

3.9 | Grenzwert von Folgen

$$a) \lim_{n \rightarrow \infty} \frac{n^2 + n + 2}{4n^3 + 1} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{1}{n^2} + \frac{2}{n^3}}{4 + \frac{1}{n^3}} \Rightarrow \frac{\lim_{n \rightarrow \infty} \frac{1}{n} + \lim_{n \rightarrow \infty} \frac{1}{n^2} + \lim_{n \rightarrow \infty} \frac{2}{n^3}}{\lim_{n \rightarrow \infty} 4 + \lim_{n \rightarrow \infty} \frac{1}{n^3}} \Rightarrow \frac{0}{4} = 0$$

$$b) \lim_{n \rightarrow \infty} \frac{(n+1)^2 - n^2}{n} = \lim_{n \rightarrow \infty} \frac{n^2 + 2n + 1 - n^2}{n} = \lim_{n \rightarrow \infty} \frac{2 + \frac{1}{n}}{1} \Rightarrow \frac{\lim_{n \rightarrow \infty} 2 + \lim_{n \rightarrow \infty} \frac{1}{n}}{1} \Rightarrow \frac{2}{1} = 2$$

$$c) \lim_{n \rightarrow \infty} \frac{4n^4 - n + 2}{2n^4 + 2n^2 + n} = \lim_{n \rightarrow \infty} \frac{4 - \frac{1}{n^3} + \frac{2}{n^4}}{2 + 2 \cdot \frac{1}{n^2} + \frac{1}{n^3}} \Rightarrow \frac{4 - \lim_{n \rightarrow \infty} \frac{1}{n^3} + \lim_{n \rightarrow \infty} \frac{2}{n^4}}{2 + 2 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^2} + \lim_{n \rightarrow \infty} \frac{1}{n^3}} \Rightarrow \frac{4}{2} = 2$$

$$d) \lim_{n \rightarrow \infty} \frac{n + n^3}{16n + n^2 + 3n^3 + 4n^4} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n^3} + \frac{1}{n}}{\frac{1}{n^3} + \frac{1}{n^2} + \frac{1}{n} + 3 + 4n} \Rightarrow \frac{0}{4} = 0$$

$$e) \lim_{n \rightarrow \infty} \frac{-4n^2 + 3n^3 + 7}{2n^3 + 5n} = \lim_{n \rightarrow \infty} \frac{-4 \frac{1}{n} + 3 + 7 \frac{1}{n^3}}{2 + 5 \frac{1}{n^2}} \Rightarrow \frac{3}{2}$$