Numerical Simulation Analysis

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We provide the summary of the numerical simulations under partial-isotropy core covariance model (see Section 7 for details).

```
################ Libraries

library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.4     v readr     2.1.6
## v forcats   1.0.1     v stringr   1.6.0
## v ggplot2   4.0.1     v tibble    3.3.1
## v lubridate 1.9.4     v tidyr     1.3.2
## v purrr     1.2.1
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(latex2exp)
library(ggpubr)

################ Loading the simulation results
file.list=list.files(path="./numerical.simul_picore",pattern=".rds")
file.list=file.list[-c(1:4)]

for(i in file.list){
  load(paste("./numerical.simul_picore/",i,sep=""))
}

################ Utensils

stat.analy=function(fit){

  kro.mle=fit$kro.mle
  cse=fit$cse
  init.ai=fit$pi.core.init.ai
  init.chol=fit$pi.core.init.chol
  fit.ai=fit$pi.core.ai
  fit.chol=fit$pi.core.chol

  kro.mle=cbind(init.ai[,1],kro.mle)
  cse=cbind(init.ai[,1],cse[,1:2])
  init.chol=cbind(init.ai[,1],init.chol)
```

```
  colnames(kro.mle)=colnames(cse)=colnames(init.chol)=c("K","C","Sigma")

  fit.ai.analy=NULL; fit.chol.analy=NULL
  for(i in 1:100){
    fit.ai.analy=rbind(fit.ai.analy,fit.ai[[i]]$eval)
    fit.chol.analy=rbind(fit.chol.analy,fit.chol[[i]]$eval)
  }
  return(list(kro.mle=kro.mle,cse=cse,init.ai=init.ai,init.chol=init.chol,fit.ai.analy=fit.ai.analy,fit
}

conv.analy=function(fit1,fit2,fit3,fit4,fit5,name){

  fit.ai1=fit1$pi.core.ai
  fit.chol1=fit1$pi.core.chol
  fit.ai2=fit2$pi.core.ai
  fit.chol2=fit2$pi.core.chol
  fit.ai3=fit3$pi.core.ai
  fit.chol3=fit3$pi.core.chol
  fit.ai4=fit4$pi.core.ai
  fit.chol4=fit4$pi.core.chol
  fit.ai5=fit5$pi.core.ai
  fit.chol5=fit5$pi.core.chol

  rel.conv.ai=NULL
  conv.ai=NULL
  rel.conv.chol=NULL
  conv.chol=NULL

  for(i in 1:100){
    temp1=c(fit.ai1[[i]]$rel.conv,fit.ai2[[i]]$rel.conv,fit.ai3[[i]]$rel.conv,fit.ai4[[i]]$rel.conv,fit
    temp2=c(fit.ai1[[i]]$convergence,fit.ai2[[i]]$convergence,fit.ai3[[i]]$convergence,fit.ai4[[i]]$con
    rel.conv.ai=cbind(rel.conv.ai,temp1)
    conv.ai=cbind(conv.ai,temp2)

    temp1=c(fit.chol1[[i]]$rel.conv,fit.chol2[[i]]$rel.conv,fit.chol3[[i]]$rel.conv,fit.chol4[[i]]$rel.
    temp2=c(fit.chol1[[i]]$convergence,fit.chol2[[i]]$convergence,fit.chol3[[i]]$convergence,fit.chol4[
    rel.conv.chol=cbind(rel.conv.chol,temp1)
    conv.chol=cbind(conv.chol,temp2)
  }
  rownames(rel.conv.ai)=rownames(conv.ai)=rownames(rel.conv.chol)=rownames(conv.chol)=name

  return(list(rel.conv.ai=rel.conv.ai,conv.ai=conv.ai,rel.conv.chol=rel.conv.chol,conv.chol=conv.chol))
}

plot.K.analy=function(fit1,fit2,fit3,fit4,fit5,samp.size,lambda){

  method=rep(c("Base-AI","PI-AI","PI-Chol"),each=100)

  dat1=tibble(stat=c(fit1$init.ai[,1],fit1$fit.ai.analy[,1],fit1$fit.chol.analy[,1]),Method=method,samp
  dat2=tibble(stat=c(fit2$init.ai[,1],fit2$fit.ai.analy[,1],fit2$fit.chol.analy[,1]),Method=method,samp
  dat3=tibble(stat=c(fit3$init.ai[,1],fit3$fit.ai.analy[,1],fit3$fit.chol.analy[,1]),Method=method,samp
  dat4=tibble(stat=c(fit4$init.ai[,1],fit4$fit.ai.analy[,1],fit4$fit.chol.analy[,1]),Method=method,samp
  dat5=tibble(stat=c(fit5$init.ai[,1],fit5$fit.ai.analy[,1],fit5$fit.chol.analy[,1]),Method=method,samp
```

```
  dat=rbind(dat1,dat2,dat3,dat4,dat5)
  dat$samp.size=factor(dat$samp.size,levels=samp.size)
  dat$Method=factor(dat$Method,levels=c("Base-AI","PI-AI","PI-Chol"))
  return(dat)
}

plot.C.analy=function(fit1,fit2,fit3,fit4,fit5,samp.size,lambda){

  method=rep(c("CSE","Base-AI","Base-Chol","PI-AI","PI-Chol"),each=100)

  dat1=tibble(stat=c(fit1$cse[,2],fit1$init.ai[,2],fit1$init.chol[,2],
                     fit1$fit.ai.analy[,2],fit1$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))
  dat2=tibble(stat=c(fit2$cse[,2],fit2$init.ai[,2],fit2$init.chol[,2],
                     fit2$fit.ai.analy[,2],fit2$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))
  dat3=tibble(stat=c(fit3$cse[,2],fit3$init.ai[,2],fit3$init.chol[,2],
                     fit3$fit.ai.analy[,2],fit3$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))
  dat4=tibble(stat=c(fit4$cse[,2],fit4$init.ai[,2],fit4$init.chol[,2],
                     fit4$fit.ai.analy[,2],fit4$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))
  dat5=tibble(stat=c(fit5$cse[,2],fit5$init.ai[,2],fit5$init.chol[,2],
                     fit5$fit.ai.analy[,2],fit5$fit.chol.analy[,2]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))

  dat=rbind(dat1,dat2,dat3,dat4,dat5)
  dat$samp.size=factor(dat$samp.size,levels=samp.size)
  dat$Method=factor(dat$Method,levels=c("KMLE","CSE","Base-AI","Base-Chol","PI-AI","PI-Chol"))
  return(dat)

}

plot.Sigma.analy=function(fit1,fit2,fit3,fit4,fit5,samp.size,lambda){

  method=rep(c("KMLE","CSE","Base-AI","PI-AI","PI-Chol"),each=100)

  dat1=tibble(stat=c(fit1$kro.mle[,3],fit1$cse[,3],fit1$init.ai[,3],
                     fit1$fit.ai.analy[,3],fit1$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500)
          )
  dat2=tibble(stat=c(fit2$kro.mle[,3],fit2$cse[,3],fit2$init.ai[,3],
                     fit2$fit.ai.analy[,3],fit2$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))
  dat3=tibble(stat=c(fit3$kro.mle[,3],fit3$cse[,3],fit3$init.ai[,3],
                     fit3$fit.ai.analy[,3],fit3$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))
  dat4=tibble(stat=c(fit4$kro.mle[,3],fit4$cse[,3],fit4$init.ai[,3],
                     fit4$fit.ai.analy[,3],fit4$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))
  dat5=tibble(stat=c(fit5$kro.mle[,3],fit5$cse[,3],fit5$init.ai[,3],
                     fit5$fit.ai.analy[,3],fit5$fit.chol.analy[,3]),Method=method,samp.size=rep(samp.si
              lambda=rep(lambda,500))
```

```
  dat=rbind(dat1,dat2,dat3,dat4,dat5)
  dat$samp.size=factor(dat$samp.size,levels=samp.size)
  dat$Method=factor(dat$Method,levels=c("KMLE","CSE","Base-AI","PI-AI","PI-Chol"))
  return(dat)

}
```

(p1,p2,r)=(16,12,3)

```
##### mean of the relative spectral norms for $K$, $C$, and $\Sigma$ (in order) across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
###lambda=0.2
### n=p/8
fit.16.12.3.24.lambda1=stat.analy(pi.core.fit.16.12.3.24.lambda1)
colMeans(fit.16.12.3.24.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.3276087 0.9838725 0.9642052
```

```
colMeans(fit.16.12.3.24.lambda1$cse)
```

```
##         K         C     Sigma
## 0.3276087 0.3714609 0.5055507
```

```
colMeans(fit.16.12.3.24.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.3276087 0.3052779 0.4942421
```

```
colMeans(fit.16.12.3.24.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.3276087 0.3383427 0.4942421
```

```
colMeans(fit.16.12.3.24.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2414748 0.1862384 0.2906121
```

```
colMeans(fit.16.12.3.24.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2449003 0.2132732 0.2979727
```

```
### n=p/4
fit.16.12.3.48.lambda1=stat.analy(pi.core.fit.16.12.3.48.lambda1)
colMeans(fit.16.12.3.48.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.2234899 0.9838725 0.9631266
```

```
colMeans(fit.16.12.3.48.lambda1$cse)
```

```
##         K         C     Sigma
## 0.2234899 0.3013176 0.3807253
```

```
colMeans(fit.16.12.3.48.lambda1$init.ai)
```

```
##         K         C     Sigma
```

```
## 0.2234899 0.2124217 0.3581320
```

```
colMeans(fit.16.12.3.48.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.2234899 0.2356304 0.3581320
```

```
colMeans(fit.16.12.3.48.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1706497 0.1233404 0.2065604
```

```
colMeans(fit.16.12.3.48.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1737263 0.1475756 0.2191892
```

```
### n=p/2
fit.16.12.3.96.lambda1=stat.analy(pi.core.fit.16.12.3.96.lambda1)
colMeans(fit.16.12.3.96.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.1567307 0.9838725 0.9638376
```

```
colMeans(fit.16.12.3.96.lambda1$cse)
```

```
##         K         C     Sigma
## 0.1567307 0.2492733 0.3078952
```

```
colMeans(fit.16.12.3.96.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.1567307 0.1490448 0.2454960
```

```
colMeans(fit.16.12.3.96.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.1567307 0.1646260 0.2454960
```

```
colMeans(fit.16.12.3.96.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.12304725 0.08730814 0.14927826
```

```
colMeans(fit.16.12.3.96.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1280300 0.1063453 0.1609864
```

```
### n=p
fit.16.12.3.192.lambda1=stat.analy(pi.core.fit.16.12.3.192.lambda1)
colMeans(fit.16.12.3.192.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.1042776 0.9838725 0.9639972
```

```
colMeans(fit.16.12.3.192.lambda1$cse)
```

```
##         K         C     Sigma
## 0.1042776 0.1854317 0.2271772
```

```
colMeans(fit.16.12.3.192.lambda1$init.ai)
```

```
##         K         C     Sigma
```

```
## 0.1042776 0.1028228 0.1693973
```

```
colMeans(fit.16.12.3.192.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.1042776 0.1130689 0.1693973
```

```
colMeans(fit.16.12.3.192.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.08612546 0.06325104 0.10789092
```

```
colMeans(fit.16.12.3.192.lambda1$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.09074193 0.07943208 0.12162029
```

```
### n=2p
fit.16.12.3.384.lambda1=stat.analy(pi.core.fit.16.12.3.384.lambda1)
colMeans(fit.16.12.3.384.lambda1$kro.mle)
```

```
##          K          C      Sigma
## 0.07192497 0.98387245 0.96373771
```

```
colMeans(fit.16.12.3.384.lambda1$cse)
```

```
##          K          C      Sigma
## 0.07192497 0.12986525 0.16009689
```

```
colMeans(fit.16.12.3.384.lambda1$init.ai)
```

```
##          K          C      Sigma
## 0.07192497 0.07156876 0.11767099
```

```
colMeans(fit.16.12.3.384.lambda1$init.chol)
```

```
##          K          C      Sigma
## 0.07192497 0.07872203 0.11767099
```

```
colMeans(fit.16.12.3.384.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.06115646 0.04812648 0.08262485
```

```
colMeans(fit.16.12.3.384.lambda1$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.06476330 0.06056848 0.09279034
```

```
###lambda=0.4
### n=p/8
fit.16.12.3.24.lambda2=stat.analy(pi.core.fit.16.12.3.24.lambda2)
colMeans(fit.16.12.3.24.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.3132777 0.9786116 0.9526806
```

```
colMeans(fit.16.12.3.24.lambda2$cse)
```

```
##         K         C     Sigma
## 0.3132777 0.5018218 0.5611585
```

```r
colMeans(fit.16.12.3.24.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.3132777 0.3868799 0.5291999
```

```r
colMeans(fit.16.12.3.24.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.3132777 0.4147466 0.5291999
```

```r
colMeans(fit.16.12.3.24.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2587529 0.3124039 0.3800776
```

```r
colMeans(fit.16.12.3.24.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2592688 0.3311026 0.3778672
```

```r
### n=p/4
fit.16.12.3.48.lambda2=stat.analy(pi.core.fit.16.12.3.48.lambda2)
colMeans(fit.16.12.3.48.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.2131655 0.9786116 0.9514639
```

```r
colMeans(fit.16.12.3.48.lambda2$cse)
```

```
##         K         C     Sigma
## 0.2131655 0.4524170 0.4824795
```

```r
colMeans(fit.16.12.3.48.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.2131655 0.2684955 0.3796573
```

```r
colMeans(fit.16.12.3.48.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.2131655 0.2881032 0.3796573
```

```r
colMeans(fit.16.12.3.48.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1814260 0.1987184 0.2605674
```

```r
colMeans(fit.16.12.3.48.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1816181 0.2145355 0.2609188
```

```r
### n=p/2
fit.16.12.3.96.lambda2=stat.analy(pi.core.fit.16.12.3.96.lambda2)
colMeans(fit.16.12.3.96.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.1501259 0.9786116 0.9520761
```

```r
colMeans(fit.16.12.3.96.lambda2$cse)
```

```
##         K         C     Sigma
## 0.1501259 0.4008564 0.4264460
```

```r
colMeans(fit.16.12.3.96.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.1501259 0.1846553 0.2582569
```

```r
