (parameter), p = 0.7 (computed from the training dataset)

√ For category C

	•••	x ⁱ	•••	у
I ₁	•••	Α	•••	I
l ₂	•••	В	•••	I
l ₃	•••	С	•••	I
I ₄	•••	Α	•••	0
I ₅	•••	В	•••	I
I ₆	•••	С	•••	ı
I ₇	•••	В	•••	0
I ₈	•••	С	•••	I
l ₉	•••	С	•••	0
I ₁₀	•••	С	•••	I

$$\hat{x}_k^C = \frac{\sum_{j=1}^n \mathbb{1}_{\{x_j^C = x_k^C\}} \cdot y_j + ap}{\sum_{j=1}^n \mathbb{1}_{\{x_j^C = x_k^C\}} + a}$$

$$= \frac{4 + 0.1 \times 0.7}{5 + 0.1} = 0.7980$$





• Greedy TS with smoothing example

	•••	xi	•••	у
I ₁	•••	Α	•••	I
l ₂	•••	В	•••	I
l ₃	•••	С	•••	I
l ₄	•••	Α	•••	0
I ₅	•••	В	•••	I
I ₆	•••	С	•••	I
I ₇	•••	В	•••	0
I ₈	•••	С	•••	I
l ₉	•••	С	•••	0
I ₁₀	•••	С	•••	I

	•••	xi(TS)	•••	у
I ₁	•••	0.5095	•••	I
l ₂	•••	0.6677	•••	I
l ₃	•••	0.7980	•••	I
I ₄	•••	0.5095	•••	0
I ₅	•••	0.6677	•••	I
I ₆	•••	0.7980	•••	I
I ₇	•••	0.6677	•••	0
I ₈	•••	0.7980	•••	I
l ₉	•••	0.7980	•••	0
I ₁₀	•••	0.7980	•••	I





Target leakage

- \checkmark feature \hat{x}_k^i is computed using y_k , the target of \mathbf{x}_k
- \checkmark leads to a conditional shift: the distribution of $\hat{x}^i|y$ differs for training and test examples

We should the use this information when we predict the target of current example

$$\hat{x}_{k}^{i} = \frac{\sum_{j=1}^{n} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} (y_{j}) + ap}{\sum_{j=1}^{n} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$





Target leakage

- \checkmark feature \hat{x}_k^i is computed using y_k , the target of \mathbf{x}_k
- \checkmark leads to a conditional shift: the distribution of $\hat{x}^i|y$ differs for training and test examples
 - $\blacksquare \ \, \text{Assume that i}^{\text{th}} \ \text{feature is categorical, all its values are unique,} \ P(y=1|x^i)=0.5, x^i \in \{A,B,C,D,E,F,\ldots\}$

	•••	x ⁱ	•••	у		•••	x ⁱ	•••	у
I ₁	•••	Α	•••	I	I ₁	•••	$\frac{1+ap}{1+a}$	•••	I
l ₂	•••	В	•••	I	l ₂	•••	$\frac{1+ap}{1+a}$	•••	I
l ₃	•••	С	•••	I	l ₃	•••	$\frac{1+ap}{1+a}$	•••	I
I ₄	•••	D	•••	0	l ₄		$\frac{0+ap}{1+a}$	•••	0
I ₅	•••	E	•••	0	I ₅	•••	$\frac{0+ap}{1+a}$	•••	0
I ₆	• • •	F	•••	0	l ₅	•••	$\frac{0+ap}{1+a}$	• • •	0

- Training data can be perfectly classified when the split criterion is set to $\ x^i = \frac{0.5 + ap}{1 + a}$





Target leakage

Training Set

Test Set

		•••	x ⁱ	•••	у			•••	X i	•••	у
	l _i	•••	Α	•••	l		l _l	•••	$\frac{1+ap}{1+a}$	•••	I
	l ₂	•••	В	•••	I		l ₂	•••	$\frac{1+ap}{1+a}$	•••	l
	l ₃	•••	С	•••	ı		l ₃	•••	$\frac{1+ap}{1+a}$	•••	I
	l ₄	•••	D	•••	0		l ₄		$\frac{0+ap}{1+a}$	•••	0
	l ₅	•••	E	•••	0		l ₅	•••	$\frac{0+ap}{1+a}$	•••	0
	l ₆	•••	F	•••	0	/	l ₆	•••	$\frac{0+ap}{1+a}$	•••	0
	l ₇	•••	G	•••	I		l ₇		p	•••	I
	l ₈	•••	Н	•••	I		I ₈		p	•••	I
	l ₉	•••	I	•••	0		l ₉		p	•••	0
I	10	•••	J	•••	0		I ₁₀	•••	p	•••	0

Perfect classification by setting the cut off value

$$x^i = \frac{0.5 + ap}{1 + a}$$

$$\hat{x}_k^i = \frac{\sum_{j=1}^n \mathbb{1}_{\{x_j^i = x_k^i\}} \cdot y_j + ap}{\sum_{j=1}^n \mathbb{1}_{\{x_j^i = x_k^i\}} + a}$$

All feature values of test instances become p



No discrimination ability





• Desired property for TS

✓ Property I:

$$\mathbb{E}(\hat{x}^i \mid y = v) = \mathbb{E}(\hat{x}_k^i \mid y_k = v)$$

$$where (\mathbf{x}_k, y_k) is the k - th training example$$

- ✓ Property 2:
 - Effective usage of all training data for calculating TS features and for learning a model





