

# Numerical Simulation Analysis

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We provide the summary of the numerical simulations under general factor type core covariance model (see Section 7.1 for details).

**##### Libraries**

```
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr      1.1.4      v readr      2.1.6
## v forcats    1.0.1      v stringr   1.6.0
## v ggplot2    4.0.1      v tibble    3.3.0
## v lubridate  1.9.4      v tidyr     1.3.1
## v purrr      1.2.0
```

```
## -- Conflicts ----- tidyverse_conflicts() --
```

```
## x dplyr::filter() masks stats::filter()
```

```
## x dplyr::lag()     masks stats::lag()
```

```
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(latex2exp)
```

**##### Loading the simulation results**

```
file.list=list.files(path="./numerical.simul_gfcore",pattern=".rds")
```

```
file.list=file.list[-c(1:4)]
```

```
for(i in file.list){
  load(paste("./numerical.simul_gfcore/",i,sep=""))
}
```

**##### Utensils**

```
stat.analy=function(fit){
```

```
  kro.mle=fit$kro.mle
```

```
  cse=fit$cse
```

```
  init.ai=fit$pi.core.init.ai
```

```
  init.chol=fit$pi.core.init.chol
```

```
  fit.ai=fit$pi.core.ai
```

```
  fit.chol=fit$pi.core.chol
```

```
  kro.mle=cbind(init.ai[,1],kro.mle)
```

```
  cse=cbind(init.ai[,1],cse[,1:2])
```

```
  init.chol=cbind(init.ai[,1],init.chol)
```

```
  colnames(kro.mle)=colnames(cse)=colnames(init.chol)=c("K","C","Sigma")
```

```

fit.ai.analy=NULL; fit.chol.analy=NULL
for(i in 1:100){
  fit.ai.analy=rbind(fit.ai.analy,fit.ai[[i]]$eval)
  fit.chol.analy=rbind(fit.chol.analy,fit.chol[[i]]$eval)
}
return(list(kro.mle=kro.mle,cse=cse,init.ai=init.ai,init.chol=init.chol,fit.ai.analy=fit.ai.analy,fit
})

conv.analy=function(fit1,fit2,fit3,fit4,fit5,name){

  fit.ai1=fit1$pi.core.ai
  fit.chol1=fit1$pi.core.chol
  fit.ai2=fit2$pi.core.ai
  fit.chol2=fit2$pi.core.chol
  fit.ai3=fit3$pi.core.ai
  fit.chol3=fit3$pi.core.chol
  fit.ai4=fit4$pi.core.ai
  fit.chol4=fit4$pi.core.chol
  fit.ai5=fit5$pi.core.ai
  fit.chol5=fit5$pi.core.chol

  rel.conv.ai=NULL
  conv.ai=NULL
  rel.conv.chol=NULL
  conv.chol=NULL

  for(i in 1:100){
    temp1=c(fit.ai1[[i]]$rel.conv,fit.ai2[[i]]$rel.conv,fit.ai3[[i]]$rel.conv,fit.ai4[[i]]$rel.conv,fit
    temp2=c(fit.ai1[[i]]$convergence,fit.ai2[[i]]$convergence,fit.ai3[[i]]$convergence,fit.ai4[[i]]$con
    rel.conv.ai=cbind(rel.conv.ai,temp1)
    conv.ai=cbind(conv.ai,temp2)

    temp1=c(fit.chol1[[i]]$rel.conv,fit.chol2[[i]]$rel.conv,fit.chol3[[i]]$rel.conv,fit.chol4[[i]]$rel.
    temp2=c(fit.chol1[[i]]$convergence,fit.chol2[[i]]$convergence,fit.chol3[[i]]$convergence,fit.chol4[
    rel.conv.chol=cbind(rel.conv.chol,temp1)
    conv.chol=cbind(conv.chol,temp2)
  }
  rownames(rel.conv.ai)=rownames(conv.ai)=rownames(rel.conv.chol)=rownames(conv.chol)=name

  return(list(rel.conv.ai=rel.conv.ai,conv.ai=conv.ai,rel.conv.chol=rel.conv.chol,conv.chol=conv.chol))
}

plot.K.analy=function(fit1,fit2,fit3,fit4,fit5,samp.size,lambda){

  method=rep(c("Base-AI", "PICSE-AI", "PICSE-Chol"),each=100)

  dat1=tibble(stat=c(fit1$init.ai[,1],fit1$fit.ai.analy[,1],fit1$fit.chol.analy[,1]),Method=method,samp
  dat2=tibble(stat=c(fit2$init.ai[,1],fit2$fit.ai.analy[,1],fit2$fit.chol.analy[,1]),Method=method,samp
  dat3=tibble(stat=c(fit3$init.ai[,1],fit3$fit.ai.analy[,1],fit3$fit.chol.analy[,1]),Method=method,samp
  dat4=tibble(stat=c(fit4$init.ai[,1],fit4$fit.ai.analy[,1],fit4$fit.chol.analy[,1]),Method=method,samp
  dat5=tibble(stat=c(fit5$init.ai[,1],fit5$fit.ai.analy[,1],fit5$fit.chol.analy[,1]),Method=method,samp

  dat=rbind(dat1,dat2,dat3,dat4,dat5)

```

```

dat$samp.size=factor(dat$samp.size,levels=samp.size)
dat$Method=factor(dat$Method,levels=c("Base-AI", "PICSE-AI", "PICSE-Chol"))
return(dat)
}

plot.C.analy=function(fit1,fit2,fit3,fit4,fit5,samp.size,lambda){

method=rep(c("CSE", "Base-AI", "BASE-Chol", "PICSE-AI", "PICSE-Chol"),each=100)

dat1=tibble(stat=c(fit1$cse[,2],fit1$init.ai[,2],fit1$init.chol[,2],
                  fit1$fit.ai.analy[,2],fit1$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))
dat2=tibble(stat=c(fit2$cse[,2],fit2$init.ai[,2],fit2$init.chol[,2],
                  fit2$fit.ai.analy[,2],fit2$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))
dat3=tibble(stat=c(fit3$cse[,2],fit3$init.ai[,2],fit3$init.chol[,2],
                  fit3$fit.ai.analy[,2],fit3$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))
dat4=tibble(stat=c(fit4$cse[,2],fit4$init.ai[,2],fit4$init.chol[,2],
                  fit4$fit.ai.analy[,2],fit4$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))
dat5=tibble(stat=c(fit5$cse[,2],fit5$init.ai[,2],fit5$init.chol[,2],
                  fit5$fit.ai.analy[,2],fit5$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))

dat=rbind(dat1,dat2,dat3,dat4,dat5)
dat$samp.size=factor(dat$samp.size,levels=samp.size)
dat$Method=factor(dat$Method,levels=c("KMLE", "CSE", "Base-AI", "BASE-Chol", "PICSE-AI", "PICSE-Chol"))
return(dat)
}

plot.Sigma.analy=function(fit1,fit2,fit3,fit4,fit5,samp.size,lambda){

method=rep(c("KMLE", "CSE", "Base-AI", "PICSE-AI", "PICSE-Chol"),each=100)

dat1=tibble(stat=c(fit1$kro.mle[,3],fit1$cse[,3],fit1$init.ai[,3],
                  fit1$fit.ai.analy[,3],fit1$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500)
)
dat2=tibble(stat=c(fit2$kro.mle[,3],fit2$cse[,3],fit2$init.ai[,3],
                  fit2$fit.ai.analy[,3],fit2$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))
dat3=tibble(stat=c(fit3$kro.mle[,3],fit3$cse[,3],fit3$init.ai[,3],
                  fit3$fit.ai.analy[,3],fit3$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))
dat4=tibble(stat=c(fit4$kro.mle[,3],fit4$cse[,3],fit4$init.ai[,3],
                  fit4$fit.ai.analy[,3],fit4$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))
dat5=tibble(stat=c(fit5$kro.mle[,3],fit5$cse[,3],fit5$init.ai[,3],
                  fit5$fit.ai.analy[,3],fit5$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.size,
                  lambda=rep(lambda,500))

```

```

dat=rbind(dat1,dat2,dat3,dat4,dat5)
dat$samp.size=factor(dat$samp.size,levels=samp.size)
dat$Method=factor(dat$Method,levels=c("KMLE","CSE","Base-AI","PICSE-AI","PICSE-Chol"))
return(dat)
}

```

$(p_1, p_2, r) = (16, 12, 3)$

```

##### mean of the relative spectral norms for $K$, $C$, and $\Sigma$ (in order) across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
### lambda=0.2
### n=p/8
fit.16.12.3.24.lambda1=stat.analy(pi.core.fit.16.12.3.24.lambda1)
colMeans(fit.16.12.3.24.lambda1$kro.mle)

##           K           C       Sigma
## 0.3265857 0.9838769 0.9641926
colMeans(fit.16.12.3.24.lambda1$cse)

##           K           C       Sigma
## 0.3265857 0.3677388 0.5053063
colMeans(fit.16.12.3.24.lambda1$init.ai)

##           K           C       Sigma
## 0.3265857 0.3052293 0.4945858
colMeans(fit.16.12.3.24.lambda1$init.chol)

##           K           C       Sigma
## 0.3265857 0.3380701 0.4945858
colMeans(fit.16.12.3.24.lambda1$fit.ai.analy)

##           K           C       Sigma
## 0.2463290 0.1862703 0.2912795
colMeans(fit.16.12.3.24.lambda1$fit.chol.analy)

##           K           C       Sigma
## 0.2479897 0.2126027 0.2961559
### n=p/4
fit.16.12.3.48.lambda1=stat.analy(pi.core.fit.16.12.3.48.lambda1)
colMeans(fit.16.12.3.48.lambda1$kro.mle)

##           K           C       Sigma
## 0.2254017 0.9838769 0.9631214
colMeans(fit.16.12.3.48.lambda1$cse)

##           K           C       Sigma
## 0.2254017 0.2922604 0.3771772
colMeans(fit.16.12.3.48.lambda1$init.ai)

##           K           C       Sigma

```

```

## 0.2254017 0.2123285 0.3583381
colMeans(fit.16.12.3.48.lambda1$init.chol)

##          K          C      Sigma
## 0.2254017 0.2356957 0.3583381
colMeans(fit.16.12.3.48.lambda1$fit.ai.analy)

##          K          C      Sigma
## 0.1721506 0.1235124 0.2073536
colMeans(fit.16.12.3.48.lambda1$fit.chol.analy)

##          K          C      Sigma
## 0.1754525 0.1476541 0.2193698
### n=p/2
fit.16.12.3.96.lambda1=stat.analy(pi.core.fit.16.12.3.96.lambda1)
colMeans(fit.16.12.3.96.lambda1$kro.mle)

##          K          C      Sigma
## 0.1560104 0.9838769 0.9638397
colMeans(fit.16.12.3.96.lambda1$cse)

##          K          C      Sigma
## 0.1560104 0.2344615 0.2971959
colMeans(fit.16.12.3.96.lambda1$init.ai)

##          K          C      Sigma
## 0.1560104 0.1488375 0.2458674
colMeans(fit.16.12.3.96.lambda1$init.chol)

##          K          C      Sigma
## 0.1560104 0.1642483 0.2458674
colMeans(fit.16.12.3.96.lambda1$fit.ai.analy)

##          K          C      Sigma
## 0.12360824 0.08780297 0.15035869
colMeans(fit.16.12.3.96.lambda1$fit.chol.analy)

##          K          C      Sigma
## 0.1279440 0.1064034 0.1614918
### n=p
fit.16.12.3.192.lambda1=stat.analy(pi.core.fit.16.12.3.192.lambda1)
colMeans(fit.16.12.3.192.lambda1$kro.mle)

##          K          C      Sigma
## 0.1043306 0.9838769 0.9639923
colMeans(fit.16.12.3.192.lambda1$cse)

##          K          C      Sigma
## 0.1043306 0.1665930 0.2117369
colMeans(fit.16.12.3.192.lambda1$init.ai)

##          K          C      Sigma

```

```

## 0.1043306 0.1029376 0.1697907
colMeans(fit.16.12.3.192.lambda1$init.chol)

##          K          C          Sigma
## 0.1043306 0.1132618 0.1697907
colMeans(fit.16.12.3.192.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.08638520 0.06316136 0.10815034
colMeans(fit.16.12.3.192.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.09093156 0.07932828 0.12169739
### n=2p
fit.16.12.3.384.lambda1=stat.analy(pi.core.fit.16.12.3.384.lambda1)
colMeans(fit.16.12.3.384.lambda1$kro.mle)

##          K          C          Sigma
## 0.07168778 0.98387694 0.96373742
colMeans(fit.16.12.3.384.lambda1$cse)

##          K          C          Sigma
## 0.07168778 0.11128662 0.14422089
colMeans(fit.16.12.3.384.lambda1$init.ai)

##          K          C          Sigma
## 0.07168778 0.07140406 0.11793333
colMeans(fit.16.12.3.384.lambda1$init.chol)

##          K          C          Sigma
## 0.07168778 0.07871915 0.11793333
colMeans(fit.16.12.3.384.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.06132030 0.04809253 0.08290504
colMeans(fit.16.12.3.384.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.06456753 0.06037039 0.09274870
###lambda=0.4
### n=p/8
fit.16.12.3.24.lambda2=stat.analy(pi.core.fit.16.12.3.24.lambda2)
colMeans(fit.16.12.3.24.lambda2$kro.mle)

##          K          C          Sigma
## 0.3136047 0.9786279 0.9526608
colMeans(fit.16.12.3.24.lambda2$cse)

##          K          C          Sigma
## 0.3136047 0.4929949 0.5573743

