Logic and Computer Design Fundamentals

Chapter 2 – Combinational Logic Circuits

Part 1 – Gate Circuits and Boolean Equations

Overview

- **Part 1 – Gate Circuits and Boolean Equations**
  - **Binary Logic and Gates**
  - **Boolean Algebra**
  - **Standard Forms**
- **Part 2 – Circuit Optimization**
  - **Two-Level Optimization**
  - **Map Manipulation**
  - **Practical Optimization**
  - **Multi-Level Circuit Optimization**
- **Part 3 – Additional Gates and Circuits**
  - **Other Gate Types**
  - **Exclusive-OR Operator and Gates**
  - **High-Impedance Outputs**

Binary Logic and Gates

- **Binary variables** take on one of two values.
- **Logical operators** operate on **binary values and binary variables**.

  Basic logical operators are the **logic functions** **AND, OR and NOT**.
- **Logic gates** implement **logic functions**.
- **Boolean Algebra**: a **useful mathematical system** for specifying and transforming logic functions.

  We study Boolean algebra as a foundation for designing and analyzing digital systems!

Binary Variables

- **Recall that the two binary values have different names:**
  - **True/False**
  - **On/Off**
  - **Yes/No**
  - **1/0**
- **We use 1 and 0 to denote the two values.**
- **Variable identifier examples:**
  - **A, B, y, z, or X_1 (Single Letter)**
  - **RESET, START_IT, or ADD, etc**

Logical Operations

- The three basic logical operations are:
  - AND
  - OR
  - NOT
- AND is denoted by a dot (·).
- OR is denoted by a plus (+).
- NOT is denoted by an overbar (¯), a single quote mark (') after, or (~) before the variable.

Logical Operation Examples

- **Examples:**
  - $Y = A \cdot B$ is read "Y is equal to A AND B."
  - $z = x + y$ is read "z is equal to x OR y."
  - $X = \overline{A}$ is read "X is equal to NOT A."
- **Note: The statement:**

  1 + 1 = 2 (read "one plus one equals two")

  is not the same as

  1 + 1 = 1 (read "1 or 1 equals 1").

Operator Definitions

- **Operations are defined on the values "0" and "1" for each operator:**

AND	OR	NOT
$0 \cdot 0 = 0$	$0 + 0 = 0$	$\overline{0} = 1$
$0 \cdot 1 = 0$	$0 + 1 = 1$	$\overline{1} = 0$
$1 \cdot 0 = 0$	$1 + 0 = 1$	
$1 \cdot 1 = 1$	$1 + 1 = 1$	

Truth Tables

- ***Truth table*** **– a tabular listing of the values of a function for all possible combinations of values on its arguments**
- **Example: Truth tables for the basic logic operations:**

AND		
X	Y	$Z = X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

OR		
X	Y	$Z = X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

NOT	
X	$Z = \overline{X}$
0	1
1	0

Construction of Truth Table

- Determine the number of Inputs (*n*) and the number of outputs (*m*) according to the design requirement.
- There should be 2^n rows and *m* output column in the Truth Table.

Input			Output
A	B	C	
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

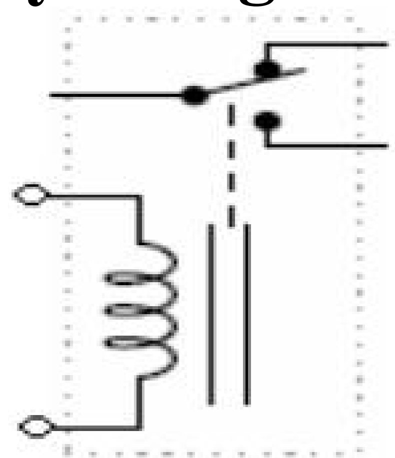
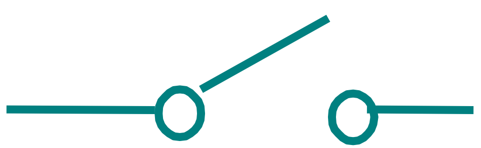
	n			m		
	A_1	…	A_n	Z_1	…	Z_m
2^{n-1}	0		0			
	0		1			
	0		0			
	0		1			
2^{n-1}	1		0			
	1		1			
	1		0			
	1		1			

Logic Function Implementation

- In 1938, Claude E. Shannon, ***A Symbolic Analysis of Relay and Switching Circuits***.

  (He suggested that using relays to implement the logic circuits)

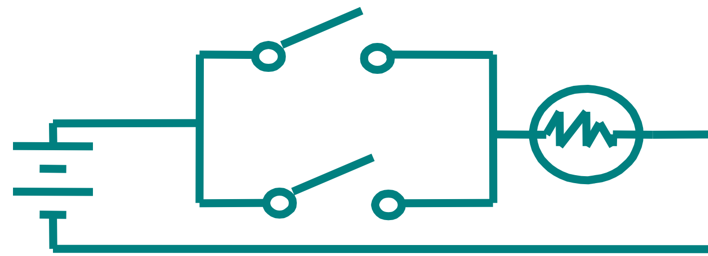
- **Switches were opened and closed by magnetic fields produced by energizing coils in *relays*.**

Logic Function Implementation

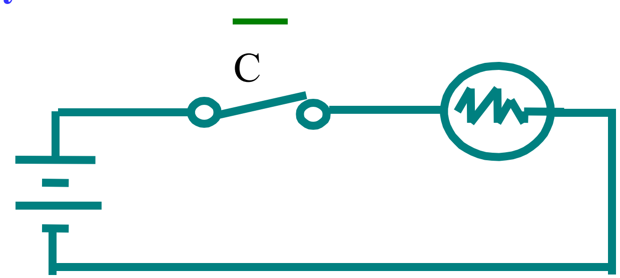
- Using Switches
  - For inputs:
    - logic 1 is switch closed
    - logic 0 is switch open
  - For outputs:
    - logic 1 is light on
    - logic 0 is light off.
  - NOT uses a switch such that:
    - logic 1 is switch open
    - logic 0 is switch closed

Switches in parallel => OR



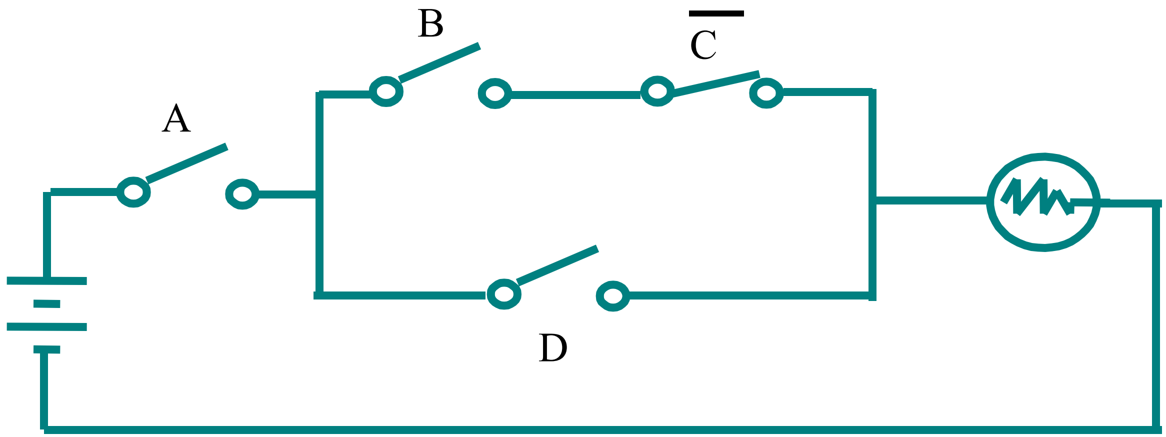
Switches in series => AND

Normally-closed switch => NOT



Logic Function Implementation (Continued)

- **Example: Logic Using Switches**



- **Light is on (L = 1) for**

  L(A, B, C, D) =A ((B C') + D) = A B C' + A D

  and off (L = 0), otherwise.

