



**“All things being equal, the simplest solution tends to be the best one.”**

**William of Ockham**

# Dimensionality Reduction

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# AGENDA

**01** Dimensionality Reduction

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**02** Variable Selection Methods

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**03** Shrinkage Methods

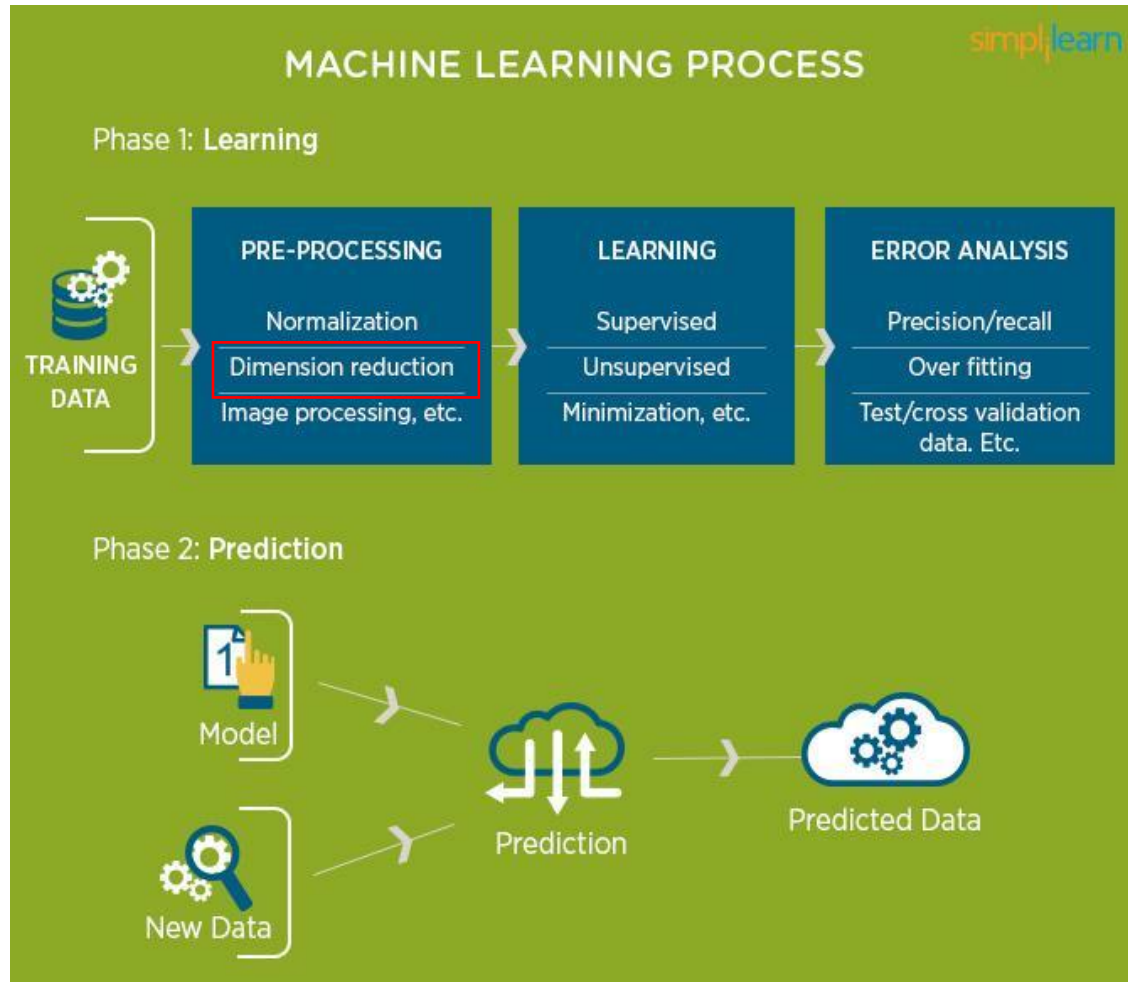
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**04** R Exercise

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# Data Analytics Process

- Process of Business Analytics with Machine Learning



# High-dimensional Data

- Examples of high dimensional data

## Document classification:

Billions of documents x Thousands/  
Millions of words/bigrams matrix



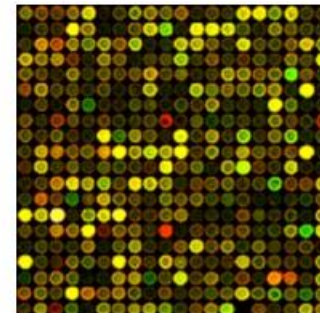
## Recommendation systems:

