

Clustering: K-Means Clustering

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AGENDA

01 Clustering: Overview

02 **K-Means Clustering**

03 Hierarchical Clustering

04 Density-based Clustering: DBSCAN

04 R Exercise

K-Means Clustering

- K-Means Clustering (KMC)

- ✓ Partitional clustering approach

- Each cluster is associated with a centroid
- Each point is assigned to the cluster with the closest centroid
- Number of cluster, K, must be specified

$$\mathbf{X} = C_1 \cup C_2 \dots \cup C_K, \quad C_i \cap C_j = \phi, \quad i \neq j$$

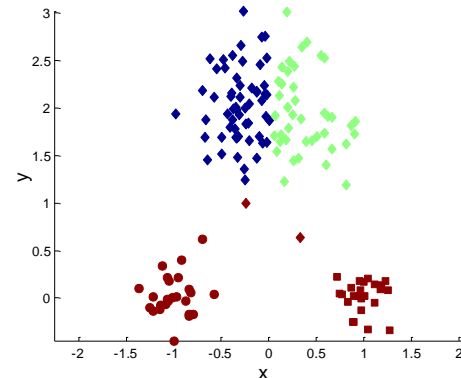
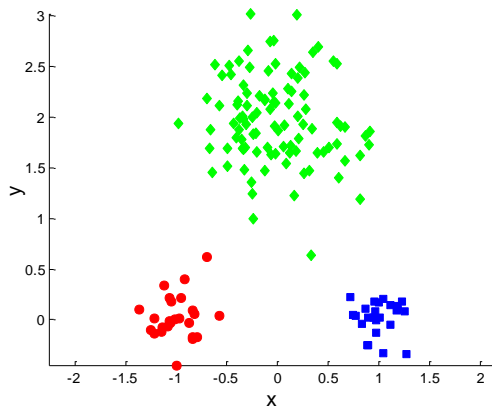
$$\arg \min_{\mathbf{C}} \sum_{i=1}^K \sum_{\mathbf{x}_j \in C_i} \|\mathbf{x}_j - \mathbf{c}_i\|^2$$

K-Means Clustering

- K-Means Clustering Procedure

-
- 1: Select K points as the initial centroids.
 - 2: **repeat**
 - 3: Form K clusters by assigning all points to the closest centroid.
 - 4: Recompute the centroid of each cluster.
 - 5: **until** The centroids don't change
-

✓ Initial centroids are often **chosen randomly**: clustering results vary according to the initial centroid selection



