Inhalt 23.06.2025

$\int \frac{1}{x} \ln x \, dx$

$\int f(g(x)) g'(x) \, dx$ $\qquad$ $\int \frac{g'(x)}{g(x)} \, dx = \ln|g(x)| + c$

$\int g(x) g'(x) \, dx = \frac{1}{2} g(x)^2 + c$

$\int (1+e^{2x})^2 e^{2x} \, dx$

$y = 1 + e^{2x}$
$dy = 2e^{2x} \, dx$
$\frac{1}{2} dy = e^{2x} \, dx$

5) $\int \frac{u}{u^2+4u+3} \, du$

1. $\checkmark$

2. Nullstellen finden von $N(x)$

$N(u) = (u+1)(u+3)$

3. $\frac{u}{u^2+4u+3} = \frac{A}{u+1} + \frac{B}{u+3} = \frac{A(u+3)+B(u+1)}{(u+1)(u+3)} \Rightarrow u = A(u+3) + B(u+1) \quad \forall u$

sei $u = -1: \; -1 = 2A \Rightarrow A = -\frac{1}{2}$ $\qquad$ Koeffizientenvergleich: $1 = A + B$
$\quad u = -3: \; -3 = -2B \Rightarrow B = \frac{3}{2}$ $\qquad$ $0 = 3A + B$

$\Rightarrow \frac{u}{u^2+4u+3} = -\frac{1}{2} \frac{1}{u+1} + \frac{3}{2} \frac{1}{u+3}$

4. $\Rightarrow \int \frac{u}{u^2+4u+3} \, du = -\frac{1}{2} \int \frac{1}{u+1} \, du + \frac{3}{2} \int \frac{1}{u+3} \, du = -\frac{1}{2} \ln|u+1| + \frac{3}{2} \ln|u+3| + c$

$\int \frac{1}{2u+2} \, du = \frac{1}{2} \ln|2u+2| + c = \frac{1}{2} \ln|2(u+1)| + c = \frac{1}{2} \ln|2| + \frac{1}{2} \ln|u+1| + c$

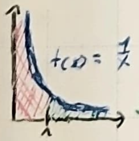
$\int \frac{1}{2} \frac{1}{u+1} \, du = \frac{1}{2} \ln|u+1| + c$

$C[a,b]$ ist Vektorraum stetiger Funktionen
$C^\infty[a,b]$ ist Vektorraum differenzierbarer Funktionen

$\int \frac{1}{1+x^2} \, dx = \int \frac{1+\tan^2 y}{1+\tan^2 y} \, dy = \int dy = y \Rightarrow \arctan x$

$x = \tan y \Leftrightarrow y = \arctan x$
$dx = 1 + \tan^2 y \, dy$

Uneigentliche Integrale



$\int_0^1 \frac{1}{x} \, dx$ unbeschränkter Integrant
$\int_1^\infty \frac{1}{x} \, dx$ unbeschränkter Integrationsbereich

8.8 g) und 8.9 i)

$\int_0^1 \frac{1}{x^\alpha} \, dx = \begin{cases} \frac{1}{1-\alpha} & \alpha < 1 \\ \infty & \alpha \geq 1 \end{cases}$ $\qquad$ $\int_{-\infty}^{-1} \frac{1}{(-x)^\alpha} \, dx$

$\int_1^\infty \frac{1}{x^\alpha} \, dx = \begin{cases} \infty & \alpha \leq 1 \\ \frac{1}{\alpha-1} & \alpha > 1 \end{cases}$

8.8 i)

$(1+x)^3 \sim x^2 \Rightarrow$ konvergiert vermutlich

$$\int_0^\infty \frac{x}{(1+x)^3}\,dx = \lim_{t\to\infty}\int_0^t \frac{x}{(1+x)^3}\,dx = \lim_{t\to\infty}\int_1^{t+1}\frac{y-1}{y^3}\,dy = \lim_{t\to\infty}\left(\int_1^{t+1}\frac{y}{y^3}\,dy - \int_1^{t+1}\frac{1}{y^3}\,dy\right)$$

Substitution: $y = 1+x$, $x = y-1$
$dy = dx$

Grenzen!!!

$$= \lim_{t\to\infty}\left(-\frac{1}{y}\Big|_1^{t+1} - \left(-\frac{1}{2}\right)\frac{1}{y^2}\Big|_1^{t+1}\right)$$

$$= \lim_{t\to\infty}\left(-\frac{1}{t+1} + 1 - \left(-\frac{1}{2}\frac{1}{(t+1)^2} + \frac{1}{2}\frac{1}{1}\right)\right)$$

$$= \lim_{t\to\infty}\left(-\underbrace{\frac{1}{t+1}}_{0} + 1 + \frac{1}{2}\underbrace{\frac{1}{(t+1)^2}}_{0} - \frac{1}{2}\right)$$

$\Rightarrow 1 - \frac{1}{2} \Rightarrow$ konvergent

$\cos^2 \leq 1$

$$\int \frac{\ln(2+\cos^2 x)}{x}\,dx$$

$\frac{1}{x}$ ist divergente Minorante