UNIVERSITY OF TRENTO - Italy

Department of Physics

Degree in Physics

Thesis

Bell's theorem without inequalities

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Preface

Quantum mechanics describes physical systems by means of a state vector, $|\psi\rangle$. For systems extended in space a description of a portion of the system (located in some space region) cannot in general be given without taking into account all other constituents of the system (no matter how far these are). This is at the basis of the phenomenon of entanglement.

In 1935 Albert Einstein, Boris Podolsky and Natan Rosen (EPR) realized that entanglement allowed a step further in the discussion on the foundations of quantum mechanics. In particular it allowed them to show that, under reasonable assumptions, $|\psi\rangle$ could not be regarded as a satisfactory description of single physical systems. Under these assumptions, $|\psi\rangle$ could provide at most correct description of statistical ensembles of systems.

The next milestone in the discussion was the contribution by John S. Bell. In 1964 he showed that the premises of the EPR argument lead to an inequality (Bell's inequality) that is violated by statistical predictions of quantum mechanics. This inequality allows experimental tests of the quantum mechanical predictions against those descending from EPR's assumptions. Since the seventies a variety of such tests have been made with no one being able to disprove quantum mechanics.

This thesis will be concerned with some elements of this debate. In Chapter 1 we will present the EPR argument. Bell's reasoning will not be illustrated in favor of a more recent result on the same matter that Daniel M. Greenberg, Michael A. Horne and Anton Zeilinger (GHZ) proposed in 1989 (Chapter 2). Finally, Chapter 3 will be concerned with some of the experimental efforts that have been made towards a test of quantum mechanics following the argument by GHZ.

We will see that the GHZ argument will lead us to conclusions analogous to those obtained by Bell but in a way that doesn't involve inequalities. We will also see, that from an experimental point of view, the argument allows, in principle, to design experiments in which a single run suffices to confirm or disprove quantum mechanics. This is in contrast with experiments that follow Bell's argument where statistical considerations on many runs of the experiment must be done.

Contents

Chapter 1

EPR "theorem"

The first chapter of this thesis will be concerned with the so called "EPR paradox". In their famous article of 1935 [1] Albert Einstein, Boris Podolsky and Natan Rosen (EPR) presented a strong logical reasoning against (the Copenhagen interpretation of) quantum mechanics. Starting from few reasonable assumptions on the locality of physical processes, some form of realism and predictions taken from quantum mechanics they derive a clear conclusion: Quantum mechanics can at most be regarded as a statistical description of ensembles of physical systems. Single systems contain more information than the theory incorporates.

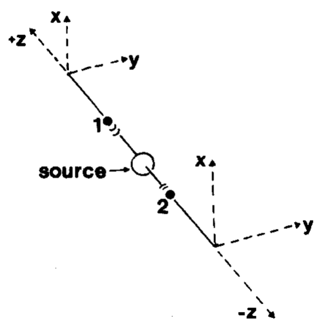

Figure 1.1: The Bohm gedankenexperiment. Credits [7].

1.1 Perfect correlations in systems of two spin-1/2 particles

In this section we will present some aspects of the quantum mechanical description of systems composed of two spin-1/2 particles. In particular we will show that there exist situations for which quantum mechanics predicts perfect correlation between the results of measurements performed on the two particles. We will also see that such correlations are independent of where and when the two measurements are performed.

Consider for instance two spin-1/2 particles emitted at a source and propagating in opposite directions along the $\hat{\mathbf{z}}$ axis (see Fig. 1.1). Each particle then enters an apparatus (e.g. a Stern-Gerlach magnet) that can measure either the S_x or S_y spin component. Suppose that the two particles are emitted in the state with total spin equal to 0, that is:

$$|\chi\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2\right) \tag{1.1}$$

where the vectors $|+\rangle_i$ and $|-\rangle_i$ represent, respectively, states of spin up and down along an arbitrary direction $\hat{\mathbf{n}} = \hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ for particle $i = 1, 2$. The state (1.1) is such as to take the same form independently of $\hat{\mathbf{n}}$, thus no ambiguity arises from omitting the direction $\hat{\mathbf{n}}$ in the notation (see Appendix A for a proof of this statement). This last statement also allows us to easily see that if the two measuring apparatuses happen to be oriented in the same direction, that is if they both measure either the S_x or S_y spin component, then the spins of the two particles will be found to have opposite sign.

Remark 1. Notice that the perfect correlation we have predicted in this section are independent on the time order with which we perform the two measurements as well as on the distance between the places where the two measuring apparatuses are located.