K-Means Clustering

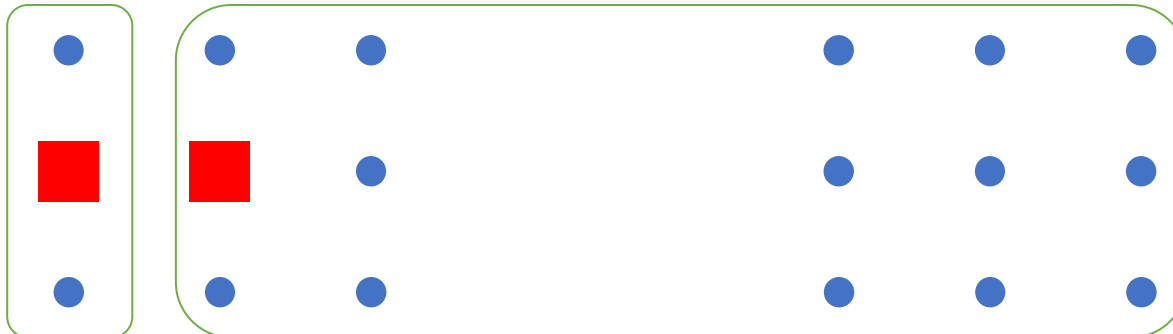
- Example

- ✓ Step 1: Initializing K centroids



- ✓ Step 2-1 (1st): Assign each instance to the closest center

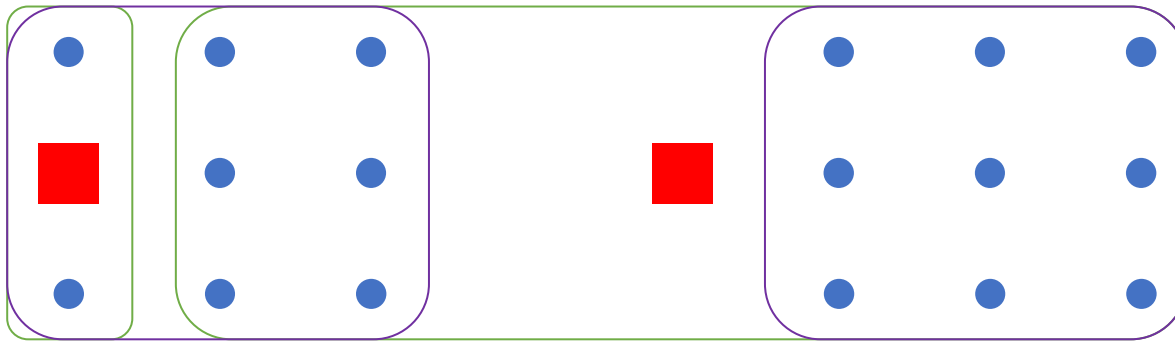
- ✓ Step 2-2 (1st): Re-compute the centroids based on the assigned instances



