海量数据流处理技术

 

童咏昕

计算机学院 软件开发环境国家重点实验室

yxtong@buaa.edu.cn

课程提纲

- 数据流简介
- 基础性问题
- 确定性算法
  - Majority算法
  - Misra-Gries(MG)算法
  - Space-Saving算法
  - Lossy Counting算法
- 随机性算法
  - Sticky Sampling算法
  - Count-Min(CM)Sketch算法

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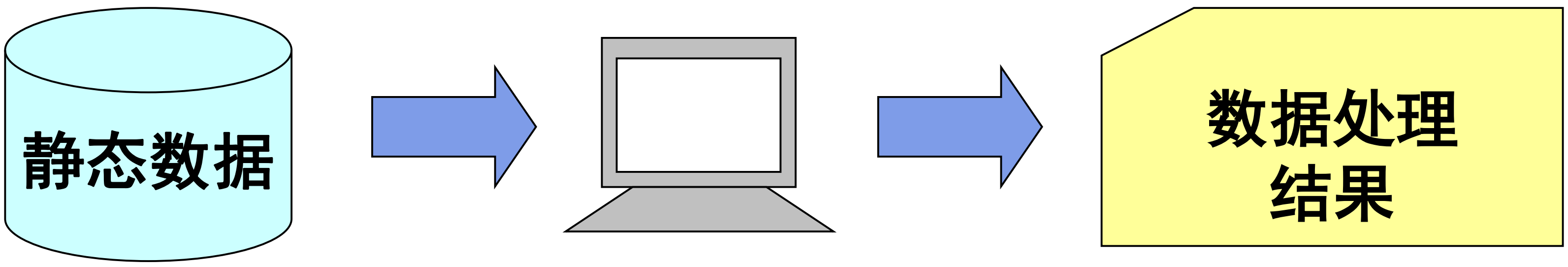
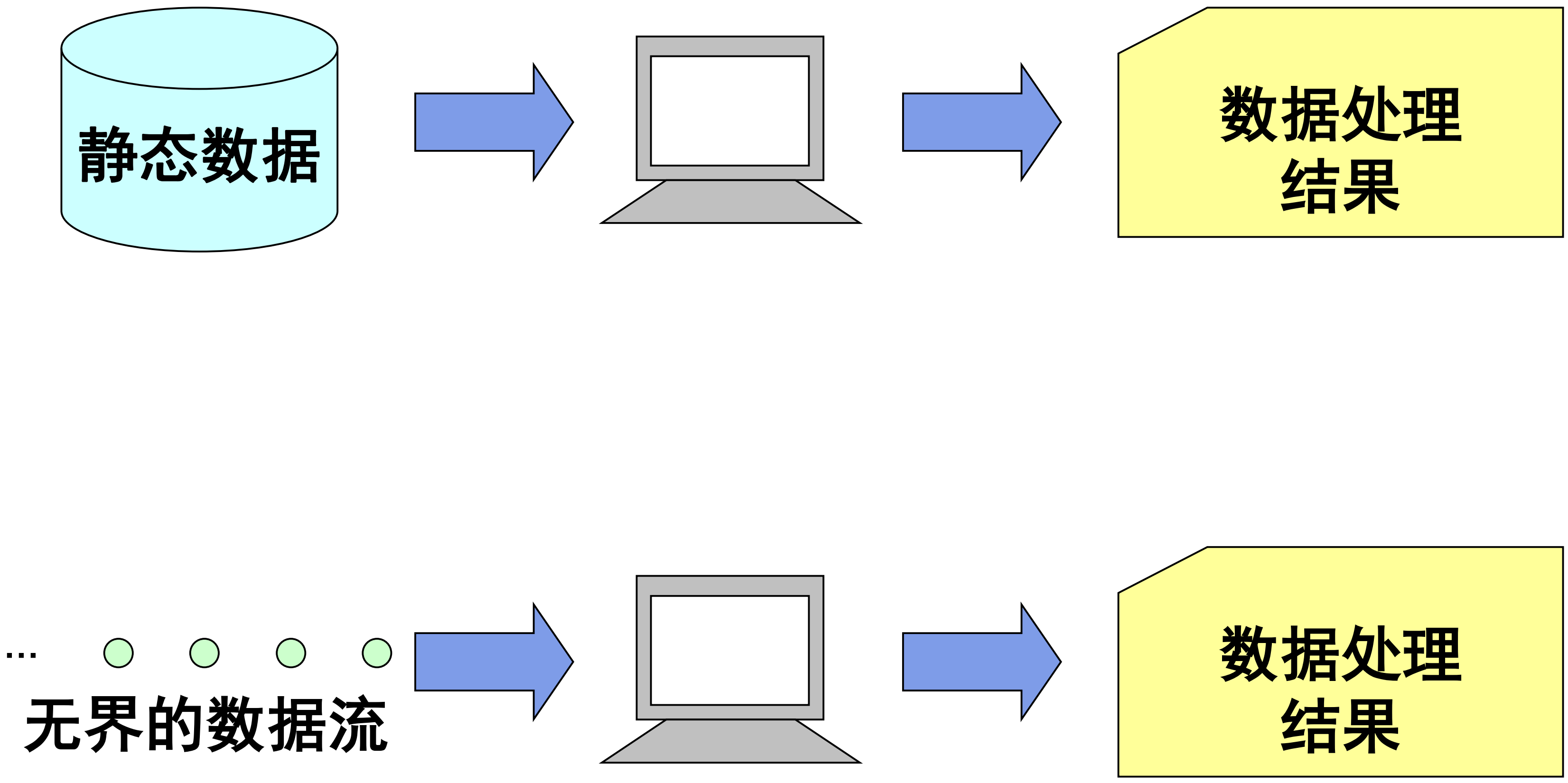
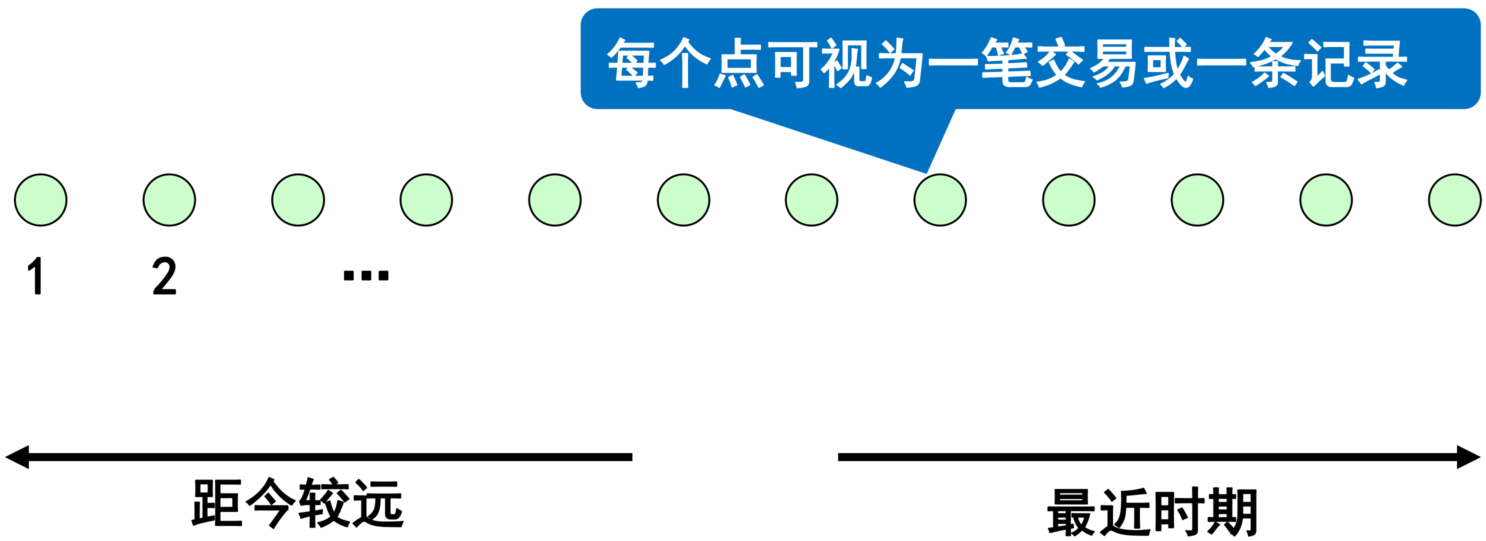
静态数据 vs. 流状数据