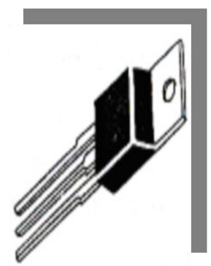
Logic Gates

- **In the earliest computers, The magnetic–control switches (*relays*) in turn opened and closed the current paths.**
- **Later, *vacuum tubes* that open and close current paths electronically replaced relays.**
- **Today, *transistors* are used as electronic switches that open and close current paths.**

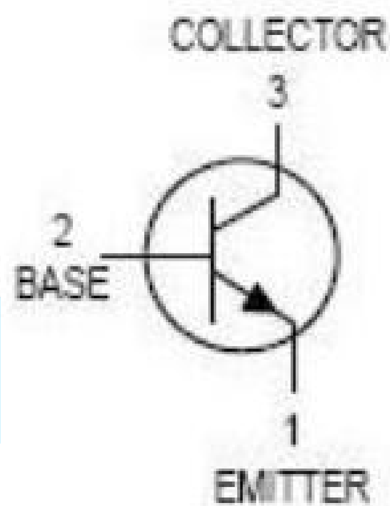
Metal Package

Plastic Package

High Power



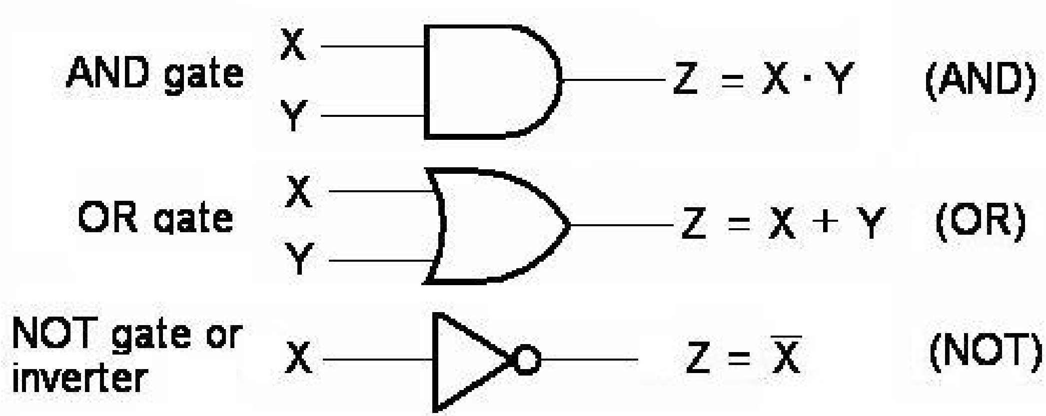
Medium Power



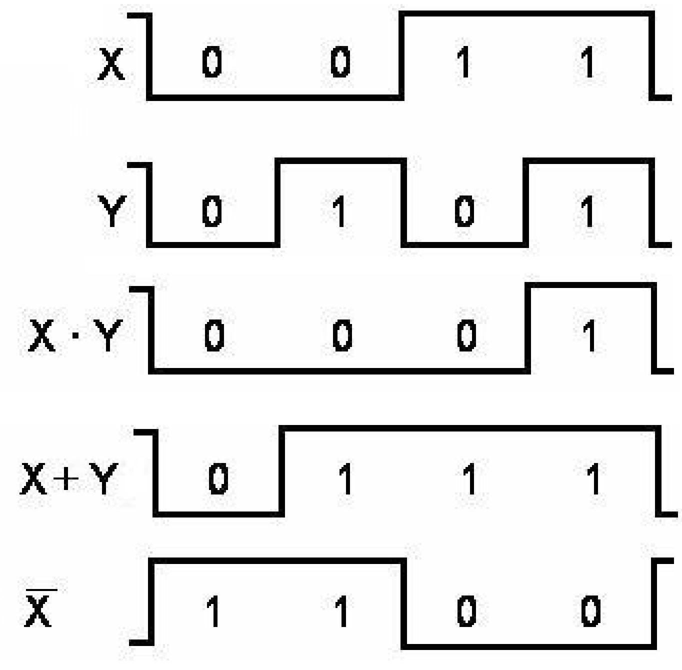
Logic Gate Symbols and waveform Behavior

- **Logic gates have special symbols and waveforms as follows:**

Inputs		Output		
X	Y	XY	X+Y	$\overline{X}$
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0