- Ways to avoid conditional shift
 - \checkmark General idea: compute the TS for \mathbf{x}_k on a subset of examples $\mathcal{D}_k \subset \mathcal{D} \backslash \{\mathbf{x}_k\}$ excluding \mathbf{x}_k

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$

- √ Holdout TS
 - lacksquare To partition the training dataset into two part $\,{\cal D}\,=\,\hat{{\cal D}}_0\cup\hat{{\cal D}}_1$
 - Use the former to compute the TS
 - Use the latter for training
 - Violates the Property 2





Ways to avoid conditional shift

✓ Leave-one-out TS

lacksquare Take $\mathcal{D}_k = \mathcal{D} ackslash \mathbf{x}_k$ for training examples

$$lacksquare$$
 Take $\mathcal{D}_k=\mathcal{D}$ for test ones

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$

Does not prevent target leakage

	•••	x ⁱ	•••	у
I ₁	•••	Α	•••	1
l ₂	•••	Α	•••	0
I ₃	•••	Α	•••	I
I ₄	•••	Α	•••	0
I ₅	•••	Α	•••	I

	•••	X ⁱ	•••	у
I ₁	•••	$\frac{2+ap}{4+a}$	•••	I
I ₂	•••	3 + ap	•••	0
I ₃	•••	$ \begin{array}{r} 4+a \\ 2+ap \\ 4+a \end{array} $	•••	I
I ₄	•••	$\frac{3+ap}{4+a}$	•••	0
l ₅	•••	$\frac{2+ap}{4+a}$	•••	I

• We can perfectly classify this training dataset by setting the cut-off value

$$\frac{2.5 + ap}{4 + a}$$





- Ways to avoid conditional shift
 - ✓ Ordered TS: introduce an artificial time
 - lacktriangleright a random permutation σ of the training examples

$$\mathcal{D}_k = \{ \mathbf{x}_j : \sigma(j) < \sigma(k) \}$$

	•••	x ⁱ	•••	TS	у
l ₁	•••	Α	•••	0.700	I
l ₂	•••	В	•••		I
l ₃	•••	С	•••		I
I ₄	•••	Α	•••		0
I ₅	•••	В	•••		I
I ₆	•••	С	•••		I
I ₇	•••	В	•••		0
I ₈	•••	С	•••		I
l ₉	•••	С	•••		0
I ₁₀	•••	С	• • •		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
$$= \frac{0 + 0.1 \times 0.7}{0 + 0.1} = 0.700$$





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	•••	x ⁱ	•••	TS	у
I ₁	•••	Α	•••	0.700	I
l ₂	•••	В	•••	0.700	I
l ₃	•••	С	•••		I
I ₄	•••	Α	•••		0
I ₅	•••	В	•••		I
I ₆	•••	С	•••		I
I ₇	•••	В	•••		0
I ₈	•••	С	•••		I
l ₉	•••	С	•••		0
I ₁₀	•••	С	•••		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
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	•••	x ⁱ	•••	TS	у
I ₁	•••	Α	•••	0.700	l
l ₂	•••	В	•••	0.700	I
l ₃	•••	С	•••	0.700	I
I ₄	•••	Α	•••		0
I ₅	•••	В	•••		I
I ₆	•••	С	•••		I
I ₇	•••	В	•••		0
I ₈	•••	С	•••		l
l ₉	•••	С	•••		0
I ₁₀	•••	С	• • •		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
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$$\mathcal{D}_k = \{ \mathbf{x}_j : \sigma(j) < \sigma(k) \}$$

	•••	x ⁱ	•••	TS	у
l ₁	•••	Α	•••	0.700	I
l ₂	•••	В	•••	0.700	I
l ₃	•••	С	•••	0.700	I
I ₄	•••	Α	•••	0.973	0
I ₅	•••	В	•••		I
I ₆	•••	С	•••		I
I ₇	•••	В	•••		0
I ₈	•••	С	•••		I
l ₉	•••	С	•••		0
I ₁₀	•••	С	• • •		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
$$= \frac{1 + 0.1 \times 0.7}{1 + 0.1} = 0.973$$





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$$\mathcal{D}_k = \{ \mathbf{x}_j : \sigma(j) < \sigma(k) \}$$

	•••	x ⁱ	•••	TS	у
I ₁	•••	Α	•••	0.700	I
l ₂	•••	В	•••	0.700	I
l ₃	•••	С	•••	0.700	I
I ₄	•••	Α	•••	0.973	0
I ₅	•••	В	•••	0.973	I
I ₆	•••	С	•••		I
I ₇	•••	В	•••		0
I ₈	•••	С	•••		I
l ₉	•••	С	•••		0
I ₁₀	•••	С	• • •		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
$$= \frac{1 + 0.1 \times 0.7}{1 + 0.1} = 0.973$$





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$$\mathcal{D}_k = \{ \mathbf{x}_j : \sigma(j) < \sigma(k) \}$$

	•••	x ⁱ	•••	TS	у
I ₁	•••	Α	•••	0.700	I
l ₂	•••	В	•••	0.700	I
l ₃	•••	С	•••	0.700	I
I ₄	•••	Α	•••	0.973	0
I ₅	•••	В	•••	0.973	l
I ₆	•••	С	•••	0.973	I
I ₇	•••	В	•••		0
I ₈	•••	С	•••		I
l ₉	•••	С	•••		0
I ₁₀	•••	С	• • •		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
$$= \frac{1 + 0.1 \times 0.7}{1 + 0.1} = 0.973$$