静态数据 vs. 流状数据

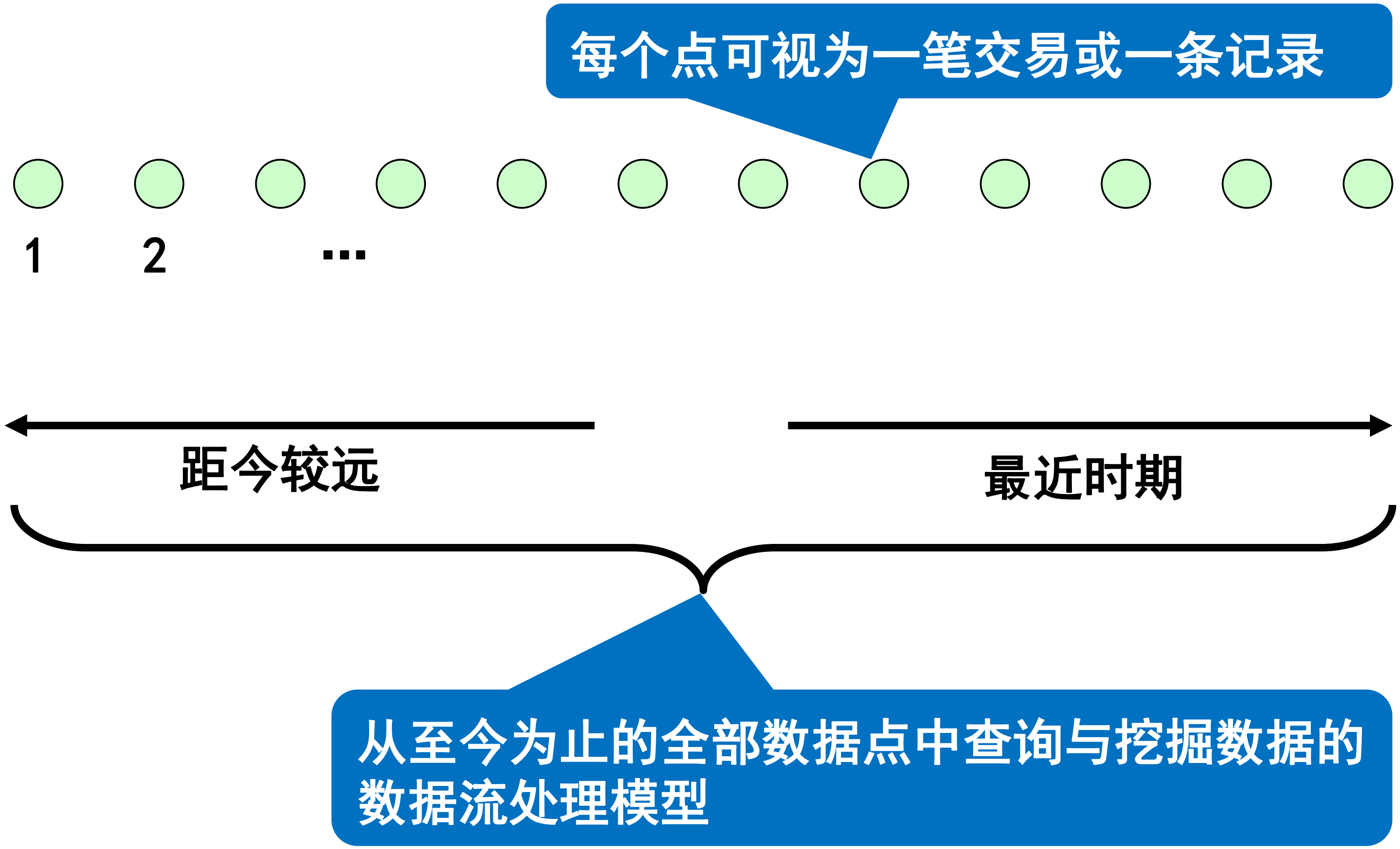


数据流概念



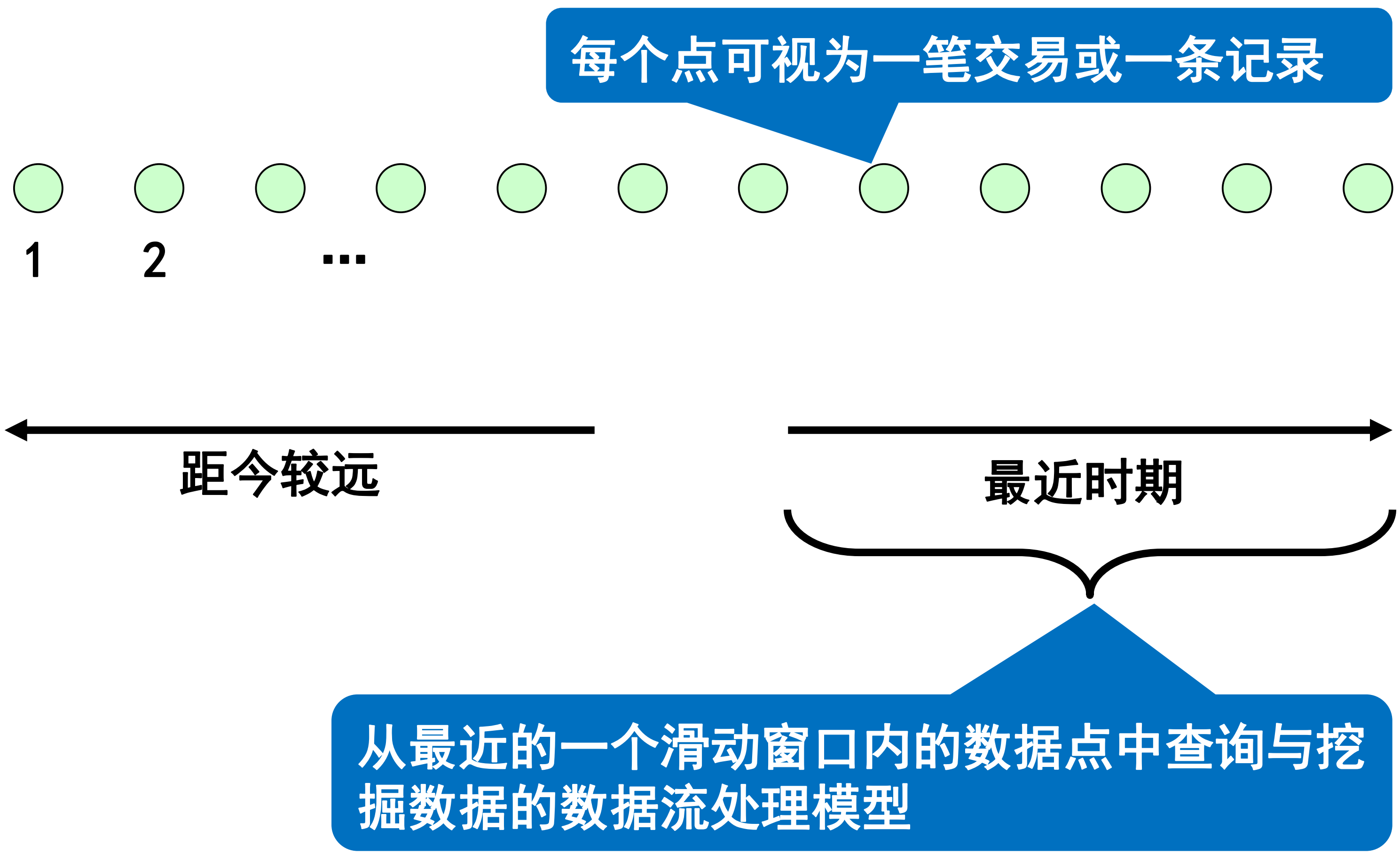
数据流模型

- 完整数据流模型(Entire Data Stream Model)



数据流模型

- 滑动窗口数据流模型(Sliding-Window Model)



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数据流处理的基础性问题

差异项种类查询问题：给定一个长度为N的数据流，该问题旨在完整数据流模型下统计差异项（即不同项）种类的个数！

数据流处理的基础性问题

差异项种类查询问题：给定一个长度为N的数据流，该问题旨在完整数据流模型下统计差异项（即不同项）种类的个数！

差异项频率查询问题：给定一个长度为N的数据流，该问题完整数据流模型下统计每个差异项（即不同项）的出现频率！

数据流处理的基础性问题

差异项种类查询问题：给定一个长度为N的数据流，该问题旨在完整数据流模型下统计差异项（即不同项）种类的个数！

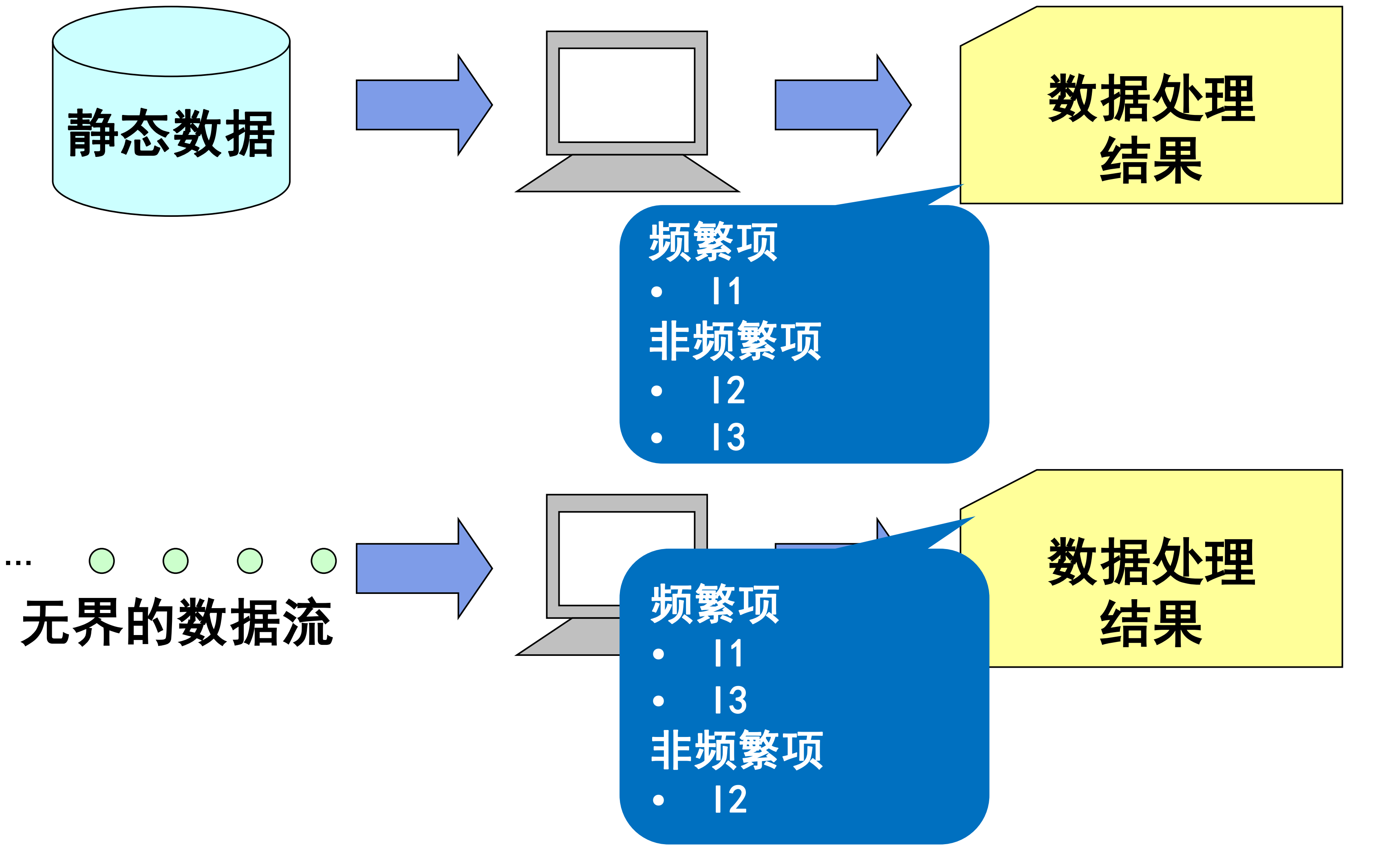
频繁项查询问题：给定一个长度为N的数据流和频繁阈值s，该问题旨在完整数据流模型下发现全部频繁项，即每个项出现频率大于sN！

差异项频率查询问题：给定一个长度为N的数据流，该问题完整数据流模型下统计每个差异项（即不同项）的出现频率！

数据流处理算法特性

	传统静态数据处理	动态数据流处理
数据类型	有限规模的静态数据	无限规模、动态且高速抵达的流状态数据
存储方式	硬盘/存储有限	存储空间不足
响应效率	非实时查询	实时查询
查询结果	精准结果	近似（或精准）结果

静态数据查询 vs. 数据流查询



假阳性(False Positive)

- E.g.
  - 期望输出
    - 频繁项
      - I_1
    - 非频繁项
      - I_2
      - I_3
  - 算法输出
    - 频繁项
      - I_1
      - I_3
    - 非频繁项
      - I_2

假阳性
-某项被分类为频繁项
-但该项事实上是非频繁项

哪个项是假阳性? I_3

还有么? **没了**

假阳性数量 = 1

如果称算法不存在假阳性

所有非频繁项在算法输出中都被分类为非频繁项

假阴性(False Negative)

- E. g.
  - 期望输出
    - 频繁项
      - I_1
    - 非频繁项
      - I_2
      - I_3
  - 算法输出
    - 频繁项
      - I_1
      - I_3
    - 非频繁项
      - I_2

假阴性
-某项被分类为非频繁项
-但该项事实上是频繁项

哪个项是假阴性? I_3

还有么? 没了

假阴性数量 = 1

假阳性数量 = 0

如果称算法不存在假阴性

所有频繁项在算法输出中都被分类为频繁项

数据流查询的近似误差

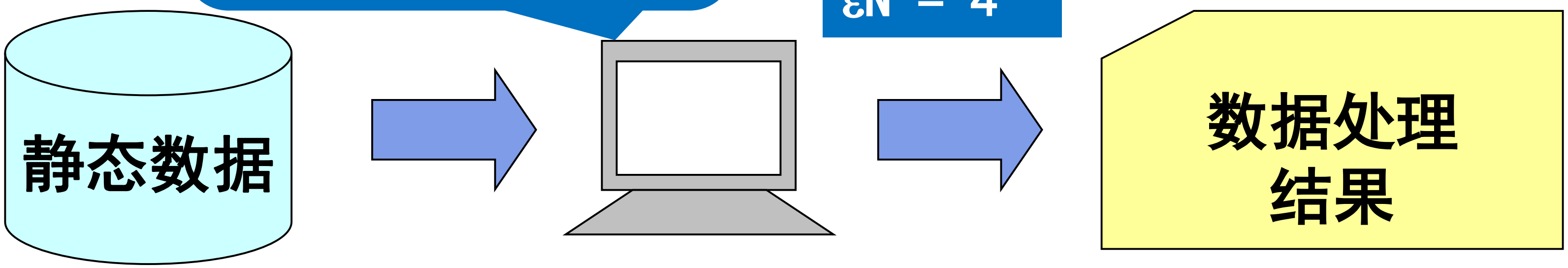
N:流中项总数

N = 20

ε = 0.2

εN = 4

所有项真实统计信息
- I1: 10
- I2: 8
- I3: 12



项	真实出现次数	估计出现次数	Diff. D	D <= εN ?
I_1	10	10	0	Yes
I_2	8	4	4	Yes
I_3	12	10	2	Yes

所有项估计统计信息
- I1: 10
- I2: 4
- I3: 10

容错概要(ε-deficient synopsis)

- 数据流长度N：为流的当前长度(或项出现的总次数)
- 近似误差ε：为输入参数(0和1之间的实数)

- 一个算法如果满足以下条件，则符合ε-deficient synopsis
  - 条件 1：不存在假阴性.

