## 6.1 Rechenregeln für komplexe Zahlen

Addition: $z_1 + z_2 = (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

Multiplikation: $z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2) = (x_1 \cdot x_2 - y_1 \cdot y_2, x_1 \cdot y_2 - x_2 \cdot y_1)$

Assoziativgesetz:

$z_1 + (z_2 + z_3) = (x_1, y_1) + ((x_2, y_2) + (x_3, y_3)) = (x_1, y_1) + (x_1 + x_2, y_1 + y_2) = (x_1 + x_2 + x_3, y_1 + y_2 + y_3)$

$(z_1 + z_2) + z_3 = ((x_1, y_1) + (x_2, y_2)) + (x_3, y_3) = (x_1 + x_2, y_1 + y_2) + (x_3, y_3) = (x_1 + x_2 + x_3, y_1 + y_2 + y_3)$

$z_1 \cdot (z_2 \cdot z_3) = (x_1, y_1) \cdot ((x_2, y_2) \cdot (x_3, y_3)) = (x_1, y_1) \cdot (x_2 \cdot x_3 - y_2 \cdot y_3, x_2 \cdot y_3 - x_3 \cdot y_2)$

$= (x_1 (x_2 \cdot x_3 - y_2 \cdot y_3) - y_1 (x_2 y_3 - x_3 y_2), x_1 (x_2 y_3 - x_3 y_2) - y_1 (x_2 \cdot x_3 - y_2 \cdot y_2))$

$= (x_1 x_2 x_3 - x_1 y_2 y_3 - x_2 y_1 y_3 - x_3 y_1 y_2, x_1 x_2 y_3 - x_1 x_3 y_2 - x_2 x_3 y_1 - y_1 y_2 y_3)$

$(z_1 \cdot z_2) \cdot z_3 = ((x_1, y_1) \cdot (x_2, y_2)) \cdot (x_3, y_3) = (x_1 x_2 - y_1 y_2 - x_1 y_2 - x_2 y_1) \cdot (x_3, y_3)$

$= (x_3 (x_1 x_2 - y_1 y_2) - y_3 (x_1 y_2 - x_2 y_1), (x_1 x_2 - y_1 y_2) y_3 + (x_1 y_2 - x_2 y_1) x_3)$

$= (x_1 x_2 x_3 - x_3 y_1 y_2 - x_1 y_2 y_3 - x_2 y_1 y_3, x_1 x_2 y_3 - y_1 y_2 y_3 - x_1 x_3 y_2 - x_2 x_3 y_1)$