480,189 users x 17,770 movies matrix



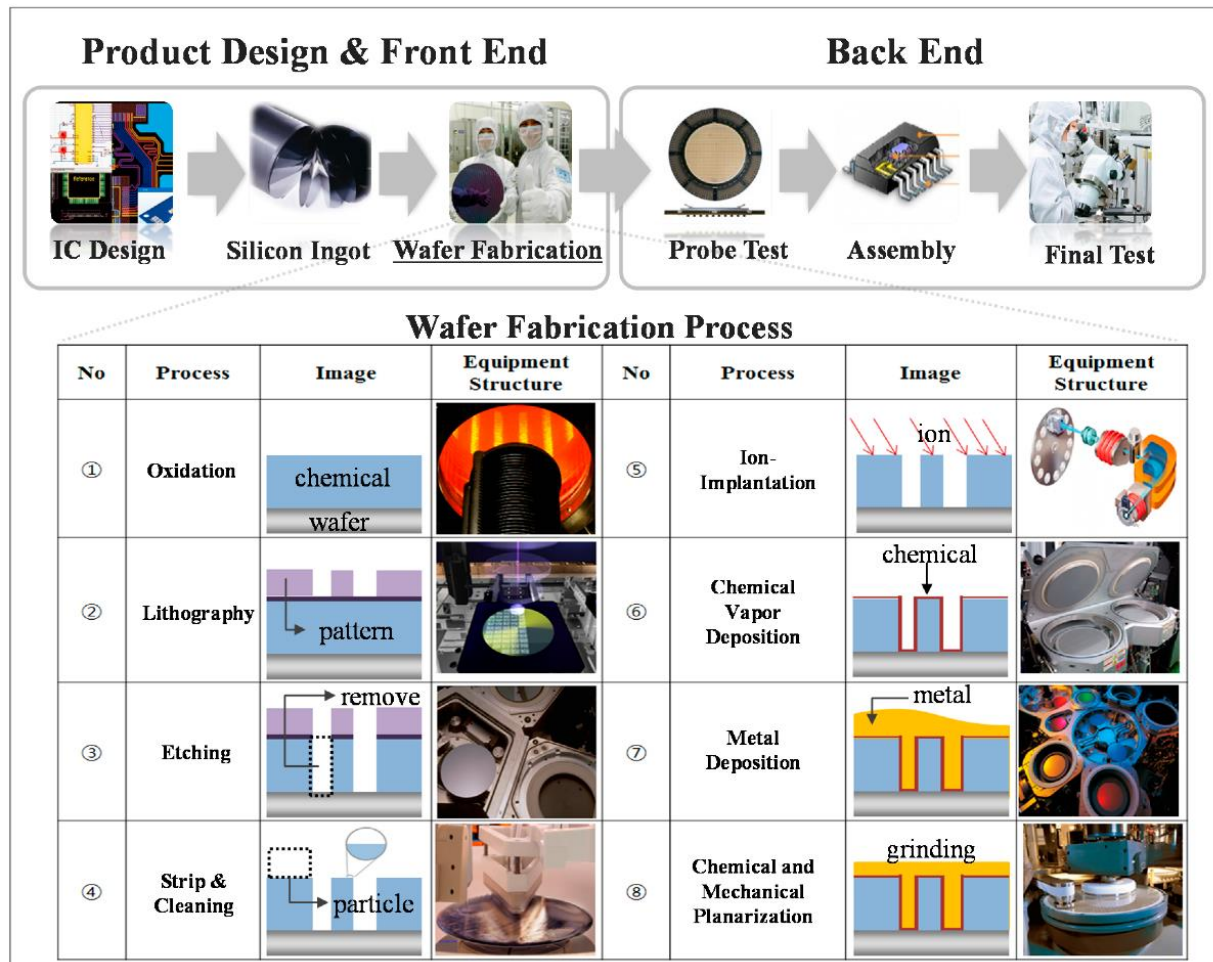
## Clustering gene expression profiles:

10,000 genes x 1,000 conditions



# High-dimensional Data

- Examples of high dimensional data

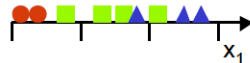
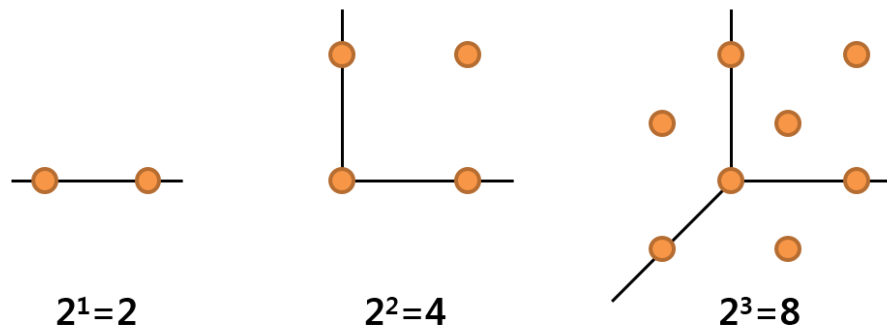


# Dimensionality Reduction: Overview

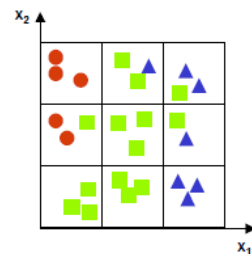
- Curse of dimensionality

- ✓ The number of instances increases exponentially to achieve the same explanation ability when the number of variables increases

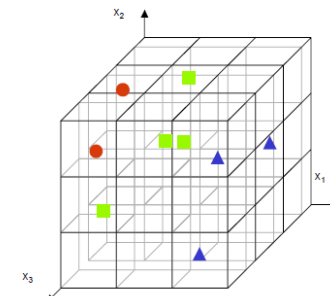
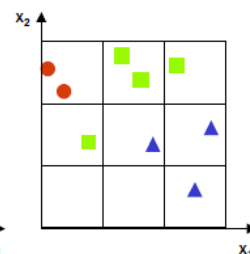
“If there are various logical ways to explain a certain phenomenon, the simplest is the best” - Occam’s Razor



Constant density



Constant # examples



# Dimensionality Reduction: Overview

- Curse of dimensionality

- ✓ Sometimes, an intrinsic dimension is relatively low compared to the original dimension.

- Ex: handwritten digits in a 16 by 16 pixel (256 dimensions)

