Projet de Fin d’Etude / Master Thesis

Semaine 4

Efficient Measurement Planning for Remote Gas Sensing with Mobile Robots,
Muhammad Asif Arain, Marcello Cirillo, Victor Hernandez Bennetts,
Erik Schaffernicht, Marco Trincavelli and Achim J. Lilienthal, Jun. 2015

- Convex relaxation: problem becomes convex when applying relaxation (dropping constraints)
- Purposes: formalize the remote gas sensing coverage problem for mobile robotics, quickly find an exploration plan that guarantees a complete coverage of the environment
- Map on the environment discretized in set A of n cells. O in A = cells of obstacles (beam blocked), S in A = cells traversable
- Solution of problem = tour within the map, obstacle free, + set of sensing actions along the tour that senses evrey cell in S.
- Pose p_j of robot, a_j in S, theta_j in Theta
- Theta = finite set of allowed orientations, equally spaced between 0 and 2pi.
- Robot position within cell is center of the cell.
- Movement of the rebot are forward motion and rotations.

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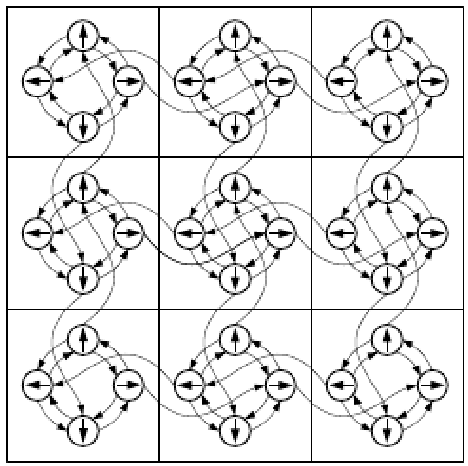

Fig. 3. The graph captures the allowed movements of the robot on a grid map when $\Theta = \{0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi\}$ and only forward movements are allowed. Small circles indicate the poses where the robot can stop (the internal arrow indicates the orientation of the robot) and the directed edges its allowed movements. Note that in the figure the poses do not correspond to the centers of the cells, but this is only for clarity reasons.

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- Time is used as measure of cost
- t^c = time to move to adjacent cell
- t^r = time to rotate
- t^m_{p_i->p_j} = movement time
- *Definition 1:* A *candidate sensing configuration* c_i is defined as a tuple (p_i, ϕ_i, r_i), where $p_i \in \mathcal{P}$, ϕ_i is the central angle of a circular sector and r_i its radius.

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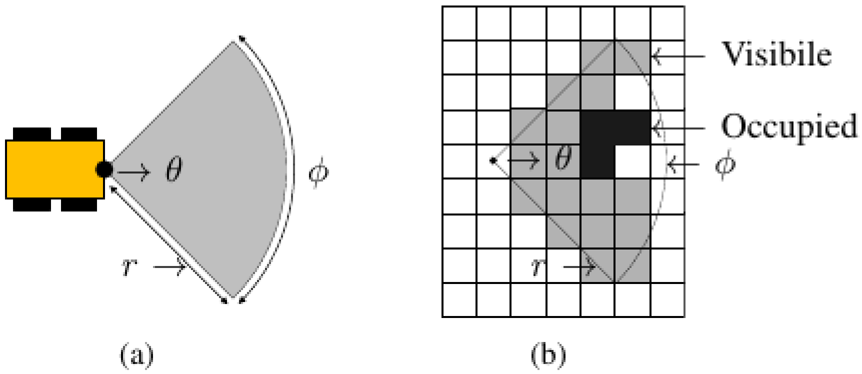
- 

Fig. 4. 4(a) A candidate sensing configuration c allows the robot to scan a circular sector of central angle ϕ and radius r. 4(b) $v_{\mathcal{P}}(c)$ is the visibility function which defines which are the cell that are observable from candidate sensing configuration c.