    在算法的输出中，所有真正频繁的项都被输出为频繁项
  - 条件 2：估计频率和真实频率的差值不超过εN.
  - 条件 3：所有真实频率少于(s-ε)N的项都会被分类为不频繁项.

近似频繁项查询问题：给定一个长度为N的数据流、频繁阈值s和近似误差ε，该问题旨在完整数据流模型下发现全部的近似频繁项，即每个项出现频率大于(s-ε)N！

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热身:过半数Majority算法(>1/2)

- 问题：如何找出数据流S中出现次数超过1/2的元素
  - |S|很大,无法记录每个元素的出现次数

热身:过半数Majority算法(>1/2)

- 输入：
  - 数据流$S=x_1, x_2, \cdots, x_n, x_i \in U$
- 输出：流S中出现频率超过s=1/2的元素

- 算法：MAJORITY
  - 如果存在多数元素,则MAJORITY一定返回该元素

Majority算法

- MAJORITY: Initialize(x)
  - key ← ∅, value ← 0
- MAJORITY: Update(x)
  - If value = 0 then
    - key ← x, value = 1
  - Else if key = x then
    - value = value + 1
  - Else
    - value = value − 1
- MAJORITY: Return()
  - If value > 0 then return key
  - Else return ∅

Majority算法

- MAJORITY: Initialize(x)
  - key ← ∅, value ← 0
- MAJORITY: Update(x)
  - If value = 0 then
    - key ← x, value
  - Else if key = x then
    - value = value + 1
  - Else
    - value = value − 1
- MAJORITY: Return()
  - If value > 0 then return key
  - Else return ∅

初始化key为空，计数器为0

Majority算法

- MAJORITY: Initialize(x)
  - key ← ∅, value ← 0
- MAJORITY: Update(x)
  - If value = 0 then
    - key ← x, value = 1
  - Else if key = x then
    - value = value + 1
  - Else
    - value = value - 1
- MAJORITY: Return()
  - If value > 0 then return key
  - Else return ∅

如果计数器为0，记录当前元素
并设计数器为1

Majority算法

- MAJORITY: Initialize(x)
  - key ← ∅, value ← 0
- MAJORITY: Update(x)
  - If value = 0 then
    - key ← x, value = 1
  - Else if key = x then
    - value = value + 1
  - Else
    - value = value
- MAJORITY: Return
  - If value > 0 then return key
  - Else return ∅

如果计数器不为0，且x与key记录元素相同，增加计数器

Majority算法

- MAJORITY: Initialize(x)
  - key ← ∅, value ← 0
- MAJORITY: Update(x)
  - If value = 0 the
    - key ← x, value
  - Else if key = x then
    - value = value + 1
  - Else
    - value = value − 1
- MAJORITY: Return()
  - If value > 0 then return key
  - Else return ∅

如果计数器不为0,且x与key记录元素不同,减小计数器

Majority算法

- MAJORITY: Initialize(x)
  - key ← ∅, value ← 0
- MAJORITY: Update(x)
  - If value = 0 then
    - key ← x, value = 1
  - Else if key = x then
    - value = value
  - Else
    - value = value - 1
- MAJORITY: Return()
  - If value > 0 then return key
  - Else return ∅

如果计数器不为0，则key中记录元素为出现次数超过1/2元素

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = ∅, value = 0

两种元素

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value − 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value − 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 0

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value − 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
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Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 0

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Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 1

- If value = 0 then
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Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 0

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  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 0

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 2

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, b, a, b, a, b, a, a, b
- key = a, value = 1
- 存在多数元素a
- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = $\varnothing$, value = 0

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

多种元素且无任意元素频率过半

MAJORITY不适用

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 0

- If value = 0 then
  - key ← x, value = 1
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Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 1

- If value = 0 then
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Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
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Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 0

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 0

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = a, value = 0

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, f, d
- key = d, value = 1

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

Majority算法实例

- a, b, a, c, a, e, a, d, a, a, d
- key = d, value = 1
- d并非多数元素
- 需要检查. 但对于流数据, 无法检查

- If value = 0 then
  - key ← x, value = 1
- Else if key = x then
  - value = value + 1
- Else
  - value = value - 1

过半数Majority算法扩展(>1/k)

- 问题：如何找出数据流S中出现频率超过s=1/k的元素
  - |S|很大，无法记录每个元素的出现次数

过半数Majority算法扩展(>1/k)

- 问题：如何找出数据流S中出现频率超过s=1/k的元素
  - |S|很大,无法记录每个元素的出现次数

- 输入：
  - 数据流$S=x_1, x_2, \cdots, x_n$, $x_i \in U$
- 输出：S中所有出现频率超过s=1/k的元素

- 算法：
  - MG算法
  - Space-Saving算法

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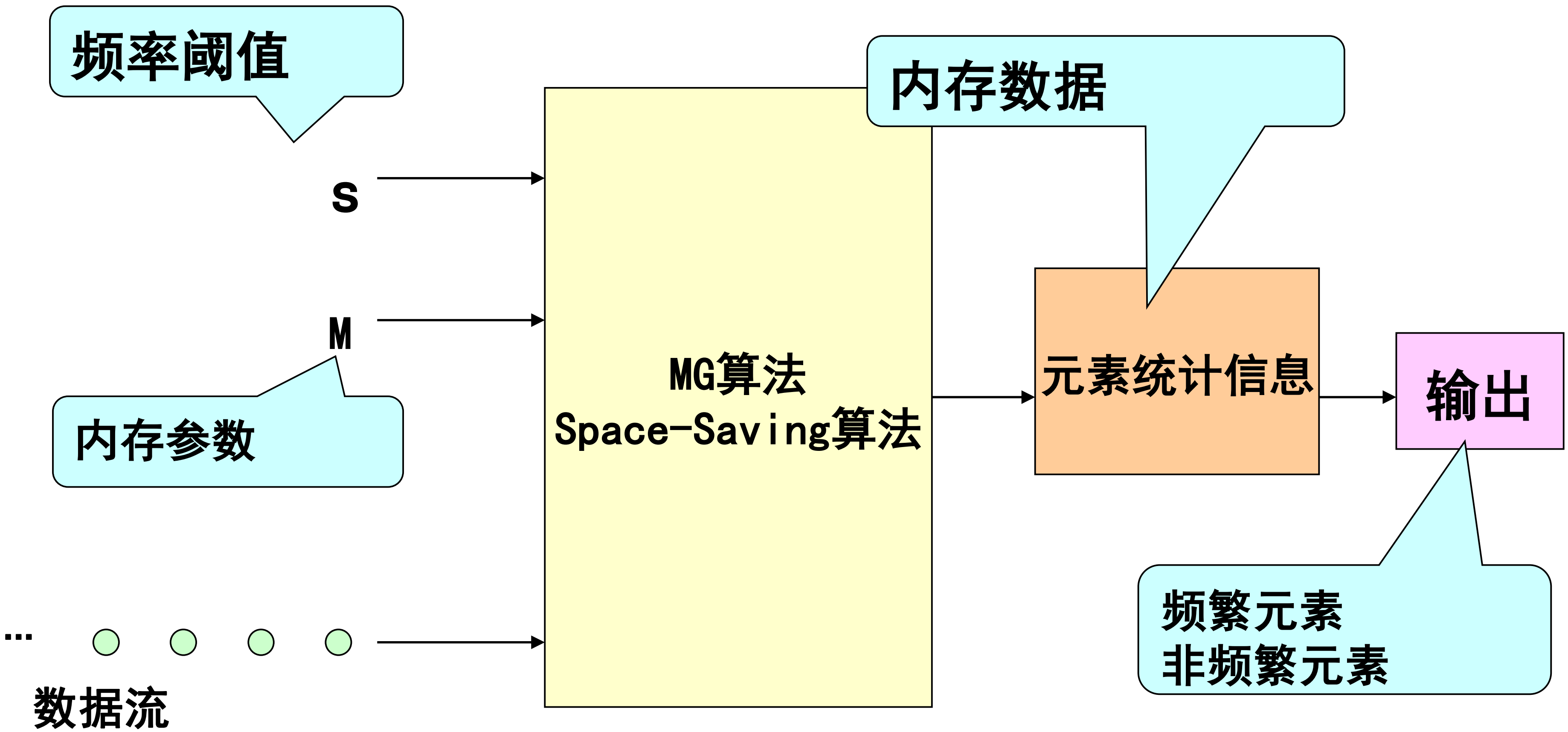
MG算法与Space-Saving算法框架



Misra-Gries(MG)算法

- MG:Update(x,w)
  - If $x \in T$
    - $w_x \leftarrow w_x + w$
  - Else
    - $T \leftarrow T \cup \{x\}$
    - $w_x \leftarrow w$
    - If $|T| > k$ then
      - $m = \min(\{w_x : x \in T\})$
      - Forall $j \in T$ do
        - $w_j \leftarrow w_j - m$
        - If $w_j = 0$ then $T \leftarrow T \setminus \{j\}$

Misra-Gries(MG)算法

- MG:Update(x, w)
  - If $x \in T$
    - $w_x \leftarrow w_x + w$
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      - Forall $j \in T$ do
        - $w_j \leftarrow w_j - m$
        - If $w_j = 0$ then $T \leftarrow T \setminus \{j\}$

对于已经在T中的元素，直接更新其权重

Misra-Gries (MG) 算法

对于不在T中的元素，将其加入T并初始化其权重

- MG:Update(x, w)
  - If $x \in T$
    - $w_x \leftarrow w_x + w$
  - Else
    - $T \leftarrow T \cup \{x\}$
    - $w_x \leftarrow w$
    - If $|T| > k$ then
      - $m = \min(\{w_x : x \in T\})$
      - Forall $j \in T$ do
        - $w_j \leftarrow w_j - m$
        - If $w_j = 0$ then $T \leftarrow T \setminus \{j\}$

