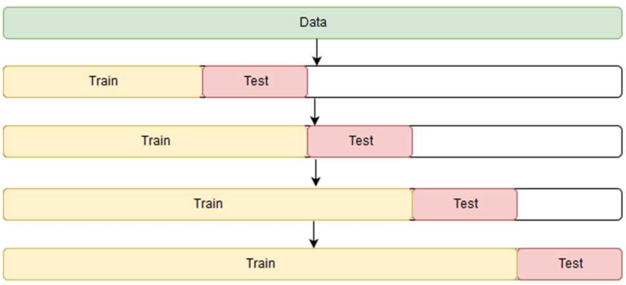
Time Series Models

```
import warnings                                             # `do not disturbe` mode
warnings.filterwarnings('ignore')

import numpy as np                                          # vectors and matrices
import pandas as pd                                         # tables and data manipulations
import matplotlib.pyplot as plt                             # plots
import seaborn as sns                                       # more plots

from dateutil.relativedelta import relativedelta # working with dates with style
from scipy.optimize import minimize                         # for function minimization

import statsmodels.formula.api as smf                       # statistics and econometrics
import statsmodels.tsa.api as smt
import statsmodels.api as sm
import scipy.stats as scs

from itertools import product                               # some useful functions
from tqdm import tqdm_notebook



%matplotlib inline
```

Cross Validation for Time Series (Cross Validation on a rolling basis)

The idea is rather simple -- we train our model on a small segment of the time series from the beginning until some t, make predictions for the next t+n steps, and calculate an error. Then, we expand our training sample to t+n value, make predictions from t+n until t+2*n , and continue moving our test segment of the time series until we hit the last available observation. As a result, we have as many folds as "n" will fit between the initial training sample and the last observation.




```
from sklearn.model_selection import TimeSeriesSplit # you have everything done for you

def timeseriesCVscore(params, series, loss_function=mean_squared_error, slen=24):
    """
        Returns error on CV

        params - vector of parameters for optimization
        series - dataset with timeseries
        slen - season length for Holt-Winters model
    """
    # errors array
    errors = []

    values = series.values
    alpha, beta, gamma = params
```

```
# set the number of folds for cross-validation
tscv = TimeSeriesSplit(n_splits=3)

# iterating over folds, train model on each, forecast and calculate error
for train, test in tscv.split(values):

    model = HoltWinters(series=values[train], slen=slen,
                        alpha=alpha, beta=beta, gamma=gamma, n_preds=len(test))
    model.triple_exponential_smoothing()

    predictions = model.result[-len(test):]
    actual = values[test]
    error = loss_function(predictions, actual)
    errors.append(error)

return np.mean(np.array(errors))
```

Autoregression (AR)
http://www.statsmodels.org/dev/generated/statsmodels.tsa.ar_model.AR.html
http://www.statsmodels.org/dev/generated/statsmodels.tsa.ar_model.ARResults.html

The autoregression (AR) method models the next step in the sequence as a linear function of the observations at prior time steps. The notation for the model involves specifying the order of the model p as a parameter to the AR function, e.g. AR(p). For example, AR(1) is a first-order autoregression model. An autoregressive (AR) model is a representation of a type of random process; as such, it is used to describe certain time-varying processes in nature, economics, etc. The autoregressive model specifies that the output variable depends linearly on its own previous values and on a stochastic term (an imperfectly predictable term); thus the model is in the form of a stochastic difference equation. The method is suitable for univariate time series without trend and seasonal components.

The basic assumption is that the current series values depend on its previous values with some lag (or several lags). The maximum lag in the model is referred to as p. To determine the initial p, you need to look at the PACF plot and find the biggest significant lag after which most other lags become insignificant.

