HO CHI MINH CITY UNIVERSITY OF TRANSPORT

Kiến thức - Kỹ năng - Sáng tạo - Hội nhập

Sứ mệnh - Tầm nhìn

Triết lý Giáo dục - Giá trị cốt lõi

Contents

0 Database and Python Resources

0.1 Database

1. SQLite: https://sqlite.org
   Tool: https://sqlitebrowser.org , https://dbeaver.io

2. PostgreSQL: https://www.postgresql.org
   Tool: https://www.pgadmin.org , https://dbeaver.io

0.2 Python Environment

1. Online: https://colab.research.google.com

2. Offline: Anaconda → Jupyter Notebook: https://www.anaconda.com/products/individual-d

0.3 Python is a programming interface

1. Python tutorial: https://pythonbasics.org/

2. Using python to connect with database to execute queries.

3. Tkinter GUI: https://docs.python.org/3/library/tk.html

4. PyQt: https://www.pythonguis.com/

1 Functional Dependencies (FDs)

Exercise 1.1

Consider relation r below:

r:	R(A	B	C	D	E)
t_1	0	0	0	0	0
t_2	0	1	1	1	0
t_3	1	0	2	2	0
t_4	1	0	3	2	0
t_5	2	1	4	0	0

A	B	C	D
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Which of the following FDs does r satisfy (why?):

a) $A \to B$
b) $AB \to D$
c) $C \to BDE$
d) $E \to A$
e) $A \to E$

a) Sai (0->0, 0->1 là sai)
b) Đúng (Có 2 cặp 1-0 AB nhưng đều có giá trị 2 ở D là đúng)
c) Đúng (Cột C khác nhau từng đôi một)
d) Sai (0->0, 0->1 là sai)
e) Đúng (Cột E giống nhau)

Exercise 1.2

Prove that r satisfies $X \to Y$ if and only if X is a key of $\pi_{XY}(r)$.

Exercise 1.3

Let r be a relation on R, with X a subset of R. Show that if $\pi_X(r)$ has the same number of tuples as r, then r satisfies $X \to Y$ for any subset Y of R.

Exercise 1.4

Prove or disprove the following inference rules for a relation $r(R)$ with W, X, Y, Z subsets of R.

a) $X \to Y$ and $Z \to W$ imply $XZ \to YW$.
b) $XY \to Z$ and $Z \to X$ imply $Z \to Y$.
c) $X \to Y$ and $Y \to Z$ imply $X \to YZ$.
d) $X \to Y$, $W \to Z$, and $Y \supseteq W$ imply $X \to Z$.

2 Amstrong's Axiom

Exercise 2.1

Consider $F = \{\ AB \to CD,\ A \to BE,\ BH \to DK,\ H \to BC\ \}$
Prove by Amstrong: $F \models AH \to CK$

Exercise 2.2

Consider $F = \{\ AB \to E,\ AG \to J,\ BE \to I,\ E \to G,\ GI \to H\ \}$
Prove by Amstrong: $F \models AB \to GH$

Exercise 2.3

Consider $F = \{\ A \to D,\ B \to CE,\ E \to H,\ D \to E,\ E \to C\ \}$
Prove by Amstrong:

a) $F \models B \to H$

b) $F \models AB \to CH$

Exercise 2.4

Consider $F = \{\ D \to BK,\ AB \to GK,\ B \to H,\ CE \to AG,\ H \to E,\ K \to G,\ EH \to K,\ G \to AH\ \}$
Prove by Amstrong:

a) $F \models AB \to GH$

b) $F \models DE \to AG$

c) $F \models BH \to EK$

3 Closure

Exercise 3.1

Show that for any set of FDs F, $F^+ = (F^+)^+$.

Exercise 3.2

Suppose $R(ABCDE)$ and set of functional dependencies:
$F = \{\ A \to BC,\ CD \to E,\ B \to D,\ E \to A\ \}$. Compute:

a) CD_F^+

b) E_F^+

Exercise 3.3

Suppose $R(ABCDEK)$ and set of functional dependencies:
$F = \{\ AB \to C,\ BC \to AD,\ D \to E,\ CK \to B\ \}$. Compute:

a) BCK_F^+

b) CD_F^+

c) D_F^+

Exercise 3.4

Suppose $R(ABCDEKGH)$ and set of functional dependencies:
$F = \{\ A \to BC,\ E \to C,\ AH \to D,\ CD \to E,\ D \to AEH,\ DH \to BC\}$. Compute:

a) AE_F^+

b) BCD_F^+

Exercise 3.5

Consider:
$F_1 = \{\ AB \to CD,\ A \to BE,\ BH \to DK,\ H \to BC\ \}$
$F_2 = \{\ AB \to E,\ AG \to J,\ BE \to I,\ E \to G,\ GI \to H\ \}$
$F_3 = \{\ A \to D,\ B \to CE,\ E \to H,\ D \to E,\ E \to C\ \}$
$F_4 = \{\ D \to BK,\ AB \to GK,\ B \to H,\ CE \to AG,\ H \to E,\ K \to G,\ EH \to K,\ G \to AH\ \}$
Compute:

a) $AH^+_{F_1}$
b) $AB^+_{F_2}$
c) $B^+_{F_3}$
d) $AB^+_{F_3}$
e) $AB^+_{F_4}$
f) $DE^+_{F_4}$
g) $BH^+_{F_4}$

Exercise 3.6

Consider $F = \{\ A \to B,\ A \to C,\ CD \to E,\ B \to D,\ E \to A\ \}$
Which of the following functional dependencies is NOT implied by the above set ?

a) $CD \to AC$
b) $BD \to CD$
c) $BC \to CD$
d) $AC \to BC$

Exercise 3.7

From Axiom 1, 2, 3 prove Axiom 4, 5 and 6.

Exercise 3.8

Prove that inference axioms 1, 2, and 6 are independent. That is, no one of them can be proved from the other two.

Exercise 3.9

$R(ABCD)$ having two FDs sets:
$F = \{\ A \to B,\ B \to C,\ AB \to D\ \}$,
$G = \{\ A \to B,\ B \to C,\ A \to C,\ A \to D\ \}$
Are the two sets equivalent ?

Exercise 3.10

$R(ABCD)$ having two FDs sets:
$F = \{\ A \to B,\ B \to C,\ A \to C\ \}$,
$G = \{\ A \to B,\ B \to C,\ A \to D\ \}$
Are the two sets equivalent ?

Exercise 3.11

$R(ACDEH)$ having two FDs sets:
$F = \{\ A \to C,\ AC \to D,\ E \to AD,\ E \to H\ \}$,
$G = \{\ A \to CD,\ E \to AH\ \}$
Are the two sets equivalent ?

Exercise 3.12

$R(ABCDE)$ having two FDs sets:
$F = \{\ A \to BC,\ A \to D,\ CD \to E\ \}$,
$G = \{\ A \to BCE,\ A \to ABD,\ CD \to E\ \}$
Are the two sets equivalent ?

Exercise 3.13

$R(ABCDE)$ having two FDs sets:
$F = \{\ AB \rightarrow C,\ A \rightarrow B,\ B \rightarrow C,\ A \rightarrow C\ \}$,
$G = \{\ AB \rightarrow C,\ A \rightarrow B,\ B \rightarrow C\ \}$
Are the two sets equivalent ?

Exercise 3.14

Consider $F = \{\ A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A,\ B \rightarrow A,\ A \rightarrow C\ \}$

a) Find a minimum cover F_c of F by loop from right to left

b) Find a minimum cover F_c of F by loop from left to right

Exercise 3.15

Consider $F = \{\ A \rightarrow BC,\ B \rightarrow C,\ A \rightarrow B,\ AB \rightarrow C\ \}$
Find a minimum cover F_c of F

Exercise 3.16

Consider $F = \{\ A \rightarrow BC,\ CD \rightarrow E,\ B \rightarrow D,\ E \rightarrow A\ \}$
Find a minimum cover F_c of F

Exercise 3.17

Consider $F = \{\ B \rightarrow A,\ AD \rightarrow BC,\ C \rightarrow ABD\ \}$
Find a minimum cover F_c of F

Exercise 3.18

Consider $R(ABC)$,
$F = \{\ AB \rightarrow C,\ A \rightarrow B\ \}$
$G = \{\ A \rightarrow B,\ B \rightarrow C\ \}$

a) Find a minimum cover F_c of F

b) Is G a minimal cover of F ? Otherwise give a data instance of R satisfy F but not G