Misra-Gries(MG)算法

如果|T|>k，将T中每个元素的权重减去T中元素的最小权重，并删除权重为0的元素

- MG:Update(x, w)
  - If $x \in T$
    - $w_x \leftarrow w_x + w$
  - Else
    - $T \leftarrow T \cup \{x\}$
    - $w_x \leftarrow w$
    - If $|T|>k$ then
      - $m=\min(\{w_x : x \in T\})$
      - Forall $j \in T$ do
        - $w_j \leftarrow w_j - m$
        - If $w_j=0$ then $T \leftarrow T \setminus \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(−, 0), (−, 0), (−, 0)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T| > k$ then
    - $m = \min(\{w_x : x \in T\})$
    - Forall $j \in T$ do
      - $w_j \leftarrow w_j - m$
      - If $w_j = 0$ then $T \leftarrow T \setminus \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 1), (–, 0), (–, 0)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T|>k$ then
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MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 1), (b, 1), (-, 0)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T|>k$ then
    - $m=\min(\{w_x : x \in T\})$
    - Forall $j \in T$ do
      - $w_j \leftarrow w_j - m$
      - If $w_j=0$ then $T \leftarrow T \setminus \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (b, 1), (-, 0)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T| > k$ then
    - $m = \min(\{w_x : x \in T\})$
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MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (b, 1), (c, 1)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
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  - $T \leftarrow T \cup \{x\}$
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      - $w_j \leftarrow w_j - m$
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MG算法实例

- a， b， a， c， d， e， a， d， f， a， d
- T={(a, 2), (b, 1), (c, 1), (d, 1)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
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MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 1), ~~(b, 0), (c, 0), (d, 0)~~}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T| > k$ then
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MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 1), (–, 0), (–, 0)}

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MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a,1),(e,1),(-,0)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
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MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (e, 1), (–, 0)}

内存参数|T|=3

- If $x \in T$
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      - If $w_j = 0$ then $T \leftarrow T \setminus \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (e, 1), (d, 1)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T|>k$ then
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    - Forall $j \in T$ do
      - $w_j \leftarrow w_j - m$
      - If $w_j=0$ then $T \leftarrow T \setminus \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a,2),(e,1),(d,1),(f,1)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T|>k$ then
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MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 1), ~~(e, 0), (d, 0), (f, 0)~~}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T|>k$ then
    - $m=\min(\{w_x : x \in T\})$
    - Forall $j \in T$ do
      - $w_j \leftarrow w_j - m$
      - If $w_j=0$ then $T \leftarrow T \backslash \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 1), (–, 0), (–, 0)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
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    - Forall $j \in T$ do
      - $w_j \leftarrow w_j - m$
      - If $w_j=0$ then $T \leftarrow T \setminus \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (−, 0), (−, 0)}

内存参数|T|=3

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If $|T|>k$ then
    - $m=\min(\{w_x : x \in T\})$
    - Forall $j \in T$ do
      - $w_j \leftarrow w_j - m$
      - If $w_j=0$ then $T \leftarrow T \setminus \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (d, 1), (−, 0)}

内存参数|T|=3

- If x∈T
  - $w_x \leftarrow w_x + w$
- Else
  - $T \leftarrow T \cup \{x\}$
  - $w_x \leftarrow w$
  - If |T|>k then
    - $m = \min(\{w_x : x \in T\})$
    - Forall j∈T do
      - $w_j \leftarrow w_j - m$
      - If $w_j = 0$ then $T \leftarrow T \setminus \{j\}$

MG算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a,2),(d,1),(-,0)}

- T中每个元素的估计误差不超过W/|T|=11/3
  - W为流中所有元素权重加和

- 使用T找出流中出现频率超过1/2的元素
- 流中出现频率超过1/2的元素为a
  - a: 2 + 11/3 > 11/2

Space-Saving算法

- Space-Saving:Update(x, w)
  - If $x \in T$
    - $w_x \leftarrow w_x + w$
  - Else
    - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$
    - $w_x \leftarrow w_y + w$
    - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法

- Space-Saving:Update(x, w)
  - If $x \in T$
    - $w_x \leftarrow w_x + w$
  - Else
    - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$
    - $w_x \leftarrow w_y + w$
    - $T \leftarrow T \cup \{x\} \setminus \{y\}$

对于已经在T中的元素，直接更新其权重

Space-Saving算法

- Space-Saving:Update(x,w)
  - If $x \in T$
    - $w_x \leftarrow w_x + w$
  - Else
    - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$
    - $w_x \leftarrow w_y + w$
    - $T \leftarrow T \cup \{x\} \setminus \{y\}$

对于不在T中的元素，将其权重加上T中元素的最小权重，并取代T中权重最小的元素

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(-, 0), (-, 0), (-, 0), (-, 0)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \operatorname{argmin}_{y \in T} w_j$
  - $w_x \leftarrow w_y + w$
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 1), (-, 0), (-, 0), (-, 0)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$ (y = -)
  - $w_x \leftarrow w_y + w$ (w_y = 0)
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 1), (b, 1), (-, 0), (-, 0)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \arg\min_{y \in T} w_j$ (y = -)
  - $w_x \leftarrow w_y + w$ (w_y = 0)
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (b, 1), (-, 0), (-, 0)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \text{argmin}_{y \in T} w_j$
  - $w_x \leftarrow w_y + w$
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (b, 1), (c, 1), (-, 0)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$ (y = -)
  - $w_x \leftarrow w_y + w$ (w_y = 0)
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (b, 1), (c, 1), (d, 1)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$ (y = -)
  - $w_x \leftarrow w_y + w$ (w_y = 0)
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 2), (e, 2), (c, 1), (d, 1)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$ (y = b)
  - $w_x \leftarrow w_y + w$ (w_y = 1)
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 3), (e, 2), (c, 1), (d, 1)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$
  - $w_x \leftarrow w_y + w$
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 3), (e, 2), (c, 1), (d, 2)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$
  - $w_x \leftarrow w_y + w$
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 3), (e, 2), ~~(c, 1)~~, (d, 2)}
- T={(a, 3), (e, 2), (f, 2), (d, 2)}
- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \arg\min_{y \in T} w_j$ (y = c)
  - $w_x \leftarrow w_y + w$ (w_y = 1)
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

内存参数|T|=4

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 4), (e, 2), (f, 2), (d, 2)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$
  - $w_x \leftarrow w_y + w$
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a, 4), (e, 2), (f, 2), (d, 3)}

内存参数|T|=4

- If $x \in T$
  - $w_x \leftarrow w_x + w$
- Else
  - $y \leftarrow \mathrm{argmin}_{y \in T} w_j$
  - $w_x \leftarrow w_y + w$
  - $T \leftarrow T \cup \{x\} \setminus \{y\}$

Space-Saving算法实例

- a, b, a, c, d, e, a, d, f, a, d
- T={(a,4),(e,2),(f,2),(d,3)}

- T中每个元素出现次数估计误差不超过W/|T|=11/4
  - W为流中所有元素权重加和

- 使用T找出流中出现频率超过1/3的元素
- 流中出现频率超过1/3的元素为a,e,f,d
  - a: 4+11/4 > 11/3
  - e: 2+11/4 > 11/3,
  - f: 2+11/3 > 11/3
  - d: 3+11/4 > 11/3

MG算法与Space-Saving算法关系

- $T_{MG}=\{(a,2),(d,1),(-,0)\}$
- $T_{Space\text{-}Saving}=\{(a,4),(e,2),(f,2),(d,3)\}$

- $T_{MG}=\{(a,2),(d,1),(-,0)\}$
- $T_{Space\text{-}Saving}=\{(a,2),(e,0),(f,0),(d,1)\}$

- $T_{MG}=\{(a,2),(d,1)\}$
- $T_{Space\text{-}Saving}=\{(a,2),(d,1)\}$

- MG算法$|T|=k$与Space-Saving算法$|T|=k+1$等价

MG算法与Space-Saving算法分析

- 内存参数为M,即|T|=M
  - 对每个元素频率的估计误差最大为1/M

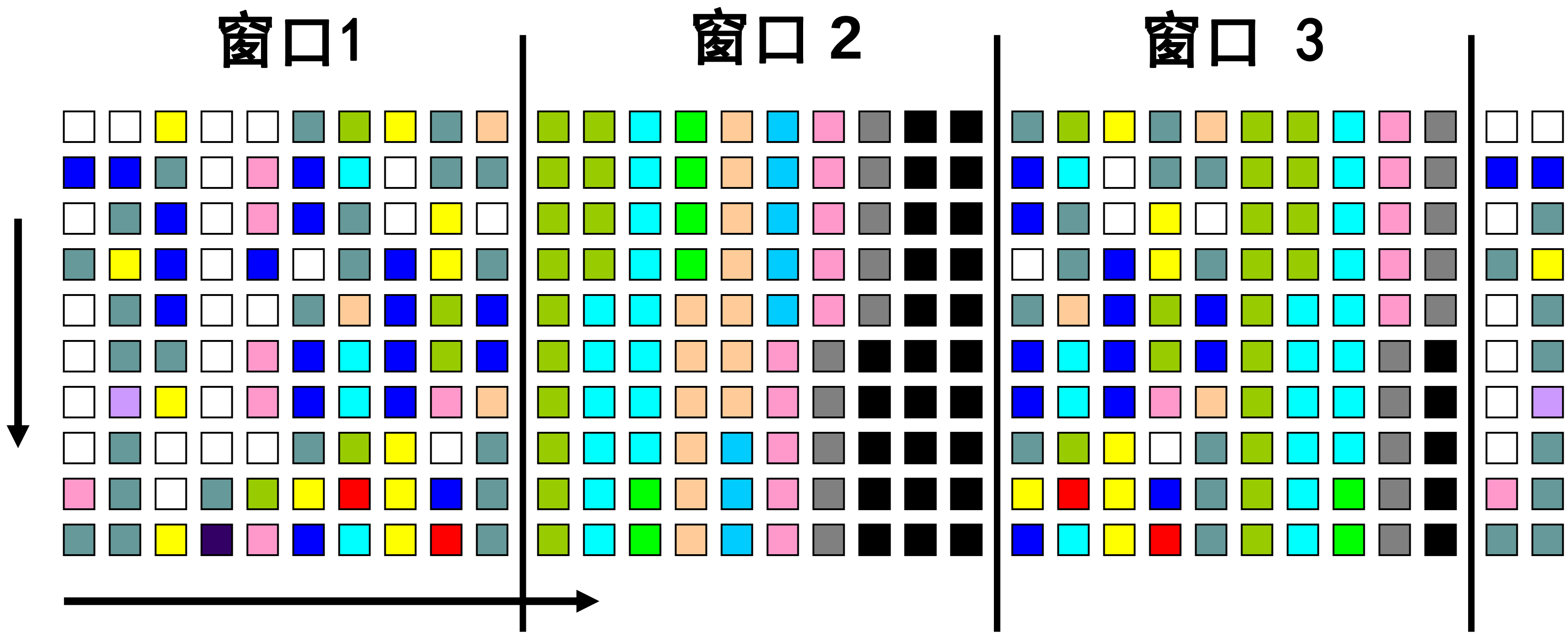
- 使用T寻找流中出现频率超过s的元素
  - 当满足E≤ 1/M ≤ ε 时
  - 流中所有出现频率超过s的元素都不会漏掉
    - 即不存在假阴性(No false negative)

课程提纲

- 数据流简介
- 基础性问题
- 确定性算法
  - Majority算法
  - Misra-Gries(MG)算法
  - Space-Saving算法
  - Lossy Counting算法
- 随机性算法
  - Sticky Sampling算法
  - Count-Min(CM)Sketch算法

