4.7 Asymptoten

a) $f(x) = \frac{4-3x^2}{5x+1}$ $\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{4-3x^2}{5x+1} - (mx+5) \overset{!}{=} 0 \Leftrightarrow \lim_{x\to\infty} \frac{4-3x^2-(mx+5)(5x+1)}{5x+1} = 0$

$\Leftrightarrow \lim_{x\to\infty} \frac{4-3x^2-(5mx^2+mx+55x+5)}{5x+1} = 0 \Leftrightarrow \lim_{x\to\infty} \frac{x^2(-3-5m)-x(m+55)+4-5}{5x+1} = 0 \Leftrightarrow \lim_{x\to\infty} \frac{\frac{1}{x}(-3-5m)-(m+55)+\frac{1}{x}(4-5)}{5+\frac{1}{x}} = 0$

$\Rightarrow -3x - 5mx = 0 \quad \wedge \quad -m - 55 = 0 \Leftrightarrow -3x = 5mx \wedge -m = 55 \Leftrightarrow m = -\frac{3}{5} \wedge b = -\frac{m}{5} = \frac{3}{25}$

$\Rightarrow g(x) = -\frac{3}{5}x + \frac{3}{25}$

b) $f(x) = \frac{5x^2-3x-2}{3x^2-5x+9}$ $\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{5x^2-3x-2}{3x^2-5x+9} - mx+5 = 0 \Leftrightarrow \lim_{x\to\infty} \frac{5x^2-3x-2-(mx+5)(3x^2-5x+9)}{3x^2-5x+9} = 0$

$\Leftrightarrow \lim_{x\to\infty} \frac{-3mx^3+x^2(5+5m-35)+x(-3-9+55)-2-45}{3x^2-5x+9} = 0 \Leftrightarrow \lim_{x\to\infty} \frac{-3mx+(5+5m-35)+\frac{1}{x}(-12+55)-\frac{2}{x^2}(2-36)}{3-5\frac{1}{x}+9\frac{1}{x^2}} = 0$

$\Rightarrow -3m = 0 \wedge 5+5m-35 = 0 \Leftrightarrow m = 0 \wedge 5 = 35 \Leftrightarrow m = 0 \wedge b = \frac{5}{3}$

$\Rightarrow g(x) = \frac{5}{3}$

c) $f(x) = \frac{x^4-6}{7x^2}$ $\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{x^4-6}{7x^2} - mx+5 = 0 \Leftrightarrow \lim_{x\to\infty} \frac{x^4-6-(mx+5)(7x^2)}{7x^2} = 0 \Leftrightarrow \lim_{x\to\infty} \frac{x^4-7mx^3+35x^2-6}{7x^2} = 0$

$\Leftrightarrow \lim_{x\to\infty} \frac{x^2-7mx+35-\frac{6}{x^2}}{7} = 0$

$\Rightarrow -7m = 0 \wedge 35 = 0 \Leftrightarrow m = 0 \wedge 5 = 0$

d) $f(x) = \frac{x^3+x+13}{7-4x}$ $\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{x^3+x+13}{7-4x} - mx+5 = 0 \Leftrightarrow \lim_{x\to\infty} \frac{x^3+x+13-(mx+5)(7-4x)}{7-4x} = 0 \Leftrightarrow \lim_{x\to\infty} \frac{x^3+4mx^2+x(-7m+45)+13-35}{7-4x} = 0$

$\Leftrightarrow \lim_{x\to\infty} \frac{x^2+4mx+(-7+45)+\frac{1}{x}(13-35)}{7\frac{1}{x}-4}$

$\Rightarrow x^2 = 0 \wedge 4m = 0 \wedge -7+45 = 0 \rightarrow$ keine Asymptote