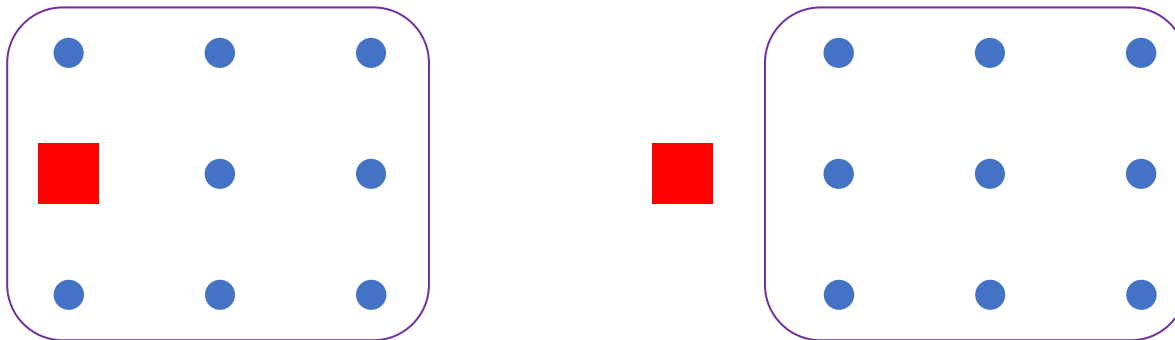
K-Means Clustering

- Example

✓ Step 2-1 (2nd): Assign each instance to the closest center



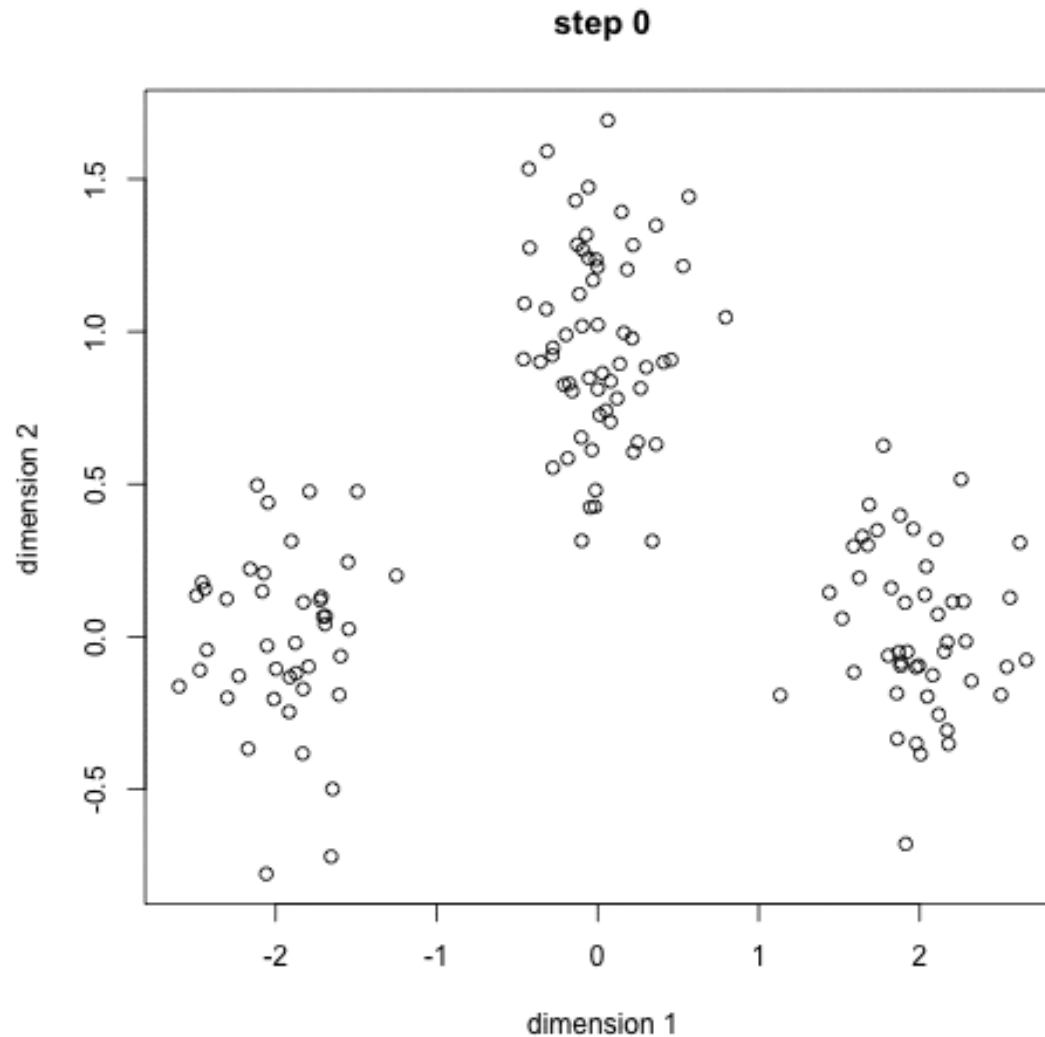
✓ Step 2-2 (2nd): Re-compute the centroids based on the assigned instances



