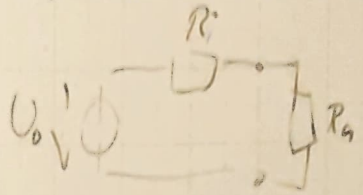
7.11

a) Urspannung U_0, Innenwiderstand R_i, Lastwiderstand R_a

R_a soll maximal werden



$$P(R_a) = \frac{U_a^2}{R_a}$$

$$U_a = U_0 \cdot \frac{R_a}{R_a + R_i}$$

$$P(R_a) = \frac{\left(U_0 \cdot \frac{R_a}{R_a + R_i}\right)^2}{R_a} = \frac{U_0^2 \cdot R_a^2}{R_a(R_a^2 + 2R_aR_i + R_i^2)} = \frac{U_0^2 \cdot R_a}{R_a^2 + 2R_aR_i + R_i^2}$$

$$P'(R_a) = \frac{(U_0^2 \cdot R_a)' \cdot (R_a^2 + 2R_aR_i + R_i^2) - U_0^2 \cdot R_a \cdot (R_a^2 + 2R_aR_i + R_i^2)'}{((R_a + R_i)^2)^2}$$

$$= \frac{U_0^2 \cdot (R_a^2 + 2R_aR_i + R_i^2) - U_0^2 R_a (2R_a + 2R_i)}{(R_a + R_i)^4}$$

$$= U_0^2 \cdot \frac{R_a^2 + 2R_aR_i + R_i^2 - 2R_a^2 + 2R_aR_i}{(R_a + R_i)^4}$$

$$= U_0^2 \frac{-R_a^2 + R_i^2}{(R_a + R_i)^4}$$

$$= \frac{U_0^2}{R_a} \cdot \frac{-R_a^2 + R_i^2}{(R_a + R_i)^3}$$

$$0 = U_0^2 \frac{-R_a^2 + R_i^2}{(R_a + R_i)^4}$$

$$\Leftrightarrow 0 = -R_a^2 + R_i^2$$

$$\Leftrightarrow R_i^2 = R_a^2$$

$$\Rightarrow R_i = R_a \quad \wedge \quad -R_i = -R_a \Leftrightarrow R_i = R_a$$

$$P''(R_a = R_i) = \frac{-2R_a (R_a + R_i)^4 - (-R_a^2)((R_a + R_i)^4)'}{(R_a + R_i)^8}$$

$$(R_a^2 + 2R_aR_i + R_i^2)(R_a^2 + 2R_aR_i + R_i^2)$$
$$= R_a^4 + 2R_a^3R_i + R_a^2R_i^2 + 2R_a^3R_i + 4R_a^2R_i^2 + 2R_aR_i^3 + R_a^2R_i^2 + 2R_aR_i^3 + R_i^4$$
$$= R_a^4 + 4R_a^3R_i + 6R_a^2R_i^2 + 4R_aR_i^3 + R_i^4$$
$$(R_a + R_i)^4 = 4R_a^3 + 12R_a^2R_i + 6R_i^2 + 4R_i^3$$

$$= \frac{-8R_a^4 - 32R_a^3R_i - 48R_a^2R_i^2 - 32R_aR_i^3 - 8R_i^4 + 4R_a^5 + 12R_a^4R_i + 12R_a^3R_i^2 + 4R_aR_i^3}{(R_a + R_i)^8}$$

$$= \frac{4R_a^5 - 8R_a^4 - 12R_a^3R_i - 6R_a^2R_i^2 + 4R_aR_i^3 - 8R_i^3R_a}{(R_a + R_i)^8}$$

$$= \frac{4R_i^5 - 8R_i^4 - 12R_i^5 - 6R_i^3 + 4R_i^4 - 8R_i^4}{R_i^{16}}$$

$$= \frac{-8R_i^5 - 12R_i^4 - 6R_i^3}{R_i^{16}} = -\frac{(8R_i^5 + 12R_i^4 + 6R_i^3)}{R_i^{16}} < 0 \Rightarrow \text{Maximum}$$