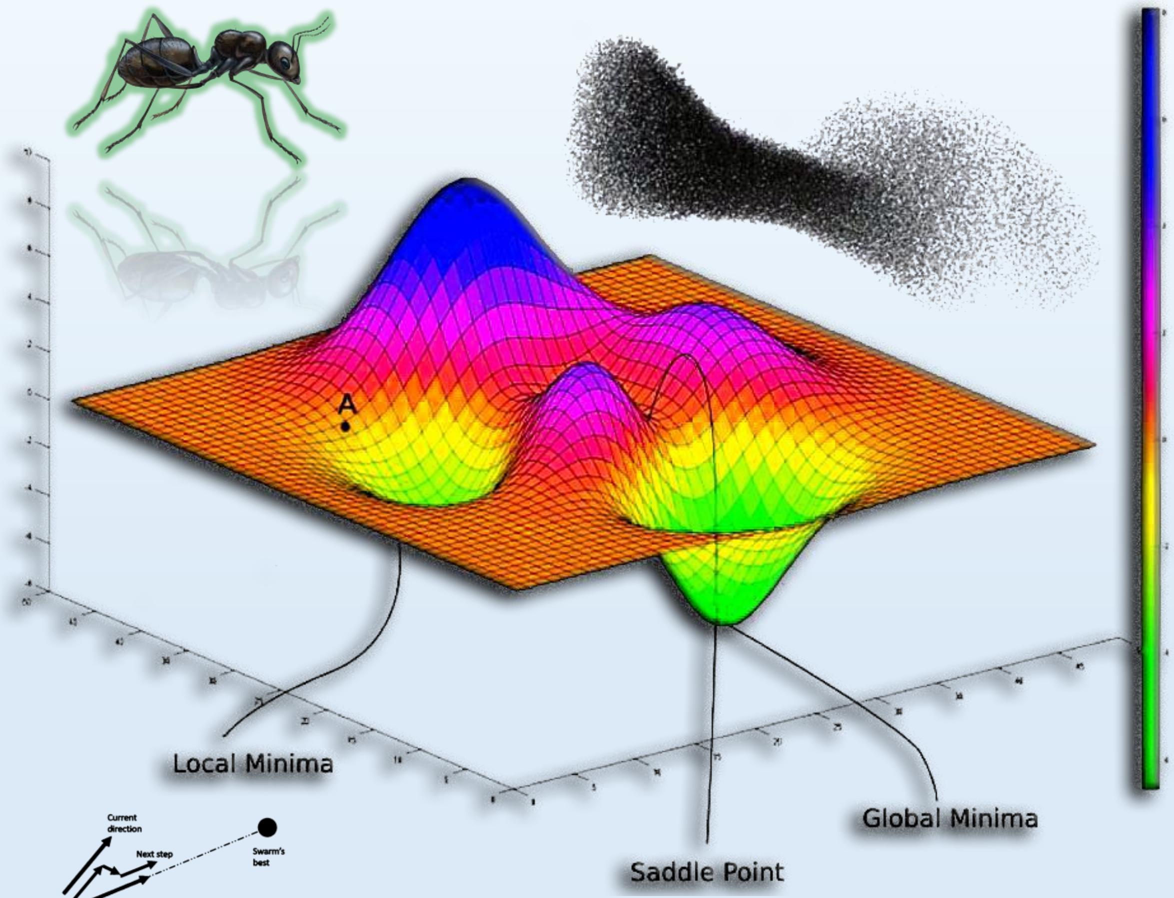

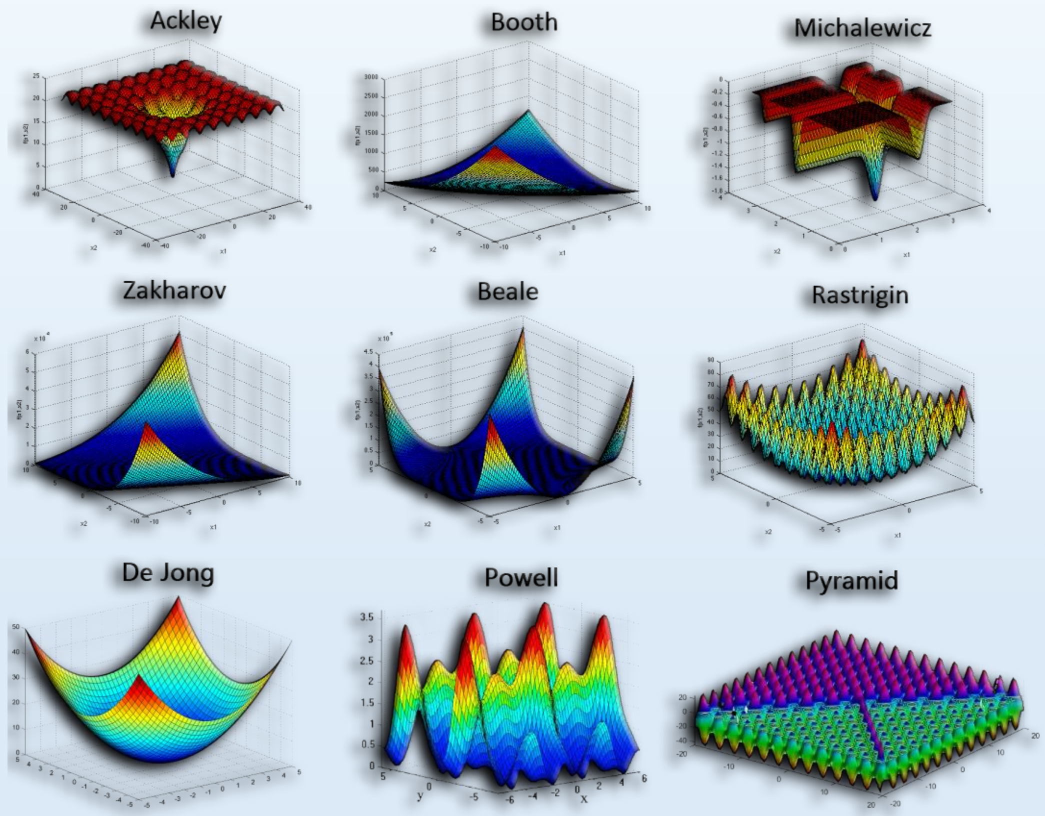

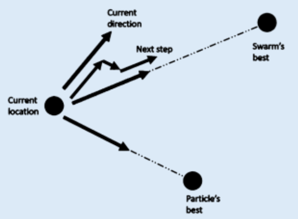

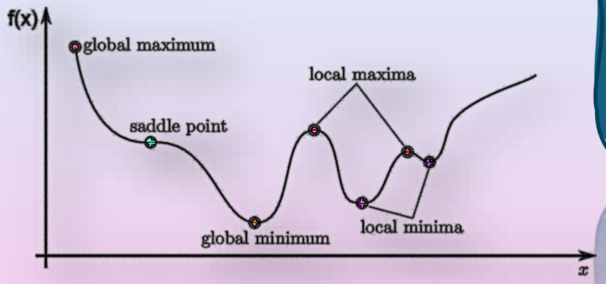

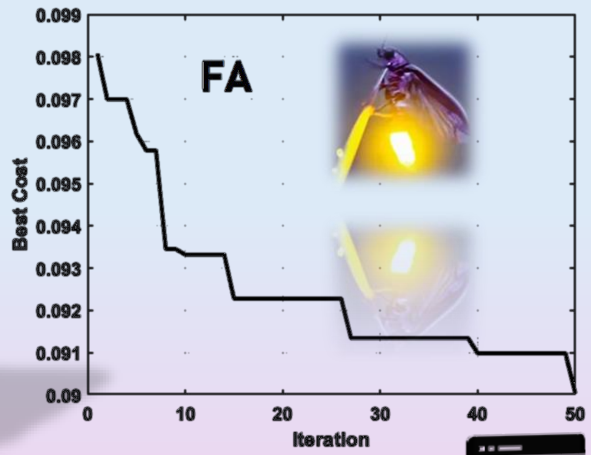

Metaheuristic Optimization: 4 Cutting-Edge Applications

Space-Time Warping by Firefly Algorithm (FA)

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2025

Outline:

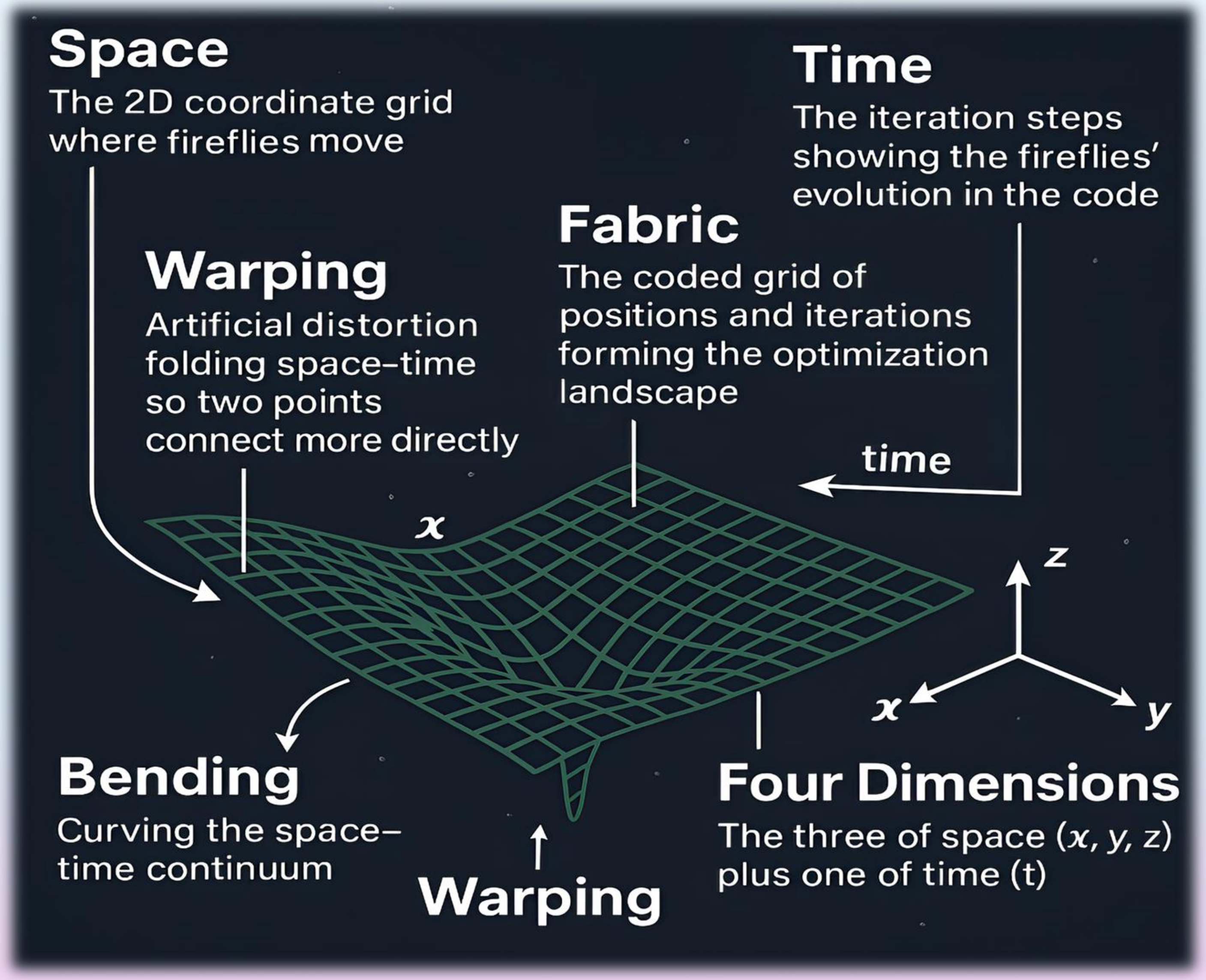
- **Optimization**
  - ❖ **Optimization Problems**
    - ➢ **Protein Folding by Differential Evolution algorithm (DE)**
    - ➢ **Space-Time Warping by Firefly Algorithm (FA)**
    - ➢ **Exoplanetary Adaptation Simulation by Genetic Algorithm (GA)**
    - ➢ **Evolved Antenna Design by Particle Swarm Optimization algorithm (PSO)**

- # Optimization Problems

❖ Space-Time Warping by Firefly Algorithm (FA)

- **Space-Time**: the four-dimensional fabric combining space and time where motion or energy can bend its geometry.
- **Space (real-world):** the three-dimensional physical area where objects exist and move.
- **Space (in the course):** the 2D coordinate system (x, y) where fireflies search for the best position.
- **Time (real world):** the continuous flow of events from past to future, measuring change and motion.
- **Time (in the course):** the iteration sequence showing how solutions evolve step by step during optimization.
- The four dimensions are x, y, z (space), and time (t).
- **Fabric:** the real world's woven structure of space and time becomes the coded grid of positions and iterations forming the optimization landscape.
- **Warping:** artificial distortion (bending) of space-time that shortens distance or time between two points.