```

```

colMeans(fit.16.12.3.24.lambda2$init.ai)

##          K          C      Sigma
## 0.3136047 0.3873812 0.5300031

colMeans(fit.16.12.3.24.lambda2$init.chol)

##          K          C      Sigma
## 0.3136047 0.4149365 0.5300031

colMeans(fit.16.12.3.24.lambda2$fit.ai.analy)

##          K          C      Sigma
## 0.2641896 0.3121133 0.3792918

colMeans(fit.16.12.3.24.lambda2$fit.chol.analy)

##          K          C      Sigma
## 0.2645486 0.3295145 0.3768277

### n=p/4
fit.16.12.3.48.lambda2=stat.analy(pi.core.fit.16.12.3.48.lambda2)
colMeans(fit.16.12.3.48.lambda2$kro.mle)

##          K          C      Sigma
## 0.2161752 0.9786279 0.9514918

colMeans(fit.16.12.3.48.lambda2$cse)

##          K          C      Sigma
## 0.2161752 0.4314219 0.4659944

colMeans(fit.16.12.3.48.lambda2$init.ai)

##          K          C      Sigma
## 0.2161752 0.2684603 0.3804179

colMeans(fit.16.12.3.48.lambda2$init.chol)

##          K          C      Sigma
## 0.2161752 0.2880863 0.3804179

colMeans(fit.16.12.3.48.lambda2$fit.ai.analy)

##          K          C      Sigma
## 0.1841089 0.1989654 0.2629906

colMeans(fit.16.12.3.48.lambda2$fit.chol.analy)

##          K          C      Sigma
## 0.1847223 0.2150795 0.2645488

### n=p/2
fit.16.12.3.96.lambda2=stat.analy(pi.core.fit.16.12.3.96.lambda2)
colMeans(fit.16.12.3.96.lambda2$kro.mle)

##          K          C      Sigma
## 0.1495447 0.9786279 0.9520875

colMeans(fit.16.12.3.96.lambda2$cse)

##          K          C      Sigma
## 0.1495447 0.3650456 0.3964961

```

```

colMeans(fit.16.12.3.96.lambda2$init.ai)

##          K          C      Sigma
## 0.1495447 0.1847706 0.2591688

colMeans(fit.16.12.3.96.lambda2$init.chol)

##          K          C      Sigma
## 0.1495447 0.1975206 0.2591688

colMeans(fit.16.12.3.96.lambda2$fit.ai.analy)

##          K          C      Sigma
## 0.1297676 0.1353599 0.1836283

colMeans(fit.16.12.3.96.lambda2$fit.chol.analy)

##          K          C      Sigma
## 0.1302317 0.1467162 0.1840015

### n=p
fit.16.12.3.192.lambda2=stat.analy(pi.core.fit.16.12.3.192.lambda2)
colMeans(fit.16.12.3.192.lambda2$kro.mle)

##          K          C      Sigma
## 0.1023154 0.9786279 0.9522382

colMeans(fit.16.12.3.192.lambda2$cse)

##          K          C      Sigma
## 0.1023154 0.2725151 0.2983929

colMeans(fit.16.12.3.192.lambda2$init.ai)

##          K          C      Sigma
## 0.1023154 0.1281013 0.1800143

colMeans(fit.16.12.3.192.lambda2$init.chol)

##          K          C      Sigma
## 0.1023154 0.1367102 0.1800143

colMeans(fit.16.12.3.192.lambda2$fit.ai.analy)

##          K          C      Sigma
## 0.08991233 0.09476181 0.12881254

colMeans(fit.16.12.3.192.lambda2$fit.chol.analy)

##          K          C      Sigma
## 0.09055613 0.10327352 0.13004171

### n=2p
fit.16.12.3.384.lambda2=stat.analy(pi.core.fit.16.12.3.384.lambda2)
colMeans(fit.16.12.3.384.lambda2$kro.mle)

##          K          C      Sigma
## 0.0707069 0.9786279 0.9519791

colMeans(fit.16.12.3.384.lambda2$cse)

##          K          C      Sigma
## 0.0707069 0.1828351 0.2052671

```



```

colMeans(fit.16.12.3.384.lambda2$init.ai)

##           K           C           Sigma
## 0.07070690 0.08857798 0.12510831

colMeans(fit.16.12.3.384.lambda2$init.chol)

##           K           C           Sigma
## 0.07070690 0.09469716 0.12510831

colMeans(fit.16.12.3.384.lambda2$fit.ai.analy)

##           K           C           Sigma
## 0.06252754 0.06631239 0.08970775

colMeans(fit.16.12.3.384.lambda2$fit.chol.analy)

##           K           C           Sigma
## 0.06280986 0.07280027 0.09175251
###lambda=0.6
### n=p/8
fit.16.12.3.24.lambda3=stat.analy(pi.core.fit.16.12.3.24.lambda3)
colMeans(fit.16.12.3.24.lambda3$kro.mle)

##           K           C           Sigma
## 0.3058981 0.9683142 0.9300139

colMeans(fit.16.12.3.24.lambda3$cse)

##           K           C           Sigma
## 0.3058981 0.6160712 0.6348471

colMeans(fit.16.12.3.24.lambda3$init.ai)

##           K           C           Sigma
## 0.3058981 0.5345289 0.6243393

colMeans(fit.16.12.3.24.lambda3$init.chol)

##           K           C           Sigma
## 0.3058981 0.5563877 0.6243393

colMeans(fit.16.12.3.24.lambda3$fit.ai.analy)

##           K           C           Sigma
## 0.2777582 0.5240068 0.5549240

colMeans(fit.16.12.3.24.lambda3$fit.chol.analy)

##           K           C           Sigma
## 0.2770631 0.5350337 0.5493208
### n=p/4
fit.16.12.3.48.lambda3=stat.analy(pi.core.fit.16.12.3.48.lambda3)
colMeans(fit.16.12.3.48.lambda3$kro.mle)

##           K           C           Sigma
## 0.2092660 0.9683142 0.9287170

colMeans(fit.16.12.3.48.lambda3$cse)

##           K           C           Sigma

```

```
## 0.2092660 0.5731654 0.5761007
colMeans(fit.16.12.3.48.lambda3$init.ai)

##          K          C      Sigma
## 0.2092660 0.3583258 0.4350614
colMeans(fit.16.12.3.48.lambda3$init.chol)

##          K          C      Sigma
## 0.2092660 0.3738246 0.4350614
colMeans(fit.16.12.3.48.lambda3$fit.ai.analy)

##          K          C      Sigma
## 0.1905991 0.3139881 0.3566524
colMeans(fit.16.12.3.48.lambda3$fit.chol.analy)

##          K          C      Sigma
## 0.1904458 0.3251092 0.3556601
### n=p/2
fit.16.12.3.96.lambda3=stat.analy(pi.core.fit.16.12.3.96.lambda3)
colMeans(fit.16.12.3.96.lambda3$kro.mle)

##          K          C      Sigma
## 0.1454748 0.9683142 0.9290889
colMeans(fit.16.12.3.96.lambda3$cse)

##          K          C      Sigma
## 0.1454748 0.5023790 0.5061985
colMeans(fit.16.12.3.96.lambda3$init.ai)

##          K          C      Sigma
## 0.1454748 0.2390687 0.2901929
colMeans(fit.16.12.3.96.lambda3$init.chol)

##          K          C      Sigma
## 0.1454748 0.2493207 0.2901929
colMeans(fit.16.12.3.96.lambda3$fit.ai.analy)

##          K          C      Sigma
## 0.1339808 0.2027266 0.2387039
colMeans(fit.16.12.3.96.lambda3$fit.chol.analy)

##          K          C      Sigma
## 0.1338445 0.2111148 0.2383885
### n=p
fit.16.12.3.192.lambda3=stat.analy(pi.core.fit.16.12.3.192.lambda3)
colMeans(fit.16.12.3.192.lambda3$kro.mle)

##          K          C      Sigma
## 0.1010763 0.9683142 0.9292238
colMeans(fit.16.12.3.192.lambda3$cse)

##          K          C      Sigma
```

```
## 0.1010763 0.3946557 0.3997634
colMeans(fit.16.12.3.192.lambda3$init.ai)

##          K          C          Sigma
## 0.1010763 0.1642501 0.2019827
colMeans(fit.16.12.3.192.lambda3$init.chol)

##          K          C          Sigma
## 0.1010763 0.1713734 0.2019827
colMeans(fit.16.12.3.192.lambda3$fit.ai.analy)

##          K          C          Sigma
## 0.09310959 0.14060300 0.16780429
colMeans(fit.16.12.3.192.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.09314804 0.14705106 0.16774213
### n=2p
fit.16.12.3.384.lambda3=stat.analy(pi.core.fit.16.12.3.384.lambda3)
colMeans(fit.16.12.3.384.lambda3$kro.mle)

##          K          C          Sigma
## 0.06970459 0.96831425 0.92895706
colMeans(fit.16.12.3.384.lambda3$cse)

##          K          C          Sigma
## 0.06970459 0.27284543 0.28250466
colMeans(fit.16.12.3.384.lambda3$init.ai)

##          K          C          Sigma
## 0.06970459 0.11368383 0.14152126
colMeans(fit.16.12.3.384.lambda3$init.chol)

##          K          C          Sigma
## 0.06970459 0.11869856 0.14152126
colMeans(fit.16.12.3.384.lambda3$fit.ai.analy)

##          K          C          Sigma
## 0.06497537 0.09826459 0.11876989
colMeans(fit.16.12.3.384.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.0649370 0.1030292 0.1187266
###lambda=0.8
### n=p/8
fit.16.12.3.24.lambda4=stat.analy(pi.core.fit.16.12.3.24.lambda4)
colMeans(fit.16.12.3.24.lambda4$kro.mle)

##          K          C          Sigma
## 0.3051467 0.9388075 0.8659407
```

```

colMeans(fit.16.12.3.24.lambda4$cse)

##          K          C      Sigma
## 0.3051467 0.7338528 0.7024756

colMeans(fit.16.12.3.24.lambda4$init.ai)

##          K          C      Sigma
## 0.3051467 0.9460708 0.9704359

colMeans(fit.16.12.3.24.lambda4$init.chol)

##          K          C      Sigma
## 0.3051467 0.9593618 0.9704359

colMeans(fit.16.12.3.24.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.2936206 0.9895081 0.9582223

colMeans(fit.16.12.3.24.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.2928567 0.9937253 0.9537380

### n=p/4
fit.16.12.3.48.lambda4=stat.analy(pi.core.fit.16.12.3.48.lambda4)
colMeans(fit.16.12.3.48.lambda4$kro.mle)

##          K          C      Sigma
## 0.2063058 0.9388075 0.8642007

colMeans(fit.16.12.3.48.lambda4$cse)

##          K          C      Sigma
## 0.2063058 0.7049302 0.6622426

colMeans(fit.16.12.3.48.lambda4$init.ai)

##          K          C      Sigma
## 0.2063058 0.5943627 0.6234684

colMeans(fit.16.12.3.48.lambda4$init.chol)

##          K          C      Sigma
## 0.2063058 0.6036444 0.6234684

colMeans(fit.16.12.3.48.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.1976472 0.5987239 0.6016877

colMeans(fit.16.12.3.48.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.1972240 0.6041615 0.5999553

### n=p/2
fit.16.12.3.96.lambda4=stat.analy(pi.core.fit.16.12.3.96.lambda4)
colMeans(fit.16.12.3.96.lambda4$kro.mle)

##          K          C      Sigma
## 0.1430761 0.9388075 0.8638360

```

```

colMeans(fit.16.12.3.96.lambda4$cse)

##          K          C      Sigma
## 0.1430761 0.6338553 0.5969361
colMeans(fit.16.12.3.96.lambda4$init.ai)

##          K          C      Sigma
## 0.1430761 0.3650225 0.3877695
colMeans(fit.16.12.3.96.lambda4$init.chol)

##          K          C      Sigma
## 0.1430761 0.3716336 0.3877695
colMeans(fit.16.12.3.96.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.1376453 0.3450316 0.3600371
colMeans(fit.16.12.3.96.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.1375037 0.3495769 0.3592209
### n=p
fit.16.12.3.192.lambda4=stat.analy(pi.core.fit.16.12.3.192.lambda4)
colMeans(fit.16.12.3.192.lambda4$kro.mle)

##          K          C      Sigma
## 0.1015027 0.9388075 0.8638388
colMeans(fit.16.12.3.192.lambda4$cse)

##          K          C      Sigma
## 0.1015027 0.5220679 0.4935599
colMeans(fit.16.12.3.192.lambda4$init.ai)

##          K          C      Sigma
## 0.1015027 0.2423196 0.2660160
colMeans(fit.16.12.3.192.lambda4$init.chol)

##          K          C      Sigma
## 0.1015027 0.2474535 0.2660160
colMeans(fit.16.12.3.192.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.09697991 0.22841660 0.25053383
colMeans(fit.16.12.3.192.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.09700115 0.23271000 0.25039783
### n=2p
fit.16.12.3.384.lambda4=stat.analy(pi.core.fit.16.12.3.384.lambda4)
colMeans(fit.16.12.3.384.lambda4$kro.mle)

##          K          C      Sigma
## 0.06905348 0.93880749 0.86351950