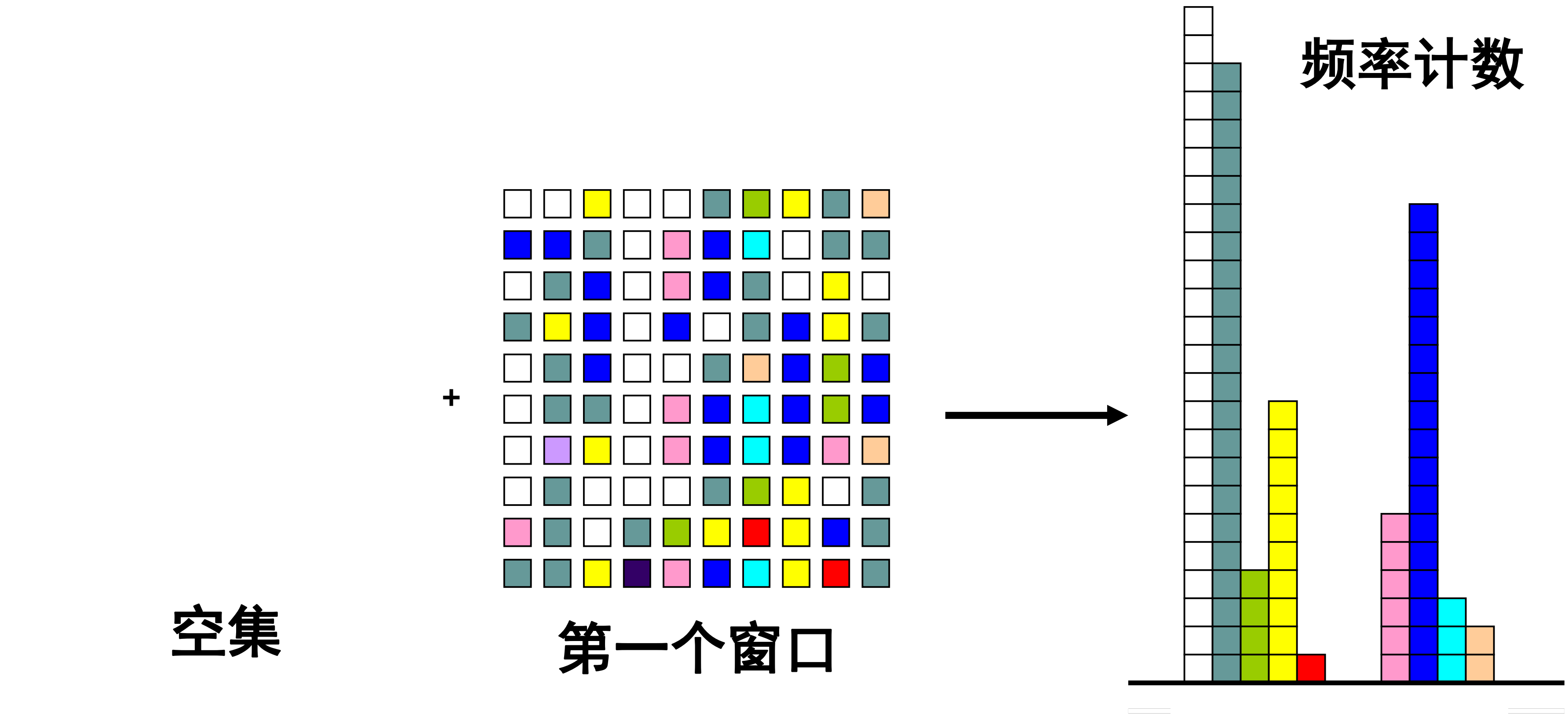
Lossy Counting算法核心思想

- 第一步：基于窗口的数据流切分



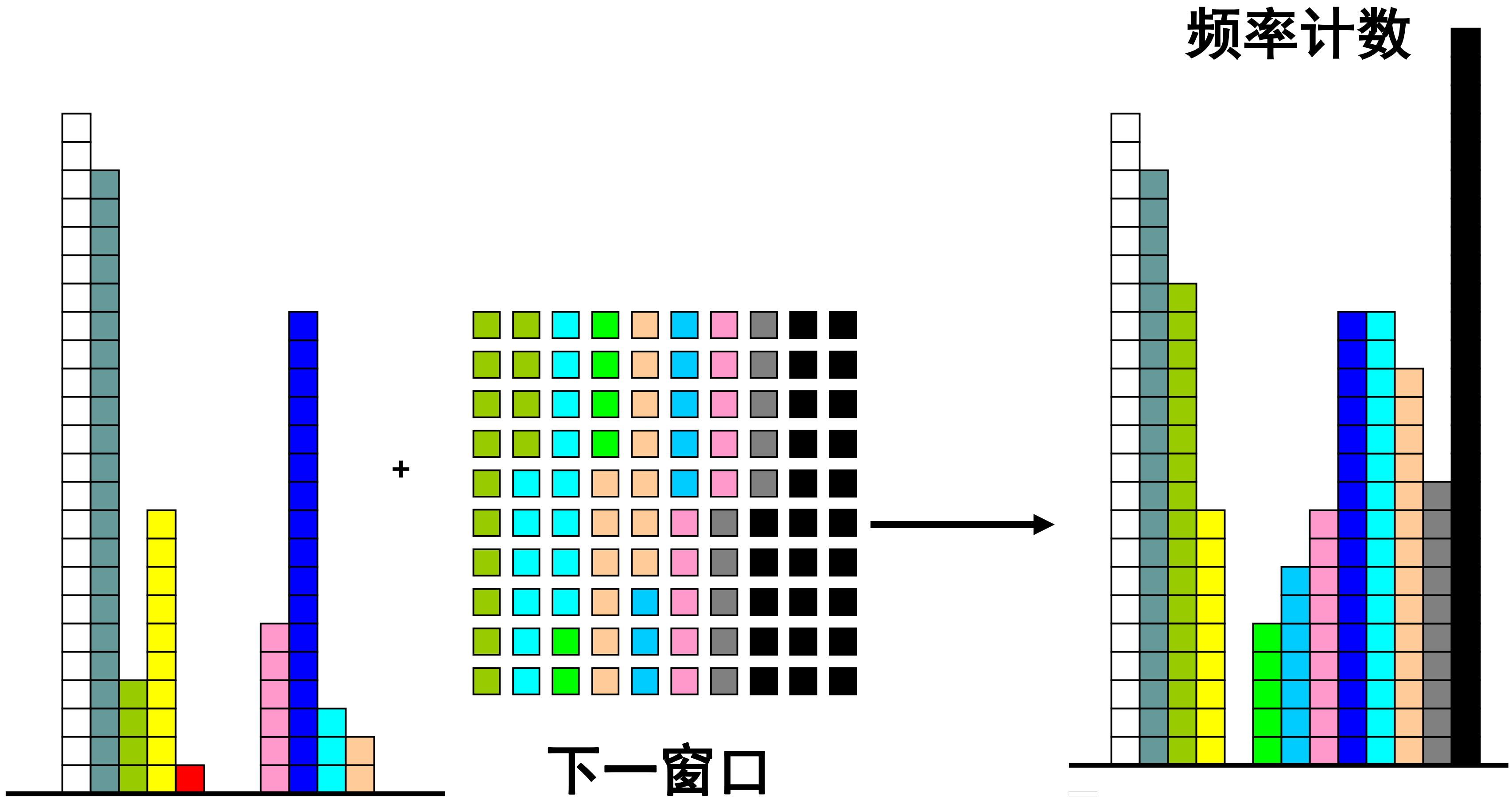
Lossy Counting算法核心思想

- 第二步：每到窗口边界，每项计数减一

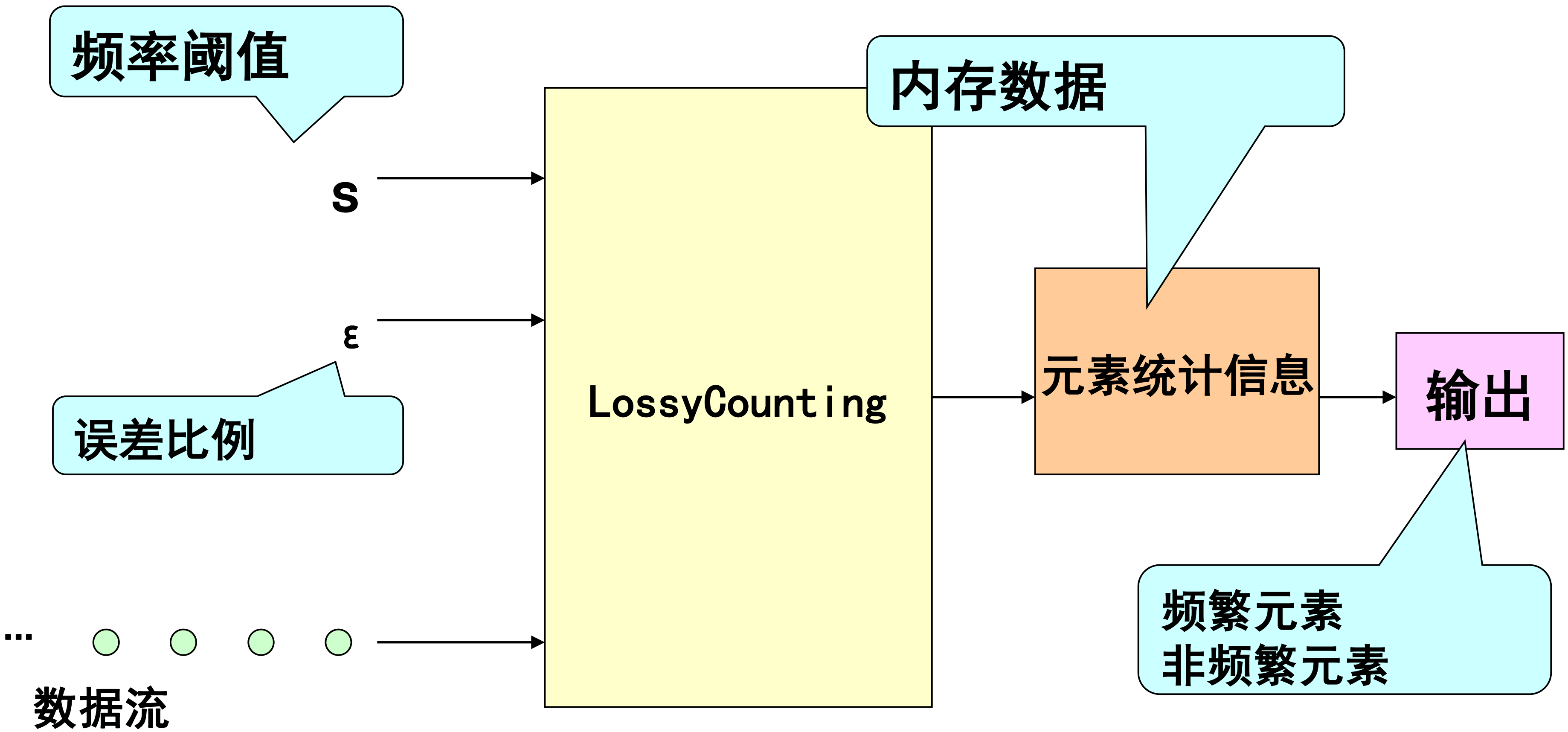


Lossy Counting算法核心思想

- 第一步：每到窗口边界，每项计数减一



Lossy Counting算法框架



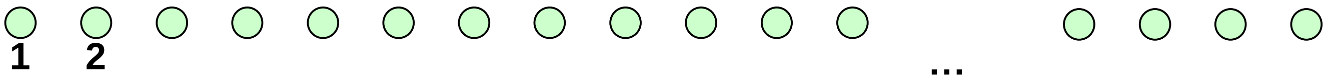
Lossy Counting算法

问题定义：统计流数据中出现频率大于给定值s的元素。

- **算法框架**
  - 根据参数设置流数据窗口大小。
  - 对于顺序到来的一个窗口内的数据更新内存中数据结构。
  - 每执行完一个窗口内的数据，更新内存中的数据结构（以减少空间），更新参数。

