8.7)

b) $\int \frac{u}{u^2+4u+3}\,du$

$N(x) = u^2+4u+3$

$u_{1/2} = -\frac{4}{2} \pm \sqrt{(\frac{4}{2})^2-3}$

$= -\frac{4}{2} \pm \sqrt{\frac{16}{4}-3}$

$= -2 \pm 1 \Rightarrow x_1 = -3, x_2 = -1 \Rightarrow N(u) = (u+3)(u+1)$

$\frac{u}{u^2+4u+3} = \frac{A}{u+3} + \frac{B}{u+1}$

$u = A(u+1) + B(u+3)$

$u = -1 \Rightarrow -1 = 2B \Rightarrow B = -\frac{1}{2}$

$u = -3 \Rightarrow -3 = -2A \Rightarrow A = \frac{3}{2}$

$\frac{u}{u^2+4u+3} = \frac{3}{2(u+3)} - \frac{1}{2(u+1)}$

$\int \frac{u}{u^2+4u+3} = \int \frac{3}{2(u+3)} - \int \frac{1}{2(u+1)} = 3\int \frac{1}{2(u+3)} - \int \frac{1}{2(u+1)} = 3\ln(|2u+6|) - \ln(|2u+2|) + c$

c) $\int (1+e^{2x})^2 e^{2x}\,dx$

Substitution:

sei $x = \frac{1}{2}\ln h$

$\int (1+e^{2\cdot\frac{1}{2}\ln h})^2 e^{2\cdot\frac{1}{2}\ln h} \cdot (\frac{1}{2}\ln h)'\,dh = \int (1+h)^2 \cdot h \cdot \frac{1}{2h}\,dh = \frac{1}{2}\int (1+h)^2 = \frac{1}{2}\int h^2+2h+1\,dh$

$= \frac{1}{2}\cdot(\frac{1}{3}h^3 + h^2 + h)$

Rücksubstitution:

$x = \frac{1}{2}\ln h \Leftrightarrow 2x = \ln h \Leftrightarrow e^{2x} = h$

$\frac{1}{2}\cdot(\frac{1}{3}\cdot(e^{2x})^3 + (e^{2x})^2 + e^{2x}) = \frac{1}{2}(\frac{1}{3}e^{6x} + e^{4x} + e^{2x})$

sei $h = t-1$

$\frac{1}{2}\int (1+t-1)^2\,dt = \frac{1}{2}\int t^2\,dt = \frac{1}{2}\cdot\frac{1}{3}t^3 = \frac{1}{6}t^3$

Rücksubstitution:

$\frac{1}{6}(h+1)^3$

$\frac{1}{6}(1+e^{2x})^3$

d) $\int \frac{1}{x}\ln x\,dx$

Partielle Integration:

$\int \frac{1}{x}\cdot\ln x\,dx = \ln x \cdot \ln x - \int \ln x \cdot \frac{1}{x}\,dx \Leftrightarrow 2\int \frac{1}{x}\cdot\ln x\,dx = \ln^2 x \Leftrightarrow \int \frac{1}{x}\cdot\ln x\,dx = \frac{\ln^2 x}{2}$