Observation 1. The correlation between the results of measurements presented here is a peculiar trait of entanglement. Let us demonstrate that the state (1.1) is entangled. If it wasn't it could be written as:

$$|\chi\rangle = |u\rangle_1|v\rangle_2,$$

since the couple $(|+\rangle_i, |-\rangle_i)$ is a basis for the spin space of particle i we would have:

$$|u\rangle_1 = a|+\rangle_1 + b|-\rangle_1,$$
$$|v\rangle_2 = c|+\rangle_2 + d|-\rangle_2$$

for some a, b, c and d, thus:

$$|\chi\rangle = ac|+\rangle_1|+\rangle_2 + ad|+\rangle_1|-\rangle_2 + bc|-\rangle_1|+\rangle_2 + bd|-\rangle_1|-\rangle_2.$$

Comparing this last equation with (1.1) we get:

$$\begin{aligned} ac &= bd = 0, \\ ad &= 1, \\ bc &= -1 \end{aligned}$$

which yield $0 = -1$. This absurd conclusion is removed admitting that the state (1.1) is entangled.

Observation 2. In this section we made use of labels for the two particles. This is for sure legitimate if the two particles are different but, with a slight change of the label's meaning, labels can be used in the case of identical particles too. It is in fact sufficient to regard 1 and 2 as labels of the measuring apparatus in which the particle is detected.

1.2 The EPR argument

In this section we will present the argument proposed by Einstein, Podolsky and Rosen [1] to prove that quantum mechanics is incomplete.

Note 1. Throughout the presentation of the argument we will refer to the experimental conditions considered in the previous section.

Let us start by stating clearly all the assumptions on which the argument rests. The first of these is the prediction of quantum mechanics we have derived in the previous section, that is:

(a) *Perfect correlation:* If the spins of the two particles are measured along the same direction, then the two spins will be found (with certainty) to have opposite sign (independently of the time order and spatial distance at which the two measurements are performed).

We then assume the validity of the following (necessary) requirement for a theory to be complete:

(b) *Completeness:* "Every element of the physical reality must have a counterpart in the [complete] physical theory." [1]

Where to identify the elements of the physical reality we assume the validity of the (sufficient) criterion that follows:

(c) *Reality:* "If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." [1]

At last we assume that the two measuring apparatuses are very far form each other, thus, again in EPR's words [1]:

(d) *Locality:* "Since at the time of measurement the two systems [(the two particles in our case)] no longer interact, no real change [[1]] can take place in the second system in consequence of anything that may be done to the first system."

The argument is then the following: Suppose a spin component, say S_x (S_y), of particle 1 is measured at some time t_1, because of the perfect correlation assumption (a), we know that if at some time $t_2 > t_1$ the same spin component of particle 2 is measured it will be found (with certainty) to have opposite sign. Locality, (d), ensures that the elements of physical reality associated to particle 2 have not changed as a consequence of the measurement performed on particle 1. Thus, by the reality assumption (c), there is an element of physical reality corresponding to the S_x (S_y) physical quantity of particle 2. Using again (d) we can state that such element of physical reality existed before the measurement on particle 1 was performed. Exchanging the role of the two particles in the previous steps leads to the conclusion that there corresponds an element of physical reality also to the S_x (S_y) physical quantity of particle 1, this element of physical reality too existed before *any* measurement was performed. We have thus obtained that the results of the two measurements were predetermined. Since the quantum state considered, (1.1), does not determine the result of the two measurements, we conclude that the theory ignores the elements of reality we have found to exist and, by completeness (b), must be considered incomplete. This completes the proof of the EPR "theorem".

Remark 2. It is worth noting here, as Laloë does in [2], that, since perfect correlation is observed only when the particles are in state (1.1), the elements of reality ignored by quantum mechanics cannot be attached to any object other than the two particles. We implicitly made use of this fact in the demonstration of the argument.

Observation 3. The original argument by Einstein, Podolsky and Rosen [1] doesn't involve spin degrees of freedom. The correlation they exploit to carry out their demon-

[1]By "real change" we mean changes to the *elements of the physical reality*

stration is between positions and momenta of two particles. The version of the argument we have presented here is due to Bohm [3].

Another point in which the argument we have presented differs from that of EPR is the final part. Their argument goes through considerations about hypothetical measurements of incompatible observables while we jumped straight to the conclusion.

Moreover, in the original article Einstein, Podolsky and Rosen assume (d) but do not specify how one can be assured that the two systems no longer interact. We have specified one such a way, namely bringing the two systems far from each other. That Einstein would have agreed with us on this ca be deduced from the following quote: "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former." [4].

Chapter 2

GHZ theorem

In this chapter we will present an argument to show that the criticism leveled against quantum mechanics by EPR is void.

The first result of this type is due to John S. Bell and is presented in his work of 1964 [5]. This thesis will not be concerned with Bell's reasoning but will rather illustrate a more recent argument that leads to the conclusion that EPR's premises are inconsistent when applied to systems of three particles. This argument was proposed by Daniel M. Greenberg, Michael A. Horne and Anton Zeilinger in 1989 [6] (see also [7]).

2.1 Bell's formalization of EPR's results

This section serves us to introduce the notation we will use to expose the main matter of this chapter, no new result will be derived here.

In the previous chapter starting from the assumptions of perfect correlation (a), completeness (b), reality (c) and locality (d) we proved the following results:

(i) *Incompleteness:* there are elements of the physical reality that have no counterpart in quantum mechanics.

