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Launch Systems

06 – Part 2
Missile Aerodynamics
Static Stability

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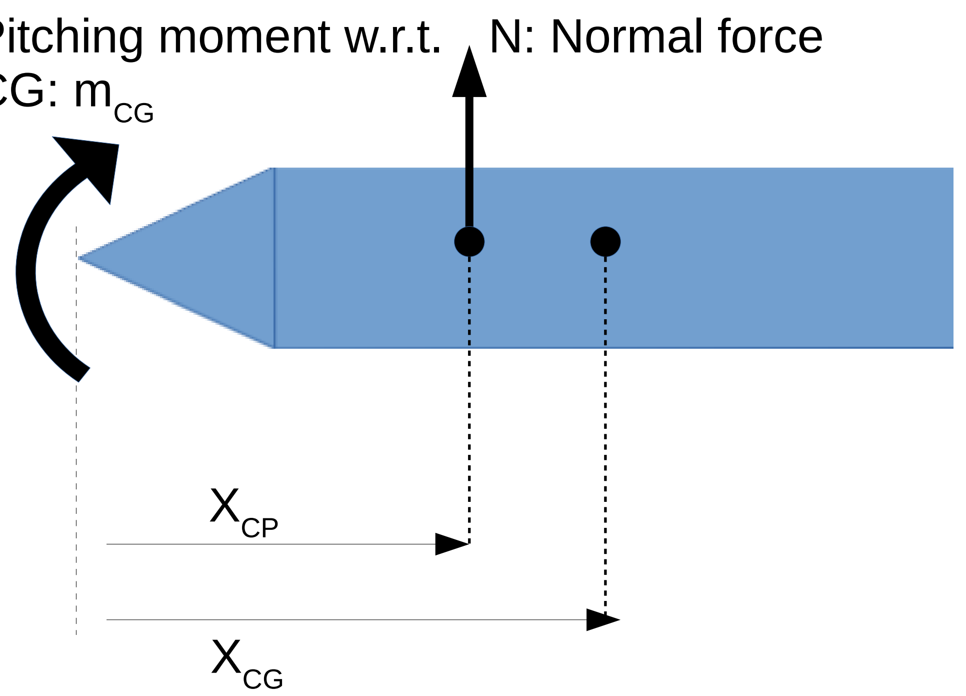
Plan and Objectives

- Properties connected to static stability (rigid body)
- Design solutions interacting with stability
- Flight control and maneuvers

Reference frame in missile aerodynamics

Reference frame



X_{CG}: position of center of gravity
X_{CP}: position of center of pressure
C_{mCG}: moment coefficient w.r.t. CG
C_N: normal force coefficient
L_B: body length
L_N: nose length
d : body diameter
S: reference cross section
S_b: base section
q: dynamic pressure
W: weight of the object
V: volume of the object

$$C_N = \frac{N}{q\,S} \qquad C_{mCG} = \frac{m_{CG}}{q\,S\,d}$$

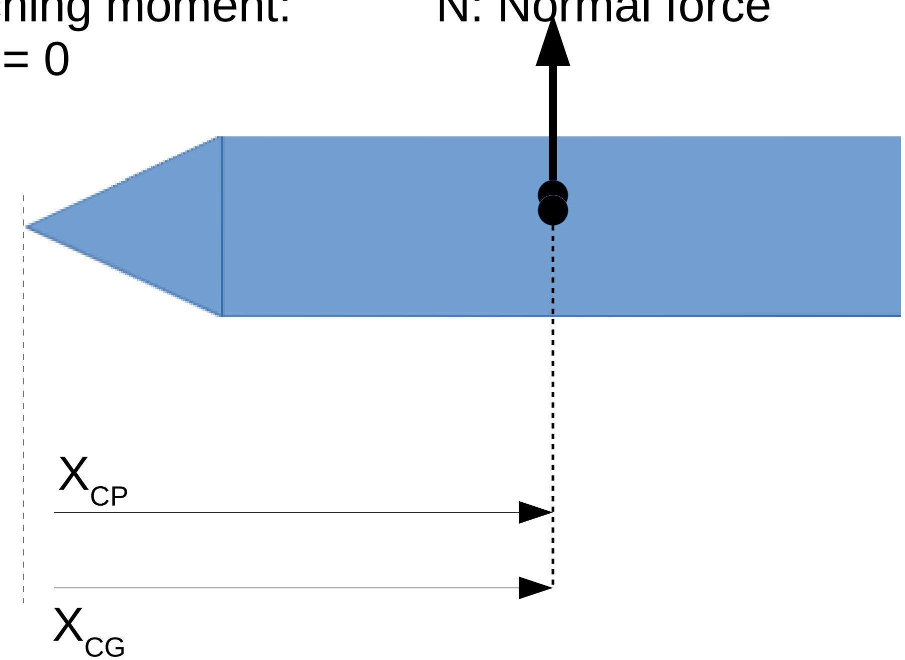
$$m_{CG} = N\left(X_{CG} - X_{CP}\right)$$

Position of CP

$$\frac{X_{CP}}{d} = \frac{X_{CG}}{d} - \frac{C_{mCG}}{C_N}$$

Reference frame

Pitching moment:
m = 0



Center of pressure is the resultant of all aerodynamic forces

The CP position changes during flight depending on the angle of attack.

The moment changes depending on the angle of attack.