- C_P = set of candidate sensing configurations for a set of poses P
- Visibility function: v_P(c) : set of cells in S visible from c
- t_c^s = cost to perform sensing action in a candidate sensing configuration c

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As defined above, given a discretized map of the environment, the sets of occupied and unoccupied cells $\mathcal{O}$ and $\mathcal{S}$ (such that $\mathcal{A} = \mathcal{O} \cup \mathcal{S}$), the set of allowed poses $\mathcal{P}$ for the robot, the set of candidate sensing configurations $\mathcal{C}_\mathcal{P}$ defined over $\mathcal{P}$, the visibility function $v_\mathcal{P}(c)$ and the cost functions $t^m_{p_i \to p_j}$ (movement time from p_i to p_j, calculated as the shortest path on the movement graph) and t^s_c (sensing time of candidate configuration c), a solution to the detection problem is an obstacle free tour defined as an ordered, finite set of sensing configurations $\pi = \{c_1, \cdots, c_k\}$ such that $\bigcup_{c_i \in \pi} v_\mathcal{P}(c_i) = \mathcal{S}$. The cost associated to a solution π is equal to the sum of the traveling costs (including the traveling cost to go from the last sensing configuration back to the first one) and sensing costs:

$$cost(\pi) = \sum_{i=1}^{k-1} t^m_{p_i \to p_{i+1}} + t^m_{p_k \to p_1} + \sum_{c_i \in \pi} t^s_{c_i} \quad (1)$$

Given the set Π of all valid solutions to a given problem instance, an optimal solution π_{opt} is the one with minimum cost:

$$\pi_{opt} = \operatorname*{argmin}_{\pi_i \in \Pi} cost(\pi_i)$$

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- Formulation inspired by [Tomioka, Y.; Takara, A.; Kitazawa, H. Generation of an Optimum Patrol Course for Mobile Surveillance Camera. IEEE Trans. Circuits Syst. Video Technol. 2012, 22, 216–224.]
- Flow variables and label variables over the movement graph of the robot
- $$\forall c_i \in \mathcal{C}_{\mathcal{P}}, \forall c_j \in \mathcal{C}_{\mathcal{P}}, c_i \neq c_j \quad f_{c_i,c_j} = \begin{cases} 1 \text{ if } & c_i = c_h \in \pi, c_j = c_{h+1} \in \pi. \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$
- $$\forall c_i \in \mathcal{C}_{\mathcal{P}} \quad \begin{cases} l_{c_h,c_i} < l_{c_i,c_j} & \text{if } c_i = c_g \in \pi, c_h = c_{g-1} \in \pi, c_j = c_{g+1} \in \pi \\ l_{c_h,c_i} > l_{c_i,c_j} & \text{if } c_i = c_g \in \pi, c_h = c_{g-1} \in \pi, c_j = c_{g+1} \in \pi, c_i \text{ special vertex of } \pi \\ l_{c_h,c_i} = l_{c_i,c_j} = 0 & \text{otherwise} \end{cases} \tag{3}$$

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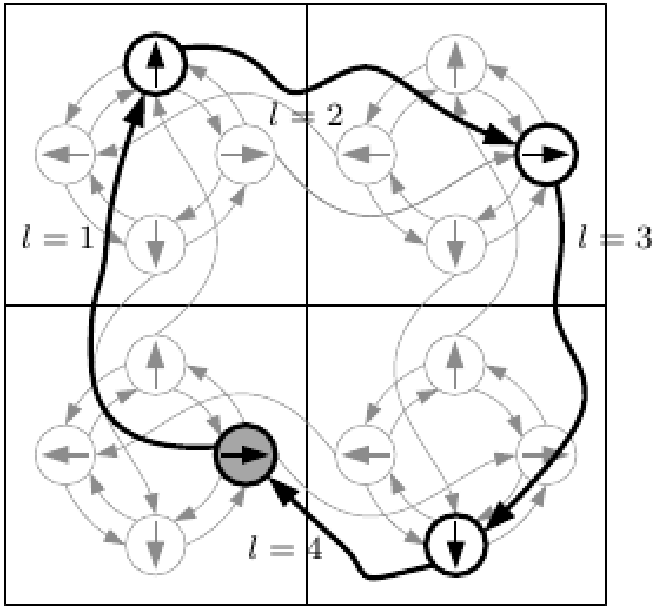

