

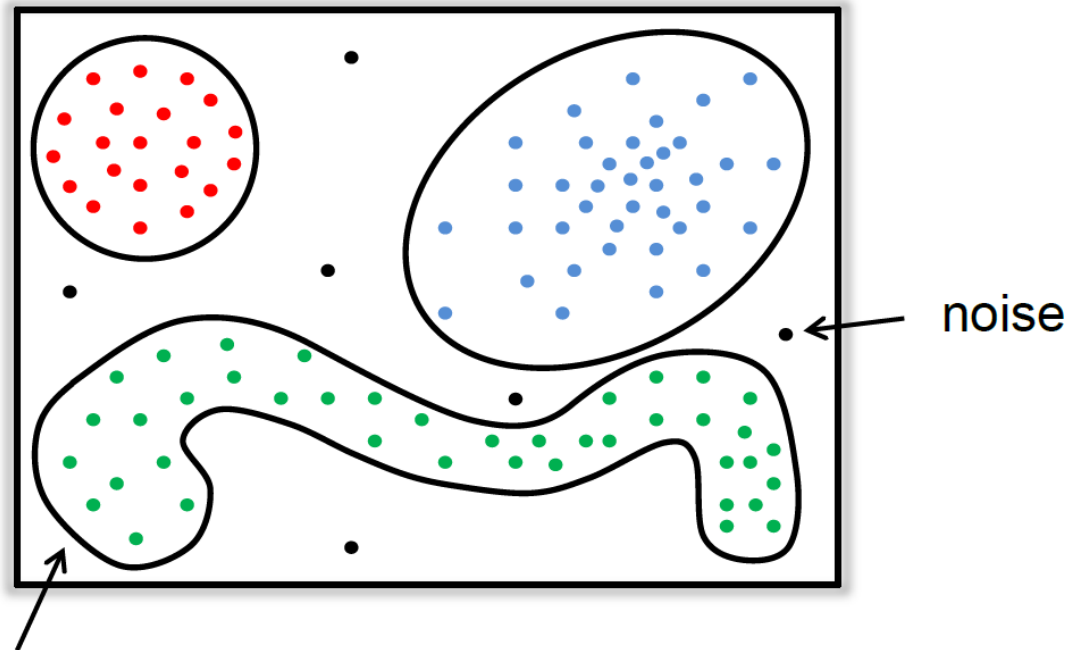
AGENDA

- 01 Clustering: Overview
- 02 K-Means Clustering
- 03 Hierarchical Clustering
- 04 **Density-based Clustering: DBSCAN**
- 04 R Exercise

Density-based Clustering

Ester et al. (1996)

- Density-based clustering
 - ✓ Conduct a clustering by considering the density of data points
 - Can find an arbitrary shape of cluster
 - Can remove noise from clustering result



arbitrarily shaped clusters

Density-based Clustering

- DBSCAN

- ✓ Most popular density-based clustering algorithm

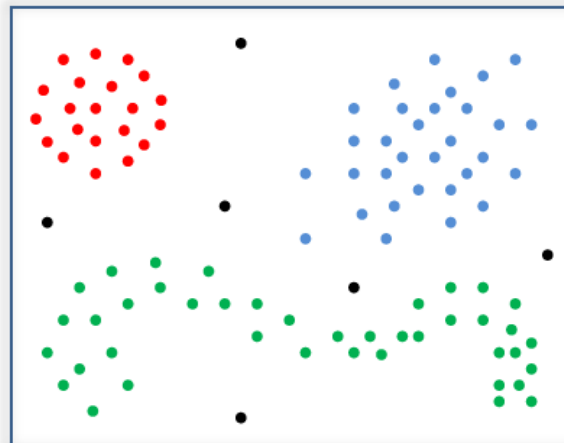
- Idea

- ✓ Clusters are the collections of data points with high density

- ✓ Density around a noise point is very low

- Purpose

- ✓ Quantify the features of clusters and noise points to find a set of valid clusters



