

Contents

1	Basic Principles	3
1.1	Problem Types	3
1.2	Derivations	3
1.3	Concepts	3
1.4	Equations	3
2	Accelerated Coordinate Systems	5
2.1	Problem Types	5
2.2	Derivations	5
2.3	Concepts	5
2.4	Equations	5
2.4.1	General Fictitious Forces Eq.	5
3	Lagrangian Dynamics	7
3.1	Problem Types	7
3.2	Derivations	7
3.3	Concepts	7
3.4	Equations	7
3.4.1	Euler-Lagrange Eq.'s	7
3.4.2	Invariance of the Lagrangian	7
3.4.3	Parallel Axis Theorem	7
3.4.4	Hamilton-Jacobi Equation	7
3.4.5	Hamilton's Eq.'s	7
4	Small Oscillations	9
4.1	Problem Types	9
4.2	Derivations	9
4.3	Concepts	9
4.4	Equations	9
5	Rigid Bodies	11
5.1	Problem Types	11
5.2	Derivations	11
5.3	Concepts	11
5.4	Equations	11
6	Hamiltonian Dynamics	13
6.1	Problem Types	13
6.2	Derivations	13
6.3	Concepts	13
6.4	Equations	13

Chapter 1

Basic Principles

1.1 Problem Types

1.2 Derivations

1.3 Concepts

1.4 Equations

Chapter 2

Accelerated Coordinate Systems

2.1 Problem Types

2.2 Derivations

2.3 Concepts

2.4 Equations

2.4.1 General Fictitious Forces Eq.

Say we have a particle, which we can observe in an inertial frame S and some other frame S' (maybe non-inertial), called the *body* frame, which is moving relative to the inertial frame. The position of S' relative to S is \mathbf{a} . The angular velocity of S' is $\boldsymbol{\omega}$ (same for both frames, as angular velocity does not depend on frame). The position of the particle in the body frame S' is \mathbf{r} and is \mathbf{r}_0 in S . Then

$$m \left(\frac{d^2 \mathbf{r}}{dt^2} \right)_{\text{body}} = m \left(\frac{d^2 \mathbf{r}_0}{dt^2} \right)_{\text{inertial}} - m \left(\frac{d^2 \mathbf{a}}{dt^2} \right)_{\text{inertial}} - 2m\boldsymbol{\omega} \times \left(\frac{d\mathbf{r}}{dt} \right) - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - m \frac{d\boldsymbol{\omega}}{dt} \times \mathbf{r}$$

Note that the $m \left(\frac{d^2 \mathbf{r}_0}{dt^2} \right)_{\text{inertial}}$ term is just the external force in the inertial frame. That is, it is the term which represents the “real” forces.

Chapter 3

Lagrangian Dynamics

3.1 Problem Types

3.2 Derivations

3.3 Concepts

3.4 Equations

3.4.1 Euler-Lagrange Eq.'s

Euler-Lagrange Equations

For one coordinate q

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = 0$$

3.4.2 Invariance of the Lagrangian

For the two Lagrangians

$$\mathcal{L} = T - V$$

and

$$\mathcal{L}' = T - V + \frac{df(x, t)}{dt}$$

the dynamics are exactly the same for *any* function $f(x, t)$.

3.4.3 Parallel Axis Theorem

Also called Steiner's Theorem

Given the moment of inertia about the center of mass, this theorem allows us to calculate the moment of inertia about an axis offset from the center (although still pointing in the same direction). For center-of-mass MoI I_c , mass M , and axis offset h , the new moment of inertia is

$$I = I_c + Mh^2$$

3.4.4 Hamilton-Jacobi Equation

For the Hamiltonian \mathcal{H}

$$H$$

3.4.5 Hamilton's Eq.'s

Chapter 4

Small Oscillations

4.1 Problem Types

4.2 Derivations

4.3 Concepts

4.4 Equations

Chapter 5

Rigid Bodies

5.1 Problem Types

5.2 Derivations

5.3 Concepts

5.4 Equations

Chapter 6

Hamiltonian Dynamics

6.1 Problem Types

6.2 Derivations

6.3 Concepts

6.4 Equations