Figure 4. An example of a solution to a problem instance. Configurations in the solution are represented by thick circles and the shortest paths between them by thick lines. Each path between two consecutive candidate sensing configurations in the solution is annotated with a label variable l, and the marked configuration corresponds to the special vertex.

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- F = matrix of flow variables: F[i, j] = f_{c_i, c_j}
- L = matrix of label variables: L[i, j] = l_{c_i, c_j}
- S = boolean vector: S[i] if c_i special vertex
- T_m = matrix of traveling costs: T_m[i, j] = t^m_{p_i, p_j}
- T_s = vector of sensing costs: T_s[i] = t^s_{c_i}
- $V[a, c] = \begin{cases} 1 & \text{if } a \in v_{\mathcal{P}}(c) \\ 0 & \text{otherwise} \end{cases}$

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$$\text{minimize } \mathbf{1}^T \left(F \circ T_m\right) \mathbf{1} + \mathbf{1}^T \left(F T_s\right) \tag{4a}$$

subject to

$$(VF)\mathbf{1} \succeq 1 \tag{4b}$$

$$F\mathbf{1} = (\mathbf{1}^T F)^T \tag{4c}$$

$$F \preceq L \preceq \mathrm{u}F \tag{4d}$$

$$-\mathrm{u}S \preceq \Big(\big(L\mathbf{1} - (\mathbf{1}^T L)^T\big) - F\mathbf{1}\Big) \preceq -S \tag{4e}$$

$$\mathbf{1}^T S = 1 \tag{4f}$$

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- (4b) Coverage constraints: require that each cell a_i in S is visible from at least one of the candidate sensing configurations selected in the solution π.
- (4c) Flow conservation constraints: ensure that the solution will consist of one or more closed paths among the selected candidate sensing configurations.
- (4d-e) Traveling route constraints: ensure that each candidate sensing configuration is visited only once in a solution, and, therefore, a solution consists of a single closed path. u is an upper limit constant for label variables.
- (4f) Special vertex constraint: restricts the number of special vertices to one and only one for each solution.

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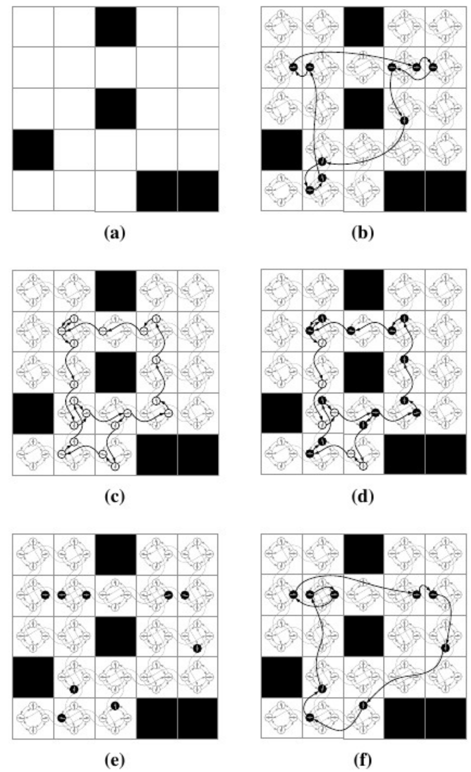