- Ways to avoid conditional shift
 - ✓ Ordered TS: introduce an artificial time
 - lacktriangleright a random permutation σ of the training examples

$$\mathcal{D}_k = \{ \mathbf{x}_j : \sigma(j) < \sigma(k) \}$$

	•••	x ⁱ	•••	TS	у
I ₁	•••	Α	•••	0.700	I
l ₂	•••	В	•••	0.700	I
l ₃	•••	С	•••	0.700	I
I ₄	•••	Α	•••	0.973	0
I ₅	•••	В	•••	0.973	I
I ₆	•••	С	•••	0.973	I
l ₇	•••	В	•••	0.986	0
I ₈	•••	С	•••		I
l ₉	•••	С	•••		0
I ₁₀	•••	С	• • •		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
$$= \frac{2 + 0.1 \times 0.7}{2 + 0.1} = 0.986$$





- Ways to avoid conditional shift
 - ✓ Ordered TS: introduce an artificial time
 - lacktriangleright a random permutation σ of the training examples

$$\mathcal{D}_k = \{ \mathbf{x}_j : \sigma(j) < \sigma(k) \}$$

	•••	x ⁱ	•••	TS	у
I_1	•••	Α	•••	0.700	l
l ₂	•••	В	•••	0.700	I
I ₃	•••	С	•••	0.700	l
I ₄	•••	Α	•••	0.973	0
I ₅	•••	В	•••	0.973	I
I ₆	•••	С	•••	0.973	I
I ₇	•••	В	•••	0.986	0
I ₈	•••	С	•••	0.986	I
l ₉	•••	С	•••		0
I ₁₀	•••	С	•••		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
$$= \frac{2 + 0.1 \times 0.7}{2 + 0.1} = 0.986$$





- Ways to avoid conditional shift
 - ✓ Ordered TS: introduce an artificial time
 - lacktriangleright a random permutation σ of the training examples

$$\mathcal{D}_k = \{ \mathbf{x}_j : \sigma(j) < \sigma(k) \}$$

	•••	x ⁱ	•••	TS	у
I ₁	•••	Α	•••	0.700	I
l ₂	•••	В	•••	0.700	I
l ₃	•••	С	•••	0.700	I
I ₄	•••	Α	•••	0.973	0
I ₅	•••	В	•••	0.973	l
I ₆	•••	С	•••	0.973	I
I ₇	•••	В	•••	0.986	0
I ₈	•••	С	•••	0.986	I
l ₉	•••	С	•••	0.990	0
I ₁₀	•••	С	• • •		I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
$$= \frac{3 + 0.1 \times 0.7}{3 + 0.1} = 0.990$$





- Ways to avoid conditional shift
 - ✓ Ordered TS: introduce an artificial time
 - lacktriangleright a random permutation σ of the training examples

$$\mathcal{D}_k = \{ \mathbf{x}_j : \sigma(j) < \sigma(k) \}$$

	•••	x ⁱ	•••	TS	у
I ₁	•••	Α	•••	0.700	l
l ₂	•••	В	•••	0.700	I
l ₃	•••	С	•••	0.700	l
I ₄	•••	Α	•••	0.973	0
I ₅	•••	В	•••	0.973	l
I ₆	•••	С	•••	0.973	l
I ₇	•••	В	•••	0.986	0
I ₈	•••	С	•••	0.986	l
l ₉	•••	С	•••	0.990	0
I ₁₀	•••	С	•••	0.749	I

$$\hat{x}_{k}^{i} = \frac{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} \cdot y_{j} + ap}{\sum_{\mathbf{x}_{j} \in \mathcal{D}_{k}} \mathbb{1}_{\{x_{j}^{i} = x_{k}^{i}\}} + a}$$
$$= \frac{3 + 0.1 \times 0.7}{4 + 0.1} = 0.749$$





Prediction Shift

$$h^t = \arg\min_{h \in H} \mathbb{E}(-g^t(\mathbf{x}, y) - h(\mathbf{x}))^2$$

 \checkmark The expectation is unknown and is usually approximated using the same dataset ${\mathcal D}$

$$h^t = \arg\min_{h \in H} \frac{1}{n} \sum_{k=1}^n \left(-g^t(\mathbf{x}, y) - h(\mathbf{x}) \right)^2$$

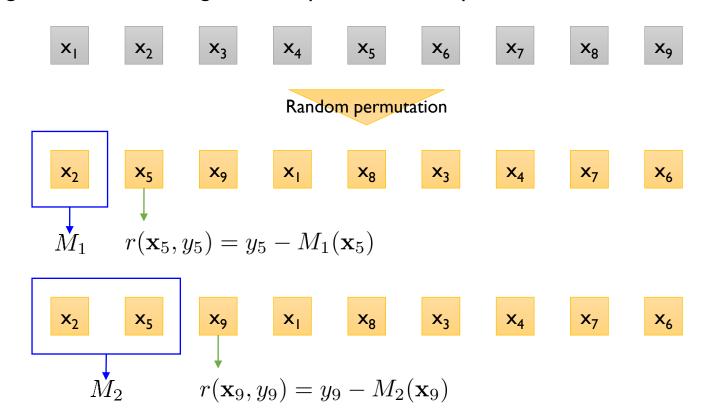
- The conditional distribution of the gradient $g^t(\mathbf{x}_k,y_k)|\mathbf{x}_k$ is shifted from that distribution on a test example $g^t(\mathbf{x},y)|\mathbf{x}$
- In turn, base predictor h^t is biased from the solution
- lacktriangle Finally it affects the generalization ability of the trained model F^t
- ✓ When we learn a model with I trees, we need to have F^{I-1} trained without the example \mathbf{x}_k to make the residual $r^{I-1}(\mathbf{x}_k,y_k)$ unshifted