(ii) *Determinism:* the result of a measurement of a spin component (of one of two particles in the state (1.1)) depends on those elements of the physical reality and is predetermined (i.e. no indeterministic process takes place when the measurement is performed).

Result (i) implies that there exists a more complete specification of the physical reality. Let us denote by λ this more complete description of a pair of particles compatible with the perfect correlation assumption (a), i.e. a description that comprises the elements of physical reality we have found to exist and are ignored by quantum mechanics. Quoting Bell's article [5], in which he introduced for the first time this notation: "It is a matter of indifference ... whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous." By (ii) we then deduce (again following Bell [5]) that the result of a spin component measurement on particle 1, A, is a function of λ, that is:

$$A = A_i(\lambda) \tag{2.1}$$

where $i = x, y$ denotes the spin component (of particle 1) measured. Analogously, for particle 2, the outcome of a measurement of the $j = x, y$ spin component will be given by the function:

$$B = B_j(\lambda). \tag{2.2}$$

In writing $A_i(\lambda)$ and $B_j(\lambda)$ this way we have implicitly fulfilled a request that comes from locality, that is: the result of a measurement of a spin component on particle 1 does not depend on the spin component of particle 2 that is measured (i.e. A depends on i only and not on j), conversely B only depends on j and not on i.

Remark 3. It is easy to see that the perfect correlation assumption "only" implies that (2.1) and (2.2) hold on a subset of λ's of measure unity. We will come back to this in due time.

Observation 4. Working on the objects just presented Bell [5] could derive an inequality that is violated by statistical predictions of quantum mechanics (i.e. Bell derived a contradiction between the EPR assumptions (a), (b), (c) and (d) and statistical predictions of quantum mechanics). Here we report both Bell's inequality and an example of contradiction with quantum mechanics without derivation as we will then focus on a more recent argument on the same matter.

Let $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ be two arbitrary unit vectors. The expectation value of the product of the two particles' spin components measured along $\hat{\mathbf{u}}$ (particle 1) and $\hat{\mathbf{v}}$ (particle 2) is given by:

$$P(\hat{\mathbf{u}}, \hat{\mathbf{v}}) = \int \rho(\lambda) A_{\hat{\mathbf{u}}}(\lambda) B_{\hat{\mathbf{v}}}(\lambda) d\lambda$$

where $\rho(\lambda)$ is the probability distribution of λ. Let $\hat{\mathbf{w}}$ be another unit vector, Bell's inequality is then the following:

$$|P(\hat{\mathbf{u}}, \hat{\mathbf{v}}) - P(\hat{\mathbf{u}}, \hat{\mathbf{w}})| - P(\hat{\mathbf{v}}, \hat{\mathbf{w}}) \leq 1 \tag{2.3}$$

If we take $\hat{\mathbf{u}}$, $\hat{\mathbf{v}}$ and $\hat{\mathbf{w}}$ to lie in the xy plane with azimuthal angles $0°$, $60°$ and $120°$

respectively, quantum mechanics predicts:

$$\begin{aligned} P(\hat{\mathbf{u}}, \hat{\mathbf{v}}) = P(\hat{\mathbf{v}}, \hat{\mathbf{w}}) &= -\frac{1}{2}, \\ P(\hat{\mathbf{u}}, \hat{\mathbf{w}}) &= \frac{1}{2}. \end{aligned}$$

Hence:

$$|P(\hat{\mathbf{u}}, \hat{\mathbf{v}}) - P(\hat{\mathbf{u}}, \hat{\mathbf{w}})| - P(\hat{\mathbf{v}}, \hat{\mathbf{w}}) = \frac{3}{2},$$

in contradiction with Bell's inequality (2.3)

2.2 Adaptation of the previous results to systems of three spin-1/2 particles

As we have anticipated in the introduction to this chapter, the EPR premises contradict quantum mechanics when applied to systems of three particles. Before moving on to present the argument that leads to a contradiction we need to adapt some of the results obtained so far to such systems.

Consider a system of three spin-1/2 particles in the state:

$$|\chi\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle_1 |+\rangle_2 |+\rangle_3 - |-\rangle_1 |-\rangle_2 |-\rangle_3\right) \tag{2.4}$$

where $|+\rangle_i$ and $|-\rangle_i$ are states of spin up and down along the $\hat{\mathbf{z}}$ direction respectively, of particle $i = 1, 2, 3$.