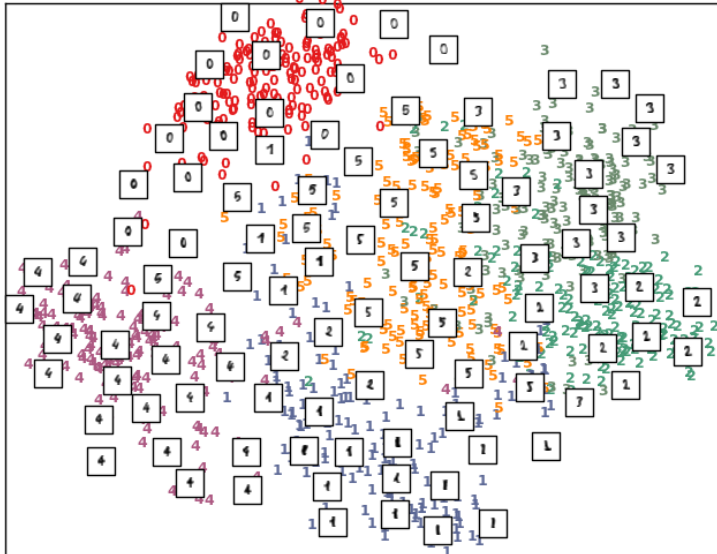




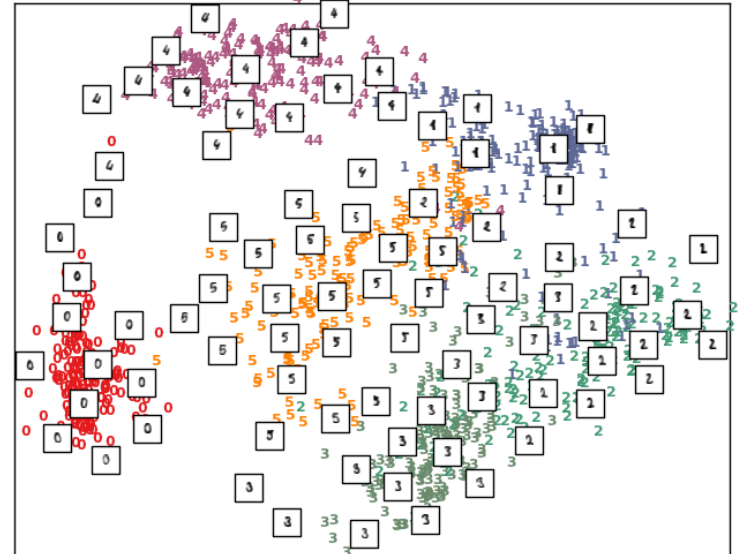
# Dimensionality Reduction: Overview

- Curse of dimensionality
  - ✓ Sometimes, an intrinsic dimension is relatively low compared to the original dimension.
    - Ex: handwritten digits in a 16 by 16 pixel (256 dimensions)
    - Reduced to two dimensions by PCA and ISOMAP

Principal Components projection of the digits (time 0.01s)



Isomap projection of the digits (time 1.51s)





# Dimensionality Reduction: Overview

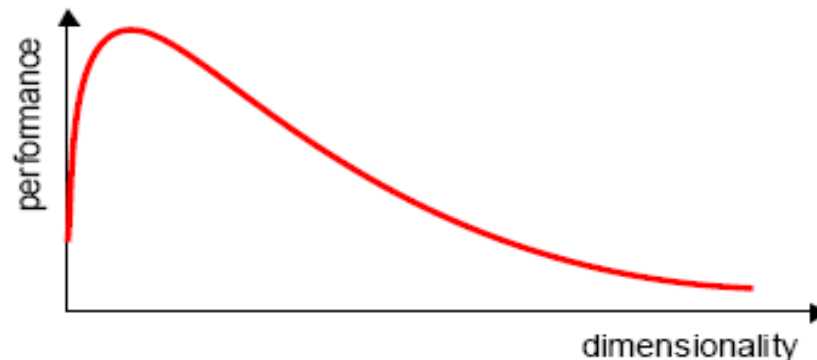
- Curse of dimensionality

- ✓ Problems caused by high-dimensionality

- Increase the probability of having noise in data → degenerate the prediction performance
- Increase computational burden for training/applying prediction models
- Require more number of examples to secure generalization ability of prediction model

- ✓ To resolve the curse of dimensionality

- Utilize domain knowledge
- Use a regularization term in objective function
- Employ a quantitative reduction technique



# Dimensionality Reduction: Overview

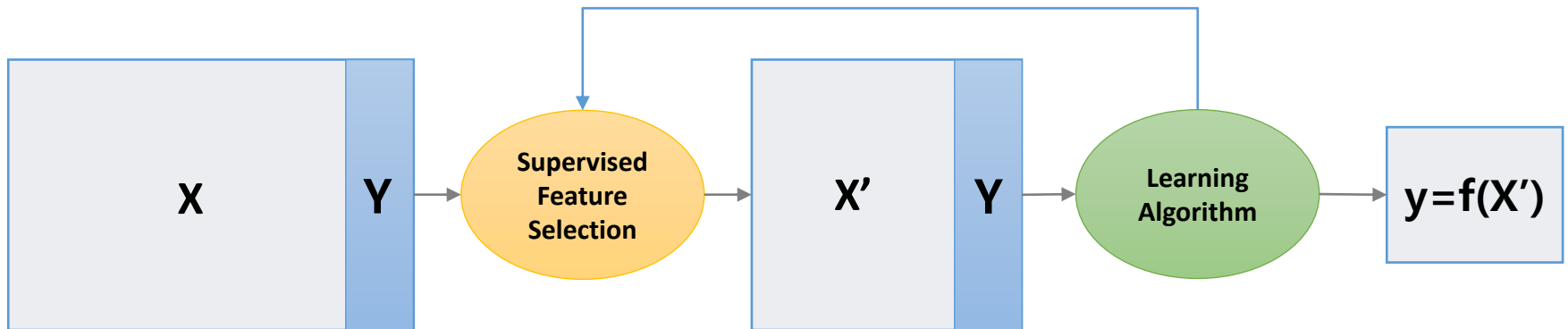
- Backgrounds
  - ✓ Theoretically, model performance improves when the number of variables increases  
(Under variable independence condition)
  - ✓ In reality, model performance degenerates due to variable dependence, existence of noise, etc.
- Purpose
  - ✓ Identify a subset of variables that best fit the model
- Effect
  - ✓ Remove correlations between variables
  - ✓ Simplified post-processing
  - ✓ Remove redundant or unnecessary variables while keeping relevant information
  - ✓ Visualization can be possible