```

```

colMeans(fit.16.12.3.384.lambda4$cse)

##           K           C           Sigma
## 0.06905348 0.37887159 0.36456528

colMeans(fit.16.12.3.384.lambda4$init.ai)

##           K           C           Sigma
## 0.06905348 0.16697357 0.19369262

colMeans(fit.16.12.3.384.lambda4$init.chol)

##           K           C           Sigma
## 0.06905348 0.17033767 0.19369262

colMeans(fit.16.12.3.384.lambda4$fit.ai.analy)

##           K           C           Sigma
## 0.06705794 0.15936713 0.18546182

colMeans(fit.16.12.3.384.lambda4$fit.chol.analy)

##           K           C           Sigma
## 0.06703104 0.16245143 0.18544336

##### boxplots of the relative spectral norms for K, C, and Sigma across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
### K

K.analy.16.12.3.lambda1=plot.K.analy(fit.16.12.3.24.lambda1,fit.16.12.3.48.lambda1,fit.16.12.3.96.lambda1)
K.analy.16.12.3.lambda2=plot.K.analy(fit.16.12.3.24.lambda2,fit.16.12.3.48.lambda2,fit.16.12.3.96.lambda2)
K.analy.16.12.3.lambda3=plot.K.analy(fit.16.12.3.24.lambda3,fit.16.12.3.48.lambda3,fit.16.12.3.96.lambda3)
K.analy.16.12.3.lambda4=plot.K.analy(fit.16.12.3.24.lambda4,fit.16.12.3.48.lambda4,fit.16.12.3.96.lambda4)

K.analy.16.12.3=rbind(K.analy.16.12.3.lambda1,K.analy.16.12.3.lambda2,K.analy.16.12.3.lambda3,K.analy.16.12.3.lambda4)
K.analy.16.12.3.lambda=factor(K.analy.16.12.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(K.analy.16.12.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$\\lambda=0.8$"))
K.analy.16.12.3$lambda=K.analy.16.12.3.lambda

png("./numerical.simul_gfcore/K.16.12.3.png",width=900,height=600)
ggplot(K.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda,layout="ncol",
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Separable Component Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_text(size=12))
dev.off()

## pdf
## 2

### C

C.analy.16.12.3.lambda1=plot.C.analy(fit.16.12.3.24.lambda1,fit.16.12.3.48.lambda1,fit.16.12.3.96.lambda1)
C.analy.16.12.3.lambda2=plot.C.analy(fit.16.12.3.24.lambda2,fit.16.12.3.48.lambda2,fit.16.12.3.96.lambda2)
C.analy.16.12.3.lambda3=plot.C.analy(fit.16.12.3.24.lambda3,fit.16.12.3.48.lambda3,fit.16.12.3.96.lambda3)
C.analy.16.12.3.lambda4=plot.C.analy(fit.16.12.3.24.lambda4,fit.16.12.3.48.lambda4,fit.16.12.3.96.lambda4)

C.analy.16.12.3=rbind(C.analy.16.12.3.lambda1,C.analy.16.12.3.lambda2,C.analy.16.12.3.lambda3,C.analy.16.12.3.lambda4)
C.analy.16.12.3.lambda=factor(C.analy.16.12.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(C.analy.16.12.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$\\lambda=0.8$"))
C.analy.16.12.3$lambda=C.analy.16.12.3.lambda

```

```

C.analy.16.12.3$lambda=C.analy.16.12.3.lambda

png("./numerical.simul_gfcore/C.16.12.3.png",width=900,height=600)
ggplot(C.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda,1)
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Core Component Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_text(hj
dev.off()

## pdf
## 2

### Sigma

Sigma.analy.16.12.3.lambda1=plot.Sigma.analy(fit.16.12.3.24.lambda1,fit.16.12.3.48.lambda1,fit.16.12.3.
Sigma.analy.16.12.3.lambda2=plot.Sigma.analy(fit.16.12.3.24.lambda2,fit.16.12.3.48.lambda2,fit.16.12.3.
Sigma.analy.16.12.3.lambda3=plot.Sigma.analy(fit.16.12.3.24.lambda3,fit.16.12.3.48.lambda3,fit.16.12.3.
Sigma.analy.16.12.3.lambda4=plot.Sigma.analy(fit.16.12.3.24.lambda4,fit.16.12.3.48.lambda4,fit.16.12.3.

Sigma.analy.16.12.3=rbind(Sigma.analy.16.12.3.lambda1,Sigma.analy.16.12.3.lambda2,Sigma.analy.16.12.3.l
Sigma.analy.16.12.3.lambda=factor(Sigma.analy.16.12.3$lambda,levels=c("lambda1","lambda2","lambda3","la
levels(Sigma.analy.16.12.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),)
Sigma.analy.16.12.3$lambda=Sigma.analy.16.12.3.lambda

png("./numerical.simul_gfcore/Sigma.16.12.3.png",width=900,height=600)
ggplot(Sigma.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambd
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Sigma Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_text(hjust=0.5))
dev.off()

## pdf
## 2

```

$(p_1,p_2,r)=(18,8,3)$

```

##### mean of the relative spectral norms for $K$, $C$, and $\Sigma$ (in order) across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
###lambda=0.2
### n=p/8
fit.18.8.3.18.lambda1=stat.analy(pi.core.fit.18.8.3.18.lambda1)
colMeans(fit.18.8.3.18.lambda1$kro.mle)

##          K          C      Sigma
## 0.4498574 0.9772074 0.9456202

colMeans(fit.18.8.3.18.lambda1$cse)

##          K          C      Sigma
## 0.4498574 0.3563862 0.5250413

colMeans(fit.18.8.3.18.lambda1$init.ai)

##          K          C      Sigma
## 0.4498574 0.2315587 0.4821864

colMeans(fit.18.8.3.18.lambda1$init.chol)

##          K          C      Sigma

```

```
## 0.4498574 0.3087617 0.4821864
colMeans(fit.18.8.3.18.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.2974022 0.1956993 0.3077567
colMeans(fit.18.8.3.18.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.2984345 0.2345839 0.3083591
### n=p/4
fit.18.8.3.36.lambda1=stat.analy(pi.core.fit.18.8.3.36.lambda1)
colMeans(fit.18.8.3.36.lambda1$kro.mle)

##          K          C          Sigma
## 0.2953311 0.9772074 0.9449694
colMeans(fit.18.8.3.36.lambda1$cse)

##          K          C          Sigma
## 0.2953311 0.2966762 0.4224137
colMeans(fit.18.8.3.36.lambda1$init.ai)

##          K          C          Sigma
## 0.2953311 0.1596066 0.3695388
colMeans(fit.18.8.3.36.lambda1$init.chol)

##          K          C          Sigma
## 0.2953311 0.2128795 0.3695388
colMeans(fit.18.8.3.36.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.2150499 0.1286145 0.2215223
colMeans(fit.18.8.3.36.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.2164757 0.1575058 0.2252247
### n=p/2
fit.18.8.3.72.lambda1=stat.analy(pi.core.fit.18.8.3.72.lambda1)
colMeans(fit.18.8.3.72.lambda1$kro.mle)

##          K          C          Sigma
## 0.1985010 0.9772074 0.9459904
colMeans(fit.18.8.3.72.lambda1$cse)

##          K          C          Sigma
## 0.1985010 0.2261084 0.3184601
colMeans(fit.18.8.3.72.lambda1$init.ai)

##          K          C          Sigma
## 0.1985010 0.1093975 0.2514982
colMeans(fit.18.8.3.72.lambda1$init.chol)

##          K          C          Sigma
```



```

## 0.1985010 0.1468992 0.2514982
colMeans(fit.18.8.3.72.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.15221467 0.08781594 0.15696918
colMeans(fit.18.8.3.72.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.1544559 0.1086482 0.1620174
### n=p
fit.18.8.3.144.lambda1=stat.analy(pi.core.fit.18.8.3.144.lambda1)
colMeans(fit.18.8.3.144.lambda1$kro.mle)

##          K          C          Sigma
## 0.1326098 0.9772074 0.9456880
colMeans(fit.18.8.3.144.lambda1$cse)

##          K          C          Sigma
## 0.1326098 0.1586276 0.2218311
colMeans(fit.18.8.3.144.lambda1$init.ai)

##          K          C          Sigma
## 0.13260978 0.07476232 0.16653618
colMeans(fit.18.8.3.144.lambda1$init.chol)

##          K          C          Sigma
## 0.1326098 0.0997750 0.1665362
colMeans(fit.18.8.3.144.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.10074511 0.05960863 0.10286507
colMeans(fit.18.8.3.144.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.10364725 0.07497804 0.10907388
### n=2p
fit.18.8.3.288.lambda1=stat.analy(pi.core.fit.18.8.3.288.lambda1)
colMeans(fit.18.8.3.288.lambda1$kro.mle)

##          K          C          Sigma
## 0.09825791 0.97720739 0.94557122
colMeans(fit.18.8.3.288.lambda1$cse)

##          K          C          Sigma
## 0.09825791 0.10684172 0.16182729
colMeans(fit.18.8.3.288.lambda1$init.ai)

##          K          C          Sigma
## 0.09825791 0.05289287 0.12233814
colMeans(fit.18.8.3.288.lambda1$init.chol)

##          K          C          Sigma

```

```
## 0.09825791 0.07146648 0.12233814
colMeans(fit.18.8.3.288.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.07340397 0.04224106 0.07599137
colMeans(fit.18.8.3.288.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.07736585 0.05447332 0.08420297
###lambda=0.4
### n=p/8
fit.18.8.3.18.lambda2=stat.analy(pi.core.fit.18.8.3.18.lambda2)
colMeans(fit.18.8.3.18.lambda2$kro.mle)

##          K          C          Sigma
## 0.4211118 0.9698372 0.9283628
colMeans(fit.18.8.3.18.lambda2$cse)

##          K          C          Sigma
## 0.4211118 0.5049964 0.5678640
colMeans(fit.18.8.3.18.lambda2$init.ai)

##          K          C          Sigma
## 0.4211118 0.3425508 0.5112613
colMeans(fit.18.8.3.18.lambda2$init.chol)

##          K          C          Sigma
## 0.4211118 0.3941785 0.5112613
colMeans(fit.18.8.3.18.lambda2$fit.ai.analy)

##          K          C          Sigma
## 0.3213489 0.3341068 0.3999743
colMeans(fit.18.8.3.18.lambda2$fit.chol.analy)

##          K          C          Sigma
## 0.3226641 0.3660907 0.3989263
### n=p/4
fit.18.8.3.36.lambda2=stat.analy(pi.core.fit.18.8.3.36.lambda2)
colMeans(fit.18.8.3.36.lambda2$kro.mle)

##          K          C          Sigma
## 0.2738012 0.9698372 0.9279139
colMeans(fit.18.8.3.36.lambda2$cse)

##          K          C          Sigma
## 0.2738012 0.4602136 0.4957487
colMeans(fit.18.8.3.36.lambda2$init.ai)

##          K          C          Sigma
## 0.2738012 0.2316718 0.3796743
```

```

colMeans(fit.18.8.3.36.lambda2$init.chol)

##          K          C      Sigma
## 0.2738012 0.2669882 0.3796743

colMeans(fit.18.8.3.36.lambda2$fit.ai.analy)

##          K          C      Sigma
## 0.2307429 0.2145546 0.2837751

colMeans(fit.18.8.3.36.lambda2$fit.chol.analy)

##          K          C      Sigma
## 0.2306523 0.2393813 0.2841754

### n=p/2
fit.18.8.3.72.lambda2=stat.analy(pi.core.fit.18.8.3.72.lambda2)
colMeans(fit.18.8.3.72.lambda2$kro.mle)

##          K          C      Sigma
## 0.1866046 0.9698372 0.9287377

colMeans(fit.18.8.3.72.lambda2$cse)

##          K          C      Sigma
## 0.1866046 0.3744440 0.4076197

colMeans(fit.18.8.3.72.lambda2$init.ai)

##          K          C      Sigma
## 0.1866046 0.1568689 0.2571139

colMeans(fit.18.8.3.72.lambda2$init.chol)

##          K          C      Sigma
## 0.1866046 0.1821242 0.2571139

colMeans(fit.18.8.3.72.lambda2$fit.ai.analy)

##          K          C      Sigma
## 0.1605010 0.1441183 0.1980337

colMeans(fit.18.8.3.72.lambda2$fit.chol.analy)

##          K          C      Sigma
## 0.1605900 0.1618087 0.1980828

### n=p
fit.18.8.3.144.lambda2=stat.analy(pi.core.fit.18.8.3.144.lambda2)
colMeans(fit.18.8.3.144.lambda2$kro.mle)

##          K          C      Sigma
## 0.1245042 0.9698372 0.9282489

colMeans(fit.18.8.3.144.lambda2$cse)

##          K          C      Sigma
## 0.1245042 0.2759209 0.3035303

colMeans(fit.18.8.3.144.lambda2$init.ai)

##          K          C      Sigma
## 0.1245042 0.1074134 0.1711953