$$C_N = \frac{N}{q\,S} \qquad C_{mCG} = \frac{m_{CG}}{q\,S\,d}$$

$$m_{CG} = N\left(X_{CG} - X_{CP}\right) = 0$$

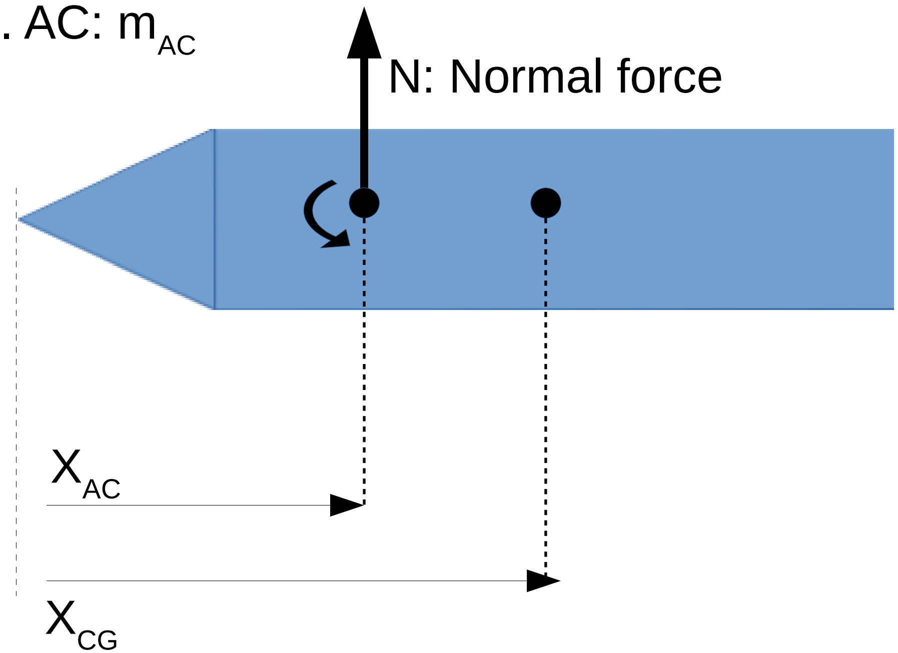
In equilibrium

Position of CP

$$X_{CP} = X_{CG}$$

Reference frame

Pitching moment
w.r.t. AC: m_{AC}



$$C_N = \frac{N}{q\,S} \qquad C_m = \frac{m}{q\,S\,d}$$

Under **equilibrium** conditions

$$0 = m_{CG} = m_{AC} - N\left(X_{CG} - X_{AC}\right)$$

X_{CG}: position of center of gravity
X_{AC}: position of aerodynamic center
C_{mAC}: moment coefficient w.r.t. AC
C_N: normal force coefficient
L_B: body length
L_N: nose length
d : body diameter
S: reference cross section
S_b: base section
q: dynamic pressure
W: weight of the object
V: volume of the object

Position of AC

$$\frac{X_{AC}}{d} = \frac{X_{CG}}{d} - \frac{C_{mAC}}{C_N}$$

Static stability

Static stability criterion

Stable condition:

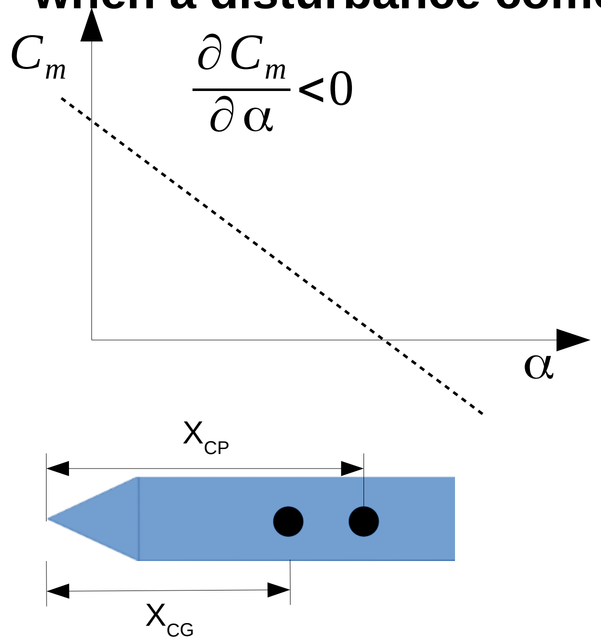
- Increment of incidence
- negative moment
- the missile backs up to the previous state

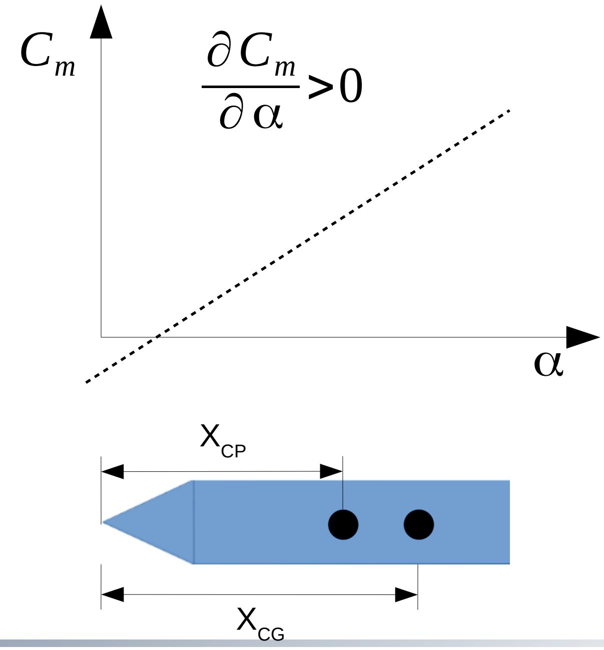
$$C_{N\alpha} = \frac{\partial C_N}{\partial \alpha}$$

Unstable condition:

- Increment of incidence
- positive moment
- the missile further increments the incidence

Driver: reciprocal position of the center of gravity and center of pressure when a disturbance comes





Considerations on stability

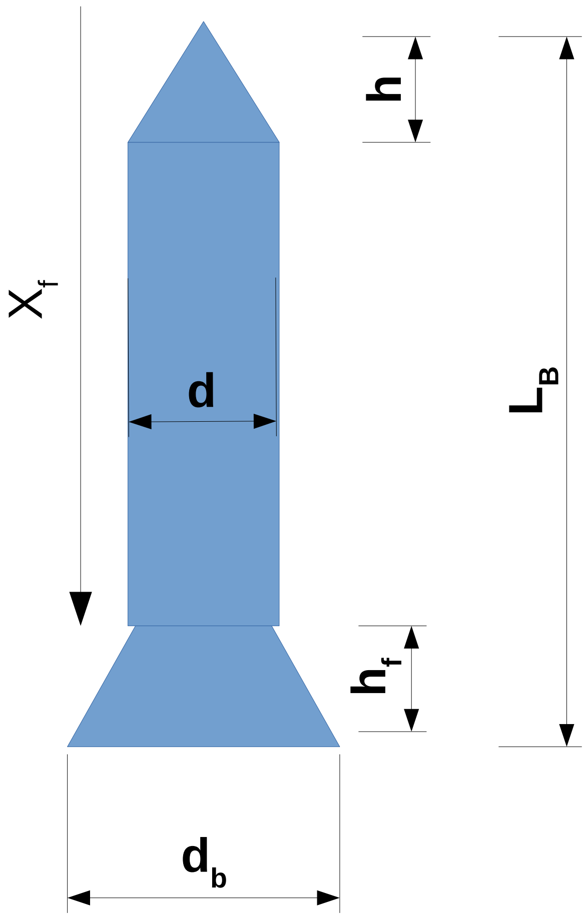
- Stable missiles
  - do not require active control for stability
  - may have oscillatory or damped behavior, after disturbance arises
- Unstable missiles
  - active control required to guide them with TVC
  - TVC response must be adequate to recover a stable operation
  - maneuverability and response capability is superior