colMeans(fit.16.12.3.96.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.1501259 0.1975371 0.2582569
```

```r
colMeans(fit.16.12.3.96.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1285462 0.1351056 0.1819678
```

```r
colMeans(fit.16.12.3.96.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1292932 0.1469014 0.1822863
```

```r
### n=p
fit.16.12.3.192.lambda2=stat.analy(pi.core.fit.16.12.3.192.lambda2)
colMeans(fit.16.12.3.192.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.1020819 0.9786116 0.9522396
```

```r
colMeans(fit.16.12.3.192.lambda2$cse)
```

```
##         K         C     Sigma
## 0.1020819 0.3228718 0.3429617
```

```r
colMeans(fit.16.12.3.192.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.1020819 0.1279600 0.1787625
```

```r
colMeans(fit.16.12.3.192.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.1020819 0.1364781 0.1787625
```

```r
colMeans(fit.16.12.3.192.lambda2$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.08970057 0.09445259 0.12733272
```

```r
colMeans(fit.16.12.3.192.lambda2$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.09026356 0.10322482 0.12897548
```

```r
### n=2p
fit.16.12.3.384.lambda2=stat.analy(pi.core.fit.16.12.3.384.lambda2)
colMeans(fit.16.12.3.384.lambda2$kro.mle)
```

```
##          K          C      Sigma
## 0.07059424 0.97861158 0.95197150
```

```r
colMeans(fit.16.12.3.384.lambda2$cse)
```

```
##          K          C      Sigma
## 0.07059424 0.23817154 0.25517422
```

```
colMeans(fit.16.12.3.384.lambda2$init.ai)
```

```
##          K          C      Sigma
## 0.07059424 0.08873395 0.12411055
```

```
colMeans(fit.16.12.3.384.lambda2$init.chol)
```

```
##          K          C      Sigma
## 0.07059424 0.09472238 0.12411055
```

```
colMeans(fit.16.12.3.384.lambda2$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.06243839 0.06558955 0.08807434
```

```
colMeans(fit.16.12.3.384.lambda2$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.06263257 0.07217430 0.09026363
```

```
###lambda=0.6
### n=p/8
fit.16.12.3.24.lambda3=stat.analy(pi.core.fit.16.12.3.24.lambda3)
colMeans(fit.16.12.3.24.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.3038568 0.9682568 0.9300386
```

```
colMeans(fit.16.12.3.24.lambda3$cse)
```

```
##         K         C     Sigma
## 0.3038568 0.6309274 0.6440176
```

```
colMeans(fit.16.12.3.24.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.3038568 0.5341362 0.6225151
```

```
colMeans(fit.16.12.3.24.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.3038568 0.5559011 0.6225151
```

```
colMeans(fit.16.12.3.24.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2710318 0.5210785 0.5477541
```

```
colMeans(fit.16.12.3.24.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2709022 0.5349312 0.5456756
```

```
### n=p/4
fit.16.12.3.48.lambda3=stat.analy(pi.core.fit.16.12.3.48.lambda3)
colMeans(fit.16.12.3.48.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.2060223 0.9682568 0.9286078
```

```
colMeans(fit.16.12.3.48.lambda3$cse)
```

```
##         K         C     Sigma
```

```
## 0.2060223 0.6067045 0.6027532
```

```
colMeans(fit.16.12.3.48.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.2060223 0.3587345 0.4322412
```

```
colMeans(fit.16.12.3.48.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.2060223 0.3741805 0.4322412
```

```
colMeans(fit.16.12.3.48.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1862933 0.3115391 0.3506887
```

```
colMeans(fit.16.12.3.48.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1862123 0.3233404 0.3503361
```

```
### n=p/2
fit.16.12.3.96.lambda3=stat.analy(pi.core.fit.16.12.3.96.lambda3)
colMeans(fit.16.12.3.96.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.1453104 0.9682568 0.9290496
```

```
colMeans(fit.16.12.3.96.lambda3$cse)
```

```
##         K         C     Sigma
## 0.1453104 0.5610646 0.5549979
```

```
colMeans(fit.16.12.3.96.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.1453104 0.2388442 0.2872865
```

```
colMeans(fit.16.12.3.96.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.1453104 0.2493549 0.2872865
```

```
colMeans(fit.16.12.3.96.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1319979 0.2032018 0.2348703
```

```
colMeans(fit.16.12.3.96.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1319990 0.2122822 0.2347553
```

```
### n=p
fit.16.12.3.192.lambda3=stat.analy(pi.core.fit.16.12.3.192.lambda3)
colMeans(fit.16.12.3.192.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.1003274 0.9682568 0.9292109
```

```
colMeans(fit.16.12.3.192.lambda3$cse)
```

```
##         K         C     Sigma
```

```
## 0.1003274 0.4860781 0.4798697
```

```
colMeans(fit.16.12.3.192.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.1003274 0.1632689 0.1983312
```

```
colMeans(fit.16.12.3.192.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.1003274 0.1704076 0.1983312
```

```
colMeans(fit.16.12.3.192.lambda3$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.09258011 0.13974902 0.16361662
```

```
colMeans(fit.16.12.3.192.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.0925522 0.1465730 0.1635821
```

```
### n=2p
fit.16.12.3.384.lambda3=stat.analy(pi.core.fit.16.12.3.384.lambda3)
colMeans(fit.16.12.3.384.lambda3$kro.mle)
```

```
##          K          C      Sigma
## 0.06930839 0.96825684 0.92892602
```

```
colMeans(fit.16.12.3.384.lambda3$cse)
```

```
##          K          C      Sigma
## 0.06930839 0.38918122 0.38564601
```

```
colMeans(fit.16.12.3.384.lambda3$init.ai)
```

```
##          K          C      Sigma
## 0.06930839 0.11309163 0.13759619
```

```
colMeans(fit.16.12.3.384.lambda3$init.chol)
```

```
##          K          C      Sigma
## 0.06930839 0.11802327 0.13759619
```

```
colMeans(fit.16.12.3.384.lambda3$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.06472019 0.09632108 0.11326857
```

```
colMeans(fit.16.12.3.384.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.0646706 0.1011153 0.1133065
```

```
###lambda=0.8
### n=p/8
fit.16.12.3.24.lambda4=stat.analy(pi.core.fit.16.12.3.24.lambda4)
colMeans(fit.16.12.3.24.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.3010243 0.9384669 0.8660539
```

```r
colMeans(fit.16.12.3.24.lambda4$cse)
```

```
##         K         C     Sigma
## 0.3010243 0.7569643 0.7171823
```

```r
colMeans(fit.16.12.3.24.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.3010243 0.9399115 0.9566858
```

```r
colMeans(fit.16.12.3.24.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.3010243 0.9532175 0.9566858
```

```r
colMeans(fit.16.12.3.24.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2852452 0.9793611 0.9489618
```

```r
colMeans(fit.16.12.3.24.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2850776 0.9833269 0.9387907
```

```r
### n=p/4
fit.16.12.3.48.lambda4=stat.analy(pi.core.fit.16.12.3.48.lambda4)
colMeans(fit.16.12.3.48.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.2013144 0.9384669 0.8639338
```

```r
colMeans(fit.16.12.3.48.lambda4$cse)
```

```
##         K         C     Sigma
## 0.2013144 0.7476402 0.6957686
```

```r
colMeans(fit.16.12.3.48.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.2013144 0.5898736 0.6126776
```

```r
colMeans(fit.16.12.3.48.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.2013144 0.5994770 0.6126776
```

```r
colMeans(fit.16.12.3.48.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1917847 0.5881735 0.5875787
```

```r
colMeans(fit.16.12.3.48.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1914133 0.5945044 0.5874324
```

```r
### n=p/2
fit.16.12.3.96.lambda4=stat.analy(pi.core.fit.16.12.3.96.lambda4)
colMeans(fit.16.12.3.96.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.1420957 0.9384669 0.8638242
```

```r
colMeans(fit.16.12.3.96.lambda4$cse)
```

```
##         K         C     Sigma
## 0.1420957 0.7087827 0.6581291
```

```r
colMeans(fit.16.12.3.96.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.1420957 0.3620333 0.3762874
```

```r
colMeans(fit.16.12.3.96.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.1420957 0.3693803 0.3762874
```

```r
colMeans(fit.16.12.3.96.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1357314 0.3443438 0.3481414
```

```r
colMeans(fit.16.12.3.96.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1356803 0.3502609 0.3477362
```

```r
### n=p
fit.16.12.3.192.lambda4=stat.analy(pi.core.fit.16.12.3.192.lambda4)
colMeans(fit.16.12.3.192.lambda4$kro.mle)
```

```
##          K          C      Sigma
## 0.09968978 0.93846694 0.86388278
```

```r
colMeans(fit.16.12.3.192.lambda4$cse)
```

```
##          K          C      Sigma
## 0.09968978 0.64645344 0.60033943
```

```r
colMeans(fit.16.12.3.192.lambda4$init.ai)
```

```
##          K          C      Sigma
## 0.09968978 0.23815211 0.25087778
```

```r
colMeans(fit.16.12.3.192.lambda4$init.chol)
```

```
##          K          C      Sigma
## 0.09968978 0.24338372 0.25087778
```

```r
colMeans(fit.16.12.3.192.lambda4$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.09549316 0.22591041 0.23512321
```

```r
colMeans(fit.16.12.3.192.lambda4$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.09548656 0.23045193 0.23496703
```

```r
### n=2p
fit.16.12.3.384.lambda4=stat.analy(pi.core.fit.16.12.3.384.lambda4)
colMeans(fit.16.12.3.384.lambda4$kro.mle)
```

```
##          K          C      Sigma
## 0.06860929 0.93846694 0.86352091
```

```r
colMeans(fit.16.12.3.384.lambda4$cse)
```

```
##          K          C      Sigma
## 0.06860929 0.56068935 0.51996021
```

```r
colMeans(fit.16.12.3.384.lambda4$init.ai)
```

```
##          K          C      Sigma
## 0.06860929 0.16246152 0.17191149
```

```r
colMeans(fit.16.12.3.384.lambda4$init.chol)
```

```
##          K          C      Sigma
## 0.06860929 0.16565813 0.17191149
```

```r
colMeans(fit.16.12.3.384.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.0665088 0.1535017 0.1603549
```

```r
colMeans(fit.16.12.3.384.lambda4$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.06648799 0.15661079 0.16031216
```

```r
##### boxplots of the relative spectral norms for K, C, and Sigma across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
### K

K.analy.16.12.3.lambda1=plot.K.analy(fit.16.12.3.24.lambda1,fit.16.12.3.48.lambda1,fit.16.12.3.96.lambda
K.analy.16.12.3.lambda2=plot.K.analy(fit.16.12.3.24.lambda2,fit.16.12.3.48.lambda2,fit.16.12.3.96.lambda
K.analy.16.12.3.lambda3=plot.K.analy(fit.16.12.3.24.lambda3,fit.16.12.3.48.lambda3,fit.16.12.3.96.lambda
K.analy.16.12.3.lambda4=plot.K.analy(fit.16.12.3.24.lambda4,fit.16.12.3.48.lambda4,fit.16.12.3.96.lambda

K.analy.16.12.3=rbind(K.analy.16.12.3.lambda1,K.analy.16.12.3.lambda2,K.analy.16.12.3.lambda3,K.analy.1
K.analy.16.12.3.lambda=factor(K.analy.16.12.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(K.analy.16.12.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX(
K.analy.16.12.3$lambda=K.analy.16.12.3.lambda

png("./numerical.simul_picore/K.16.12.3.png",width=900,height=600)
ggplot(K.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda,la
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Separable Component Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_tex
dev.off()
```

```
## pdf
##   2
```

```r
### C

C.analy.16.12.3.lambda1=plot.C.analy(fit.16.12.3.24.lambda1,fit.16.12.3.48.lambda1,fit.16.12.3.96.lambda
C.analy.16.12.3.lambda2=plot.C.analy(fit.16.12.3.24.lambda2,fit.16.12.3.48.lambda2,fit.16.12.3.96.lambda
C.analy.16.12.3.lambda3=plot.C.analy(fit.16.12.3.24.lambda3,fit.16.12.3.48.lambda3,fit.16.12.3.96.lambda
C.analy.16.12.3.lambda4=plot.C.analy(fit.16.12.3.24.lambda4,fit.16.12.3.48.lambda4,fit.16.12.3.96.lambda

C.analy.16.12.3=rbind(C.analy.16.12.3.lambda1,C.analy.16.12.3.lambda2,C.analy.16.12.3.lambda3,C.analy.1
C.analy.16.12.3.lambda=factor(C.analy.16.12.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(C.analy.16.12.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX(
```

```
C.analy.16.12.3$lambda=C.analy.16.12.3.lambda

png("./numerical.simul_picore/C.16.12.3.png",width=900,height=600)
ggplot(C.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda,la
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Core Component Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_text(hju
