Regression Analysis Assignment2

November 1, 2021

Problem1. Prove $\frac{1}{n} \leq h_{ii} + \frac{\hat{\mathbf{e}}_i^2}{\hat{\mathbf{e}}^t\hat{\mathbf{e}}} \leq 1$ referring to slide3 in the lecture.(Hint: Consider the augmented matrix $\mathbf{X}^* = (\mathbf{X}, \mathbf{Y})$ and put $\mathbf{H}^* = \mathbf{X}^*(\mathbf{X}^{*t}\mathbf{X}^*)^{-1}\mathbf{X}^{*t}$. Then apply the similar argument to $\mathbf{H}^*$ as in the proof of $\frac{1}{n} \leq h_{ii} \leq 1$.)

Problem2. Observe $(\mathbf{X}_1, Y_1), \ldots (\mathbf{X}_n, Y_n)$ for $p+1$-dimensional vector $\mathbf{X}_i^t$. We consider the multiple linear regression model. Let $\hat{\sigma}^2$ be the MSE of the full model and $\hat{\sigma}_i^2$ be the MSE without i-th observation. Prove that

$$(n-p-2)\hat{\sigma}_i^2 = (n-p-1)\hat{\sigma}^2 - \frac{e_i^2}{1-h_{ii}}$$

where e_i be the i-th residual and h_{ii} be a i-th diagonal element of the hat matrix.