```

```

colMeans(fit.18.8.3.144.lambda2$init.chol)

##          K          C          Sigma
## 0.1245042 0.1244917 0.1711953

colMeans(fit.18.8.3.144.lambda2$fit.ai.analy)

##          K          C          Sigma
## 0.10860704 0.09784872 0.13240060

colMeans(fit.18.8.3.144.lambda2$fit.chol.analy)

##          K          C          Sigma
## 0.1088086 0.1101353 0.1326317

### n=2p
fit.18.8.3.288.lambda2=stat.analy(pi.core.fit.18.8.3.288.lambda2)
colMeans(fit.18.8.3.288.lambda2$kro.mle)

##          K          C          Sigma
## 0.09094657 0.96983724 0.92815287

colMeans(fit.18.8.3.288.lambda2$cse)

##          K          C          Sigma
## 0.09094657 0.19003553 0.21643343

colMeans(fit.18.8.3.288.lambda2$init.ai)

##          K          C          Sigma
## 0.09094657 0.07616145 0.12542944

colMeans(fit.18.8.3.288.lambda2$init.chol)

##          K          C          Sigma
## 0.09094657 0.08862987 0.12542944

colMeans(fit.18.8.3.288.lambda2$fit.ai.analy)

##          K          C          Sigma
## 0.07874344 0.06936801 0.09765789

colMeans(fit.18.8.3.288.lambda2$fit.chol.analy)

##          K          C          Sigma
## 0.07894997 0.07799759 0.09834846

### lambda=0.6
### n=p/8
fit.18.8.3.18.lambda3=stat.analy(pi.core.fit.18.8.3.18.lambda3)
colMeans(fit.18.8.3.18.lambda3$kro.mle)

##          K          C          Sigma
## 0.4104129 0.9554282 0.8950440

colMeans(fit.18.8.3.18.lambda3$cse)

##          K          C          Sigma
## 0.4104129 0.6307429 0.6280749

colMeans(fit.18.8.3.18.lambda3$init.ai)

##          K          C          Sigma

```

```
## 0.4104129 0.5411863 0.6235305
colMeans(fit.18.8.3.18.lambda3$init.chol)

##          K          C      Sigma
## 0.4104129 0.5730460 0.6235305
colMeans(fit.18.8.3.18.lambda3$fit.ai.analy)

##          K          C      Sigma
## 0.3550601 0.5622683 0.5806514
colMeans(fit.18.8.3.18.lambda3$fit.chol.analy)

##          K          C      Sigma
## 0.3519299 0.5872006 0.5753340

### n=p/4
fit.18.8.3.36.lambda3=stat.analy(pi.core.fit.18.8.3.36.lambda3)
colMeans(fit.18.8.3.36.lambda3$kro.mle)

##          K          C      Sigma
## 0.2700667 0.9554282 0.8948009
colMeans(fit.18.8.3.36.lambda3$cse)

##          K          C      Sigma
## 0.2700667 0.6028948 0.5848852
colMeans(fit.18.8.3.36.lambda3$init.ai)

##          K          C      Sigma
## 0.2700667 0.3434887 0.4306280
colMeans(fit.18.8.3.36.lambda3$init.chol)

##          K          C      Sigma
## 0.2700667 0.3673228 0.4306280
colMeans(fit.18.8.3.36.lambda3$fit.ai.analy)

##          K          C      Sigma
## 0.2443865 0.3384581 0.3797775
colMeans(fit.18.8.3.36.lambda3$fit.chol.analy)

##          K          C      Sigma
## 0.2443401 0.3580142 0.3798059

### n=p/2
fit.18.8.3.72.lambda3=stat.analy(pi.core.fit.18.8.3.72.lambda3)
colMeans(fit.18.8.3.72.lambda3$kro.mle)

##          K          C      Sigma
## 0.1815981 0.9554282 0.8953597
colMeans(fit.18.8.3.72.lambda3$cse)

##          K          C      Sigma
## 0.1815981 0.5163395 0.5074762
colMeans(fit.18.8.3.72.lambda3$init.ai)

##          K          C      Sigma
```

```
## 0.1815981 0.2263502 0.2863259
colMeans(fit.18.8.3.72.lambda3$init.chol)

##          K          C          Sigma
## 0.1815981 0.2439588 0.2863259
colMeans(fit.18.8.3.72.lambda3$fit.ai.analy)

##          K          C          Sigma
## 0.1654531 0.2190650 0.2534906
colMeans(fit.18.8.3.72.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.1655234 0.2333235 0.2534680
### n=p
fit.18.8.3.144.lambda3=stat.analy(pi.core.fit.18.8.3.144.lambda3)
colMeans(fit.18.8.3.144.lambda3$kro.mle)

##          K          C          Sigma
## 0.1225344 0.9554282 0.8945109
colMeans(fit.18.8.3.144.lambda3$cse)

##          K          C          Sigma
## 0.1225344 0.4016269 0.3967448
colMeans(fit.18.8.3.144.lambda3$init.ai)

##          K          C          Sigma
## 0.1225344 0.1532243 0.1926403
colMeans(fit.18.8.3.144.lambda3$init.chol)

##          K          C          Sigma
## 0.1225344 0.1652233 0.1926403
colMeans(fit.18.8.3.144.lambda3$fit.ai.analy)

##          K          C          Sigma
## 0.1137948 0.1469625 0.1734070
colMeans(fit.18.8.3.144.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.1137909 0.1569719 0.1733690
### n=2p
fit.18.8.3.288.lambda3=stat.analy(pi.core.fit.18.8.3.288.lambda3)
colMeans(fit.18.8.3.288.lambda3$kro.mle)

##          K          C          Sigma
## 0.08894988 0.95542823 0.89447983
colMeans(fit.18.8.3.288.lambda3$cse)

##          K          C          Sigma
## 0.08894988 0.28576214 0.28785822
colMeans(fit.18.8.3.288.lambda3$init.ai)

##          K          C          Sigma
```

```
## 0.08894988 0.10870413 0.14371344
colMeans(fit.18.8.3.288.lambda3$init.chol)

##          K          C          Sigma
## 0.08894988 0.11703857 0.14371344
colMeans(fit.18.8.3.288.lambda3$fit.ai.analy)

##          K          C          Sigma
## 0.08177674 0.10449885 0.13199318
colMeans(fit.18.8.3.288.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.08177845 0.11114489 0.13196083
###lambda=0.8
### n=p/8
fit.18.8.3.18.lambda4=stat.analy(pi.core.fit.18.8.3.18.lambda4)
colMeans(fit.18.8.3.18.lambda4$kro.mle)

##          K          C          Sigma
## 0.4067328 0.9147403 0.8045424
colMeans(fit.18.8.3.18.lambda4$cse)

##          K          C          Sigma
## 0.4067328 0.7305303 0.6673306
colMeans(fit.18.8.3.18.lambda4$init.ai)

##          K          C          Sigma
## 0.4067328 1.1306747 1.0779444
colMeans(fit.18.8.3.18.lambda4$init.chol)

##          K          C          Sigma
## 0.4067328 1.1429158 1.0779444
colMeans(fit.18.8.3.18.lambda4$fit.ai.analy)

##          K          C          Sigma
## 0.3794338 1.1591675 1.0618951
colMeans(fit.18.8.3.18.lambda4$fit.chol.analy)

##          K          C          Sigma
## 0.3795983 1.1631404 1.0575969
### n=p/4
fit.18.8.3.36.lambda4=stat.analy(pi.core.fit.18.8.3.36.lambda4)
colMeans(fit.18.8.3.36.lambda4$kro.mle)

##          K          C          Sigma
## 0.2708083 0.9147403 0.8038019
colMeans(fit.18.8.3.36.lambda4$cse)

##          K          C          Sigma
## 0.2708083 0.7169463 0.6396534
```

```

colMeans(fit.18.8.3.36.lambda4$init.ai)

##          K          C      Sigma
## 0.2708083 0.6687164 0.6489945

colMeans(fit.18.8.3.36.lambda4$init.chol)

##          K          C      Sigma
## 0.2708083 0.6814926 0.6489945

colMeans(fit.18.8.3.36.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.2592537 0.6695038 0.6185169

colMeans(fit.18.8.3.36.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.2592278 0.6805267 0.6223999

### n=p/2
fit.18.8.3.72.lambda4=stat.analy(pi.core.fit.18.8.3.72.lambda4)
colMeans(fit.18.8.3.72.lambda4$kro.mle)

##          K          C      Sigma
## 0.1799346 0.9147403 0.8033260

colMeans(fit.18.8.3.72.lambda4$cse)

##          K          C      Sigma
## 0.1799346 0.6400597 0.5773397

colMeans(fit.18.8.3.72.lambda4$init.ai)

##          K          C      Sigma
## 0.1799346 0.4046526 0.3971683

colMeans(fit.18.8.3.72.lambda4$init.chol)

##          K          C      Sigma
## 0.1799346 0.4148062 0.3971683

colMeans(fit.18.8.3.72.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.1721204 0.3887763 0.3801580

colMeans(fit.18.8.3.72.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.1722787 0.3967741 0.3783453

### n=p
fit.18.8.3.144.lambda4=stat.analy(pi.core.fit.18.8.3.144.lambda4)
colMeans(fit.18.8.3.144.lambda4$kro.mle)

##          K          C      Sigma
## 0.1218588 0.9147403 0.8013305

colMeans(fit.18.8.3.144.lambda4$cse)

##          K          C      Sigma
## 0.1218588 0.5254340 0.4735317

```



```

colMeans(fit.18.8.3.144.lambda4$init.ai)

##          K          C          Sigma
## 0.1218588 0.2549076 0.2730879

colMeans(fit.18.8.3.144.lambda4$init.chol)

##          K          C          Sigma
## 0.1218588 0.2620751 0.2730879

colMeans(fit.18.8.3.144.lambda4$fit.ai.analy)

##          K          C          Sigma
## 0.1183634 0.2451186 0.2667792

colMeans(fit.18.8.3.144.lambda4$fit.chol.analy)

##          K          C          Sigma
## 0.1183298 0.2511977 0.2666631

#### n=2p
fit.18.8.3.288.lambda4=stat.analy(pi.core.fit.18.8.3.288.lambda4)
colMeans(fit.18.8.3.288.lambda4$kro.mle)

##          K          C          Sigma
## 0.08849394 0.91474029 0.80138339

colMeans(fit.18.8.3.288.lambda4$cse)

##          K          C          Sigma
## 0.08849394 0.39006185 0.35754660

colMeans(fit.18.8.3.288.lambda4$init.ai)

##          K          C          Sigma
## 0.08849394 0.18025084 0.23552895

colMeans(fit.18.8.3.288.lambda4$init.chol)

##          K          C          Sigma
## 0.08849394 0.18381900 0.23552895

colMeans(fit.18.8.3.288.lambda4$fit.ai.analy)

##          K          C          Sigma
## 0.08499674 0.17618698 0.23460397

colMeans(fit.18.8.3.288.lambda4$fit.chol.analy)

##          K          C          Sigma
## 0.08497281 0.17941423 0.23459973

##### boxplots of the relative spectral norms for K, C, and Sigma across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
### K

K.analy.18.8.3.lambda1=plot.K.analy(fit.18.8.3.18.lambda1,fit.18.8.3.36.lambda1,fit.18.8.3.72.lambda1,f
K.analy.18.8.3.lambda2=plot.K.analy(fit.18.8.3.18.lambda2,fit.18.8.3.36.lambda2,fit.18.8.3.72.lambda2,f
K.analy.18.8.3.lambda3=plot.K.analy(fit.18.8.3.18.lambda3,fit.18.8.3.36.lambda3,fit.18.8.3.72.lambda3,f
K.analy.18.8.3.lambda4=plot.K.analy(fit.18.8.3.18.lambda4,fit.18.8.3.36.lambda4,fit.18.8.3.72.lambda4,f

K.analy.18.8.3=rbind(K.analy.18.8.3.lambda1,K.analy.18.8.3.lambda2,K.analy.18.8.3.lambda3,K.analy.18.8.