Figure 5. (**a**) a simple test map, where obstructed cells are represented in black and traversable ones in white. Candidate sensing configurations are defined over poses in the cells such that $\Theta = \{0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi\}$ and have identical ϕ and r ($r = 2$ cells, $\phi = \frac{\pi}{2}$). In this example setup, the movement from one cell to the next requires 1 s, the rotation of $\frac{\pi}{2}$ requires 0.5 s, and a sensing action takes 4 s; (**b**) the optimal solution, when traveling and sensing costs are considered at the same time. Here, the curved arrows represent the minimum distance from a sensing configuration to the next on the underlying graph. In this case, the total exploration time is 52.5 s (16.5 s for traveling and 36 s for sensing). (**c**,**d**): Here, traveling time is minimized first (see Section 5.1). (c) the minimum cost closed path from which all cells can be observed is calculated; (d) and then the minimum set of sensing configurations is selected, yielding an overall exploration time of 66.5 s. (**e**,**f**): Here, sensing time is minimized first (see Section 5.2). (e) the set of minimum cost sensing configurations is selected from which all cells are observable; (f) and then connecting them with the shortest closed path. This approach yields to an overall exploration time of 55 s.

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- Decomposition of the problem:
  - Minimizing the Traveling Time first, Minimizing the Sensing Time then.
  - Minimizing the Sensing Time first, Minimizing the Traveling Time then.
    - $$\underset{C}{\text{minimize}}\ C^T T_s \quad (5a)$$
      subject to
      $$VC \succeq 1 \quad (5b)$$
      $$C \in \{0, 1\} \quad (5c)$$
    - C = vector whose elements are binary variables representing if a given candidate sensing configuration is selected or not.

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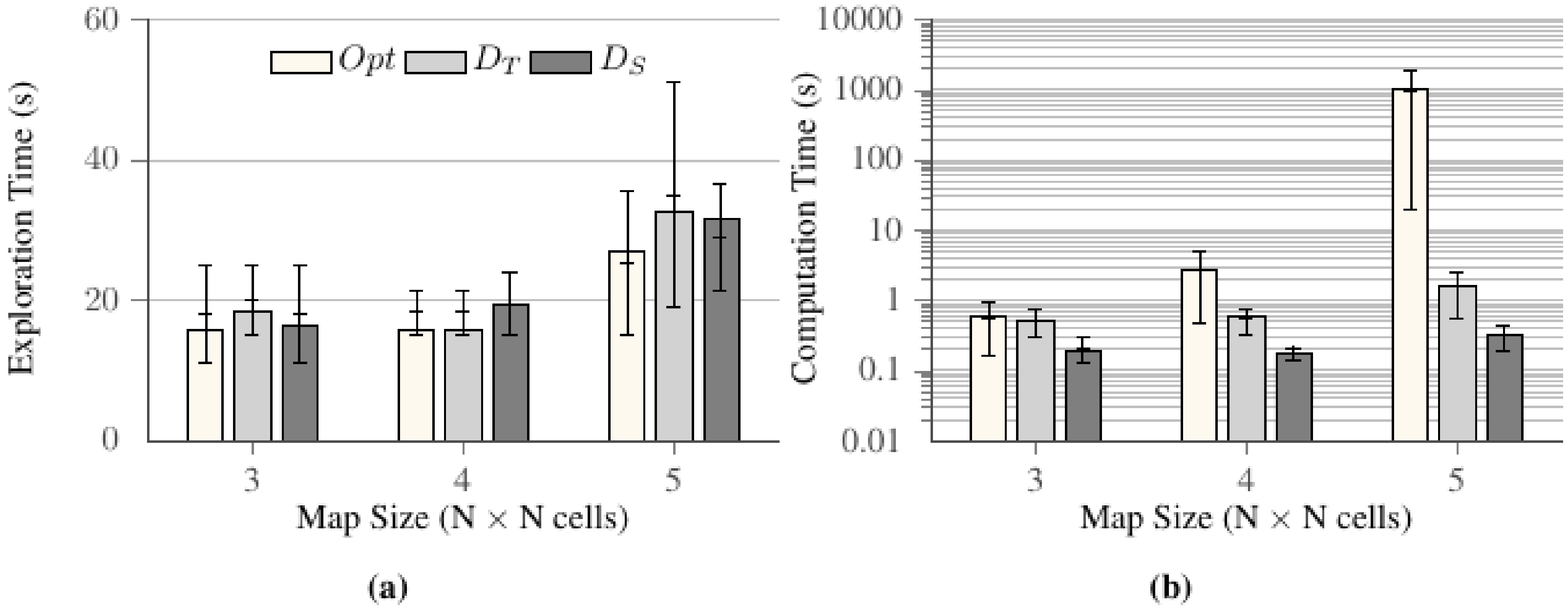