We are interested in the predictions of quantum mechanics for the results of experiments that measure a spin component of each particle. In particular we are interested in measurements of the S_y spin component of two particles and the S_x component of the remaining one. Such experiments correspond to the measurement of each observable in (one of) the following triplets:

$$\begin{aligned} &(S_{1x} \times \mathbb{1}_2 \times \mathbb{1}_3, \quad \mathbb{1}_1 \times S_{2y} \times \mathbb{1}_3, \quad \mathbb{1}_1 \times \mathbb{1}_2 \times S_{3y}) \\ &(S_{1y} \times \mathbb{1}_2 \times \mathbb{1}_3, \quad \mathbb{1}_1 \times S_{2x} \times \mathbb{1}_3, \quad \mathbb{1}_1 \times \mathbb{1}_2 \times S_{3y}) \\ &(S_{1y} \times \mathbb{1}_2 \times \mathbb{1}_3, \quad \mathbb{1}_1 \times S_{2y} \times \mathbb{1}_3, \quad \mathbb{1}_1 \times \mathbb{1}_2 \times S_{3x}) \end{aligned} \tag{2.5}$$

where the $\times$ sign denotes the tensor product of operators and $\mathbb{1}_i$ is the identity operator on the spin space of particle $i = 1, 2, 3$.

It is easy to show that the result of a measurement on one of the particles can be predicted with certainty by appropriate measurements on the other two particles (see Appendix A). If the particles are emitted at a source and move away from it in different directions, let's say in the xy plane at 120° form each other, the measurements can be made in places very far from each other. We are thus in a situation analogous to that envisaged by EPR for their argument and the assumptions (a), (b), (c) and (d) can now be written in the form:

(a′) *Perfect correlation:* If we measure the S_y spin component of any two of the three particles and the S_x component of the remaining one then, the result of the measurement on one of the particles can be predicted with certainty by performing the measurements on the other two.

(b′) *Completeness:* Same as (b).

(c′) *Reality:* Same as (c).

(d′) *Locality:* Since at the time of measurement the three particles no longer interact, no real change can take place in one particle in consequence of anything that may be done to the other two.

The EPR argument presented in §1.2 can then be adapted to these assumptions and thus we conclude that in this case too there exists a more complete specification of the physical reality. If once again we denote this more complete specification by λ we also conclude that the results of spin component measurements on particles 1, 2 and 3, respectively A, B and C, are functions of the spin component being measured and of λ:

$$\begin{aligned} A &= A_i(\lambda) \\ B &= B_j(\lambda) \\ C &= C_k(\lambda) \end{aligned} \tag{2.6}$$

where $i, j, k = x, y$ denote the spin component.

In this case also, locality demands that A doesn't depend on the spin components of particles 2 and 3 that are measured, i.e. A cannot depend on j and k. Similarly B cannot depend on i and k and C cannot depend on i and j.

Remark 4. The same as in Remark 3 holds for the functions in (2.6).

2.3 The GHZ argument

We are now ready to prove the main result of this chapter. With the machinery introduced so far it will be a matter of a few lines.

The EPR argument adapted to the case of three particles in state (2.4) led us to the conclusion that there exists a more complete specification of the physical reality, λ, and three functions A, B and C that determine the results of spin component measurements on the particles.

Consider now a measurement of the product[1] of three spin components, one for each particle. In particular we are interested in the following observables:

$$\begin{aligned} &S_{1x} \times S_{2y} \times S_{3y} \\ &S_{1y} \times S_{2x} \times S_{3y} \\ &S_{1y} \times S_{2y} \times S_{3x}. \end{aligned} \tag{2.7}$$

State (2.4) is an eigenstate of the three observables (2.7) with eigenvalue $+1$ (see Appendix A). Thus (assuming again that quantum mechanics gives correct predictions) we must have:

$$\begin{aligned} A_x B_y C_y &= 1 \\ A_y B_x C_y &= 1 \\ A_y B_y C_x &= 1 \end{aligned} \tag{2.8}$$

where we have dropped the λ in the notation because we are holding it fixed from now on.

If we considered a measurement of the product of the three particles' S_x spin component the result of such a measurement would be given by $A_x B_x C_x$. Because $A_y^2 = 1$, and the same goes for B and C, this can be obtained by multiplying together the three equations in (2.8) yielding:

$$A_x B_x C_x = 1. \tag{2.9}$$

However, the quantum mechanical prediction for the observable:

$$S_{x1} \times S_{2x} \times S_{3x}, \tag{2.10}$$

on the state of equation (2.4), is -1 with probability equal to 1 (refer again to Appendix

[1]Let us remark that what is measured is only the product of three spin components and not individual components.

A) in contrast to equation (2.9).

We have thus obtained a contradiction that can be summarized in the following:

Theorem 1. *One (at least) of the following statements is false:*

(i) Nature is compatible with assumption (d′)

(ii) Nature is compatible with the notion of "element of physical reality" implied by (c′).

(iii) Quantum mechanics gives correct predictions for perfect correlations.

Remark 5. We have seen in Remark 4 that the functions A, B and C need to be well defined "only" on a subset of λ's of measure unity. However, since finite unions of sets of measure zero also have measure zero, the equations in (2.8) and (2.9) also hold on a set of λ's of measure one.

Note 2. (i) and (ii) together are referred to as local-realism.

Chapter 3

Experimental tests

This last chapter will be concerned with some of the experimental efforts that have been made to test quantum mechanics against local-realism following the GHZ argument.

We will illustrate the ideal experimental conditions first, then we will report an experiment that has been realized to observe GHZ entanglement and finally we will give reference to a test of quantum mechanics against local-realism.