Exercise 3.19

Consider $R(ABCDE)$, $F = \{\ AB \rightarrow CD,\ B \rightarrow CD,\ CD \rightarrow AE,\ DE \rightarrow AB,\ D \rightarrow E\ \}$
Compute Projected Functional Dependencies:

a) $\pi_{R_1(ABC)}(F)$
b) $\pi_{R_2(BCD)}(F)$
c) $\pi_{R_3(CDE)}(F)$
d) $\pi_{R_4(ADE)}(F)$
e) $\pi_{R_5(BDE)}(F)$
f) $\pi_{R_6(AE)}(F)$
g) $\pi_{R_7(DE)}(F)$

Exercise 3.20

Consider $R(ABCDEGH)$,
$F = \{\ AB \rightarrow CD,\ E \rightarrow D,\ ABC \rightarrow DE,\ E \rightarrow AB,\ D \rightarrow AG,\ ACD \rightarrow BE\ \}$
Compute Projected Functional Dependencies:

a) $\pi_{R_1(ABCD)}(F)$
b) $\pi_{R_2(DEGH)}(F)$
c) $\pi_{R_3(CDE)}(F)$
d) $\pi_{R_4(ADE)}(F)$
e) $\pi_{R_5(BDE)}(F)$
f) $\pi_{R_6(AE)}(F)$
g) $\pi_{R_7(DE)}(F)$

4 Keys

B1: L = ABCED, R = BDCA
NF = H, OL = E, LR = ABCD
B2: 0, A, B, C, D, AB, AC, AD, BC, BD, CD
ABC, ABD, ACD, BCD, ABCD

Exercise 4.1

Consider $R(ABCDEH)$ with a set of FDs
$F = \{ A \to B,\ BC \to D,\ E \to C,\ D \to A \}$
What are the candidate keys of R

a) $AE,\ BE$
b) $AE,\ BE,\ DE$
c) $AEH,\ BEH,\ BCH$
d) $AEH,\ BEH,\ DEH$

Exercise 4.2

Consider $R(DEGHIJKLMN)$ with a set of FDs
$F = \{ DE \to G,\ D \to IJ,\ EH \to KL,\ K \to M,\ L \to N \}$
What is the key for R ?

a) EF
b) DEH
c) $DEHKL$
d) E

Exercise 4.3

Consider $R(ABCDEKGH)$ with a set of FDs
$F = \{ ABC \to DE,\ AB \to D,\ DE \to ABCK,\ E \to C \}$
Find all the candidate keys of R

Exercise 4.4

Consider $R(ABCDEGHK)$ with a set of FDs
$F = \{ CD \to A,\ EC \to H,\ GHB \to AB,\ C \to D,\ EG \to A,\ H \to B,\ BE \to CD,\ EC \to B \}$
Find all the candidate keys of R

5 Normal Form by FDs

Exercise 5.1

Which normal form of relational scheme below:

a) $R_1(ABC),\ F_1 = \{ A \to C \}$
b) $R_2(ABC),\ F_2 = \{ C \to B \}$
c) $R_3(ABCD),\ F_3 = \{ A \to B,\ B \to A \}$
d) $R_4(ABCD),\ F_4 = \{ D \to C,\ B \to A \}$
e) $R_5(ABCD),\ F_5 = \{ B \to D,\ C \to D \}$
f) $R_6(ABCDE),\ F_6 = \{ AB \to C,\ B \to A,\ D \to A \}$
g) $R_7(ABCDE),\ F_7 = \{ AB \to C,\ C \to D,\ D \to A \}$
h) $R_8(ABCDE),\ F_8 = \{ AB \to CD,\ CD \to AE,\ D \to A \}$
i) $R_9(ABCDE),\ F_9 = \{ D \to A,\ BC \to E,\ A \to C \}$
j) $R_{10}(ABCDEG),\ F_{10} = \{ AB \to CG,\ G \to D,\ B \to D \}$
k) $R_{11}(ABCDE),\ F_{11} = \{ E \to D,\ C \to B,\ A \to E\ B \to A,\ D \to C \}$
l) $R_{12}(ABCDE),\ F_{12} = \{ AC \to B,\ BD \to C,\ CE \to D \}$
m) $R_{13}(ABCD),\ F_{13} = \emptyset$

Exercise 5.2

Consider $R(ABCD)$, $F = \{\ A \to C,\ B \to D\ \}$

a) Keys and Normal form?

b) Decompose R

Exercise 5.3

Consider $R(ABCD)$, $F = \{\ AC \to D\ \}$

a) Keys and Normal form?