dev.off()
```

```
## pdf
##   2
```

```
### Sigma

Sigma.analy.16.12.3.lambda1=plot.Sigma.analy(fit.16.12.3.24.lambda1,fit.16.12.3.48.lambda1,fit.16.12.3.9
Sigma.analy.16.12.3.lambda2=plot.Sigma.analy(fit.16.12.3.24.lambda2,fit.16.12.3.48.lambda2,fit.16.12.3.9
Sigma.analy.16.12.3.lambda3=plot.Sigma.analy(fit.16.12.3.24.lambda3,fit.16.12.3.48.lambda3,fit.16.12.3.9
Sigma.analy.16.12.3.lambda4=plot.Sigma.analy(fit.16.12.3.24.lambda4,fit.16.12.3.48.lambda4,fit.16.12.3.9

Sigma.analy.16.12.3=rbind(Sigma.analy.16.12.3.lambda1,Sigma.analy.16.12.3.lambda2,Sigma.analy.16.12.3.la
Sigma.analy.16.12.3.lambda=factor(Sigma.analy.16.12.3$lambda,levels=c("lambda1","lambda2","lambda3","lam
levels(Sigma.analy.16.12.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),T
Sigma.analy.16.12.3$lambda=Sigma.analy.16.12.3.lambda

png("./numerical.simul_picore/Sigma.16.12.3.png",width=900,height=600)
ggplot(Sigma.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambd
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Sigma Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_text(hjust=0.5))
dev.off()
```

```
## pdf
##   2
```

(p1,p2,r)=(18,8,3)

```
##### mean of the relative spectral norms for $K$, $C$, and $\Sigma$ (in order) across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
###lambda=0.2
### n=p/8
fit.18.8.3.18.lambda1=stat.analy(pi.core.fit.18.8.3.18.lambda1)
colMeans(fit.18.8.3.18.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.4535699 0.9772086 0.9456013
```

```
colMeans(fit.18.8.3.18.lambda1$cse)
```

```
##         K         C     Sigma
## 0.4535699 0.3585475 0.5245121
```

```
colMeans(fit.18.8.3.18.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.4535699 0.2316037 0.4829898
```

```
colMeans(fit.18.8.3.18.lambda1$init.chol)
```

```
##         K         C     Sigma
```

```
## 0.4535699 0.3088223 0.4829898
```

```
colMeans(fit.18.8.3.18.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2968641 0.1961544 0.3054970
```

```
colMeans(fit.18.8.3.18.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2982491 0.2356472 0.3070351
```

```
### n=p/4
fit.18.8.3.36.lambda1=stat.analy(pi.core.fit.18.8.3.36.lambda1)
colMeans(fit.18.8.3.36.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.2962833 0.9772086 0.9448899
```

```
colMeans(fit.18.8.3.36.lambda1$cse)
```

```
##         K         C     Sigma
## 0.2962833 0.3060811 0.4247198
```

```
colMeans(fit.18.8.3.36.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.2962833 0.1589771 0.3683848
```

```
colMeans(fit.18.8.3.36.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.2962833 0.2118604 0.3683848
```

```
colMeans(fit.18.8.3.36.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2148790 0.1272021 0.2199813
```

```
colMeans(fit.18.8.3.36.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2160711 0.1563340 0.2233129
```

```
### n=p/2
fit.18.8.3.72.lambda1=stat.analy(pi.core.fit.18.8.3.72.lambda1)
colMeans(fit.18.8.3.72.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.1976135 0.9772086 0.9459828
```

```
colMeans(fit.18.8.3.72.lambda1$cse)
```

```
##         K         C     Sigma
## 0.1976135 0.2411374 0.3271667
```

```
colMeans(fit.18.8.3.72.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.1976135 0.1087699 0.2513820
```

```
colMeans(fit.18.8.3.72.lambda1$init.chol)
```

```
##         K         C     Sigma
```

```
## 0.1976135 0.1467277 0.2513820
```

```
colMeans(fit.18.8.3.72.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.15050501 0.08669645 0.15583156
```

```
colMeans(fit.18.8.3.72.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1535911 0.1087210 0.1636012
```

```
### n=p
fit.18.8.3.144.lambda1=stat.analy(pi.core.fit.18.8.3.144.lambda1)
colMeans(fit.18.8.3.144.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.1333728 0.9772086 0.9456527
```

```
colMeans(fit.18.8.3.144.lambda1$cse)
```

```
##         K         C     Sigma
## 0.1333728 0.1774281 0.2357276
```

```
colMeans(fit.18.8.3.144.lambda1$init.ai)
```

```
##          K          C      Sigma
## 0.13337278 0.07435535 0.16636929
```

```
colMeans(fit.18.8.3.144.lambda1$init.chol)
```

```
##          K          C      Sigma
## 0.13337278 0.09945355 0.16636929
```

```
colMeans(fit.18.8.3.144.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.10099063 0.05946159 0.10171468
```

```
colMeans(fit.18.8.3.144.lambda1$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.10361894 0.07487122 0.10783875
```

```
### n=2p
fit.18.8.3.288.lambda1=stat.analy(pi.core.fit.18.8.3.288.lambda1)
colMeans(fit.18.8.3.288.lambda1$kro.mle)
```

```
##          K          C      Sigma
## 0.09857484 0.97720864 0.94552034
```

```
colMeans(fit.18.8.3.288.lambda1$cse)
```

```
##          K          C      Sigma
## 0.09857484 0.12622776 0.17482013
```

```
colMeans(fit.18.8.3.288.lambda1$init.ai)
```

```
##          K          C      Sigma
## 0.09857484 0.05290276 0.12174085
```

```
colMeans(fit.18.8.3.288.lambda1$init.chol)
```

```
##          K          C      Sigma
```

```
## 0.09857484 0.07166078 0.12174085
```

```
colMeans(fit.18.8.3.288.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.07336390 0.04213031 0.07469104
```

```
colMeans(fit.18.8.3.288.lambda1$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.07731757 0.05460479 0.08281965
```

```
###lambda=0.4
### n=p/8
fit.18.8.3.18.lambda2=stat.analy(pi.core.fit.18.8.3.18.lambda2)
colMeans(fit.18.8.3.18.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.4229994 0.9698406 0.9283627
```

```
colMeans(fit.18.8.3.18.lambda2$cse)
```

```
##         K         C     Sigma
## 0.4229994 0.5100803 0.5687081
```

```
colMeans(fit.18.8.3.18.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.4229994 0.3422842 0.5117979
```

```
colMeans(fit.18.8.3.18.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.4229994 0.3939295 0.5117979
```

```
colMeans(fit.18.8.3.18.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.3220319 0.3355863 0.3957423
```

```
colMeans(fit.18.8.3.18.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.3233173 0.3684800 0.3985522
```

```
### n=p/4
fit.18.8.3.36.lambda2=stat.analy(pi.core.fit.18.8.3.36.lambda2)
colMeans(fit.18.8.3.36.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.2740226 0.9698406 0.9277144
```

```
colMeans(fit.18.8.3.36.lambda2$cse)
```

```
##         K         C     Sigma
## 0.2740226 0.4798283 0.5087085
```

```
colMeans(fit.18.8.3.36.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.2740226 0.2300408 0.3765423
```

```r
colMeans(fit.18.8.3.36.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.2740226 0.2648668 0.3765423
```

```r
colMeans(fit.18.8.3.36.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2302647 0.2123672 0.2810870
```

```r
colMeans(fit.18.8.3.36.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2301340 0.2373735 0.2812600
```

```r
### n=p/2
fit.18.8.3.72.lambda2=stat.analy(pi.core.fit.18.8.3.72.lambda2)
colMeans(fit.18.8.3.72.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.1851278 0.9698406 0.9286999
```

```r
colMeans(fit.18.8.3.72.lambda2$cse)
```

```
##         K         C     Sigma
## 0.1851278 0.4086897 0.4350283
```

```r
colMeans(fit.18.8.3.72.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.1851278 0.1553816 0.2561868
```

```r
colMeans(fit.18.8.3.72.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.1851278 0.1809308 0.2561868
```

```r
colMeans(fit.18.8.3.72.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1584535 0.1421937 0.1951205
```

```r
colMeans(fit.18.8.3.72.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1583483 0.1603131 0.1952540
```

```r
### n=p
fit.18.8.3.144.lambda2=stat.analy(pi.core.fit.18.8.3.144.lambda2)
colMeans(fit.18.8.3.144.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.1253902 0.9698406 0.9281315
```

```r
colMeans(fit.18.8.3.144.lambda2$cse)
```

```
##         K         C     Sigma
## 0.1253902 0.3239413 0.3440278
```

```r
colMeans(fit.18.8.3.144.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.1253902 0.1065170 0.1697475
```

```r
colMeans(fit.18.8.3.144.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.1253902 0.1236677 0.1697475
```

```r
colMeans(fit.18.8.3.144.lambda2$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.10960111 0.09717525 0.12980031
```

```r
colMeans(fit.18.8.3.144.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1096622 0.1097393 0.1300263
```

```r
### n=2p
fit.18.8.3.288.lambda2=stat.analy(pi.core.fit.18.8.3.288.lambda2)
colMeans(fit.18.8.3.288.lambda2$kro.mle)
```

```
##          K          C      Sigma
## 0.09107433 0.96984064 0.92801293
```

```r
colMeans(fit.18.8.3.288.lambda2$cse)
```

```
##          K          C      Sigma
## 0.09107433 0.24509205 0.26335271
```

```r
colMeans(fit.18.8.3.288.lambda2$init.ai)
```

```
##          K          C      Sigma
## 0.09107433 0.07558191 0.12268984
```

```r
colMeans(fit.18.8.3.288.lambda2$init.chol)
```

```
##          K          C      Sigma
## 0.09107433 0.08832115 0.12268984
```

```r
colMeans(fit.18.8.3.288.lambda2$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.07808453 0.06872589 0.09377958
```

```r
colMeans(fit.18.8.3.288.lambda2$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.07842278 0.07778535 0.09417346
```

```r
###lambda=0.6
### n=p/8
fit.18.8.3.18.lambda3=stat.analy(pi.core.fit.18.8.3.18.lambda3)
colMeans(fit.18.8.3.18.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.4095622 0.9554330 0.8949674
```

```r
colMeans(fit.18.8.3.18.lambda3$cse)
```

```
##         K         C     Sigma
## 0.4095622 0.6398102 0.6321163
```

```r
colMeans(fit.18.8.3.18.lambda3$init.ai)
```

```
##         K         C     Sigma
```

```
## 0.4095622 0.5411593 0.6243103
```

```
colMeans(fit.18.8.3.18.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.4095622 0.5731826 0.6243103
```

```
colMeans(fit.18.8.3.18.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.3524875 0.5676037 0.5807449
```

```
colMeans(fit.18.8.3.18.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.3557469 0.5854364 0.5754777
```

```
### n=p/4
fit.18.8.3.36.lambda3=stat.analy(pi.core.fit.18.8.3.36.lambda3)
colMeans(fit.18.8.3.36.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.2686489 0.9554330 0.8943091
```

```
colMeans(fit.18.8.3.36.lambda3$cse)
```

```
##         K         C     Sigma
## 0.2686489 0.6310991 0.6064512
```

```
colMeans(fit.18.8.3.36.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.2686489 0.3402006 0.4241858
```

```
colMeans(fit.18.8.3.36.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.2686489 0.3645264 0.4241858
```

```
colMeans(fit.18.8.3.36.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2424089 0.3354218 0.3769658
```

```
colMeans(fit.18.8.3.36.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2424973 0.3555934 0.3769794
```

```
### n=p/2
fit.18.8.3.72.lambda3=stat.analy(pi.core.fit.18.8.3.72.lambda3)
colMeans(fit.18.8.3.72.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.1800082 0.9554330 0.8952053
```

```
colMeans(fit.18.8.3.72.lambda3$cse)
```

```
##         K         C     Sigma
## 0.1800082 0.5690311 0.5510604
```

```
colMeans(fit.18.8.3.72.lambda3$init.ai)
```

```
##         K         C     Sigma
```

```
## 0.1800082 0.2226079 0.2816749
```

```
colMeans(fit.18.8.3.72.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.1800082 0.2404252 0.2816749
```

```
colMeans(fit.18.8.3.72.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1638078 0.2155079 0.2470155
```

```
colMeans(fit.18.8.3.72.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1638077 0.2301732 0.2469917
```

```
### n=p
fit.18.8.3.144.lambda3=stat.analy(pi.core.fit.18.8.3.144.lambda3)
colMeans(fit.18.8.3.144.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.1236703 0.9554330 0.8941865
```

```
colMeans(fit.18.8.3.144.lambda3$cse)
```

```
##         K         C     Sigma
## 0.1236703 0.4848014 0.4680974
```

```
colMeans(fit.18.8.3.144.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.1236703 0.1508806 0.1870749
```

```
colMeans(fit.18.8.3.144.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.1236703 0.1631288 0.1870749
```

```
