UNIVERSITY OF TRENTO - Italy
Department of Physics

Degree in Physics
Thesis

Bell's theorem without inequalities

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Perfect correlations in systems of two spin-1/2 particles

For systems of two spin-1/2 particles in the state (singlet state):

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2\right)$$

QM predicts opposite results if the same spin component is measured on both particles.

EPR argument

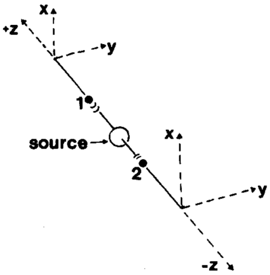

Figure: The Bohm gedankenexperiment. Credits [1].

- **Perfect correlation:** see previous slide.
- **Locality:** if the two particles are far from each other, measurements on one particle do not change the elements of physical reality of the other particle.
- **Reality:** If the result of a measurement can be predicted with certainty, there exists an element of physical reality corresponding to the measured quantity.

$$\Downarrow$$

QM is incomplete

Bell's formalization of EPR's results

Let λ be the complete description of pairs in the singlet state, then:

$$\text{Results of measurements on particle 1} \quad \longrightarrow \quad A = A_{\hat{\mathbf{u}}}(\lambda)$$

$$\text{Results of measurements on particle 2} \quad \longrightarrow \quad B = B_{\hat{\mathbf{v}}}(\lambda)$$

where $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ indicate the spin component measured.

Because of locality A depends on $\hat{\mathbf{u}}$ and not on $\hat{\mathbf{v}}$, B depends on $\hat{\mathbf{v}}$ and not on $\hat{\mathbf{u}}$.

Bell's inequality

Let $E(\hat{\mathbf{u}}, \hat{\mathbf{v}})$ be the expectation value of the product of the two particles' spin components. Using the objects just introduced:

$$E(\hat{\mathbf{u}}, \hat{\mathbf{v}}) = \int \rho(\lambda) A_{\hat{\mathbf{u}}}(\lambda) B_{\hat{\mathbf{v}}}(\lambda) d\lambda$$

where $\rho(\lambda)$ is the probability distribution of the λ's.
Then Bell proved that, for any theory that obeys the EPR assumptions:

$$|E(\hat{\mathbf{u}}, \hat{\mathbf{v}}) - E(\hat{\mathbf{u}}, \hat{\mathbf{w}})| - E(\hat{\mathbf{v}}, \hat{\mathbf{w}}) \leq 1$$

But for $\hat{\mathbf{u}}$, $\hat{\mathbf{v}}$ and $\hat{\mathbf{w}}$ in the xy plane with azimuthal angles 0°, 60° and 120° respectively, QM predicts:

$$|E(\hat{\mathbf{u}}, \hat{\mathbf{v}}) - E(\hat{\mathbf{u}}, \hat{\mathbf{w}})| - E(\hat{\mathbf{v}}, \hat{\mathbf{w}}) = \frac{3}{2},$$

Thus, *statistical predictions* of QM contradict EPR's assumptions.

Perfect correlations in systems of three spin-1/2 particles

Consider a system of three spin-1/2 particles in the state:

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_1|+\rangle_2|+\rangle_3 - |-\rangle_1|-\rangle_2|-\rangle_3\right)$$

where $|+\rangle_i$ and $|-\rangle_i$ are taken along the $\hat{\mathbf{z}}$ direction.
If we measure a spin component of each particle, in particular the triples of observables:

$$\begin{array}{l}
(S_{1x} \times \mathbb{1}_2 \times \mathbb{1}_3, \quad \mathbb{1}_1 \times S_{2y} \times \mathbb{1}_3, \quad \mathbb{1}_1 \times \mathbb{1}_2 \times S_{3y}) \\
(S_{1y} \times \mathbb{1}_2 \times \mathbb{1}_3, \quad \mathbb{1}_1 \times S_{2x} \times \mathbb{1}_3, \quad \mathbb{1}_1 \times \mathbb{1}_2 \times S_{3y}) \\
(S_{1y} \times \mathbb{1}_2 \times \mathbb{1}_3, \quad \mathbb{1}_1 \times S_{2y} \times \mathbb{1}_3, \quad \mathbb{1}_1 \times \mathbb{1}_2 \times S_{3x})
\end{array}$$

QM predicts that the result of the measurement of any observable in a triple is completely determined once the other two observables in the triple are measured.

