Mathematical Data Science HW7

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Problem 1

Solution. This project is carried out by two people. We have not decided on a clear topic yet, but we expect to select the topic below.

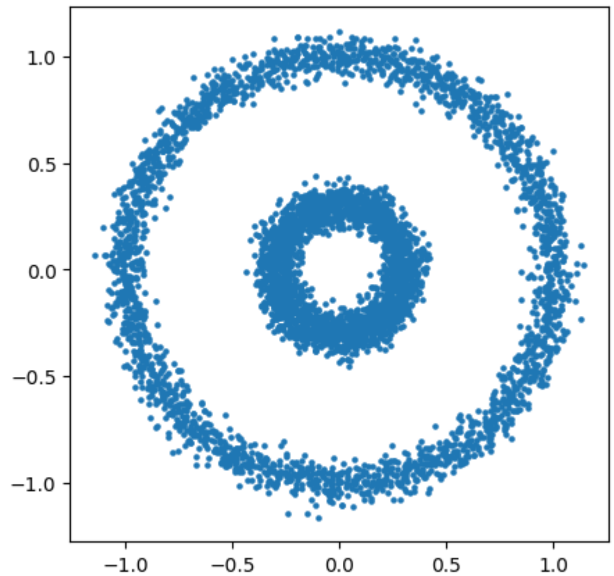
- Fine dust diffusion simulation based on 2D CA

For each cell, we would like to define the interaction of various objects involved in the generation and disappearance of dust and determine the state after a certain period. We plan to select our country as a 2D map and compare it with fine dust concentration distribution data provided by the National Institute of Environmental Research(NIER). Other data can also be used for objects that affect the state of each cell(e.g. distribution of factories, urban density, state of nature conservation, wind direction and speed, etc.). Therefore, the next step is to clearly define the objects and start designing.

□

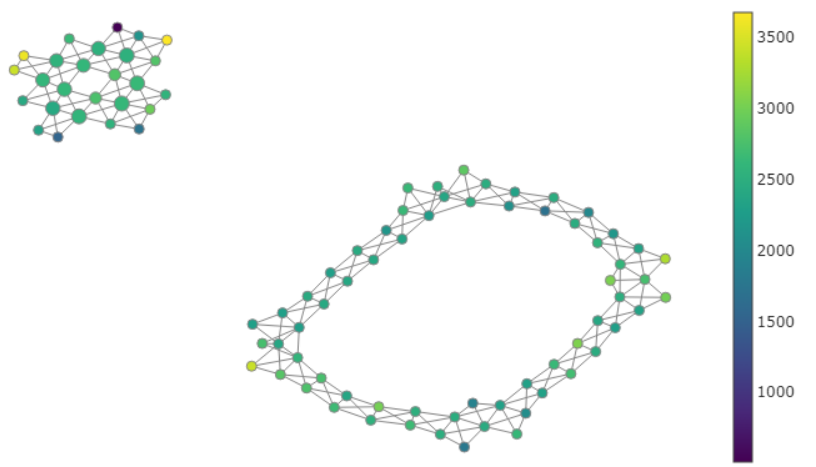
Problem 2

Solution. I choose 2D annulus large and small circles as following:



Use the filter functions f_1 and f_2, then we get the below mapper graph.

$$f_1(x, y) = x$$
$$f_2(x, y) = y$$



$\square$

Problem 3

Proof. First, denote

$$[P_0, P_1, \cdots, P_{k-1}, P_{k+1}, \cdots, P_n] = \widehat{\mathcal{P}}_k$$

By the definition of the map,

$$\begin{aligned}
\delta_{n-1} \circ \delta_n(\sigma^n) &= \delta_{n-1}\left(\delta_n(\sigma^n)\right) \\
&= \delta_{n-1}\left(\sum_{i=0}^{n}(-1)^i \widehat{\mathcal{P}}_i\right) \\
&= \sum_{i=0}^{n}(-1)^i \delta_{n-1}\left(\widehat{\mathcal{P}}_i\right) \qquad (1)
\end{aligned}$$

Here, we have

$$\delta_{n-1}\left(\widehat{\mathcal{P}}_i\right) = \begin{cases} \sum_{j=0}^{i-1}(-1)^j \widehat{\mathcal{P}}_{j,i} - \sum_{j=i+1}^{n}(-1)^j \widehat{\mathcal{P}}_{i,j}, & \text{if } 1 \le i \le n-1 \\ -\sum_{j=i+1}^{n}(-1)^j \widehat{\mathcal{P}}_{i,j}, & \text{if } i = 0 \\ \sum_{j=0}^{i-1}(-1)^j \widehat{\mathcal{P}}_{j,i}, & \text{if } i = n \end{cases} \qquad (2)$$

Note that $\widehat{\mathcal{P}}_{i,j}$ means there is a defect in ith and jth position with $i < j$. Put (2) into (1). Then

$$\delta_{n-1} \circ \delta_n(\sigma^n) = \sum_{i=1}^{n}\sum_{j=0}^{i-1}(-1)^{i+j}\widehat{\mathcal{P}}_{j,i} - \sum_{i=0}^{n-1}\sum_{j=i+1}^{n}(-1)^{i+j}\widehat{\mathcal{P}}_{i,j}$$

For the second-term, we can switch the indices i and j. This gives

$$\sum_{i=0}^{n-1}\sum_{j=i+1}^{n}(-1)^{i+j}\widehat{\mathcal{P}}_{i,j} = \sum_{i=1}^{n}\sum_{j=0}^{i-1}(-1)^{i+j}\widehat{\mathcal{P}}_{j,i}$$

Therefore, $\delta_{n-1} \circ \delta_n(\sigma^n) = 0$.

□