colMeans(fit.18.8.3.144.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1154549 0.1451078 0.1667954
```

```
colMeans(fit.18.8.3.144.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1154154 0.1554376 0.1667262
```

```
### n=2p
fit.18.8.3.288.lambda3=stat.analy(pi.core.fit.18.8.3.288.lambda3)
colMeans(fit.18.8.3.288.lambda3$kro.mle)
```

```
##          K          C      Sigma
## 0.08861275 0.95543302 0.89412493
```

```
colMeans(fit.18.8.3.288.lambda3$cse)
```

```
##          K          C      Sigma
## 0.08861275 0.39431049 0.38151153
```

```
colMeans(fit.18.8.3.288.lambda3$init.ai)
```

```
##          K          C      Sigma
```

```
## 0.08861275 0.10575868 0.13332895
```

```
colMeans(fit.18.8.3.288.lambda3$init.chol)
```

```
##          K          C      Sigma
## 0.08861275 0.11473346 0.13332895
```

```
colMeans(fit.18.8.3.288.lambda3$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.08055589 0.10181545 0.11948627
```

```
colMeans(fit.18.8.3.288.lambda3$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.08055097 0.10913713 0.11943916
```

```
###lambda=0.8
### n=p/8
fit.18.8.3.18.lambda4=stat.analy(pi.core.fit.18.8.3.18.lambda4)
colMeans(fit.18.8.3.18.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.4039592 0.9146690 0.8046165
```

```
colMeans(fit.18.8.3.18.lambda4$cse)
```

```
##         K         C     Sigma
## 0.4039592 0.7453415 0.6714559
```

```
colMeans(fit.18.8.3.18.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.4039592 1.1090417 1.0651170
```

```
colMeans(fit.18.8.3.18.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.4039592 1.1205895 1.0651170
```

```
colMeans(fit.18.8.3.18.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.3740523 1.1384606 1.0194404
```

```
colMeans(fit.18.8.3.18.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.3765526 1.1374090 1.0186328
```

```
### n=p/4
fit.18.8.3.36.lambda4=stat.analy(pi.core.fit.18.8.3.36.lambda4)
colMeans(fit.18.8.3.36.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.2678410 0.9146690 0.8029721
```

```
colMeans(fit.18.8.3.36.lambda4$cse)
```

```
##         K         C     Sigma
## 0.2678410 0.7535399 0.6658741
```

```r
colMeans(fit.18.8.3.36.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.2678410 0.6470325 0.6372995
```

```r
colMeans(fit.18.8.3.36.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.2678410 0.6603274 0.6372995
```

```r
colMeans(fit.18.8.3.36.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2558827 0.6419838 0.6126136
```

```r
colMeans(fit.18.8.3.36.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2558363 0.6563310 0.6121150
```

```r
### n=p/2
fit.18.8.3.72.lambda4=stat.analy(pi.core.fit.18.8.3.72.lambda4)
colMeans(fit.18.8.3.72.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.1769366 0.9146690 0.8033119
```

```r
colMeans(fit.18.8.3.72.lambda4$cse)
```

```
##         K         C     Sigma
## 0.1769366 0.7073533 0.6296845
```

```r
colMeans(fit.18.8.3.72.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.1769366 0.3881658 0.3805409
```

```r
colMeans(fit.18.8.3.72.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.1769366 0.3988277 0.3805409
```

```r
colMeans(fit.18.8.3.72.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1689755 0.3795337 0.3607000
```

```r
colMeans(fit.18.8.3.72.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1689807 0.3870873 0.3592558
```

```r
### n=p
fit.18.8.3.144.lambda4=stat.analy(pi.core.fit.18.8.3.144.lambda4)
colMeans(fit.18.8.3.144.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.1241679 0.9146690 0.8009681
```

```r
colMeans(fit.18.8.3.144.lambda4$cse)
```

```
##         K         C     Sigma
## 0.1241679 0.6379728 0.5647694
```

```
colMeans(fit.18.8.3.144.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.1241679 0.2451775 0.2455398
```

```
colMeans(fit.18.8.3.144.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.1241679 0.2529320 0.2455398
```

```
colMeans(fit.18.8.3.144.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1207516 0.2394517 0.2353054
```

```
colMeans(fit.18.8.3.144.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1207037 0.2459291 0.2352128
```

```
### n=2p
fit.18.8.3.288.lambda4=stat.analy(pi.core.fit.18.8.3.288.lambda4)
colMeans(fit.18.8.3.288.lambda4$kro.mle)
```

```
##          K          C      Sigma
## 0.08732347 0.91466899 0.80096834
```

```
colMeans(fit.18.8.3.288.lambda4$cse)
```

```
##          K          C      Sigma
## 0.08732347 0.55502785 0.49220974
```

```
colMeans(fit.18.8.3.288.lambda4$init.ai)
```

```
##          K          C      Sigma
## 0.08732347 0.16657691 0.16970641
```

```
colMeans(fit.18.8.3.288.lambda4$init.chol)
```

```
##          K          C      Sigma
## 0.08732347 0.17180408 0.16970641
```

```
colMeans(fit.18.8.3.288.lambda4$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.08338246 0.16302269 0.16471119
```

```
colMeans(fit.18.8.3.288.lambda4$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.08337211 0.16778033 0.16467443
```

```
##### boxplots of the relative spectral norms for K, C, and Sigma across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
### K

K.analy.18.8.3.lambda1=plot.K.analy(fit.18.8.3.18.lambda1,fit.18.8.3.36.lambda1,fit.18.8.3.72.lambda1,fi
K.analy.18.8.3.lambda2=plot.K.analy(fit.18.8.3.18.lambda2,fit.18.8.3.36.lambda2,fit.18.8.3.72.lambda2,fi
K.analy.18.8.3.lambda3=plot.K.analy(fit.18.8.3.18.lambda3,fit.18.8.3.36.lambda3,fit.18.8.3.72.lambda3,fi
K.analy.18.8.3.lambda4=plot.K.analy(fit.18.8.3.18.lambda4,fit.18.8.3.36.lambda4,fit.18.8.3.72.lambda4,fi

K.analy.18.8.3=rbind(K.analy.18.8.3.lambda1,K.analy.18.8.3.lambda2,K.analy.18.8.3.lambda3,K.analy.18.8.3
```

```
K.analy.18.8.3.lambda=factor(K.analy.18.8.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(K.analy.18.8.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$
K.analy.18.8.3$lambda=K.analy.18.8.3.lambda

png("./numerical.simul_picore/K.18.8.3.png",width=900,height=600)
ggplot(K.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda,lab
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Separable Component Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text
dev.off()
```

```
## pdf
##   2
```

```
### C

C.analy.18.8.3.lambda1=plot.C.analy(fit.18.8.3.18.lambda1,fit.18.8.3.36.lambda1,fit.18.8.3.72.lambda1,fi
C.analy.18.8.3.lambda2=plot.C.analy(fit.18.8.3.18.lambda2,fit.18.8.3.36.lambda2,fit.18.8.3.72.lambda2,fi
C.analy.18.8.3.lambda3=plot.C.analy(fit.18.8.3.18.lambda3,fit.18.8.3.36.lambda3,fit.18.8.3.72.lambda3,fi
C.analy.18.8.3.lambda4=plot.C.analy(fit.18.8.3.18.lambda4,fit.18.8.3.36.lambda4,fit.18.8.3.72.lambda4,fi

C.analy.18.8.3=rbind(C.analy.18.8.3.lambda1,C.analy.18.8.3.lambda2,C.analy.18.8.3.lambda3,C.analy.18.8.3
C.analy.18.8.3.lambda=factor(C.analy.18.8.3$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(C.analy.18.8.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$
C.analy.18.8.3$lambda=C.analy.18.8.3.lambda

png("./numerical.simul_picore/C.18.8.3.png",width=900,height=600)
ggplot(C.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda,lab
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Core Component Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text(hjus
dev.off()
```

```
## pdf
##   2
```

```
### Sigma

Sigma.analy.18.8.3.lambda1=plot.Sigma.analy(fit.18.8.3.18.lambda1,fit.18.8.3.36.lambda1,fit.18.8.3.72.la
Sigma.analy.18.8.3.lambda2=plot.Sigma.analy(fit.18.8.3.18.lambda2,fit.18.8.3.36.lambda2,fit.18.8.3.72.la
Sigma.analy.18.8.3.lambda3=plot.Sigma.analy(fit.18.8.3.18.lambda3,fit.18.8.3.36.lambda3,fit.18.8.3.72.la
Sigma.analy.18.8.3.lambda4=plot.Sigma.analy(fit.18.8.3.18.lambda4,fit.18.8.3.36.lambda4,fit.18.8.3.72.la

Sigma.analy.18.8.3=rbind(Sigma.analy.18.8.3.lambda1,Sigma.analy.18.8.3.lambda2,Sigma.analy.18.8.3.lambda
Sigma.analy.18.8.3.lambda=factor(Sigma.analy.18.8.3$lambda,levels=c("lambda1","lambda2","lambda3","lambd
levels(Sigma.analy.18.8.3.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),Te
Sigma.analy.18.8.3$lambda=Sigma.analy.18.8.3.lambda

png("./numerical.simul_picore/Sigma.18.8.3.png",width=900,height=600)
ggplot(Sigma.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Sigma Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text(hjust=0.5))
dev.off()
```

```
## pdf
##   2
```

(p1,p2,r)=(16,12,5)

```
##### mean of the relative spectral norms for $K$, $C$, and $\Sigma$ (in order) across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
###lambda=0.2
### n=p/8
fit.16.12.5.24.lambda1=stat.analy(pi.core.fit.16.12.5.24.lambda1)
colMeans(fit.16.12.5.24.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.3091739 0.9734962 0.9362426
```

```
colMeans(fit.16.12.5.24.lambda1$cse)
```

```
##         K         C     Sigma
## 0.3091739 0.4955638 0.5913930
```

```
colMeans(fit.16.12.5.24.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.3091739 0.5149124 0.6519917
```

```
colMeans(fit.16.12.5.24.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.3091739 0.5418039 0.6519917
```

```
colMeans(fit.16.12.5.24.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2331178 0.3250231 0.3775793
```

```
colMeans(fit.16.12.5.24.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2358000 0.3421524 0.3847671
```

```
### n=p/4
fit.16.12.5.48.lambda1=stat.analy(pi.core.fit.16.12.5.48.lambda1)
colMeans(fit.16.12.5.48.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.2252796 0.9734962 0.9342197
```

```
colMeans(fit.16.12.5.48.lambda1$cse)
```

```
##         K         C     Sigma
## 0.2252796 0.4054231 0.4549263
```

```
colMeans(fit.16.12.5.48.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.2252796 0.3602018 0.4842630
```

```
colMeans(fit.16.12.5.48.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.2252796 0.3771686 0.4842630
```

```
colMeans(fit.16.12.5.48.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
```

```
## 0.1641200 0.2111237 0.2681259
```

```
colMeans(fit.16.12.5.48.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1675930 0.2296375 0.2823582
```

```
### n=p/2
fit.16.12.5.96.lambda1=stat.analy(pi.core.fit.16.12.5.96.lambda1)
colMeans(fit.16.12.5.96.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.1491571 0.9734962 0.9354857
```

```
colMeans(fit.16.12.5.96.lambda1$cse)
```

```
##         K         C     Sigma
## 0.1491571 0.3230452 0.3593768
```

```
colMeans(fit.16.12.5.96.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.1491571 0.2536165 0.3221161
```

```
colMeans(fit.16.12.5.96.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.1491571 0.2641958 0.3221161
```

```
colMeans(fit.16.12.5.96.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1123478 0.1490446 0.1935808
```

```
colMeans(fit.16.12.5.96.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1175731 0.1676212 0.2084122
```

```
### n=p
fit.16.12.5.192.lambda1=stat.analy(pi.core.fit.16.12.5.192.lambda1)
colMeans(fit.16.12.5.192.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.1029342 0.9734962 0.9353505
```

```
colMeans(fit.16.12.5.192.lambda1$cse)
```

```
##         K         C     Sigma
## 0.1029342 0.2415532 0.2670691
```

```
colMeans(fit.16.12.5.192.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.1029342 0.1737256 0.2226171
```

```
colMeans(fit.16.12.5.192.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.1029342 0.1809073 0.2226171
```

```
colMeans(fit.16.12.5.192.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
```

```
## 0.08249349 0.10584354 0.13792959
```

```
colMeans(fit.16.12.5.192.lambda1$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.08800202 0.12327021 0.15449087
```

```
### n=2p
fit.16.12.5.384.lambda1=stat.analy(pi.core.fit.16.12.5.384.lambda1)
colMeans(fit.16.12.5.384.lambda1$kro.mle)
```

```
##          K          C      Sigma
## 0.07188587 0.97349621 0.93538779
```

```
colMeans(fit.16.12.5.384.lambda1$cse)
```

```
##          K          C      Sigma
## 0.07188587 0.17188504 0.19035342
```

```
colMeans(fit.16.12.5.384.lambda1$init.ai)
```

```
##          K          C      Sigma
## 0.07188587 0.12104576 0.15762287
```

```
colMeans(fit.16.12.5.384.lambda1$init.chol)
```

```
##          K          C      Sigma
## 0.07188587 0.12620822 0.15762287
```

```