AR(p) = $$X_t = c + \sum_{i=1}^{p} \varphi_i X_{t-i} + \varepsilon_t$$

where \phi s are the parameters of the model, c is a constant, and \epsilon_t is white noise

Because each shock affects X values infinitely far into the future from when they occur, any given value Xt is affected by shocks occurring infinitely far into the past (Intertemporal Effect of shocks)

```
from statsmodels.tsa.ar_model import AR
from random import random

# contrived dataset
data = [x + random( ) for x in range(1, 100)]

# fit model
model = AR(data)
model_fit = model.fit()

# make prediction
yhat = model_fit.predict(len(data), len(data))
print(yhat)
```

```
import statsmodels.api as sm
from numpy.random import normal, seed
from scipy.stats import norm
from statsmodels.tsa.arima_model import ARMA
from statsmodels.tsa.stattools import adfuller
```

```
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.arima_model import ARIMA
import math
from sklearn.metrics import mean_squared_error

ar1 = np.array([1, -0.9]) # We choose -0.9 as AR parameter is +0.9
ma1 = np.array([1])
AR1 = ArmaProcess(ar1, ma1)
sim1 = AR1.generate_sample(nsample=1000)

model = ARMA(sim1, order=(1,0))
result = model.fit()
print(result.summary())
print("μ={} ,φ={}".format(result.params[0],result.params[1]))

φ is around 0.9 which is what we chose as AR parameter in our first simulated model.

# Predicting simulated AR(1) model
result.plot_predict(start=900, end=1010)
plt.show()

rmse = math.sqrt(mean_squared_error(sim1[900:1011], result.predict(start=900,end=999)))
print("The root mean squared error is {}.".format(rmse))

# Predicting humidity level of Montreal
humid = ARMA(humidity["Montreal"].diff().iloc[1:].values, order=(1,0))
res = humid.fit()
res.plot_predict(start=1000, end=1100)
plt.show()

rmse =
math.sqrt(mean_squared_error(humidity["Montreal"].diff().iloc[900:1000].values,
result.predict(start=900,end=999)))
print("The root mean squared error is {}.".format(rmse))

# Predicting closing prices of google
humid = ARMA(google["Close"].diff().iloc[1:].values, order=(1,0))
res = humid.fit()
res.plot_predict(start=900, end=1010)
```

Moving Average (MA)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.arima_model.ARMA.html
http://www.statsmodels.org/dev/generated/statsmodels.tsa.arima_model.ARMAResults.html

The moving average (MA) method models the next step in the sequence as a linear function of the residual errors from a mean process at prior time steps. A moving average model is different from calculating the moving average of the time series. The notation for the model involves specifying the order of the model q as a parameter to the MA function, e.g. MA(q). For example, MA(1) is a first-order moving average model. The method is suitable for univariate time series without trend and seasonal components.

Again with the assumption (similarly to AR model) that the current error depends on the previous with some lag, which is referred to as q. The initial value can be found on the ACF plot with the same logic as before.

```
# MA example
from statsmodels.tsa.arima_model import ARMA
from random import random
# contrived dataset
```

```
data = [x + random() for x in range(1, 100)]
# fit model
model = ARMA(data, order=(0, 1))
model_fit = model.fit(disp=False)
# make prediction
yhat = model_fit.predict(len(data), len(data))
print(yhat)
```

```
import statsmodels.api as sm
from numpy.random import normal, seed
from scipy.stats import norm
from statsmodels.tsa.arima_model import ARMA
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.arima_model import ARIMA
import math
from sklearn.metrics import mean_squared_error

ar1 = np.array([1])
ma1 = np.array([1, -0.5])
MA1 = ArmaProcess(ar1, ma1)
sim1 = MA1.generate_sample(nsample=1000)

# Forecasting and predicting montreal humidity
model = ARMA(humidity["Montreal"].diff().iloc[1:].values, order=(0,3))
result = model.fit()
print(result.summary())
print("μ={} ,θ={}".format(result.params[0],result.params[1]))
result.plot_predict(start=1000, end=1100)
plt.show()

rmse = \
math.sqrt(mean_squared_error(humidity["Montreal"].diff().iloc[1000:1101].values,
result.predict(start=1000,end=1100)))
print("The root mean squared error is {}.".format(rmse))
```

Autoregressive Moving Average (ARMA)
The Autoregressive Moving Average (ARMA) method models the next step in the sequence as a linear function of the observations and resiudal errors at prior time steps.

