2.2 Intervalle

a) $\{x \in \mathbb{R} \mid |x-1| = |x-3|\}$

$\{2\}$

b) $\{x \in \mathbb{R} \mid \frac{|x-1|}{|x+1|} = 2\}$

$\frac{|x-1|}{|x+1|} = 2 \Leftrightarrow |x-1| = 2|x+1|$

1. $x-1 \geq 0 \rightarrow x \geq 1$

$x-1 = 2(x+1) \Leftrightarrow x-1 = 2x+2 \Leftrightarrow x = -3 \quad \{-3\} \notin \{\mathbb{R}^{\geq 1}\}$

2. $x-1 < 0 \rightarrow x < 1$

$-(x-1) = 2(x+1) \Leftrightarrow -x+1 = 2x+2 \Leftrightarrow -1 = 3x \Leftrightarrow x = -\frac{1}{3}$

$-(x-1) = -2(x+1) \Leftrightarrow -x+1 = -2x-2 \Leftrightarrow 3 = -x \Leftrightarrow x = -3$

$[-3, -\frac{1}{3}]$

c) $\{x \in \mathbb{R} \mid x^2 - x + 10 > 16\}$

$x^2 - x + 10 = 16 \Leftrightarrow 0 = x^2 - x - 6 \Leftrightarrow x_{1,2} = -\frac{-1}{2} \pm \sqrt{(\frac{-1}{2})^2 + 6} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + 6} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{24}{4}} = \frac{1}{2} \pm \frac{5}{2} = \begin{cases} \frac{-4}{2} = -2 \\ \frac{6}{2} = 3 \end{cases}$

Es ist eine nach oben geöffnete Parabel:

$]-\infty, -2[,]3, \infty[$ oder $\mathbb{R} \setminus [-2, 3]$

d) $\bigcup_{n \in \mathbb{N}} [1+\frac{1}{n}, 3-\frac{1}{n}]$

$\lim_{n \to \infty} 1 + \frac{1}{n} \Rightarrow 1$

$\lim_{n \to \infty} 3 - \frac{1}{n} \Rightarrow n$

$\Rightarrow]1, 3[$