colMeans(fit.16.12.5.384.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.06000010 0.08294023 0.11086012
```

```
colMeans(fit.16.12.5.384.lambda1$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.06563495 0.09925281 0.12734464
```

```
###lambda=0.4
### n=p/8
fit.16.12.5.24.lambda2=stat.analy(pi.core.fit.16.12.5.24.lambda2)
colMeans(fit.16.12.5.24.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.3029550 0.9649711 0.9158266
```

```
colMeans(fit.16.12.5.24.lambda2$cse)
```

```
##         K         C     Sigma
## 0.3029550 0.6003698 0.6113231
```

```
colMeans(fit.16.12.5.24.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.3029550 0.6377679 0.7212338
```

```
colMeans(fit.16.12.5.24.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.3029550 0.6612437 0.7212338
```

```
colMeans(fit.16.12.5.24.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2579420 0.5776648 0.5714714
```

```
colMeans(fit.16.12.5.24.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2548037 0.5796871 0.5549188
```

```
### n=p/4
fit.16.12.5.48.lambda2=stat.analy(pi.core.fit.16.12.5.48.lambda2)
colMeans(fit.16.12.5.48.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.2132995 0.9649711 0.9135912
```

```
colMeans(fit.16.12.5.48.lambda2$cse)
```

```
##         K         C     Sigma
## 0.2132995 0.5297800 0.5266318
```

```
colMeans(fit.16.12.5.48.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.2132995 0.4326629 0.5147321
```

```
colMeans(fit.16.12.5.48.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.2132995 0.4472773 0.5147321
```

```
colMeans(fit.16.12.5.48.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1781782 0.3380584 0.3633912
```

```
colMeans(fit.16.12.5.48.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1777204 0.3435558 0.3592819
```

```
### n=p/2
fit.16.12.5.96.lambda2=stat.analy(pi.core.fit.16.12.5.96.lambda2)
colMeans(fit.16.12.5.96.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.1445895 0.9649711 0.9148267
```

```
colMeans(fit.16.12.5.96.lambda2$cse)
```

```
##         K         C     Sigma
## 0.1445895 0.4522669 0.4557357
```

```
colMeans(fit.16.12.5.96.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.1445895 0.2952019 0.3403058
```

```
colMeans(fit.16.12.5.96.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.1445895 0.3040244 0.3403058
```

```r
colMeans(fit.16.12.5.96.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1206214 0.2170909 0.2440099
```

```r
colMeans(fit.16.12.5.96.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1213462 0.2250259 0.2451298
```

```r
### n=p
fit.16.12.5.192.lambda2=stat.analy(pi.core.fit.16.12.5.192.lambda2)
colMeans(fit.16.12.5.192.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.1015111 0.9649711 0.9145897
```

```r
colMeans(fit.16.12.5.192.lambda2$cse)
```

```
##         K         C     Sigma
## 0.1015111 0.3622414 0.3621516
```

```r
colMeans(fit.16.12.5.192.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.1015111 0.2016169 0.2356860
```

```r
colMeans(fit.16.12.5.192.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.1015111 0.2075604 0.2356860
```

```r
colMeans(fit.16.12.5.192.lambda2$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.08723428 0.14822368 0.16625071
```

```r
colMeans(fit.16.12.5.192.lambda2$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.08790297 0.15386775 0.16752224
```

```r
### n=2p
fit.16.12.5.384.lambda2=stat.analy(pi.core.fit.16.12.5.384.lambda2)
colMeans(fit.16.12.5.384.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.0706898 0.9649711 0.9147048
```

```r
colMeans(fit.16.12.5.384.lambda2$cse)
```

```
##         K         C     Sigma
## 0.0706898 0.2710376 0.2729559
```

```r
colMeans(fit.16.12.5.384.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.0706898 0.1413166 0.1662560
```

```r
colMeans(fit.16.12.5.384.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.0706898 0.1456214 0.1662560
```

```r
colMeans(fit.16.12.5.384.lambda2$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.06015923 0.10260472 0.11731223
```

```r
colMeans(fit.16.12.5.384.lambda2$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.06084969 0.10685352 0.11934052
```

```r
###lambda=0.6
### n=p/8
fit.16.12.5.24.lambda3=stat.analy(pi.core.fit.16.12.5.24.lambda3)
colMeans(fit.16.12.5.24.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.2997765 0.9483611 0.8764817
```

```r
colMeans(fit.16.12.5.24.lambda3$cse)
```

```
##         K         C     Sigma
## 0.2997765 0.6955758 0.6633756
```

```r
colMeans(fit.16.12.5.24.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.2997765 0.8661288 0.8868167
```

```r
colMeans(fit.16.12.5.24.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.2997765 0.8842904 0.8868167
```

```r
colMeans(fit.16.12.5.24.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2787755 0.9924472 0.9483571
```

```r
colMeans(fit.16.12.5.24.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2778115 1.0143933 0.9707471
```

```r
### n=p/4
fit.16.12.5.48.lambda3=stat.analy(pi.core.fit.16.12.5.48.lambda3)
colMeans(fit.16.12.5.48.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.2057594 0.9483611 0.8738484
```

```r
colMeans(fit.16.12.5.48.lambda3$cse)
```

```
##         K         C     Sigma
## 0.2057594 0.6483876 0.6075207
```

```r
colMeans(fit.16.12.5.48.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.2057594 0.5597434 0.5900638
```

```r
colMeans(fit.16.12.5.48.lambda3$init.chol)
```

```
##         K         C     Sigma
```

```
## 0.2057594 0.5715882 0.5900638
```

```
colMeans(fit.16.12.5.48.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1883634 0.5520283 0.5378725
```

```
colMeans(fit.16.12.5.48.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1878954 0.5558266 0.5332654
```

```
### n=p/2
fit.16.12.5.96.lambda3=stat.analy(pi.core.fit.16.12.5.96.lambda3)
colMeans(fit.16.12.5.96.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.1423404 0.9483611 0.8749452
```

```
colMeans(fit.16.12.5.96.lambda3$cse)
```

```
##         K         C     Sigma
## 0.1423404 0.5826208 0.5525694
```

```
colMeans(fit.16.12.5.96.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.1423404 0.3664189 0.3832459
```

```
colMeans(fit.16.12.5.96.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.1423404 0.3736224 0.3832459
```

```
colMeans(fit.16.12.5.96.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1281699 0.3283758 0.3297969
```

```
colMeans(fit.16.12.5.96.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1286290 0.3353292 0.3314896
```

```
### n=p
fit.16.12.5.192.lambda3=stat.analy(pi.core.fit.16.12.5.192.lambda3)
colMeans(fit.16.12.5.192.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.1011636 0.9483611 0.8744600
```

```
colMeans(fit.16.12.5.192.lambda3$cse)
```

```
##         K         C     Sigma
## 0.1011636 0.4991198 0.4673402
```

```
colMeans(fit.16.12.5.192.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.1011636 0.2475918 0.2622432
```

```
colMeans(fit.16.12.5.192.lambda3$init.chol)
```

```
##         K         C     Sigma
```

```
## 0.1011636 0.2523591 0.2622432
```

```
colMeans(fit.16.12.5.192.lambda3$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.09192584 0.21593685 0.22113141
```

```
colMeans(fit.16.12.5.192.lambda3$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.09202567 0.22023841 0.22084636
```

```
### n=2p
fit.16.12.5.384.lambda3=stat.analy(pi.core.fit.16.12.5.384.lambda3)
colMeans(fit.16.12.5.384.lambda3$kro.mle)
```

```
##          K          C      Sigma
## 0.06983756 0.94836106 0.87468588
```

```
colMeans(fit.16.12.5.384.lambda3$cse)
```

```
##          K          C      Sigma
## 0.06983756 0.40144327 0.37863215
```

```
colMeans(fit.16.12.5.384.lambda3$init.ai)
```

```
##          K          C      Sigma
## 0.06983756 0.17283071 0.18395181
```

```
colMeans(fit.16.12.5.384.lambda3$init.chol)
```

```
##          K          C      Sigma
## 0.06983756 0.17623268 0.18395181
```

```
colMeans(fit.16.12.5.384.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.0635183 0.1482818 0.1544959
```

```
colMeans(fit.16.12.5.384.lambda3$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.06350798 0.15098481 0.15435301
```

```
###lambda=0.8
### n=p/8
fit.16.12.5.24.lambda4=stat.analy(pi.core.fit.16.12.5.24.lambda4)
colMeans(fit.16.12.5.24.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.2995362 0.9017934 0.7693394
```

```
colMeans(fit.16.12.5.24.lambda4$cse)
```

```
##         K         C     Sigma
## 0.2995362 0.7716264 0.6715925
```

```
colMeans(fit.16.12.5.24.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.2995362 1.4598665 1.3494509
```

```
colMeans(fit.16.12.5.24.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.2995362 1.4704381 1.3494509
```

```
colMeans(fit.16.12.5.24.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2863311 1.4983850 1.3087723
```

```
colMeans(fit.16.12.5.24.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2869906 1.5451894 1.3562507
```

```
### n=p/4
fit.16.12.5.48.lambda4=stat.analy(pi.core.fit.16.12.5.48.lambda4)
colMeans(fit.16.12.5.48.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.2008463 0.9017934 0.7651468
```

```
colMeans(fit.16.12.5.48.lambda4$cse)
```

```
##         K         C     Sigma
## 0.2008463 0.7531600 0.6418491
```

```
colMeans(fit.16.12.5.48.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.2008463 0.8935841 0.8263429
```

```
colMeans(fit.16.12.5.48.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.2008463 0.9006421 0.8263429
```

```
colMeans(fit.16.12.5.48.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1968527 1.0132363 0.9008423
```

```
colMeans(fit.16.12.5.48.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1976936 1.0394686 0.9211055
```

```
### n=p/2
fit.16.12.5.96.lambda4=stat.analy(pi.core.fit.16.12.5.96.lambda4)
colMeans(fit.16.12.5.96.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.1405082 0.9017934 0.7654498
```

```
colMeans(fit.16.12.5.96.lambda4$cse)
```

```
##         K         C     Sigma
## 0.1405082 0.7028001 0.6049841
```

```
colMeans(fit.16.12.5.96.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.1405082 0.5518102 0.5055872
```

```
colMeans(fit.16.12.5.96.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.1405082 0.5564456 0.5055872
```

```
colMeans(fit.16.12.5.96.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1354613 0.5803303 0.5138790
```

```
colMeans(fit.16.12.5.96.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1354233 0.5813312 0.5107205
```

```
### n=p
fit.16.12.5.192.lambda4=stat.analy(pi.core.fit.16.12.5.192.lambda4)
colMeans(fit.16.12.5.192.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.1007144 0.9017934 0.7641495
```

```
colMeans(fit.16.12.5.192.lambda4$cse)
```

```
##         K         C     Sigma
## 0.1007144 0.6372794 0.5407862
```

```
colMeans(fit.16.12.5.192.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.1007144 0.3589174 0.3365885
```

```
colMeans(fit.16.12.5.192.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.1007144 0.3623157 0.3365885
```

```
colMeans(fit.16.12.5.192.lambda4$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.09602196 0.35677069 0.32397714
```

```
colMeans(fit.16.12.5.192.lambda4$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.09604628 0.35943771 0.32366672
```

```
### n=2p
fit.16.12.5.384.lambda4=stat.analy(pi.core.fit.16.12.5.384.lambda4)
colMeans(fit.16.12.5.384.lambda4$kro.mle)
```

```
##          K          C      Sigma
## 0.06894272 0.90179340 0.76449448
```

```
colMeans(fit.16.12.5.384.lambda4$cse)
```

```
##          K          C      Sigma
## 0.06894272 0.55022078 0.47281682
```

```
colMeans(fit.16.12.5.384.lambda4$init.ai)
```

```
##          K          C      Sigma
## 0.06894272 0.24302046 0.22641851
```

```r
colMeans(fit.16.12.5.384.lambda4$init.chol)
```

```
##          K          C      Sigma
## 0.06894272 0.24519274 0.22641851
```

```r
colMeans(fit.16.12.5.384.lambda4$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.06641907 0.23567894 0.21726926
```

```r
colMeans(fit.16.12.5.384.lambda4$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.06636957 0.23766972 0.21705889
```

```r