Figure 6. Comparison of all approaches, optimal and disjoint (Opt, D_T, D_S), on three sets of maps. Each set contains maps of varying size, from 3×3 to 5×5. The solid bars indicate the average values over the 3 maps for 3×3 and 10 maps for the rest, and error bars show the minimum and maximum values observed during the trial. (**a**) shows the solution quality; and (**b**) shows the computation time taken by the all three approaches on a logarithmic scale.

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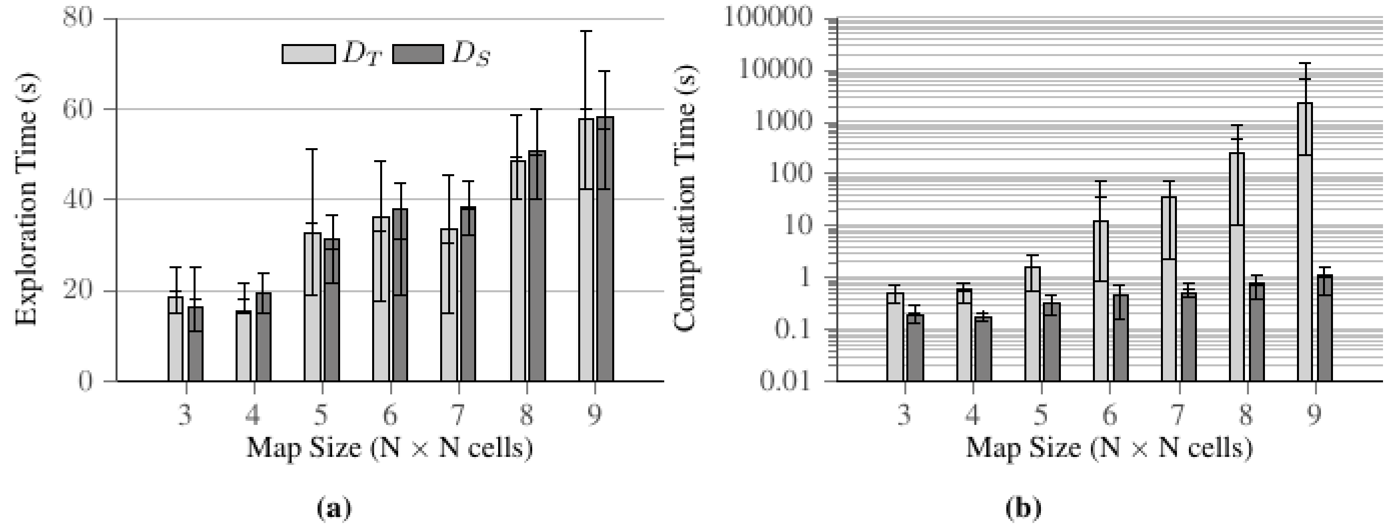

Figure 7. Comparison of two disjoint approaches up to maps of size 9×9. Notations are similar as in the previous figure, *i.e.*, bars indicate the average values over the 3 maps for 3×3 and 10 maps for the rest, and error bars indicate the minimum and maximum values observed during the trial. (**a**) shows the solution quality of two disjoint approaches (D_T, D_S); and (**b**) shows the average computation time on a logarithmic scale for both approaches.

