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HIGH RESOLUTION GUIDED WAVE TOMOGRAPHY

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ABSTRACT. Guided wave tomography uses the dispersive nature of Lamb waves to enable a map of thickness to be obtained from a map of wave velocity. Iterative HARBUT, the Hybrid Algorithm for Robust Breast Ultrasound Tomography has been introduced for this problem, and here it is demonstrated to accurately converge in the presence of artificial noise with a signal to noise ratio of 6dB. The method is shown to be robust with experimental data too. The issue of the more realistic array configuration of a pair of parallel ultrasonic arrays is also considered. VISCIT (the Virtual Image Space Component Iterative Technique) was introduced to solve this, leading to images with significantly reduced artefacts and more accurate contrast reconstructions.

Keywords: Guided Wave Tomography, Inversion, Limited View
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INTRODUCTION

Corrosion is a significant problem in the petrochemical industry. Large, often inaccessible areas must be inspected to establish the extent of the damage caused. This is slow and difficult using conventional thickness-gauging methods which can only measure thickness values directly beneath the probe.

Guided wave tomography has been proposed as a solution to accurately estimate the remaining wall thicknesses of corrosion patches in plates and large diameter pipes. By utilising a guided wave mode which is dispersive, if frequency is fixed then the wave velocity will become a function of thickness. One approach to guided wave tomography is therefore to reconstruct the velocity map from a series of ultrasonic measurements, then convert this back to thickness[1].

To establish the minimum wall thickness, an accurate, reliable velocity reconstruction method of sufficient resolution must be used. This paper utilises the HARBUT method[2] which initially uses bent-ray tomography (BRT) to estimate a low-resolution map of velocity, then uses this as a background when performing a higher-resolution diffraction tomography

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(DT) reconstruction, allowing the finer details of the scatterer to be reconstructed. Reference [3] explored the potential of HARBUT for guided wave tomography, developing iterative HARBUT. Iterative HARBUT recognises that further improvements are possible by repeating the DT stage on the latest reconstruction, particularly if the image has not been well captured by a single DT stage. This is a possibility for guided wave tomography where the defects of interest are typically small yet deep (and hence high contrast), making both BRT and DT perform poorly.

One significant challenge with using iterative algorithms in practice is the tendancy to overfit errors in the data. The previous study of [3] only considered clean, numerical data; here we test the algorithm's robustness in the presence of noise by adding random errors to the numerical data. This is also verified through an experimental data set.

A circular, full view configuration is used for the initial study. While this makes imaging more straightforward, it is rarely practical for guided wave tomography. It is more convenient to place a pair of rings of transducers, one on each side of the region of interest, and take measurements between them. Unwrapping this into a flat plate leads to the problem becoming a reconstruction between a parallel pair of linear arrays.

We therefore investigate the parallel array problem, which is a limited view problem compared to the full view circular array. We test the standard reconstruction methods, and compare them to a new method, VISCIT (the Virtual Image Space Component Iterative Technique) which compensates for the missing information from a limited view configuration by a thresholding regularisation method [4].

THEORY

This section outlines the scattering theory and from that, how reconstructions are performed with HARBUT.

We consider the acoustic wave equation

$$\rho(\mathbf{r})\nabla.\left[\frac{1}{\rho(\mathbf{r})}\nabla p(\mathbf{r})\right]-\frac{1}{c(\mathbf{r})^2}\frac{\partial^2 p(\mathbf{r})}{\partial t^2}=0 \tag{1}$$

where $p(\mathbf{r})$ is the pressure at point $\mathbf{r}$, $\rho(\mathbf{r})$ is the density, and $c(\mathbf{r})$ is the sound speed. We simplify the stress models of guided wave tomography to the acoustic problem. This is valid within our assumption of no mode conversion when the Lamb waves interact with the features. By converting the equation to the temporal frequency domain, it can be rewritten as

$$\left(\nabla^2+k_w^2\right)\psi=-O\psi \tag{2}$$

where ψ is the scalar potential of the field (equal to the Fourier transform of the pressure) and $k_w=2\pi f/c_w$ is the wavenumber of the background. $O(\mathbf{r})$ is the object function, and (ignoring density variations) is related to sound speed by

$$O(\mathbf{r})=k_w^2\left[\left(\frac{c_w}{c(\mathbf{r})}\right)^2-1\right]. \tag{3}$$

The aim is to reconstruct this object function.