It combines both Autoregression (AR) and Moving Average (MA) models.

The notation for the model involves specifying the order for the AR(p) and MA(q) models as parameters to an ARMA function, e.g. ARMA(p, q). An ARIMA model can be used to develop AR or MA models.

The method is suitable for univariate time series without trend and seasonal components.

Autoregressive Moving Average (ARMA)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.arima_model.ARMA.html
http://www.statsmodels.org/dev/generated/statsmodels.tsa.arima_model.ARMAResults.html

AR(p)+MA(q)=ARMA(p,q)

The Autoregressive Moving Average (ARMA) method models the next step in the sequence as a linear function of the observations and residual errors at prior time steps. It combines both Autoregression (AR) and Moving Average (MA) models. The notation for the model involves specifying the order for the AR(p) and MA(q) models as parameters to an ARMA function, e.g. ARMA(p, q). An ARIMA model can be used to develop AR or MA models. The method is suitable for univariate time series without trend and seasonal components.
Autoregressive–moving-average (ARMA) models provide a parsimonious description of a (weakly) stationary

stochastic process in terms of two polynomials, one for the autoregression and the second for the moving average. It's the fusion of AR and MA models.

ARMA(1,1) model

$R_t = \mu + \phi R_{t-1} + \epsilon_t + \theta\epsilon_{t-1}$

Basically, Today's return = mean + Yesterday's return + noise + yesterday's noise.

```
from statsmodels.tsa.arima_model import ARMA
from random import random
# contrived dataset
data = [random() for x in range(1, 100)]
# fit model
model = ARMA(data, order=(2, 1))
model_fit = model.fit(disp=False)
# make prediction
yhat = model_fit.predict(len(data), len(data))
print(yhat)
```

```
import statsmodels.api as sm
from numpy.random import normal, seed
from scipy.stats import norm
from statsmodels.tsa.arima_model import ARMA
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.arima_model import ARIMA
import math
from sklearn.metrics import mean_squared_error


# Forecasting and predicting microsoft stocks volume
model = ARMA(microsoft["Volume"].diff().iloc[1:].values, order=(3,3))
result = model.fit()
print(result.summary())
print("μ={}, φ={}, θ={}".format(result.params[0],result.params[1],result.params[2]))
result.plot_predict(start=1000, end=1100)
plt.show()

rmse = math.sqrt(mean_squared_error(microsoft["Volume"].diff().iloc[1000:1101].values,
result.predict(start=1000,end=1100)))
print("The root mean squared error is {}.".format(rmse))
```

Autoregressive Integrated Moving Average (ARIMA)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.arima_model.ARIMA.html#statsmodels.tsa.arima_model.ARIMA
http://www.statsmodels.org/dev/generated/statsmodels.tsa.arima_model.ARIMAResults.html

The Autoregressive Integrated Moving Average (ARIMA) method models the next step in the sequence as a linear function of the differenced observations and residual errors at prior time steps. It combines both Autoregression (AR) and Moving Average (MA) models as well as a differencing pre-processing step of the sequence to make the sequence stationary, called integration (I). The notation for the model involves specifying the order for the AR(p), I(d), and MA(q) models as parameters to an ARIMA function, e.g. **ARIMA(p, d, q)**. An ARIMA model can also be used to develop AR, MA, and ARMA models.
An autoregressive integrated moving average (ARIMA) model is a generalization of an autoregressive moving average (ARMA) model. Both of these models are fitted to time series data either to better understand the data or to predict future points in the series (forecasting). ARIMA models are applied in some cases where data show evidence of non-stationarity, where an initial differencing step (corresponding to the "integrated" part of the model) can be applied one or more times to eliminate the non-stationarity.

p,d : params from AR and MA models
I(d) - order of integration. This is simply the number of nonseasonal differences needed to make the series stationary. (In our case, it's just 1 because we used first differences.

