W5 Systematic Selection

5.1 Systematic Selection – How it's done

Systematic Selection
• A simple method of selecting a sample from a list (e.g. population of transactions)
• Take every so many elements….
• Choose the start – first, any, any one up to the interval
• say taking every 10th, starting with first … (Interval k = 10)
• When do we stop? When we have n selections?
• This list has N elements
• If n = 50, stop with element 501 … and elements 502 to 1,000 have zero chance of selection
• As do elements 2,3,4….,10…and 12, 13, 14…. , 20 and so on
• Two remedies needed:
  - spread the selection out over the whole list
  - vary the selection start
• For example, for n= 50, don't take every 10th, but every 20th
• And don't start with the first, but start from any element from 1 to 20 … at random
• That is, adapt the selection process to the size of sample and size of list
• Calculate an interval of k = N / n (k= 1,000 / 50 = 20)
• And choose to start anywhere from 1 to k = 20 … at random
• Conceptually, this is taking the population, dividing it into k samples, and choosing one of them: it's kinda like stratified sampling in a way
• This is equivalent to cluster sampling – each possible systematic sample is a cluster of n elements

5.2 What happens if the interval is not an interval?

• To repeat the process, first determine the sampling interval k = N / n
• Select a random number (RN) from 1 to k
• Add k repeatedly
• suppose, for example, there were N = 12,000 dwellings in a city and a sample of n = 500 is required
  - k = 12,000 / 500 = 24
  - take a RN from 01 to 24, say 03
  - Take the 3rd dwelling, and every 24th thereafter: 3, 27, 51, etc.
• But what do we do in the more common situation where k is not an integer/
  - examples
    - N = 9, n = 2 and k = 4.5
    - N = 952, n = 200, and k = 4.76
    - N = 170,345, n = 1,250, and k = 136.272
• Consider three alternatives…
  • First round the fractional interval
    - for example, when N = 9, n = 2, take k = 4 or 5
    - if k = 4 and RN = 1, the sample is the **three** element 1, 5, 9
    - if RN = 2,3, or 4 the sample has **only two** elements
    - if k = 5 and RN = 1,2,3 or 4 the sample has **two** elements
    - if RN = 5, the sample has **only one** element
    - What would happen if N = 952 and n = 200?
      - rounding k to 5, RN's 1,2,3 & 4 select 191, and RN 5 selects 190 – neither sample size is 200!
    - what about for N = 170,345 and n = 1,250? ➔ the sample size can be either 1252 or 1253
    - Rounding thus has the problem that the sample size is not fixed, and we don't get the target sample size!
  • Second solution is one some people prefer
    • treat the list as circular
      - As before, calculate the interval k = N / n, and round up or down, say to k*
      - choose a RN anywhere from 1 to N at random
      - then start counting every k*th thereafter
      - keep going until exactly n elements are selected

• But what if you get to the end of the list before you have n elements?
- we do "wrap"
- think about the list like it is a clock
- suppose n = 5 and N = 12, or k = 12/5 = 2.4
- round to [k] = 2, and choose random start 7:
- take every 2 after starting at 7 …
- and then 9… and then 11 …. And then …
- …. And then … 1
- and then 3 … and then … STOP because n = 5
- remember, start anywhere on the list … and wrap

• Here use the **fractional interval**…
- choose a random start from 0.1 to 4.6
- but how do you do that?

• One way is with a table of random numbers
• Since we need a number from 0.1 to 4.6, why not choose a random number from 01 to 46
• Suppose the number is 35
• "Insert" a decimal to make it fractional: 3.5
• Alternatively, generate a UNIFROM random number from zero to 1 in statistical software, say 0.76087
• Multiply by 4.6, and get 3.5
• But then what?
- do systematic counting …
- but "count" every 4.6 ….
- starting with (1) 3.5, we 'count' to (2) 3.5 + 4.6 = 8.1
- and again, (3) 8.1 + 4.6 = 12.7 …
- and again, (4) 12.7 + 4.6 = 17.3 …
- and again, (5) 17.3 + 4.6 = 21.9 …
- and just to be sure, one more time gives us, (5) 21.9 + 4.6 = 26.5
- oops! We are off the list

• But before we got off the list, we had n = 5 "selections"
- 3.5, 8.1, 12.7, 17.3, and 21.9

• What do we do with the decimals though?
- truncate to the whole number
- that is our selections are 3, 8, 12, 17, and 21 (truncation: drop decimals at the end)

• What does this all mean?