Results from the slender body theory/1

For cases **when sin α ≈ α and cos α ≈ 1**

The theory finds that $$\frac{C_m}{C_N}=\left(\frac{V}{S_b d}-\frac{L_b-X_{CG}}{d}\right)$$

Where S_b is the vehicle base area. Check Sforza Chp. 7.6.2 for complete formulation



The center of pressure is: $$\frac{X_{CP}}{d}=\frac{L_B}{d}-\frac{V}{S_b d}$$

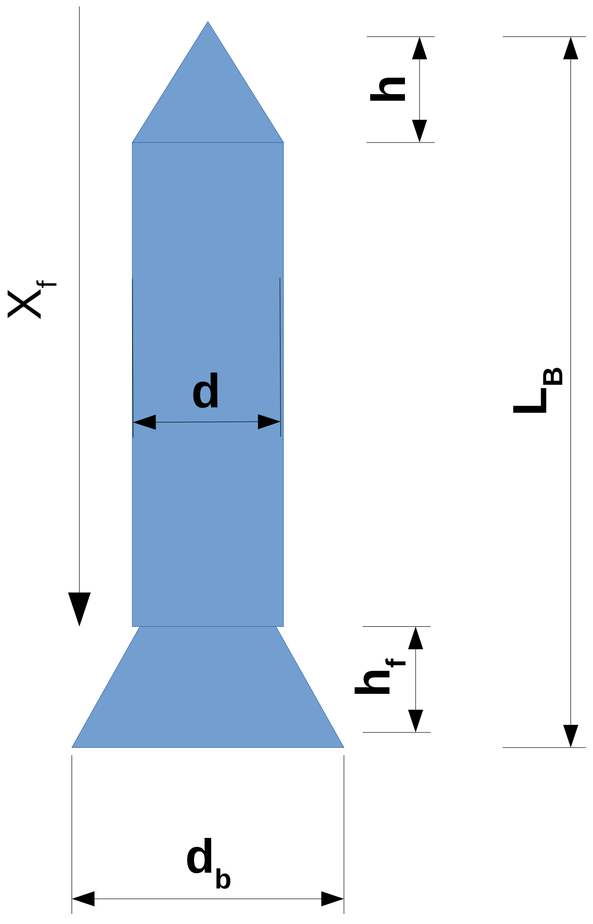
Nondimensional volume

The non-dimensional volume of this simple missile is:

$$\frac{V}{S_b d}=\left[\frac{L_B}{d}-\frac{2}{3}\frac{h}{d}-\underbrace{\frac{1}{3}\frac{h_f}{d}\left(2-\frac{d_b^2}{d^2}-\frac{d_b}{d}\right)}_{}\right]\frac{S}{S_b}$$

A change of the flare shape can control CP

Results from the slender body theory/2



Afterbody (flare) slope should be small

$$\frac{d_b - d}{2 h_f} \ll 1$$

The center of pressure is:

$$\frac{X_{CP}}{d} \approx \frac{2}{3}\frac{h}{d}\frac{S}{S_b} + \left(1 - \frac{S}{S_b}\right)\frac{L_B}{d} - \frac{h_f}{d}\left(\frac{d_m^2}{d^2} - 1\right)\frac{S}{S_b}$$

where d_m is the mean radius of the flare cone

If a flare does not exist, $S_b = S$ and the equation is:

$$X_{CP} = \frac{2}{3} h$$

The aerodynamic center is a **property of the nose only**

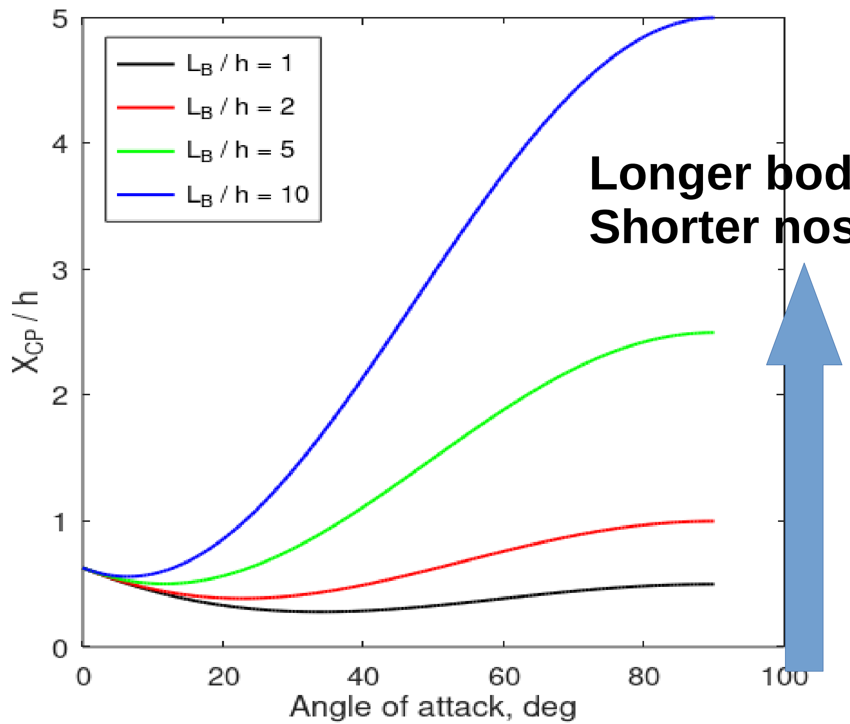
Semi-empirical relations

For generic cases **when sin α ≠ α and cos α < 1**

- Mix of slender body theory and cross-flow theory
- Also in this case the CP prediction is a property of the nose only
- Simple body: nose + cylinder

$$\frac{X_{CP}}{h} \approx 0{,}63\left(1-\sin^2\alpha\right)+0{,}5\,\frac{L_B}{h}\sin^2\alpha$$