The method is suitable for univariate time series with trend and without seasonal components

ARIMA(1,0,0)

$y_t = a_1 y_{t-1} + \epsilon_t$

ARIMA(1,0,1)

$y_t = a_1 y_{t-1} + \epsilon_t + b_1 \epsilon_{t-1}$

ARIMA(1,1,1)

$\Delta y_t = a_1 \Delta y_{t-1} + \epsilon_t + b_1 \epsilon_{t-1}$ where $\Delta y_t = y_t - y_{t-1}$

```
from statsmodels.tsa.arima_model import ARIMA
from random import random
# contrived dataset
data = [x + random() for x in range(1, 100)]
# fit model
model = ARIMA(data, order=(1, 1, 1))
model_fit = model.fit(disp=False)
# make prediction
yhat = model_fit.predict(len(data), len(data), typ='levels')
print(yhat)
```

```
import statsmodels.api as sm
from numpy.random import normal, seed
from scipy.stats import norm
from statsmodels.tsa.arima_model import ARMA
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.arima_model import ARIMA
import math
from sklearn.metrics import mean_squared_error

# Predicting the microsoft stocks volume
rcParams['figure.figsize'] = 16, 6
model = ARIMA(microsoft["Volume"].diff().iloc[1:].values, order=(2,1,0))
result = model.fit()
print(result.summary())
result.plot_predict(start=700, end=1000)
plt.show()

rmse = math.sqrt(mean_squared_error(microsoft["Volume"].diff().iloc[700:1001].values, result.predict(start=700,end=1000)))
print("The root mean squared error is {}.".format(rmse))
```

Seasonal Autoregressive Integrated Moving-Average (SARIMA)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.statespace.sarimax.SARIMAX.html
http://www.statsmodels.org/dev/generated/statsmodels.tsa.statespace.sarimax.SARIMAXResults.html

The Seasonal Autoregressive Integrated Moving Average (SARIMA) method models the next step in the sequence as a linear function of the differenced observations, errors, differenced seasonal observations, and seasonal errors at prior time steps. It combines the ARIMA model with the ability to perform the same autoregression, differencing, and moving average modeling at the seasonal level. The notation for the model involves specifying the order for the AR(p), I(d), and MA(q) models as parameters to an ARIMA function and AR(P), I(D), MA(Q) and m parameters at the seasonal level, e.g. SARIMA(p, d, q)(P, D, Q)m where "m" is the number of time steps in each season (the seasonal period). A SARIMA model can be used to develop AR, MA, ARMA and ARIMA models.

P - order of autoregression for the seasonal component of the model, which can be derived from PACF. But you need to look at the number of significant lags, which are the multiples of the season period length. For example, if the period equals 24 and we see the 24-th and 48-th lags are significant in the PACF, that means the initial P should be 2.

Q - similar logic using the ACF plot instead.

D - order of seasonal integration. This can be equal to 1 or 0, depending on whether seasonal differeces were applied or not.ARIMA model is of the form: ARIMA(p,d,q): p is AR parameter, d is differential parameter, q is MA parameter

SARIMA models are **useful for modeling seasonal time series, in which the mean and other statistics for a given season are not stationary across the years.** The SARIMA model defined constitutes a straightforward extension of the nonseasonal autoregressive-moving average (ARMA) and autoregressive integrated moving average (ARIMA) models presented