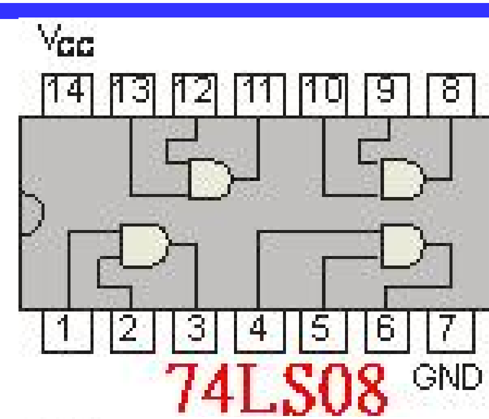
(a) Graphic symbols

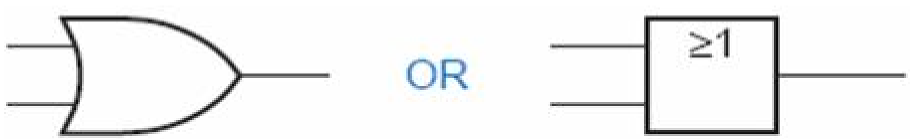


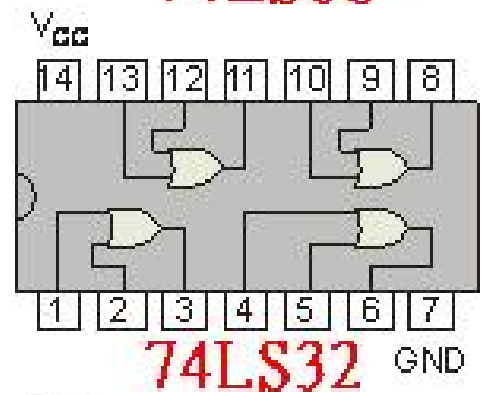
(b) Timing diagram

Basic Gates in TTL74 Series

International China

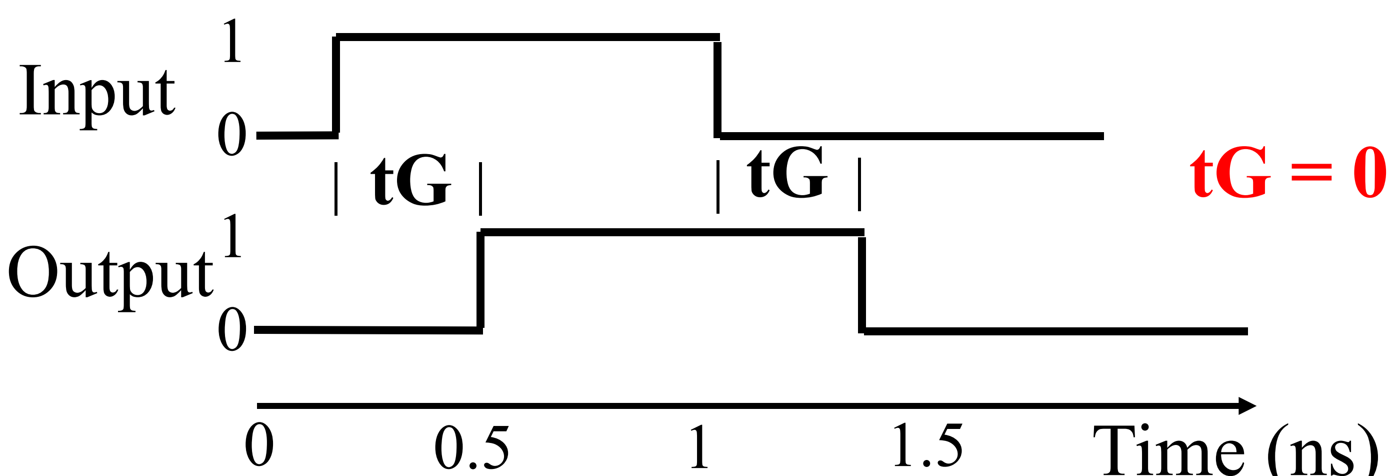






Gate Delay

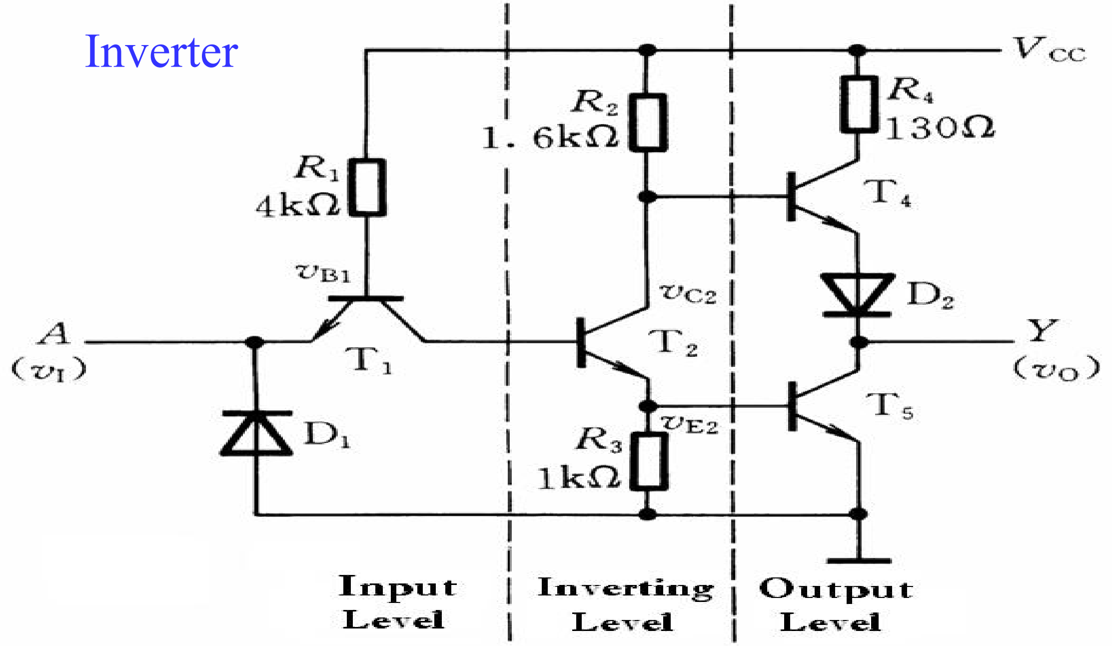
- In actual physical gates, if one or more input changes causes the output to change, the output change **does not occur instantaneously**.
- The delay between an input change(s) and the resulting output change is the ***gate delay*** denoted by t_G:



tG = 0.3 ns

TTL Logic Series

- 74S Series
- 74LS Series
- 74AS Series
- 74ALS Series
- 74F Series



		Family				
Description	*Symbol*	*74S*	*74LS*	*74AS*	*74ALS*	*74F*
Maximum propagation delay (ns)		3	9	1.7	4	3
Power consumption per gate (mW)		19	2	8	1.2	4
Speed-power product (pJ)		57	18	13.6	4.8	12

Truth Table, Boolean Equation and Logic Diagram

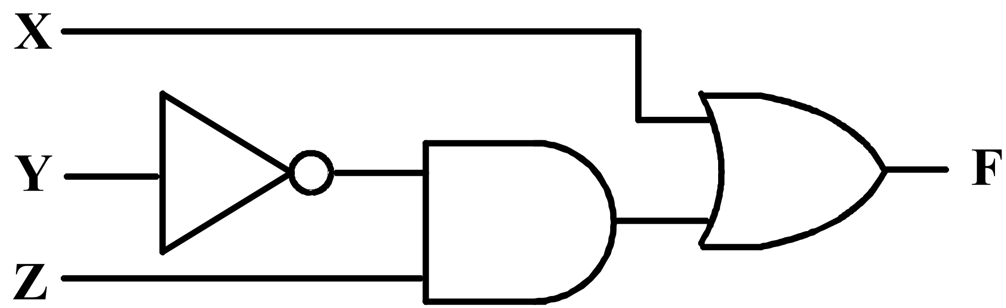
Truth Table

X Y Z	$F = X + \overline{Y} \times Z$
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	1
1 1 0	1
1 1 1	1

Boolean Equation

$$F = X + \overline{Y}\,Z$$

Logic Diagram



- **Boolean equations, truth tables and logic diagrams describe the same function!**
- **Truth tables are unique; Boolean equations and logic diagrams are not. This gives flexibility in implementing functions.**

Boolean Algebra

In 1854, George Boole, ***An Investigation of the Laws of Thought, on which are Founded the Mathematical Theories of Logic and Probabilities*** (Idea=>0,1 symbols)

Boolean Algebra is an algebraic structure defined on a set of at least two elements, together with three binary operators (denoted +, · and ¯)

For example, $AB(C+D)+\overline{E}$, $F = \overline{A}B+C\overline{D}$ are **Boolean Expressions.**

$F= AB(C+D)+E$ is called **Boolean Equation**

Boolean Operator Precedence

- **The order of evaluation in a Boolean Expression is:**
  1. Parentheses (bracket)
  2. NOT
  3. AND
  4. OR
- **NOTE: Parentheses appear around OR expressions**
- **Example:** $F = A(B + C)(C + \overline{D})$

Examples of Boolean Expression

Example Determine the values of A, B, and C that make the sum term of the expression $\overline{A} + B + \overline{C} = 0$?