- The length of the body does not influence the small angle of attack but has a strong influence when the angle grows



Longer body
Shorter nose

Flare stabilization

- The center of pressure of the afterbody, for small angles of attack (slender body theory), is given by:

$$(X_{CP})_f = X_f + 0{,}33\, h_f \frac{2\frac{d_b}{d}+1}{\frac{d_b}{d}+1} \qquad (C_{N\alpha})_f = 2\left[\left(\frac{d_b}{d}\right)^2 - 1\right] = 2\left[\left(\frac{S_b}{S}\right) - 1\right]$$

- The static margin of an object with a flare is obtained by averaging the contributions of the body and the afterbody:

$$\frac{X_{CP}-X_{CG}}{d} = -\frac{(C_{N\alpha})_B \frac{X_{CG}-(X_{CP})_B}{d} + (C_{N\alpha})_F \frac{X_{CG}-(X_{CP})_F}{d}}{(C_{N\alpha})_B + (C_{N\alpha})_F}$$

For symmetric bodies CP = AC

Flare Trade-off

- Flare advantages
  - lower span to obtain stabilization
  - lower aerodynamic heating (best for hypersonic regime)
  - minor variation in global aerodynamics
  - good compatibility with launch platform
  - w.r.t. TVC, reduced weight due to the absence of moving parts and reduced propellant required
- Disvantages
  - higher drag due to incremented front section w.r.t. wings
  - tail wings supply also control capability

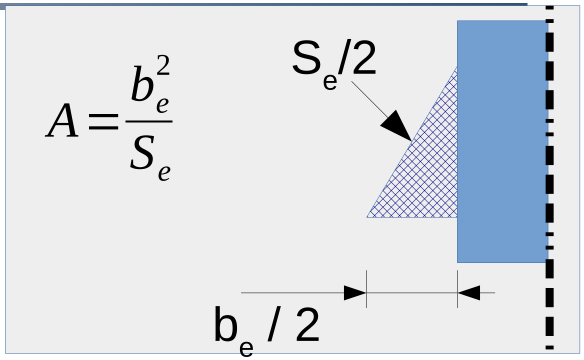
Wings

Wings trade-off

Measure of merit	No wing	Strake small wings	Large wings
Range in high Ma, high q	↑↑	↔	↓
Range in Moderate Ma, and q	↔	↑↑	↓
Range in subsonic Ma, low q	↓	↔	↑↑
Max α	↑↑	↑	↓
Compatibility launch platform	↑↑	↑	↓
Radar cross section	↑↑	↑	↓
Volume/weight for propellant	↑↑	↑	↔
Lower guidance time constant	↓	↔	↑↑
Normal acceleration	↓	↑	↑↑
High altitude capability	↓	↔	↑
Less body bending aeroelasticity (wing stiffens body)	↓	↑	↑↑

Normal force on surfaces

- Wings: close to the CG (fixed or movable)
- Tail/canard: aft/forward the CG (fixed or movable)
- Strakes: small aspect ratio A; they can be anywhere



$$\text{for } M > \left(1 + \left(\frac{8}{(\pi A)}\right)^2\right)^{1/2} \qquad \text{for } M < \left(1 + \left(\frac{8}{(\pi A)}\right)^2\right)^{1/2}$$

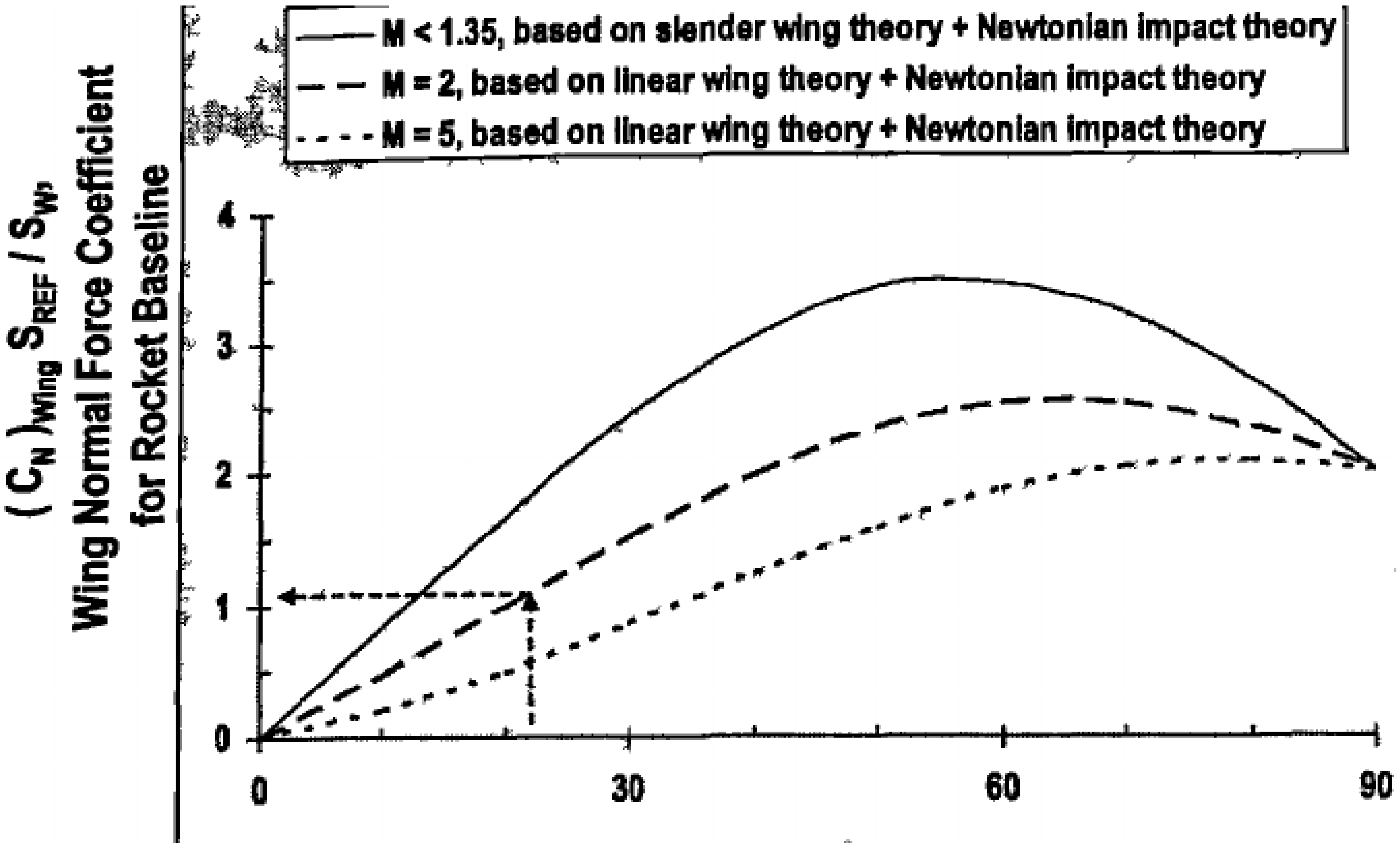
$$\frac{\partial C_N}{\partial \alpha} = \frac{4}{(M^2 - 1)^{1/2}} \frac{S_{surf}}{S_{ref}} \qquad \frac{\partial C_N}{\partial \alpha} = \frac{\pi A}{2} \frac{S_{surf}}{S_{ref}}$$