The method is suitable for **univariate time series with trend and/or seasonal components.**

```
# SARIMA example
from statsmodels.tsa.statespace.sarimax import SARIMAX
from random import random
# contrived dataset
data = [x + random() for x in range(1, 100)]
# fit model
model = SARIMAX(data, order=(1, 1, 1), seasonal_order=(1, 1, 1, 1))
model_fit = model.fit(disp=False)
# make prediction
yhat = model_fit.predict(len(data), len(data))
print(yhat)
```

```
import statsmodels.api as sm
from numpy.random import normal, seed
from scipy.stats import norm
from statsmodels.tsa.arima_model import ARMA
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.arima_model import ARIMA
import math
from sklearn.metrics import mean_squared_error

# Predicting closing price of Google'
train_sample = google["Close"].diff().iloc[1:].values
model = sm.tsa.SARIMAX(train_sample,order=(4,0,4),trend='c')
result = model.fit(maxiter=1000,disp=False)
print(result.summary())
predicted_result = result.predict(start=0, end=500)
result.plot_diagnostics()
# calculating error
rmse = math.sqrt(mean_squared_error(train_sample[1:502], predicted_result))
print("The root mean squared error is {}.".format(rmse))

plt.plot(train_sample[1:502],color='red')
plt.plot(predicted_result,color='blue')
plt.legend(['Actual','Predicted'])
plt.title('Google Closing prices')
plt.show( )
```

Finding Optimal Params for SARIMA

```
# setting initial values and some bounds for them
ps = range(2, 5)
d=1
qs = range(2, 5)
Ps = range(0, 2)
D=1
Qs = range(0, 2)
s = 24 # season length is still 24

# creating list with all the possible combinations of parameters
parameters = product(ps, qs, Ps, Qs)
parameters_list = list(parameters)
len(parameters_list)

def optimizeSARIMA(parameters_list, d, D, s):
    """
        Return dataframe with parameters and corresponding AIC

        parameters_list - list with (p, q, P, Q) tuples
        d - integration order in ARIMA model
        D - seasonal integration order
        s - length of season
    """

    results = []
    best_aic = float("inf")

    for param in tqdm_notebook(parameters_list):
        # we need try-except because on some combinations model fails to converge
        try:
            model=sm.tsa.statespace.SARIMAX(ads.Ads, order=(param[0], d, param[1]),
                                            seasonal_order=(param[2], D, param[3],
s)).fit(disp=-1)
        except:
            continue
        aic = model.aic
        # saving best model, AIC and parameters
        if aic < best_aic:
            best_model = model
            best_aic = aic
            best_param = param
        results.append([param, model.aic])

    result_table = pd.DataFrame(results)
    result_table.columns = ['parameters', 'aic']
    # sorting in ascending order, the lower AIC is - the better
    result_table = result_table.sort_values(by='aic', ascending=True).reset_index(drop=True)

    return result_table

%%time
result_table = optimizeSARIMA(parameters_list, d, D, s)

# set the parameters that give the lowest AIC
p, q, P, Q = result_table.parameters[0]
```

```
best_model=sm.tsa.statespace.SARIMAX(ads.Ads, order=(p, d, q),
                                        seasonal_order=(P, D, Q, s)).fit(disp=-1)
print(best_model.summary())

## Inspect Residuals of best model
tsplot(best_model.resid[24+1:], lags=60) # refer to the visualization of time series doc for tsplot

## Plot SARIMA
~~Evaluation Metrics~~
from sklearn.metrics import r2_score, median_absolute_error, mean_absolute_error
from sklearn.metrics import median_absolute_error, mean_squared_error, mean_squared_log_error

def mean_absolute_percentage_error(y_true, y_pred):
    return np.mean(np.abs((y_true - y_pred) / y_true)) * 100

def plotSARIMA(series, model, n_steps):
    """
        Plots model vs predicted values

        series - dataset with timeseries
        model - fitted SARIMA model
        n_steps - number of steps to predict in the future

    """
    # adding model values
    data = series.copy()
    data.columns = ['actual']
    data['arima_model'] = model.fittedvalues
    # making a shift on s+d steps, because these values were unobserved by the model
    # due to the differentiating
    data['arima_model'][:s+d] = np.NaN

    # forecasting on n_steps forward
    forecast = model.predict(start = data.shape[0], end = data.shape[0]+n_steps)
    forecast = data.arima_model.append(forecast)
    # calculate error, again having shifted on s+d steps from the beginning
    error = mean_absolute_percentage_error(data['actual'][s+d:], data['arima_model'][s+d:])

    plt.figure(figsize=(15, 7))
    plt.title("Mean Absolute Percentage Error: {0:.2f}%".format(error))
    plt.plot(forecast, color='r', label="model")
    plt.axvspan(data.index[-1], forecast.index[-1], alpha=0.5, color='lightgrey')
    plt.plot(data.actual, label="actual")
    plt.legend()
    plt.grid(True);

plotSARIMA(ads, best_model, 50)
```