Ordered Boosting

✓ A boosting algorithm not suffering from the prediction shift problem

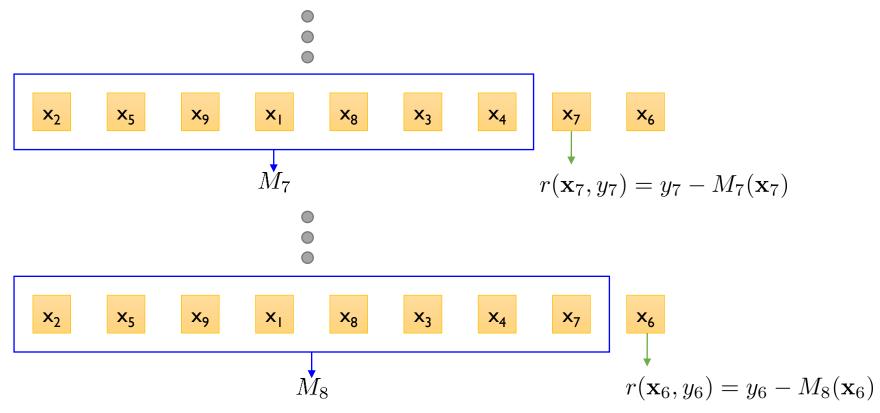






Ordered Boosting

✓ A boosting algorithm not suffering from the prediction shift problem



√ The same permutation is used for both TS computing and ordered boosting





Ordered Boosting and Building a Tree in CatBoost

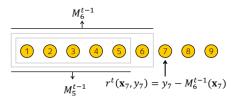


Figure 1: Ordered boosting principle, examples are ordered according to σ .

$\begin{tabular}{ll} \textbf{Algorithm 1:} & Ordered boosting \\ \hline \textbf{input} & : \{(\mathbf{x}_k,y_k)\}_{k=1}^n,I; \\ \hline $\sigma \leftarrow$ random permutation of $[1,n]$; \\ $M_i \leftarrow 0$ for $i=1..n$; \\ \textbf{for } t \leftarrow 1$ \textbf{ to } I$ \textbf{ do} \\ & & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & \\ & & & & \\$

```
Algorithm 2: Building a tree in CatBoost
input: M, \{(\mathbf{x}_i, y_i)\}_{i=1}^n, \alpha, L, \{\sigma_i\}_{i=1}^s, Mode
grad \leftarrow CalcGradient(L, M, y);
r \leftarrow random(1, s);
if Mode = Plain then
     G \leftarrow (grad_r(i) \text{ for } i = 1..n);
if Mode = Ordered then
  G \leftarrow (grad_{r,\sigma_r(i)-1}(i) \text{ for } i=1..n);
T \leftarrow \text{empty tree};
foreach step of top-down procedure do
     foreach candidate split c do
          T_c \leftarrow \text{add split } c \text{ to } T;
          if Mode = Plain then
               \Delta(i) \leftarrow \operatorname{avg}(\operatorname{grad}_r(p)) for
                p: leaf_r(p) = leaf_r(i) for i = 1..n;
          if Mode = Ordered then
               \Delta(i) \leftarrow \operatorname{avg}(grad_{r,\sigma_r(i)-1}(p)) for
                p: leaf_r(p) = leaf_r(i), \sigma_r(p) < \sigma_r(i)
                 for i = 1..n:
          loss(T_c) \leftarrow cos(\Delta, G)
     T \leftarrow \arg\min_{T_c}(loss(T_c))
if Mode = Plain then
     M_{r'}(i) \leftarrow M_{r'}(i) - \alpha \arg(grad_{r'}(p)) for
      p: leaf_{r'}(p) = leaf_{r'}(i) for r' = 1..s, i = 1..n;
if Mode = Ordered then
     M_{r',j}(i) \leftarrow M_{r',j}(i) - \alpha \operatorname{avg}(\operatorname{grad}_{r',j}(p)) for
      p: leaf_{r'}(p) = leaf_{r'}(i), \sigma_{r'}(p) \leq j for r' = 1...s,
      i = 1..n, j > \sigma_{r'}(i) - 1;
```





CatBoost: Procedure

CatBoost Procedure

- ✓ Initialization: generate (s+1) independent random permutations of the training dataset
 - s permutations to evaluate the split
 - I permutation to compute the leaf value of the obtained trees
- ✓ Both TS and predictions used by ordered boosting have a high variance
- ✓ Using only one permutation may increase the variance of the final model predictions, while several permutations can reduce it