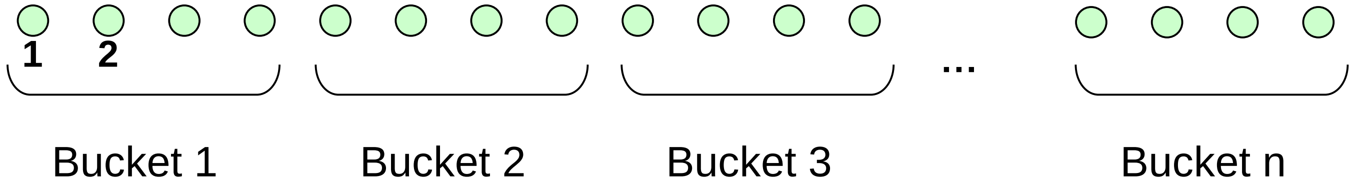
Lossy Counting算法

问题定义：统计流数据中出现频率大于给定值s的元素。



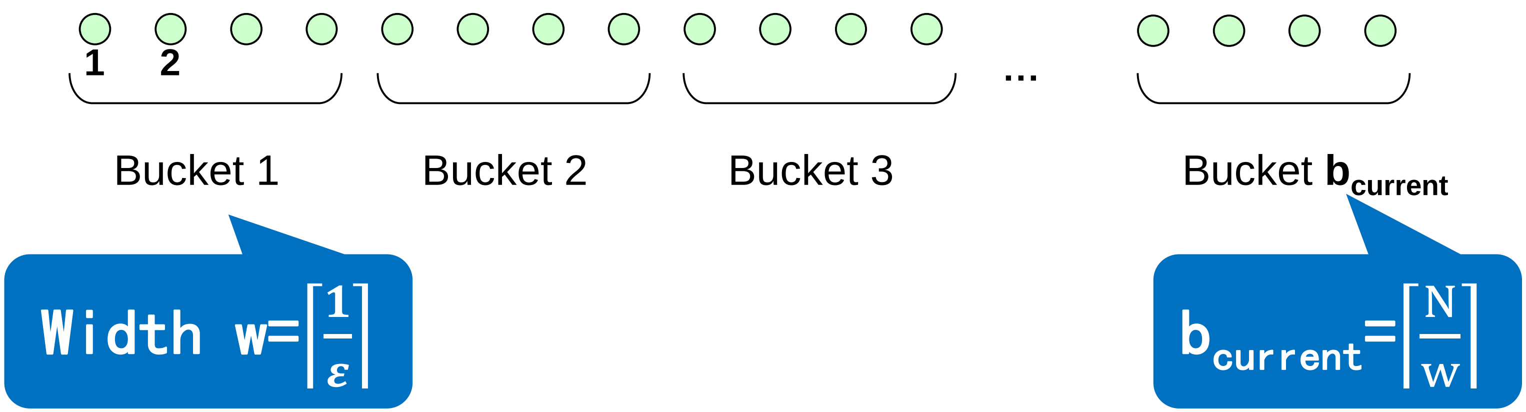
Lossy Counting算法

问题定义：统计流数据中出现频率大于给定值s的元素。



Lossy Counting算法

问题定义：统计流数据中出现频率大于给定值s的元素。



Lossy Counting算法

问题定义：统计流数据中出现频率大于给定值s的元素。

- **算法介绍**
  - 内存中保存部分元素、其出现次数以及该元素的最大误差(e, f, Δ)，对于数据流顺序到来的元素，若数据出现的元素已存在，将次数加一；若不存在，则将(e, 1, Δ)放入内存。
    - 窗口大小不变，动态更新内存存储信息。
    - 数据结构：字典
    - 返回数据结构中频度大于sN-εN的元素。
    - 确定性算法
    - 至多使用$\frac{1}{\varepsilon}\log(\varepsilon N)$个key-value对
- **相关参数**
    - 窗口大小参数$w = \lceil 1/\varepsilon \rceil$

Lossy Counting算法

问题定义：统计流数据中出现频率大于给定值s的元素。

元素

元素自被添加之后出现的次数

1. D: Empty set
   - Will contain (e, f, Δ)

f的最大可能误差

2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever
   $N \equiv 0 \bmod w$
   (i.e., whenever it reaches the bucket boundary)
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=1

a

s

元素	频度	误差

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever $N \equiv 0 \mod w$
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=1

a b

s

元素	频度	误差
a	1	0

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever
   N $\equiv$ 0 mod w
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=1

a b a

s

元素	频度	误差
a	2	0
b	1	0

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever $N \equiv 0 \bmod w$
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=1

a b a c

s

元素	频度	误差
a	2	0
b	1	0

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
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4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=1

a b a c d

s

元素	频度	误差
a	2	0
b	1	0
c	1	0

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
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   (e, f, Δ) is deleted if
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Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=1

a b a c d

s

元素	频度	误差
a	2	0
b	1	0
c	1	0
d	1	0

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever $N \equiv 0 \bmod w$
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=1

a b a c d

s

元素	频度	误差	
a	2	0	2+0>1
b	1	0	1+0<=1, drop
c	1	0	1+0<=1, drop
d	1	0	1+0<=1, drop

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever $N \equiv 0 \bmod w$
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=2

a b a c d | e

s

元素	频度	误差
a	2	0

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever $N \equiv 0 \bmod w$
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=2

a b a c d | e a

s

元素	频度	误差
a	2	0
e	1	1

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
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   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=2

a b a c d | e a d

s

元素	频度	误差
a	3	0
e	1	1

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever $N \equiv 0 \bmod w$
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=2

a b a c d | e a d f

s

元素	频度	误差
a	3	0
e	1	1
d	1	1

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
3. s, Remove some entries in D whenever $N \equiv 0 \bmod w$
   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=2

a b a c d | e a d f a

s

元素	频度	误差
a	3	0
e	1	1
d	1	1
f	1	1

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
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   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=2

a b a c d | e a d f a

s

元素	频度	误差
a	4	0
e	1	1
d	1	1
f	1	1

3+0>2

1+1<=2,drop

1+1>=2,drop

1+1>=2,drop

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
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   The rule of deletion is:
   (e, f, Δ) is deleted if
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4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=2

a b a c d | e a d f a | d

s

元素	频度	误差
a	4	0

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
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   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=3

a b a c d | e a d f a | d

s

元素	频度	误差
a	4	0
d	1	2

1. D: Empty set
   - Will contain (e, f, Δ)
2. When data e arrives,
   - If e exists in D,
     - Increment f in (e, f, Δ)
   - If e does not exist in D,
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   The rule of deletion is:
   (e, f, Δ) is deleted if
   $f + \Delta <= b_{current}$
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Lossy Counting算法

ε=0.2, 窗口大小w=[1/ε]=5,窗口个数b=3

a b a c d | e a d f a | d

s

元素	频度	误差
a	4	0
d	1	2

[Output] Get a list of items where
f + εN >= sN

s=0.3, 取使得f>=N(s- ε)的元素，为**a**

课程提纲

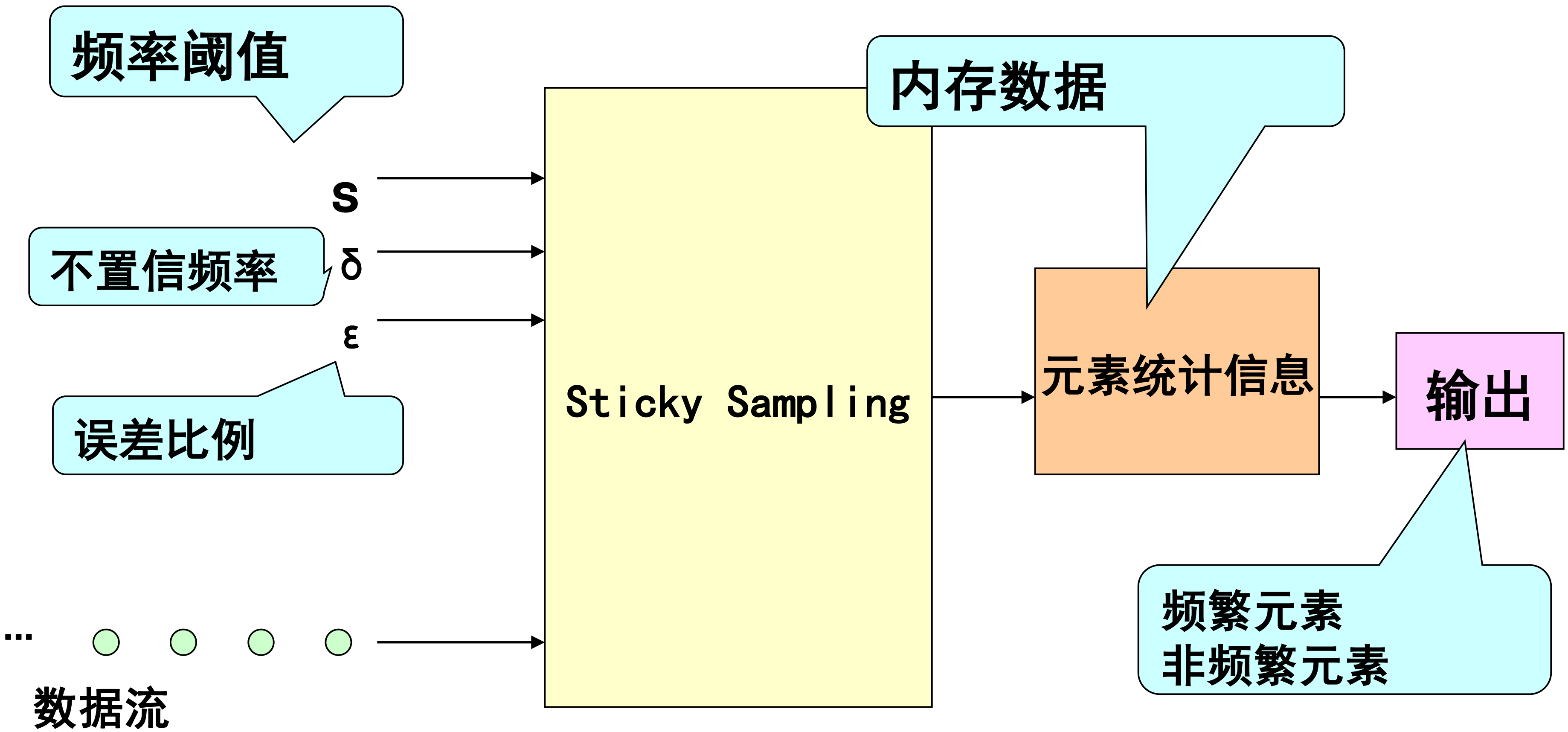
- 数据流简介
- 基础性问题
- 确定性算法
  - Majority算法
  - Misra-Gries(MG)算法
  - Space-Saving算法
  - Lossy Counting算法
- 随机性算法
  - Sticky Sampling算法
  - Count-Min(CM)Sketch算法