Seasonal Autoregressive Integrated Moving-Average with Exogenous Regressors (SARIMAX)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.statespace.sarimax.SARIMAX.html
http://www.statsmodels.org/dev/generated/statsmodels.tsa.statespace.sarimax.SARIMAXResults.html

The Seasonal Autoregressive Integrated Moving-Average with Exogenous Regressors (SARIMAX) is an extension of

the SARIMA model that also includes the modeling of exogenous variables.
Exogenous variables are also called covariates and can be thought of as parallel input sequences that have observations at the same time steps as the original series. The primary series may be referred to as endogenous data to contrast it from the exogenous sequence(s). The observations for exogenous variables are included in the model directly at each time step and are not modeled in the same way as the primary endogenous sequence (e.g. as an AR, MA, etc. process). The SARIMAX method can also be used to model the subsumed models with exogenous variables, such as ARX, MAX, ARMAX, and ARIMAX.

The method is suitable for univariate time series with trend and/or seasonal components and exogenous variables.

```
# SARIMAX example
from statsmodels.tsa.statespace.sarimax import SARIMAX
from random import random
# contrived dataset
data1 = [x + random() for x in range(1, 100)]
data2 = [x + random() for x in range(101, 200)]
# fit model
model = SARIMAX(data1, exog=data2, order=(1, 1, 1), seasonal_order=(0, 0, 0, 0))
model_fit = model.fit(disp=False)
# make prediction
exog2 = [200 + random()]
yhat = model_fit.predict(len(data1), len(data1), exog=[exog2])
print(yhat)
```

Vector Autoregression (VAR)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.vector_ar.var_model.VAR.html
http://www.statsmodels.org/dev/generated/statsmodels.tsa.vector_ar.var_model.VARResults.html

The Vector Autoregression (VAR) method models the next step in each time series using an AR model. It is the generalization of AR to multiple parallel time series, e.g. multivariate time series.
Vector autoregression (VAR) is a stochastic process model used to capture the linear interdependencies among multiple time series. VAR models generalize the univariate autoregressive model (AR model) by allowing for more than one evolving variable. All variables in a VAR enter the model in the same way: each variable has an equation explaining its evolution based on its own lagged values, the lagged values of the other model variables, and an error term. VAR modeling does not require as much knowledge about the forces influencing a variable as do structural models with simultaneous equations: The only prior knowledge required is a list of variables which can be hypothesized to affect each other intertemporally.
The notation for the model involves specifying the order for the AR(p) model as parameters to a VAR function, e.g. VAR(p). The method is suitable for multivariate time series without trend and seasonal components.

A general VAR(p) Model:

GENERAL NOTATION HERE

A two-series, VAR(1) Model:

$$y_{1,t} = c_1 + \phi_{11,1} y_{1,t-1} + \phi_{12,1} y_{2,t-1} + u_{1,t}$$
$$y_{2,t} = c_2 + \phi_{21,1} y_{1,t-1} + \phi_{22,1} y_{2,t-1} + u_{2,t}$$