```

```

K.analy.18.8.3.lambda=factor(K.analy.18.8.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(K.analy.18.8.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$\\lambda=0.8$"))
K.analy.18.8.3$lambda=K.analy.18.8.3.lambda

png("./numerical.simul_gfcore/K.18.8.3.png",width=900,height=600)
ggplot(K.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda,lab
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Separable Component Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text
dev.off()

## pdf
## 2

### C

C.analy.18.8.3.lambda1=plot.C.analy(fit.18.8.3.18.lambda1,fit.18.8.3.36.lambda1,fit.18.8.3.72.lambda1,f
C.analy.18.8.3.lambda2=plot.C.analy(fit.18.8.3.18.lambda2,fit.18.8.3.36.lambda2,fit.18.8.3.72.lambda2,f
C.analy.18.8.3.lambda3=plot.C.analy(fit.18.8.3.18.lambda3,fit.18.8.3.36.lambda3,fit.18.8.3.72.lambda3,f
C.analy.18.8.3.lambda4=plot.C.analy(fit.18.8.3.18.lambda4,fit.18.8.3.36.lambda4,fit.18.8.3.72.lambda4,f

C.analy.18.8.3=rbind(C.analy.18.8.3.lambda1,C.analy.18.8.3.lambda2,C.analy.18.8.3.lambda3,C.analy.18.8.3.lambda4)
C.analy.18.8.3.lambda=factor(C.analy.18.8.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(C.analy.18.8.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$\\lambda=0.8$"))
C.analy.18.8.3$lambda=C.analy.18.8.3.lambda

png("./numerical.simul_gfcore/C.18.8.3.png",width=900,height=600)
ggplot(C.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda,lab
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Core Component Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text(hjust=0.5))
dev.off()

## pdf
## 2

### Sigma

Sigma.analy.18.8.3.lambda1=plot.Sigma.analy(fit.18.8.3.18.lambda1,fit.18.8.3.36.lambda1,fit.18.8.3.72.lambda1,f
Sigma.analy.18.8.3.lambda2=plot.Sigma.analy(fit.18.8.3.18.lambda2,fit.18.8.3.36.lambda2,fit.18.8.3.72.lambda2,f
Sigma.analy.18.8.3.lambda3=plot.Sigma.analy(fit.18.8.3.18.lambda3,fit.18.8.3.36.lambda3,fit.18.8.3.72.lambda3,f
Sigma.analy.18.8.3.lambda4=plot.Sigma.analy(fit.18.8.3.18.lambda4,fit.18.8.3.36.lambda4,fit.18.8.3.72.lambda4,f

Sigma.analy.18.8.3=rbind(Sigma.analy.18.8.3.lambda1,Sigma.analy.18.8.3.lambda2,Sigma.analy.18.8.3.lambda3,Sigma.analy.18.8.3.lambda4)
Sigma.analy.18.8.3.lambda=factor(Sigma.analy.18.8.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(Sigma.analy.18.8.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$\\lambda=0.8$"))
Sigma.analy.18.8.3$lambda=Sigma.analy.18.8.3.lambda

png("./numerical.simul_gfcore/Sigma.18.8.3.png",width=900,height=600)
ggplot(Sigma.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda,lab
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Sigma Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text(hjust=0.5))
dev.off()

## pdf
## 2

```

$(p_1, p_2, r) = (16, 12, 5)$

```
##### mean of the relative spectral norms for  $K$ ,  $C$ , and  $\Sigma$  (in order) across 100 iterations
##### under different choices of the sample size  $n$  and the value of non-spiked eigenvalue  $\lambda$ .
###  $\lambda = 0.2$ 
###  $n = p/8$ 
fit.16.12.5.24.lambda1=stat.analy(pi.core.fit.16.12.5.24.lambda1)
colMeans(fit.16.12.5.24.lambda1$kro.mle)

##           K           C           Sigma
## 0.3104290 0.9734961 0.9362013

colMeans(fit.16.12.5.24.lambda1$cse)

##           K           C           Sigma
## 0.3104290 0.4938516 0.5927379

colMeans(fit.16.12.5.24.lambda1$init.ai)

##           K           C           Sigma
## 0.3104290 0.5158918 0.6527251

colMeans(fit.16.12.5.24.lambda1$init.chol)

##           K           C           Sigma
## 0.3104290 0.5423546 0.6527251

colMeans(fit.16.12.5.24.lambda1$fit.ai.analy)

##           K           C           Sigma
## 0.2351154 0.3325902 0.3872793

colMeans(fit.16.12.5.24.lambda1$fit.chol.analy)

##           K           C           Sigma
## 0.2368727 0.3467676 0.3907216

###  $n = p/4$ 
fit.16.12.5.48.lambda1=stat.analy(pi.core.fit.16.12.5.48.lambda1)
colMeans(fit.16.12.5.48.lambda1$kro.mle)

##           K           C           Sigma
## 0.2260572 0.9734961 0.9342592

colMeans(fit.16.12.5.48.lambda1$cse)

##           K           C           Sigma
## 0.2260572 0.3997424 0.4535451

colMeans(fit.16.12.5.48.lambda1$init.ai)

##           K           C           Sigma
## 0.2260572 0.3612767 0.4842754

colMeans(fit.16.12.5.48.lambda1$init.chol)

##           K           C           Sigma
## 0.2260572 0.3777756 0.4842754

colMeans(fit.16.12.5.48.lambda1$fit.ai.analy)

##           K           C           Sigma
```

```
## 0.1644800 0.2136253 0.2703601
colMeans(fit.16.12.5.48.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.1676738 0.2297647 0.2812347

### n=p/2
fit.16.12.5.96.lambda1=stat.analy(pi.core.fit.16.12.5.96.lambda1)
colMeans(fit.16.12.5.96.lambda1$kro.mle)

##          K          C          Sigma
## 0.1501236 0.9734961 0.9355565
colMeans(fit.16.12.5.96.lambda1$cse)

##          K          C          Sigma
## 0.1501236 0.3125959 0.3516430
colMeans(fit.16.12.5.96.lambda1$init.ai)

##          K          C          Sigma
## 0.1501236 0.2532748 0.3220968
colMeans(fit.16.12.5.96.lambda1$init.chol)

##          K          C          Sigma
## 0.1501236 0.2638246 0.3220968
colMeans(fit.16.12.5.96.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.1138481 0.1486114 0.1943679
colMeans(fit.16.12.5.96.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.1195041 0.1671750 0.2089434

### n=p
fit.16.12.5.192.lambda1=stat.analy(pi.core.fit.16.12.5.192.lambda1)
colMeans(fit.16.12.5.192.lambda1$kro.mle)

##          K          C          Sigma
## 0.1022652 0.9734961 0.9353922
colMeans(fit.16.12.5.192.lambda1$cse)

##          K          C          Sigma
## 0.1022652 0.2270632 0.2556009
colMeans(fit.16.12.5.192.lambda1$init.ai)

##          K          C          Sigma
## 0.1022652 0.1735713 0.2227232
colMeans(fit.16.12.5.192.lambda1$init.chol)

##          K          C          Sigma
## 0.1022652 0.1805594 0.2227232
colMeans(fit.16.12.5.192.lambda1$fit.ai.analy)

##          K          C          Sigma
```

```
## 0.08185204 0.10625991 0.13962643
colMeans(fit.16.12.5.192.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.08731071 0.12314650 0.15497967
### n=2p
fit.16.12.5.384.lambda1=stat.analy(pi.core.fit.16.12.5.384.lambda1)
colMeans(fit.16.12.5.384.lambda1$kro.mle)

##          K          C          Sigma
## 0.07149767 0.97349609 0.93543888
colMeans(fit.16.12.5.384.lambda1$cse)

##          K          C          Sigma
## 0.07149767 0.15690850 0.17751254
colMeans(fit.16.12.5.384.lambda1$init.ai)

##          K          C          Sigma
## 0.07149767 0.12121670 0.15784446
colMeans(fit.16.12.5.384.lambda1$init.chol)

##          K          C          Sigma
## 0.07149767 0.12637367 0.15784446
colMeans(fit.16.12.5.384.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.05951156 0.08290782 0.11109733
colMeans(fit.16.12.5.384.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.06519095 0.09938809 0.12771482
###lambda=0.4
### n=p/8
fit.16.12.5.24.lambda2=stat.analy(pi.core.fit.16.12.5.24.lambda2)
colMeans(fit.16.12.5.24.lambda2$kro.mle)

##          K          C          Sigma
## 0.3044358 0.9649725 0.9157926
colMeans(fit.16.12.5.24.lambda2$cse)

##          K          C          Sigma
## 0.3044358 0.5955302 0.6100796
colMeans(fit.16.12.5.24.lambda2$init.ai)

##          K          C          Sigma
## 0.3044358 0.6395278 0.7234977
colMeans(fit.16.12.5.24.lambda2$init.chol)

##          K          C          Sigma
## 0.3044358 0.6623373 0.7234977
colMeans(fit.16.12.5.24.lambda2$fit.ai.analy)
```

```

##           K           C           Sigma
## 0.2568204 0.5821725 0.5715966
colMeans(fit.16.12.5.24.lambda2$fit.chol.analy)

##           K           C           Sigma
## 0.2592574 0.5843323 0.5631781
### n=p/4
fit.16.12.5.48.lambda2=stat.analy(pi.core.fit.16.12.5.48.lambda2)
colMeans(fit.16.12.5.48.lambda2$kro.mle)

##           K           C           Sigma
## 0.2150196 0.9649725 0.9137175
colMeans(fit.16.12.5.48.lambda2$cse)

##           K           C           Sigma
## 0.2150196 0.5169921 0.5156825
colMeans(fit.16.12.5.48.lambda2$init.ai)

##           K           C           Sigma
## 0.2150196 0.4345164 0.5156927
colMeans(fit.16.12.5.48.lambda2$init.chol)

##           K           C           Sigma
## 0.2150196 0.4489509 0.5156927
colMeans(fit.16.12.5.48.lambda2$fit.ai.analy)

##           K           C           Sigma
## 0.1805681 0.3461878 0.3720314
colMeans(fit.16.12.5.48.lambda2$fit.chol.analy)

##           K           C           Sigma
## 0.1798905 0.3515555 0.3671819
### n=p/2
fit.16.12.5.96.lambda2=stat.analy(pi.core.fit.16.12.5.96.lambda2)
colMeans(fit.16.12.5.96.lambda2$kro.mle)

##           K           C           Sigma
## 0.1466835 0.9649725 0.9150164
colMeans(fit.16.12.5.96.lambda2$cse)

##           K           C           Sigma
## 0.1466835 0.4282396 0.4344362
colMeans(fit.16.12.5.96.lambda2$init.ai)

##           K           C           Sigma
## 0.1466835 0.2948627 0.3405850
colMeans(fit.16.12.5.96.lambda2$init.chol)

##           K           C           Sigma
## 0.1466835 0.3035842 0.3405850

```

```

colMeans(fit.16.12.5.96.lambda2$fit.ai.analy)

##          K          C          Sigma
## 0.1243395 0.2206655 0.2500256

colMeans(fit.16.12.5.96.lambda2$fit.chol.analy)

##          K          C          Sigma
## 0.1244847 0.2247570 0.2472811

### n=p
fit.16.12.5.192.lambda2=stat.analy(pi.core.fit.16.12.5.192.lambda2)
colMeans(fit.16.12.5.192.lambda2$kro.mle)

##          K          C          Sigma
## 0.1012752 0.9649725 0.9147030

colMeans(fit.16.12.5.192.lambda2$cse)

##          K          C          Sigma
## 0.1012752 0.3261250 0.3305807

colMeans(fit.16.12.5.192.lambda2$init.ai)

##          K          C          Sigma
## 0.1012752 0.2015946 0.2367018

colMeans(fit.16.12.5.192.lambda2$init.chol)

##          K          C          Sigma
## 0.1012752 0.2074128 0.2367018

colMeans(fit.16.12.5.192.lambda2$fit.ai.analy)

##          K          C          Sigma
## 0.08672925 0.14981423 0.17184528

colMeans(fit.16.12.5.192.lambda2$fit.chol.analy)

##          K          C          Sigma
## 0.08763299 0.15453220 0.17205159

### n=2p
fit.16.12.5.384.lambda2=stat.analy(pi.core.fit.16.12.5.384.lambda2)
colMeans(fit.16.12.5.384.lambda2$kro.mle)

##          K          C          Sigma
## 0.06985333 0.96497254 0.91483943

colMeans(fit.16.12.5.384.lambda2$cse)

##          K          C          Sigma
## 0.06985333 0.22863624 0.23495745

colMeans(fit.16.12.5.384.lambda2$init.ai)

##          K          C          Sigma
## 0.06985333 0.14183750 0.16752617

colMeans(fit.16.12.5.384.lambda2$init.chol)

##          K          C          Sigma
## 0.06985333 0.14611897 0.16752617