3.1 Ideal experimental conditions

In this brief section we will present what the ideal experiment to verify quantum mechanics against local-realism would be within the argument presented in Chapter 2.

We have noticed in §2.3 that the state (2.4) is an eigenstate of all observables (2.7) and (2.10), moreover it is easy to prove that all these observables commute with each other. This implies that, in principle, we could verify the quantum mechanical predictions with a single apparatus that measures subsequently the observables (2.7) and (2.10). In addition, a single run of the experiment would suffice to verify such predictions.

Realizing this experiment would involve putting three particles in state (2.4) and designing apparatuses that measure the product of three spin components.

To our knowledge no attempt on this approach has been made.

3.2 Actual experiments

In this section we will present an experimental approach that has actually been pursued to verify the quantum mechanical predictions against local-realism.

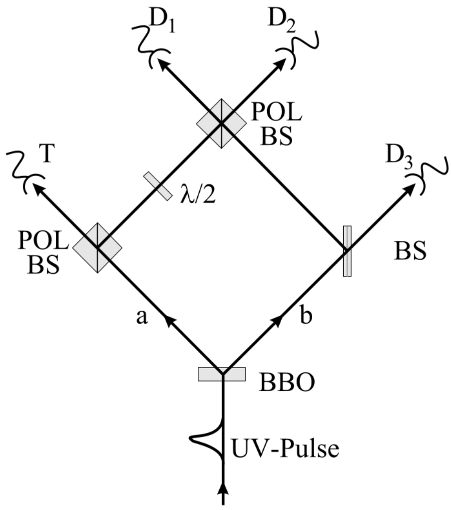

Figure 3.1: Experimental set-up for GHZ entanglement demonstration. Credits [8].

3.2.1 Observation of GHZ entanglement

We begin this section by exposing a method that can be used to obtain systems of three particles exhibiting GHZ entanglement [8].

Consider the experimental apparatus schematized in Fig. 3.1. Short pulses of ultraviolet light, which passes through a nonlinear crystal, generate pairs of polarization entangled photons in the state:

$$\frac{1}{\sqrt{2}}\left(|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b\right), \tag{3.1}$$

where the state $|H\rangle_a|V\rangle_b$ indicates a horizontally polarized photon in arm a and a vertically polarized photon in arm b, conversely, the state $|V\rangle_a|H\rangle_b$ indicates a vertically polarized photon in arm a and a horizontally polarized photon in arm b.

Continuing along arm a we find a polarizing beam splitter that reflects vertically polarized photons and transmits horizontally polarized photons towards detector T. Reflected (vertically polarized) photons go then through a $\lambda/2$ plate that rotates polarization to 45°, then to a second polarizing beam splitter (analogous to the one already encountered) and finally to detectors D_1 (V polarized photons) and D_2 (H polarized photons).

Along arm b we find a 50/50 (polarization independent) beam splitter; transmitted photons go to detector D_3 while reflected photons go the second polarizing beam splitter of arm a and finally to detectors D_1 (H photons) and D_2 (V photons).

All detectors, are behind narrow interference filters, the reason of which will be clarified later.

We want to show here that if two pairs are generated by a single pulse and each of the four photons is detected by one of the detectors T, D_1, D_2 and D_3 (exactly one photon per detector) then (with an additional hypothesis we will present in due time) a three photon GHZ state is recorded by detectors D_1, D_2 and D_3. Here is the reasoning: When such a fourfold detection is observed one photon in arm a must have been H polarized. Its companion in arm b, V polarized, is either transmitted by the (non polarizing) beam splitter and detected by D_3 or reflected and detected by D_2 (each possibility with 50% chance). Consider the former case. Then the other photon in arm b, that must be H polarized, is detected by D_1 and the last photon, coming through arm a, is detected by D_2 and is thus H polarized. This situation corresponds to the detection of the state:

$$|H\rangle_1|H\rangle_2|V\rangle_3 \tag{3.2}$$

by detectors D_1, D_2 and D_3. In the latter case an analogous argument leads us to conclude that the situation corresponds to the detection of the state:

$$|V\rangle_1|V\rangle_2|H\rangle_3. \tag{3.3}$$

The two possible states, (3.2) and (3.3), that correspond to a fourfold detection will not in general superpose coherently, thus will not give a GHZ state.