# Dimensionality Reduction: Overview

- Supervised vs. Unsupervised Dimensionality Reduction

- ✓ Supervised dimensionality reduction

- Use data mining models to verify the reduced dimensions
    - Dimensionality reduction results can be different according to the data mining algorithms employed

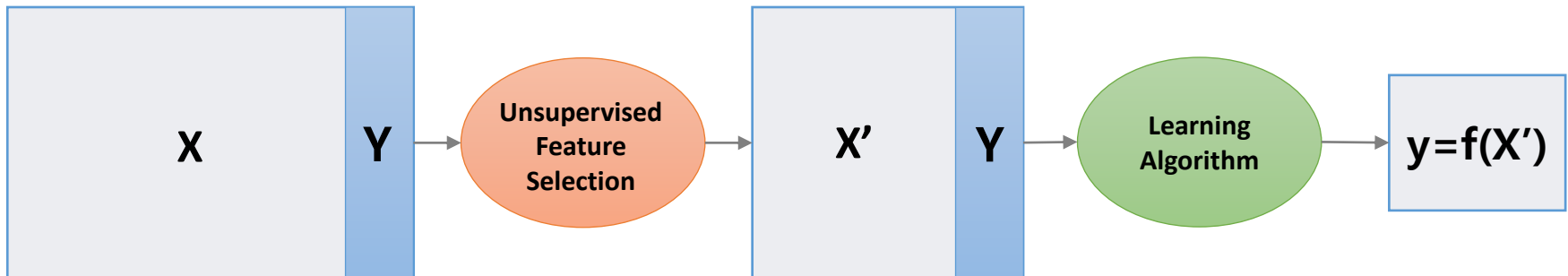


# Dimensionality Reduction: Overview

- Supervised vs. Unsupervised Dimensionality Reduction

- ✓ Unsupervised dimensionality reduction

- Find a set of coordinate systems in a lower dimension that preserve the information (e.g., variance, distance, etc.) in the original input space as much as possible
    - Do not use data mining models during the process
    - Dimensionality reduction results are identical if the data and method is same



# Dimensionality Reduction: Overview

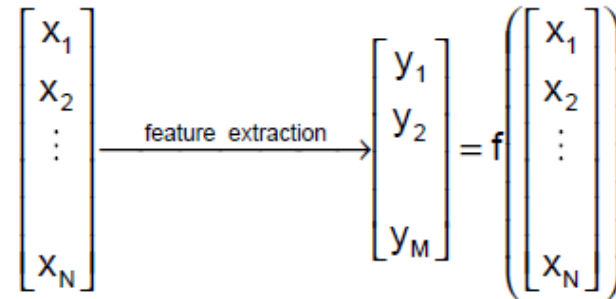
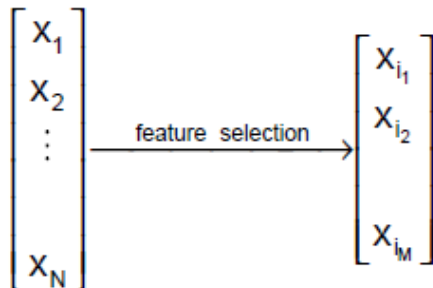
- Dimensionality reduction techniques

- ✓ Variable/feature selection

- Select a subset of variables from the original variable set
    - Filter – Variable selection and model training are independent
    - Wrapper – Variable selection is done to optimize the result of the considered data mining model

- ✓ Variable/feature extraction

- Extract a new smaller set of variables that preserve the characteristics of the original data
    - Performance metric that is independent from data mining models is used



# Dimensionality Reduction: Overview

- Selection vs. Extraction

✓ Conceptual difference between variable selection and variable extraction

$X_1$	$X_2$	$X_3$	...	$X_n$
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...

Variable selection

$X_1$	$X_5$	$X_8$
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...

Variable extraction

$Z_1$	$Z_2$	$Z_3$
...	...	...
...	...	...
...	...	...
...	...	...
...	...	...

$$\begin{aligned} Z_1 &= X_1 + 0.2 * X_2 \\ Z_2 &= X_3 - 2 * X_5 \\ Z_3 &= X_4 + X_6 - X_9 \end{aligned}$$