```

```

colMeans(fit.16.12.5.384.lambda2$fit.ai.analy)

##           K           C           Sigma
## 0.05965283 0.10417473 0.12033358
colMeans(fit.16.12.5.384.lambda2$fit.chol.analy)

##           K           C           Sigma
## 0.06028816 0.10872521 0.12214493
###lambda=0.6
### n=p/8
fit.16.12.5.24.lambda3=stat.analy(pi.core.fit.16.12.5.24.lambda3)
colMeans(fit.16.12.5.24.lambda3$kro.mle)

##           K           C           Sigma
## 0.3028983 0.9483784 0.8765024
colMeans(fit.16.12.5.24.lambda3$cse)

##           K           C           Sigma
## 0.3028983 0.6868453 0.6554850
colMeans(fit.16.12.5.24.lambda3$init.ai)

##           K           C           Sigma
## 0.3028983 0.8699324 0.8881842
colMeans(fit.16.12.5.24.lambda3$init.chol)

##           K           C           Sigma
## 0.3028983 0.8876704 0.8881842
colMeans(fit.16.12.5.24.lambda3$fit.ai.analy)

##           K           C           Sigma
## 0.2877193 0.9922696 0.9515841
colMeans(fit.16.12.5.24.lambda3$fit.chol.analy)

##           K           C           Sigma
## 0.2836092 1.0051523 0.9407846
### n=p/4
fit.16.12.5.48.lambda3=stat.analy(pi.core.fit.16.12.5.48.lambda3)
colMeans(fit.16.12.5.48.lambda3$kro.mle)

##           K           C           Sigma
## 0.2078564 0.9483784 0.8741131
colMeans(fit.16.12.5.48.lambda3$cse)

##           K           C           Sigma
## 0.2078564 0.6273561 0.5891855
colMeans(fit.16.12.5.48.lambda3$init.ai)

##           K           C           Sigma
## 0.2078564 0.5638015 0.5925996
colMeans(fit.16.12.5.48.lambda3$init.chol)

##           K           C           Sigma

```



```
## 0.2078564 0.5754675 0.5925996
colMeans(fit.16.12.5.48.lambda3$fit.ai.analy)

##          K          C      Sigma
## 0.1906219 0.5606743 0.5482304
colMeans(fit.16.12.5.48.lambda3$fit.chol.analy)

##          K          C      Sigma
## 0.1901923 0.5648567 0.5438706
### n=p/2
fit.16.12.5.96.lambda3=stat.analy(pi.core.fit.16.12.5.96.lambda3)
colMeans(fit.16.12.5.96.lambda3$kro.mle)

##          K          C      Sigma
## 0.1452901 0.9483784 0.8753477
colMeans(fit.16.12.5.96.lambda3$cse)

##          K          C      Sigma
## 0.1452901 0.5441668 0.5186403
colMeans(fit.16.12.5.96.lambda3$init.ai)

##          K          C      Sigma
## 0.1452901 0.3666798 0.3842529
colMeans(fit.16.12.5.96.lambda3$init.chol)

##          K          C      Sigma
## 0.1452901 0.3735314 0.3842529
colMeans(fit.16.12.5.96.lambda3$fit.ai.analy)

##          K          C      Sigma
## 0.1314784 0.3302199 0.3338574
colMeans(fit.16.12.5.96.lambda3$fit.chol.analy)

##          K          C      Sigma
## 0.1318877 0.3351211 0.3337000
### n=p
fit.16.12.5.192.lambda3=stat.analy(pi.core.fit.16.12.5.192.lambda3)
colMeans(fit.16.12.5.192.lambda3$kro.mle)

##          K          C      Sigma
## 0.1008365 0.9483784 0.8746988
colMeans(fit.16.12.5.192.lambda3$cse)

##          K          C      Sigma
## 0.1008365 0.4358806 0.4127736
colMeans(fit.16.12.5.192.lambda3$init.ai)

##          K          C      Sigma
## 0.1008365 0.2481951 0.2669292
colMeans(fit.16.12.5.192.lambda3$init.chol)

##          K          C      Sigma
```

```
## 0.1008365 0.2526943 0.2669292
colMeans(fit.16.12.5.192.lambda3$fit.ai.analy)

##          K          C          Sigma
## 0.09119008 0.21975786 0.23046286
colMeans(fit.16.12.5.192.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.0911462 0.2234025 0.2299500
### n=2p
fit.16.12.5.384.lambda3=stat.analy(pi.core.fit.16.12.5.384.lambda3)
colMeans(fit.16.12.5.384.lambda3$kro.mle)

##          K          C          Sigma
## 0.06882021 0.94837839 0.87495462
colMeans(fit.16.12.5.384.lambda3$cse)

##          K          C          Sigma
## 0.06882021 0.31608363 0.30333192
colMeans(fit.16.12.5.384.lambda3$init.ai)

##          K          C          Sigma
## 0.06882021 0.17425233 0.18921746
colMeans(fit.16.12.5.384.lambda3$init.chol)

##          K          C          Sigma
## 0.06882021 0.17762619 0.18921746
colMeans(fit.16.12.5.384.lambda3$fit.ai.analy)

##          K          C          Sigma
## 0.06271681 0.15136129 0.16210931
colMeans(fit.16.12.5.384.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.06271812 0.15404777 0.16201927
###lambda=0.8
### n=p/8
fit.16.12.5.24.lambda4=stat.analy(pi.core.fit.16.12.5.24.lambda4)
colMeans(fit.16.12.5.24.lambda4$kro.mle)

##          K          C          Sigma
## 0.3041913 0.9020679 0.7678933
colMeans(fit.16.12.5.24.lambda4$cse)

##          K          C          Sigma
## 0.3041913 0.7566865 0.6591615
colMeans(fit.16.12.5.24.lambda4$init.ai)

##          K          C          Sigma
## 0.3041913 1.4754942 1.3498422
```

```

colMeans(fit.16.12.5.24.lambda4$init.chol)

##          K          C      Sigma
## 0.3041913 1.4856620 1.3498422

colMeans(fit.16.12.5.24.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.2900649 1.5162239 1.3136048

colMeans(fit.16.12.5.24.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.2898473 1.5354981 1.3175943

### n=p/4
fit.16.12.5.48.lambda4=stat.analy(pi.core.fit.16.12.5.48.lambda4)
colMeans(fit.16.12.5.48.lambda4$kro.mle)

##          K          C      Sigma
## 0.2036019 0.9020679 0.7639503

colMeans(fit.16.12.5.48.lambda4$cse)

##          K          C      Sigma
## 0.2036019 0.7225537 0.6165195

colMeans(fit.16.12.5.48.lambda4$init.ai)

##          K          C      Sigma
## 0.2036019 0.9120027 0.8330766

colMeans(fit.16.12.5.48.lambda4$init.chol)

##          K          C      Sigma
## 0.2036019 0.9191349 0.8330766

colMeans(fit.16.12.5.48.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.1999062 1.0354435 0.9047564

colMeans(fit.16.12.5.48.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.2000332 1.0541739 0.9149034

### n=p/2
fit.16.12.5.96.lambda4=stat.analy(pi.core.fit.16.12.5.96.lambda4)
colMeans(fit.16.12.5.96.lambda4$kro.mle)

##          K          C      Sigma
## 0.1443709 0.9020679 0.7646570

colMeans(fit.16.12.5.96.lambda4$cse)

##          K          C      Sigma
## 0.1443709 0.6496015 0.5613273

colMeans(fit.16.12.5.96.lambda4$init.ai)

##          K          C      Sigma
## 0.1443709 0.5595856 0.5123391

```

```

colMeans(fit.16.12.5.96.lambda4$init.chol)

##          K          C          Sigma
## 0.1443709 0.5642255 0.5123391
colMeans(fit.16.12.5.96.lambda4$fit.ai.analy)

##          K          C          Sigma
## 0.1388005 0.5964871 0.5281645
colMeans(fit.16.12.5.96.lambda4$fit.chol.analy)

##          K          C          Sigma
## 0.1389816 0.5980790 0.5253968
### n=p
fit.16.12.5.192.lambda4=stat.analy(pi.core.fit.16.12.5.192.lambda4)
colMeans(fit.16.12.5.192.lambda4$kro.mle)

##          K          C          Sigma
## 0.1002124 0.9020679 0.7630015
colMeans(fit.16.12.5.192.lambda4$cse)

##          K          C          Sigma
## 0.1002124 0.5460538 0.4676843
colMeans(fit.16.12.5.192.lambda4$init.ai)

##          K          C          Sigma
## 0.1002124 0.3641955 0.3509813
colMeans(fit.16.12.5.192.lambda4$init.chol)

##          K          C          Sigma
## 0.1002124 0.3674308 0.3509813
colMeans(fit.16.12.5.192.lambda4$fit.ai.analy)

##          K          C          Sigma
## 0.09485962 0.36689647 0.34472112
colMeans(fit.16.12.5.192.lambda4$fit.chol.analy)

##          K          C          Sigma
## 0.09494741 0.36948894 0.34390920
### n=2p
fit.16.12.5.384.lambda4=stat.analy(pi.core.fit.16.12.5.384.lambda4)
colMeans(fit.16.12.5.384.lambda4$kro.mle)

##          K          C          Sigma
## 0.06818306 0.90206794 0.76334113
colMeans(fit.16.12.5.384.lambda4$cse)

##          K          C          Sigma
## 0.06818306 0.41542549 0.36177254
colMeans(fit.16.12.5.384.lambda4$init.ai)

##          K          C          Sigma
## 0.06818306 0.24884379 0.25686814

```

```

colMeans(fit.16.12.5.384.lambda4$init.chol)

##           K           C           Sigma
## 0.06818306 0.25120927 0.25686814

colMeans(fit.16.12.5.384.lambda4$fit.ai.analy)

##           K           C           Sigma
## 0.06570237 0.24455042 0.25152637

colMeans(fit.16.12.5.384.lambda4$fit.chol.analy)

##           K           C           Sigma
## 0.06568136 0.24647748 0.25142503

##### boxplots of the relative spectral norms for K, C, and Sigma across 100 iterations
##### under different choices of the sample size  $n$  and the value of non-spiked eigenvalue  $\lambda$ .
### K

K.analy.16.12.5.lambda1=plot.K.analy(fit.16.12.5.24.lambda1,fit.16.12.5.48.lambda1,fit.16.12.5.96.lambda1)
K.analy.16.12.5.lambda2=plot.K.analy(fit.16.12.5.24.lambda2,fit.16.12.5.48.lambda2,fit.16.12.5.96.lambda2)
K.analy.16.12.5.lambda3=plot.K.analy(fit.16.12.5.24.lambda3,fit.16.12.5.48.lambda3,fit.16.12.5.96.lambda3)
K.analy.16.12.5.lambda4=plot.K.analy(fit.16.12.5.24.lambda4,fit.16.12.5.48.lambda4,fit.16.12.5.96.lambda4)

K.analy.16.12.5=rbind(K.analy.16.12.5.lambda1,K.analy.16.12.5.lambda2,K.analy.16.12.5.lambda3,K.analy.16.12.5.lambda4)
K.analy.16.12.5.lambda=factor(K.analy.16.12.5$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(K.analy.16.12.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$\\lambda=0.8$"))
K.analy.16.12.5$lambda=K.analy.16.12.5.lambda

png("./numerical.simul_gfcore/K.16.12.5.png",width=900,height=600)
ggplot(K.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda,1)
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Separable Component Consistency :  $(p_1,p_2,r)=(16,12,5)$ "))+theme(plot.title=element_text(hjust=0))
dev.off()

## pdf
## 2

### C

C.analy.16.12.5.lambda1=plot.C.analy(fit.16.12.5.24.lambda1,fit.16.12.5.48.lambda1,fit.16.12.5.96.lambda1)
C.analy.16.12.5.lambda2=plot.C.analy(fit.16.12.5.24.lambda2,fit.16.12.5.48.lambda2,fit.16.12.5.96.lambda2)
C.analy.16.12.5.lambda3=plot.C.analy(fit.16.12.5.24.lambda3,fit.16.12.5.48.lambda3,fit.16.12.5.96.lambda3)
C.analy.16.12.5.lambda4=plot.C.analy(fit.16.12.5.24.lambda4,fit.16.12.5.48.lambda4,fit.16.12.5.96.lambda4)

C.analy.16.12.5=rbind(C.analy.16.12.5.lambda1,C.analy.16.12.5.lambda2,C.analy.16.12.5.lambda3,C.analy.16.12.5.lambda4)
C.analy.16.12.5.lambda=factor(C.analy.16.12.5$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(C.analy.16.12.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$\\lambda=0.8$"))
C.analy.16.12.5$lambda=C.analy.16.12.5.lambda

png("./numerical.simul_gfcore/C.16.12.5.png",width=900,height=600)
ggplot(C.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda,1)
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Core Component Consistency :  $(p_1,p_2,r)=(16,12,5)$ "))+theme(plot.title=element_text(hjust=0))
dev.off()

## pdf

```

```
## 2
### Sigma

Sigma.analy.16.12.5.lambda1=plot.Sigma.analy(fit.16.12.5.24.lambda1,fit.16.12.5.48.lambda1,fit.16.12.5.
Sigma.analy.16.12.5.lambda2=plot.Sigma.analy(fit.16.12.5.24.lambda2,fit.16.12.5.48.lambda2,fit.16.12.5.
Sigma.analy.16.12.5.lambda3=plot.Sigma.analy(fit.16.12.5.24.lambda3,fit.16.12.5.48.lambda3,fit.16.12.5.
Sigma.analy.16.12.5.lambda4=plot.Sigma.analy(fit.16.12.5.24.lambda4,fit.16.12.5.48.lambda4,fit.16.12.5.

Sigma.analy.16.12.5=rbind(Sigma.analy.16.12.5.lambda1,Sigma.analy.16.12.5.lambda2,Sigma.analy.16.12.5.l
Sigma.analy.16.12.5.lambda=factor(Sigma.analy.16.12.5$lambda,levels=c("lambda1","lambda2","lambda3","la
levels(Sigma.analy.16.12.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),'
Sigma.analy.16.12.5$lambda=Sigma.analy.16.12.5.lambda

png("./numerical.simul_gfcore/Sigma.16.12.5.png",width=900,height=600)
ggplot(Sigma.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Sigma Consistency : $(p_1,p_2,r)=(16,12,5)$"))+theme(plot.title=element_text(hjust=0.5))
dev.off()

## pdf
## 2
```