For α < 10°

Extension of linear wing theory with Newtonian impact theory

$$(C_N)_{surf} = \left(\frac{4|\sin\alpha' \cos\alpha'|}{(M^2 - 1)^{1/2}} + 2\sin^2\alpha'\right) \frac{S_{surf}}{S_{ref}} \qquad (C_N)_{surf} = \left(\frac{\pi A}{2} |\sin\alpha' \cos\alpha'| + 2\sin^2\alpha'\right) \frac{S_{surf}}{S_{ref}}$$

Normal force



Lower Mach number: wings are more effective

Source: Fleeman

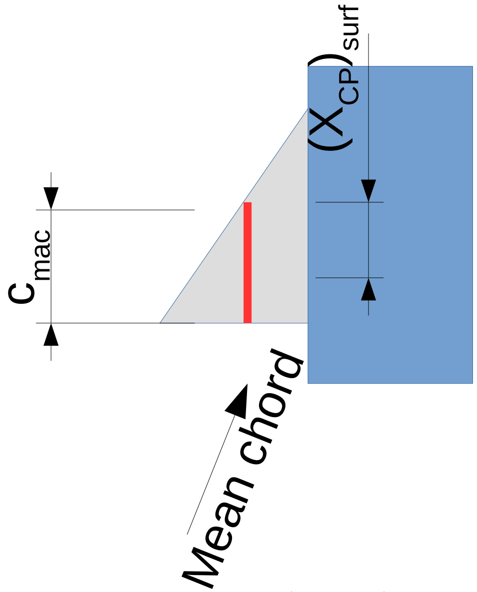
Global coefficients

- The coefficients can be summed up only when they refer to the same reference surface
- The reference surface is the missile cross section
- Flare formula is already corrected with the proper reference surface
- Wing formula changes.

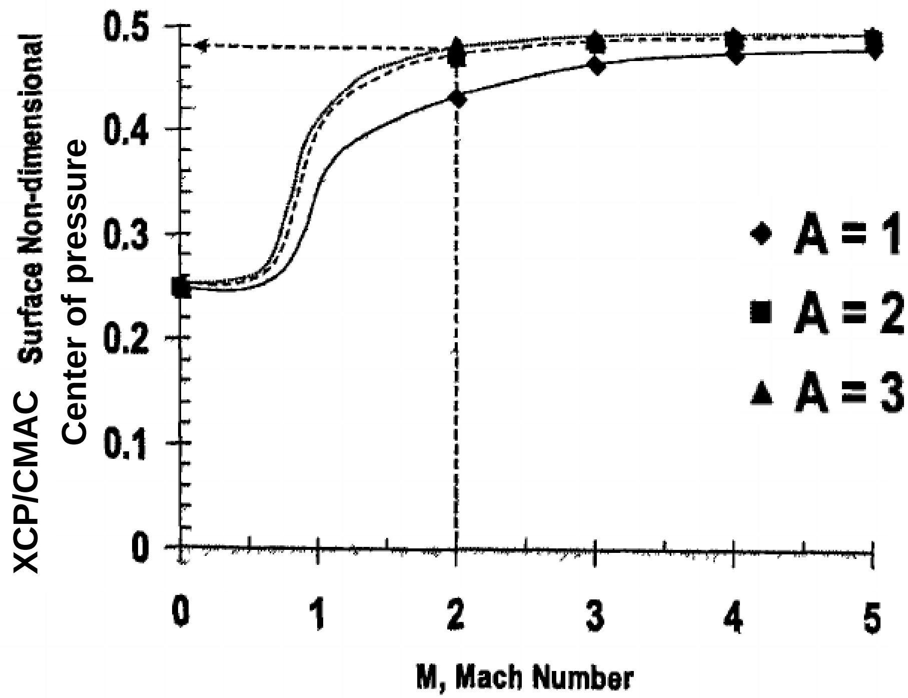
$$N = (C_{N\alpha})_{\text{wing}} \alpha q S_{\text{wing}} = (C_{N\alpha})_{\text{wing}} \frac{S_{\text{wing}}}{S_{\text{ref}}} q \alpha S_{\text{ref}}$$

$$(C_{N\alpha})_{\text{total}} = (C_{N\alpha})_{\text{body}} + (C_{N\alpha})_{\text{wing}} \frac{S_{\text{wing}}}{S_{\text{ref}}}$$

Position of the CP in a wing



Measured from the leading edge of the mean chord



Subsonic
Ma < 0.7

$$\left(\frac{X_{CP}}{c_{mac}}\right)_{surf} = 0{,}25$$

Supersonic
Ma > 2

$$\left(\frac{X_{CP}}{c_{mac}}\right)_{surf} = \frac{A\left(M^2-1\right)^{1/2}-0{,}67}{2A\left(M^2-1\right)^{1/2}-1}$$