Under the Born approximation it can be shown that each send-receive measurement from a far-field scattering experiment corresponds to a single component of the 2D Fourier transform of the object function [5]

$$\psi_s(r\hat{\mathbf{s}},r\hat{\mathbf{s}}_\mathbf{0})=\Pi\frac{\exp(ik_w r)}{\sqrt{r}}\tilde{O}\left[k_w(\hat{\mathbf{s}}-\hat{\mathbf{s}}_\mathbf{0})\right] \tag{4}$$

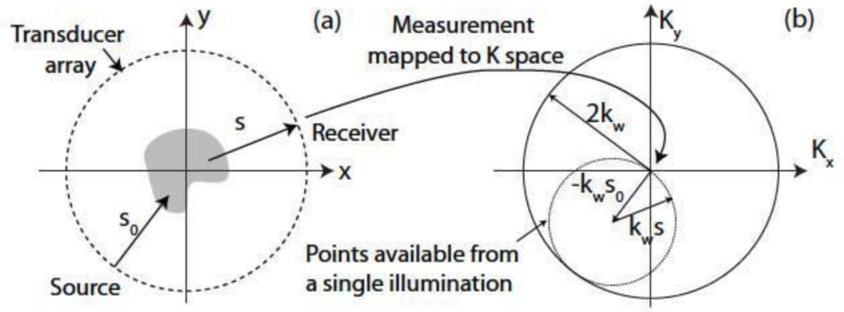

FIGURE 1. Mapping under the Born approximation of each measurement from the transducer array to a point in **K** space. The combination of all available source and receiver directions enables all points in the disk of radius $2k_w$ to be determined.

where $\psi_s(r\hat{\mathbf{s}}, r\hat{\mathbf{s}}_0)$ is the scattering measurement taken at a large radius r in direction $\hat{\mathbf{s}}$, for a plane wave illumination in direction $\hat{\mathbf{s}}_0$. Π is a constant defined as $\Pi = \exp(i\pi/4)/\sqrt{8\pi k_w}$, and $\tilde{O}$ is the 2D Fourier transformed object function. As illustrated in Fig. 1, by collecting measurements from all directions, it is possible to completely fill a disk of radius $2k_w$ in the spatial frequency domain, which allows a low-pass filtered version of the object to be reconstructed.

The Born approximation restricts the objects that can be imaged to small, low contrast scatterers, such that the phase shift through the object relative to the background is less than π. This leads to the limitation [5]

$$an_\delta < \frac{\lambda}{4} \tag{5}$$

where a is the radius of the feature, n_δ is the contrast and λ is the wavelength. The features we wish to image rarely fulfil this criterion, so we consider an alternative approach.

In [2] we introduced HARBUT, an algorithm for breast ultrasound tomography. This method modifies standard diffraction tomography to work with an inhomogeneous background. By choosing a background which is sufficiently accurate, the remaining perturbation will be small enough that the Born approximation is valid. The paper showed that for the breast problem, the lower resolution bent-ray method provided a sufficiently accurate background. The HARBUT algorithm consists of three stages.

The first stage of HARBUT is to perform a ray-based velocity reconstruction using the arrival times of the waves. In contrast to the straight ray methods commonly used [6], we use an iterative bent-ray method. This accounts for the refraction of the waves as they pass through the feature, allowing for more accurate reconstructions.

The concept of the algorithm can be outlined as:

1. Get initial estimate of sound speed (typically set everything to a constant background value)

2. Run forward eikonal (ray-based) solver to calculate arrival times for current model for each send-receive pair in the array

3. Calculate errors in each modelled arrival time compared to the target arrival time from experimental measurements

4. Account for these errors by projecting them back along their respective rays

5. Repeat from 2 until satisfactory convergence is achieved.

Full details of the algorithm used are included in [7].