✓ Stop the algorithm because there is no change for centroids and membership

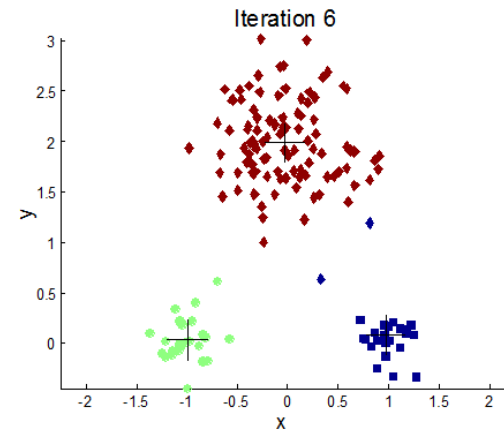
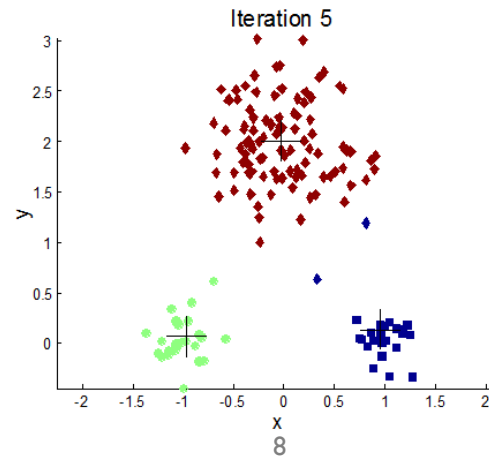
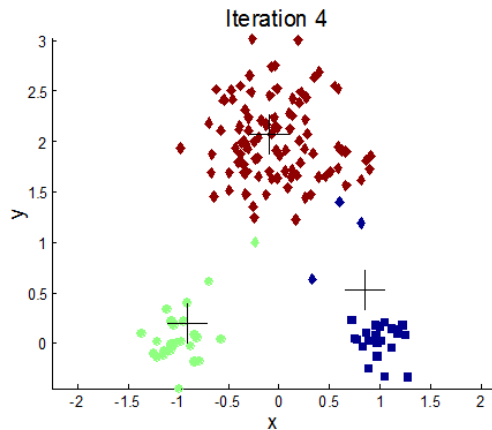
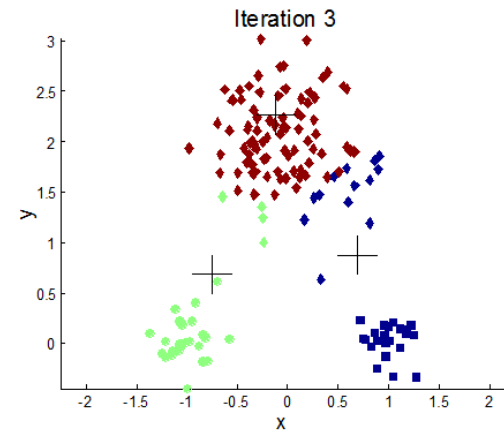
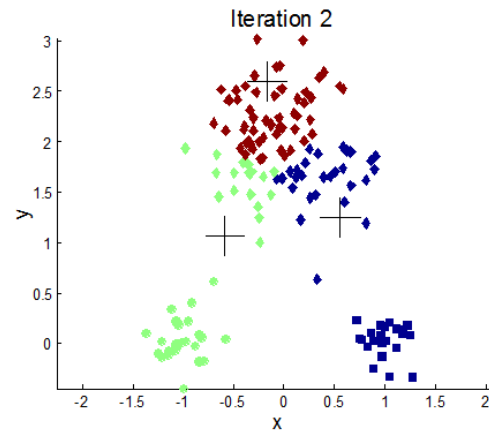
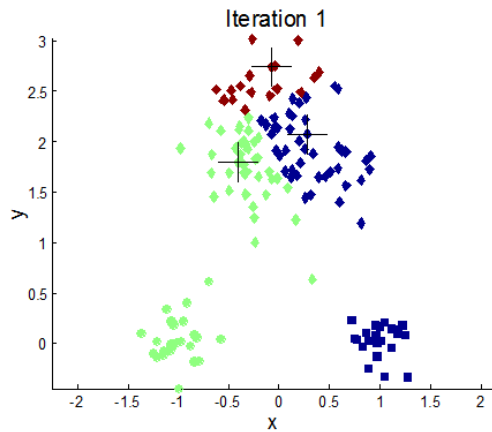
K-Means Clustering