Solution Each literal must = 0; therefore $A = 1$, $B = 0$ and $C = 1$.

Example What are the values of the A, B and C if the product term of $A.\overline{B}.\overline{C} = 1$?

Solution Each literal must = 1; therefore $A = 1$, $B = 0$ and $C = 0$.

Basic Identities in Boolean Algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A + A = A$
4. $A + \bar{A} = 1$
5. $A \cdot 1 = A$
6. $A \cdot 0 = 0$
7. $A \cdot A = A$
8. $A \cdot \bar{A} = 0$
9. $\bar{\bar{A}} = A$

The Dual of an algebraic expression

- The Dual of an algebraic expression is obtained by **interchanging + and · and interchanging 0's and 1's.**
- **The identities appear in dual pairs.**

1. $A + 0 = A$	5. $A \,.\, 1 = A$	=>*self-dual*
2. $A + 1 = 1$	6. $A \,.\, 0 = 0$	
3. $A + A = A$	7. $A \,.\, A = A$	=>*self-dual*
4. $A + \bar{A} = 1$	8. $A \,.\, \bar{A} = 0$	

- **If the dual expression = the original expression, then we call them *self-dual*.**

Examples of the Dual Expression

Usually, the dual of an expression does not equal the expression itself.

Example 1 **:** $F = (A + \overline{C}) \cdot B + 0$

dual $F = (A \cdot \overline{C} + B) \cdot 1 = A \cdot \overline{C} + B$

Example 2 **:** $G = X \cdot Y + (\overline{W + Z})$

dual $G = ((X + Y) \cdot (\overline{W \cdot Z}))$

Example 3**:** $H = A \cdot B + A \cdot C + B \cdot C$

dual $H = (A + B)(A + C)(B + C).$

Useful Theorem in Boolean Algebra

1. **Absorption Theorem**

$$A + AB = A, \quad A + \overline{A}B = A + B$$

2. **Commutative Law**

$$A+B = B+A, \qquad AB = BA$$

3. **Associative Law**

$$A+(B+C) = (A+B)+C, \quad A(BC) = (AB)C$$

4. **Distributive Law**

$$A(B+C) = AB+AC, \; A+BC=(A+B)(A+C)$$

5. **DeMorgan Law**

$$\overline{AB} = \overline{A} + \overline{B} \qquad \overline{A+B} = \overline{A}\,\overline{B}$$

Application of DeMorgan Law

$$\overline{ABC} = \overline{A} + \overline{B} + \overline{C}$$

$$\overline{\overline{A}\,\overline{B}\,\overline{C}} = A + B + C$$

$$\overline{\overline{A + B\overline{C}} + D(\overline{E + \overline{F}})}$$

$$= (A + B\overline{C})\overline{D(\overline{E + \overline{F}})}$$

$$= (A + B\overline{C})(\overline{D} + E + \overline{F})$$

Boolean Algebraic Proof

- There are two ways to do the Proof for Boolean Identity:

1. Use Truth Table, draw out the truth tables for expressions at left and right side.

   If all terms of both truth tables are equal, then the expressions are identical.

2. Use Algebraic Manipulation, that is to use the identities and theorems of Boolean algebra to make two sides equal.

Expression to Truth Table

$$F1 = xy\bar{z}$$
$$F2 = x + \bar{y}z$$
$$F3 = \bar{x}\bar{y}\bar{z} + \bar{x}\,y\,z + x\bar{y}$$
$$F4 = x\bar{y} + \bar{x}\,z$$

x	y	z	F1	F2	F3	F4
0	0	0	0	0	1	0
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0

Proof Example of Using Truth Table

1. Demonstrate by means of truth table the validity of the identity: $A + \overline{A}B = A + B$

Proof:

Write the truth tables for two sides of the Expression.

A	B	$A + \overline{A}B$	$A + B$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

Proof Example of Using Truth Table

2. Demonstrate by means of truth table the validity of the identity: $\overline{A+B+C} = \overline{A}\,\overline{B}\,\overline{C}$

Proof: Write the truth tables for two sides of the Expression.

A	B	C	$\overline{A+B+C}$	$\overline{A}\,\overline{B}\,\overline{C}$
0	0	0	1	1
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

Proof Example of Using Algebraic Manipulation

1. **Prove the identity of the Boolean equations, using algebraic manipulation: $A + A \cdot B = A$**

Proof:

$$\mathrm{L} = A \cdot 1 + A \cdot B \qquad (\mathrm{X} = \mathrm{X} \cdot 1)$$

$$= A \cdot (1 + B) \qquad (\mathrm{X} \cdot \mathrm{Y} + \mathrm{X} \cdot \mathrm{Z} = \mathrm{X} \cdot (\mathrm{Y} + \mathrm{Z}))$$

$$= A \cdot 1 \qquad (1 + \mathrm{X} = 1)$$

$$= A \qquad (\mathrm{X} \cdot 1 = \mathrm{X})$$

$$= \mathrm{R}$$

Proof Example of Using Algebraic Manipulation

2. **Prove the identity of the Boolean equations, using algebraic manipulation:**

$$(A + B)(A + C) = A + BC$$

Proof:

$\mathbf{L} = AA + AC + AB + BC$	**(Distributive Law)**
$= A + AC + AB + BC$	**(X · X=1)**
$= A(1 + C + B) + BC$	**(Distributive Law)**
$= A \cdot 1 + BC$	**(X + 1 = 1)**
$= A + BC$	**(X ·1=X)**
$= \mathbf{R}$	

Proof Example of Using Algebraic Manipulation

3. **Prove the identity of the Boolean equations, using algebraic manipulation:**

$$(A\overline{B}(C+BD)+\overline{A}\,\overline{B})C=\overline{B}C$$

Proof:

$$(A\overline{B}(C+BD)+\overline{A}\,\overline{B})C$$

$$=(A\overline{B}C+\overline{A}\,\overline{B})C$$

$$=(AC+\overline{A})\overline{B}C$$

$$=(\overline{A}+C)\overline{B}C$$

$$=\overline{A}\,\overline{B}C+C\overline{B}C$$

$$=\overline{B}C(\overline{A}+1)$$

$$=\overline{B}C$$

Exercise 1

1. Demonstrate by means of truth tables the validity of the following identities:

   (a) $\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$ (b) $X + YZ = (X + Y)(X + Z)$