The statement we make now is that the lack of coherence is due to the possibility of obtaining information on the pair to which each photon belongs. If we can erase such information then a coherent superposition of the states (3.2) and (3.3) will be observed. Let us see why. When a single pulse generates a double pair, these come out from the source in the product state:

$$\frac{1}{2}\left(|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b\right)\left(|H\rangle'_a|V\rangle'_b - |V\rangle'_a|H\rangle'_b\right). \tag{3.4}$$

The evolution of single components from the source to the detectors T, D_1, D_2 and D_3

is given by:

$$
\begin{aligned}
&|H\rangle_a \to |H\rangle_T, \\
&|V\rangle_b \to \frac{1}{\sqrt{2}}\left(|V\rangle_2 + |V\rangle_3\right), \\
&|V\rangle_a \to \frac{1}{\sqrt{2}}\left(|V\rangle_1 + |H\rangle_2\right), \\
&|H\rangle_b \to \frac{1}{\sqrt{2}}\left(|H\rangle_1 + |H\rangle_3\right),
\end{aligned} \tag{3.5}
$$

the same holds for the primed states. Substituting these expressions into (3.4) and dropping the terms that do not correspond to fourfold detection we obtain, after normalization:

$$
\frac{1}{2}\left(|H\rangle_T\left(|H\rangle_1'|H\rangle_2'|V\rangle_3 + |V\rangle_1'|V\rangle_2|H\rangle_3'\right) + |H\rangle_T'\left(|H\rangle_1|H\rangle_2|V\rangle_3' + |V\rangle_1|V\rangle_2'|H\rangle_3\right)\right)
$$

Finally, assuming that primed and unprimed photons are indistinguishable we obtain the state:

$$
\frac{1}{\sqrt{2}}|H\rangle_T\left(|H\rangle_1|H\rangle_2|V\rangle_3 + |V\rangle_1|V\rangle_2|H\rangle_3\right). \tag{3.6}
$$

In the experiment considered primed and unprimed photons could, in principle, be distinguished by measuring their energy (the four photon could all have different energies and the sum of the energies of the photons in each pair must be the same for both pairs) and by measuring the delay with which the photons are detected. As an example of this consider the case in which T and D_3 fire simultaneously and D_1 and D_2 also fire simultaneously, this corresponds to one pair being detected by T and D_3 and the other pair by D_1 and D_2. However, if the pulse that generates the two pairs is short and the interference filters behind which the photons are detected have a narrow bandwidth, such possibility of distinction is lost due to the spreading in time of the wave packets. The possibility of distinction by energy measurement is also lost because of the filters.

Experimental demonstration of GHZ entanglement obtained by means of the procedure just presented is reported in the same letter [8] which we summarize here.

First of all it is demonstrated that only the two components (3.2) and (3.3) are observed. This is done by comparison of the count rates of the eight possible combinations of polarization measurements ($H_1H_2H_3$, $H_1H_2V_3$, ..., $V_1V_2V_3$). A ratio $12:1$ between the expected and unexpected counts is reported. Second, it is demonstrated that the two components (3.2) and (3.3) form a coherent superposition (and not a statistical mixture). This is done by measuring the linear polarization of the photon detected by

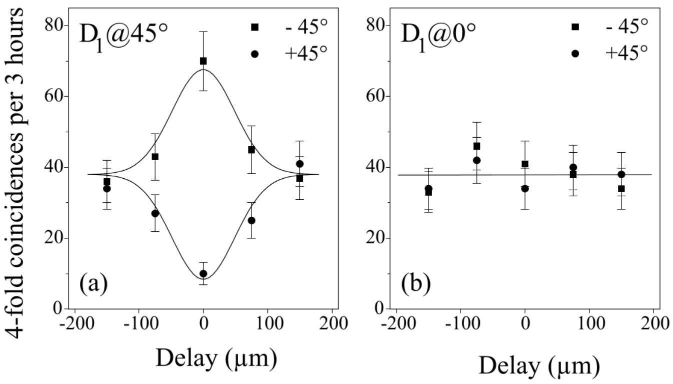

Figure 3.2: "Experimental confirmation of GHZ entanglement. Graph (a) shows the results obtained for polarization analysis of the photon at D_3, conditioned on the trigger, and the detection of one photon at D_1 polarized at $+45°$ and one photon at D_2 polarized at $-45°$. The two curves show the fourfold coincidences for a polarizer oriented at $+45°$ and $-45°$, respectively, in front of detector D_3 as a function of the spatial delay in path a. The difference between the two curves at zero delay confirms the GHZ entanglement. By comparison [graph (b)] no such intensity difference is predicted if the polarizer in front of the detector D_1 is set at $0°$." Credits [8].

D_1 along the $+45°$ bisector between the H and V directions. It is easy to show that such a measurement projects the state (3.6) into:

$$\frac{1}{\sqrt{2}}|H\rangle_T|+45°\rangle_1\left(|H\rangle_2|V\rangle_3+|V\rangle_2|H\rangle_3\right),$$

that takes the following form when written on the $(|+45°\rangle_i, |-45°\rangle_i), i=2,3$ basis:

$$\frac{1}{\sqrt{2}}|H\rangle_T|+45°\rangle_1\left(|+45°\rangle_2|+45°\rangle_3-|-45°\rangle_2|-45°\rangle_3\right).$$

The terms involving $|+45°\rangle_2|-45°\rangle_3$ and $|-45°\rangle_2|+45°\rangle_3$ interfere destructively, thus their absence indicates the coherent superposition of terms in (3.6).

To show that it is the "which-path" information that destroys coherence measures at different delays are taken. This is obtained by varying path a (and thus gradually restoring the possibility of pair distinction).