Sticky Sampling算法

问题定义：统计流数据中出现频率大于给定值s的元素。

- **算法框架**
  - 基本数据流模型
  - 根据参数设置流数据窗口大小。
  - 对于顺序到来的一个窗口内的数据更新内存中数据结构。
  - 每执行完一个窗口内的数据，更新内存中的数据结构（以减少空间），更新参数。

Sticky Sampling算法框架



Sticky Sampling算法

问题定义：统计流数据中出现频率大于给定值s的元素。

- **算法介绍**
  - 内存中保存部分元素及其出现次数(e, f)，对于数据流顺序到来的元素，数据出现的元素已存在，将次数加一；若不存在，做一次抽样决定是否将其放入内存。
    - 每处理完一个窗口的数据，抽样频率减半，窗口大小加倍，更新数据结构。
    - 返回数据结构中频度大于sN-εN的元素。
    - 至多使用$\frac{1}{\varepsilon}\log(s^{-1}\delta^{-1})$个key-value对

Sticky Sampling算法

问题定义：统计流数据中出现频率大于给定值s的元素。

- **相关参数**
  - 抽样率频率参数r，值随数据流的增加而变化。
  - 置信频率参数δ
  - 样窗口参数 $t = \frac{1}{\varepsilon}\log(s^{-1}\delta^{-1})$，窗口大小依次为2t, 2t, 4t, 8t, …

2t	**1**
2t	**2**
4t	**4**
…	**…**

Sticky Sampling算法

问题定义：统计流数据中出现频率大于给定值s的元素。

- **相关参数**
  - 抽样率频率参数r，值随数 … 而变化。
  - 置信频率参数δ
  - 样窗口参数 $t = \frac{1}{\varepsilon}\log(s^{-1}\delta^{-1})$，窗口大小依次为 2t, 2t, 4t, 8t, …

e.g.
s = 0.02
ε = 0.01
δ= 0.1

t = 622

1~1244	**2t**	**1**
1245~2488	**2t**	**2**
2489~4976	**4t**	**4**
	…	**…**

Sticky Sampling算法

问题定义：统计流数据中出现频率大于给定值s的元素。

元素 估计频度

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r (where r: sampling rate)
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)

4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第1个窗口r=1, 窗口大小2, 抽样频率1/r=1

a

0.3<1/r, insert

S

元素	频度

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)

4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第1个窗口r=1, 窗口大小2, 抽样频率1/r=1

a b

S

元素	频度
a	1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第1个窗口r=1, 窗口大小2, 抽样频率1/r=1

a b

s

元素	频度
a	1
b	1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第1个窗口r=1, 窗口大小2, 抽样频率1/r=1

a b

s

元素	频度
a	1
b	1

0<1

0<1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第2个窗口r=2, 窗口大小2, 抽样频率1/r=1/2

a b | a

不需要sample

s

元素	频度
a	1
b	1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第2个窗口r=2, 窗口大小2, 抽样频率1/r=1/2

a b | a c

0.4<1/r, insert

s

元素	频度
a	2
b	1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where
   f + εN >= sN

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第2个窗口r=2, 窗口大小2, 抽样频率1/r=1/2

a b | a c

s

元素	频度
a	2
b	1
c	1

1<2

1>=1,drop

0<1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第3个窗口r=4, 窗口大小4, 抽样频率1/r=1/4

a b | a c | d

0.1<1/r, insert

s

元素	频度
a	1
c	1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第3个窗口r=4, 窗口大小4, 抽样频率1/r=1/4

a b | a c | d e

0.4>1/r, drop

S

元素	频度
a	1
c	1
d	1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第3个窗口r=4, 窗口大小4, 抽样频率1/r=1/4

a b | a c | d e a

不需要sample

s

元素	频度
a	1
c	1
d	1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where
   f + εN >= sN

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第3个窗口r=4, 窗口大小4, 抽样频率1/r=1/4

a b | a c | d e a d

不需要sample

s

元素	频度
a	2
c	1
d	1

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第3个窗口r=4, 窗口大小4, 抽样频率1/r=1/4

a b | a c | d e a d

S

元素	频度	
a	2	**0**<2
c	1	**1>=**1, drop
d	2	**0**<2

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第4个窗口r=8, 窗口大小8, 抽样频率1/r=1/8

a b | a c | d e a d | f

0.2>1/r, drop

s

元素	频度
a	2
c	1
d	2

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第4个窗口r=8, 窗口大小8, 抽样频率1/r=1/8

a b | a c | d e a d | f a

不需要sample

s

元素	频度
a	2
c	1
d	2

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
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第4个窗口r=8, 窗口大小8, 抽样频率1/r=1/8

a b | a c | d e a d | f a d

不需要sample

S

元素	频度
a	3
c	1
d	2

1. S: empty list
   → will contain (e, f)
2. When data e arrives,
   - if e exists in S, increment f in (e, f)
   - if e does not exist in S, add entry (e, 1) with prob. 1/r
3. When r changes,
   - For each entry (e, f),
     - Repeatedly toss a coin with P(head) = 1/r until the outcome of the coin toss is head
     - If the outcome of the toss is tail,
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4. [Output] Get a list of items where
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抽样窗口参数t=1, ε=0.5
第4个窗口r=8, 窗口大小8, 抽样频率1/r=1/8

a b | a c | d e a d | f a d

1. S: empty list
   → will contain (e, f)
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     - If the outcome of the toss is tail,
       - Decrement f in (e, f)
4. [Output] Get a list of items where
   $f + \varepsilon N >= sN$

S

元素	频度
a	3
c	1
d	3

Sticky Sampling算法

抽样窗口参数t=1, ε=0.5
第4个窗口r=8, 窗口大小8, 抽样频率1/r=1/8

a b | a c | d e a d | f a d

[Output] Get a list of items where
f + εN >= sN

s=0.6, 取使得f>=(s- ε)N的元素，结果为a,d

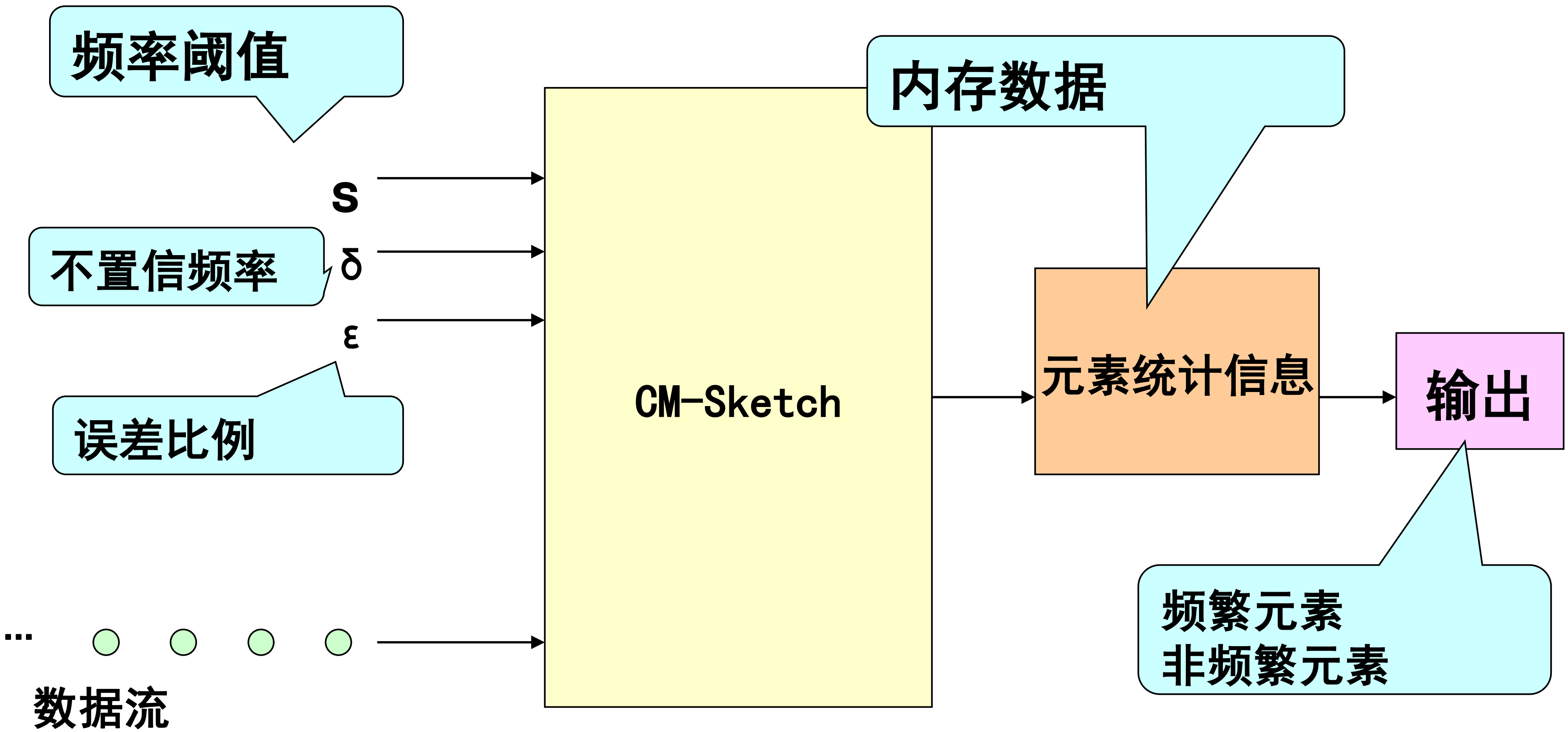
S

元素	频度
a	3
c	1
d	3

课程提纲

- 数据流简介
- 基础性问题
- 确定性算法
  - Majority算法
  - Misra-Gries(MG)算法
  - Space-Saving算法
  - Lossy Counting算法
- 随机性算法
  - Sticky Sampling算法
  - Count-Min(CM)Sketch算法

CM Sketch算法框架



CM Sketch

问题定义：估计流数据中元素出现的总次数。

- 算法设计
  - 使用Hash函数将元素映射到长度更小的数组中。
  - 使用多个独立的Hash函数减小Hash碰撞导致的误差。
  - 多个映射结果中最小的一个作为估计结果(Count Min).
  - 要求哈希函数h:{1,2,⋯,n}→{1,2,⋯,w}碰撞频率不超过1/m.

$$\mathrm{Pr}\{x_i, x_j \epsilon \{1,2,\ldots,n\} \wedge x_i \neq x_j \wedge P(x_i) = \mathrm{P}(x_j)\} < 1/\mathrm{m}$$

  - 数据结构：二维数组
  - 元素的估计值$\mathrm{E}st(e) = \min_{1 \le i \le d}\{CM[i, h_i(e)]\}$

m为元素种类

- 相关参数
  - 数组深度$w = \lceil e/\varepsilon \rceil$为数组列数
  - 哈希函数个数$d = \lceil ln(1/\delta) \rceil$为数组行数

CM Sketch

问题定义：统计流数据中出现频率大于给定值s的元素。

- CM伪代码

```
CM_Sketch():
h1,h2,…,hd //Hash Function
CM←array[d,w]
For every element (e,f):
    for i from 1 to d:
        CM[i,hi(e)]←CM[i,hi(e)]+f
```