2. Prove the identity of each of the following Boolean equations, using algebraic manipulation:

   (a) $\overline{X}\,\overline{Y} + \overline{X}Y + XY = \overline{X} + Y$ (b) $\overline{A}B + \overline{B}\,\overline{C} + AB + \overline{B}C = 1$

3. Given that AB = 0 and A+B =1, use algebraic manipulation to prove that : $(A + C)(\overline{A} + B) = BC$

Do it manually and fill the answers in

EIE130_Chapter2Part1_Exercise1 via 電子作業 on

examcoo.com

Assignment 2

2-1, **2-2**, **2-4**

Due date:

D1:Next Wednesday

D2:Next Friday

During the class

Assignment 2

2-1. Demonstrate by means of truth tables the validity of the following identities:

(a) $\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$ (b) $X + YZ = (X + Y)(X + Z)$

(c) $\overline{X}Y + \overline{Y}Z + X\overline{Z} = X\overline{Y} + Y\overline{Z} + \overline{X}Z$

2-2. Prove the identity of each of the following Boolean equations, using algebraic manipulation:

(a) $\overline{X} \cdot \overline{Y} + \overline{X}Y + XY = \overline{X} + Y$ (b) $\overline{A}B + \overline{B} \cdot \overline{C} + AB + \overline{B}C = 1$

(c) $Y + \overline{X}Z + X\overline{Y} = X + Y + Z$ (d) $\overline{X} \cdot \overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X} \cdot \overline{Y} + XZ + Y\overline{Z}$

2-4 Given that AB = 0 and A+B =1, use algebraic manipulation to prove that $(A + C)(\overline{A} + B)(B + C) = BC$

Do it manually and fill the answers in EIE130_Chapter2Part1_Assignment2 via 班級考試 on examcoo.com

Expression Simplification

- **The reasons for Expression Simplification:**

1. Reduce the Judgement and Program Switching if using the software to implement the logic.
2. Reduce the number of Gates and Wiring when using the hardware to implement the logic

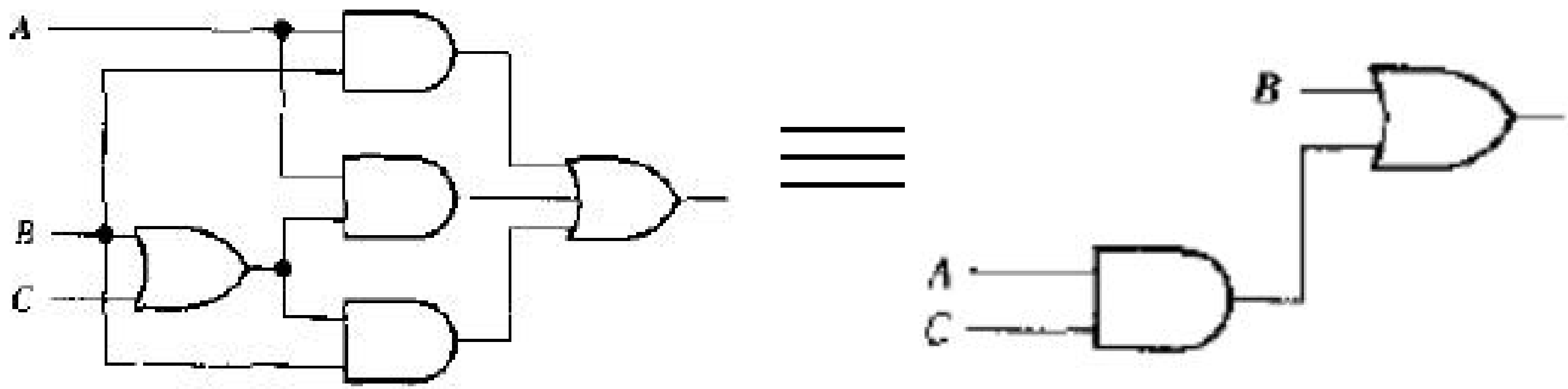
- **Methods of Simplification**

1. Using the Identities and Theorem in Boolean Algebra
2. Using the K-map

In this section, we only use the first method.

Expression Simplification

- AB+A(B+C)+B(B+C)=B+AC



Example of Expression Simplification

1. Simplify the Boolean expression to the expression containing a minimum number of <u>literals</u> (letter or variable)

$$A\,B + \overline{A}\,C\,D + \overline{A}\,B\,D + \overline{A}\,C\,\overline{D} + A\,B\,C\,D$$

$$= AB + ABCD + \overline{A}\,C\,D + \overline{A}\,C\,\overline{D} + \overline{A}\,B\,D$$

$$= AB + AB(CD) + \overline{A}\,C\,(D + \overline{D}) + \overline{A}\,B\,D$$

$$= AB + \overline{A}\,C + \overline{A}\,B\,D = B(A + \overline{A}\overline{D}) + \overline{A}C$$

$$= B\,(A + D) + \overline{A}\,C$$

5 literals

Example of Expression Simplification

2. Simplify the Boolean expression to the expression containing a minimum number of literals

$$AB+A(B+C)+B(B+C)$$

$$=AB+AB+AC+BB+BC$$

$$=AB+AC+B+BC$$

$$=B(A+1+C)+AC$$

$$=B+AC$$

3 literals

Complementing Functions

- Use DeMorgan's Theorem to complement a function:

$$\overline{AB} = \overline{A} + \overline{B} \qquad \overline{A + B} = \overline{A}\,\overline{B}$$

1. Interchange AND and OR operators
2. Complement each constant value and literal

Example of Complementing Functions

***Example 1*: Complement** $F = \bar{x}y\bar{z} + x\bar{y}\bar{z}$

$$\bar{F} = (x + \bar{y} + z)(\bar{x} + y + z)$$

***Example 2*: Complement** $G = (\bar{a} + bc)\bar{d} + e$

$$\bar{G} = \overline{(\bar{a} + bc)\bar{d} + e}$$

$$= \overline{(\bar{a} + bc)\bar{d}} \cdot \bar{e}$$

$$= (\overline{\bar{a} + bc} + d) \cdot \bar{e}$$

$$= (a \cdot \overline{bc} + d) \cdot \bar{e}$$

$$= (a \cdot (\bar{b} + \bar{c}) + d) \cdot \bar{e}$$

Exercise 2

1. Simplify the following Boolean expressions to expressions containing a minimum number of literals:
   (a) $\overline{A}\,\overline{C} + \overline{A}BC + \overline{B}C$ (b) $(\overline{A + B + C}) \cdot \overline{ABC}$
2. Using DeMorgan's theorem, express the function
   $$F = A\overline{B}C + \overline{A}\,\overline{C} + AB$$
   with only OR and complement operation

Do it manually and fill the answers in EIE130_Chapter2Part1_Exercise2 via 電子作業 on examcoo.com

