

Figure 1: Example of the  $\epsilon$ -excellence property not being monotonic. On the left, a bipartite graph with two independent sets A and B. A simple exhaustive check shows that A is  $\frac{1}{5}$ -excellent. On the other hand, raising the  $\epsilon$ -value up to  $\frac{2}{5}$  introduces a new  $\frac{2}{5}$ -good set B witnessing that A is not excellent, as half of the vertices of A have one truth value, and half the other. On the right is the corresponding bi-adjacency matrix.

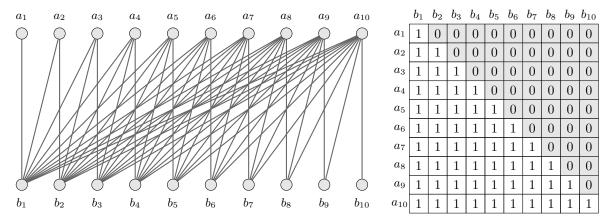
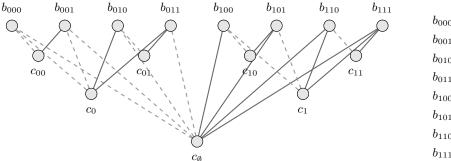


Figure 2: On the left, a half-graph with  $2 \times 10$  vertices. On the right, the corresponding bi-adjacency matrix.



|           | $c_{00}$ | $c_0$ | $c_{01}$ | $c_{\emptyset}$ | $c_{10}$ | $c_1$ | $c_{11}$ |
|-----------|----------|-------|----------|-----------------|----------|-------|----------|
| $b_{000}$ | 0        | 0     | -        | 0               | -        | -     | -        |
| $b_{001}$ | 1        | 0     | -        | 0               | -        | -     | -        |
| $b_{010}$ | -        | 1     | 0        | 0               | -        | -     | -        |
| $b_{011}$ | 1        | 1     | 1        | 0               | 1        | -     | -        |
| $b_{100}$ | 1        | -     | -        | 1               | 0        | 0     | -        |
| $b_{101}$ | -        | -     | -        | 1               | 1        | 0     | -        |
| $b_{110}$ | -        | -     | -        | 1               | -        | 1     | 0        |
| $b_{111}$ | -        | -     | -        | 1               | -        | 1     | 1        |

Figure 3: On the left, example of a 3-tree. Solid lines show adjacent vertices, and dashed lines show non-adjacent vertices. Pairs of vertices without a line may or may not be connected. In particular, notice that connections between disjoint sub-trees are not defined, and may be edges or non-edges in any combination. On the right, the corresponding bi-adjacency matrix.

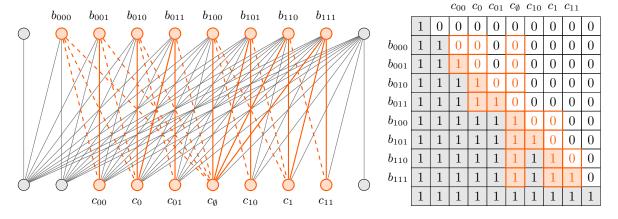


Figure 4: On the left, example of a 3-tree in a half-graph with  $2 \times 10$  vertices. Orange lines and nodes highlight the 3-tree structure, with dashed orange lines remarking the relevant non-edges. On the right is the corresponding bi-adjacency matrix. Again, orange cells highlight edges relative to the 3-tree structure.