- KMC example



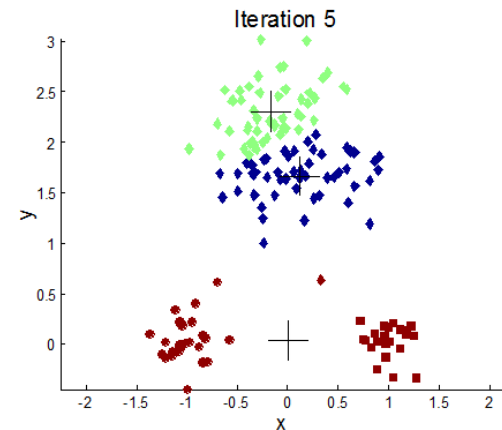
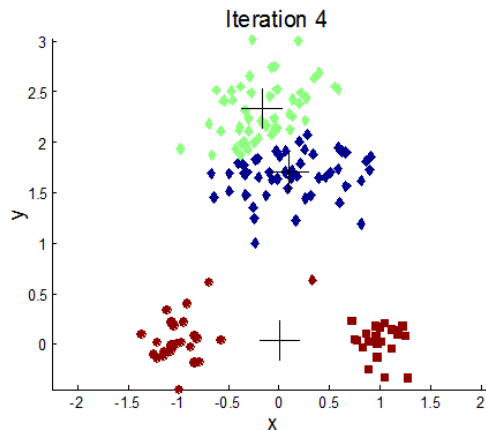
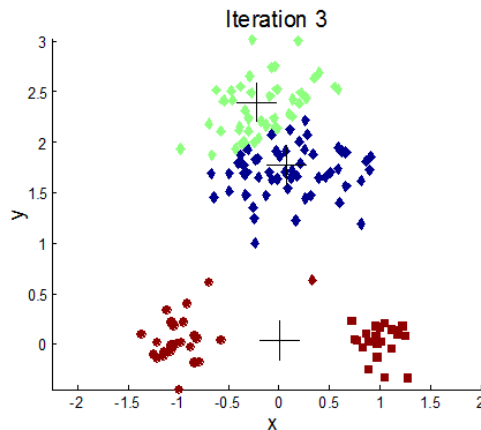
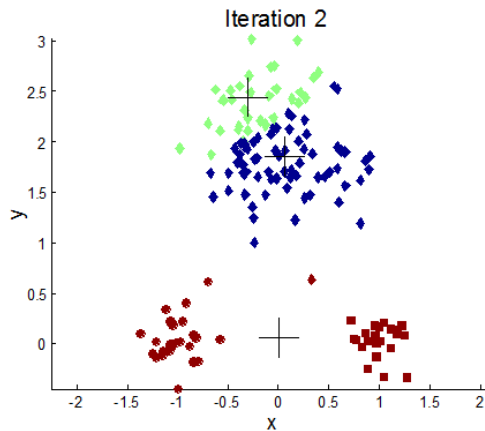
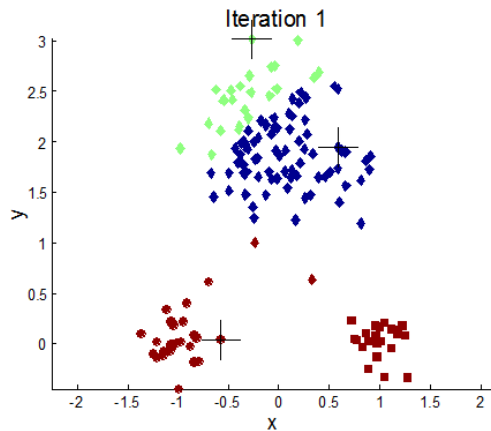
K-Means Clustering

- Effect of initial centroids
 - ✓ Desirable centroid selection



K-Means Clustering

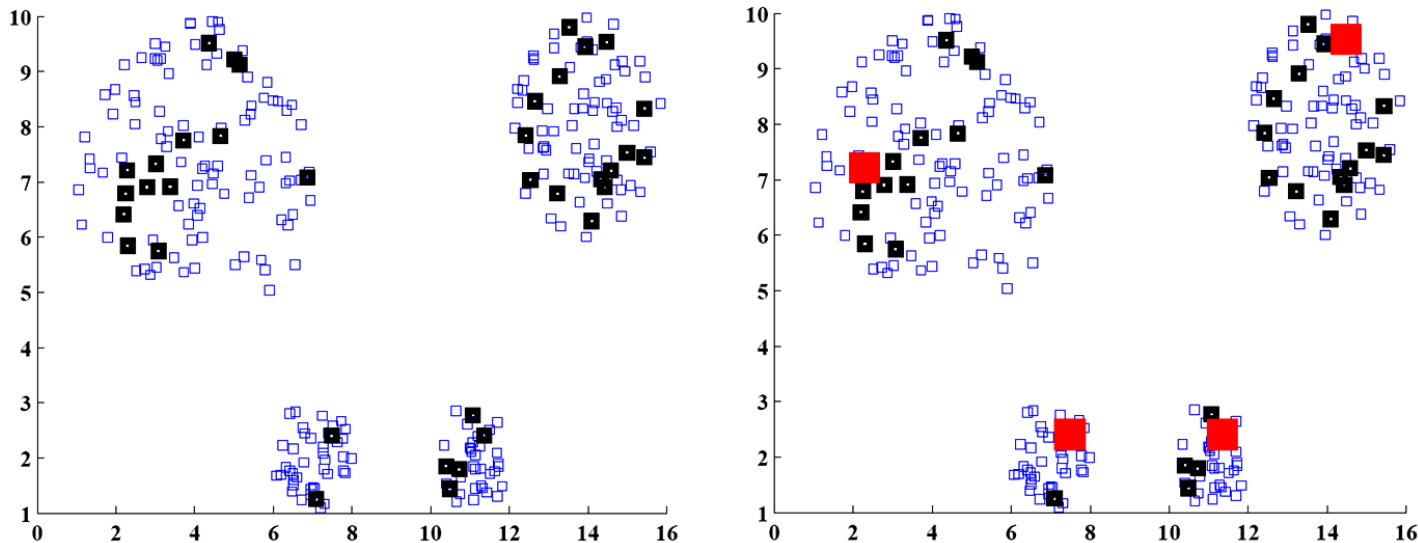
- Effects of initial centroids
 - ✓ Undesirable centroid selection



