Seminar on Algebra, Geometry and Discrete Mathematics

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1 Proof of asymptotic behaviour of eigenvalues in families of expanders

Remark 1.1. Unless otherwise specified, all rings we discuss will be commutative with unit.

Definition 1.2. Let $f : A \to B$ be a ring homomorphism, and let $J \subset B$ be an ideal. Then, the *contraction* of J to A is $J^c := \{a \in A \mid f(a) \in J\}$.

Proposition 1.3. *In the above situation, J^c is an ideal of A.*

Proof. It is an additive subgroup of A as f is an additive group homomorphism. Furthermore, if $a \in J^c$ and $r \in A$, then $f(a) \in J$ so $f(ra) = f(r)f(a) \in J$ as J is an ideal. $\square$