EPR premises adapted to three particles

The EPR premises hold also in the case of three particles:

- **Perfect correlation:** see previous slide.
- **Locality:** if the particles are far from each other, measurements on one particle do not change the elements of physical reality of the other particles.
- **Reality:** If the result of a measurement can be predicted with certainty, there exists an element of physical reality corresponding to the measured quantity.

$$\Downarrow$$

There exists a more complete state specification λ;
Results of spin component measurements are given by:

$$A = A_i(\lambda),\ B = B_j(\lambda),\ C = C_k(\lambda)$$

for particle 1, 2 and 3 respectively.

GHZ argument

QM predicts for the product of three spin components (on the state of two slides ago):

$$\begin{array}{lcl} \mathcal{P}(S_{1x} \times S_{2y} \times S_{3y} = 1) = 1 & \Rightarrow & A_x B_y C_y = 1 \\ \mathcal{P}(S_{1y} \times S_{2x} \times S_{3y} = 1) = 1 & \Rightarrow & A_y B_x C_y = 1 \\ \mathcal{P}(S_{1y} \times S_{2y} \times S_{3x} = 1) = 1 & \Rightarrow & A_y B_y C_x = 1 \\ & & \Downarrow \\ & & A_x B_x C_x = 1 \end{array}$$

But QM also predicts:

$$\mathcal{P}(S_{1x} \times S_{2x} \times S_{3x} = -1) = 1 \quad \Rightarrow \quad A_x B_x C_x = -1$$

Thus *certain predictions* of QM contradict EPR's assumptions.

Statement of the GHZ theorem

Theorem

One (at least) of the following statements is false:

- *Nature is compatible with the* **locality** *assumption.*
- *Nature is compatible with the* **reality** *assumption.*
- *QM gives correct predictions for* **perfect correlations**.

Ideal experiment

The state $\frac{1}{\sqrt{2}}(|+\rangle_1|+\rangle_2|+\rangle_3 - |-\rangle_1|-\rangle_2|-\rangle_3)$ is an eigenstate of all the observables considered:

$$\begin{aligned} &S_{1x} \times S_{2y} \times S_{3y} \\ &S_{1y} \times S_{2x} \times S_{3y} \\ &S_{1y} \times S_{2y} \times S_{3x} \\ &S_{1x} \times S_{2x} \times S_{3x}. \end{aligned}$$

Moreover, all these observables commute with each other.

Thus, it is possible to measure all four products at the same time, i.e. a single run (with a single triplet) of the experiment suffices to confirm or disprove QM against local-realism.

To our knowledge no experiment of this type has been performed.

Interferometer

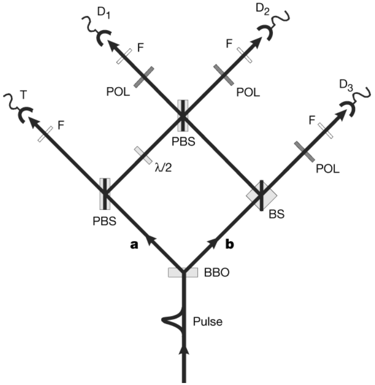

Figure: Schematic drawing of the interferometer. Credits [2].

UV pulses generate pairs of photons in (polarization) state:

$$\frac{1}{\sqrt{2}}\left(|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b\right).$$

If two pairs are generated by a single UV pulse and all four detectors fire, then at D_1, D_2 and D_3 we observe either of the following states:

$$|H\rangle_1|H\rangle_2|V\rangle_3$$
$$|V\rangle_1|V\rangle_2|H\rangle_3$$

An intensity ratio of 12 : 1 is observed between the desired and undesired states.