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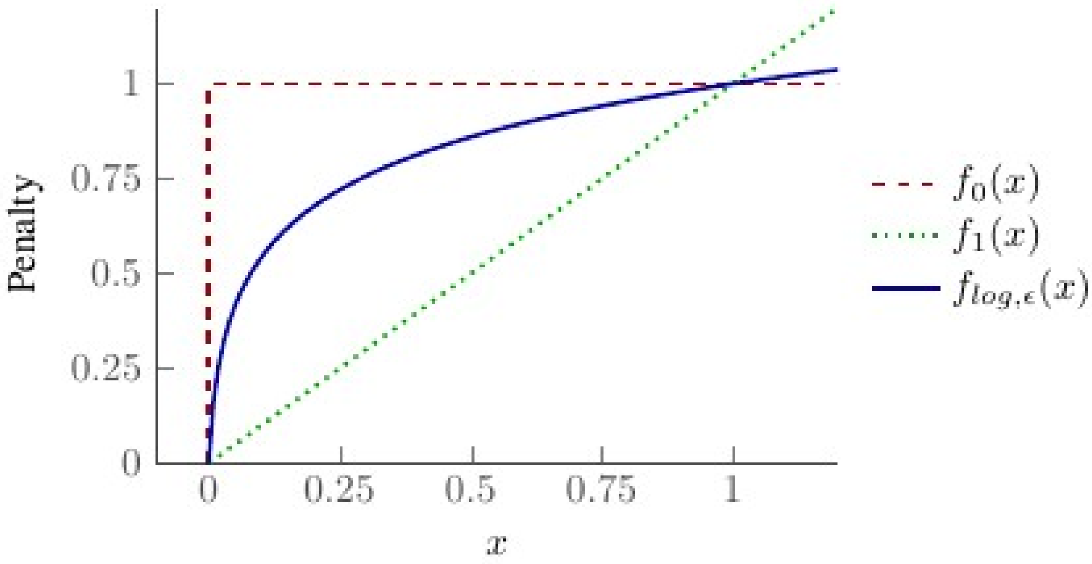

Figure 8. The concave loss function $f_{log,\epsilon}(x)$ approximates better the ℓ_0 sparsity count $f_0(x)$ than by the traditional convex ℓ_1 relaxation $f_1(x)$ [48].

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$$\underset{C}{\text{minimize}}\ (W \circ C)^T T_s \tag{6a}$$

subject to

$$VC \succeq 1 \tag{6b}$$

$$0 \preceq C \preceq 1 \tag{6c}$$

W is a weight vector of cardinality $|\mathcal{C}_\mathcal{P}|$. At the beginning, all the elements of W are equal to 1, *i.e.*, $W = \mathbf{1}$. In subsequent iterations, the weights are updated according to Equation (7).

$$W_{(i)} = \frac{\epsilon}{C_{(i)} + \epsilon} \quad i = 1, ..., |W| \tag{7}$$

Epsilon = convergence rate: high at beginning, decreases exponentially

$$\epsilon_{(i)} = \left(\frac{1}{e^1 - 1}\right)^{1+\left((i-1)\times 10^{-1}\right)} \tag{8}$$

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Algorithm 1 *conv*-SPP

1: Set $W = \mathbf{1}$;
2: Solve Equation (6);
3: Update W as in Equation (7) and ϵ as in Equation (8);
4: Go back to step 2, if none of the stopping criteria is true;
5: Discard zero elements of C;
6: Solve Equation (5) with updated C;