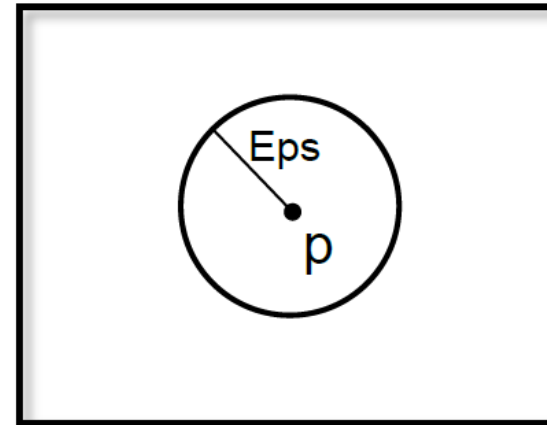
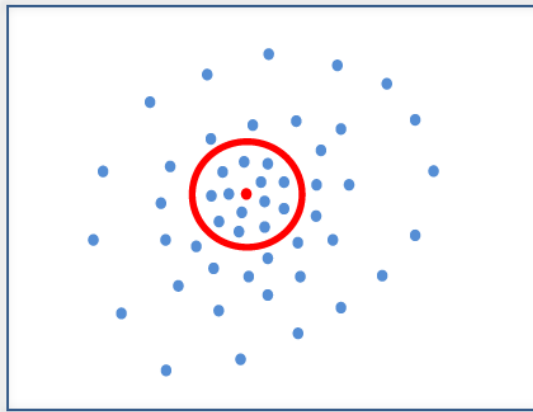
Density-based Clustering

- DBSCAN

- ✓ Definition 1: ϵ -neighborhood of a point

- The ϵ -neighborhood of a point, denoted by $N_\epsilon(p)$, is defined by

$$N_\epsilon(p) = \{q \in D \mid \text{dist}(p, q) \leq \epsilon\}$$



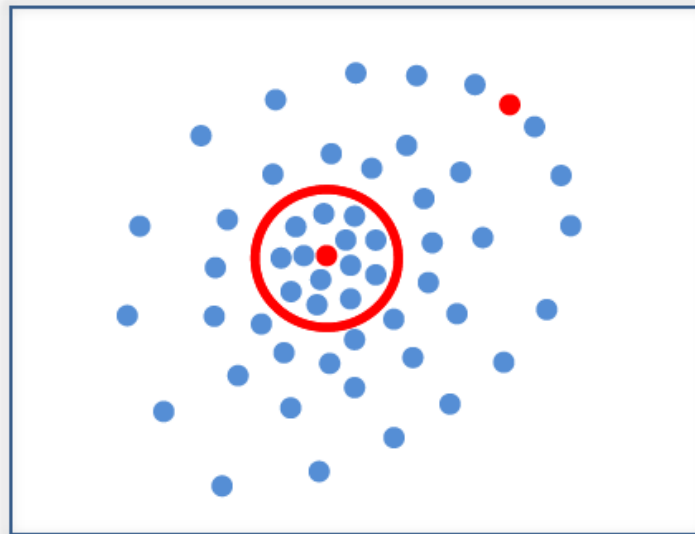
- ✓ Naïve Approach: require for each point in a cluster that there are at least a minimum number (MinPts) of points in an ϵ -neighborhood of that point

Density-based Clustering

- DBSCAN

- ✓ Problem of Naïve Approach

- There are two kinds of points in a cluster
 - Points inside of the cluster (core points)
 - Points on the border of the cluster (border points)
 - An ϵ -neighborhood of a border point contains significantly less points than an ϵ -neighborhood of a core point

