Prior Information Based Segmentation: A 3D Level Set Surface Matching Approach

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Abstract—This paper presents a level set based segmentation method with shape priors. The shape priors guide the level set deformations so that the contour extraction process is affected not only from the local image properties, but also from the expert knowledge in the form of manual contours. The method does not need an explicit training phase and it does not complicate the level set functional because level set deformations and incorporation of prior information are done separately. The system uses manual expert contours to produce new level set surfaces which are warped into the surface from the level set process. The prior information is incorporated into the level sets by re-initializing these warped surfaces as new level set surfaces. The resulting system is validated by running experiments on synthetic data and real MR and ultrasound cardiac images.

I. Introduction

The level set method [1], [2] is a popular segmentation technique based on embedding the shapes of objects as the zero level set of a higher dimensional surface. The higher dimensional surface evolves according to the image and surface features, which results in localized image features in the zero level set. The level set methods can be extended to any higher dimension. Moreover, the contours in the zero level set can change topology such as merging or breaking into parts.

The level set methods are very robust in taking image properties into account and the zero level interfaces can deform to extract contours in pixel-wise detail. However, in some cases global or prior information must also be considered with local properties during evolution. The prior information is essential especially in medical imaging applications where the images are very noisy, low contrast and some parts of organ contours are missing. In medical imaging applications, human organs and even pathological cases have similar contours, however the level set method cannot take advantage of common shapes without using prior information.

There are a number of proposals for using shape priors in the literature. Leventon *et al.* [4] incorporated prior knowledge about the intensity and curvature of the structure based on training data modeled through a Gaussian distribution and principle component analysis to recover the covariance matrix of probability density function of shapes and alternate between segmentation and imposing prior knowledge. Chen *et al.* [5], [6] used an average model as a prior in its implicit function and for a given curve, and they found the transformation that projects it closer to the zero-level set of the implicit represen-

tation of the prior. The prior knowledge is modeled through a Gaussian distribution on the space of distance functions by performing a singular value decomposition on the set of registered training set and objects are recovered according to various data-driven terms [8]. Rousson and Paragios [3] constrained the level set to follow a shape global consistency by creating a shape model with Gaussian density function and shape prior is imposed by the comparison between the model and evolving contour.

The common technique in the above methods is creating the shape prior using a collection of samples by training. The training phase is usually time consuming, requires many samples, and has overtraining problem. After creating the shape prior, the level set segmentation is then corrected according to the trained shape prior by adding a new term into the main level set functional which makes the functional more complex and increases the computational cost. In order to address above problems, we previously proposed a method for echocardiographic images [9] that is based on incorporating prior information into the level sets by stopping the deformations and re-initializing the level set surface under the prior shape influence. Although the method worked favorably, the prior information is incorporated into the level set by only 2D contour matching approach between the zero level set and the expert models. This approach does not take the advantage of the information contained in the higher dimensional surface.

In this paper, we present a novel extension of our previous work for recovering shape boundaries with level set method. The prior knowledge is incorporated into the level set method by re-initializing the surface under the influence of shape priors which are taken as 3D level set surfaces from the human experts. During the re-initializing process, the surface is affected regularly by the best matching 3D prior model surface and then it continues to evolve on the image. In our method, the surface evolution and the influence of prior information are separated; therefore the method developed can be used with any level set formulation. Furthermore, the method does not significantly increase the computational power requirements and no explicit training phase is required because the model knowledge is incorporated directly during the level set surface evolution process. The employment of 3D prior surfaces made the system perform much better than our previous system, which was verified by experiments on real data.

The method is tested and verified by both synthetic and real images containing dual concentric contours. Experiments on the synthetic images are performed to show the robustness of our method with various shape models under various controlled noise levels and missing sections in the shape boundaries. The real data includes the cardiac images taken by magnetic resonance (MR) and ultrasound modalities.

MR cardiac and echocardiographic images provide crucial information about the heart walls (the endocardium and the epicardium), ventricular blood volume, the ventricular wall mass, the ventricular wall motion, and wall thickening properties. However, it is a challenging task to extract the endocardium and the epicardium from cardiac images automatically because of the modality related problems in the images. In the cardiac MR images, there are unrelated high contrast structures such as papillary muscles (Fig. 1(a)). The echocardiographic images are very noisy and parts of cardiac walls are missing (Fig. 1(b)). Employing prior information for the automatic extraction of heart walls is considered as a practical solution. Therefore, we used cardiac images as the test bed for 3D prior information based level set method. Note that the cardiac images are only a test bed environment and our method can be generalized into any medical or nonmedical application.

