7.1) (Ableitung)

a) $f(x) = \sqrt{x}$ an $x_0 = 5$

$$f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{\sqrt{x} - \sqrt{x_0}}{x - x_0} \Rightarrow \lim_{x \to 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5} \Rightarrow \lim_{x \to 5} \frac{x-5}{(x-5)(\sqrt{x} + \sqrt{5})} \Rightarrow \lim_{x \to 5} \frac{1}{\sqrt{x} + \sqrt{5}}$$

$$\Rightarrow \frac{1}{2\sqrt{5}}$$

b) $f(x) = \ln x$ an $x_0 = 1$

$$f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{\ln x - \ln x_0}{x - x_0} = \lim_{x \to x_0} \frac{\ln \frac{x}{x_0}}{x - x_0} = \lim_{x \to x_0} \frac{1}{x - x_0} \ln \frac{x}{x_0} = \lim_{x \to x_0} \frac{x}{x_0}^{\frac{1}{x-x_0}}$$

$$\Rightarrow \lim_{x \to 1} \ln x^{\frac{1}{x-1}} = \lim_{x \to 1} \ln (1 + x - 1)^{\frac{1}{x-1}} = \lim_{x \to 1} \ln \left(1 + \frac{1}{\frac{1}{x-1}}\right)^{\frac{1}{x-1}}$$

sei $\frac{1}{x-1} = h$, $\lim_{x \to 1} h = \lim_{x \to 1} \frac{1}{x-1} = \infty$

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$$= \lim_{h \to \infty} \ln \left(1 + \frac{1}{h}\right)^h = \ln e = 1$$

c) $f(x) = \sin x$ an $x_0 = \frac{\pi}{2}$

$$f'(x) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \to x_0} \frac{\sin x - \sin x_0}{x - x_0} \Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{\sin x - \sin \frac{\pi}{2}}{x - \frac{\pi}{2}} = \lim_{x \to \frac{\pi}{2}} \frac{(\sin x - \sin \frac{\pi}{2})(x - \frac{\pi}{2})}{(x - \frac{\pi}{2})^2}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{(\sin x - \sin \frac{\pi}{2})(x - \frac{\pi}{2})}{x^2 - 2\frac{\pi}{2}x + \frac{\pi^2}{4}} = \lim_{x \to \frac{\pi}{2}} \frac{(\sin x - \sin \frac{\pi}{2})(x - \frac{\pi}{2})}{x^2 - \pi x + \frac{\pi^2}{4}} \Rightarrow \frac{0}{\frac{\pi^2}{4} - \frac{\pi^2}{2} + \frac{\pi^2}{4}} = \frac{0}{\frac{\pi^2}{2} - \frac{\pi^2}{2}} = \frac{0}{0}$$