Overview – Canonical Forms

- **What are Canonical Forms?**
- **Minterms and Maxterms**
- **Index Representation of Minterms and Maxterms**
- **Sum-of-Minterm (SOM) Representations**
- **Product-of-Maxterm (POM) Representations**
- **Representation of Complements of Functions**
- **Conversions between Representations**

Canonical Forms

- **It is useful to specify Boolean functions in a form that:**
  - **Allows comparison for equality.**
  - **Has a correspondence to the truth tables**
- **Canonical Forms in common usage:**
  - **Sum of Minterms (SOM), normally use.**
  - **Product of Maxterms (POM)**

Minterms

- **Minterms are AND terms with every variable present in either true or complemented form.**
- **Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n minterms for n variables.**
- **Example: Two variables (X and Y) produce 2 x 2 = 4 combinations:**

  XY (both normal)

  $X\overline{Y}$ (X normal, Y complemented)

  $\overline{X}Y$ (X complemented, Y normal)

  $\overline{X}\,\overline{Y}$ (both complemented)
- **Thus there are four minterms of two variables.**

Maxterms

- **<u>Maxterms</u> are OR terms with every variable in true or complemented form.**
- **Given that each binary variable may appear normal (e.g., x) or complemented (e.g., $\overline{x}$), there are 2^n maxterms for n variables.**
- **<u>Example:</u> Two variables (X and Y) produce $2 \times 2 = 4$ combinations:**

  $X + Y$ **(both normal)**

  $X + \overline{Y}$ **(x normal, y complemented)**

  $\overline{X} + Y$ **(x complemented, y normal)**

  $\overline{X} + \overline{Y}$ **(both complemented)**

Maxterms and Minterms

- **Examples: Two variable minterms and maxterms.**

Index	Minterm	Maxterm
0	$\overline{x}\,\overline{y}$	$x + y$
1	$\overline{x}\,y$	$x + \overline{y}$
2	$x\,\overline{y}$	$\overline{x} + y$
3	$x\,y$	$\overline{x} + \overline{y}$

- **The index above is important for describing which variables in the terms are true and which are complemented.**

Standard Order

- **All variables will be present in a minterm or maxterm and will be listed in the <u>same order</u> (usually alphabetically)**

- ***Example*: For variables a, b, c:**
  - **Maxterms: $(a + b + \bar{c})$, $(a + b + c)$**
  - **Terms: $(b + a + c)$, $a\,\bar{c}\,b$, and $(c + b + a)$ are NOT in standard order.**
  - **Minterms: $a\,b\,c$, $a\,\bar{b}\,c$, $a\,b\,\bar{c}$**
  - **Terms: $(\bar{a} + c)$, $\bar{b}\,c$, and $(a + \bar{b})$ do not contain all variables**

Purpose of the Index

- The index for the minterm or maxterm, expressed as a binary number, is used to determine whether the variable is shown in the true form or complemented form.
- For Minterms:
  - "1" means the variable is "Not Complemented" and
  - "0" means the variable is "Complemented".
- For Maxterms:
  - "0" means the variable is "Not Complemented" and
  - "1" means the variable is "Complemented".

Index Example in Three Variables

Example for three variables:

- **Assume the variables are called *X*, *Y*, and *Z*.**
- **The standard order is *X*, then *Y*, then *Z*.**
- **The Index 0 (base 10) = 000 (base 2) for three variables). All three variables are complemented for minterm 0 ($\overline{X},\overline{Y},\overline{Z}$) and no variables are complemented for Maxterm 0 (*X*,*Y*,*Z*).**
  - **Minterm 0, called m_0 is $\overline{X}\,\overline{Y}\,\overline{Z}$.**
  - **Maxterm 0, called M_0 is $(X+Y+Z)$.**
  - **Minterm 6 ?** $=> m_6 = m_{110} => (XY\overline{Z})$
  - **Maxterm 6 ?** $=> M_6 = M_{110} => (\overline{X}+\overline{Y}+Z)$

Index Examples in Four Variables

Index i	Binary Pattern	Minterm m_i	Maxterm M_i
0	0000	$\bar{a}\,\bar{b}\,\bar{c}\,\bar{d}$	$a + b + c + d$
1	0001	$\bar{a}\,\bar{b}\,\bar{c}\,d$	?
3	0011	?	$a + b + \bar{c} + \bar{d}$
5	0101	$\bar{a}\,b\,\bar{c}\,d$	$a + \bar{b} + c + \bar{d}$
7	0111	?	$a + \bar{b} + \bar{c} + \bar{d}$
10	1010	$a\,\bar{b}\,c\,\bar{d}$	$\bar{a} + b + \bar{c} + d$
13	1101	$a\,b\,\bar{c}\,d$	?
15	1111	$a\,b\,c\,d$	$\bar{a} + \bar{b} + \bar{c} + \bar{d}$

Minterm and Maxterm Relationship

- **Review:** **DeMorgan's Theorem**
  $\overline{xy} = \overline{x} + \overline{y}$ **and** $\overline{x + y} = \overline{x} \cdot \overline{y}$

- **Two-variable example (*x*,*y*) :**
  $M_2 = \overline{x} + y$ **and** $m_2 = x \cdot \overline{y}$

  $$\overline{m_2} = \overline{x \cdot \overline{y}} = \overline{x} + y = M_2$$

  Thus M_2 is the complement of m_2 and vice-versa.

- **Since DeMorgan's Theorem holds for *n* variables, the above holds for terms of *n* variables**
- **giving:**

  $$M_i = \overline{m_i} \text{ and } m_i = \overline{M_i}$$

  Thus M_i is the complement of m_i.

Function Tables for Both

- **Minterms of 2 variables**

x y	m_0	m_1	m_2	m_3
0 0	1	0	0	0
0 1	0	1	0	0
1 0	0	0	1	0
1 1	0	0	0	1

Maxterms of 2 variables

x y	M_0	M_1	M_2	M_3
0 0	0	1	1	1
0 1	1	0	1	1
1 0	1	1	0	1
1 1	1	1	1	0

Column Complemented

- **Each minterm has one and only one 1 present in the 2^n terms (a minimum of 1s). All other entries are 0.**
- **Each maxterm has one and only one 0 present in the 2^n terms All other entries are 1 (a maximum of 1s).**

All functions formed by two variables

x	y	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

F0=0

F1=m0=M1xM2xM3

F2=m1=M0xM2xM3

F3=m0+m1= M2xM3

F4=m2=M0xM1xM3

…….

F15=1

Observations

According to the function tables, we find that

- **We can implement any function by "ORing" the minterms corresponding to "1" entries in the function table. These are called the minterms of the function.**
- **We can implement any function by "ANDing" the maxterms corresponding to "0" entries in the function table. These are called the maxterms of the function.**
- **This gives us two canonical forms:**
  - **Sum of Minterms (SOM)**
  - **Product of Maxterms (POM)**

for stating any Boolean function.