CM Sketch

ε=1,δ=0.1

w=$\lceil e/\varepsilon \rceil$=4, d=$\lceil ln(1/\delta) \rceil$=3

CM

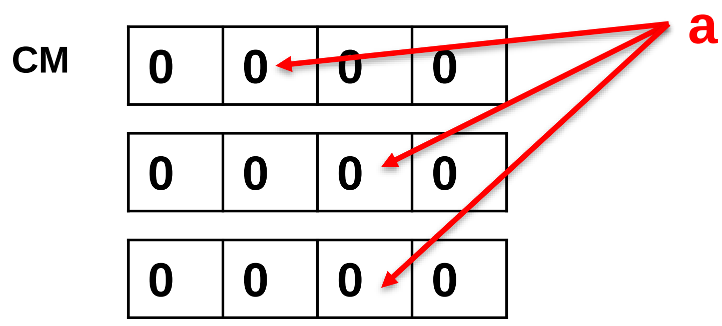
0	0	0	0
0	0	0	0
0	0	0	0

a

$$CM[i, h_i(e)] = CM[i, h_i(e)] + 1$$

CM Sketch

ε=1,δ=0.1
w=⌈e/ε⌉=4, d=⌈$ln(1/\delta)$⌉=3

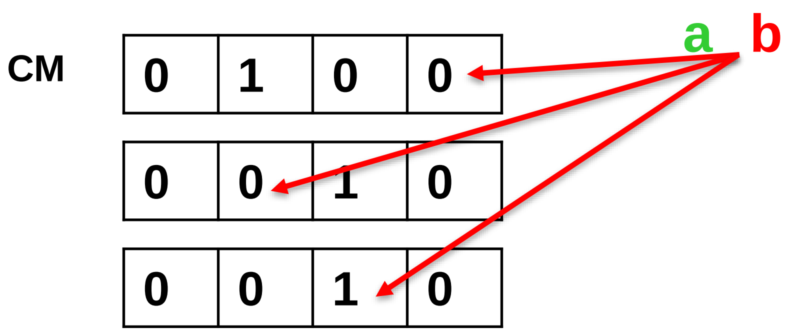


$$CM[i, h_i(e)] = CM[i, h_i(e)] + 1$$

CM Sketch

$\varepsilon=1, \delta=0.1$

$w=\lceil e/\varepsilon \rceil=4$, $d=\lceil ln(1/\delta) \rceil=3$

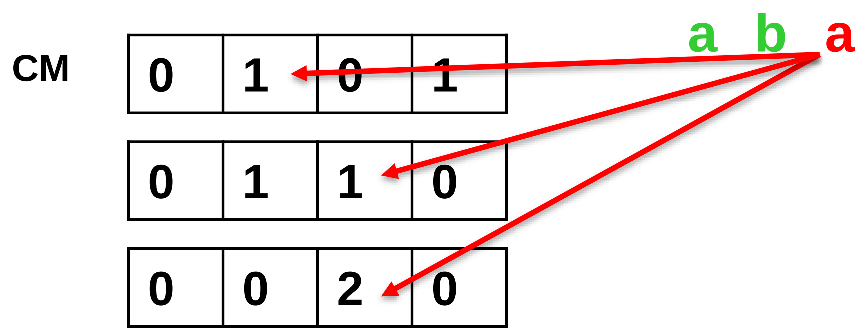


$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$

CM Sketch

ε=1,δ=0.1

w=$\lceil e/\varepsilon \rceil$=4, d=$\lceil ln(1/\delta) \rceil$=3



$CM[i, h_i(e)] = CM[i, h_i(e)]$+1

CM Sketch

ε=1,δ=0.1

$w = \lceil e/\varepsilon \rceil$=4, $d = \lceil \ln(1/\delta) \rceil$=3

a b a c

CM

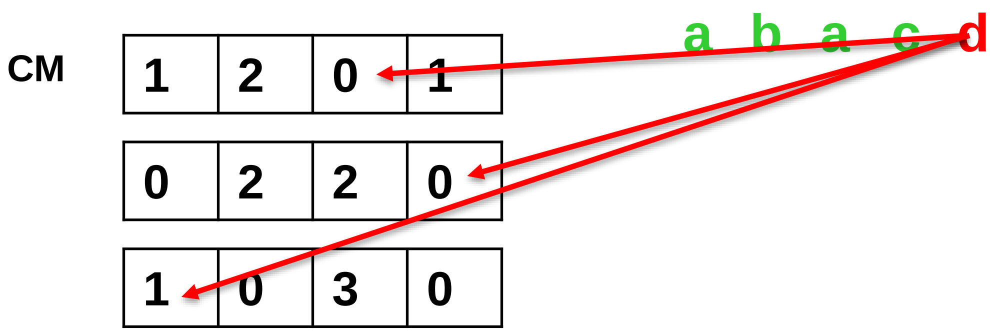
0	2	0	1
0	1	2	0
0	0	3	0

$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$

CM Sketch

ε=1,δ=0.1

$w=\lceil e/\varepsilon \rceil$=4, $d=\lceil ln(1/\delta) \rceil$=3

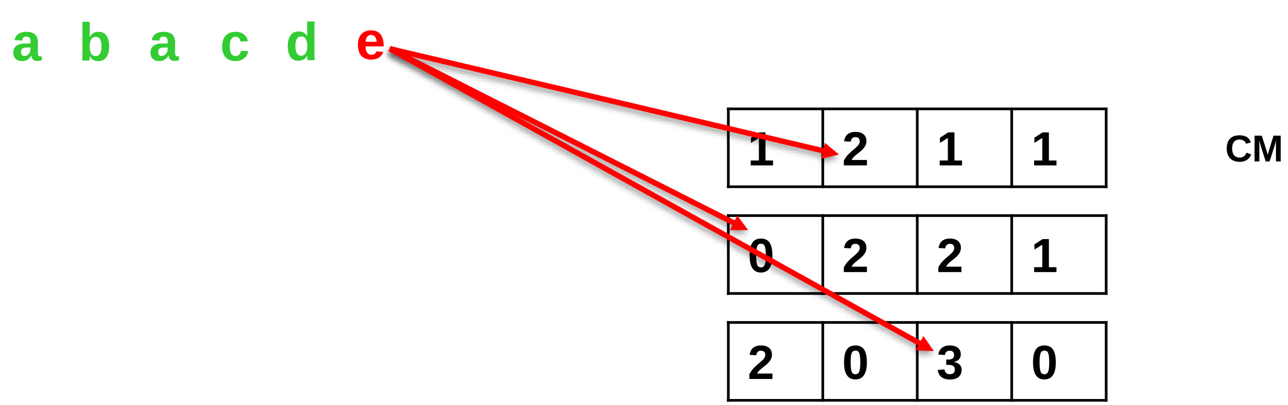


CM[i,hi(e)]←CM[i,hi(e)]+f

CM Sketch

$\varepsilon=1, \delta=0.1$

$w=\lceil e/\varepsilon \rceil=4,\ d=\lceil \ln(1/\delta) \rceil=3$

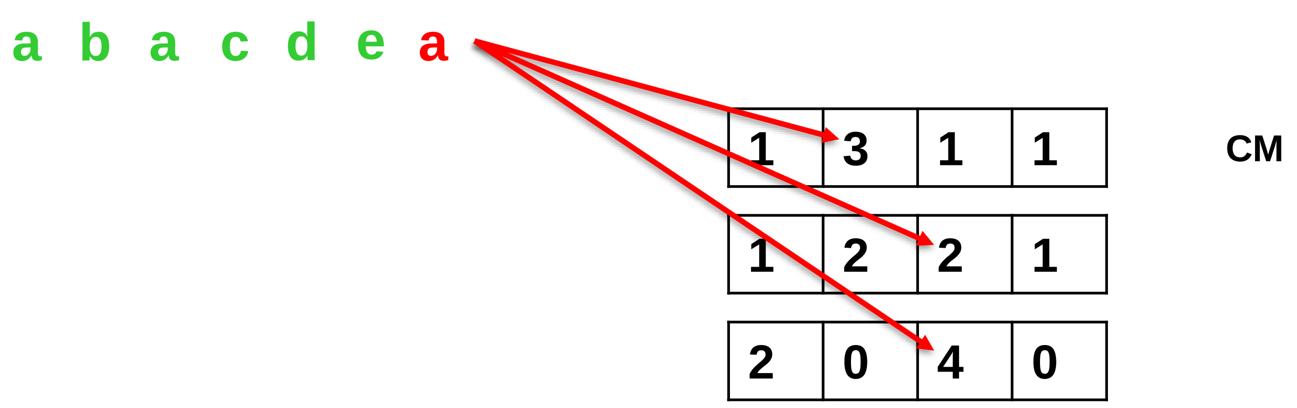


$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$

CM Sketch

$\varepsilon=1, \delta=0.1$

$w=\lceil e/\varepsilon \rceil=4$, $d=\lceil \ln(1/\delta) \rceil=3$



$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$

CM Sketch

ε=1,δ=0.1

$w=\lceil e/\varepsilon \rceil$=4, $d=\lceil ln(1/\delta) \rceil$=3

a b a c d e a d

1	4	1	1	CM
1	2	3	1	
2	0	5	0	

$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$

CM Sketch

ε=1,δ=0.1

$w=\lceil e/\varepsilon \rceil$=4, $d=\lceil ln(1/\delta) \rceil$=3

a b a c d e a d f

CM

1	4	2	1
1	2	3	4
1	0	5	0

$CM[i,h_i(e)] = CM[i,h_i(e)]$+1

CM Sketch

ε=1,δ=0.1
w=$\lceil e/\varepsilon \rceil$=4, d=$\lceil ln(1/\delta) \rceil$=3

a b a c d e a d f a

1	4	2	2
1	3	3	4
2	0	5	0

CM

$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$

CM Sketch

ε=1,δ=0.1

$w=\lceil e/\varepsilon \rceil$=4, $d=\lceil ln(1/\delta) \rceil$=3

a b a c d e a d f a d

1	5	2	2
1	3	4	4
2	0	6	0

CM

$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$

CM Sketch

ε=1,δ=0.1
w=$\lceil e/\varepsilon \rceil$=4, d=$\lceil ln(1/\delta) \rceil$=3

a b a c d e a d f a d

1	5	3	2	CM
1	3	4	5	
3	0	6	0	

$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$
$$Est(e) = \min_{1 \le i \le d}\{CM[i, h_i(e)]\}$$

CM Sketch

ε=1,δ=0.1

w=$\lceil e/\varepsilon \rceil$=4, d=$\lceil ln(1/\delta) \rceil$=3

a b a c d e a d f a d

1	5	3	2
1	3	4	5
3	0	6	0

CM

Est(a) = 4

$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$

$$Est(e) = \min_{1 \le i \le d}\{CM[i, h_i(e)]\}$$

CM Sketch

ε=1,δ=0.1
$w=\lceil e/\varepsilon \rceil$=4, $d=\lceil ln(1/\delta) \rceil$=3

a b a c d e a d f a d

1	5	3	2	CM
1	3	4	5	
3	0	6	0	

Est(d) = 3

$$CM[i, h_i(e)] = CM[i, h_i(e)]+1$$
$$Est(e) = \min_{1 \le i \le d} \{CM[i, h_i(e)]\}$$

数据流算法算法比较

	算法	ε-Deficient Synopsis	内存消耗
确定性算法	Majority [1]	100%	1
	Misra-Gries [5]	100%(E≤ε)	M
	Space-Saving [4]	100%(E≤ε)	M
	Lossy Counting [3]	100%	$\lceil 1/\varepsilon \log(\varepsilon N) \rceil$
随机性算法	Sticky Sampling [3]	1-δ	$\lceil 2/\varepsilon \ln(s^{-1}\delta^{-1}) \rceil$
	CM-Sketch [2]	1-δ	$\lceil e/\varepsilon \rceil \times \lceil \ln(1/\delta) \rceil$

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谢谢