##### boxplots of the relative spectral norms for K, C, and Sigma across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
### K

K.analy.16.12.5.lambda1=plot.K.analy(fit.16.12.5.24.lambda1,fit.16.12.5.48.lambda1,fit.16.12.5.96.lambda
K.analy.16.12.5.lambda2=plot.K.analy(fit.16.12.5.24.lambda2,fit.16.12.5.48.lambda2,fit.16.12.5.96.lambda
K.analy.16.12.5.lambda3=plot.K.analy(fit.16.12.5.24.lambda3,fit.16.12.5.48.lambda3,fit.16.12.5.96.lambda
K.analy.16.12.5.lambda4=plot.K.analy(fit.16.12.5.24.lambda4,fit.16.12.5.48.lambda4,fit.16.12.5.96.lambda

K.analy.16.12.5=rbind(K.analy.16.12.5.lambda1,K.analy.16.12.5.lambda2,K.analy.16.12.5.lambda3,K.analy.1
K.analy.16.12.5.lambda=factor(K.analy.16.12.5$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(K.analy.16.12.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX(
K.analy.16.12.5$lambda=K.analy.16.12.5.lambda

png("./numerical.simul_picore/K.16.12.5.png",width=900,height=600)
ggplot(K.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda,la
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Separable Component Consistency : $(p_1,p_2,r)=(16,12,5)$"))+theme(plot.title=element_tex
dev.off()
```

```
## pdf
##   2
```

```r
### C

C.analy.16.12.5.lambda1=plot.C.analy(fit.16.12.5.24.lambda1,fit.16.12.5.48.lambda1,fit.16.12.5.96.lambda
C.analy.16.12.5.lambda2=plot.C.analy(fit.16.12.5.24.lambda2,fit.16.12.5.48.lambda2,fit.16.12.5.96.lambda
C.analy.16.12.5.lambda3=plot.C.analy(fit.16.12.5.24.lambda3,fit.16.12.5.48.lambda3,fit.16.12.5.96.lambda
C.analy.16.12.5.lambda4=plot.C.analy(fit.16.12.5.24.lambda4,fit.16.12.5.48.lambda4,fit.16.12.5.96.lambda

C.analy.16.12.5=rbind(C.analy.16.12.5.lambda1,C.analy.16.12.5.lambda2,C.analy.16.12.5.lambda3,C.analy.1
C.analy.16.12.5.lambda=factor(C.analy.16.12.5$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(C.analy.16.12.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX(
C.analy.16.12.5$lambda=C.analy.16.12.5.lambda

png("./numerical.simul_picore/C.16.12.5.png",width=900,height=600)
ggplot(C.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda,la
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Core Component Consistency : $(p_1,p_2,r)=(16,12,5)$"))+theme(plot.title=element_text(hju
dev.off()
```

```
## pdf
```

```
##   2
```

```
### Sigma

Sigma.analy.16.12.5.lambda1=plot.Sigma.analy(fit.16.12.5.24.lambda1,fit.16.12.5.48.lambda1,fit.16.12.5.9
Sigma.analy.16.12.5.lambda2=plot.Sigma.analy(fit.16.12.5.24.lambda2,fit.16.12.5.48.lambda2,fit.16.12.5.9
Sigma.analy.16.12.5.lambda3=plot.Sigma.analy(fit.16.12.5.24.lambda3,fit.16.12.5.48.lambda3,fit.16.12.5.9
Sigma.analy.16.12.5.lambda4=plot.Sigma.analy(fit.16.12.5.24.lambda4,fit.16.12.5.48.lambda4,fit.16.12.5.9

Sigma.analy.16.12.5=rbind(Sigma.analy.16.12.5.lambda1,Sigma.analy.16.12.5.lambda2,Sigma.analy.16.12.5.la
Sigma.analy.16.12.5.lambda=factor(Sigma.analy.16.12.5$lambda,levels=c("lambda1","lambda2","lambda3","lam
levels(Sigma.analy.16.12.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),T
Sigma.analy.16.12.5$lambda=Sigma.analy.16.12.5.lambda

png("./numerical.simul_picore/Sigma.16.12.5.png",width=900,height=600)
ggplot(Sigma.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambd
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Sigma Consistency : $(p_1,p_2,r)=(16,12,5)$"))+theme(plot.title=element_text(hjust=0.5))
dev.off()
```

```
## pdf
##   2
```

(p1,p2,r)=(18,8,5)

```
##### mean of the relative spectral norms for $K$, $C$, and $\Sigma$ (in order) across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
###lambda=0.2
### n=p/8
fit.18.8.5.18.lambda1=stat.analy(pi.core.fit.18.8.5.18.lambda1)
colMeans(fit.18.8.5.18.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.4172801 0.9638411 0.9100496
```

```
colMeans(fit.18.8.5.18.lambda1$cse)
```

```
##         K         C     Sigma
## 0.4172801 0.5393024 0.6905302
```

```
colMeans(fit.18.8.5.18.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.4172801 0.5109531 0.7487002
```

```
colMeans(fit.18.8.5.18.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.4172801 0.5540172 0.7487002
```

```
colMeans(fit.18.8.5.18.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2975550 0.3867494 0.4532416
```

```
colMeans(fit.18.8.5.18.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
```

```
## 0.2992438 0.4056617 0.4573998
```

```
### n=p/4
fit.18.8.5.36.lambda1=stat.analy(pi.core.fit.18.8.5.36.lambda1)
colMeans(fit.18.8.5.36.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.2775269 0.9638411 0.9076400
```

```
colMeans(fit.18.8.5.36.lambda1$cse)
```

```
##         K         C     Sigma
## 0.2775269 0.4309824 0.5081950
```

```
colMeans(fit.18.8.5.36.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.2775269 0.3537759 0.5239512
```

```
colMeans(fit.18.8.5.36.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.2775269 0.3826955 0.5239512
```

```
colMeans(fit.18.8.5.36.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2079164 0.2319186 0.3111689
```

```
colMeans(fit.18.8.5.36.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2089332 0.2490611 0.3142217
```

```
### n=p/2
fit.18.8.5.72.lambda1=stat.analy(pi.core.fit.18.8.5.72.lambda1)
colMeans(fit.18.8.5.72.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.1834581 0.9638411 0.9090289
```

```
colMeans(fit.18.8.5.72.lambda1$cse)
```

```
##         K         C     Sigma
## 0.1834581 0.3376902 0.3890844
```

```
colMeans(fit.18.8.5.72.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.1834581 0.2510779 0.3626156
```

```
colMeans(fit.18.8.5.72.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.1834581 0.2714186 0.3626156
```

```
colMeans(fit.18.8.5.72.lambda1$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1357514 0.1543235 0.2109915
```

```
colMeans(fit.18.8.5.72.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
```

```
## 0.1386133 0.1728470 0.2200965
```

```
### n=p
fit.18.8.5.144.lambda1=stat.analy(pi.core.fit.18.8.5.144.lambda1)
colMeans(fit.18.8.5.144.lambda1$kro.mle)
```

```
##         K         C     Sigma
## 0.1299825 0.9638411 0.9071619
```

```
colMeans(fit.18.8.5.144.lambda1$cse)
```

```
##         K         C     Sigma
## 0.1299825 0.2437429 0.2845580
```

```
colMeans(fit.18.8.5.144.lambda1$init.ai)
```

```
##         K         C     Sigma
## 0.1299825 0.1716670 0.2529815
```

```
colMeans(fit.18.8.5.144.lambda1$init.chol)
```

```
##         K         C     Sigma
## 0.1299825 0.1847605 0.2529815
```

```
colMeans(fit.18.8.5.144.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.09781666 0.10547038 0.14785439
```

```
colMeans(fit.18.8.5.144.lambda1$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1017532 0.1214730 0.1594581
```

```
### n=2p
fit.18.8.5.288.lambda1=stat.analy(pi.core.fit.18.8.5.288.lambda1)
colMeans(fit.18.8.5.288.lambda1$kro.mle)
```

```
##          K          C      Sigma
## 0.09132618 0.96384111 0.90755468
```

```
colMeans(fit.18.8.5.288.lambda1$cse)
```

```
##          K          C      Sigma
## 0.09132618 0.17700234 0.21019075
```

```
colMeans(fit.18.8.5.288.lambda1$init.ai)
```

```
##          K          C      Sigma
## 0.09132618 0.12225269 0.17685358
```

```
colMeans(fit.18.8.5.288.lambda1$init.chol)
```

```
##          K          C      Sigma
## 0.09132618 0.13202447 0.17685358
```

```
colMeans(fit.18.8.5.288.lambda1$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.07072603 0.07737039 0.10961293
```

```
colMeans(fit.18.8.5.288.lambda1$fit.chol.analy)
```

```
##          K          C      Sigma
```

```
## 0.07584646 0.09222807 0.12287848
```

```
###lambda=0.4
### n=p/8
fit.18.8.5.18.lambda2=stat.analy(pi.core.fit.18.8.5.18.lambda2)
colMeans(fit.18.8.5.18.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.4127019 0.9523623 0.8821432
```

```
colMeans(fit.18.8.5.18.lambda2$cse)
```

```
##         K         C     Sigma
## 0.4127019 0.6406200 0.6837192
```

```
colMeans(fit.18.8.5.18.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.4127019 0.6609726 0.8157232
```

```
colMeans(fit.18.8.5.18.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.4127019 0.6968389 0.8157232
```

```
colMeans(fit.18.8.5.18.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.3334993 0.7008597 0.7128635
```

```
colMeans(fit.18.8.5.18.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.3325513 0.7172629 0.7011103
```

```
### n=p/4
fit.18.8.5.36.lambda2=stat.analy(pi.core.fit.18.8.5.36.lambda2)
colMeans(fit.18.8.5.36.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.2702994 0.9523623 0.8792663
```

```
colMeans(fit.18.8.5.36.lambda2$cse)
```

```
##         K         C     Sigma
## 0.2702994 0.5629076 0.5666911
```

```
colMeans(fit.18.8.5.36.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.2702994 0.4411067 0.5540492
```

```
colMeans(fit.18.8.5.36.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.2702994 0.4649701 0.5540492
```

```
colMeans(fit.18.8.5.36.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2288367 0.3833101 0.4200576
```

```r
colMeans(fit.18.8.5.36.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2281946 0.3960083 0.4149750
```

```r
### n=p/2
fit.18.8.5.72.lambda2=stat.analy(pi.core.fit.18.8.5.72.lambda2)
colMeans(fit.18.8.5.72.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.1800736 0.9523623 0.8807200
```

```r
colMeans(fit.18.8.5.72.lambda2$cse)
```

```
##         K         C     Sigma
## 0.1800736 0.4750083 0.4801332
```

```r
colMeans(fit.18.8.5.72.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.1800736 0.3055384 0.3771486
```

```r
colMeans(fit.18.8.5.72.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.1800736 0.3217268 0.3771486
```

```r
colMeans(fit.18.8.5.72.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1510176 0.2404174 0.2758502
```

```r
colMeans(fit.18.8.5.72.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1511447 0.2532600 0.2766827
```

```r
### n=p
fit.18.8.5.144.lambda2=stat.analy(pi.core.fit.18.8.5.144.lambda2)
colMeans(fit.18.8.5.144.lambda2$kro.mle)
```

```
##         K         C     Sigma
## 0.1266627 0.9523623 0.8784410
```

```r
colMeans(fit.18.8.5.144.lambda2$cse)
```

```
##         K         C     Sigma
## 0.1266627 0.3687995 0.3744336
```

```r
colMeans(fit.18.8.5.144.lambda2$init.ai)
```

```
##         K         C     Sigma
## 0.1266627 0.2088284 0.2618771
```

```r
colMeans(fit.18.8.5.144.lambda2$init.chol)
```

```
##         K         C     Sigma
## 0.1266627 0.2189775 0.2618771
```

```r
colMeans(fit.18.8.5.144.lambda2$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1077317 0.1624071 0.1910572
```

```r
colMeans(fit.18.8.5.144.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1077386 0.1696699 0.1907468
```

```r
### n=2p
fit.18.8.5.288.lambda2=stat.analy(pi.core.fit.18.8.5.288.lambda2)
colMeans(fit.18.8.5.288.lambda2$kro.mle)
```

```
##          K          C      Sigma
## 0.09038642 0.95236232 0.87883212
```

```r
colMeans(fit.18.8.5.288.lambda2$cse)
```

```
##          K          C      Sigma
## 0.09038642 0.28170324 0.28836337
```

```r
colMeans(fit.18.8.5.288.lambda2$init.ai)
```

```
##          K          C      Sigma
## 0.09038642 0.14719019 0.18313677
```

```r
colMeans(fit.18.8.5.288.lambda2$init.chol)
```

```
##          K          C      Sigma
## 0.09038642 0.15444919 0.18313677
```

```r
colMeans(fit.18.8.5.288.lambda2$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.07635137 0.11347895 0.13683146
```

```r
colMeans(fit.18.8.5.288.lambda2$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.0767057 0.1193988 0.1365827
```

```r
###lambda=0.6
### n=p/8
fit.18.8.5.18.lambda3=stat.analy(pi.core.fit.18.8.5.18.lambda3)
colMeans(fit.18.8.5.18.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.4091656 0.9302059 0.8298927
```

```r
colMeans(fit.18.8.5.18.lambda3$cse)
```

```
##         K         C     Sigma
## 0.4091656 0.7215266 0.6940905
```

```r
colMeans(fit.18.8.5.18.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.4091656 0.9523289 1.0119461
```

```r
colMeans(fit.18.8.5.18.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.4091656 0.9793062 1.0119461
```

```r
colMeans(fit.18.8.5.18.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
```

```
## 0.3773818 1.0945865 1.0692541
```

```