Minterm Function Example

Given the truth table for F_1 below

We can prove that $F_1 = m_1 + m_4 + m_7$ by substituting all the 0 and 1 in it as in the right table!

x y z	F_1
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	0
1 1 0	0
1 1 1	1

$$F_1 = m_1 + m_4 + m_7$$
$$= \overline{x}\,\overline{y}\,z + x\,\overline{y}\,\overline{z} + x\,y\,z\ ?$$

x y z	index	m1	+	m4	+	m7	= F1
0 0 0	0	0	+	0	+	0	= 0
0 0 1	1	1	+	0	+	0	= 1
0 1 0	2	0	+	0	+	0	= 0
0 1 1	3	0	+	0	+	0	= 0
1 0 0	4	0	+	1	+	0	= 1
1 0 1	5	0	+	0	+	0	= 0
1 1 0	6	0	+	0	+	0	= 0
1 1 1	7	0	+	0	+	1	= 1

Minterm Function Example

Find F(A, B, C, D, E) = $m_2 + m_9 + m_{17} + m_{23}$

Solution:

F(A, B, C, D, E) =

$$\overline{A}\,\overline{B}\,\overline{C}D\overline{E} + \overline{A}B\overline{C}\,\overline{D}E + A\overline{B}\,\overline{C}\,\overline{D}E + A\overline{B}CDE$$

Maxterm Function Example

Given the truth table for F_1 below

We can prove that $F_1 = M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$

by substituting all the 0 and 1 in it as in the right table!

x y z	F_1
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	0
1 1 0	0
1 1 1	1

$$F_1 = M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$$
$$= (x+y+z)\cdot(x+\bar{y}+z)\cdot(x+\bar{y}+\bar{z})?$$
$$\cdot(\bar{x}+y+\bar{z})\cdot(\bar{x}+\bar{y}+z)$$

x y z	i	M0 ·	M2 ·	M3 ·	M5 ·	M6	= F1
0 0 0	0	0 ·	1 ·	1 ·	1 ·	1	= 0
0 0 1	1	1 ·	1 ·	1 ·	1 ·	1	= 1
0 1 0	2	1 ·	0 ·	1 ·	1 ·	1	= 0
0 1 1	3	1 ·	1 ·	0 ·	1 ·	1	= 0
1 0 0	4	1 ·	1 ·	1 ·	1 ·	1	= 1
1 0 1	5	1 ·	1 ·	1 ·	0 ·	1	= 0
1 1 0	6	1 ·	1 ·	1 ·	1 ·	0	= 0
1 1 1	7	1 ·	1 ·	1 ·	1 ·	1	= 1

Maxterm Function Example

- $\mathrm{F}(\mathrm{A},\mathrm{B},\mathrm{C},\mathrm{D}) = \mathrm{M}_3 \times \mathrm{M}_8 \times \mathrm{M}_{11} \times \mathrm{M}_{14}$

Solution:

F(A, B,C,D) =

$$(A+B+\overline{C}+\overline{D})\cdot(\overline{A}+B+C+D)\cdot(\overline{A}+B+\overline{C}+\overline{D})\cdot(\overline{A}+\overline{B}+\overline{C}+D)$$

Canonical Sum of Minterms

- **Any Boolean function can be expressed as a <u>Sum of Minterms (SOM)</u>.**
- **To change an expression to Canonical Sum of Minterms, we can expand it by "ANDing" any term missing a variable v with a term ($v + \overline{v}$).**
- ***Example*: Expand $f = x + \overline{x}\ \overline{y}$ as a sum of minterms.**

  First expand terms: $f = x(y + \overline{y}) + \overline{x}\ \overline{y}$

  Then distribute terms: $f = xy + x\overline{y} + \overline{x}\ \overline{y}$

  Express as sum of minterms: $f = m_3 + m_2 + m_0$

Canonical Product of Maxterms

- **Any Boolean Function can be expressed as a <u>Product of Maxterms (POM)</u>.**
- **To change an expression to Canonical Sum of Maxterms, we can expand it by "ORing" terms missing variable v with a term ($v \cdot \bar{v}$) and then applying the distributive law again.**
- ***Example*: Convert to product of maxterms:**

$$f(x, y, z) = x + \bar{x}\,\bar{y}$$

Apply the distributive law:

$$x + \bar{x}\,\bar{y} = (x + \bar{x})(x + \bar{y}) = 1 \cdot (x + \bar{y}) = x + \bar{y}$$

Add missing variable z:

$$x + \bar{y} + z \cdot \bar{z} = (x + \bar{y} + z)(x + \bar{y} + \bar{z})$$

Express as POM: $f = M_2 \cdot M_3$

Shorthand SOM & POM Form

- To simplify the writing of the SOM and POM, we can use the Shorthand Form of them.
- Shorthand SOM Form

$$F(X_1,\ldots,X_n)=\sum\nolimits_m(a_{1,}\ldots,a_n)$$

If F = $m_1+m_4+m_5+m_6+m_7$

Then $F(A,B,C)=\Sigma m(1,4,5,6,7)$

- Shorthand POM Form

$$F(X_1,\ldots,X_n)=\prod\nolimits_M(a_1,\ldots,a_n)$$

If F = $M_2M_3M_5M_7$

Then $F(x,y,z)=\Pi_M(2,3,5,7)$

Function Complements

- **The complement of a function expressed as a sum of minterms is constructed by selecting the minterms missing in the sum-of-minterms canonical forms.**
- **Alternatively, the complement of a function expressed by a Sum of Minterms form is simply the Product of Maxterms with the same indices.**
- ***Example*: Given** $F(x, y, z) = \Sigma_m(1,3,5,7)$

$$\overline{F}(x, y, z) = \Sigma_m(0,2,4,6)$$

$$\overline{F}(x, y, z) = \Pi_M(1,3,5,7)$$

Conversion Between Forms

- To **convert** between **sum-of-minterms and product-of-maxterms form** (or vice-versa) we follow these steps:
  - **Find the function complement** by swapping terms in the list with terms not in the list.
  - **Change from products to sums, or vice versa.**
- *Example*: Given F as before: $F(x, y, z) = \Sigma_m(1,3,5,7)$
- Form the Complement: $\overline{F}(x, y, z) = \Sigma_m(0,2,4,6)$
- Then use the other form with the same indices – this forms the complement again, giving the other form of the original function: $F(x, y, z) = \Pi_M(0,2,4,6)$

Standard Forms

- **Standard Sum-of-Products (SOP) form:** equations are written as an **OR of AND terms**
- **Standard Product-of-Sums (POS) form:** equations are written as an **AND of OR terms**
- **Examples:**
  - SOP: $A\,B\,C + \overline{A}\,\overline{B}\,C + B$
  - POS: $(A+B)\cdot(A+\overline{B}+\overline{C})\cdot C$
- **These "mixed" forms are neither SOP nor POS**
  - $(A\,B + C)\,(A + C)$
  - $A\,B\,\overline{C} + A\,C\,(A + B)$