where

- $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are white noise processes that may be contemporaneously correlated.
- $\phi_{ii,l}$ captures the effect of the l^{th} lag of series y_i on itself
- $\phi_{ij,l}$ captures the effect of the l^{th} lag of series y_j on y_i

```
# VAR example
```

```
from statsmodels.tsa.vector_ar.var_model import VAR
from random import random
# contrived dataset with dependency
data = list()
for i in range(100):
v1 = i + random()
v2 = v1 + random()
row = [v1, v2]
data.append(row)
# fit model
model = VAR(data)
model_fit = model.fit()
# make prediction
yhat = model_fit.forecast(model_fit.y, steps=1)
print(yhat)
```

```
import statsmodels.api as sm
from numpy.random import normal, seed
from scipy.stats import norm
from statsmodels.tsa.arima_model import ARMA
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.arima_model import ARIMA
import math
from sklearn.metrics import mean_squared_error

# Predicting closing price of Google and microsoft
train_sample = pd.concat([google["Close"].diff().iloc[1:],microsoft["Close"].diff().iloc[1:]],axis=1)
model = sm.tsa.VARMAX(train_sample,order=(2,1),trend='c')
result = model.fit(maxiter=1000,disp=False)
print(result.summary())
predicted_result = result.predict(start=0, end=1000)
result.plot_diagnostics()
# calculating error
rmse = math.sqrt(mean_squared_error(train_sample.iloc[1:1002].values, predicted_result.values))
print("The root mean squared error is {}.".format(rmse))
```

Vector Autoregression Moving-Average (VARMA)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.statespace.varmax.VARMAX.html
http://www.statsmodels.org/dev/generated/statsmodels.tsa.statespace.varmax.VARMAXResults.html

The Vector Autoregression Moving-Average (VARMA) method models the next step in each time series using an ARMA model. It is the generalization of ARMA to multiple parallel time series, e.g. multivariate time series.

The notation for the model involves specifying the order for the AR(p) and MA(q) models as parameters to a VARMA function, e.g. VARMA(p, q). A VARMA model can also be used to develop VAR or VMA models.

The method is suitable for multivariate time series without trend and seasonal components.

```
# VARMA example
from statsmodels.tsa.statespace.varmax import VARMAX
from random import random
# contrived dataset with dependency
data = list()
for i in range(100):
v1 = random()
v2 = v1 + random()
row = [v1, v2]
data.append(row)
# fit model
```

```
model = VARMAX(data, order=(1, 1))
model_fit = model.fit(disp=False)
# make prediction
yhat = model_fit.forecast()
print(yhat)
```

Simple Exponential Smoothing (SES)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.holtwinters.SimpleExpSmoothing.html

The Simple Exponential Smoothing (SES) method models the next time step as an exponentially weighted linear function of observations at prior time steps.

The method is suitable for univariate time series without trend and seasonal components.

```
# SES example
from statsmodels.tsa.holtwinters import SimpleExpSmoothing
from random import random
# contrived dataset
data = [x + random() for x in range(1, 100)]
# fit model
model = SimpleExpSmoothing(data)
model_fit = model.fit()
# make prediction
yhat = model_fit.predict(len(data), len(data))
print(yhat)
```

Holt Winter's Exponential Smoothing (HWES)

http://www.statsmodels.org/dev/generated/statsmodels.tsa.holtwinters.HoltWintersResults.html

The Holt Winter's Exponential Smoothing (HWES) also called the Triple Exponential Smoothing method models the next time step as an exponentially weighted linear function of observations at prior time steps, taking trends and seasonality into account.