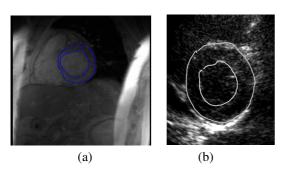


Fig. 1. (a) Example cardiac MR image (b) Example echocardiographic image. The epicardium and endocardium contours are marked by an expert in both images

The outline of this paper is as follows: The level set formulation and surface evolution is presented in Section II-A. The best matching surface technique for shape prior information incorporation is introduced in Section II-B. The Section II-C includes the algorithm and implementation details. Section III presents the experiments performed on synthetic and real images to test the validity of our method. Finally, we conclude in Section IV.

II. SHAPE PRIORS WITH LEVEL SETS

The level set methods extract the object boundaries in zero level set of a higher dimensional surface under the influence of local image properties. However, paying too much attention to local image properties causes problems in cases where global or prior information needs to be imposed especially in medical imaging applications. For example, the cardiac images (both MR and ultrasound) are very noisy and include missing parts and unrelated structures around the organs and sometimes they may have low gray level contrast. The images by themselves

do not include enough information to extract the cardiac walls. Therefore, prior information about the shapes of objects which share a common form must be incorporated into the level set method for a successful segmentation.

Our method incorporates prior information into the level set method by re-initializing the evolving surface. There are two repeating stages in the method: In the first stage, the level set surface evolves according to the classical level set formulation. The second stage directly employs the expert contours as 3D level set surfaces with prior information. It re-initializes the evolving level set surface so that it is similar to the best matching level set surface that includes the prior information. The local image properties are considered in the first stage and the global or prior information is considered in the second stage. These stages follow each other until the desired contours are found.

A. Stage I: Surface Evolution and Level Set Formulation

In this study we use the variational level set formulation [7] because of its advantages like easy implementation and computational efficiency. However, our system can also be used with any level set approaches with simple modifications. The following formulations are developed for the cardiac wall extraction problem, but the method is extensible to any other application. We used a similar formulation in [9].

Consider two closed curves $c_1(t)$ and $c_2(t)$ evolving on the plane \Re^2 with time t where $c_1(t)$ is used for extracting the inner contour and $c_2(t)$ is used for extracting the outer contour of the cardiac wall. Let C be the set of points on $c_1(0)$ and $c_2(0)$. Consider ϕ as a signed distance function and let \mathbf{x} be the position vector:

$$\phi(\mathbf{x}) = \begin{cases} 0, & \text{if } \mathbf{x} \in C \\ -d(x), & \text{if } \mathbf{x} \text{ is outside } c_1 \text{ but inside } c_2 \\ d(x), & \text{otherwise,} \end{cases}$$
 (1)

where d is the shortest Euclidian distance to C from point x. The contours c_1 and c_2 are the zero level set of ϕ at t = 0.

$$C = (\mathbf{x} | \phi(\mathbf{x}, t = 0) = 0). \tag{2}$$

According to the variational level set formulation, the 3D surface ϕ is evolved under the influence of the internal energy term $P(\phi)$ and the external energy term $\varepsilon_m(\phi)$. The variational energy function $\varepsilon(\phi)$ is defined as:

$$\varepsilon(\phi) = \mu P(\phi) + \varepsilon_m(\phi),$$
 (3)

where μ is a parameter controlling the weight of the internal energy term in the overall contour extraction process. The internal energy term forces the level set function not to deviate from the signed distance function which is desired to satisfy $|\nabla \phi| = 1$ in $\Omega \subset \Re^2$. Internal energy term function is:

$$P(\phi) = \int_{\Omega} \frac{1}{2} (|\nabla \phi| - 1)^2 dx dy. \tag{4}$$

The external energy term is used for moving the contours towards the object boundaries. It uses the edge indicator function g defined as

$$g = \frac{1}{1 + |\nabla G_{\sigma} * I|^2},\tag{5}$$

where G is the two dimensional Gaussian with variance σ . The external energy term consists of the image length and area of the zero level contour of ϕ :

