8.6 I

a) $\int \frac{1}{x^2-x}\,dx$

Zerlege $N(x)$ in Linearfaktoren und bestimme Nullstellen:

$N(x) = x^2 - x$

pq-Formel:

$x_{1/2} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2} \Rightarrow x_1 = 1,\ x_2 = 0$

$$x^2 - x : x - 1 = x$$
$$\underline{-x^2 - x}$$
$$0$$

Linearfaktoren:

$x^2 - x = x\,(x-1)^1$

Partialbrüche Nullstellen zuordnen:

$\frac{1}{x^2-x} = \frac{A}{x+0} + \frac{B}{x-1}$

Brüche auf Hauptnenner bringen:

$1 = A(x-1) + B(x+0)$
$x = 1 \Rightarrow A(1-1) + B(1+0) \Rightarrow B = 1$
$x = 0 \Rightarrow A(0-1) + B(0+0) \Rightarrow A = -1$

$\frac{1}{x^2-x} = -\frac{1}{x} + \frac{1}{x-1}$

Integrieren:

$\int \frac{1}{x^2-x} = -\int \frac{1}{x}\,dx + \int \frac{1}{x-1}\,dx = -\ln(|x|) + \ln(|x-1|) + c$

b) $\int \frac{u}{u+4}\,du$

Polynomdivision:

$$u : u+4 = 1 - \frac{4}{u+4}$$
$$\underline{-(u+4)}$$
$$-4$$

$N(x)$ in Linearfaktoren und bestimme Nullstellen: → ist schon Linearfaktor

Integrieren:

$\int \frac{u}{u+4}\,du = -\int \frac{4}{u+4}\,du = -4\int \frac{1}{u+4}\,du = -4\ln(|u+4|) + c$

c) $\int \frac{1}{x^2+3x-10}\,dx$

Polynomdivision unnötig

Linearfaktoren und Nullstellen:

$N(x) = x^2+3x-10$

pq-Formel:

$$x_{1/2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2+10}$$
$$= -\frac{3}{2} \pm \sqrt{\frac{49}{4}}$$
$$= -\frac{3}{2} \pm \frac{7}{2}$$
$$\Rightarrow x_1 = -5,\ x_2 = 2$$

$N(x) = (x+5)(x-2)$

Partialbrüche:

$$\frac{1}{x^2+3x-10} = \frac{A}{x+5} + \frac{B}{x-2}$$

Hauptnenner:

$$1 = A(x-2) + B(x+5)$$
$$x=2 \Rightarrow 1 = 7B \Rightarrow B = \frac{1}{7}$$
$$x=-5 \Rightarrow 1 = -7A \Rightarrow A = -\frac{1}{7}$$

$$\frac{1}{x^2+3x-10} = -\frac{1}{7(x-2)} + \frac{1}{7(x+5)}$$

Integrieren:

$$\int \frac{1}{x^2+3x-10} = -\int \frac{1}{7(x-2)} + \int \frac{1}{7(x+5)} = -\ln(|7(x-2)|) + \ln(|7(x+5)|) = -\ln(|7x-14|) + \ln(|7x+35|)$$

d) $\int \frac{1}{t^3+3t^2-4}\,dt$

$N(t) = t^3+3t^2-4 \quad N(t) = 0$ bei $t=1$

$$t^3+3t^2-4 : t-1 = t^2+4t+4 = (t+2)^2 \Rightarrow N(t) = (t-1)(t+2)^2$$
$$-(t^3-t^2)$$
$$4t^2-4$$
$$-(4t^2-4t)$$
$$4t-4$$
$$-(4t-4)$$
$$0$$

$$\frac{1}{t^3+3t^2-4} = \frac{A}{t-1} + \frac{B}{t+2} + \frac{C}{(t+2)^2}$$

$$1 = A(t+2)^2 + B(t-1)(t+2) + C(t-1)$$
$$x=-2 \Rightarrow 1 = -3C \Rightarrow C = -\frac{1}{3}$$
$$x=1 \Rightarrow 1 = 9A \Rightarrow A = \frac{1}{9}$$
$$x=0 \Rightarrow 1 = \frac{1}{9}\cdot 4 - 2B + \frac{1}{3}$$
$$\Leftrightarrow \frac{2}{9} = -2B \Rightarrow B = -\frac{1}{9}$$

$$\frac{1}{t^3+3t^2-4} = \frac{1}{9(t-1)} - \frac{1}{9(t+2)} - \frac{1}{3(t+2)^2}$$

$$\int \frac{1}{t^3+3t^2-4} = \int \frac{1}{9(t-1)} - \int \frac{1}{9(t+2)} - \int \frac{1}{3(t+2)^2} = \ln(|9t-9|) - \ln(|9t+18|) - \ln(|3(t+2)^2|)$$