K-Means Clustering

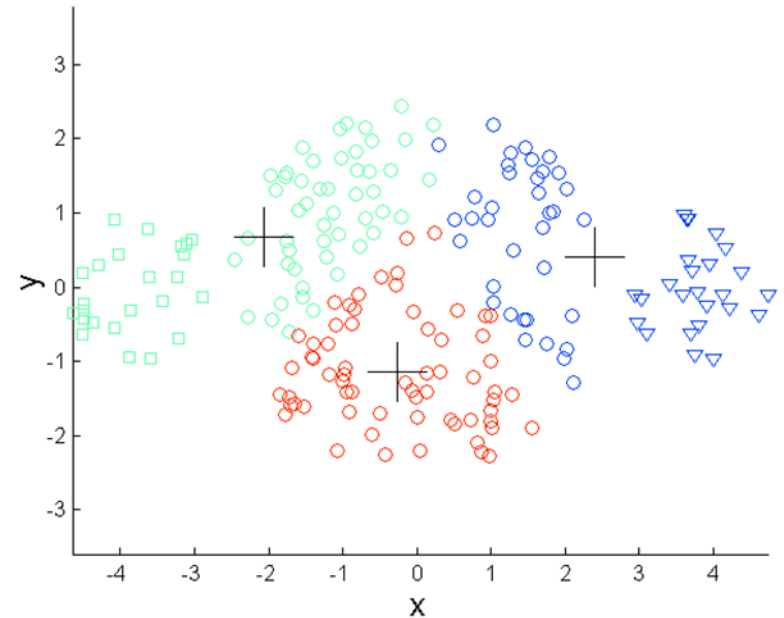
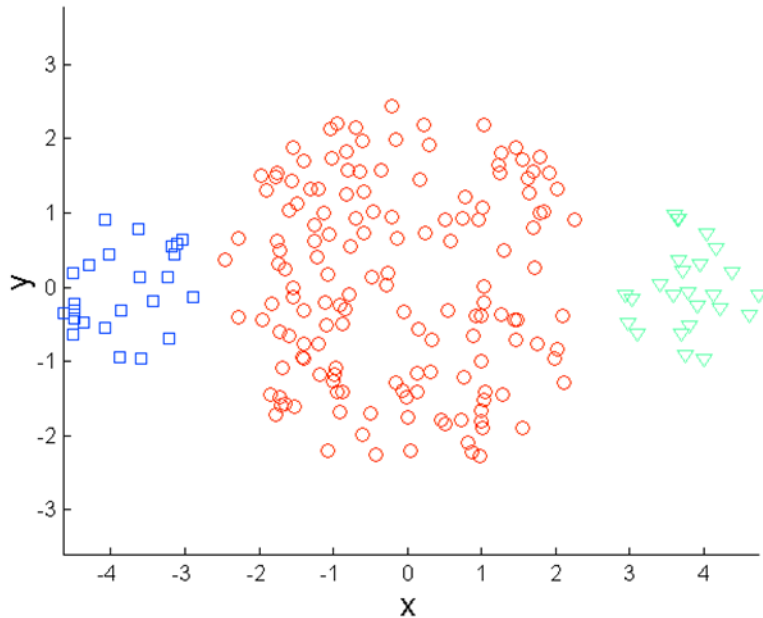
- Some remedies for initial centroid selection
 - ✓ Multiple runs
 - ✓ Sample and use hierarchical clustering to determine initial centroids
 - ✓ Preprocessing & Postprocessing

$$\mathcal{L}(\mathbf{x}_s | \mathbf{S}, \mathbf{C}) = d_G(\mathbf{x}_s, \mathbf{S}) \times \frac{1}{1 + \exp(-d_R(\mathbf{x}_s, \mathbf{S}))}$$