CatBoost: Procedure

CatBoost Procedure

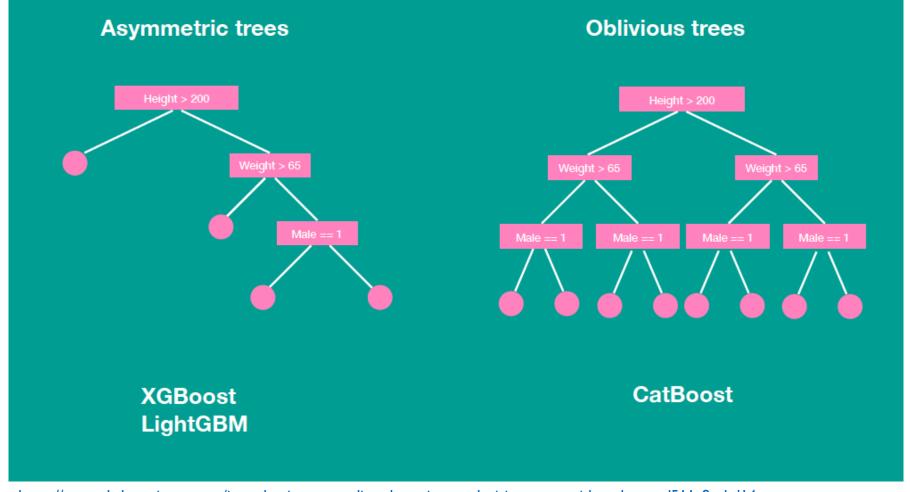
- ✓ Building an oblivious tree^{the same splitting criterion is used across an entire level of the tree} in the ordered boosting mode
 - Maintain the supporting models $M_{r,j}$
 - $M_{r,j}(i)$: the current prediction for the ith example based on the first j examples in the permutation r
 - At each iteration t, sample a random permutation σ_r and construct a tree T_t
 - TS are computed according to this permutation σ_r
 - The permutation affects the tree learning procedure
 - For each example i, take the gradient $grad_{r,\sigma_r(i)-1}(i)$
 - While constructing a tree, the loss of the candidate split c is computed by the cosine similarity between the leaf value and the gradient of each example
 - At the candidate splits evaluation step, the leaf value $\Delta(i)$ for example i is obtained individually by averaging the gradients $grad_{r,\sigma_r(i)-1}$ of the preceding example p lying in the same leaf node of i





CatBoost: Procedure

Oblivious tree



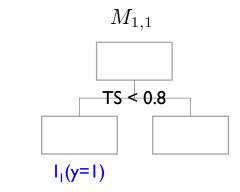






- Ordered Boosting Example
 - \checkmark Assumption: squared loss (gradient: f(x) y)
 - √ random permutation is conducted

		x ⁱ	•••	TS	у	G	Δ
I _I		Α	•••	0.700	I	0	0
l ₂		В	•••	0.700	I		
l ₃		С	•••	0.700	I		
I ₄		Α	•••	0.973	0		
I ₅		В	•••	0.973	I		
I ₆	•••	С	•••	0.973	I		
l ₇	•••	В	•••	0.986	0		
I ₈	•••	С	•••	0.986	I		
l ₉		С		0.990	0		
I ₁₀	•••	С	•••	0.749	I		



$$G(I_1) = 0$$

$$\Delta(I_1) = 0$$

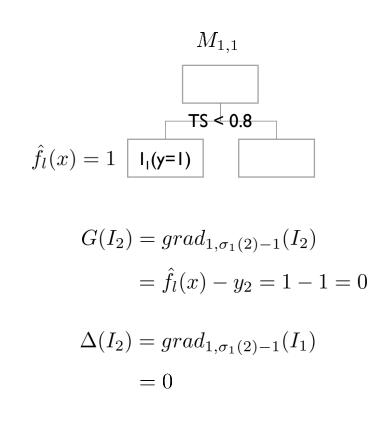
Lecturer's guess

 Initialize both the first gradient and leaf value to 0



- Ordered Boosting Example
 - \checkmark Assumption: squared loss (gradient: f(x) y)
 - √ random permutation is conducted

		x ⁱ		TS	у	G	Δ
I ₁	•••	Α	•••	0.700	I	0	0
l ₂	•••	В	•••	0.700	I	0	0
l ₃		U	•••	0.700	I		
I ₄		Α	•••	0.973	0		
I ₅		В		0.973	I		
I ₆		U	•••	0.973	I		
I ₇		В	•••	0.986	0		
I ₈		U	•••	0.986	I		
l ₉	•••	U	•••	0.990	0		
I ₁₀	•••	C	•••	0.749	I		

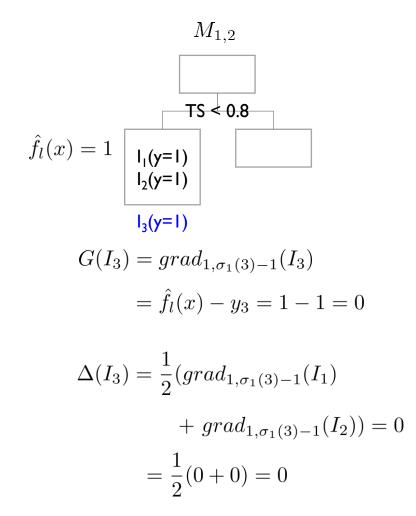






- \checkmark Assumption: squared loss (gradient: f(x) y)
- √ random permutation is conducted

	•••	χ ⁱ	•••	TS	у	G	Δ
I ₁		Α		0.700	I	0	0
l ₂	•••	В	•••	0.700	I	0	0
l ₃		С		0.700	I	0	0
I ₄		Α		0.973	0		
I ₅		В		0.973	I		
I ₆		U		0.973	I		
I ₇		В		0.986	0		
I ₈		U		0.986	I		
l ₉	•••	U	•••	0.990	0		
I ₁₀		U	•••	0.749	I		

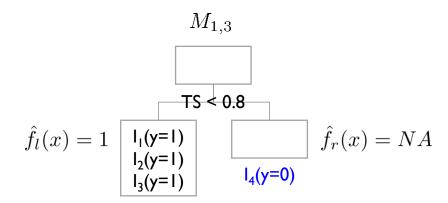






- ✓ Assumption: squared loss (gradient: f(x) y)
- √ random permutation is conducted