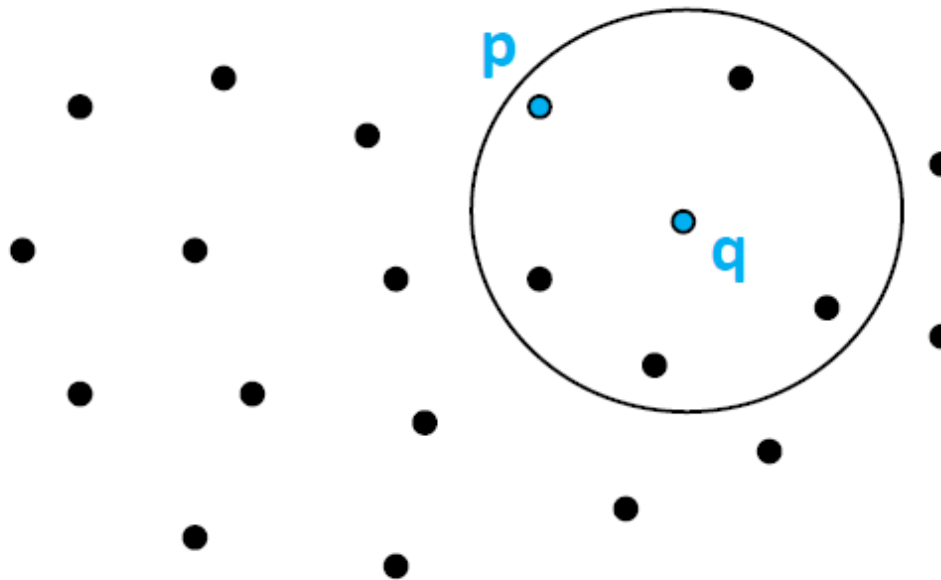


Density-based Clustering

- DBSCAN

- ✓ Better idea

- For every point p in a cluster C , there is a point q in C so that p is inside of the ϵ -neighborhood of q (Border points are connected to core points)
 - $N_\epsilon(q)$ contains at least MinPts points (Core points = high density)



Density-based Clustering

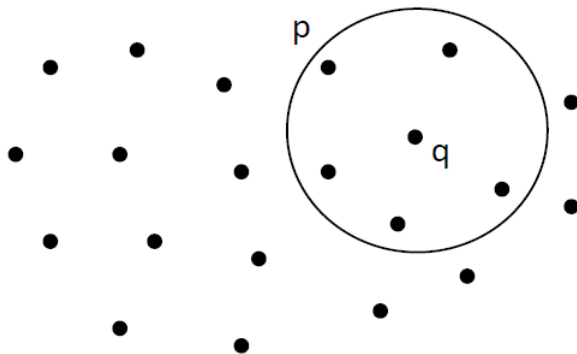
- DBSCAN

- ✓ Definition 2: directly density-reachable

- A point p is directly density-reachable from a point q with regard to the parameters ϵ and $MinPts$, if

1) $p \in N_{\epsilon}(q)$ (*reachability*)

2) $|N_{\epsilon}(q)| \geq MinPts$ (*core point condition*)



$MinPts = 5$

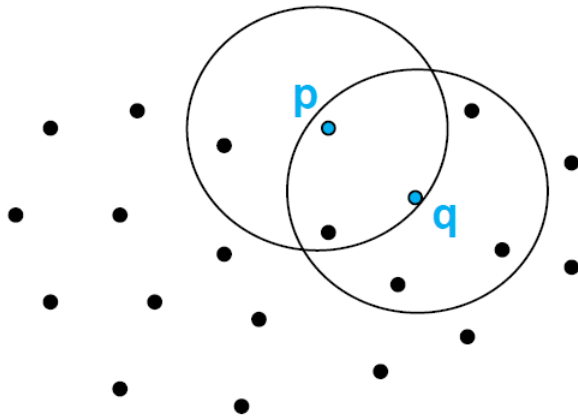
$|N_{Eps}(q)| = 6 \geq 5 = MinPts$ (core point condition)

Density-based Clustering

- DBSCAN

- ✓ Property

- Directly density-reachable is symmetric for **pairs of core points**
 - It is not symmetric if **one core point** and **one border point** are involved



Parameter: MinPts = 5

p directly density reachable from q

$$p \in N_{Eps}(q)$$

$$|N_{Eps}(q)| = 6 \geq 5 = \text{MinPts} \quad (\text{core point condition})$$

q **not** directly density reachable from p

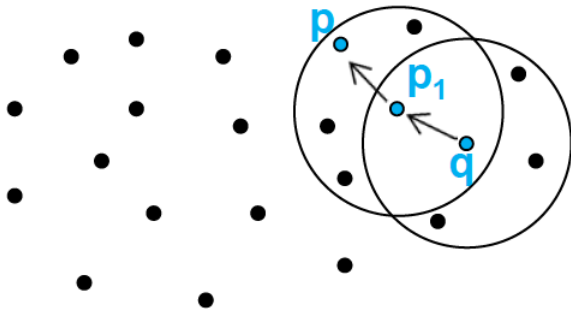
$$|N_{Eps}(p)| = 4 < 5 = \text{MinPts} \quad (\text{core point condition})$$

Density-based Clustering

- DBSCAN

- ✓ Definition 3: density-reachable

- A point p is density-reachable from a point q with regard to the parameters ϵ and MinPts , if there is a chain of points p_1, p_2, \dots, p_s with $p_1 = q$ and $p_s = p$ such that p_{i+1} is directly density-reachable from p_i for all $1 < i < s-1$