b) Decompose R

Exercise 5.4

Consider $R(ABCDE)$, $F = \{\ AB \to C,\ B \to A,\ D \to A\ \}$

a) Keys and Normal form?

b) Decompose R

Exercise 5.5

Consider $R(ABCDE)$, $F = \{\ CD \to A,\ EC \to B,\ AD \to C\ \}$

a) Keys and Normal form?

b) Decompose R

Exercise 5.6

Consider $R(ABCDEGH)$,
$F = \{\ CD \to A,\ EC \to H,\ GHB \to AB,\ C \to D,\ EG \to A,\ H \to B,\ BE \to CD,\ EC \to B\ \}$

a) Keys and Normal form ?

b) Decompose R

Exercise 5.7

Consider $R(ABCD)$, $F = \{\ A \to B,\ B \to C,\ D \to B\ \}$

a) Normal form of R ?

b) If R is not good, let try to find a good decomposition for R

Exercise 5.8

Consider $R(ABCD)$, $F = \{\ A \to B,\ B \to C,\ A \to D,\ D \to C\ \}$
One decomposition ρ of R:

$R_1(AB)$, F_1
$R_2(AC)$, F_2
$R_3(BD)$, F_3

a) F_i ?

b) Keys and Normal form of R_i ?

Exercise 5.9

Consider $R(A\ B\ D\ E\ M\ N\ O\ P\ X\ Y\ Z\ V\ W)$,
$F = \{\ D \to XMNPE,\ MPN \to EYABO,\ MN \to ZO,\ O \to V,\ P \to ABW,\ AB \to P,\ NE \to MP\ \}$
One decomposition ρ of R:

$R_1(DXMNPE)$, F_1
$R_2(MNPEYABO)$, F_2
$R_3(MNZO)$, F_3
$R_4(OV)$, F_4
$R_5(PABW)$, F_5

a) F_i ?

b) Keys and Normal form of R_i ?

c) Evaluate the quality of ρ (Normal form, Conserve information, Conserve FDs)

d) If ρ is not good, let make a improvement of ρ

Exercise 5.10

Consider $R(ABCDEGH)$,
$F = \{\ CD \to A,\ EC \to H,\ GHB \to AB,\ C \to D,\ EG \to A,\ H \to B,\ BE \to CD,\ EC \to B\ \}$
Evaluate the decomposition below (Normal form, Conserve information, Conserve FDs)
$\rho = \{\ R_1(ABC),\ R_2(CDEG),\ R_3(EGH)\ \}$

Exercise 5.11

Give an example of a relation in 3NF that has some prime attribute transitively dependent upon a key

Exercise 5.12

Let R_1 and R_2 be relation schemes with $R_1 \cap R_2 = X$. Show that for any relation $r(R_1R_2)$ that satisfies $X \to R_2$,

$$r = \pi_{R_1}(r) \bowtie \pi_{R_2}(r)$$

6 Multivalued Dependencies (MVDs)

Exercise 6.1

Consider relation r below:

r:	R(A	B	C	D	E)
t_1	0	0	1	0	0
t_2	0	0	2	1	0
t_3	0	2	2	0	1

From data instance R above make R satisfies each MVD below:

a) $AB \twoheadrightarrow C$
b) $AB \twoheadrightarrow E$
c) $D \twoheadrightarrow C$
d) $AD \twoheadrightarrow C$
e) $C \twoheadrightarrow DE$

Exercise 6.2

Let $R(ABCDE)$, $\mathfrak{D} = \{\ A \twoheadrightarrow BC,\ A \twoheadrightarrow E,\ E \twoheadrightarrow CD\ \}$
Proving by MVDs axiom:

a) $\mathfrak{D} \models A \twoheadrightarrow C$
b) $\mathfrak{D} \models A \twoheadrightarrow BD$
c) $\mathfrak{D} \models AC \twoheadrightarrow BD$
d) $\mathfrak{D} \models AC \twoheadrightarrow BE$
e) $\mathfrak{D} \models DE \twoheadrightarrow AC$
f) $\mathfrak{D} \models DE \twoheadrightarrow AB$

Exercise 6.3

Let $R(ABCDGH)$, $\mathfrak{D} = \{\ A \twoheadrightarrow B,\ B \twoheadrightarrow GH,\ CD \twoheadrightarrow G\ \}$
Proving by MVDs axiom:

a) $\mathfrak{D} \models BC \twoheadrightarrow AD$
b) $\mathfrak{D} \models BC \twoheadrightarrow GH$
c) $\mathfrak{D} \models BC \twoheadrightarrow DG$
d) $\mathfrak{D} \models CD \twoheadrightarrow AB$
e) $\mathfrak{D} \models CD \twoheadrightarrow BG$
f) $\mathfrak{D} \models CD \twoheadrightarrow GH$