Condition for coherent superposition

The couple of pairs is generated in the state:

$$\frac{1}{2}\left(|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b\right)\left(|H\rangle'_a|V\rangle'_b - |V\rangle'_a|H\rangle'_b\right). \tag{1}$$

Single component's evolution in the interferometer is given by:

$$\begin{aligned} |H\rangle_a &\to |H\rangle_T & |V\rangle_a &\to \tfrac{1}{\sqrt{2}}\left(|V\rangle_1 + |H\rangle_2\right) \\ |V\rangle_b &\to \tfrac{1}{\sqrt{2}}\left(|V\rangle_2 + |V\rangle_3\right) & |H\rangle_b &\to \tfrac{1}{\sqrt{2}}\left(|H\rangle_1 + |H\rangle_3\right) \end{aligned}$$

Substituting these into (1) and dropping unwanted terms leads to:

$$|H\rangle_T\left(|H\rangle'_1|H\rangle'_2|V\rangle_3 + |V\rangle'_1|V\rangle_2|H\rangle'_3\right) + |H\rangle'_T\left(|H\rangle_1|H\rangle_2|V\rangle'_3 + |V\rangle_1|V\rangle'_2|H\rangle_3\right)$$

Finally, if unprimed and primed photons are indistinguishable:

$$\frac{1}{\sqrt{2}}|H\rangle_T\left(|H\rangle_1|H\rangle_2|V\rangle_3 + |V\rangle_1|V\rangle_2|H\rangle_3\right).$$

How to render the photons indistinguishable?

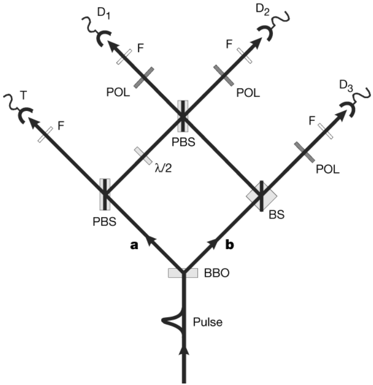

Figure: Credits [2].

Primed and unprimed photons can be distinguished by:

- Energy conservation.
- Timing of detection.

The use of narrow interference filters in front of the detectors cancels the possibility of distinguishing the photons.

Method to experimentally confirm GHZ entanglement

A measurement of $+45^\circ$ polarization on photon 1 projects the coherent state into:

$$\frac{1}{\sqrt{2}}|H\rangle_T|+45^\circ\rangle_1\left(|H\rangle_2|V\rangle_3+|V\rangle_2|H\rangle_3\right),$$

that written on the $(|+45^\circ\rangle_i, |-45^\circ\rangle)_i$ bases, for photon 2 and 3 gives:

$$\frac{1}{2\sqrt{2}}\left(|+\rangle_2|+\rangle_3-\cancel{|+\rangle_2|-\rangle_3}+\cancel{|-\rangle_2|+\rangle_3}-|-\rangle_2|-\rangle_3+\right.$$
$$\left.+|+\rangle_2|+\rangle_3+\cancel{|+\rangle_2|-\rangle_3}-\cancel{|-\rangle_2|+\rangle_3}-|-\rangle_2|-\rangle_3\right).$$

The terms $|+\rangle_2|-\rangle_3$ and $|-\rangle_2|+\rangle_3$ interfere destructively. Hence, the observation of the terms $|+\rangle_2|+\rangle_3$ and $|-\rangle_2|-\rangle_3$ is a proof of coherence.

Experimental confirmation of GHZ entanglement

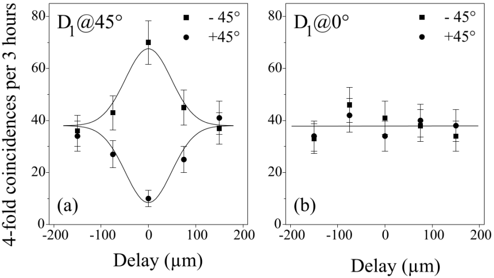

Figure: Results for polarization analysis of photon 3, conditioned on the detection of photon 1 polarized at $+45^\circ$ and photon 2 polarized at -45°. The (spatial) delay refers to a change in path *a* introduced to show that coherence is lost if photons can be distinguished. By comparison (graph (b)) no intensity difference is predicted if the polarizer in front of the detector D_1 is set at 0°. Credits [3].

Conclusion - Claims regarding local realism

The experimental procedure and analysis presented doesn't allow claims in favor or against QM to be made.

However, such claims are present in the literature. It has been shown that a more refined analysis of the data, coming from an improved version of the interferometer presented, allows to make claims against local-realism. However, the analysis of these claims is beyond the scope of this thesis.

Conclusion - By-product

The study of the matter with which this thesis is concerned allowed us to observe:

- Entanglement does not necessarily arise from interaction of the entangled systems.

Summary

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