$\text{MinPts} = 5$

$|N_{\text{Eps}}(q)| = 5 = \text{MinPts}$ (core point condition)

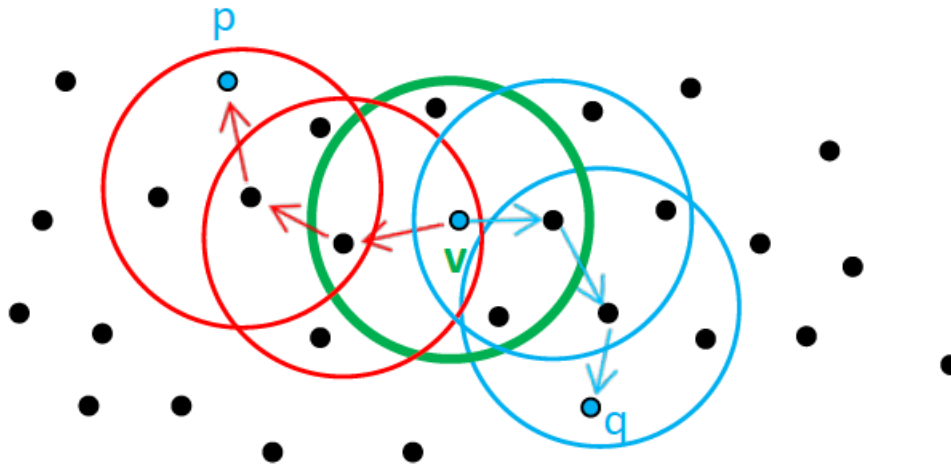
$|N_{\text{Eps}}(p_1)| = 6 \geq 5 = \text{MinPts}$ (core point condition)

Density-based Clustering

- DBSCAN

- ✓ Definition 4: density-connected

- A point p is density-connected to a point q with regard to the parameters ϵ and MinPts, if there is a point v such that both p and q are density-reachable from v



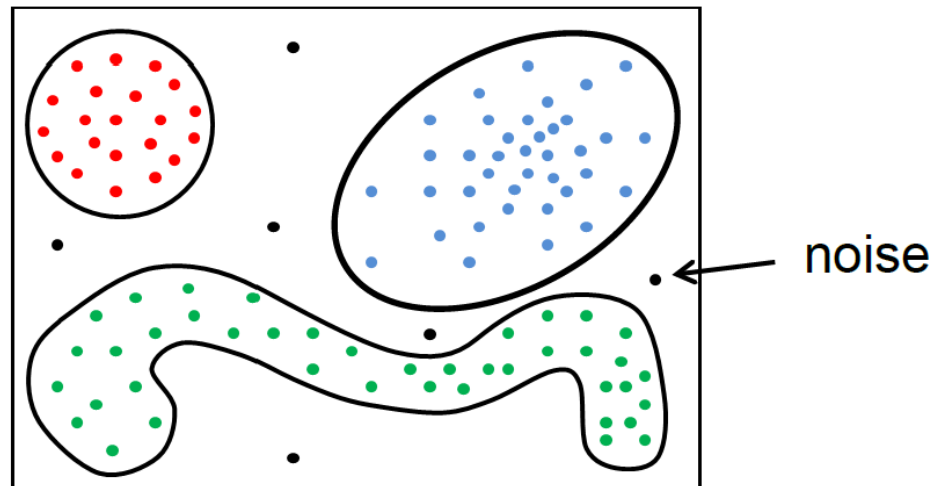
MinPts = 5

Density-based Clustering

- DBSCAN

- ✓ Definition 5: Cluster

- A cluster with regard to the parameters ϵ and MinPts is a non-empty subset C of the database D with
 - (1) For all $p, q \in D$: If $p \in C$ and q is density-reachable from p with regard to the parameters ϵ and MinPts, then $q \in C$ (**Maximality**)
 - (2) For all $p, q \in C$: The point p is density-connected to q with regard to the parameters ϵ and MinPts (**Connectivity**)

