5.5) Exponentielle Darstellung komplexer Zahlen

a) $1+i\sqrt{3} = r\cos\varphi + i\sin\varphi \Rightarrow 2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) \Rightarrow 2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right) = 2e^{i\frac{\pi}{3}}$

b) $-5 = r(\cos\varphi - i\sin\varphi) = -5(\overset{1}{\cos\varphi} + i\overset{0}{\sin\varphi}) \Rightarrow -5e^{i2\pi}$

c) $-5-i5 = r(\cos\varphi + i\sin\varphi) \Rightarrow -5\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) \Rightarrow -5\sqrt{2}e^{i\frac{\pi}{4}}$

d) $(1-i\sqrt{3})^3 = (1,-\sqrt{3})^3 = (1,-\sqrt{3})(1,-\sqrt{3})(1,-\sqrt{3}) = (1\cdot1-(-\sqrt{3})\cdot(-\sqrt{3}),\ 1\cdot(-\sqrt{3})+1\cdot(-\sqrt{3}))(1,-\sqrt{3})$
$= (-2,-2\sqrt{3})(1,-\sqrt{3}) = (-2\cdot1-(-2\sqrt{3})\cdot(-\sqrt{3}),\ -2\cdot(-\sqrt{3})+1\cdot(-2\sqrt{3}))$
$= (-2-6,0) = (-8,0) \Rightarrow -8 \Rightarrow -8e^{i2\pi}$

e) $(1+i)^5 = (1+i)(1+i)(1+i)^2(1+i) = (1,1)(1,1)(1,1)^2(1,1) = (1\cdot1-1\cdot1,\ 1\cdot1+1\cdot1)(1,1)^2(1,1)$
$= (0,2)(0,2)(1,1) = (0\cdot0-2\cdot2,\ 2\cdot0+2\cdot0)(1,1) = (-4,0)(1,1) = (-4\cdot1-0\cdot1,\ -4\cdot1+0\cdot1) = (-4,-4)$
$= -4-i4 = -4(1+i) = -4\sqrt{2}\,e^{i\frac{\pi}{4}}$

f) $(\sqrt{3}+i^3)(1-i) = (\sqrt{3}-i)(1-i) = (\sqrt{3}\cdot1-(-1)\cdot(-1),\ \sqrt{3}\cdot(-1)+1\cdot(-1)) = (\sqrt{3}-1,\ -\sqrt{3}-1)$
$\frac{\pi}{12}$ sind doof

g) $\dfrac{2-i\frac{6}{\sqrt{3}}}{2+i\frac{6}{\sqrt{3}}}$ NR: $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ $\quad \dfrac{2-i2\sqrt{3}}{2+i2\sqrt{3}} \Rightarrow \dfrac{(2,-2\sqrt{3})}{(2,2\sqrt{3})} = (2,-2\sqrt{3})\left(\frac{2}{2^2+2^2\cdot3},\ \frac{-2\sqrt{3}}{2^2+2^2\cdot3}\right)$
$= (2,-2\sqrt{3})\left(\frac{1}{8},\frac{-\sqrt{3}}{8}\right) = \left(2\cdot\frac{1}{8}-(-2\sqrt{3})\cdot\left(\frac{-\sqrt{3}}{8}\right),\ 2\cdot\frac{(-\sqrt{3})}{8}+\frac{1}{8}\cdot(-2\sqrt{3})\right) = \left(\frac{1}{4}-\frac{3}{4},\ \frac{-2\sqrt{3}}{8}+\frac{-2\sqrt{3}}{8}\right)$
$= \left(-\frac{1}{2},\frac{-\sqrt{3}}{2}\right) = -\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right) \Rightarrow -e^{i\frac{\pi}{3}}$

h) $\dfrac{\sqrt{2}}{7+i7} \Rightarrow \dfrac{(\sqrt{2},0)}{(7,7)} = (\sqrt{2},0)\left(\frac{7}{7^2+7^2},\frac{-7}{7^2+7^2}\right) = (\sqrt{2},0)\left(\frac{7}{98},\frac{-7}{98}\right) = (\sqrt{2},0)\left(\frac{1}{14},\frac{-1}{14}\right)$
$= \left(\sqrt{2}\cdot\frac{1}{14}-0,\ \sqrt{2}\cdot\frac{-1}{14}+0\right) = \left(\frac{\sqrt{2}}{14},\frac{-\sqrt{2}}{14}\right) \Rightarrow \frac{\sqrt{2}}{14}-i\frac{\sqrt{2}}{14} = \frac{1}{7}\left(\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right) = \frac{1}{7}e^{i\frac{3\pi}{4}}$