4.4 Grenzwerte $x \to \pm\infty$

a) $\lim\limits_{x\to\pm\infty} \frac{x^2-3}{2x^2+5} = \lim\limits_{x\to\pm\infty} \frac{1-3\frac{1}{x^2}}{2+5\frac{1}{x^2}} \Rightarrow \frac{1-0}{2+0} = \frac{1}{2}$

b) $\lim\limits_{x\to\pm\infty} \left(\frac{x^4}{x^3-1} - x\right) = \lim\limits_{x\to\pm\infty} \frac{x^4-x^4+x}{x^3-1} = \lim\limits_{x\to\pm\infty} \frac{x}{x^3-1} \Rightarrow \lim\limits_{x\to\infty} \frac{\frac{1}{x^2}}{1-\frac{1}{x^3}} \to 0$

$\Rightarrow \lim\limits_{x\to-\infty} \frac{\frac{1}{x^2}}{1-\frac{1}{x^3}} \to 0$

c) $\lim\limits_{x\to\pm\infty} \frac{7+13x^2-6x^3}{3x^3-5x} = \lim\limits_{x\to\pm\infty} \frac{-6+13\frac{1}{x}+7\frac{1}{x^3}}{3-5\frac{1}{x^2}} \to \frac{-6}{3} = -2$

d) $\lim_{x\to\infty}\left(\sqrt{x+2}-\sqrt{x}\right)=\lim_{x\to\infty}\sqrt{x}\left(\sqrt{1+\frac{2}{x}}-\sqrt{1}\right)\Rightarrow\sqrt{x}\cdot(1-1)=0$

e) $\lim_{x\to\pm\infty}\frac{x^3-4x}{x^2+1}=\lim_{x\to\pm\infty}\frac{1-4\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^3}}\Rightarrow\infty$

f) $\lim_{x\to\pm\infty}\left(\frac{x^2+2}{2x^2-3}\right)^2\Rightarrow\left(\lim_{x\to\pm\infty}\frac{1+2\frac{1}{x^2}}{2-3\frac{1}{x^2}}\right)^2\Rightarrow 0^2=0$

g) $\lim_{x\to\pm\infty}\sqrt{\frac{18x^2+2x}{2x^2-1}}=\lim_{x\to\pm\infty}\sqrt{\frac{18+\frac{2}{x}}{2-\frac{1}{x^2}}}\Rightarrow\sqrt{\frac{18}{2}}=\sqrt{9}=3$

h) $\lim_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{5}}{\sqrt{4x^2+x}-\frac{1}{2}x}=\lim_{x\to\infty}\frac{x\sqrt{1+\frac{1}{x^2}}+x\sqrt{\frac{5}{x^2}}}{x\sqrt{4+\frac{1}{x}}-\frac{1}{2}x}=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x^2}}+\sqrt{\frac{5}{x^2}}}{\sqrt{4+\frac{1}{x}}-\frac{1}{2}}\Rightarrow\frac{1}{-\frac{1}{2}}=-2$

i) $\lim_{x\to\pm\infty}\frac{|x|}{x}\Rightarrow\lim_{x\to\infty}\frac{x}{x}\Rightarrow 1$

$\Rightarrow\lim_{x\to-\infty}\frac{-x}{x}\Rightarrow 1$

j) $\lim_{x\to\pm\infty}\frac{\sqrt{x^2+1}}{x+1}=\lim_{x\to\pm\infty}\frac{x\sqrt{1+\frac{1}{x^2}}}{x(1+\frac{1}{x})}=\lim_{x\to\pm\infty}\frac{\sqrt{1+\frac{1}{x^2}}}{1+\frac{1}{x}}\Rightarrow\lim_{x\to+\infty}\frac{1}{1}=1$

$\Rightarrow\lim_{x\to-\infty}\frac{1}{-1}=-1$

k) $\lim_{x\to\infty}x\left(\sqrt{x^2+1}-x\right)=\lim_{x\to\infty}\frac{x^4+x^2-x^4}{\sqrt{x^4+x^2}+x^2}=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1}\Rightarrow\frac{1}{2}$

l) $\lim_{x\to\pm\infty}\frac{x}{1+x^2}=\lim_{x\to\pm\infty}\frac{\frac{1}{x}}{\frac{1}{x^2}+1}\Rightarrow 0$

m) $\lim_{x\to\pm\infty}\left(\frac{6x+2}{7x^2-3}\right)\left(\frac{4}{3}x-5\frac{1}{x^2}\right)=\lim_{x\to\pm\infty}\left(\frac{6\frac{1}{x}+2\frac{1}{x^2}}{7-3\frac{1}{x^2}}\right)\left(\frac{4x^3-15}{3x^2}\right)=\lim_{x\to\pm\infty}\frac{24x^2+8x-90\frac{1}{x}-30\frac{1}{x^2}}{21x^2-9}=\lim_{x\to\pm\infty}\frac{24+8\frac{1}{x}-90\frac{1}{x^2}-30\frac{1}{x^4}}{21-9\frac{1}{x^2}}$

$\Rightarrow\frac{24}{21}=\frac{8}{7}$

n) $\lim_{x\to\pm\infty}\frac{x^2}{(x-3)^2}=\lim_{x\to\pm\infty}\frac{x^2}{x^2-6x+9}=\lim_{x\to\pm\infty}\frac{1}{1-6\frac{1}{x}+9\frac{1}{x^2}}\Rightarrow 1$

o) $\lim_{x\to\pm\infty}\sqrt[3]{\frac{(2x-1)(4x+7)}{1+x^2}}=\lim_{x\to\pm\infty}\sqrt[3]{\frac{8x^2+14x-4x-7}{1+x^2}}=\lim_{x\to\pm\infty}\sqrt[3]{\frac{8+10\frac{1}{x}-7\frac{1}{x^2}}{1+\frac{1}{x^2}}}\Rightarrow\sqrt[3]{8}=2$