colMeans(fit.18.8.5.18.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.3752557 1.1086683 1.0470619
```

```
### n=p/4
fit.18.8.5.36.lambda3=stat.analy(pi.core.fit.18.8.5.36.lambda3)
colMeans(fit.18.8.5.36.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.2685218 0.9302059 0.8260480
```

```
colMeans(fit.18.8.5.36.lambda3$cse)
```

```
##         K         C     Sigma
## 0.2685218 0.6766907 0.6301858
```

```
colMeans(fit.18.8.5.36.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.2685218 0.6051182 0.6474310
```

```
colMeans(fit.18.8.5.36.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.2685218 0.6231734 0.6474310
```

```
colMeans(fit.18.8.5.36.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2432527 0.6317347 0.6174331
```

```
colMeans(fit.18.8.5.36.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2443648 0.6420952 0.6159369
```

```
### n=p/2
fit.18.8.5.72.lambda3=stat.analy(pi.core.fit.18.8.5.72.lambda3)
colMeans(fit.18.8.5.72.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.1783995 0.9302059 0.8272623
```

```
colMeans(fit.18.8.5.72.lambda3$cse)
```

```
##         K         C     Sigma
## 0.1783995 0.6045142 0.5623266
```

```
colMeans(fit.18.8.5.72.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.1783995 0.3971746 0.4244738
```

```
colMeans(fit.18.8.5.72.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.1783995 0.4097690 0.4244738
```

```
colMeans(fit.18.8.5.72.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
```

```
## 0.1604783 0.3750941 0.3809277
```

```
colMeans(fit.18.8.5.72.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1604511 0.3838957 0.3794144
```

```
### n=p
fit.18.8.5.144.lambda3=stat.analy(pi.core.fit.18.8.5.144.lambda3)
colMeans(fit.18.8.5.144.lambda3$kro.mle)
```

```
##         K         C     Sigma
## 0.1245711 0.9302059 0.8242088
```

```
colMeans(fit.18.8.5.144.lambda3$cse)
```

```
##         K         C     Sigma
## 0.1245711 0.5019423 0.4687490
```

```
colMeans(fit.18.8.5.144.lambda3$init.ai)
```

```
##         K         C     Sigma
## 0.1245711 0.2674655 0.2886684
```

```
colMeans(fit.18.8.5.144.lambda3$init.chol)
```

```
##         K         C     Sigma
## 0.1245711 0.2750915 0.2886684
```

```
colMeans(fit.18.8.5.144.lambda3$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1149084 0.2438868 0.2515029
```

```
colMeans(fit.18.8.5.144.lambda3$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1148513 0.2500948 0.2512079
```

```
### n=2p
fit.18.8.5.288.lambda3=stat.analy(pi.core.fit.18.8.5.288.lambda3)
colMeans(fit.18.8.5.288.lambda3$kro.mle)
```

```
##          K          C      Sigma
## 0.08929326 0.93020589 0.82448603
```

```
colMeans(fit.18.8.5.288.lambda3$cse)
```

```
##          K          C      Sigma
## 0.08929326 0.41024164 0.38296587
```

```
colMeans(fit.18.8.5.288.lambda3$init.ai)
```

```
##          K          C      Sigma
## 0.08929326 0.18584040 0.20094519
```

```
colMeans(fit.18.8.5.288.lambda3$init.chol)
```

```
##          K          C      Sigma
## 0.08929326 0.19160212 0.20094519
```

```
colMeans(fit.18.8.5.288.lambda3$fit.ai.analy)
```

```
##          K          C      Sigma
```

```
## 0.08035171 0.16598911 0.17820922
```

```
colMeans(fit.18.8.5.288.lambda3$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.08036163 0.17083454 0.17799992
```

```
###lambda=0.8
### n=p/8
fit.18.8.5.18.lambda4=stat.analy(pi.core.fit.18.8.5.18.lambda4)
colMeans(fit.18.8.5.18.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.4044047 0.8695186 0.7006921
```

```
colMeans(fit.18.8.5.18.lambda4$cse)
```

```
##         K         C     Sigma
## 0.4044047 0.7658713 0.6495351
```

```
colMeans(fit.18.8.5.18.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.4044047 1.7286244 1.5689222
```

```
colMeans(fit.18.8.5.18.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.4044047 1.7409566 1.5689222
```

```
colMeans(fit.18.8.5.18.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.3751211 1.7621006 1.4895496
```

```
colMeans(fit.18.8.5.18.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.3739814 1.7615759 1.4754447
```

```
### n=p/4
fit.18.8.5.36.lambda4=stat.analy(pi.core.fit.18.8.5.36.lambda4)
colMeans(fit.18.8.5.36.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.2684566 0.8695186 0.6924481
```

```
colMeans(fit.18.8.5.36.lambda4$cse)
```

```
##         K         C     Sigma
## 0.2684566 0.7539836 0.6198450
```

```
colMeans(fit.18.8.5.36.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.2684566 1.0509550 0.9304660
```

```
colMeans(fit.18.8.5.36.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.2684566 1.0612334 0.9304660
```

```r
colMeans(fit.18.8.5.36.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.2599114 1.1196220 0.9563287
```

```r
colMeans(fit.18.8.5.36.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.2605555 1.1294546 0.9588298
```

```r
### n=p/2
fit.18.8.5.72.lambda4=stat.analy(pi.core.fit.18.8.5.72.lambda4)
colMeans(fit.18.8.5.72.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.1769550 0.8695186 0.6907545
```

```r
colMeans(fit.18.8.5.72.lambda4$cse)
```

```
##         K         C     Sigma
## 0.1769550 0.7090968 0.5779726
```

```r
colMeans(fit.18.8.5.72.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.1769550 0.6458334 0.5752331
```

```r
colMeans(fit.18.8.5.72.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.1769550 0.6539764 0.5752331
```

```r
colMeans(fit.18.8.5.72.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1689296 0.6840990 0.5931713
```

```r
colMeans(fit.18.8.5.72.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1682688 0.6918243 0.5947441
```

```r
### n=p
fit.18.8.5.144.lambda4=stat.analy(pi.core.fit.18.8.5.144.lambda4)
colMeans(fit.18.8.5.144.lambda4$kro.mle)
```

```
##         K         C     Sigma
## 0.1245396 0.8695186 0.6852707
```

```r
colMeans(fit.18.8.5.144.lambda4$cse)
```

```
##         K         C     Sigma
## 0.1245396 0.6281662 0.5129079
```

```r
colMeans(fit.18.8.5.144.lambda4$init.ai)
```

```
##         K         C     Sigma
## 0.1245396 0.4127109 0.3680053
```

```r
colMeans(fit.18.8.5.144.lambda4$init.chol)
```

```
##         K         C     Sigma
## 0.1245396 0.4166182 0.3680053
```

```r
colMeans(fit.18.8.5.144.lambda4$fit.ai.analy)
```

```
##         K         C     Sigma
## 0.1209916 0.4161859 0.3609913
```

```r
colMeans(fit.18.8.5.144.lambda4$fit.chol.analy)
```

```
##         K         C     Sigma
## 0.1209658 0.4194778 0.3603245
```

```r
### n=2p
fit.18.8.5.288.lambda4=stat.analy(pi.core.fit.18.8.5.288.lambda4)
colMeans(fit.18.8.5.288.lambda4$kro.mle)
```

```
##          K          C      Sigma
## 0.08774049 0.86951861 0.68485271
```

```r
colMeans(fit.18.8.5.288.lambda4$cse)
```

```
##          K          C      Sigma
## 0.08774049 0.54795862 0.44761088
```

```r
colMeans(fit.18.8.5.288.lambda4$init.ai)
```

```
##          K          C      Sigma
## 0.08774049 0.27388212 0.24723975
```

```r
colMeans(fit.18.8.5.288.lambda4$init.chol)
```

```
##          K          C      Sigma
## 0.08774049 0.27762017 0.24723975
```

```r
colMeans(fit.18.8.5.288.lambda4$fit.ai.analy)
```

```
##          K          C      Sigma
## 0.08397386 0.26824765 0.24175876
```

```r
colMeans(fit.18.8.5.288.lambda4$fit.chol.analy)
```

```
##          K          C      Sigma
## 0.08396603 0.27148341 0.24157851
```

```r
##### boxplots of the relative spectral norms for K, C, and Sigma across 100 iterations
##### under different choices of the sample size $n$ and the value of non-spiked eigenvalue $\lambda$.
### K

K.analy.18.8.5.lambda1=plot.K.analy(fit.18.8.5.18.lambda1,fit.18.8.5.36.lambda1,fit.18.8.5.72.lambda1,fi
K.analy.18.8.5.lambda2=plot.K.analy(fit.18.8.5.18.lambda2,fit.18.8.5.36.lambda2,fit.18.8.5.72.lambda2,fi
K.analy.18.8.5.lambda3=plot.K.analy(fit.18.8.5.18.lambda3,fit.18.8.5.36.lambda3,fit.18.8.5.72.lambda3,fi
K.analy.18.8.5.lambda4=plot.K.analy(fit.18.8.5.18.lambda4,fit.18.8.5.36.lambda4,fit.18.8.5.72.lambda4,fi

K.analy.18.8.5=rbind(K.analy.18.8.5.lambda1,K.analy.18.8.5.lambda2,K.analy.18.8.5.lambda3,K.analy.18.8.5
K.analy.18.8.5.lambda=factor(K.analy.18.8.5$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(K.analy.18.8.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$
K.analy.18.8.5$lambda=K.analy.18.8.5.lambda

png("./numerical.simul_picore/K.18.8.5.png",width=900,height=600)
ggplot(K.analy.18.8.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda,lab
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
```

```
  ggtitle(TeX("Separable Component Consistency : $(p_1,p_2,r)=(18,8,5)$"))+theme(plot.title=element_tex
dev.off()
```

```
## pdf
##   2
```

```
### C

C.analy.18.8.5.lambda1=plot.C.analy(fit.18.8.5.18.lambda1,fit.18.8.5.36.lambda1,fit.18.8.5.72.lambda1,fi
C.analy.18.8.5.lambda2=plot.C.analy(fit.18.8.5.18.lambda2,fit.18.8.5.36.lambda2,fit.18.8.5.72.lambda2,fi
C.analy.18.8.5.lambda3=plot.C.analy(fit.18.8.5.18.lambda3,fit.18.8.5.36.lambda3,fit.18.8.5.72.lambda3,fi
C.analy.18.8.5.lambda4=plot.C.analy(fit.18.8.5.18.lambda4,fit.18.8.5.36.lambda4,fit.18.8.5.72.lambda4,fi

C.analy.18.8.5=rbind(C.analy.18.8.5.lambda1,C.analy.18.8.5.lambda2,C.analy.18.8.5.lambda3,C.analy.18.8.5
C.analy.18.8.5.lambda=factor(C.analy.18.8.5$lambda,levels=c("lambda1","lambda2","lambda3","lambda4"))
levels(C.analy.18.8.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),TeX("$
C.analy.18.8.5$lambda=C.analy.18.8.5.lambda

png("./numerical.simul_picore/C.18.8.5.png",width=900,height=600)
ggplot(C.analy.18.8.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda,lab
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Core Component Consistency : $(p_1,p_2,r)=(18,8,5)$"))+theme(plot.title=element_text(hjus
dev.off()
```

```
## pdf
##   2
```

```
### Sigma

Sigma.analy.18.8.5.lambda1=plot.Sigma.analy(fit.18.8.5.18.lambda1,fit.18.8.5.36.lambda1,fit.18.8.5.72.la
Sigma.analy.18.8.5.lambda2=plot.Sigma.analy(fit.18.8.5.18.lambda2,fit.18.8.5.36.lambda2,fit.18.8.5.72.la
Sigma.analy.18.8.5.lambda3=plot.Sigma.analy(fit.18.8.5.18.lambda3,fit.18.8.5.36.lambda3,fit.18.8.5.72.la
Sigma.analy.18.8.5.lambda4=plot.Sigma.analy(fit.18.8.5.18.lambda4,fit.18.8.5.36.lambda4,fit.18.8.5.72.la

Sigma.analy.18.8.5=rbind(Sigma.analy.18.8.5.lambda1,Sigma.analy.18.8.5.lambda2,Sigma.analy.18.8.5.lambda
Sigma.analy.18.8.5.lambda=factor(Sigma.analy.18.8.5$lambda,levels=c("lambda1","lambda2","lambda3","lambd
levels(Sigma.analy.18.8.5.lambda)=c(TeX("$\\lambda=0.2$"),TeX("$\\lambda=0.4$"),TeX("$\\lambda=0.6$"),Te
Sigma.analy.18.8.5$lambda=Sigma.analy.18.8.5.lambda

png("./numerical.simul_picore/Sigma.18.8.5.png",width=900,height=600)
ggplot(Sigma.analy.18.8.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("Sigma Consistency : $(p_1,p_2,r)=(18,8,5)$"))+theme(plot.title=element_text(hjust=0.5))
dev.off()
```

```
## pdf
##   2
```

Combining figures

```
### K

K1=ggplot(K.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$K$ Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_text(hjust=0.5))
```

```
K2=ggplot(K.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$K$ Consistency : $(p_1,p_2,r)=(16,12,5)$"))+theme(plot.title=element_text(hjust=0.5))
K3=ggplot(K.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$K$ Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text(hjust=0.5))
K4=ggplot(K.analy.18.8.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$K$ Consistency : $(p_1,p_2,r)=(18,8,5)$"))+theme(plot.title=element_text(hjust=0.5))

png("./numerical.simul_picore/K.rank3.png",width=900,height=600)
ggarrange(K1,K3,ncol=2,common.legend=TRUE)
dev.off()
```

```
## pdf
##   2
```

```
png("./numerical.simul_picore/K.rank5.png",width=900,height=600)
ggarrange(K2,K4,ncol=2,common.legend=TRUE)
dev.off()
```

```
## pdf
##   2
```

```
### C

C1=ggplot(C.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$C$ Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_text(hjust=0.5))
C2=ggplot(C.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$C$ Consistency : $(p_1,p_2,r)=(16,12,5)$"))+theme(plot.title=element_text(hjust=0.5))
C3=ggplot(C.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$C$ Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text(hjust=0.5))