The second stage is to perform the beamforming algorithm, correcting for the ray tomography background. The corrected beamforming image is generated by [2]

$$I(\mathbf{z}) = \int_{\mathbf{S}}\int_{\mathbf{S}} \frac{\psi(\mathbf{x},\mathbf{y})}{G_b(\mathbf{x},\mathbf{z})G_b(\mathbf{z},\mathbf{y})} d\mathbf{x}d\mathbf{y} \tag{6}$$

where $\psi(\mathbf{x},\mathbf{y})$ is the field measured at source position $\mathbf{x}$ and receiver position $\mathbf{y}$ and the Green's function $G_b(\mathbf{x}',\mathbf{y}) = G_0(\mathbf{x}',\mathbf{y})\,e^{i\omega\Delta t}$, with G_0 being the free space Green's function, ω being the angular frequency and Δt the relative time correction component for the background field, as calculated by the eikonal solver.

The final stage is to convert the beamforming image to a diffraction tomography image by using the filter (via the spatial frequency domain) [8]

$$F(\mathbf{K}) = \frac{k_w |\mathbf{K}| \sqrt{1 - |\mathbf{K}|/4k_w^2}}{8\pi^2\Pi} \Lambda(\mathbf{K}) \tag{7}$$

where $\mathbf{K}$ is the spatial frequency and

$$\Lambda(\mathbf{K}) = \begin{cases} 1, & |\mathbf{K}| \leq 2k_w \\ 0, & |\mathbf{K}| > 2k_w \end{cases}.$$

This filtered image gives the object function perturbation, which is combined with the background image to give a full sound speed map. We now apply HARBUT to guided wave tomography.

ITERATIVE HARBUT FOR GUIDED WAVE TOMOGRAPHY

Guided wave tomography has been performed using ray-based methods [6] which are good for large scale objects, and Born approximation type methods [9] which are most suitable for small scale objects. In this section, we test HARBUT [2], which combines the complementary strengths of these algorithms together.

Lamb waves are guided waves travelling in a plate of finite thickness. They have known dispersion relationships, where both phase and group velocities vary with the product of frequency and thickness. Figure 2(a) shows the dispersion curves for an aluminium plate. From this, we can see that any thinning of the plate can be identified and quantified by determining the velocity variation.

Transducers can be placed around the feature to transmit and receive waves through the plate. By using a circular configuration of transducers, a full matrix of scattering data (i.e. one measurement for each combination of sending and receiving transducers) can be taken. The goal is then to build up a map of sound speed from these measurements, which can be used to calculate the thickness.

A numerical model was performed to test HARBUT, as shown in Fig. 2(b). The model was run using Abaqus Explicit, using 3D stress elements. The material was aluminium with nominal thickness of 10mm and a region of the plate was thinned to match the thickness map in Fig. 3(a). This is a slowly varying region of corrosion, which has a minimum thickness of 5mm. 128 transducers were used in the array, with a 5 cycle Hann-windowed 50kHz toneburst as the input signal, exciting the A0 mode. The array was 800mm in diameter.

The arrival times were extracted from the data by enveloping each signal using a Hilbert transform, then taking the arrival time as the point at which the signal exceeded a threshold

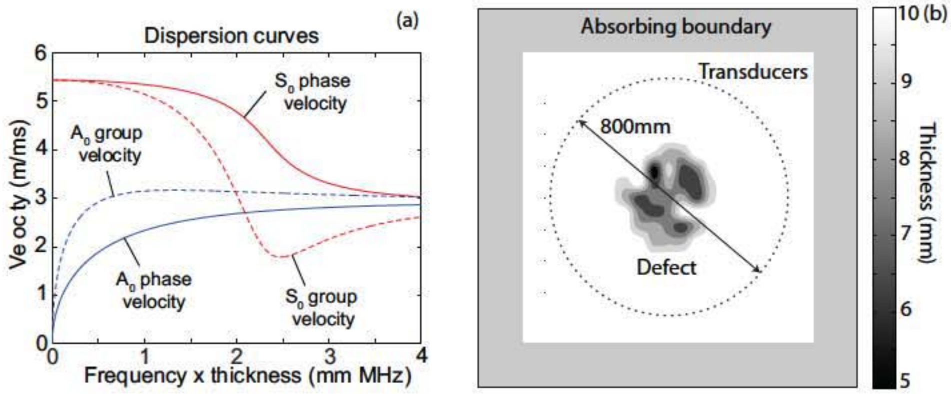

FIGURE 2. Guided wave tomography. (a) shows the dispersion curves for Lamb waves in an aluminium plate. For a fixed frequency, as the thickness varies so does the velocity. Therefore reconstructing velocity allows the thickness to be determined. (b) gives the 3D stress model run using Abaqus.