Stopping criteria: cfr paper

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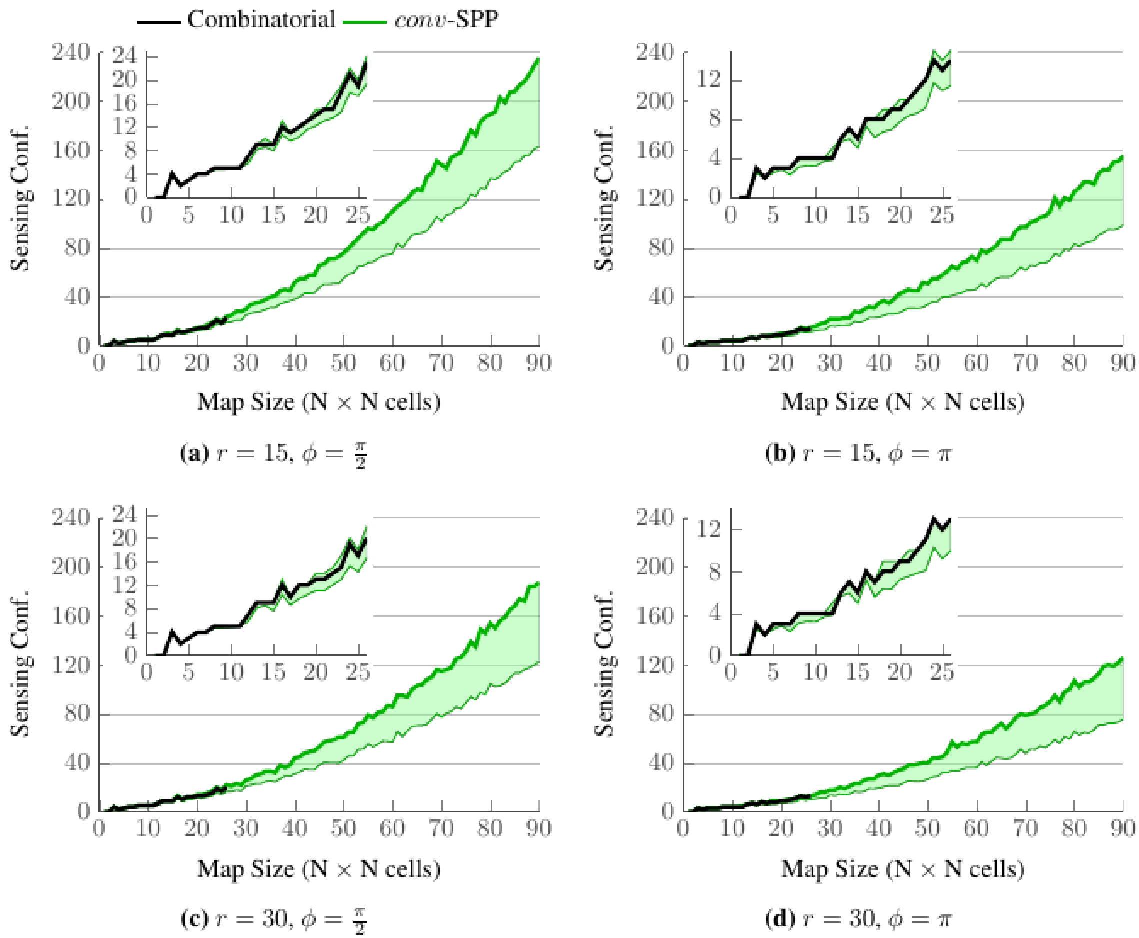

Figure 11. Comparison of the quality of the solutions obtained with *conv*-SPP and the combinatorial method on randomly generated maps. Sensing parameters are fixed to $\Theta = \{0, \frac{\pi}{2}, \pi, \frac{3}{2}\pi\}$, $r = \{15, 30\}$, and $\phi = \{\pi, \pi/2\}$. The optimal solutions (thick black lines) are close to the solutions provided by *conv*-SPP (thick green lines) with respect to the number of sensing configurations selected. The lower bounds of the green intervals represent the result of a single ℓ_1-minimization step.

