5.2 | Rechenregeln für die komplexe Konjugation

wenn $z = x + iy$, dann $\bar{z} = x - iy$

a)

$$\left.\begin{aligned} z &= x + iy \\ \bar{z} &= x - iy \\ \bar{\bar{z}} &= x + iy \end{aligned}\right\} =$$

b)

$$\mathrm{Re}\, z \overset{!}{=} \frac{1}{2}(z + \bar{z}) = \frac{1}{2}(x + iy + x - iy) = \frac{1}{2}(2x) = x = \mathrm{Re}\, z \quad \square$$

$$\mathrm{Im}\, z \overset{!}{=} \frac{1}{2i}(z - \bar{z}) = \frac{1}{2i}(x + iy - (x - iy)) = \frac{1}{2i}(x + iy - x + iy) = \frac{1}{2i}(2iy) = y = \mathrm{Im}\, z \quad \square$$

c)

$$z \cdot \bar{z} \overset{!}{=} (\mathrm{Re}\, z)^2 + (\mathrm{Im}\, z)^2 \overset{!}{\geq} 0 \Rightarrow (x + iy)(x - iy) = x^2 - ixy + ixy + (iy)^2 = x^2 + y^2 = (\overset{\geq 0}{\mathrm{Re}\, z})^2 + (\overset{\geq 0}{\mathrm{Im}\, z})^2 \geq 0 \quad \square$$

d)

$$\overline{z_1 + z_2} \overset{!}{=} \bar{z_1} + \bar{z_2}$$

$$\left.\begin{aligned} \overline{z_1 + z_2} &= \overline{x_1 + x_2 + i(y_1 + y_2)} = x_1 + x_2 - i(y_1 + y_2) = x_1 + x_2 - iy_1 - iy_2 \\ \bar{z_1} + \bar{z_2} &= x_1 - iy_1 + x_2 - iy_2 = x_1 + x_2 - iy_1 - iy_2 \end{aligned}\right\} = \quad \square$$

$$\overline{z_1 \cdot z_2} \overset{!}{=} \bar{z_1} \cdot \bar{z_2}$$

$$\left.\begin{aligned} \overline{z_1 \cdot z_2} &= \overline{(x_1 + iy_1) \cdot (x_2 + iy_2)} = \overline{x_1 x_2 + i x_1 y_2 + i x_2 y_1 + y_1 y_2} = x_1 x_2 + y_1 y_2 - i(x_1 y_2 + x_2 y_1) \\ \bar{z_1} \cdot \bar{z_2} &= (x_1 - iy_1) \cdot (x_2 - iy_2) = x_1 x_2 - i x_1 y_2 - i x_2 y_1 + y_1 y_2 = x_1 x_2 + y_1 y_2 - i(x_1 y_2 + x_2 y_1) \end{aligned}\right\} = \quad \square$$

e)

$$z = \bar{z} \overset{!}{\Leftrightarrow} z \in \mathbb{R}$$

$$z = \bar{z} \Leftrightarrow x + iy = x - iy \overset{y=0}{\Rightarrow} x = x \Rightarrow z \in \mathbb{R}, \text{ da } \mathrm{Im}\, z = 0 \quad \square$$