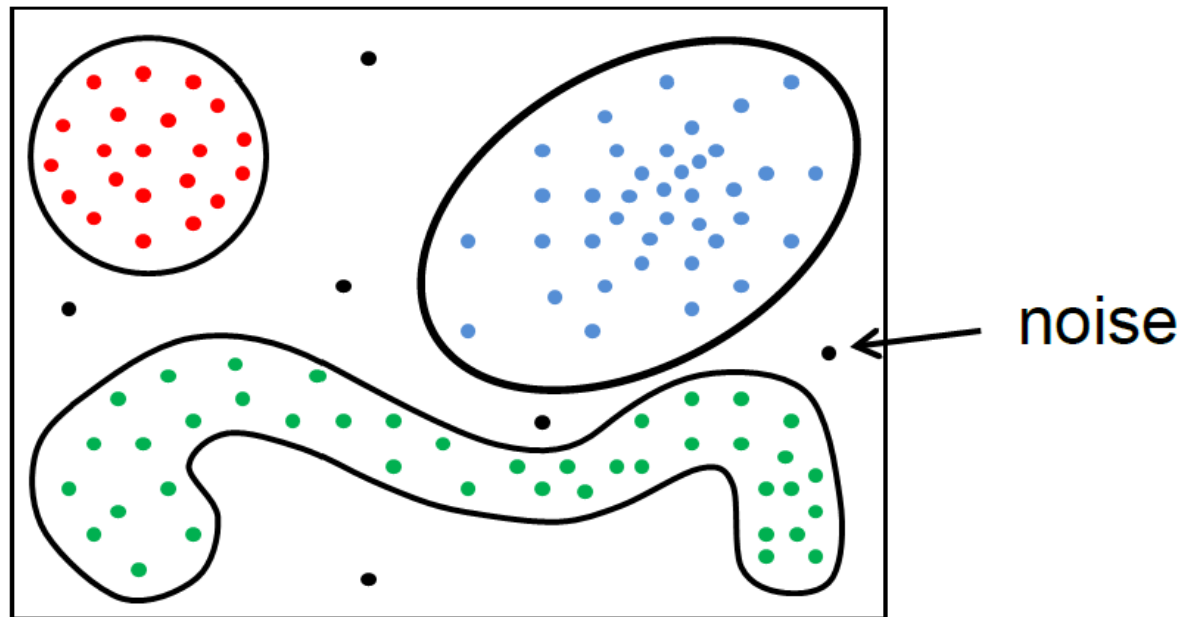


Density-based Clustering

- DBSCAN

- ✓ Definition 6: Noise

- Let C_1, \dots, C_k be the clusters of the database D with regard to the parameters ε and MinPts
 - The set of points in the database D not belonging to any cluster C_1, \dots, C_k is called noise