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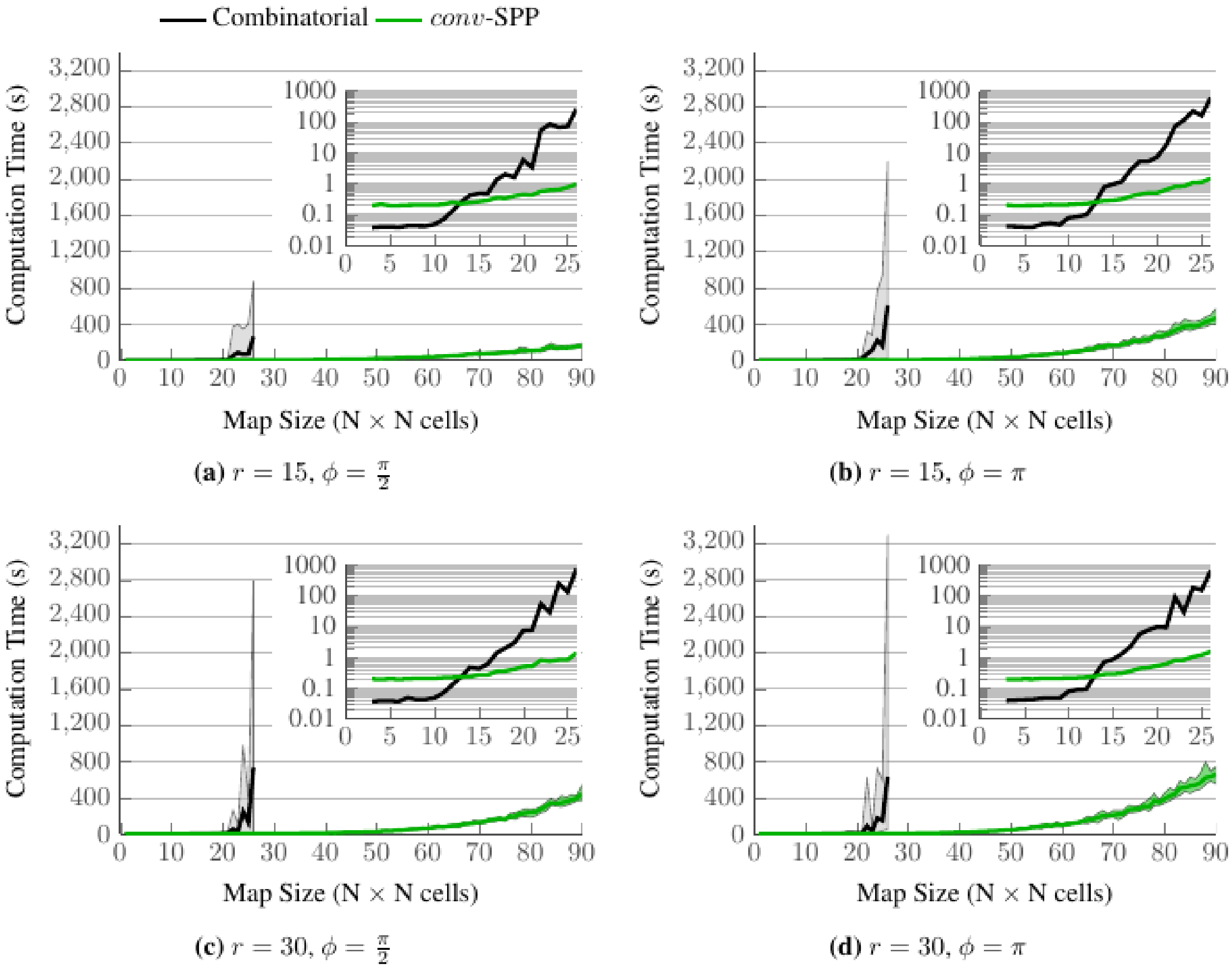

Figure 13. Computation times to calculate the optimal solutions and to solve the instances with *conv*-SPP. The thick lines represent the average computation times for each set of maps, while the colored intervals are bounded by the minimum and maximum times it took to solve all the instances in one set. The inset graphs show the computation times on a logarithmic scale.