Fig. 3.2 reports experimental results regarding the polarization of photon 3 for fourfold events in which photon 1 and 2 are polarized at $+45°$ and $-45°$ respectively. Squares refer to $-45°$ polarization and circles to $+45°$ polarization.

Observation 5. The argument presented above and the subsequent experimental demonstration show that entanglement doesn't necessarily arise from interaction of the entangled subsystems. We want to give here a general illustration of such a fact taken from [9].

Consider the factorized two particle state:

$$|\psi\rangle = |\alpha\rangle_1 |\beta\rangle_2 \tag{3.7}$$

and the entangled state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|a\rangle_1 |b\rangle_2 + |c\rangle_1 |d\rangle_2 \right) \tag{3.8}$$

where, for simplicity, $\langle a|c\rangle = \langle b|d\rangle = 0$. Then, if (3.7) is not orthogonal to (3.8), it is possible to obtain an entangled state from (3.7) using the projection operator:

$$P = |\Psi\rangle\langle\Psi|. \tag{3.9}$$

The experiment presented in this section can be seen as an actual realization of a projector of the type (3.9). Another example of such a projector that has been experimentally realized is the one used in entanglement swapping experiments which is described in [9].

3.2.2 Claims regarding local-realism and quantum mechanics

The experimental observations and analysis we have reported in the previous subsection are not sufficient to make statements against or in favor of local-realism (or quantum mechanics). For example the experimental procedure might be put into question because of the selection needed to isolate fourfold coincidences.

However it has been shown in [10] that a more refined analysis of the experiment we have presented, together with some additional operational requirements, leads to the possibility of observing violations of local-realism (or of quantum mechanics). In fact claims of having observed such violations (of local-realism) have been made [11].

The matter of these two last references [10] and [11] is nevertheless beyond the scope of this thesis and will not be discussed further.

Note 3. It is interesting to note the history of publication of references [8], [10] and [11].

In reference [8] no claims regarding local-realism were made. Marek Żukowski, informed of the results in [8], carried out a more detailed analysis of the experiment. He found that minor adjustments were to be made in order to be in a position to make such claims [10]. The same group of [8] published [11] less than a month later than [10].

Conclusion

In this thesis we have presented EPR's critique of quantum mechanics, we have seen that, under the assumptions of perfect correlation, reality and locality, quantum mechanics is not a complete description of reality (according to EPR's criterion of completeness).

We briefly mentioned the Bell's inequality, that can be derived from the EPR assumptions, and we have seen that the inequality is violated by statistical predictions of quantum mechanics.

Then we have illustrated the GHZ argument showing that the EPR reasoning doesn't apply to systems of three particles. We have seen that for such systems the EPR premises are inconsistent and, in contrast with Bell's argument, the contradiction arises for perfect correlations predicted by quantum mechanics[1]. Hence, the demonstration of the conflict between EPR's assumptions and quantum mechanics doesn't involve inequalities.

In the last chapter we have seen that, in principle, the GHZ reasoning allows to design experiments in which a single run (of the experiment) is sufficient to perform a test of quantum mechanics against local-realism. To our knowledge no experiment of this type has been performed.

Finally, we have discussed some actual experimental effort towards a test of quantum mechanics following the GHZ argument. We have presented an experiment that has been performed to observe GHZ entanglement, but the analysis we have reported didn't allow us to make claims in favor or against quantum mechanics. However, we have given reference to a more refined analysis of a slightly modified version of the experiment that allowed such claims to be made.

The techniques employed in the experiment we have presented also allowed us to observe that entanglement doesn't necessarily arise from interaction of the entangled subsystems but can also be obtained by a suitable projection (onto an entangled state) of a factorized state.

[1] In [7] the authors note: "There is an irony in this result in that perfect correlations are central to EPR's argument for the existence of states more complete than those of quantum mechanics."

Appendix A

A.1 Singlet state representation on different bases

We prove here that the state of equation (1.1) takes the same form on all orthonormal bases of the S_x, S_y and S_z observables.

Let us prove that the state:

$$|\chi\rangle_{\hat{\mathbf{z}}} = \frac{1}{\sqrt{2}} \left(|\hat{\mathbf{z}}, +\rangle_1 |\hat{\mathbf{z}}, -\rangle_2 - |\hat{\mathbf{z}}, -\rangle_1 |\hat{\mathbf{z}}, +\rangle_2 \right) \tag{A.1}$$

has the same form on the tensor basis obtained from the two bases $\left(|\hat{\mathbf{x}}, +\rangle_i, |\hat{\mathbf{x}}, -\rangle_i\right), i = 1, 2$ the same argument can be adapted to the $\hat{\mathbf{y}}$ case.

