4.13 Ganzrationale Funktionen

a) $f(x) = x^3 - x^2 - x + 1$

1. Linearfaktor durch sehen: $(x-1)$

$x^3 - x^2 - x + 1 : x - 1 = x^2 - 1$

$-(x^3 - x^2)$

$-x + 1$

$-(-x + 1)$

0

$\Rightarrow (x-1)(x^2-1)$

trivial: $\Rightarrow (x-1)^2(x+1)$ ✓

b) $f(x) = 2x^3 + 4x^2 + 4x + 2$

1. Linearfaktor durch sehen: $x+1$

$2x^3 + 4x^2 + 4x + 2 : x + 1 = 2x^2 + 2x + 2$

$-(2x^3 + 2x^2)$

$2x^2 + 4x + 2$

$-(2x^2 + 2x)$

$2x + 2$

$-(2x + 2)$

0

pq-Formel: $0 = 2x^2 + 2x + 2 \Leftrightarrow 0 = x^2 + x + 1 \Rightarrow x_{1/2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 1}$ ⇒ Wurzel von negativ ↯

$\Rightarrow f(x) = (x+1)(2x^2+2x+2)$ ✓

c) $f(x) = -x^2 + x + 6$

pq-Formel: $0 = -x^2 + x + 6 \Leftrightarrow 0 = x^2 - x - 6 \Rightarrow x_{1/2} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 6} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{24}{4}} = \frac{1}{2} \pm \frac{5}{2} \Rightarrow \begin{cases} 3 \\ -2 \end{cases}$

$\Rightarrow f(x) = (x-3)(x+2)(-1)$ ✓

d) $f(x) = x^4 - 191x^2 - 980$

Sei $x^2 = t$: pq-Formel: $0 = t^2 - 191t - 980 \Rightarrow t_{1/2} = \frac{191}{2} \pm \sqrt{\left(\frac{191}{2}\right)^2 + 980} = \frac{191}{2} \pm \sqrt{\frac{36481}{4} + \frac{3920}{4}} = \frac{191}{2} \pm \frac{201}{2}$

$= \begin{cases} 196 \\ -5 \end{cases} \Rightarrow \begin{cases} x = \sqrt{196} = 14 \\ \nu = \sqrt{-5} \notin \end{cases} \Rightarrow x^2 = -5 \Leftrightarrow 0 = x^2 + 5$

$\Rightarrow f(x) = (x+14)(x-14)(x^2+5)$

e) $f(x) = 2x^2 + 4x + 20$

pq-Formel: $0 = 2x^2 + 4x + 20 \Leftrightarrow 0 = x^2 + 2x + 10 \Rightarrow x_{1/2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 10} = -1 \pm \sqrt{-9}$ ↯

$\Rightarrow f(x) = 2x^2 + 4x + 20$ ✓

$4x^2 - 1 : x - \frac{1}{\sqrt{2}} = 4x + \frac{4}{\sqrt{2}}$

$-(4x^2 - \frac{4x}{\sqrt{2}})$

$\frac{4x}{\sqrt{2}} - 1$

$-(\frac{4x}{\sqrt{2}} - 2)$

1

f) $f(x) = 4x^4 - 1$

Sei $x^2 = t \Rightarrow 4t^2 - 1$, pq-Formel: $0 = 4t^2 - 1 \Leftrightarrow 0 = t^2 - \frac{1}{4} \Rightarrow t_{1/2} = \pm\sqrt{\frac{1}{4}} = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases} \Rightarrow x_{1/2} = \begin{cases} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{cases}$