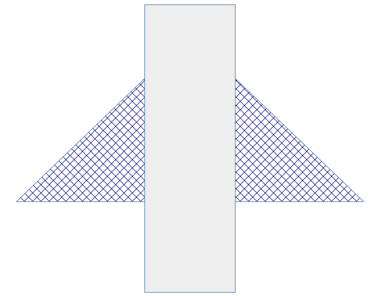
Source: Fleeman

Drag on planar surfaces

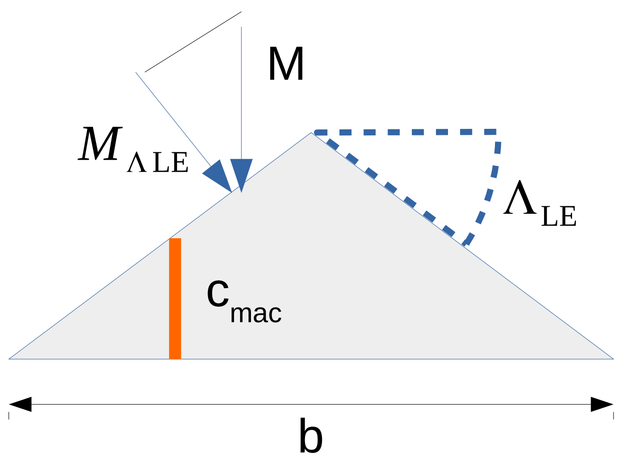
- Contributions: friction + wave

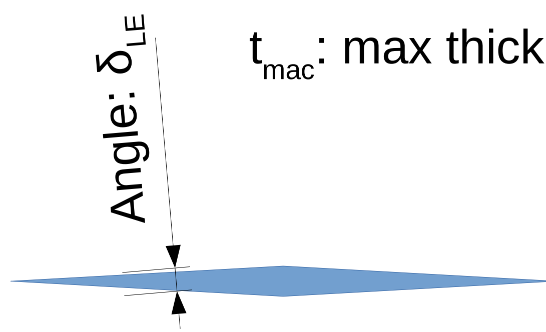
$$\left(C_{D0}\right)_{\text{Surf-friction}} = 0{,}0133\left(\frac{M}{q\,c_{mac}}\right)^{0{,}2} 2\,\frac{S_{\text{surf}}}{S_{\text{ref}}}$$

For two half-wings



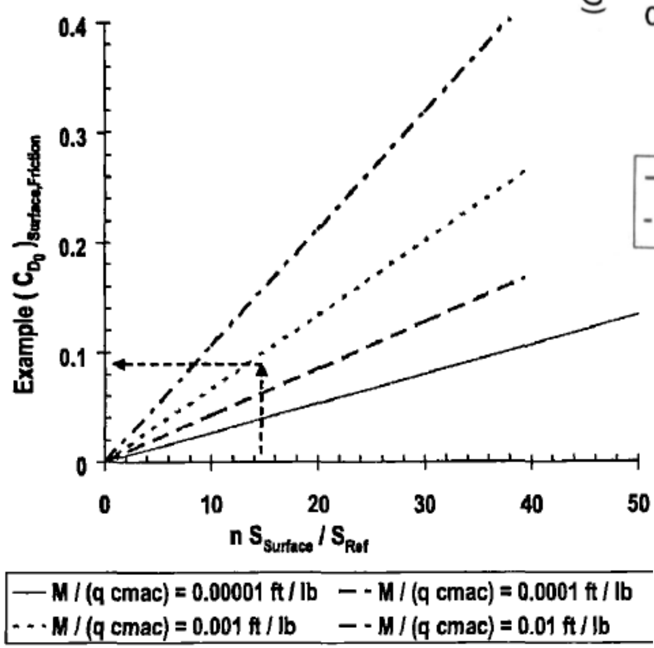
$$\left(C_{D0}\right)_{\text{Surf-wave}} = \frac{1{,}429}{M_{\Lambda\,\text{LE}}^{2}}\left(\left(1{,}2\,M_{\Lambda\,\text{LE}}^{2}\right)^{3{,}5}\left(\frac{2{,}4}{2{,}8\,M_{\Lambda\,\text{LE}}^{2}-0{,}4}\right)^{2{,}5}-1\right)\frac{\left(\sin^{2}\delta_{\text{LE}}\cos\Lambda_{\text{LE}}\,t_{mac}\,b\right)}{S_{ref}}$$