Simple method that is "epsem"

Element	Random start	No. of RS' s	*f*
1	10-19	10	1/4.6
2	20-29	10	1/4.6
3	30-39	10	1/4.6
4	40-46,01-03	10	1/4.6
5	04-13	10	1/4.6
…	…	…	…
23	01-09,46	10	1/4.6

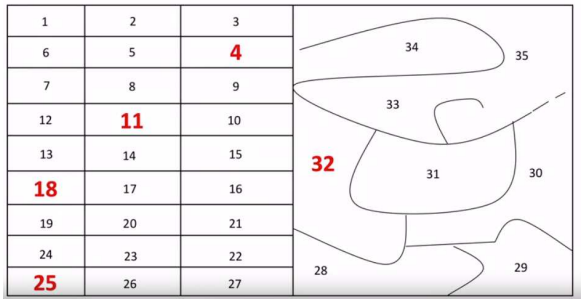
5.3 Systematic selection and implicit stratification

• List order combined with systematic selection can improve the efficiency (in terms of variance) of systematic sample designs
• Arrange the list order in advance
• Determines which kind of samples are selected
• random order: SRS (e.g. if transaction data comes in by time order and then there is no association between "amount" of transaction which is our variable of interest and time of transaction, that the list is effectively random)
• stratified order (e.g. sort by transaction list by subcategory such as Business Services, Conferences & training etc.)
- Systematic selection from a list ordered by a categorical auxiliary variable is implicitly equivalent to proportionately allocated stratified sampling. Systematic sampling applied to ordered lists gives implicit stratification of the population

following the order of the list. Since systematic sampling is epsem, the systematic sample isimplicitly proportionately allocated stratified random sampling.

- gains in precision due to proportionately allocated stratified sampling … implicitly

• Serpentine order (geographic representation)



• Linear trend order

- strong gains in precision

• Periodic trend order

• Generally, list order other than random is a useful property in combination with systematic selection

• Implicit stratification: gains in precision

5.4 How to estimate standard errors for systematic samples

- **Estimation of the sample mean:** $\bar{y} = \frac{\sum y_i}{n}$
- **Sampling variance cannot be estimated using only survey data**
  - **Only a single random start used**
  - **Two approaches to dealing with the problem**
    - **Use additional random starts**
    - **Model the variance**
- **Use c random starts and**

$$\bar{y} = \frac{\sum\sum y_{\gamma i}}{cn} = \frac{1}{c}\sum_{\gamma}\bar{y}_{\gamma}$$

$$\mathrm{var}(\bar{y}) = \frac{1}{c}\sum_{\gamma}\left(\bar{y}_{\gamma} - \bar{y}\right)^2$$

- **Model the population (sample selection process)**
- **SRS model**
  - **Are elements in the list are ordered at random?**
  - **Yes?**

$$\mathrm{var}(\bar{y}) = (1-f)\frac{s^2}{n}$$

- **Are elements in the list are ordered at random?**
  - **Yes?**
  - **Can we assume homogeneity across 'rows' (zones), in groups of rows?**
  - **Yes?**
  - **Assume random ordering within zones**
  - **Proportionately allocated selection with n_h = 1 selected per zone**
- **Collapse neighboring zones to create "pseudo strata" that have multiple selections, and using**

$$\mathrm{var}(\bar{y}) = \frac{1-f}{n}\sum W_h s_h^2$$

$$W_h = \frac{n_h}{n}$$

- **Is the ordering really almost continuous?**
- **Yes?**
- **Stratified random model special case: pair successive rows**

$$\mathrm{var}(\bar{y}) = \frac{1-f}{n^2}\sum_{h}\left(y_{h1} - y_{h2}\right)^2$$

Example / illustration

Block	# Rental	# HUs	i
240	23	30	1
278	25	33	2
288	42	61	3
377	0	3	4
388	16	27	5
398	37	47	6

- ***Epsem* sample**

$$\bar{y}_{\#rental} = \frac{\sum y_i}{n}$$

$$= (23 + 25 + 42 + 0 + 16 + 37)/6 = 23.83$$

- **This list is probably continuously ordered with respect to Y.**
- **Paired selection model, even # elements**

$$\text{var}(\bar{y}) = \frac{(1-f)}{n^2}\sum_{h}^{n/2}(y_{ha} - y_{hb})^2$$

$$= \left(1 - \frac{6}{60}\right)\left(\frac{1}{6^2}\right)\left[(23-25)^2 + (42-0)^2 + (16-37)^2\right]$$

$$= (0.9)(0.0278)(4 + 1764 + 441) = 55.225$$

- **Is the list order random?**
- **SRS model**

$$\text{var}(\bar{y}) = (1-f)\frac{s^2}{n} = \left(1 - \frac{6}{60}\right)\left(\frac{1}{6}\right)\frac{\left(4543 - 6 * 23.83^2\right)}{6-1}$$

$$= (0.90)(0.1667)(226.97) = 34.045$$