C4=ggplot(C.analy.18.8.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(.~lambda
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$C$ Consistency : $(p_1,p_2,r)=(18,8,5)$"))+theme(plot.title=element_text(hjust=0.5))

png("./numerical.simul_picore/C.rank3.png",width=900,height=600)
ggarrange(C1,C3,ncol=2,common.legend=TRUE)
dev.off()
```

```
## pdf
##   2
```

```
png("./numerical.simul_picore/C.rank5.png",width=900,height=600)
ggarrange(C2,C4,ncol=2,common.legend=TRUE)
dev.off()
```

```
## pdf
##   2
```

```
### Sigma

sigma1=ggplot(Sigma.analy.16.12.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
```

```
  ggtitle(TeX("$\\Sigma$ Consistency : $(p_1,p_2,r)=(16,12,3)$"))+theme(plot.title=element_text(hjust=0
sigma2=ggplot(Sigma.analy.16.12.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$\\Sigma$ Consistency : $(p_1,p_2,r)=(16,12,5)$"))+theme(plot.title=element_text(hjust=0
sigma3=ggplot(Sigma.analy.18.8.3,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$\\Sigma$ Consistency : $(p_1,p_2,r)=(18,8,3)$"))+theme(plot.title=element_text(hjust=0.5
sigma4=ggplot(Sigma.analy.18.8.5,mapping=aes(x=samp.size,y=stat,col=Method))+geom_boxplot()+facet_wrap(
  theme_bw()+xlab("Sample Size")+ylab("Relative Norm")+
  ggtitle(TeX("$\\Sigma$ Consistency : $(p_1,p_2,r)=(18,8,5)$"))+theme(plot.title=element_text(hjust=0.5

png("./numerical.simul_picore/Sigma.rank3.png",width=900,height=600)
ggarrange(sigma1,sigma3,ncol=2,common.legend=TRUE)
dev.off()
```

```
## pdf
##   2
```

```
png("./numerical.simul_picore/Sigma.rank5.png",width=900,height=600)
ggarrange(sigma2,sigma4,ncol=2,common.legend=TRUE)
dev.off()
```

```
## pdf
##   2
```

Lambda

```
### Analysis of lambda estimation for CSE, Base-AI, Base-Chol
lambda1=0.2; lambda2=0.4; lambda3=0.6; lambda4=0.8


lambda.analy=function(fit1,fit2,fit3,fit4,fit5,lambda){

  result=matrix(0,3,5)

  pi.lambda.ai=function(fit){

    fit=fit$pi.core.ai
    temp=NULL
    for(i in 1:100){
      temp=c(temp,fit[[i]]$lambda)
    }
    return(temp)
  }

  pi.lambda.chol=function(fit){

    fit=fit$pi.core.chol
    temp=NULL
    for(i in 1:100){
      temp=c(temp,fit[[i]]$lambda)
    }
    return(temp)
  }
```

```
  fit.summary=function(fit){
    temp1=mean(abs(fit$cse[,"w"]-lambda))
    temp2=mean(abs(pi.lambda.ai(fit)-lambda))
    temp3=mean(abs(pi.lambda.chol(fit)-lambda))
    return(c(temp1,temp2,temp3))
  }

  result[,1]=fit.summary(fit1); result[,2]=fit.summary(fit2)
  result[,3]=fit.summary(fit3); result[,4]=fit.summary(fit4)
  result[,5]=fit.summary(fit5)
  rownames(result)=c("CSE","Base-AI","Base-Chol")
  colnames(result)=c("p/8","p/4","p/2","p","2p")
  return(result)
}

lambda.16.12.3.lambda1=lambda.analy(pi.core.fit.16.12.3.24.lambda1,pi.core.fit.16.12.3.48.lambda1,
             pi.core.fit.16.12.3.96.lambda1,pi.core.fit.16.12.3.192.lambda1,
             pi.core.fit.16.12.3.384.lambda1,lambda1)
lambda.16.12.3.lambda2=lambda.analy(pi.core.fit.16.12.3.24.lambda2,pi.core.fit.16.12.3.48.lambda2,
             pi.core.fit.16.12.3.96.lambda2,pi.core.fit.16.12.3.192.lambda2,
             pi.core.fit.16.12.3.384.lambda2,lambda2)
lambda.16.12.3.lambda3=lambda.analy(pi.core.fit.16.12.3.24.lambda3,pi.core.fit.16.12.3.48.lambda3,
             pi.core.fit.16.12.3.96.lambda3,pi.core.fit.16.12.3.192.lambda3,
             pi.core.fit.16.12.3.384.lambda3,lambda3)
lambda.16.12.3.lambda4=lambda.analy(pi.core.fit.16.12.3.24.lambda4,pi.core.fit.16.12.3.48.lambda4,
             pi.core.fit.16.12.3.96.lambda4,pi.core.fit.16.12.3.192.lambda4,
             pi.core.fit.16.12.3.384.lambda4,lambda4)
lambda.16.12.3=rbind(lambda.16.12.3.lambda1,lambda.16.12.3.lambda2,lambda.16.12.3.lambda3,lambda.16.12.3

lambda.18.8.3.lambda1=lambda.analy(pi.core.fit.18.8.3.18.lambda1,pi.core.fit.18.8.3.36.lambda1,
             pi.core.fit.18.8.3.72.lambda1,pi.core.fit.18.8.3.144.lambda1,
             pi.core.fit.18.8.3.288.lambda1,lambda1)
lambda.18.8.3.lambda2=lambda.analy(pi.core.fit.18.8.3.18.lambda2,pi.core.fit.18.8.3.36.lambda2,
             pi.core.fit.18.8.3.72.lambda2,pi.core.fit.18.8.3.144.lambda2,
             pi.core.fit.18.8.3.288.lambda2,lambda2)
lambda.18.8.3.lambda3=lambda.analy(pi.core.fit.18.8.3.18.lambda3,pi.core.fit.18.8.3.36.lambda3,
             pi.core.fit.18.8.3.72.lambda3,pi.core.fit.18.8.3.144.lambda3,
             pi.core.fit.18.8.3.288.lambda3,lambda3)
lambda.18.8.3.lambda4=lambda.analy(pi.core.fit.18.8.3.18.lambda4,pi.core.fit.18.8.3.36.lambda4,
             pi.core.fit.18.8.3.72.lambda4,pi.core.fit.18.8.3.144.lambda4,
             pi.core.fit.18.8.3.288.lambda4,lambda4)
lambda.18.8.3=rbind(lambda.18.8.3.lambda1,lambda.18.8.3.lambda2,lambda.18.8.3.lambda3,lambda.18.8.3.lam

lambda.16.12.5.lambda1=lambda.analy(pi.core.fit.16.12.5.24.lambda1,pi.core.fit.16.12.5.48.lambda1,
             pi.core.fit.16.12.5.96.lambda1,pi.core.fit.16.12.5.192.lambda1,
             pi.core.fit.16.12.5.384.lambda1,lambda1)
lambda.16.12.5.lambda2=lambda.analy(pi.core.fit.16.12.5.24.lambda2,pi.core.fit.16.12.5.48.lambda2,
             pi.core.fit.16.12.5.96.lambda2,pi.core.fit.16.12.5.192.lambda2,
             pi.core.fit.16.12.5.384.lambda2,lambda2)
lambda.16.12.5.lambda3=lambda.analy(pi.core.fit.16.12.5.24.lambda3,pi.core.fit.16.12.5.48.lambda3,
             pi.core.fit.16.12.5.96.lambda3,pi.core.fit.16.12.5.192.lambda3,
             pi.core.fit.16.12.5.384.lambda3,lambda3)
lambda.16.12.5.lambda4=lambda.analy(pi.core.fit.16.12.5.24.lambda4,pi.core.fit.16.12.5.48.lambda4,
```

```
            pi.core.fit.16.12.5.96.lambda4,pi.core.fit.16.12.5.192.lambda4,
            pi.core.fit.16.12.5.384.lambda4,lambda4)
lambda.16.12.5=rbind(lambda.16.12.5.lambda1,lambda.16.12.5.lambda2,lambda.16.12.5.lambda3,lambda.16.12.5

lambda.18.8.5.lambda1=lambda.analy(pi.core.fit.18.8.5.18.lambda1,pi.core.fit.18.8.5.36.lambda1,
            pi.core.fit.18.8.5.72.lambda1,pi.core.fit.18.8.5.144.lambda1,
            pi.core.fit.18.8.5.288.lambda1,lambda1)
lambda.18.8.5.lambda2=lambda.analy(pi.core.fit.18.8.5.18.lambda2,pi.core.fit.18.8.5.36.lambda2,
            pi.core.fit.18.8.5.72.lambda2,pi.core.fit.18.8.5.144.lambda2,
            pi.core.fit.18.8.5.288.lambda2,lambda2)
lambda.18.8.5.lambda3=lambda.analy(pi.core.fit.18.8.5.18.lambda3,pi.core.fit.18.8.5.36.lambda3,
            pi.core.fit.18.8.5.72.lambda3,pi.core.fit.18.8.5.144.lambda3,
            pi.core.fit.18.8.5.288.lambda3,lambda3)
lambda.18.8.5.lambda4=lambda.analy(pi.core.fit.18.8.5.18.lambda4,pi.core.fit.18.8.5.36.lambda4,
            pi.core.fit.18.8.5.72.lambda4,pi.core.fit.18.8.5.144.lambda4,
            pi.core.fit.18.8.5.288.lambda4,lambda4)
lambda.18.8.5=rbind(lambda.18.8.5.lambda1,lambda.18.8.5.lambda2,lambda.18.8.5.lambda3,lambda.18.8.5.lam

lambda.16.12.3
```

```
##                  p/8        p/4        p/2          p          2p
## CSE       0.03538270 0.02133617 0.04015967 0.076752814 0.114729866
## Base-AI   0.02222243 0.01636412 0.01166016 0.007340000 0.004891518
## Base-Chol 0.02225342 0.01635482 0.01164942 0.007341136 0.004892377
## CSE       0.05301751 0.02622539 0.05249068 0.116660801 0.189538761
## Base-AI   0.04136241 0.02796982 0.01814328 0.011012466 0.007376469
## Base-Chol 0.04144792 0.02801447 0.01815513 0.011016130 0.007376061
## CSE       0.05560110 0.02385413 0.05436044 0.127566663 0.217854674
## Base-AI   0.06589558 0.03725924 0.02052015 0.011521836 0.007675272
## Base-Chol 0.06590791 0.03722325 0.02049095 0.011509301 0.007674393
## CSE       0.04342032 0.02071966 0.06064738 0.129278976 0.215141606
## Base-AI   0.09898363 0.04913388 0.02168106 0.010355263 0.006319690
## Base-Chol 0.09897153 0.04897957 0.02165705 0.010348740 0.006319442
```

```
lambda.18.8.3
```

```
##                  p/8        p/4        p/2          p          2p
## CSE       0.04446121 0.02141966 0.03648475 0.075796807 0.113615550
## Base-AI   0.02540540 0.01897363 0.01210418 0.007950951 0.005992746
## Base-Chol 0.02573281 0.01911913 0.01202682 0.007951151 0.005988229
## CSE       0.07695659 0.03055652 0.04769451 0.116094828 0.188200667
## Base-AI   0.04589151 0.03098166 0.01815907 0.012159680 0.008886152
## Base-Chol 0.04691995 0.03127381 0.01816156 0.012180935 0.008885094
## CSE       0.08973885 0.03033203 0.04968001 0.128709474 0.218863107
## Base-AI   0.07508902 0.04042822 0.02026400 0.012907903 0.008978096
## Base-Chol 0.07534737 0.04054606 0.02029274 0.012905697 0.008968727
## CSE       0.07152020 0.02364335 0.04880453 0.124662279 0.213863196
## Base-AI   0.12348171 0.05726715 0.02490459 0.012482782 0.007474412
## Base-Chol 0.12352325 0.05752111 0.02458944 0.012483399 0.007470539
```

```
lambda.16.12.5
```

```
##                  p/8        p/4        p/2          p          2p
## CSE       0.02443301 0.01582839 0.03699319 0.075405649 0.114217749
## Base-AI   0.03376575 0.01954232 0.01043469 0.005571645 0.004268891
## Base-Chol 0.03354677 0.01943467 0.01039791 0.005561228 0.004256985
```

```
## CSE       0.03550207 0.02075124 0.04845246 0.116307467 0.190439736
## Base-AI   0.07416175 0.03861345 0.01856872 0.009294752 0.006920942
## Base-Chol 0.07421391 0.03847154 0.01854771 0.009291183 0.006917340
## CSE       0.03342905 0.02110330 0.04983830 0.130200840 0.223861669
## Base-AI   0.12321108 0.06127366 0.02683594 0.012527200 0.008345038
## Base-Chol 0.12395858 0.06116120 0.02669856 0.012490034 0.008332595
## CSE       0.02350530 0.01766943 0.04362316 0.121914939 0.216369065
## Base-AI   0.18075674 0.09060620 0.04049118 0.018321569 0.009727469
## Base-Chol 0.18174192 0.09097264 0.04038085 0.018287477 0.009723832
```

```
lambda.18.8.5
```

```
##                   p/8        p/4        p/2          p          2p
## CSE       0.02912499 0.01641330 0.03271426 0.07419846 0.112880903
## Base-AI   0.04339008 0.02227774 0.01086976 0.00780461 0.004956155
## Base-Chol 0.04325750 0.02223689 0.01078567 0.00777840 0.004952488
## CSE       0.04874709 0.02749823 0.04199857 0.11485285 0.188558728
## Base-AI   0.09744070 0.04516585 0.02180704 0.01361989 0.008114598
## Base-Chol 0.09814851 0.04526297 0.02175785 0.01358660 0.008116168
## CSE       0.05263695 0.03275118 0.04032541 0.12786943 0.222481274
## Base-AI   0.16314152 0.07533571 0.03496031 0.01864111 0.009982332
## Base-Chol 0.16265644 0.07535093 0.03488349 0.01859681 0.009946017
## CSE       0.04016418 0.03440620 0.02567971 0.10910240 0.207408938
## Base-AI   0.24655959 0.11897415 0.05442674 0.02640191 0.012708451
## Base-Chol 0.24598897 0.11887893 0.05412087 0.02634993 0.012688541
```

C Consistency for KMLE

```
### Consistency of KMLE with respect to C
### In fact, this is given by ||C-I||/||C|| for true C

library(RSpectra)
load("./numerical.simul_picore/para.16.12.3.rds")
load("./numerical.simul_picore/para.18.8.3.rds")
load("./numerical.simul_picore/para.16.12.5.rds")
load("./numerical.simul_picore/para.18.8.5.rds")

## (p1,p2,r)=(16,12,3)
A=para.16.12.3$A; A.sig=svds(A,1)$d
(A.sig^2*0.8-0.8)/(A.sig^2*0.8+0.2)
```

```
## [1] 0.9838725
```

```
(A.sig^2*0.6-0.6)/(A.sig^2*0.6+0.4)
```

```
## [1] 0.9786116
```

```
(A.sig^2*0.4-0.4)/(A.sig^2*0.4+0.6)
```

```
## [1] 0.9682568
```

```
(A.sig^2*0.2-0.2)/(A.sig^2*0.2+0.8)
```

```
## [1] 0.9384669
```

```
## (p1,p2,r)=(18,8,3)
A=para.18.8.3$A; A.sig=svds(A,1)$d
```

```
(A.sig^2*0.8-0.8)/(A.sig^2*0.8+0.2)
```

```
## [1] 0.9772086
```

```
(A.sig^2*0.6-0.6)/(A.sig^2*0.6+0.4)
```

```
## [1] 0.9698406
```

```
(A.sig^2*0.4-0.4)/(A.sig^2*0.4+0.6)
```

```
## [1] 0.955433
```

```
(A.sig^2*0.2-0.2)/(A.sig^2*0.2+0.8)
```

```
## [1] 0.914669
```

```
## (p1,p2,r)=(16,12,5)
A=para.16.12.5$A; A.sig=svds(A,1)$d
(A.sig^2*0.8-0.8)/(A.sig^2*0.8+0.2)
```

```
## [1] 0.9734962
```

```
(A.sig^2*0.6-0.6)/(A.sig^2*0.6+0.4)
```

```
## [1] 0.9649711
```

```
(A.sig^2*0.4-0.4)/(A.sig^2*0.4+0.6)
```

```
## [1] 0.9483611
```

```
(A.sig^2*0.2-0.2)/(A.sig^2*0.2+0.8)
```

```
## [1] 0.9017934
```

```
## (p1,p2,r)=(18,8,5)
A=para.18.8.5$A; A.sig=svds(A,1)$d
(A.sig^2*0.8-0.8)/(A.sig^2*0.8+0.2)
```

```
## [1] 0.9638411
```

```
(A.sig^2*0.6-0.6)/(A.sig^2*0.6+0.4)
```

```
## [1] 0.9523623
```

```
(A.sig^2*0.4-0.4)/(A.sig^2*0.4+0.6)
```

```
## [1] 0.9302059
```

```
(A.sig^2*0.2-0.2)/(A.sig^2*0.2+0.8)
```

```
## [1] 0.8695186
```