Let us write the S_x operator as a 2×2 matrix on the $(|\hat{\mathbf{z}}, +\rangle_i, |\hat{\mathbf{z}}, -\rangle_i)$ basis:

$$S_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

in units $\frac{\hbar}{2} = 1$. This operator can be diagonalized leading to the eigenvectors:

$$\begin{aligned} |\hat{\mathbf{x}}, +\rangle_i &= \frac{1}{\sqrt{2}} \left(|\hat{\mathbf{z}}, +\rangle_i + |\hat{\mathbf{z}}, -\rangle_i \right), \\ |\hat{\mathbf{x}}, -\rangle_i &= \frac{1}{\sqrt{2}} \left(|\hat{\mathbf{z}}, +\rangle_i - |\hat{\mathbf{z}}, -\rangle_i \right) \end{aligned} \tag{A.2}$$

with eigenvalues $+1$ and -1 respectively. Inverting the two equations (A.2) and substituting into (A.1) leads to the desired result.

A.2 Quantum mechanical predictions on GHZ states

A.2.1 Perfect correlation in systems of three spin-1/2 particles

Consider three particles in the state:

$$|\chi\rangle = |+\rangle_1|+\rangle_2|+\rangle_3 - |-\rangle_1|-\rangle_2|-\rangle_3, \tag{A.3}$$

we will show here that the result of a measurement of either the S_x or S_y spin component on any of the three particles can be predicted with certainty by appropriate measurements on the other two.

Let us take, for example, a S_y spin component measurement on particle 3, analogous arguments can be applied to the other combinations of particles and spin components.

To predict the result of this measurement we consider the first of the triples in (2.5), that is:

$$(S_{1x} \times \mathbb{1}_2 \times \mathbb{1}_3, \quad \mathbb{1}_1 \times S_{2y} \times \mathbb{1}_3, \quad \mathbb{1}_1 \times \mathbb{1}_2 \times S_{3y})\,.$$

We will show that after the first two observables in the triple are measured the third particle will be left in an eigenstate of the $\mathbb{1}_1 \times \mathbb{1}_2 \times S_{3y}$ observable.

Writing the state (A.3) on a basis of eigenstates of the three observables in the triplet considered leads to:

$$\begin{aligned}|\chi\rangle = \frac{1}{2}\,(&i|\hat{\mathbf{x}},+\rangle_1|\hat{\mathbf{y}},+\rangle_2|\hat{\mathbf{y}},-\rangle_3 + i|\hat{\mathbf{x}},+\rangle_1|\hat{\mathbf{y}},-\rangle_2|\hat{\mathbf{y}},+\rangle_3 + \\ &+|\hat{\mathbf{x}},-\rangle_1|\hat{\mathbf{y}},+\rangle_2|\hat{\mathbf{y}},+\rangle_3 - |\hat{\mathbf{x}},-\rangle_1|\hat{\mathbf{y}},-\rangle_2|\hat{\mathbf{y}},-\rangle_3)\,.\end{aligned}$$

Thus to any of the four possible combinations of results for the first two measurements there corresponds only one possible result for the third observable.

A.2.2 Measurements of the product of spin components on GHZ states

We will show here that the state (2.4), i.e.:

$$|\chi\rangle = |+\rangle_1|+\rangle_2|+\rangle_3 - |-\rangle_1|-\rangle_2|-\rangle_3$$

is an eigenstate of all of the observables (2.7), i.e.:

$$\begin{aligned}&S_{1x} \times S_{2y} \times S_{3y}\\ &S_{1y} \times S_{2x} \times S_{3y}\\ &S_{1y} \times S_{2y} \times S_{3x}\end{aligned} \tag{A.4}$$

with eigenvalue $+1$ and it is also eigenstate of the observable (2.10), i.e.:

$$S_{x1} \times S_{2x} \times S_{3x}$$

this time with eigenvalue -1.

Let us prove this statement for the first observable in (A.4) the same argument can be adapted to the other observables. We begin with the spin of particle 1:

$$\begin{aligned} S_{1-}|\chi\rangle &= 2|-\rangle_1|+\rangle_2|+\rangle_3 \\ S_{1+}|\chi\rangle &= -2|+\rangle_1|-\rangle_2|-\rangle_3 \end{aligned}$$

that gives:

$$|\chi\rangle' := S_{1x}|\chi\rangle = |-\rangle_1|+\rangle_2|+\rangle_3 - |+\rangle_1|-\rangle_2|-\rangle_3.$$

For particle 2 we have:

$$\begin{aligned} S_{2-}|\chi\rangle' &= 2|-\rangle_1|-\rangle_2|+\rangle_3 \\ S_{2+}|\chi\rangle' &= -2|+\rangle_1|+\rangle_2|-\rangle_3 \end{aligned}$$

thus:

$$|\chi\rangle'' := S_{2y}|\chi\rangle' = -\frac{1}{i}\left(|+\rangle_1|+\rangle_2|-\rangle_3 + |-\rangle_1|-\rangle_2|+\rangle_3\right).$$

The same goes through for particle 3 and leads to:

$$|\chi\rangle''' := S_{3y}|\chi\rangle'' = |+\rangle_1|+\rangle_2|+\rangle_3 - |-\rangle_1|-\rangle_2|-\rangle_3 = |\chi\rangle.$$

which is the desired result.

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