4.6 Vermischte Grenzwerte

a) $\lim_{x \uparrow 1} 2^{\frac{1}{x-1}} \Rightarrow 0$

b) $\lim_{x \downarrow \frac{\pi}{4}} 3^{\tan 2x} \Rightarrow \lim_{x \downarrow \frac{\pi}{2}} 3^{\tan x} \Rightarrow 0$

c) $\lim_{x \to 0} \frac{\sin(\sin x)}{x} = \lim_{x \to 0} \frac{\sin(\sin x)}{x} \cdot \frac{\sin x}{\sin x} \Rightarrow \lim_{x \to 0} \frac{\sin(\sin x)}{\sin x} \cdot \lim_{x \to 0} \frac{\sin x}{x} \Rightarrow \lim_{x \to 0} \frac{\sin(\sin x)}{\sin x}$

$\sin x = y \Rightarrow \lim_{y \to 0} \frac{\sin y}{y} \Rightarrow 1$

d) $\lim_{x \downarrow 0} \frac{\sin x}{\sqrt{x^3}} = \lim_{x \downarrow 0} \frac{\sin x}{\sqrt{x^3}} \cdot \frac{x}{x} \Rightarrow \lim_{x \downarrow 0} \frac{\sin x}{x} \cdot \lim_{x \downarrow 0} \frac{x}{\sqrt{x^3}} \Rightarrow \lim_{x \downarrow 0} \frac{x}{\sqrt{x^3}} = \lim_{x \to} \frac{x^1}{x^{\frac{3}{2}}} = \lim_{x \to} x^{-\frac{1}{2}} = \frac{\lim 1}{\sqrt{x}} \Rightarrow \infty$

e) $\lim_{x \uparrow \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x$, sei $y = \frac{\pi}{2} - x \Rightarrow \lim_{y \downarrow 0}$ u. $\tan\left(\frac{\pi}{2} - y\right) = \lim_{y \downarrow 0} y \cdot \frac{\sin\left(\frac{\pi}{2} - y\right)}{\cos\left(\frac{\pi}{2} - y\right)}$

$= \lim_{y \downarrow 0} \frac{\sin\frac{\pi}{2}\cos y - \sin y \cos\frac{\pi}{2}}{\cos\frac{\pi}{2}\cos y + \sin\frac{\pi}{2}\sin y} \cdot y = \lim_{x \downarrow 0} \frac{\cos y}{\sin y} \cdot y = \lim_{y \downarrow 0} \cos y \cdot \frac{x}{\sin y}$ $\lim_{y \downarrow 0} \cos y \Rightarrow 1$

f) $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x \to 0} \frac{2\sin x \cos x}{3\sin x \cos x} = \lim_{x \to 0} \frac{2}{3} \Rightarrow \frac{2}{3}$

g) $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \lim_{x \to 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)} = \lim_{x \to 0} \frac{1 - \frac{1}{2}(1 + \cos 2x)}{x^2(1 + \cos x)}$?

h) $\lim_{x \to \infty} \left(1 + \frac{\alpha}{x}\right)^x \quad (\alpha > 0)$