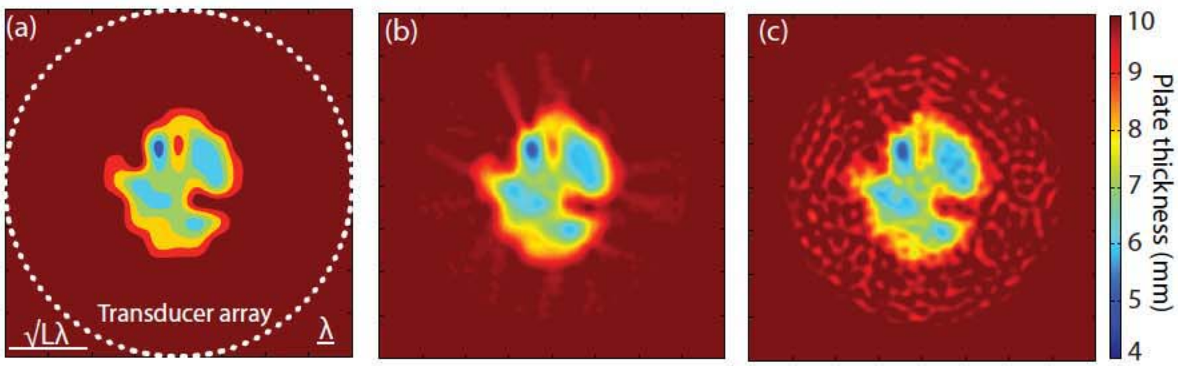

FIGURE 3. Reconstructions of thickness using HARBUT. The wavelength, λ is 37mm. (a) is the original thickness map. (b) gives the standard iterative HARBUT reconstruction, as presented in [3]. This is generated directly from numerical data and required 8 iterations. (c) presents the same image as (b), except that here, noise has been added to the frequency domain data prior to the iterations beginning. While this adds speckle to the image it does not affect convergence and there is little evidence of overfitting. The minimum thickness can be estimated reasonably accurately in either reconstruction.

of 10% of its peak. The frequency domain data was obtained by performing a Fast Fourier Transform (FFT) on the data and taking the component closest to the 50kHz input signal. This data was then imaged as shown in Fig. 3.

Figure 3(a) presents the original thickness map, which was used to generate the Abaqus model. In Fig. 3(b) we present the iterative HARBUT reconstruction, which iteratively applies DT until the sound speed had stopped changing and was taken to have converged – in this case eight iterations were required. The image shows almost no artefacts and very accurately captures all the features of the original thickness map.

Figure 3(c) presents the same as (b), except now random noise has been added to the data prior to performing the iterations. This noise has a normally distributed amplitude with RMS 50% that of the original matrix of send-receive data, and phase uniformly distributed from 0 to 2π. This corresponds to a very poor signal to noise ratio of 6dB, confirming that the algorithm still converges and avoids overfitting even with these significant errors.

Figure 4 presents the results from an experimental data set. A pair of simple flat bottomed holes were machined onto the 10mm aluminium plate. The remaining thicknesses are 5mm

