5.4) Kartesische Darstellung komplexer Zahlen

a) $\frac{3+4i}{1-2i} = \frac{(3,4)}{(1,-2)} = (3,4)\cdot(1,-2)^{-1} = (3,4)\cdot\left(\frac{1}{1^2+2^2}, \frac{-(-2)}{1^2+2^2}\right) = (3,4)\left(\frac{1}{5}, \frac{2}{5}\right) = \left(3\cdot\frac{1}{5} - 4\cdot\frac{2}{5}, 3\cdot\frac{2}{5}+4\cdot\frac{1}{5}\right)$

$= \left(\frac{3}{5} - \frac{8}{5}, \frac{6}{5}+\frac{4}{5}\right) = (-1,2) = -1+2i$

b) $\frac{i(2+i)}{(1+i)(2-i)} = \frac{(0,1)((2,0)+(0,1))}{((1,0)+(0,1))((2,0)-(0,1))} = \frac{(0,1)(2,1)}{(1,1)(2,-1)} = \frac{(0\cdot 2-1\cdot 1, 0\cdot 1+1\cdot 2)}{(1\cdot 2-1\cdot(-1), 1\cdot(-1)+1\cdot 2)} = \frac{(-1,2)}{(3,1)} = (-1,2)\cdot\left(\frac{3}{3^2+1^2}, \frac{-1}{3^2+1^2}\right)$

$= (-1,2)\left(\frac{3}{10}, \frac{-1}{10}\right) = \left(-1\cdot\frac{3}{10} - 2\cdot\frac{-1}{10}, -1\cdot\frac{-1}{10} + 2\cdot\frac{3}{10}\right) = \left(-\frac{3}{10} - \frac{2}{10}, -\frac{1}{10}+\frac{6}{10}\right) = \left(-\frac{1}{2}, \frac{1}{2}\right) = -\frac{1}{10} + i\frac{7}{10}$

c) $2i\frac{(1+i)^2}{(1+i)(2-i)} = (0,2)\frac{((1,0)+(0,1))^2}{((1,0)+(0,1))((2,0)-(0,1))} = (0,2)\frac{(1,1)^2}{(1,1)(2,-1)} = (0,2)\frac{(1\cdot 1-1\cdot 1, 1\cdot 1+1\cdot 1)}{(1\cdot 2-1\cdot(-1), 1\cdot(-1)+1\cdot 2)}$

$= (0,2)\left((0,2)(3,1)^{-1}\right) = (0,2)\left(\left(\frac{3}{3^2+1^2}, -\frac{1}{3^2+1^2}\right)(0,2)\right) = (0,2)\left((0,2)\left(\frac{3}{10}, -\frac{1}{10}\right)\right)$

$= (0,2)\left(0\cdot\frac{3}{10} - 2\cdot\left(-\frac{1}{10}\right), 0\cdot\left(-\frac{1}{10}\right) + 2\cdot\frac{3}{10}\right) = (0,2)\left(\frac{1}{5}, \frac{3}{5}\right) = \left(0\cdot\frac{1}{5} - 2\cdot\frac{3}{5}, 0\cdot\frac{3}{5} + 2\cdot\frac{1}{5}\right) = \left(-\frac{6}{5}, \frac{2}{5}\right)$

$= -\frac{6}{5} + i\frac{2}{5}$

d) $4e^{i\frac{\pi}{6}} = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 4\cdot\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = 2\sqrt{3} + i2$

e) $\sqrt{2}e^{i\frac{25}{4}\pi} = \sqrt{2}\left(\cos 25\frac{\pi}{4} + i\sin 25\frac{\pi}{4}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 1+i$

f) $ie^{i\frac{11\pi}{4}} = i\left(\cos 11\frac{\pi}{4} + i\sin 11\frac{\pi}{4}\right) = i\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = (0,1)\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$= \left(0\cdot\left(-\frac{\sqrt{2}}{2}\right) - 1\cdot\frac{\sqrt{2}}{2}, 0\cdot\frac{\sqrt{2}}{2} + 1\cdot\left(-\frac{\sqrt{2}}{2}\right)\right) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2} = -\frac{2}{2\sqrt{2}} - i\frac{2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

g) $3e^{i4\pi} + 2e^{i7\pi} = 3(\cos 4\pi + i\sin 4\pi) + 2(\cos 7\pi + i\sin 7\pi) = 3(1+i0) + 2(-1+i0)$

$= (3,0) + (-2,0) = (1,0)$

h) $\sqrt{2}e^{-i\frac{\pi}{4}} = \sqrt{2}\left(\cos -\frac{\pi}{4} + i\sin -\frac{\pi}{4}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{-\sqrt{2}}{2}\right) = (1-i)$

i) $(1-i)^9 = ((1,-1)(1,-1))^4(1,-1) = (1\cdot 1-(-1)\cdot(-1), 1\cdot(-1)+1\cdot(-1))^4(1,-1) = ((0,-2)(0,-2))^2(1,-1)$

$= (0\cdot 0-(-2)(-2), 0\cdot(-2)+(-2)\cdot 0)^2(1,-1) = (-4,0)^2(1,-1) = (-4\cdot(-4)-0\cdot 0, 0\cdot(-4)+0\cdot(-4))(1,-1)$

$= (16,0)(1,-1) = (16\cdot 1-0\cdot(-1), 16(-1)+0\cdot 1i) = (16,-16) = 16-i16$

j) $e^{i1\frac{\pi}{2}}(\cos 1 + i\sin 1)$