

Let the joint pmf of (X, Y) be given by the table below:

$x \backslash y$	0	1	2	3
1	$1/15$ $2/30$	$1/15$ $2/30$	$2/15$ $4/30$	$1/15$ $2/30$
2	$1/10$ $3/30$	$1/10$ $3/30$	$1/5$ $6/30$	$1/10$ $3/30$
3	$1/30$	$1/30$	0	$1/10$ $3/30$

Find $\text{Cov}(X, Y)$

Find $\text{Corr}(X, Y)$

X	1	2	3
	$\frac{5}{15}$	$\frac{5}{10}$	$\frac{5}{30}$

$$E(X) = \left(1 \cdot \frac{5}{15} \right) + \left(2 \cdot \frac{5}{10} \right) + \left(3 \cdot \frac{5}{30} \right)$$

$$\frac{5}{15} + \frac{10}{10} + \frac{15}{30}$$

(2) (3)

$$= \frac{10}{30} + \frac{30}{30} + \frac{15}{30}$$

$$E(X) = \frac{55}{30} = \frac{11}{6}$$

y	0	1	2	3
	$\frac{6}{30}$	$\frac{6}{30}$	$\frac{10}{30}$	$\frac{8}{30}$

$$E(y) = (0 \times \frac{6}{30}) + (1 \times \frac{6}{30}) + (2 \times \frac{10}{30}) + (3 \times \frac{8}{30})$$

$$= 0 + \frac{6}{30} + \frac{20}{30} + \frac{24}{30}$$

$$E(y) = \frac{50}{30} = \frac{5}{3}$$

$$\text{cov}(x, y) = E(xy) - E(x) \cdot E(y)$$

$$\begin{aligned} E(xy) &= (1 \times \frac{2}{30}) + (2 \times \frac{4}{30}) + (3 \times \frac{2}{30}) + \\ &\quad (2 \times \frac{3}{30}) + (4 \times \frac{6}{30}) + (6 \times \frac{3}{30}) + \\ &\quad (3 \times \frac{1}{30}) + (6 \times 0) + (9 \times \frac{2}{30}) \\ &= \frac{2}{30} + \frac{8}{30} + \frac{6}{30} + \frac{6}{30} + \frac{24}{30} + \frac{18}{30} \\ &\quad + \frac{3}{30} + 0 + \frac{27}{30} \end{aligned}$$

$$E(xy) = \frac{94}{30}$$

$$\text{cov}(x, y) = E(xy) - E(x) \cdot E(y) = \frac{94}{30} - \frac{11}{6} \cdot \frac{5}{3}$$

$$= \frac{94}{30} - \frac{55}{18}$$

(3) (5)

$$= \frac{282 - 275}{90}$$

$$\text{cov}(x, y) = \frac{7}{90} //$$

$$= 0,0777$$

$$\text{corr}(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

$$\sigma_x^2 = E(x^2) - \bar{E}(x)^2$$

x	1	2	3
	$\frac{5}{15}$	$\frac{5}{10}$	$\frac{5}{30}$

$$E(x^2) = (1^2 \times \frac{5}{15}) + (2^2 \times \frac{5}{10}) + (3^2 \times \frac{5}{30})$$

$$= \frac{5}{15} + \frac{20}{10} + \frac{45}{30}$$

(2) (3)

$$= \frac{10}{30} + \frac{60}{30} + \frac{45}{30}$$

$$E(x^2) = \frac{115}{30} //$$

y	0	1	2	3
	$\frac{6}{30}$	$\frac{6}{30}$	$\frac{10}{30}$	$\frac{8}{30}$

$$E(y^2) = (0^2 \times \frac{6}{30}) + (1^2 \times \frac{6}{30}) + (2^2 \times \frac{10}{30}) + (3^2 \times \frac{8}{30})$$

$$= 0 + \frac{6}{30} + \frac{40}{30} + \frac{72}{30}$$

$$E(y^2) = \frac{118}{30} //$$

$$\sigma_x^2 = \frac{115}{30} - (\frac{11}{6})^2$$

$$= \frac{115}{30} - \frac{121}{36} \Rightarrow \frac{690}{180} - \frac{605}{180}$$

(6) (5)

$$\sigma_x^2 = \frac{85}{180} //$$

$$\sigma_x = 0,6871$$

$$\sigma_y = 1,0749$$

$$\sigma_y^2 = \frac{118}{30} - (\frac{5}{3})^2 \Rightarrow \frac{118}{30} - \frac{25}{9}$$

(3) (10)

$$= \frac{354}{90} - \frac{250}{90}$$

$$\sigma_y^2 = \frac{104}{90} //$$

$$\text{Corr}(x,y) = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y} \Rightarrow \frac{7}{90}$$

$$0,6871 \times 1,0749$$

$$\text{Corr}(x,y) = \frac{0,0777}{0,7385} \Rightarrow 0,1052 //$$