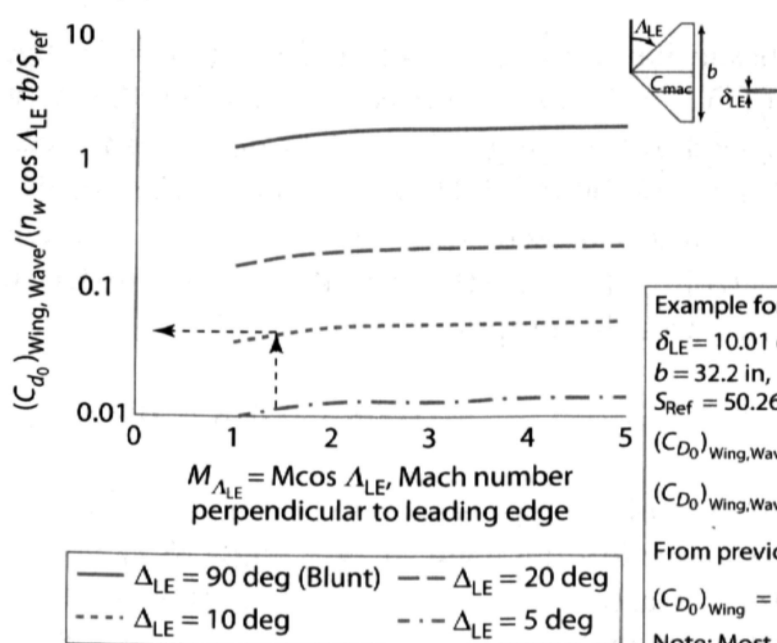




t_{mac}: max thickness on mac

Some examples





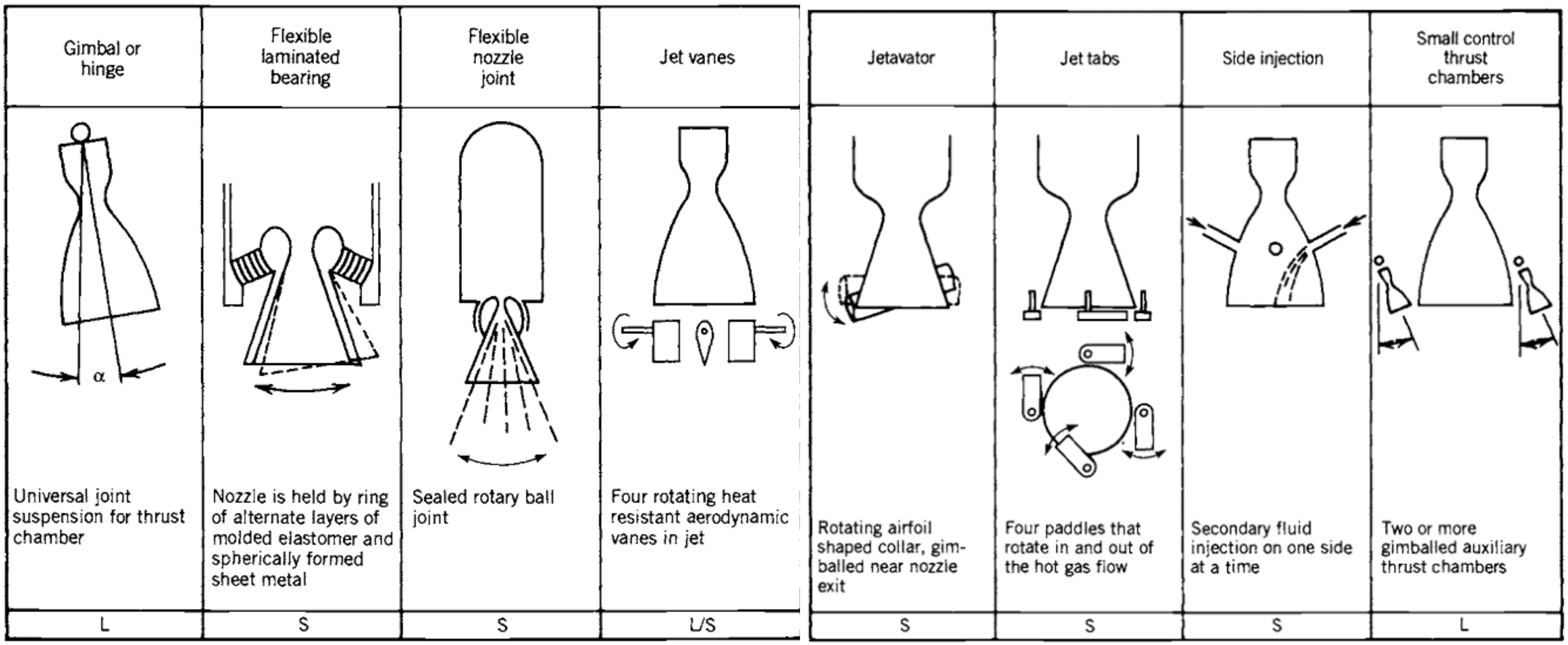
Note:
n_W = number of wings (cruciform = 2)
δ_{LE} = leading edge section angle
Λ_{LE} = leading edge sweep angle
t_{mac} = max thickness of mac
b = span
S_{Ref} = reference area

Example for Rocket Baseline Missile Wing:
$\delta_{LE} = 10.01$ deg, $\Lambda_{LE} = 45$ deg, $t_{mac} = 0.585$ in,
$b = 32.2$ in,
$S_{Ref} = 50.26$ in^2, $M = 2$ ($M_{\Lambda_{LE}} = 2\cos 45° = 1.41$)
$(C_{D_0})_{\text{Wing,Wave}} / (n_W \cos \Lambda_{LE}\, t_{mac} b / S_{Ref}) = 0.0457$
$(C_{D_0})_{\text{Wing,Wave}} = 0.0457(2)(0.707)(0.585)(32.2)/50.26 = 0.024$
From previous figure, $(C_{D_0})_{\text{Wing,Friction}} = 0.090$
$(C_{D_0})_{\text{Wing}} = 0.024 + 0.090 = 0.11$
Note: Most of baseline wing drag is skin friction drag

Thrust Vector Control

Thrust Vector Control options

- Nozzle management enables the variation of the thrust vector, useful for maneuvers and stability