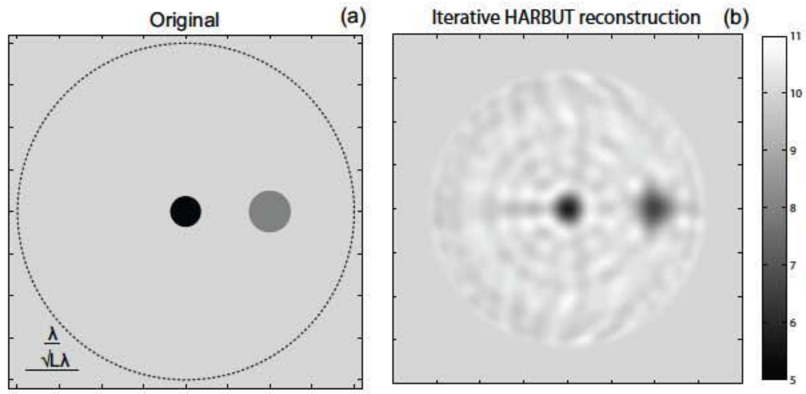

FIGURE 4. Experimental thickness reconstructions with 50kHz A_0 Lamb waves. The wavelength, λ is 37mm. (a) is the original thickness map. (b) gives the iterative HARBUT reconstruction, confirming that iterating in this way does not cause significant overfitting of experimental errors.

and 7mm and sizes are 60mm and 100mm respectively as shown in Fig. 4(a); a full matrix of data was then obtained from a circular array of A_0 transducers at a diameter of 400mm. While there are small artefacts present in the reconstruction in Fig. 4(b), the image has converged and there is no evidence of the algorithm attempting to overfit the experimental errors.

Having demonstrated the suitability of HARBUT for full-view guided wave tomography, we now consider a more realistic limited view configuration with a pair of parallel linear transducer arrays.

LIMITED VIEW GUIDED WAVE TOMOGRAPHY

Guided wave tomography is potentially most useful when direct access to the plate surface is limited, such as beneath a support. This problem is particularly important since water can collect around the support, accelerating corrosion damage in this region. Although in the previous section, circular transducer arrays were used to simplify the analysis, these are rarely practical for imaging beneath a support. Instead, the preferred configuration is a pair of ring arrays either side of the support, stretching around the circumference of the pipe.

As before, we simplify the problem by unwrapping the pipe to a flat plate, and consider the problem as reconstructing the thickness map between a pair of parallel arrays. This is a limited view problem since there is a range of angles where we do not have any transducers and hence cannot provide illuminations or measure the wavefield. The missing angles reduce as the distance between the parallel arrays reduces, or if the length of the arrays increases.

As shown by Volker and Bloom [10], it is possible to extend the effective lengths of the arrays by considering helical paths travelling around the pipe, however this depends on the ability to separate out each individual component wavepacket, which is likely to be challenging in practice. For our analysis we therefore consider a fixed array length.

One approach to dealing with the limited view problem is to simply set the unknown values to zero and generate a standard full view image with this data. This has a filtering effect on the image, introducing significant artefacts and reducing the reconstructed contrast.

In [4] we presented a solution to this problem, VISCIT (the Virtual Image Space Component Iterative Technique). Better values were estimated for the missing components. These were determined by applying a threshold to the limited view reconstruction then using that to populate the unknown components. This process was iterated until convergence.

In Fig. 5 we compare a pair of reconstructions from limited view data. The problem con-

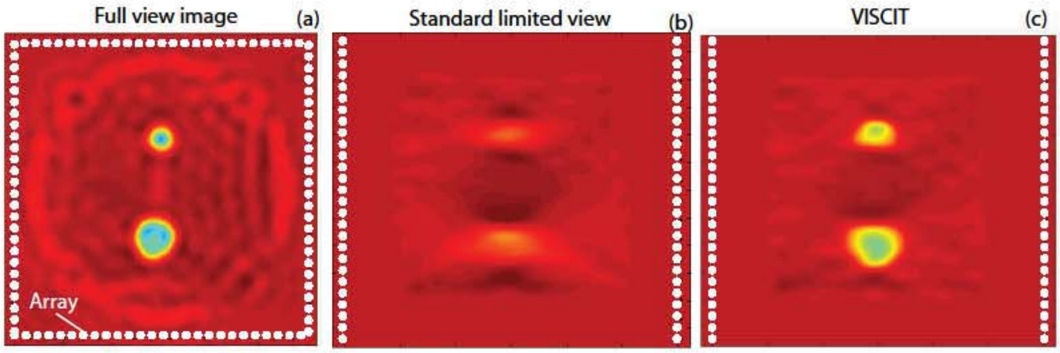

FIGURE 5. Reconstructions of thickness for a plate with two idealised defects (flat-bottomed holes) machined into its surface. The defects have 5mm and 7mm remaining wall thickness and are 60mm and 100mm in diameter respectively. The data is obtained using S_0 Lamb waves at 150kHz. (a) shows the full view reconstruction from a square array. The data from the top and bottom arrays is removed for the limited view reconstructions in (b) and (c). (b) gives the standard reconstruction using a full view method. (c) uses the VISCIT method to significantly improve the reconstruction.