	•••	X ⁱ	•••	TS	у	G	Δ
I _I		Α	•••	0.700	I	0	0
l ₂		В		0.700	I	0	0
l ₃		C		0.700	I	0	0
I ₄		Α		0.973	0	0	0
I ₅	•••	В	•••	0.973	I		
I ₆	•••	U	•••	0.973	I		
I ₇		В		0.986	0		
I ₈		C		0.986	I		
l ₉		C		0.990	0		
I ₁₀	•••	C	•••	0.749	I		



$$G(I_4) = \operatorname{grad}_{1,\sigma_1(4)-1}(I_4)$$

$$= 0$$

$$\Delta(I_4) = 0$$

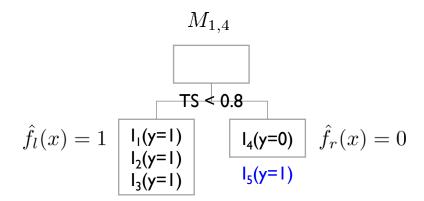






- \checkmark Assumption: squared loss (gradient: f(x)-y)
- √ random permutation is conducted

	•••	χ ⁱ		TS	у	G	Δ
I ₁		Α	•••	0.700	I	0	0
l ₂	•••	В	•••	0.700	I	0	0
l ₃	•••	U	•••	0.700	I	0	0
I ₄	•••	Α	•••	0.973	0	0	0
I ₅	•••	В	•••	0.973	I	-1	0
I ₆	•••	U	•••	0.973	I		
I ₇	•••	В	•••	0.986	0		
I ₈		C		0.986	I		
l ₉		C		0.990	0		
I ₁₀	•••	C	•••	0.749	I		



$$G(I_5) = grad_{1,\sigma_1(5)-1}(I_5)$$
$$= \hat{f}_r(x) - y_5 = -1$$

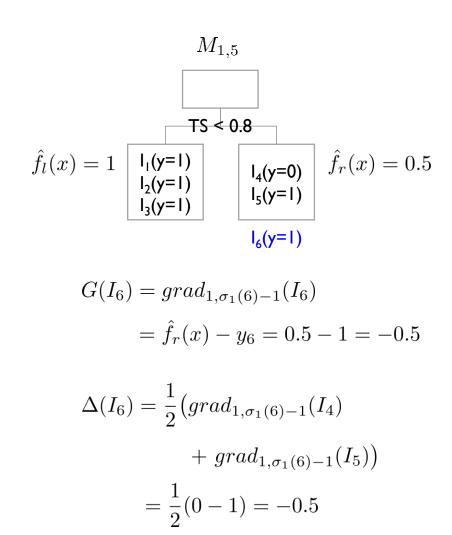
$$\Delta(I_5) = \operatorname{grad}_{1,\sigma_1(5)-1}(I_4)$$
$$= 0$$





- \checkmark Assumption: squared loss (gradient: f(x) y)
- √ random permutation is conducted

		x ⁱ		TS	у	G	Δ
I _I		Α		0.700	I	0	0
l ₂		В		0.700	I	0	0
l ₃		С		0.700	I	0	0
I ₄		Α		0.973	0	0	0
l ₅	•••	В	•••	0.973	I	-1	0
I ₆		С	•••	0.973	I	-0.5	-0.5
I ₇		В		0.986	0		
I ₈		С	•••	0.986	I		
l ₉	•••	С	•••	0.990	0		
I ₁₀	•••	С	•••	0.749	I		

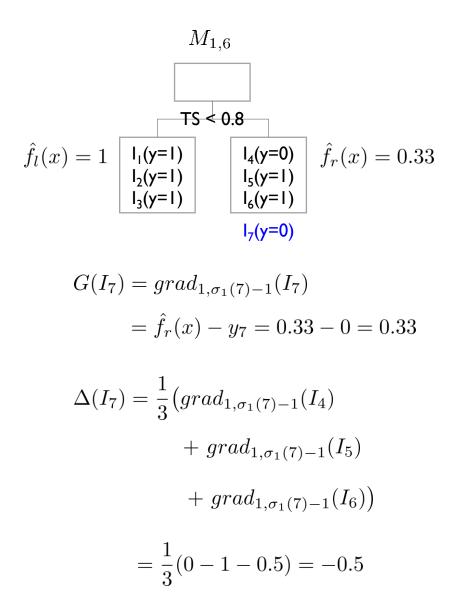






- \checkmark Assumption: squared loss (gradient: f(x) y)
- √ random permutation is conducted

	 χ ⁱ	•••	TS	у	G	Δ
I ₁	 Α	•••	0.700	I	0	0
l ₂	 В		0.700	I	0	0
l ₃	 C		0.700	I	0	0
I ₄	 Α		0.973	0	0	0
I ₅	 В		0.973	I	-1	0
I ₆	 C		0.973	I	-0.5	-0.5
I ₇	 В		0.986	0	0.33	-0.5
I ₈	 C		0.986	I		
l ₉	 C		0.990	0		
I ₁₀	 C	•••	0.749	I		

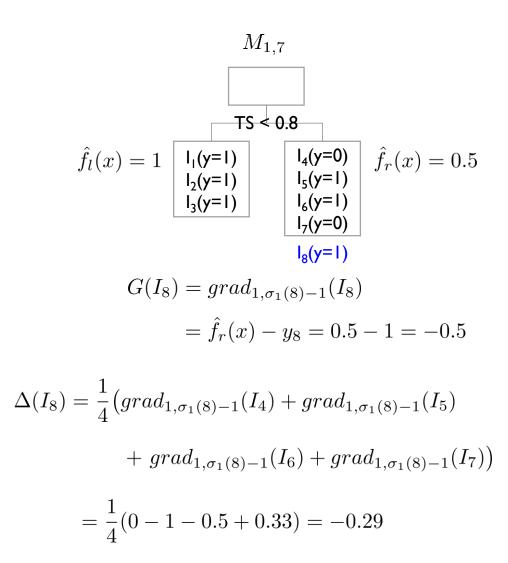






- \checkmark Assumption: squared loss (gradient: f(x) y)
- √ random permutation is conducted

