7.8 | Regel von L'Hospital

a) $\lim\limits_{x \to \pi} \frac{\sin 3x}{\tan 5x}$

Regel von L'Hospital: $\lim\limits_{x \to u} \frac{f(x)}{g(x)} = \lim\limits_{x \to u} \frac{f'(x)}{g'(x)} = \lim\limits_{x \to u} \frac{f^{(n)}(x)}{g^{(n)}(x)}$, wenn $\lim\limits_{x \to u} f(x) = \lim\limits_{x \to u} g^{(n)}(x)$

$$\left.\begin{array}{l} \lim\limits_{x \to \pi} \sin 3x \Rightarrow \sin 3\pi = 0 \\ \lim\limits_{x \to \pi} \tan 5x \Rightarrow \tan 5\pi = 0 \end{array}\right\} \text{sind gleich}$$

$(\sin 3x)' = 3\cos 3x$

$(\tan 5x)' = 5(1 + \tan^2 5x)$

$\lim\limits_{x \to \pi} \frac{3\cos 3x}{5(1+\tan^2 5x)} \Rightarrow \frac{3\cos 3\pi}{5(1+\tan^2 5\pi)} = \frac{0}{5} = 0$ //

b) $\lim\limits_{x \to 0} 2x \frac{\sin 2x}{\sinh^2 x}$ $\quad \sinh = \frac{1}{2}(e^x - e^{-x})$

$$\left.\begin{array}{l} \lim\limits_{x \to 0} 2x \sin 2x \Rightarrow 2 \cdot 0 \cdot \sin(2 \cdot 0) = 0 \\ \lim\limits_{x \to 0} \sinh^2 x = \lim\limits_{x \to 0} \left(\frac{1}{2}(e^x - e^{-x})\right)^2 \Rightarrow \left(\frac{1}{2}(e^0 - e^0)\right)^2 = 0 \end{array}\right\} \text{sind gleich}$$

$(2x \sin 2x)' = (2x)' \sin 2x + 2x(\sin 2x)' = 2\sin 2x + 4x\cos 2x$

$(\sinh^2 x)' = \left(\left(\frac{1}{2}(e^x - e^{-x})\right)^2\right)' = \frac{1}{2}(e^x + e^{-x}) \cdot (e^x - e^{-x}) = \frac{1}{2}(e^{2x} - e^{-2x})$

$$\left.\begin{array}{l} \lim\limits_{x \to 0} 2\sin 2x + 4x\cos 2x \rightarrow 2\sin 0 + 0 = 0 \\ \lim\limits_{x \to 0} \frac{1}{2}(e^{2x} - e^{-2x}) \Rightarrow \frac{1}{2}(e^0 - e^0) = 0 \end{array}\right\} \text{sind gleich}$$

$(2x \sin 2x)'' = (2\sin 2x)' + (4x)'\cos 2x + 4x(\cos 2x)' = 4\cos 2x + 4\cos 2x - 8\sin 2x$

$(\sinh^2 x)'' = \frac{1}{2}(e^{2x} - e^{-2x})' = \frac{1}{2}(2e^{2x} + 2e^{-2x}) = e^{2x} + e^{-2x}$

$\lim\limits_{x \to 0} \frac{\sin 2x}{\sinh^2 x} = \lim\limits_{x \to 0} \frac{4\cos 2x + 4\cos 2x - 8\sin 2x}{e^{2x} + e^{2x}} \Rightarrow \frac{4+4-0}{1+1} = 4$ //