$(p_1,p_2,r)=(18,8,5)$

```
##### mean of the relative spectral norms for  $K$ ,  $C$ , and  $\Sigma$  (in order) across 100 iterations
##### under different choices of the sample size  $n$  and the value of non-spiked eigenvalue  $\lambda$ .
###lambda=0.2
### n=p/8
fit.18.8.5.18.lambda1=stat.analy(pi.core.fit.18.8.5.18.lambda1)
colMeans(fit.18.8.5.18.lambda1$kro.mle)

##          K          C      Sigma
## 0.4144783 0.9638268 0.9099092
colMeans(fit.18.8.5.18.lambda1$cse)

##          K          C      Sigma
## 0.4144783 0.5369506 0.6936303
colMeans(fit.18.8.5.18.lambda1$init.ai)

##          K          C      Sigma
## 0.4144783 0.5110386 0.7482357
colMeans(fit.18.8.5.18.lambda1$init.chol)

##          K          C      Sigma
## 0.4144783 0.5545233 0.7482357
colMeans(fit.18.8.5.18.lambda1$fit.ai.analy)

##          K          C      Sigma
## 0.2936365 0.3924721 0.4602142
colMeans(fit.18.8.5.18.lambda1$fit.chol.analy)

##          K          C      Sigma
```

```
## 0.2942171 0.4073266 0.4563046

### n=p/4
fit.18.8.5.36.lambda1=stat.analy(pi.core.fit.18.8.5.36.lambda1)
colMeans(fit.18.8.5.36.lambda1$kro.mle)

##          K          C          Sigma
## 0.2760968 0.9638268 0.9076500

colMeans(fit.18.8.5.36.lambda1$cse)

##          K          C          Sigma
## 0.2760968 0.4232770 0.5076847

colMeans(fit.18.8.5.36.lambda1$init.ai)

##          K          C          Sigma
## 0.2760968 0.3531599 0.5232493

colMeans(fit.18.8.5.36.lambda1$init.chol)

##          K          C          Sigma
## 0.2760968 0.3818112 0.5232493

colMeans(fit.18.8.5.36.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.2078077 0.2322876 0.3115257

colMeans(fit.18.8.5.36.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.2090947 0.2498853 0.3158213

### n=p/2
fit.18.8.5.72.lambda1=stat.analy(pi.core.fit.18.8.5.72.lambda1)
colMeans(fit.18.8.5.72.lambda1$kro.mle)

##          K          C          Sigma
## 0.1818612 0.9638268 0.9088839

colMeans(fit.18.8.5.72.lambda1$cse)

##          K          C          Sigma
## 0.1818612 0.3243982 0.3822353

colMeans(fit.18.8.5.72.lambda1$init.ai)

##          K          C          Sigma
## 0.1818612 0.2511521 0.3626540

colMeans(fit.18.8.5.72.lambda1$init.chol)

##          K          C          Sigma
## 0.1818612 0.2710691 0.3626540

colMeans(fit.18.8.5.72.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.1349511 0.1548443 0.2114134

colMeans(fit.18.8.5.72.lambda1$fit.chol.analy)

##          K          C          Sigma
```

```

## 0.1377418 0.1718151 0.2201081
### n=p
fit.18.8.5.144.lambda1=stat.analy(pi.core.fit.18.8.5.144.lambda1)
colMeans(fit.18.8.5.144.lambda1$kro.mle)

##          K          C          Sigma
## 0.1292909 0.9638268 0.9070238
colMeans(fit.18.8.5.144.lambda1$cse)

##          K          C          Sigma
## 0.1292909 0.2261573 0.2731532
colMeans(fit.18.8.5.144.lambda1$init.ai)

##          K          C          Sigma
## 0.1292909 0.1716378 0.2531209
colMeans(fit.18.8.5.144.lambda1$init.chol)

##          K          C          Sigma
## 0.1292909 0.1848695 0.2531209
colMeans(fit.18.8.5.144.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.09809259 0.10594491 0.14970630
colMeans(fit.18.8.5.144.lambda1$fit.chol.analy)

##          K          C          Sigma
## 0.1016628 0.1208852 0.1611105
### n=2p
fit.18.8.5.288.lambda1=stat.analy(pi.core.fit.18.8.5.288.lambda1)
colMeans(fit.18.8.5.288.lambda1$kro.mle)

##          K          C          Sigma
## 0.09276499 0.96382683 0.90741171
colMeans(fit.18.8.5.288.lambda1$cse)

##          K          C          Sigma
## 0.09276499 0.15927818 0.19766765
colMeans(fit.18.8.5.288.lambda1$init.ai)

##          K          C          Sigma
## 0.09276499 0.12248783 0.17721627
colMeans(fit.18.8.5.288.lambda1$init.chol)

##          K          C          Sigma
## 0.09276499 0.13231778 0.17721627
colMeans(fit.18.8.5.288.lambda1$fit.ai.analy)

##          K          C          Sigma
## 0.07223125 0.07843427 0.11286105
colMeans(fit.18.8.5.288.lambda1$fit.chol.analy)

##          K          C          Sigma

```



```

## 0.07712413 0.09271966 0.12463097
###lambda=0.4
### n=p/8
fit.18.8.5.18.lambda2=stat.analy(pi.core.fit.18.8.5.18.lambda2)
colMeans(fit.18.8.5.18.lambda2$kro.mle)

##          K          C      Sigma
## 0.4057991 0.9523191 0.8819741
colMeans(fit.18.8.5.18.lambda2$cse)

##          K          C      Sigma
## 0.4057991 0.6365949 0.6861166
colMeans(fit.18.8.5.18.lambda2$init.ai)

##          K          C      Sigma
## 0.4057991 0.6648972 0.8144429
colMeans(fit.18.8.5.18.lambda2$init.chol)

##          K          C      Sigma
## 0.4057991 0.7008474 0.8144429
colMeans(fit.18.8.5.18.lambda2$fit.ai.analy)

##          K          C      Sigma
## 0.3323418 0.7127249 0.7176785
colMeans(fit.18.8.5.18.lambda2$fit.chol.analy)

##          K          C      Sigma
## 0.3268141 0.7153985 0.6932218
### n=p/4
fit.18.8.5.36.lambda2=stat.analy(pi.core.fit.18.8.5.36.lambda2)
colMeans(fit.18.8.5.36.lambda2$kro.mle)

##          K          C      Sigma
## 0.2700714 0.9523191 0.8791812
colMeans(fit.18.8.5.36.lambda2$cse)

##          K          C      Sigma
## 0.2700714 0.5473665 0.5570896
colMeans(fit.18.8.5.36.lambda2$init.ai)

##          K          C      Sigma
## 0.2700714 0.4407324 0.5521882
colMeans(fit.18.8.5.36.lambda2$init.chol)

##          K          C      Sigma
## 0.2700714 0.4640202 0.5521882
colMeans(fit.18.8.5.36.lambda2$fit.ai.analy)

##          K          C      Sigma
## 0.2263432 0.3871662 0.4265793

```

```
colMeans(fit.18.8.5.36.lambda2$fit.chol.analy)

##           K           C           Sigma
## 0.2278224 0.3993820 0.4247423

### n=p/2
fit.18.8.5.72.lambda2=stat.analy(pi.core.fit.18.8.5.72.lambda2)
colMeans(fit.18.8.5.72.lambda2$kro.mle)

##           K           C           Sigma
## 0.1791041 0.9523191 0.8802166

colMeans(fit.18.8.5.72.lambda2$cse)

##           K           C           Sigma
## 0.1791041 0.4466527 0.4565753

colMeans(fit.18.8.5.72.lambda2$init.ai)

##           K           C           Sigma
## 0.1791041 0.3064962 0.3773873

colMeans(fit.18.8.5.72.lambda2$init.chol)

##           K           C           Sigma
## 0.1791041 0.3220549 0.3773873

colMeans(fit.18.8.5.72.lambda2$fit.ai.analy)

##           K           C           Sigma
## 0.1513856 0.2450460 0.2827887

colMeans(fit.18.8.5.72.lambda2$fit.chol.analy)

##           K           C           Sigma
## 0.1506006 0.2535995 0.2783266

### n=p
fit.18.8.5.144.lambda2=stat.analy(pi.core.fit.18.8.5.144.lambda2)
colMeans(fit.18.8.5.144.lambda2$kro.mle)

##           K           C           Sigma
## 0.1263310 0.9523191 0.8779679

colMeans(fit.18.8.5.144.lambda2$cse)

##           K           C           Sigma
## 0.1263310 0.3263830 0.3383861

colMeans(fit.18.8.5.144.lambda2$init.ai)

##           K           C           Sigma
## 0.1263310 0.2096952 0.2629223

colMeans(fit.18.8.5.144.lambda2$init.chol)

##           K           C           Sigma
## 0.1263310 0.2195196 0.2629223

colMeans(fit.18.8.5.144.lambda2$fit.ai.analy)

##           K           C           Sigma
## 0.1083000 0.1641962 0.1956011
```

```
colMeans(fit.18.8.5.144.lambda2$fit.chol.analy)

##           K           C           Sigma
## 0.1083325 0.1714500 0.1949438

### n=2p
fit.18.8.5.288.lambda2=stat.analy(pi.core.fit.18.8.5.288.lambda2)
colMeans(fit.18.8.5.288.lambda2$kro.mle)

##           K           C           Sigma
## 0.09209303 0.95231913 0.87836726
colMeans(fit.18.8.5.288.lambda2$cse)

##           K           C           Sigma
## 0.09209303 0.23310270 0.24608279
colMeans(fit.18.8.5.288.lambda2$init.ai)

##           K           C           Sigma
## 0.09209303 0.14831331 0.18505801
colMeans(fit.18.8.5.288.lambda2$init.chol)

##           K           C           Sigma
## 0.09209303 0.15577123 0.18505801
colMeans(fit.18.8.5.288.lambda2$fit.ai.analy)

##           K           C           Sigma
## 0.07909111 0.11615252 0.14261143
colMeans(fit.18.8.5.288.lambda2$fit.chol.analy)

##           K           C           Sigma
## 0.07931654 0.12171446 0.14271111

### lambda=0.6
### n=p/8
fit.18.8.5.18.lambda3=stat.analy(pi.core.fit.18.8.5.18.lambda3)
colMeans(fit.18.8.5.18.lambda3$kro.mle)

##           K           C           Sigma
## 0.4021752 0.9301092 0.8298895
colMeans(fit.18.8.5.18.lambda3$cse)

##           K           C           Sigma
## 0.4021752 0.7147875 0.6952151
colMeans(fit.18.8.5.18.lambda3$init.ai)

##           K           C           Sigma
## 0.4021752 0.9693051 1.0104817
colMeans(fit.18.8.5.18.lambda3$init.chol)

##           K           C           Sigma
## 0.4021752 0.9949916 1.0104817
colMeans(fit.18.8.5.18.lambda3$fit.ai.analy)

##           K           C           Sigma
```

```

## 0.3736036 1.1031034 1.0567785
colMeans(fit.18.8.5.18.lambda3$fit.chol.analy)

##          K          C      Sigma
## 0.3718204 1.1481597 1.0754789

### n=p/4
fit.18.8.5.36.lambda3=stat.analy(pi.core.fit.18.8.5.36.lambda3)
colMeans(fit.18.8.5.36.lambda3$kro.mle)

##          K          C      Sigma
## 0.2678544 0.9301092 0.8258900
colMeans(fit.18.8.5.36.lambda3$cse)

##          K          C      Sigma
## 0.2678544 0.6536238 0.6146057
colMeans(fit.18.8.5.36.lambda3$init.ai)

##          K          C      Sigma
## 0.2678544 0.6074925 0.6437303
colMeans(fit.18.8.5.36.lambda3$init.chol)

##          K          C      Sigma
## 0.2678544 0.6246319 0.6437303
colMeans(fit.18.8.5.36.lambda3$fit.ai.analy)

##          K          C      Sigma
## 0.2444111 0.6550876 0.6436697
colMeans(fit.18.8.5.36.lambda3$fit.chol.analy)

##          K          C      Sigma
## 0.2441419 0.6622649 0.6414292

### n=p/2
fit.18.8.5.72.lambda3=stat.analy(pi.core.fit.18.8.5.72.lambda3)
colMeans(fit.18.8.5.72.lambda3$kro.mle)

##          K          C      Sigma
## 0.1786960 0.9301092 0.8259245
colMeans(fit.18.8.5.72.lambda3$cse)

##          K          C      Sigma
## 0.1786960 0.5613413 0.5271455
colMeans(fit.18.8.5.72.lambda3$init.ai)

##          K          C      Sigma
## 0.1786960 0.4014418 0.4247961
colMeans(fit.18.8.5.72.lambda3$init.chol)

##          K          C      Sigma
## 0.1786960 0.4129383 0.4247961
colMeans(fit.18.8.5.72.lambda3$fit.ai.analy)

##          K          C      Sigma