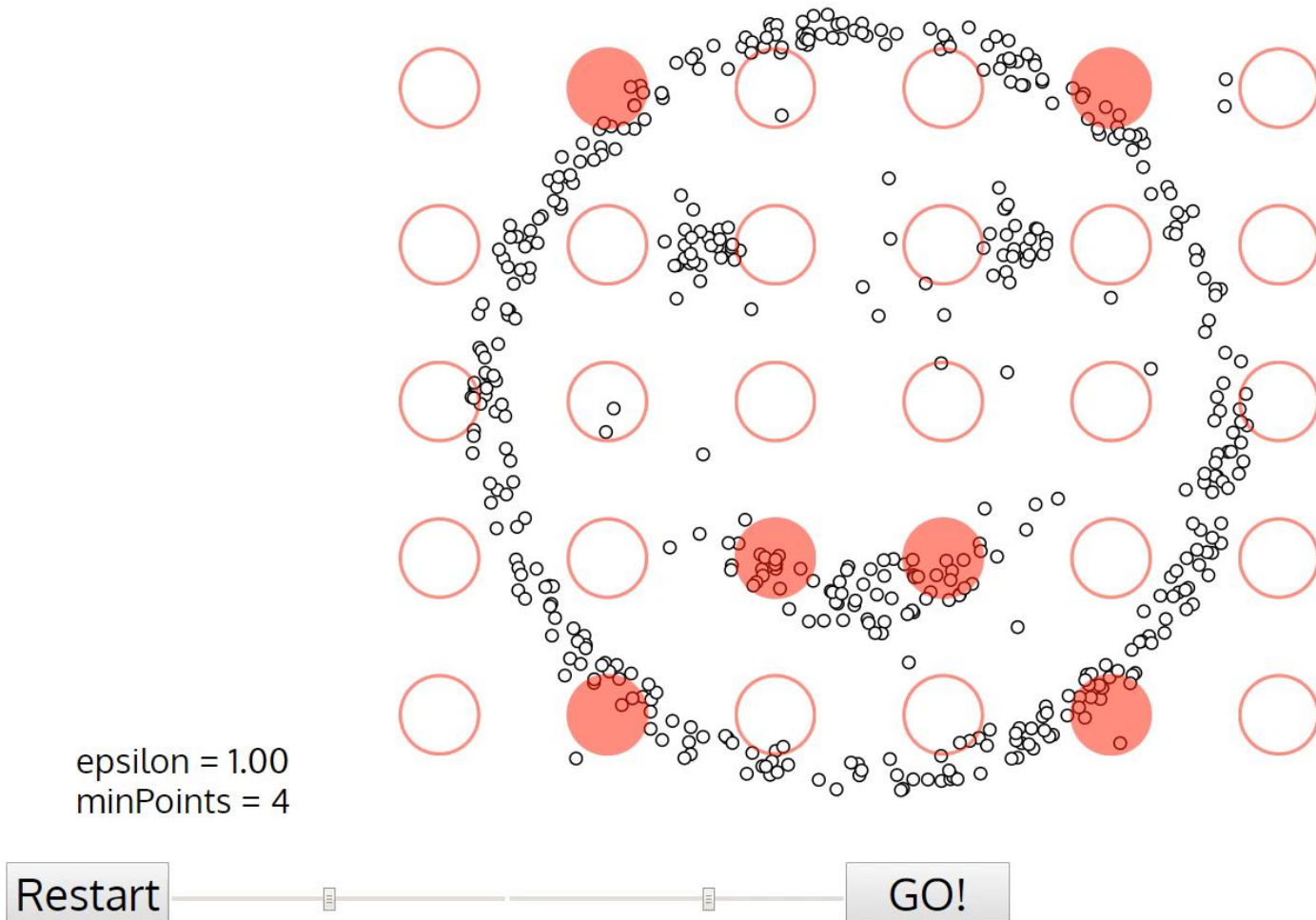


Density-based Clustering

- DBSCAN: Algorithm
 - ✓ Input: N objects to be clustered and global parameter, ϵ and MinPts
 - ✓ Output: Cluster of objects
- Algorithm
 - ✓ Arbitrary select a point p
 - ✓ Retrieve all points density-reachable from p w.r.t. ϵ and MinPts
 - ✓ If p is a core points, a cluster is formed
 - ✓ If p is a border point, no points are density reachable from p and DBSCAN visits the next point of the database
 - ✓ Continue the process until all of the points have been processed

Density-based Clustering

- DBSCAN example

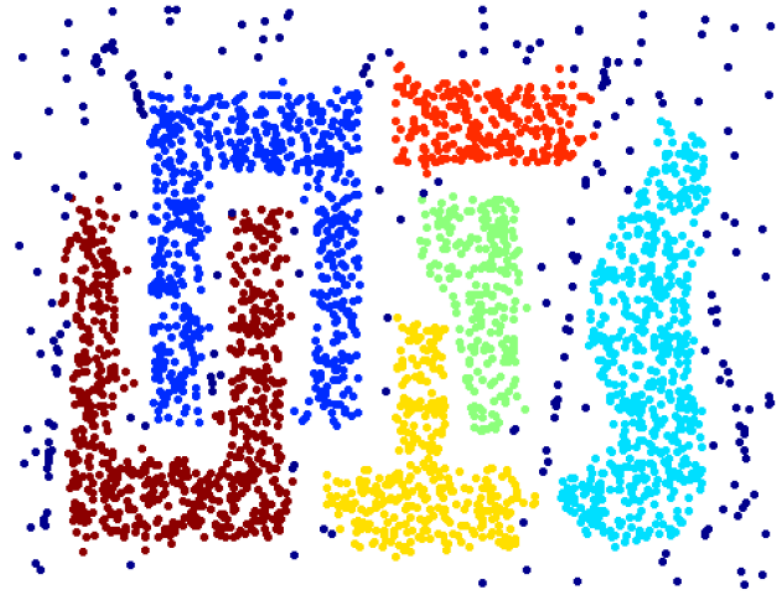


Density-based Clustering

- DBSCAN example



Original Points



Clusters