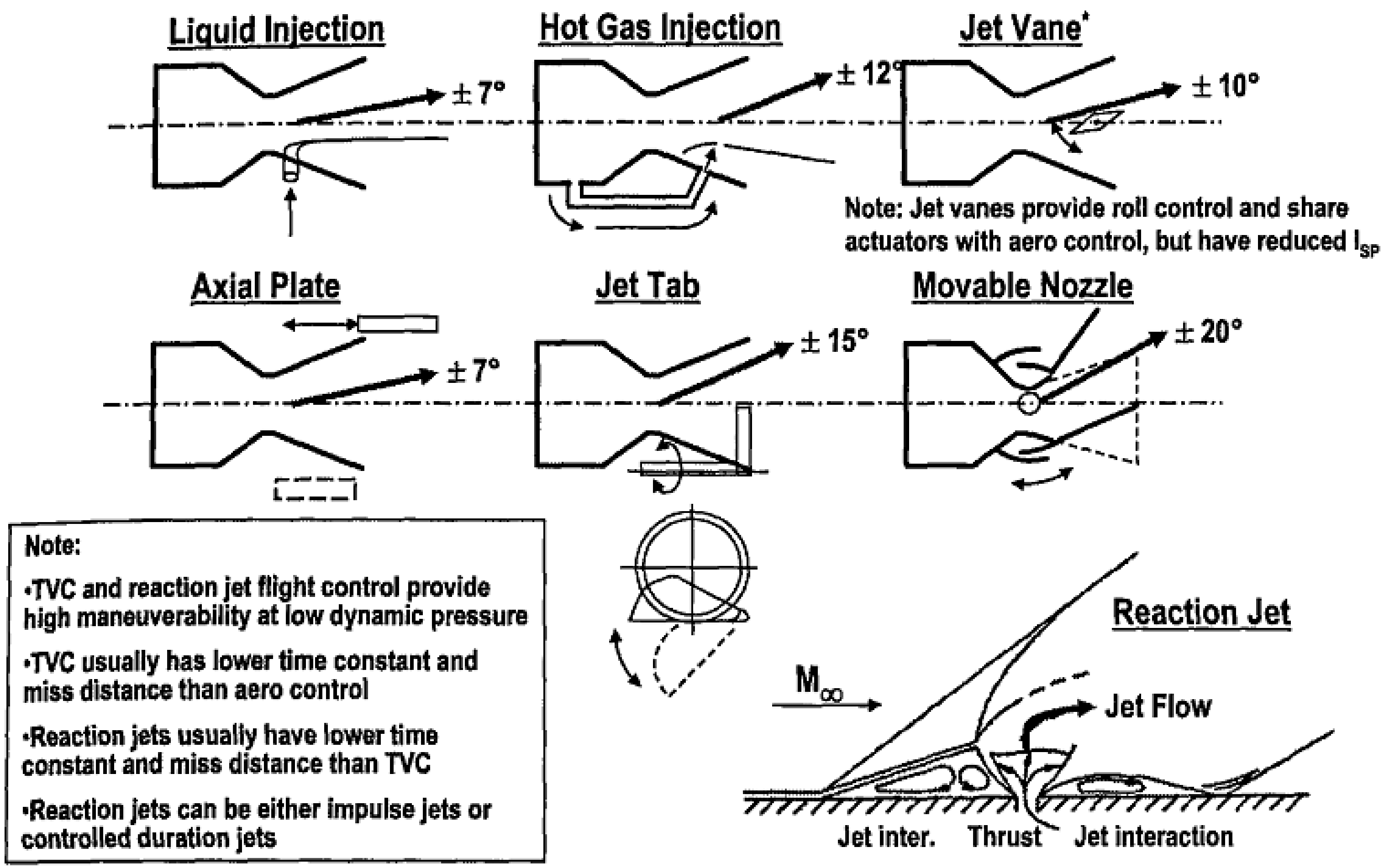


L: predominant use in liquid rockets S: predominant use in solid rockets

TVC: moving the nozzle

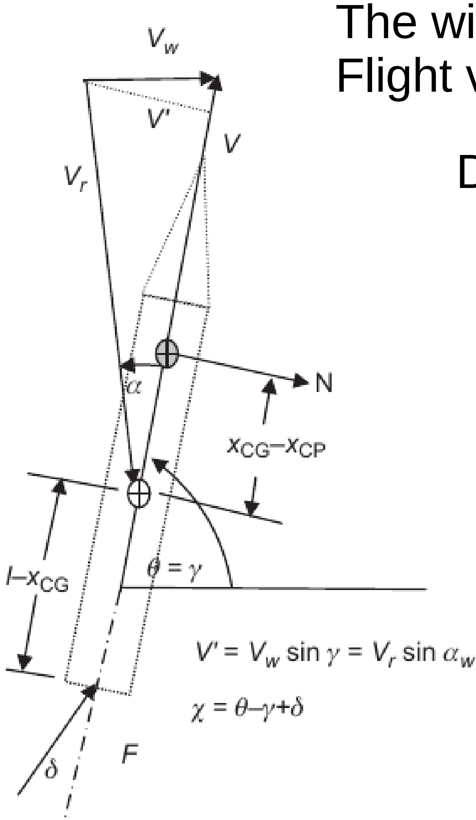
- Gimbal: the whole combustion chamber moves.
  - Pros: easy, proven, minor thrust loss, +/- 12°
  - Cons: high inertia/slow, large actuators, requires compact combustion chambers
  - Used in liquid space rockets
- Movable nozzle: flexible joint
  - Pros: flight proven, predictable power requirement, +/- 12°
  - High torque requested
  - Used in large SRMs
- Movable nozzle: ball joint
  - Pros: proven, no thrust loss if the whole nozzle is moved, +/- 20°
  - Limited duration, seal moved in presence of hot gases
  - Used in large SRMs

TVC: jet deflection



Cross-wind example / 1

- Missile in cross-wind
- Find the correction of the TVC to maintain the flight path angle



The wind is parallel to the Earth
Flight velocity parallel to missile axis

Different viewpoint: missile vs wind $\alpha=-\alpha_w$

Moment balance:

$$\sin\delta=-\frac{X_{CG}-X_{CP}}{F\left(L_B-X_{CG}\right)}\left(C_{N\alpha}\right)\alpha q S$$

If a fin is added: $(N)_F=\left(C_{N\alpha}\right)_F\frac{S_{\text{fin}}}{S}\alpha q S$

Frame of reference for vector sum: the moving missile

Cross-wind example / 2

- The TVC deflection becomes:

$$\sin\delta = \frac{\frac{qS}{W_0}}{\frac{T}{W_0}} \left(C_{N\alpha} \left(\frac{\frac{L_B}{d} - \frac{X_{CP}}{d}}{\frac{L_B}{d} - \frac{X_{CG}}{d}} - 1 \right) - \left(C_{N\alpha}\right)_{\text{fin}} \frac{S_{\text{fin}}}{S} \right) \frac{V_w}{V_R} \sin\gamma$$

$$W_0 = m_0 g \rightarrow \left(m_0 \text{ in tons}\right)$$

$\frac{qS}{W_0}$: dyn. pressure to initial weight ratio

$\frac{T}{W_0}$: thrust to weight ratio

Fitting parameter for launchers (q in kPa) $\frac{qS}{W_0} \approx 0{,}1191\, q \left[m_0 \left(\frac{L_B}{d} \right)^2 \right]^{-1/3}$

For typical launcher data we find: $0{,}0015 \leq \frac{qS}{W_0} \leq 0{,}006$

Useful formula for simple rocket under slender body theory $\left(C_{N\alpha}\right) = \frac{\partial C_N}{\partial \alpha} = 2\frac{S_b}{S}$

References

- E. Fleeman. Missile design and system engineering. AIAA Educational Series, 2012
- P.M. Sforza. Manned Spacecraft Design Principles, Butterworth Heinemann, 2016
- J.P. Sutton and O. Biblarz. Rocket Propulsion Elements. Seventh Edition, 2001