```

```

## 0.1614605 0.3826035 0.3845216
colMeans(fit.18.8.5.72.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.1615469 0.3907311 0.3839501
### n=p
fit.18.8.5.144.lambda3=stat.analy(pi.core.fit.18.8.5.144.lambda3)
colMeans(fit.18.8.5.144.lambda3$kro.mle)

##          K          C          Sigma
## 0.1259822 0.9301092 0.8229050
colMeans(fit.18.8.5.144.lambda3$cse)

##          K          C          Sigma
## 0.1259822 0.4310073 0.4092298
colMeans(fit.18.8.5.144.lambda3$init.ai)

##          K          C          Sigma
## 0.1259822 0.2711719 0.2938958
colMeans(fit.18.8.5.144.lambda3$init.chol)

##          K          C          Sigma
## 0.1259822 0.2782223 0.2938958
colMeans(fit.18.8.5.144.lambda3$fit.ai.analy)

##          K          C          Sigma
## 0.1163362 0.2490371 0.2619241
colMeans(fit.18.8.5.144.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.1163361 0.2547225 0.2619640
### n=2p
fit.18.8.5.288.lambda3=stat.analy(pi.core.fit.18.8.5.288.lambda3)
colMeans(fit.18.8.5.288.lambda3$kro.mle)

##          K          C          Sigma
## 0.09152124 0.93010925 0.82320524
colMeans(fit.18.8.5.288.lambda3$cse)

##          K          C          Sigma
## 0.09152124 0.31660668 0.30349057
colMeans(fit.18.8.5.288.lambda3$init.ai)

##          K          C          Sigma
## 0.09152124 0.19085376 0.21021350
colMeans(fit.18.8.5.288.lambda3$init.chol)

##          K          C          Sigma
## 0.09152124 0.19647315 0.21021350
colMeans(fit.18.8.5.288.lambda3$fit.ai.analy)

##          K          C          Sigma

```

```
## 0.08397158 0.17354527 0.19325987
colMeans(fit.18.8.5.288.lambda3$fit.chol.analy)

##          K          C          Sigma
## 0.08393267 0.17755959 0.19317341
###lambda=0.8
### n=p/8
fit.18.8.5.18.lambda4=stat.analy(pi.core.fit.18.8.5.18.lambda4)
colMeans(fit.18.8.5.18.lambda4$kro.mle)

##          K          C          Sigma
## 0.4020173 0.8697154 0.7045681
colMeans(fit.18.8.5.18.lambda4$cse)

##          K          C          Sigma
## 0.4020173 0.7560302 0.6553992
colMeans(fit.18.8.5.18.lambda4$init.ai)

##          K          C          Sigma
## 0.4020173 1.7584930 1.5358708
colMeans(fit.18.8.5.18.lambda4$init.chol)

##          K          C          Sigma
## 0.4020173 1.7712599 1.5358708
colMeans(fit.18.8.5.18.lambda4$fit.ai.analy)

##          K          C          Sigma
## 0.3726658 1.7827162 1.4502935
colMeans(fit.18.8.5.18.lambda4$fit.chol.analy)

##          K          C          Sigma
## 0.3779525 1.8247102 1.4893652
### n=p/4
fit.18.8.5.36.lambda4=stat.analy(pi.core.fit.18.8.5.36.lambda4)
colMeans(fit.18.8.5.36.lambda4$kro.mle)

##          K          C          Sigma
## 0.2681783 0.8697154 0.6956824
colMeans(fit.18.8.5.36.lambda4$cse)

##          K          C          Sigma
## 0.2681783 0.7238837 0.6062452
colMeans(fit.18.8.5.36.lambda4$init.ai)

##          K          C          Sigma
## 0.2681783 1.0801473 0.9188810
colMeans(fit.18.8.5.36.lambda4$init.chol)

##          K          C          Sigma
## 0.2681783 1.0893361 0.9188810
```

```

colMeans(fit.18.8.5.36.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.2601817 1.1308325 0.9241194
colMeans(fit.18.8.5.36.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.2609636 1.1471661 0.9355235
### n=p/2
fit.18.8.5.72.lambda4=stat.analy(pi.core.fit.18.8.5.72.lambda4)
colMeans(fit.18.8.5.72.lambda4$kro.mle)

##          K          C      Sigma
## 0.1790717 0.8697154 0.6902683
colMeans(fit.18.8.5.72.lambda4$cse)

##          K          C      Sigma
## 0.1790717 0.6515751 0.5398188
colMeans(fit.18.8.5.72.lambda4$init.ai)

##          K          C      Sigma
## 0.1790717 0.6783766 0.5910795
colMeans(fit.18.8.5.72.lambda4$init.chol)

##          K          C      Sigma
## 0.1790717 0.6850662 0.5910795
colMeans(fit.18.8.5.72.lambda4$fit.ai.analy)

##          K          C      Sigma
## 0.1722436 0.7420626 0.6265671
colMeans(fit.18.8.5.72.lambda4$fit.chol.analy)

##          K          C      Sigma
## 0.1736625 0.7471213 0.6258211
### n=p
fit.18.8.5.144.lambda4=stat.analy(pi.core.fit.18.8.5.144.lambda4)
colMeans(fit.18.8.5.144.lambda4$kro.mle)

##          K          C      Sigma
## 0.1272286 0.8697154 0.6840101
colMeans(fit.18.8.5.144.lambda4$cse)

##          K          C      Sigma
## 0.1272286 0.5293522 0.4432124
colMeans(fit.18.8.5.144.lambda4$init.ai)

##          K          C      Sigma
## 0.1272286 0.4378673 0.3901200
colMeans(fit.18.8.5.144.lambda4$init.chol)

##          K          C      Sigma
## 0.1272286 0.4415391 0.3901200

```

```

colMeans(fit.18.8.5.144.lambda4$fit.ai.analy)

##           K           C           Sigma
## 0.1241308 0.4475903 0.3922296

colMeans(fit.18.8.5.144.lambda4$fit.chol.analy)

##           K           C           Sigma
## 0.1240305 0.4494564 0.3905622

### n=2p
fit.18.8.5.288.lambda4=stat.analy(pi.core.fit.18.8.5.288.lambda4)
colMeans(fit.18.8.5.288.lambda4$kro.mle)

##           K           C           Sigma
## 0.09088225 0.86971541 0.68370502

colMeans(fit.18.8.5.288.lambda4$cse)

##           K           C           Sigma
## 0.09088225 0.40560740 0.34441138

colMeans(fit.18.8.5.288.lambda4$init.ai)

##           K           C           Sigma
## 0.09088225 0.32840957 0.29652039

colMeans(fit.18.8.5.288.lambda4$init.chol)

##           K           C           Sigma
## 0.09088225 0.32938751 0.29652039

colMeans(fit.18.8.5.288.lambda4$fit.ai.analy)

##           K           C           Sigma
## 0.0881248 0.3256475 0.2956644

colMeans(fit.18.8.5.288.lambda4$fit.chol.analy)

##           K           C           Sigma
## 0.08809078 0.32644097 0.29551683

##### boxplots of the relative spectral norms for K, C, and Sigma across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
### K

K.analy.18.8.5.lambda1=plot.K.analy(fit.18.8.5.18.lambda1,fit.18.8.5.36.lambda1,fit.18.8.5.72.lambda1,f
K.analy.18.8.5.lambda2=plot.K.analy(fit.18.8.5.18.lambda2,fit.18.8.5.36.lambda2,fit.18.8.5.72.lambda2,f
K.analy.18.8.5.lambda3=plot.K.analy(fit.18.8.5.18.lambda3,fit.18.8.5.36.lambda3,fit.18.8.5.72.lambda3,f
K.analy.18.8.5.lambda4=plot.K.analy(fit.18.8.5.18.lambda4,fit.18.8.5.36.lambda4,fit.18.8.5.72.lambda4,f

K.analy.18.8.5=rbind(K.analy.18.8.5.lambda1,K.analy.18.8.5.lambda2,K.analy.18.8.5.lambda3,K.analy.18.8.
K.analy.18.8.5.lambda=factor(K.analy.18.8.5$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(K.analy.18.8.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$\\lambda=0.8$"))
K.analy.18.8.5$lambda=K.analy.18.8.5.lambda

png("./numerical.simul_gfcore/K.18.8.5.png",width=900,height=600)
ggplot(K.analy.18.8.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(~lambda,lab
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+

```



```
  ggtitle(TeX("Separable Component Consistency :  $(p_1, p_2, r) = (18, 8, 5)$ ")) + theme(plot.title = element_text(
dev.off()
```

```
## pdf
```

```
## 2
```

```
### C
```

```
C.analy.18.8.5.lambda1=plot.C.analy(fit.18.8.5.18.lambda1, fit.18.8.5.36.lambda1, fit.18.8.5.72.lambda1, f
C.analy.18.8.5.lambda2=plot.C.analy(fit.18.8.5.18.lambda2, fit.18.8.5.36.lambda2, fit.18.8.5.72.lambda2, f
C.analy.18.8.5.lambda3=plot.C.analy(fit.18.8.5.18.lambda3, fit.18.8.5.36.lambda3, fit.18.8.5.72.lambda3, f
C.analy.18.8.5.lambda4=plot.C.analy(fit.18.8.5.18.lambda4, fit.18.8.5.36.lambda4, fit.18.8.5.72.lambda4, f
```

```
C.analy.18.8.5=rbind(C.analy.18.8.5.lambda1, C.analy.18.8.5.lambda2, C.analy.18.8.5.lambda3, C.analy.18.8.5.lambda4)
C.analy.18.8.5.lambda=factor(C.analy.18.8.5.lambda, levels=c("lambda1", "lambda2", "lambda3", "lambda4"))
levels(C.analy.18.8.5.lambda)=c(TeX(" $\\lambda=0.2$ "), TeX(" $\\lambda=0.4$ "), TeX(" $\\lambda=0.6$ "), TeX(" $\\lambda=0.8$ "))
C.analy.18.8.5.lambda=C.analy.18.8.5.lambda
```

```
png("./numerical.simul_gfcore/C.18.8.5.png", width=900, height=600)
ggplot(C.analy.18.8.5, mapping=aes(x=samp.size, y=stat, col=Method)) + geom_boxplot() + facet_wrap(~lambda, lab
  theme_bw() + xlab("Sample Size") + ylab("Relative Norm") +
  ggtitle(TeX("Core Component Consistency :  $(p_1, p_2, r) = (18, 8, 5)$ ")) + theme(plot.title = element_text(hjust=0.5))
dev.off()
```

```
## pdf
```

```
## 2
```

```
### Sigma
```

```
Sigma.analy.18.8.5.lambda1=plot.Sigma.analy(fit.18.8.5.18.lambda1, fit.18.8.5.36.lambda1, fit.18.8.5.72.lambda1, f
Sigma.analy.18.8.5.lambda2=plot.Sigma.analy(fit.18.8.5.18.lambda2, fit.18.8.5.36.lambda2, fit.18.8.5.72.lambda2, f
Sigma.analy.18.8.5.lambda3=plot.Sigma.analy(fit.18.8.5.18.lambda3, fit.18.8.5.36.lambda3, fit.18.8.5.72.lambda3, f
Sigma.analy.18.8.5.lambda4=plot.Sigma.analy(fit.18.8.5.18.lambda4, fit.18.8.5.36.lambda4, fit.18.8.5.72.lambda4, f
```

```
Sigma.analy.18.8.5=rbind(Sigma.analy.18.8.5.lambda1, Sigma.analy.18.8.5.lambda2, Sigma.analy.18.8.5.lambda3, Sigma.analy.18.8.5.lambda4)
Sigma.analy.18.8.5.lambda=factor(Sigma.analy.18.8.5.lambda, levels=c("lambda1", "lambda2", "lambda3", "lambda4"))
levels(Sigma.analy.18.8.5.lambda)=c(TeX(" $\\lambda=0.2$ "), TeX(" $\\lambda=0.4$ "), TeX(" $\\lambda=0.6$ "), TeX(" $\\lambda=0.8$ "))
Sigma.analy.18.8.5.lambda=Sigma.analy.18.8.5.lambda
```

```
png("./numerical.simul_gfcore/Sigma.18.8.5.png", width=900, height=600)
ggplot(Sigma.analy.18.8.5, mapping=aes(x=samp.size, y=stat, col=Method)) + geom_boxplot() + facet_wrap(~lambda, lab
  theme_bw() + xlab("Sample Size") + ylab("Relative Norm") +
  ggtitle(TeX("Sigma Consistency :  $(p_1, p_2, r) = (18, 8, 5)$ ")) + theme(plot.title = element_text(hjust=0.5))
dev.off()
```

```
## pdf
```

```
## 2
```