The method is suitable for univariate time series with trend and/or seasonal components.

```
# HWES example
from statsmodels.tsa.holtwinters import ExponentialSmoothing
from random import random
# contrived dataset
data = [x + random() for x in range(1, 100)]
# fit model
model = ExponentialSmoothing(data)
model_fit = model.fit()
# make prediction
yhat = model_fit.predict(len(data), len(data))
print(yhat)
```

State Space Methods

A general state space model is of the form

$y_t = Z_t \alpha_t + d_t + \varepsilon_t$

$\alpha_t = T_t \alpha_t - 1 + c_t + R_t \eta_t$

where y_t refers to the observation vector at time t, α_t refers to the (unobserved) state vector at time t, and where the irregular components are defined as

$\varepsilon_t \sim N(0, H_t)$

$\eta_t \sim N(0, Q_t)$

The remaining variables (Z_t,d_t,H_t,T_t,c_t,R_t,Q_t) in the equations are matrices describing the process. Their variable names and dimensions are as follows

Z : design (k_endog×k_states×nobs)

d : obs_intercept (k_endog×nobs)

H : obs_cov (k_endog×k_endog×nobs)

T : transition (k_states×k_states×nobs)

c : state_intercept (k_states×nobs)

R : selection (k_states×k_posdef×nobs)

Q : state_cov (k_posdef×k_posdef×nobs)

In the case that one of the matrices is time-invariant (so that, for example, Z_t=Z_t+1 ∀ $_t$), its last dimension may be of size 1 rather than size nobs.

This generic form encapsulates many of the most popular linear time series models (see below) and is very flexible, allowing estimation with missing observations, forecasting, impulse response functions, and much more.

Unobserved Components

A UCM decomposes the response series into components such as trend, seasons, cycles, and the regression effects due to predictor series. The following model shows a possible scenario:

$$\begin{aligned} y_t &= \mu_t + \gamma_t + \psi_t + \sum_{j=1}^{m} \beta_j x_{jt} + \epsilon_t \\ \epsilon_t &\sim i.i.d.\ N(0, \sigma_\epsilon^2) \end{aligned}$$

Source: http://support.sas.com/documentation/cdl/en/etsug/66840/HTML/default/viewer.htm#etsug_ucm_details01.htm

```
import statsmodels.api as sm
from numpy.random import normal, seed
from scipy.stats import norm
from statsmodels.tsa.arima_model import ARMA
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.arima_model import ARIMA
import math
from sklearn.metrics import mean_squared_error

# Predicting closing price of Google'
train_sample = google["Close"].diff().iloc[1:].values
model = sm.tsa.UnobservedComponents(train_sample,'local level')
result = model.fit(maxiter=1000,disp=False)
print(result.summary())
predicted_result = result.predict(start=0, end=500)
result.plot_diagnostics()
# calculating error
rmse = math.sqrt(mean_squared_error(train_sample[1:502], predicted_result))
print("The root mean squared error is {}.".format(rmse))

plt.plot(train_sample[1:502],color='red')
plt.plot(predicted_result,color='blue')
plt.legend(['Actual','Predicted'])
plt.title('Google Closing prices')
plt.show()
```

Dynamic Factor Models

Dynamic-factor models are flexible models for multivariate time series in which the observed endogenous variables are linear functions of exogenous covariates and unobserved factors, which have a vector autoregressive structure.

The unobserved factors may also be a function of exogenous covariates. The disturbances in the equations for the dependent variables may be autocorrelated.

```
import statsmodels.api as sm
from numpy.random import normal, seed
from scipy.stats import norm
from statsmodels.tsa.arima_model import ARMA
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima_process import ArmaProcess
from statsmodels.tsa.arima_model import ARIMA
import math
from sklearn.metrics import mean_squared_error

# Predicting closing price of Google and microsoft
train_sample = pd.concat([google["Close"].diff().iloc[1:],microsoft["Close"].diff().iloc[1:]],axis=1)
model = sm.tsa.DynamicFactor(train_sample, k_factors=1, factor_order=2)
result = model.fit(maxiter=1000,disp=False)
print(result.summary())
predicted_result = result.predict(start=0, end=1000)
result.plot_diagnostics()
# calculating error
rmse = math.sqrt(mean_squared_error(train_sample.iloc[1:1002].values, predicted_result.values))
print("The root mean squared error is {}.".format(rmse))
```

LSTM
https://www.kaggle.com/thebrownviking20/intro-to-recurrent-neural-networks-lstm-gru</u>**