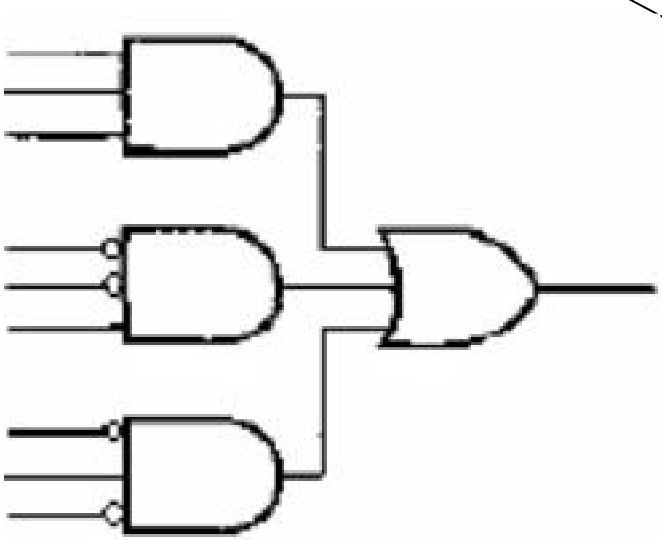
Standard Sum-of-Products (SOP)

- **A sum of minterms form for *n* variables can be written down directly from a truth table.**
  - **Implementation of this form is a two-level network of gates such that:**
  - **The first level consists of *n*-input AND gates, and**
  - **The second level is a single OR gate (with fewer than 2^n inputs).**
- **This form often can be simplified so that the corresponding circuit is simpler.**

Truth Table to Standard SOP(SOM)

- Truth Table → Standard SOP(SOM)

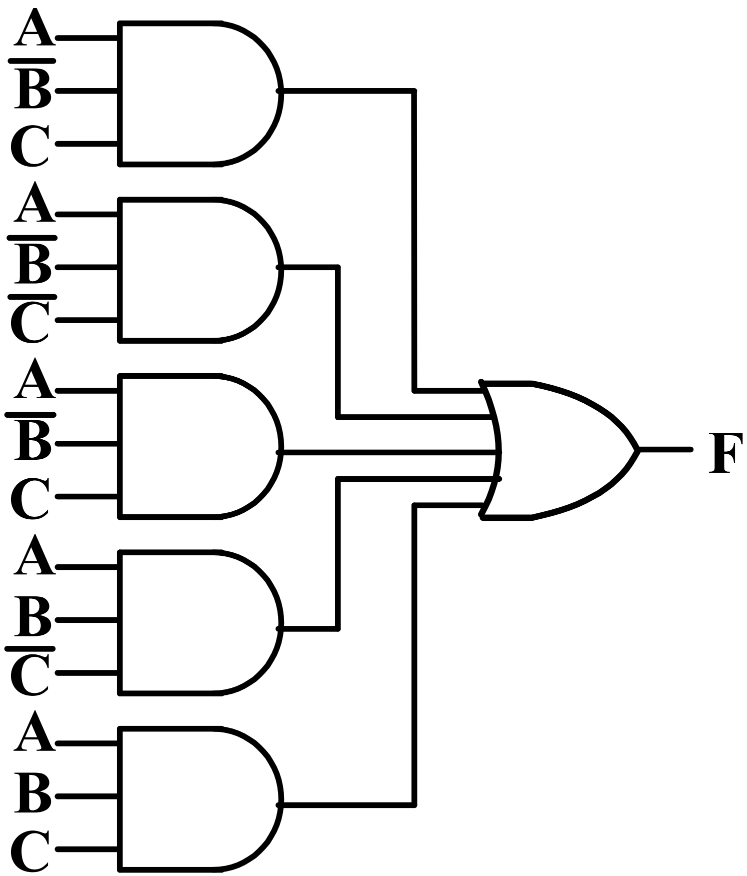
$$ABC + \overline{A}\,\overline{B}C + \overline{A}B\overline{C}$$

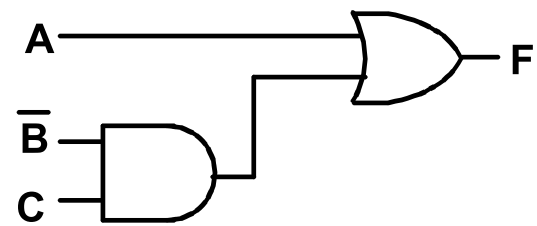


Input			Output
A	B	C	
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

AND/OR Two-level Implementation of SOP Expression

- The two level implementations for F are shown below – it is quite apparent which is simpler!





Exercise 3

1. Expand $\mathbf{F = A + \overline{B}\,C}$ as a Sum of Minterms
2. Expand $\mathbf{f(A, B, C) = A\,\overline{C} + B\,C + \overline{A}\,\overline{B}}$ as a Product of Maxterms
3. Simplify the expression given by

$$\mathbf{F(A, B, C) = \Sigma m(1,4,5,6,7)}$$

Do it manually and fill the answers in EIE130_Chapter2Part1_Exercise3 via 電子作業 on examcoo.com

Assignment 3

2-6 Simplify the following Boolean expressions to expressions containing a minimum number of literals:

(a) $\overline{A}\cdot\overline{C}+\overline{A}BC+\overline{B}C$ (b) $(\overline{A+B+C})\cdot\overline{ABC}$ (c) $AB\overline{C}+AC$

(d) $\overline{A}\cdot\overline{B}D+\overline{A}\cdot\overline{C}D+BD$ (e) $(\overline{\overline{A}+B})(\overline{\overline{A}+\overline{C}})(\overline{A\overline{B}C})$

2-8 Using DeMorgan's theorem, express the function $F=A\overline{B}C+\overline{A}\cdot\overline{C}+AB$

(a) with only OR and complement operations.

(b) with only AND and complement operations.

2-9 Find the complement of the following expressions:

(a) $A\overline{B}+\overline{A}B$ (b) $(\overline{V}W+X)Y+\overline{Z}$

(c) $WX(\overline{Y}Z+Y\overline{Z})+\overline{W}\cdot\overline{X}(\overline{Y}+Z)(Y+\overline{Z})$ (d) $(A+\overline{B}+C)(\overline{A}\cdot\overline{B}+C)(A+\overline{B}\cdot\overline{C})$

Assignment 3

2-11 For the Boolean functions E and F, as given in the following truth table:

(a) List the minterms and maxterms of each functions.

(b) List the minterms of $\overline{E}$ and $\overline{F}$

(c) List the minterms of $E+F$ and EF

(d) Express E and F in sum-of-minterms algebraic form.

(e) Simplify E and F to expressions with a minimum of literals.

X	Y	Z		E	F
0	0	0		0	1
0	0	1		1	0
0	1	0		1	1
0	1	1		0	0
1	0	0		1	1
1	0	1		0	0
1	1	0		1	0
1	1	1		0	1

Due date:

D1:Next Wednesday D2:Next Friday

Do it manually and fill the answers in EIE130_Chapter2Part1_Assignment3 via 班級考試 on examcoo.com