5.6) Elementare Zusammenhänge bei komplexen Zahlen

a)

$\cos\alpha \overset{!}{=} \frac{1}{2}(e^{i\alpha}+e^{-i\alpha}) = \frac{1}{2}((\cos\alpha + i\sin\alpha)+(\cos-\alpha + i\sin\alpha))$

$= \frac{1}{2}(\cos\alpha + \cos-\alpha,\ i(\sin\alpha + \sin-\alpha)) \Rightarrow \frac{1}{2}(2\cos\alpha, 0) \Rightarrow \cos\alpha$

b)

$\sin\alpha \overset{!}{=} \frac{1}{2i}(e^{i\alpha}-e^{-i\alpha}) = \frac{1}{2i}((\cos\alpha, i\sin\alpha)-(\cos-\alpha, i\sin-\alpha)) = \frac{1}{2i}(\cos\alpha-\cos-\alpha,\ i(\sin\alpha-\sin-\alpha))$

$\Rightarrow \frac{1}{2i}(\cos\alpha-\cos\alpha,\ i(\sin\alpha-(-\sin\alpha))) = \frac{1}{2i}(0, i2\sin\alpha) \Rightarrow \sin\alpha$

c) $|e^{i\alpha}| \overset{!}{=} 1 \Rightarrow |\cos\alpha + i\sin\alpha| = \sqrt{\cos^2\alpha+\sin^2\alpha} = \sqrt{1} = 1$

d) $\mathrm{Re}\, z \overset{!}{=} \frac{1}{2}(z+\bar{z}) \Rightarrow \frac{1}{2}((x,y)+(x,-y)) = \frac{1}{2}(x+x, y-y) = \frac{1}{2}(2x,0) = x = \mathrm{Re}\, z$