Mathematical Data Science HW5

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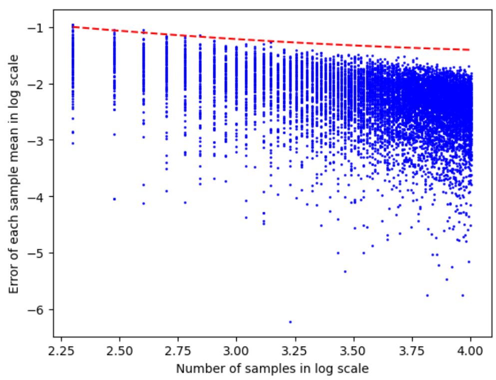
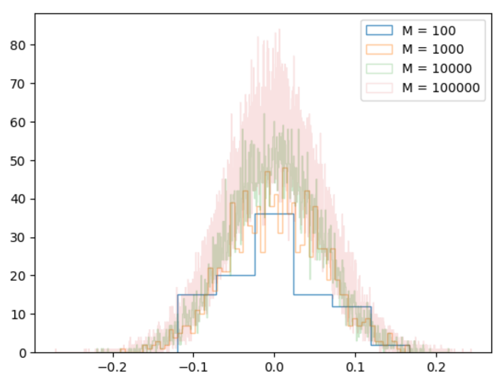
Problem 1

1) *Solution.* The below figure shows the convergence of the sample mean. Decreasing with the rate of $\frac{1}{\sqrt{N}}$.

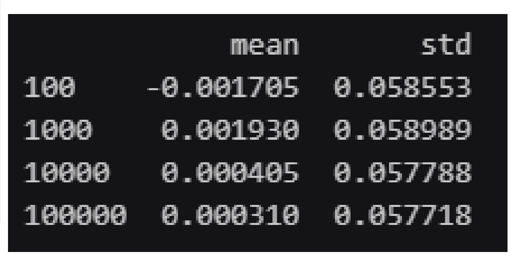


$\square$

2) *Solution.* Following histograms show the distribution of $\mu_M(N)$ with increasing M. We can check it becomes similar to a normal distribution as M goes to ∞. Since the exact distribution is a uniform distribution from -1 to 1, $\mu_{\text{exact}} = 0$ and $\sigma_{\text{exact}} = \sqrt{\frac{1}{3}} \simeq 0.577$. Here is the error.



(a) sample histogram



	mean	std
100	-0.001705	0.058553
1000	0.001930	0.058989
10000	0.000405	0.057788
100000	0.000310	0.057718

(b) sample value and error

From this, we can check that $\mu_M \simeq \mu_{\text{exact}}$ and $\sigma_M \times 10 \simeq \sigma_{\text{exact}}$. Note that the coefficient '10' comes from the sample size $N = 100$.

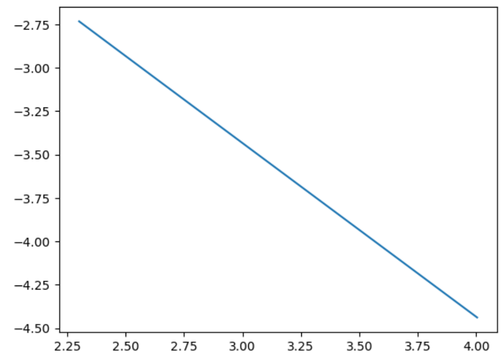
$\square$

3) *Solution.* From 2), we can check that σ_{exact} is related to N. i.e. $\sigma_M \propto \frac{1}{\sqrt{N}}$. Because of this, the error in 1) converges with rate $\frac{1}{\sqrt{N}}$. This is why the errors of the most trials decay with the rate of $\frac{1}{\sqrt{N}}$.

$\square$

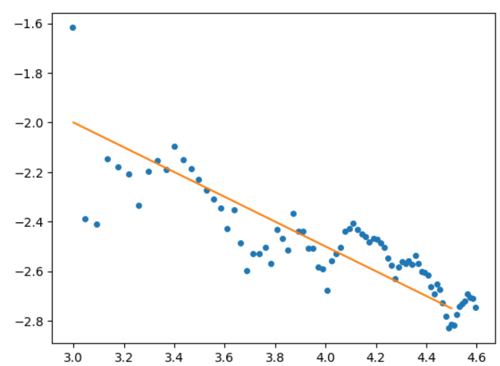
Problem 2

1) *Solution.* Here is the result. I use uniform samples for X_i. The below graph shows log-scale axis with number of samples versus errors. This graph has slope -1. i.e. The value $\alpha = -1$, rate of convergence is $O\left(\frac{1}{N}\right)$.



□

2) *Solution.* Here is the result. The blue dots represent $\mu_M(N)$ for each M and the orange line has slope $-\frac{1}{2}$. Note that this also has the similar log-scale axis. It shows that μ_M also converges, but not as fast as the above problem. Because it has the rate of convergence $O\left(\frac{1}{\sqrt{N}}\right)$.



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