

Figure 1: Example of the  $\epsilon$ -excellence property not being monotonic. On the left, a bipartite graph with two independent sets A and B. A simple exhaustive check shows that A is  $\frac{1}{5}$ -excellent. On the other hand, raising the value of  $\epsilon$  up to  $\frac{2}{5}$  introduces a new  $\frac{2}{5}$ -good set B witnessing that A is not  $\frac{2}{5}$ -excellent, as half of the vertices of A have one truth value, and half the other. On the right is the corresponding bi-adjacency matrix.

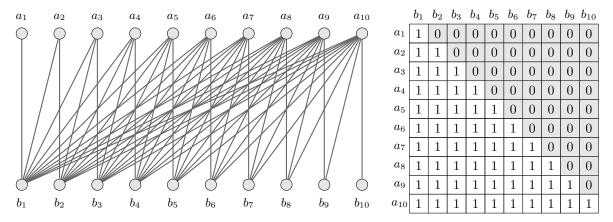


Figure 2: On the left, a half-graph with  $2 \times 10$  vertices. On the right, the corresponding bi-adjacency matrix.

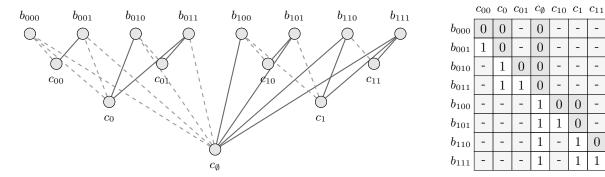


Figure 3: On the left, example of a 3-tree. Solid lines show adjacent vertices, and dashed lines show non-adjacent vertices. Pairs of vertices without a line may or may not be connected. In particular, notice that connections between disjoint sub-trees are not defined, and may be edges or non-edges in any combination (e.g. the pair  $(c_1, c_{01})$ ). On the right, the corresponding bi-adjacency matrix.

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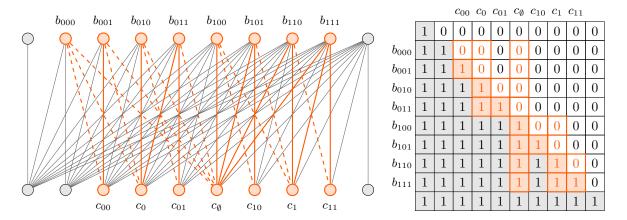
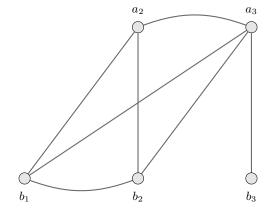


Figure 4: On the left, example of a 3-tree in a half-graph with  $2 \times 10$  vertices. Orange lines and nodes highlight the 3-tree structure, with dashed orange lines remarking the relevant non-edges. On the right is the corresponding bi-adjacency matrix. Again, orange cells highlight edges relative to the 3-tree structure.



	$a_2$	$a_3$	$b_1$	$b_2$	$b_3$
$a_2$	0	1	1	1	0
$a_3$	1	0	1	1	1
$b_1$	1	1	0	1	0
$b_2$	1	1	1	0	0
$b_3$	0	1	0	0	0

Figure 5: On the left, an example of a graph smaller than a  $3 \times 3$  half-graph thatfor which no induced copies can be found in a 3-stable graph. It is basically a  $3 \times 3$  half-graph in which  $a_1$  and  $b_2$  are the same vertex, and an edge is added between  $a_2$  and  $a_3$ . On the right, the corresponding adjacency matrix. Orange cells highlight edges relative to the 3-order structure