- **Optimization Problems**
  - ❖ **Space-Time Warping by Firefly Algorithm (FA)**



- # Optimization Problems

  - ## Space-Time Warping by Firefly Algorithm (FA)

    - **Geodesic**: the shortest possible path between two points in curved space-time.
    - **Bending Effort (λ/lambda bend):** resistance against curvature, represents the energy needed to change a path's geometry.
    - **Warp Field**: a simulated distortion field that alters how distances and time are measured.
    - **Objective Function**: mathematical rule measuring how **costly or good** a candidate solution is.
    - **Firefly Algorithm**: a metaheuristic inspired by fireflies' light attraction, brighter ones attract others.
    - **Attractiveness (β/beta)**: strength of movement toward a better (brighter) solution; decays with distance.
    - **Absorption Coefficient (γ/gamma):** controls how fast attractiveness decreases as distance grows.
    - **Randomness (α)**: exploratory noise allowing escape from local minima.
    - **Intensity**: brightness equivalent of a solution's quality (**fitness**).
    - **Iteration**: one full update cycle where all fireflies move and fitness is recalculated.
    - **Global Best**: the position with the lowest total cost found so far.
    - **Distance Metric**: Euclidean norm used to measure how far two solutions are.

- # Optimization Problems

❖ Space-Time Warping by Firefly Algorithm (FA)

- **Our goal or objective here:** To find the **optimal point (path endpoint)** that minimizes total energy, the combined cost of distance, bending, traversal effort, and warp-field energy.
- So we do the process to **bend space-time efficiently, finding a point (or path) where the energy cost to travel through warped space is minimal.**
- **In simple terms: Finding the smoothest, most energy-efficient, least-curved path from a start point to a target in warped space-time.**

Optimization Problems

❖ Space-Time Warping by Firefly Algorithm (FA)

- **Why does it matter?**
  - Space-time warping is important because it enables theoretical concepts like **faster-than-light travel, gravitational navigation, and the discovery of minimum-energy paths** in extreme physics environments.
  - By simulating it computationally, we gain insights into **how curvature, energy, and distance interact**, which is valuable for optimization, robotics, astrophysics, and even AI path planning.
- **Why is FA a better solver in this context?**
  - Firefly Algorithm (FA) excels in solving this problem because it naturally **balances exploration and exploitation, helping escape local minima in complex, warped landscapes.**
  - Its **distance-based attraction makes it ideal for space-time scenarios**, where curvature and energy fields influence pathfinding non-linearly.

- **Optimization Problems**

❖ Space-Time Warping by Firefly Algorithm (FA)

- **Inputs and Initialization**
- Each firefly represents a candidate solution (**possible location or path endpoint**):

$$x_i = [x_{i1}, x_{i2}, \dots, x_{iD}]$$

- Where D is the number of dimensions (2 here).

We start with:

- $\boldsymbol{N}$: number of fireflies. **Number of possible space-time paths being explored in parallel**.
- $\boldsymbol{D}$: number of dimensions
- $\boldsymbol{x_i(0)}$: initial random positions within bounds $[-5,15]$.
  - ➢ They represent different initial guesses for where the minimum-energy point could be in warped space.