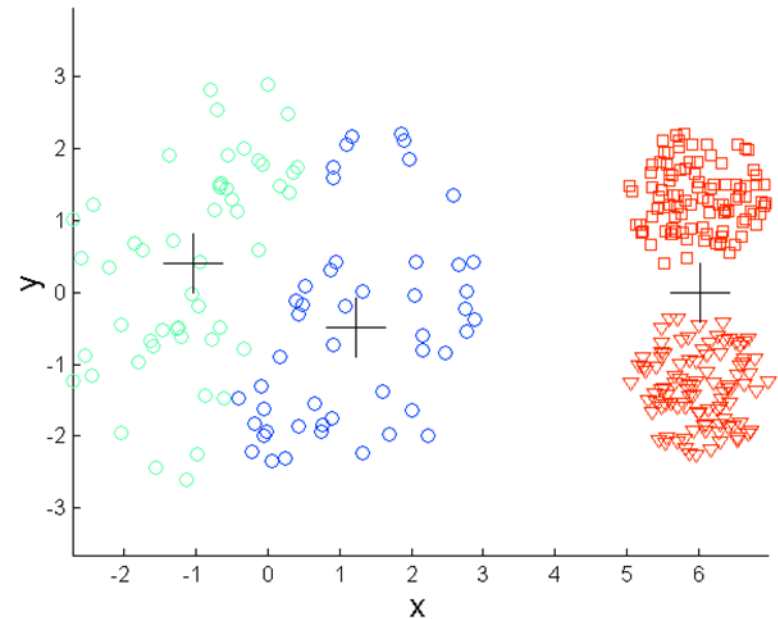
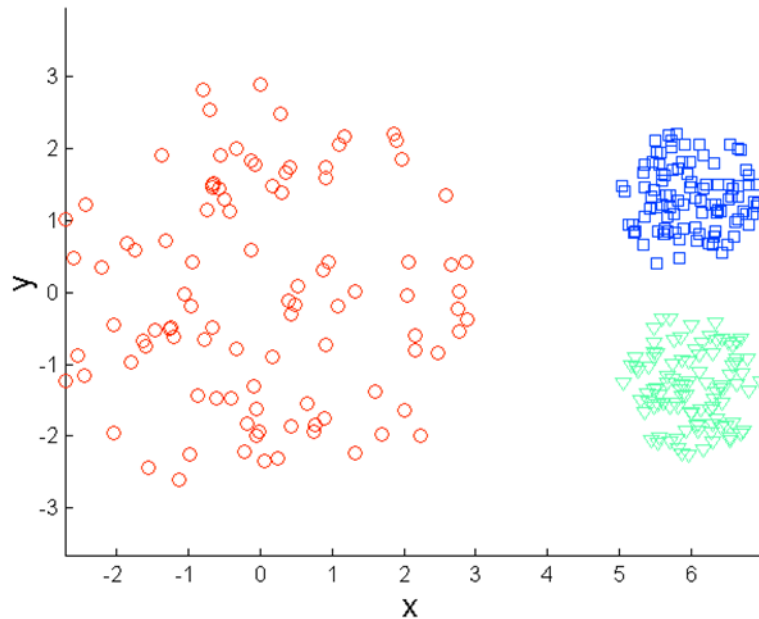
K-Means Clustering

- Limitations of K-Means Clustering
 - ✓ Cannot cope with different sizes



K-Means Clustering

- Limitations of K-Means Clustering
 - ✓ Cannot cope with different densities



K-Means Clustering

- Limitations of K-Means Clustering
 - ✓ Cannot cope with non-globular shapes