Exercise 6.4

Let $R(ABCGHI)$, $\mathfrak{D} = \{\ A \twoheadrightarrow B,\ B \twoheadrightarrow HI,\ CG \twoheadrightarrow H\ \}$
Compute $X^{++}_{\mathfrak{D}}$:

a) $A^{++}_{\mathfrak{D}}$

b) $AG^{++}_{\mathfrak{D}}$

c) $BG^{++}_{\mathfrak{D}}$

d) $BC^{++}_{\mathfrak{D}}$

e) $HG^{++}_{\mathfrak{D}}$

Exercise 6.5

Prove the correctness of inference axioms M1 and M2.

Exercise 6.6

Prove the correctness of inference axiom M3.

Exercise 6.7

We know axiom M7 is correct from Lemma 8.3
Prove the correctness of inference axiom M4 using axioms M3 and M7.

Exercise 6.8

Prove the correctness of inference axiom M5 using axioms M4.

Exercise 6.9

Prove the correctness of inference axiom M6 using axioms M1-M5 and M7

7 Tableau Chase Test

Exercise 7.1

Consider $\mathfrak{D} = \{\ AB \to CD,\ A \to BE,\ BH \to DK,\ H \to BC\ \}$
Prove by Tableau Chase test: $\mathfrak{D} \models AH \to CK$

Exercise 7.2

Consider $\mathfrak{D} = \{\ AB \to E,\ AG \to J,\ BE \to I,\ E \to G,\ GI \to H\ \}$
Prove by Tableau Chase test: $\mathfrak{D} \models AB \to GH$

Exercise 7.3

Consider $\mathfrak{D} = \{\ A \to D,\ B \to CE,\ E \to H,\ D \to E,\ E \to C\ \}$
Prove by Tableau Chase test:

a) $\mathfrak{D} \models B \to H$

b) $\mathfrak{D} \models AB \to CH$

Exercise 7.4

Consider $\mathfrak{D} = \{\ D \to BK,\ AB \to GK,\ B \to H,\ CE \to AG,\ H \to E,\ K \to G,\ EH \to K,\ G \to AH\ \}$
Prove by Tableau Chase test:

a) $\mathfrak{D} \models AB \to GH$

b) $\mathfrak{D} \models DE \to AG$

c) $\mathfrak{D} \models BH \to EK$

Exercise 7.5

Suppose $R(ABCDE)$ and set of functional dependencies:
$\mathfrak{D} = \{\ A \to BC,\ CD \to E,\ B \to D,\ E \to A\ \}$. **Using Tableau Chase test to compute**:

a) $CD^+_{\mathfrak{D}}$

b) $E^+_{\mathfrak{D}}$

Exercise 7.6

Suppose $R(ABCDEK)$ and set of functional dependencies:
$\mathfrak{D} = \{\ AB \to C,\ BC \to AD,\ D \to E,\ CK \to B\ \}$. **Using Tableau Chase test to compute**:

a) $BCK^+_{\mathfrak{D}}$

b) $CD^+_{\mathfrak{D}}$

c) $D^+_{\mathfrak{D}}$

Exercise 7.7

Suppose $R(ABCDEKGH)$ and set of functional dependencies:
$\mathfrak{D} = \{\ A \to BC,\ E \to C,\ AH \to D,\ CD \to E,\ D \to AEH,\ DH \to BC\}$. **Using Tableau Chase test to compute**:

a) $AE^+_{\mathfrak{D}}$

b) $BCD^+_{\mathfrak{D}}$

Exercise 7.8

Consider:
$\mathfrak{D}_1 = \{\ AB \to CD,\ A \to BE,\ BH \to DK,\ H \to BC\ \}$
$\mathfrak{D}_2 = \{\ AB \to E,\ AG \to J,\ BE \to I,\ E \to G,\ GI \to H\ \}$
$\mathfrak{D}_3 = \{\ A \to D,\ B \to CE,\ E \to H,\ D \to E,\ E \to C\ \}$
$\mathfrak{D}_4 = \{\ D \to BK,\ AB \to GK,\ B \to H,\ CE \to AG,\ H \to E,\ K \to G,\ EH \to K,\ G \to AH\ \}$
Using Tableau Chase test to compute:

a) $AH^+_{\mathfrak{D}_1}$

b) $AB^+_{\mathfrak{D}_2}$

c) $B^+_{\mathfrak{D}_3}$

d) $AB^+_{\mathfrak{D}_3}$

e) $AB^+_{\mathfrak{D}_4}$

f) $DE^+_{\mathfrak{D}_4}$

g) $BH^+_{\mathfrak{D}_4}$

Exercise 7.9

Consider $\mathfrak{D} = \{\ A \to B,\ A \to C,\ CD \to E,\ B \to D,\ E \to A\ \}$
Using Tableau Chase test to compute: Which of the following functional dependencies is NOT implied by the above set ?

a) $CD \to AC$

b) $BD \to CD$

c) $BC \to CD$

d) $AC \to BC$

Exercise 7.10

Let $R(ABCDE)$, $\mathfrak{D} = \{\ A \twoheadrightarrow BC,\ A \twoheadrightarrow E,\ E \twoheadrightarrow CD\ \}$
Using Tableau Chase test to compute:

a) $\mathfrak{D} \models A \twoheadrightarrow C$
b) $\mathfrak{D} \models A \twoheadrightarrow BD$
c) $\mathfrak{D} \models AC \twoheadrightarrow BD$
d) $\mathfrak{D} \models AC \twoheadrightarrow BE$
e) $\mathfrak{D} \models DE \twoheadrightarrow AC$
f) $\mathfrak{D} \models DE \twoheadrightarrow AB$

Exercise 7.11

Let $R(ABCDGH)$, $\mathfrak{D} = \{\ A \twoheadrightarrow B,\ B \twoheadrightarrow GH,\ CD \twoheadrightarrow G\ \}$
Using Tableau Chase test to compute:

a) $\mathfrak{D} \models BC \twoheadrightarrow AD$
b) $\mathfrak{D} \models BC \twoheadrightarrow GH$
c) $\mathfrak{D} \models BC \twoheadrightarrow DG$
d) $\mathfrak{D} \models CD \twoheadrightarrow AB$
e) $\mathfrak{D} \models CD \twoheadrightarrow BG$
f) $\mathfrak{D} \models CD \twoheadrightarrow GH$

8 More Normal form and Dependencies

Exercise 8.1

Modify the relation r below to satisfy the MVDs $A \twoheadrightarrow BC$ and $CD \twoheadrightarrow BE$ by adding rows.

r:	R(A	B	C	D	E)
t_1	0	0	0	0	0
t_2	0	1	0	1	0
t_3	1	0	0	0	1

Exercise 8.2

Prove that if a relation $r(R)$ satisfies the MVDs $X \twoheadrightarrow Y_1,\ X \twoheadrightarrow Y_2\ ...\ X \twoheadrightarrow Y_k$, where $R = XY_1Y_2...Y_k$, then r decomposes converse information onto the relation schemes $XY_1,\ XY_2,\ ...,\ XY_k$.

Exercise 8.3

Let $r(R)$ be a relation where $R_1 \subseteq R,\ R_2 \subseteq R$ and $R = R_1R_2$. Prove that $r = \pi_{R_1}(r) \bowtie \pi_{R_2}(r)$ if and only if: $Count(\pi_R([X = x](r))) = Count(\pi_{R_1}([X = x](r))) \times Count(\pi_{R_2}([X = x](r)))$ for every X-value x in r

Exercise 8.4

Prove that if relation $r(R)$ satisfies $X \twoheadrightarrow Y$ and $Z = R - XY$, then

$$\pi_Z(\sigma_{X=x}(r)) = \pi_Z(\sigma_{XY=xy}(r))$$

for every XY-value xy in r

Exercise 8.5

Let relation scheme R and let $W,\ X,\ Y,\ Z \subseteq R$. Show that:
$\{\ X \twoheadrightarrow Y\ ,\ Z \subseteq W\ \} \models XW \twoheadrightarrow YZ$

Exercise 8.6

Prove the correctness of inference axiom M6 using axioms M1-M5 and M7

Exercise 8.7

Let relation scheme R and let $X,\ Y,\ Z \subseteq R$. Show that:
$\{\ X \twoheadrightarrow Y\ ,\ XY \rightarrow Z\ \} \models X \rightarrow Z - Y$