	•••	x i	•••	TS	у	G	Δ
I ₁		Α		0.700	I	0	0
l ₂		В		0.700	I	0	0
l ₃		С		0.700	I	0	0
I ₄		Α		0.973	0	0	0
I ₅		В		0.973	I	-1	0
I ₆		С		0.973	I	-0.5	-0.5
l ₇		В		0.986	0	0.33	-0.5
I ₈		С		0.986	I	-0.5	-0.29
l ₉		С		0.990	0		
I ₁₀		С		0.749	I		
	•	•			•	•	

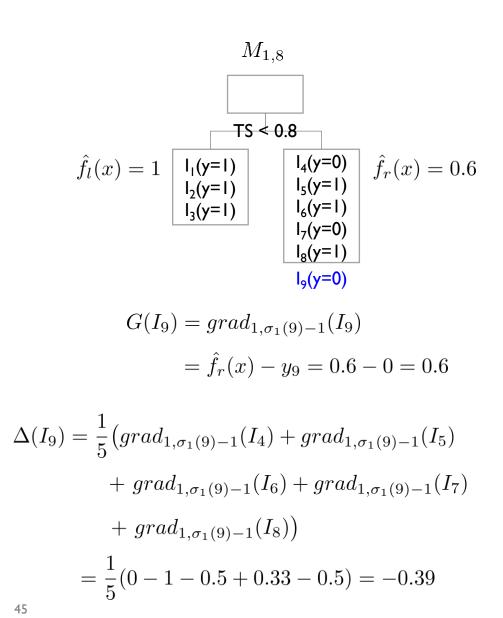






- \checkmark Assumption: squared loss (gradient: f(x) y)
- √ random permutation is conducted

		x ⁱ		TS	у	G	Δ
I _I		Α	•••	0.700	I	0	0
l ₂		В	•••	0.700	I	0	0
l ₃		U	•••	0.700	I	0	0
I ₄		Α	•••	0.973	0	0	0
I ₅		В	•••	0.973	I	-1	0
I ₆		U	•••	0.973	I	-0.5	-0.5
I ₇		В		0.986	0	0.33	-0.5
l ₈		U		0.986	I	-0.5	-0.29
l ₉		С		0.990	0	0.6	-0.39
I ₁₀	•••	C	•••	0.749	I		

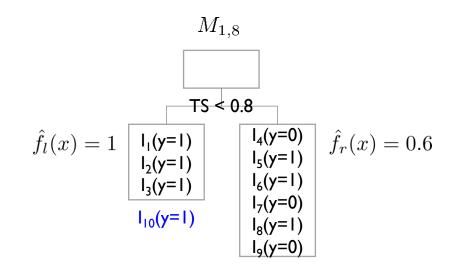






- \checkmark Assumption: squared loss (gradient: f(x) y)
- √ random permutation is conducted

		x ⁱ		TS	у	G	Δ
I ₁	•••	Α	•••	0.700	I	0	0
l ₂	•••	В	•••	0.700	I	0	0
l ₃	•••	U	•••	0.700	I	0	0
I ₄	•••	Α	•••	0.973	0	0	0
I ₅	•••	В	•••	0.973	I	-1	0
I ₆	•••	C		0.973	I	-0.5	-0.5
l ₇	•••	В	•••	0.986	0	0.33	-0.5
l ₈	•••	U	•••	0.986	I	-0.5	-0.29
l ₉	•••	U	•••	0.990	0	0.6	-0.39
I ₁₀	•••	С	•••	0.749	l	0	0



$$G(I_10) = \operatorname{grad}_{1,\sigma_1(10)-1}(I_{10})$$
$$= \hat{f}_l(x) - y_{10} = 1 - 1 = 0$$

$$\Delta(I_10) = \frac{1}{3} \left(grad_{1,\sigma_1(10)-1}(I_1) + grad_{1,\sigma_1(10)-1}(I_2) + grad_{1,\sigma_1(10)-1}(I_3) \right)$$
$$= \frac{1}{3} (0+0+0) = 0$$





- Ordered Boosting Example
 - \checkmark Assumption: squared loss (gradient: f(x) y)
 - √ random permutation is conducted

		x ⁱ		TS	у	G	Δ
I ₁	•••	Α	•••	0.700	I	0	0
l ₂	•••	В	•••	0.700	I	0	0
l ₃	•••	U	•••	0.700	I	0	0
I ₄	•••	A	•••	0.973	0	0	0
I ₅	•••	В		0.973	I	- l	0
I ₆	•••	U	•••	0.973	I	-0.5	-0.5
I ₇	•••	В	•••	0.986	0	0.33	-0.5
I ₈		C		0.986	I	-0.5	-0.29
l ₉		С		0.990	0	0.6	-0.39
I ₁₀		С		0.749	I	0	0

$$loss(T_c) = cos(\Delta,G) = -0.2003$$
 (TS < 0.8)





Choosing leaf values

- ✓ Given all the trees constructed, the leaf values of the final model F are calculated by the standard gradient boosting procedure
- \checkmark Training examples i are matched to leaves $leaf_0(i)$ using permutation σ_0 to calculate TS
- ✓ When the final model F is applied to a new example at testing time, TS is calculated on the whole training data





• Experimental Results

Table 8: Comparison with baselines: logloss / zero-one loss, relative increase is presented in the brackets.