sists of two idealised defects (flat-bottomed holes) of remaining thicknesses 5mm and 7mm and diameters 60mm and 100mm respectively; Fig. 5(a) shows the full view reconstruction using a square array. For the limited view images we remove the top and bottom arrays and use only the components transmitted from left to right (by reciprocity there is no additional information encoded by waves travelling the opposite direction).

Figure 5(b) presents the reconstruction which is obtained with a standard reconstruction algorithm, which does not account for the missing information. It is clear that the reconstructed contrast is reduced and there are significant artefacts in the image. VISCIT, shown in Fig. 5(c), presents a significant improvement, allowing much better estimation of the depths of the defects over the standard method in Fig. 5(b).

To verify the performance of VISCIT we have tested several different separation distances – in effect varying the range of available viewing angles. Figure 6 presents the results. It is clear that VISCIT performs well at a wide range of separation distances. Even with the largest separation distance we only underestimate the wall loss by 1mm.

CONCLUSIONS

The iterative HARBUT method has been applied to a simulated data set with significant levels of noise added (corresponding to a 6dB signal to noise ratio), confirming that the method is robust, avoiding overfitting errors in the reconstruction and not affecting convergence. It also performs well with experimental data, with little effect from experimental uncertainties.

Using a more realistic configuration of a pair of parallel arrays presents challenges to traditional algorithms, because information necessary to accurately reconstruct thickness maps is unavailable from the limited view arrays. We have introduced VISCIT (the Virtual Image Space Component Iterative Technique) which obtains estimates for the missing information through a threshold-based regularisation method. VISCIT was shown to perform well across a range of problems, minimising the artefacts and contrast loss associated with the limited view problem.

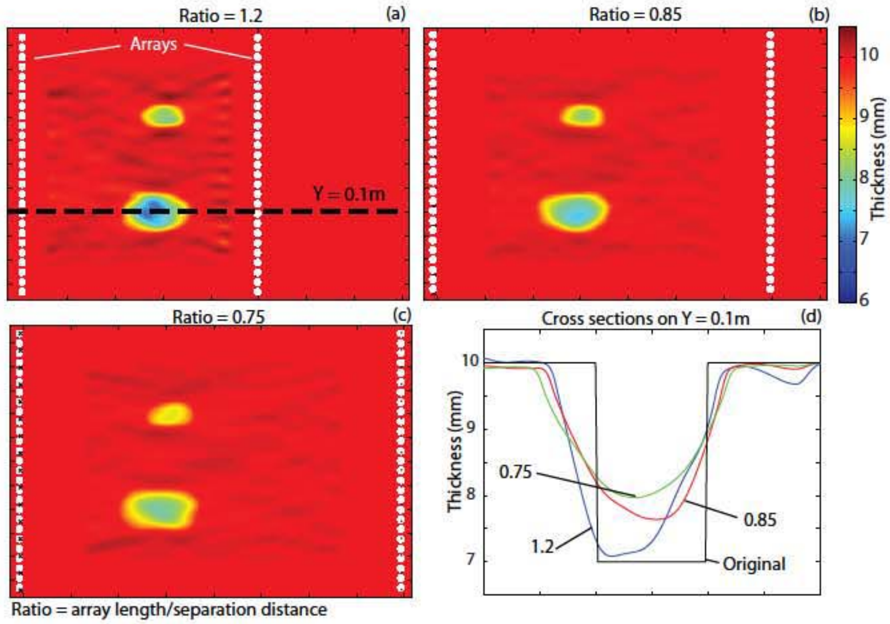

FIGURE 6. Thickness reconstructions for different array separation distances. The ratios given are the array length divided by the separation distance. (a) - (c) present the reconstructions for several different separation distances. (d) gives the cross sections along the line marked in (a). It is clear that as the ratio reduces, the reconstructed wall loss reduces and the inclusions extend in the horizontal direction.

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