$$\varepsilon_{q,\lambda,v} = \lambda L_q(\phi) + vA_q(\phi),$$
 (6)

where λ and ν are weighting parameters. $L_g(\phi)$ is the length of the zero level curve of ϕ and $A_g(\phi)$ is the area term that speeds up the curve evolution. The length term $L_g(\phi)$ is defined as

$$L_g(\phi) = \int_{\Omega} g\delta(\phi) |\nabla \phi| \, dx dy, \tag{7}$$

and the area term $A_g(\phi)$ is defined as

$$A_g(\phi) = \int_{\Omega} gH(-\phi)dxdy, \tag{8}$$

where δ is the univariate Dirac function and H is the Heaviside function.

In stage I, the surface ϕ evolves under the influence of internal and external energy terms as defined by Equation 3. It is expected that the zero level contours c_1 and c_2 move towards the object boundaries for some time under the image forces. The next section explains how the shape prior information is integrated into the surface evolution.

B. Stage II: The Best Matching Surface as Prior Information

In stage II, the evolving surface ϕ is taken from the image and it is influenced by the shape prior. The shape prior information prevents the contours in the zero level of the surface to merge or break into parts. Also the missing or unrelated part problems in the boundaries can be addressed with the shape prior mechanism.

Our method does not contain any explicit training phases to construct a shape model before the level set evolution. The 3D surfaces that have model shape boundaries in their zero level sets are directly used as prior knowledge by re-initializing the surfaces. After the re-initialization, the new surface ϕ can be evolved with any level set algorithms because it separates the surface evolution and prior knowledge incorporation stages.

For the dual contour extraction process, consider ϕ^{in} as a level set surface and c_1^{in} and c_2^{in} as the inner and outer contours of the zero level set C^{in} of the surface ϕ^{in} . Let m_1^i be the inner contour and let m_2^i be the outer contour of the model boundaries that will be used as shape prior where $0 < i \le n$ and n is the number of models. In our case, the model contours m_1^i and m_2^i are the cardiac walls detected by experts. Our aim is finding the best matching ϕ_i^m and ϕ^{in} where ϕ_i^m is the surface that includes m_1^i and m_2^i as the zero level set.

However, the best matching surface ϕ_i^m cannot be found directly because of the scale, rotation, and translation differences between the zero level contours $m_1^i - c_1^{in}$ and $m_2^i - c_2^{in}$. Therefore, we have to match the contours m_1^i and m_2^i with c_1^{in} and c_2^{in} , respectively. This match will also produce a transformation that would warp m_1^i and m_2^i into wm_1^i and wm_2^i . These contours are called the wrapped model contours and they supply prior information from the expert contours to the contours extracted by the level set method. We need to find the best matching $c_1^{in} - wm_1^i$ and $c_2^{in} - wm_2^i$ so that the most relevant prior information is incorporated into the level set method. Instead of measuring the match scores between $c_1^{in} - wm_1^i$ and $c_2^{in} - wm_2^i$, we propose to measure the match score between the surfaces ϕ^{in} and $w\phi_i^m$, where $w\phi_i^m$ is the level set surface that includes wm_1^i and wm_2^i as the zero level set.

In this surface matching technique, we first translate the center of all contours (model and C^{in}) to origin and then we convert the contours to Polar coordinate (θ,r) space. We define two functions $R_i(c)$ and $\Theta_i(c)$ that returns the r value and θ value of the i^{th} point of contour c, respectively. These steps remove the translation differences between the contours.

To remove scale differences between two given contours c_j and c_k , we propose a piecewise uniform scaling approach by using only a neighborhood of contour positions of size h. These local average r values are compared to the contours for calculating the local scaling amount.

$$S(c_j, c_k, \theta) = \frac{\sum_{i=-h/2}^{h/2} R_i(c_j)}{\sum_{i=-h/2}^{h/2} R_i(c_k)},$$
(9)

where $\theta = \Theta_p(c_j) = \Theta_q(c_k)$ for some p and q.

We transformed one contour to another by multiplying the r positions of the contour with the local scaling factors. Given two contours c_j and c_k , we can produce the warped contour wc_j by deforming c_j to c_k using the following formula

$$R_i(wc_j) = R_p(c_k)S(c_j, c_k, \theta), \tag{10}$$

where $\theta = \Theta_p(c_i) = \Theta_q(c_k)$ for $\theta = 0...2\pi$.

