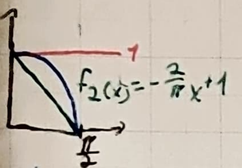
Integrationsmethoden

Noch zu letzter Woche (Wiederholung):

$\frac{\pi}{4} \leq \int_0^{\pi/2} \cos x \, dx \leq \frac{\pi}{2}$ ohne Flächenbetrachtung, ohne explizite Integrierung



$\forall x: \cos x \leq 1$

$\Rightarrow \int_0^{\pi/2} \cos x \, dx \leq \int_0^{\pi/2} 1 \, dx = x \Big|_0^{\pi/2} = \frac{\pi}{2}$

Skizze: $\cos x \geq f_2(x), \ \forall x \in [0, \frac{\pi}{2}]$

$$\Rightarrow \int_0^{\pi/2} \cos x \, dx \geq -\frac{2}{\pi}\int_0^{\pi/2} x \, dx + \overbrace{\int_0^{\pi/2} 1 \, dx}^{\pi/2} = -\frac{2}{\pi} \cdot \frac{1}{2}x^2 \Big|_0^{\pi/2} + \frac{\pi}{2} = \ldots = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$$

8.4 | Substitution

Form: $\int f(g(t)) \cdot g'(t) \, dt \overset{\text{sei } x = g(t),\ dx = g'(t)dt}{=} \int f(x) \, dx$

Ableitung: $\frac{df}{dx}(x)$

$\int \frac{\sin x}{\cos x} \, dx$

$\int \frac{g'(t)}{g(t)} \, dt = \ln|g(t)| + c$, „Logarithmisches integrieren“ $\int \frac{3t^2+6t}{t^3+3t^2-4} \, dt = \ln|t^3+3t^2-4| + c$

$\int f(x) \cdot f'(x) \, dx = \frac{1}{2} f^2(x) + c \quad (g(f(x)) = f(x))$

a) $\int_1^e \frac{1}{u(1+\ln u)} \, du$

Sei $y = 1 + \ln u$

$\Rightarrow \frac{dy}{du} = \frac{1}{u} \Rightarrow dy = \frac{1}{u} du$ „formal“

~~$u\,dy$~~ du f

Grenzen: $y(1) = 1 + \ln 1 = 1$

$y(e) = 1 + \ln e = 2$

$\int_1^2 \frac{1}{y} \, dy = \ln y \Big|_1^2 = \ln 2 - \ln 1 = \ln 2$ //

Alternativ:

$\int_1^e \frac{1}{u(1+\ln u)} \, du = \int_0^1 \frac{1}{e^x(1+x)} e^x \, dx = \int_0^1 \frac{1}{x+1} \, dx = \ln|x+1| \Big|_0^1 = \ln 2$

Sei $u = g(x) = e^x \Rightarrow du = e^x dx$

$x = g^{-1}(u) = \ln u$

Grenzen:

$g^{-1}(1) = \ln 1 = 0$

$g^{-1}(e) = \ln e = 1$

$\int_0^{\pi/2} \sin^5(x) \, dx = \int_0^{\pi/2} \sin^4 x \cdot \sin x \, dx = \int_0^{\pi/2} (1 - \cos^2 x)^2 \sin x \, dx$

$\cos x = u, \ du = -\sin x \, dx$

8.5 | Partielle Integration

$(uv)' = u' \cdot v + u \cdot v' \quad | \int \ldots dx$

$\int u'v = uv - \int uv'$

d) $\int \sin(\ln x)dx = \int \overset{u'}{1} \cdot \overset{v}{\sin(\ln x)}\, dx = x \cdot \sin(\ln x) - \int x \cos(\ln x) \cdot \frac{1}{x} dx$

$= x\sin(\ln x) - \left(x \cdot \cos(\ln x) - \int x \cdot \left(-\sin(\ln x)\right) \cdot \frac{1}{x} dx\right) \quad | + \int \sin \ln x \, dx$

$\Leftrightarrow 2\int \sin(\ln x)dx = x \cdot \sin(\ln x) - x \cdot \cos(\ln x) + c$

$\Rightarrow \int \sin(\ln x) dx = \frac{1}{2} x \cdot \sin \ln x - \frac{1}{2} x \cdot \cos \ln x + c, \; c \in \mathbb{R}$

8.6) Partialbruchzerlegung

i) $\int \frac{1}{t^3+3t^2-4} dt$

1.) Entfällt, weil echt gebrochen rational

2.) Nullstellen des Nenners finden:

a) $t=1$ Polynomdivision

$$\begin{array}{l} (t^3+3t^2-4) : (t-1) = t^2+4t+4 \\ \underline{-(t^3-t^2)} \\ \quad 4t^2-4 \\ \quad \underline{-(4t^2-4t)} \\ \qquad 4t-4 \\ \qquad \underline{-(4t-4)} \\ \qquad\quad 0 \end{array}$$

Satz von Vieta

$\Rightarrow N(t) = (t-1)(t^2+4t+4) = (t-1)(t+2)(t+2) = (t-1)(t+2)^2$

b) $\frac{1}{t^3+3t^2-4} = \frac{A}{t-1} + \frac{B}{(t+2)} + \frac{C}{(t+2)^2}$

$\Rightarrow 1 = A(t+2)^2 + B(t+2)(t-1) + C(t-1) \quad \forall t$

$t=1 \Rightarrow 1 = A \cdot 9$

$t=2 \qquad \vdots$

$t=0$