	CatBoost	LightGBM	XGBoost
Adult	0.2695 / 0.1267	0.2760 (+2.4%) / 0.1291 (+1.9%)	0.2754 (+2.2%) / 0.1280 (+1.0%)
Amazon	0.1394 / 0.0442	0.1636 (+17%) / 0.0533 (+21%)	0.1633 (+17%) / 0.0532 (+21%)
Click	0.3917 / 0.1561	0.3963 (+1.2%) / 0.1580 (+1.2%)	0.3962 (+1.2%) / 0.1581 (+1.2%)
Epsilon	0.2647 / 0.1086	0.2703 (+1.5%) / 0.114 (+4.1%)	0.2993 (+11%) / 0.1276 (+12%)
Appetency	0.0715 / 0.01768	0.0718 (+0.4%) / 0.01772 (+0.2%)	0.0718 (+0.4%) / 0.01780 (+0.7%)
Churn	0.2319 / 0.0719	0.2320 (+0.1%) / 0.0723 (+0.6%)	0.2331 (+0.5%) / 0.0730 (+1.6%)
Internet	0.2089 / 0.0937	0.2231 (+6.8%) / 0.1017 (+8.6%)	0.2253 (+7.9%) / 0.1012 (+8.0%)
Upselling	0.1662 / 0.0490	0.1668 (+0.3%) / 0.0491 (+0.1%)	0.1663 (+0.04%) / 0.0492 (+0.3%)
Kick	0.2855 / 0.0949	0.2957 (+3.5%) / 0.0991 (+4.4%)	0.2946 (+3.2%) / 0.0988 (+4.1%)





• XGBoost vs. LightGBM vs. CatBoost

Function	XGBoost	CatBoost	Light GBM
Important parameters which control overfitting	 learning_rate or eta optimal values lie between 0.01-0.2 max_depth min_child_weight: similar to min_child leaf; default is 1 	 Learning_rate Depth - value can be any integer up to 16. Recommended - [1 to 10] No such feature like min_child_weight I2-leaf-reg: L2 regularization coefficient. Used for leaf value calculation (any positive integer allowed) 	 learning_rate max_depth: default is 20. Important to note that tree still grows leaf-wise. Hence it is important to tune num_leaves (number of leaves in a tree) which should be smaller than 2^(max_depth). It is a very important parameter for LGBM min_data_in_leaf: default=20, alias= min_data, min_child_samples
Parameters for categorical values	Not Available	 cat_features: It denotes the index of categorical features one_hot_max_size: Use one-hot encoding for all features with number of different values less than or equal to the given parameter value (max – 255) 	categorical_feature: specify the categorical features we want to use for training our model
Parameters for controlling speed	 colsample_bytree: subsample ratio of columns subsample: subsample ratio of the training instance n_estimators: maximum number of decision trees; high value can lead to overfitting 	 rsm: Random subspace method. The percentage of features to use at each split selection No such parameter to subset data iterations: maximum number of trees that can be built; high value can lead to overfitting 	 feature_fraction: fraction of features to be taken for each iteration bagging_fraction: data to be used for each iteration and is generally used to speed up the training and avoid overfitting num_iterations: number of boosting iterations to be performed; default=100





Anghel et al. (2019)

• XGBoost vs. LightGBM vs. CatBoost

Table 3: Best test scores across algorithms and datasets.

Dataset	Baseline		XGBoost		LightGBM		Catboost	
	Test	Val	Test	Val	Test	Val	Test	Val
Higgs	0.4996	0.5005	0.8353	0.8512	0.8573	0.8577	0.8498	0.8496
Epsilon	0.4976	0.5008	_	_	0.9513	0.9518	0.9537	0.9538
Microsoft	0.2251	0.3974	0.4917	0.6443	0.4871	0.6473	0.3782	0.5492
Yahoo	0.5802	0.8106	0.7983	0.9146	0.7965	0.9142	0.7351	0.8849





Anghel et al. (2019)

• XGBoost vs. LightGBM vs. CatBoost

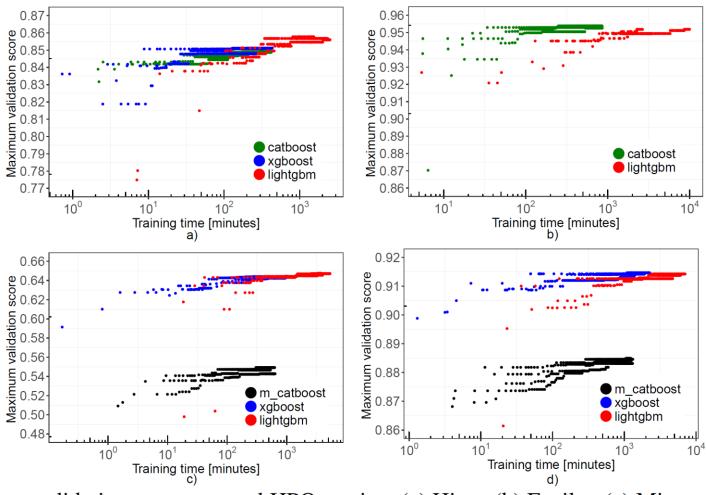


Figure 2: Max. validation score vs. total HPO runtime (a) Higgs (b) Epsilon (c) Microsoft (d) Yahoo.