• Optimization Problems

❖ Space-Time Warping by Firefly Algorithm (FA)

- **Constants:**
- $\boldsymbol{\alpha}$: randomness factor
  - ➢ **Physical Interpretation**: Quantum or thermal noise, helps exploration beyond local curvature wells.
- $\boldsymbol{\beta_0}$: base attractiveness
  - ➢ **Physical Interpretation**: Intrinsic gravitational strength, how strongly bright fireflies attract others.
- $\boldsymbol{\gamma}$: light absorption coefficient.
  - ➢ **Physical Interpretation**: Curvature decay, how fast attraction weakens with distance.
- $\boldsymbol{\lambda_{\text{bend}}}$: bending penalty factor. **the stiffness of space-time; how resistant it is to bending**.
  - ➢ **Physical Interpretation**: Rigidity of space-time fabric, how resistant the curvature is.
- **warp_field**: static modifier simulating space +time distortion. higher values mean stronger curvature.
  - ➢ **Physical Interpretation**: Artificial distortion of the metric, simulates warp drive curvature.

• Optimization Problems

❖ Space-Time Warping by Firefly Algorithm (FA)

➢ Objective Function:

$$Objective = f(x) = d(x, \text{end}) + \lambda_{\text{bend}} \sum (x - \text{start})^2 + 0.5 \sum |x - \text{start}|(1 + w) + 0.2 \sum w^2$$

- Now let's connect each term to both math and space-time meaning:

Term	Mathematical Role	Physical / Conceptual Meaning
$d(x, \text{end}) = \|(x - \text{end})(1 + w)\|$	Measures distance distorted by warp.	**Geodesic distance through warped space - how long it takes light or an object to reach the goal.**
$\lambda_{\text{bend}} \sum(x - \text{start})^2$	Quadratic penalty on deviation.	**Curvature resistance - bending spacetime costs energy.**
(0.5 \sum	x - \start	(1+w))
$0.2\sum w^2$	Constant energy term.	**Warp-field maintenance energy - keeping the warp active consumes power.**

- So $f(x)$ measures **total energy required** for a path or location in curved space-time.
  → **Lower $f(x)$** = smoother curvature, shorter distance, lower energy - exactly what physics wants for efficiency.

- **Optimization Problems**
  - **Space-Time Warping by Firefly Algorithm (FA)**

- **Firefly Brightness** - Energy Becomes Light

In the Firefly Algorithm:

$$I_i = f(x_i)$$

is the intensity (brightness) of firefly i.

- **Low energy (low $f(x)$) = bright firefly (efficient point).**
- High energy (high $f(x)$) = dim firefly (inefficient or costly point).

So "brightness" literally encodes energy efficiency in warped space-time.

- **Firefly Attraction** - Gravitational-Like Pull

Between any two fireflies i and j :

$$r_{ij} = \|x_i - x_j\|, \beta = \beta_0 e^{-\gamma r_{ij}^2}$$

- The closer they are, the stronger the attraction - like gravitational pull or curvature influence in general relativity.
- Bright fireflies (low energy) pull dim ones toward them - mimicking matter bending space-time so others move along that curvature.

• Optimization Problems

❖ Space-Time Warping by Firefly Algorithm (FA)

○ Movement - Evolution of Space-Time Paths

The update rule:

$$x_i^{(t+1)} = x_i^{(t)} + \beta\left(x_j^{(t)} - x_i^{(t)}\right) + \alpha(\text{rand} - 0.5)$$

means:

- Firefly i moves toward brighter ones (**lower-energy zones**) - physically like matter following curvature toward an energy minimum.
- The random term (α) adds small quantum-like fluctuations, creating exploration of the warped field.
- After each move, positions are clipped to remain within defined boundaries - our cosmic box.

- # Optimization Problems

❖ Space-Time Warping by Firefly Algorithm (FA)

- **Iteration - Time in the Simulation**

- Each iteration corresponds to time evolution - a **discrete tick of cosmic time**. The swarm keeps moving, and space-time gradually stabilizes as the energy landscape smooths out.

- **Convergence - Finding the Stable Warp**

After many iterations:

$$f_{\text{best}} = \min_i f(x_i)$$

- The **swarm settles near the lowest-energy point**, which corresponds to:
- The most efficient configuration of curvature and distance is the optimal warp path from start to end through space-time.
- That **final objective value (last iteration fitness)** is **the minimum energy**, proving the algorithm found the smoothest, least-resistant space-time configuration.
- **It is the most efficient way to traverse that curved universe.**
- This value corresponds to a 2D space like: **Global Best Position: [1.62744009 1.01394335]** in the our problem.

- # Optimization Problems
  - ## Space-Time Warping by Firefly Algorithm (FA)