Initial conditions (at first time slice $t = t_1$): $\lambda(t = t_1) = 0$, choose $B(t = t_1, u)$

Get the subtracted fields $\sigma, \dot{\sigma}, \dot{b}$ and a **at first time slice**:

- By solving the 4 radial differential equations. Spectral method. With Eigen library (LU decomposition with Full Pivoting).

Get λ and b at the second time slice:

- Calculate $\partial_t b$ and $\partial_t \lambda$ at first time slice.
- Integrate the interpolation of $\partial_t b$ from $t = t_1$ to $t = t_2$ to get $b(t = t_2, u)$. Same for λ.

Get the subtracted fields at the second time slice:

- By solving the 4 radial differential equations. Spectral method. With Eigen library (LU decomposition with Full Pivoting).

Get λ and b at the 3rd time slice:

m = min (6, number of the current time slice)

- Calculate $\partial_t b$ and $\partial_t \lambda$ at second time slice.
- Interpolation of $\partial_t b$ using the values of $\partial_t b$ at the last m previous time slices. Same for λ.
- Integrate interpolation of $\partial_t b$ from $t = t_2$ to $t = t_3$ to get $b(t = t_3, u)$. Same for λ.

and so on ...

How to get λ **and** b at next time slice after a couple of time slices:

- Use Adams Bashforth method **of 5 steps**.

How to get $\boldsymbol{\lambda}$ **and** $\boldsymbol{b}$ at next time slice:

(in more detail)

Adaptive time stepping:

- **Define table dts** of 10 elements **:**

  10^{-4} , $10^{-3.5}$, 10^{-3}, $10^{-2.5}$, 10^{-2}, $10^{-1.5}$, 10^{-1}, $10^{-0.5}$, (1- sum of first part), 1

- $\boldsymbol{dt = 1/2000}$
- $t = t + dt *$ **dts[[Min[Length[dts] , Length[timelist]]]];**
- Equidistant time steps starting from 10th time slice.

Store the t_values in timelist. (timelist[[-1]] is the current time.)

How to get $\boldsymbol{\lambda}$ **and** $\boldsymbol{b}$ at next time slice:

If

- we are <u>at least at 6th time slice</u>
- AND <u>(timelist[[-6]]-timelist[[-5]])-(timelist[[-3]]-timelist[[-2]])<10^{-12}</u>

then use Adams Bashforth method of 5 steps (to get $\boldsymbol{\lambda}$ **and** $\boldsymbol{b}$ at next time slice).

Else integrate a polynomial Interpolation of $\partial_t b$ to get b at the next time slice. Same for λ

Horizon gauge fixing

Fix $\lambda(t = t_1) = 0$ at the first next slice.

Solve the eoms for the subtracted fields.

Find the position u_h of the horizon (u_h is the root of $\dot{\Sigma}$. Root found using Newton Raphson method.)

The horizon stationarity condition fixes $\partial_t \lambda$. Use $\partial_t \lambda$ to get λ at the next slice.