6.6 | Vektorprodukt

$\underline{a} = \begin{pmatrix}4\\2\\1\end{pmatrix}$, $\underline{b} = \begin{pmatrix}2\\3\\3\end{pmatrix}$, $\underline{c} = \begin{pmatrix}3\\-2\\0\end{pmatrix}$

a) $\underline{a} \times \underline{b}$

$$\begin{pmatrix}4\\2\\1\end{pmatrix} \times \begin{pmatrix}2\\3\\3\end{pmatrix} = \begin{pmatrix}2\cdot3-3\cdot1\\1\cdot2-3\cdot4\\4\cdot3-2\cdot2\end{pmatrix} = \begin{pmatrix}6-3\\2-12\\12-4\end{pmatrix} = \begin{pmatrix}3\\-10\\8\end{pmatrix}$$

$$\begin{pmatrix}4\\2\\1\end{pmatrix} \begin{pmatrix}2\\3\\3\end{pmatrix}$$

b) $\underline{a} \times \underline{c}$

$$\begin{pmatrix}4\\2\\1\end{pmatrix} \times \begin{pmatrix}3\\-2\\0\end{pmatrix} = \begin{pmatrix}2\cdot0-1\cdot(-2)\\1\cdot3-4\cdot0\\4\cdot(-2)-2\cdot3\end{pmatrix} = \begin{pmatrix}0+2\\3-0\\-8-6\end{pmatrix} = \begin{pmatrix}2\\3\\-14\end{pmatrix}$$

$$\begin{pmatrix}4\\2\\1\end{pmatrix} \begin{pmatrix}3\\-2\\0\end{pmatrix}$$

c) $\underline{a} \times (\underline{b} \times \underline{c})$

$\underline{b} \times \underline{c}$: $\begin{pmatrix}2\\3\\3\end{pmatrix} \times \begin{pmatrix}3\\-2\\0\end{pmatrix} = \begin{pmatrix}3\cdot0-3\cdot(-2)\\3\cdot3-2\cdot0\\2\cdot(-2)-3\cdot3\end{pmatrix} = \begin{pmatrix}0+6\\9-0\\-4-9\end{pmatrix} = \begin{pmatrix}6\\9\\-13\end{pmatrix}$

$\underline{a} \times (\underline{b} \times \underline{c})$: $\begin{pmatrix}4\\2\\1\end{pmatrix} \times \begin{pmatrix}6\\9\\-13\end{pmatrix} = \begin{pmatrix}2\cdot(-13)-1\cdot9\\1\cdot6-4\cdot(-13)\\4\cdot9-2\cdot6\end{pmatrix} = \begin{pmatrix}-26-9\\6+52\\36-12\end{pmatrix} = \begin{pmatrix}-35\\68\\24\end{pmatrix}$

f) Die Fläche des Parallelogramms aus $\underline{a} \times \underline{b}$: $|\underline{a} \times \underline{b}|$

$\underline{a} \times \underline{b}$: $\begin{pmatrix}4\\2\\1\end{pmatrix} \times \begin{pmatrix}2\\3\\3\end{pmatrix} = \begin{pmatrix}2\cdot3-1\cdot3\\1\cdot2-4\cdot3\\4\cdot3-1\cdot3\end{pmatrix} = \begin{pmatrix}6-3\\2-12\\12-3\end{pmatrix} = \begin{pmatrix}3\\-10\\8\end{pmatrix}$

$|\underline{a} \times \underline{b}| = \sqrt{3^2+(-10)^2+8^2} = \sqrt{9+100+64} = \sqrt{173}$