Each model contour c_i^m is warped to C^{in} using the transformation defined by Equation 10. Then we construct the new warped surfaces $w\phi_i^m$ from the warped contours wm_1^i and wm_2^i by using the following equation:

$$w\phi_i^m(x) = \begin{cases} -d(x), & \text{if } \mathbf{x} \text{ is outside } wm_1^i \text{ but inside } wm_2^i \\ 0, & \text{if } \mathbf{x} \in wm_1^i \text{ or } wm_2^i \\ d(x), & \text{otherwise.} \end{cases}$$
 (11)

Finally, we compare each model surface $w\phi_i^m$ with ϕ^{in} by

$$D(\phi_1, \phi_2) = \sum_{x} |\phi_1(x) - \phi_2(x)|, \qquad (12)$$

for all possible x positions. The surface $w\phi_i^m$ that has the smallest $D(w\phi_i^m,\phi^{in})$ value is chosen as ϕ^{out} which is the level set surface that will be re-initialized on the image for deformation process.

C. Implementation and Algorithm Details

In the beginning $c_1^{in}(0)$ and $c_2^{in}(0)$ are initialized on the image at t=0. c_1^{in} should be placed inside the endocardium and c_2^{in} should be placed outside the epicardium.

In stage I, the surface ϕ is constructed using Equation 1 by embedding c_1^{in} and c_2^{in} into the zero level of ϕ . Then ϕ is evolved on image by calculating the level set functional including the internal and external energy terms (Equation 3) until t reaches a threshold value.

In stage II, the surface ϕ^{in} having the contours $c_1^{in}(t)$ and $c_2^{in}(t)$ in its zero level is borrowed from stage I. The aim in Stage II is selecting the best matching 3D model surface ϕ_i^m . The model contours are scaled according to Equation 10 and the new model surfaces $w\phi_i^m$ are formed by Equation 11. Then the best matching surface $w\phi_i^m$ is selected by Equation 12 as ϕ^{out} . The new surface ϕ^{out} is re-initialized on the image and stage I repeats. If the contours stop deforming, the evolution is complete and the object boundaries are the zero level set of the final ϕ ; otherwise stage II repeats.

III. EXPERIMENTAL RESULTS

The system is validated using real and synthetic images. The experiments on synthetic images indicated the robustness of our system under controlled levels of varying noise. They also provided a mechanism of comparing the system results with ground truth. The experiments of the real images were performed on the images of left ventricle taken by MR and ultrasound modalities. These experiments were very useful to show the practical applicability of our system in real life.

A. Synthetic Images

The proposed contour extraction system uses expert contours as prior knowledge. In order to test the ability to recover the boundaries of the objects accurately in a noisy environment, various shaped objects are used as shape priors.

Two kinds of synthetic images containing a ring shaped object and a rectangular shaped object were used. The images are 100 by 100 pixels in size and have 50 gray levels contrast. They were corrupted with missing sections and various amounts of noise (Gaussian, salt and pepper, and speckle). For all experiments, we used the same model contours as the prior knowledge which included a concentric circle pair and a concentric rectangle pair.

We ran our system by initializing one contour inside the ring and another outside the ring. Some examples of the synthetic images, the initial contours, and the final boundaries are shown in Fig. 2. The first two columns show the level set method without using shape priors. The missing parts in the object boundaries cause the contours go into topological changes and it is obviously seen that the shapes with missing parts cannot be segmented without prior information. The third

and fourth columns show the successful segmentations when the shapes of the initial contours are same with the detected object contours. In the last two columns, initial contours were deliberately chosen different from the objects contained in the image. The results showed that even in this challenging test our system can recover the objects in images under the influence of prior information.

Table I includes the numerical results for the synthetic images segmented with different shaped initial contours and various shape priors. The results show that our level set with shape prior behaves very stable under challenging image noise and missing object sections.

B. Application to Echocardiograms and Cardiac MRI

Recovering cardiac walls from MR and ultrasound cardiac images produces crucial information. We tested our system on the MR and echocardiographic images in this real world task.

Our previous system [9] was only tested with echocardiographic images of the left-ventricular (LV) short-axis transthoracic views during the cardiac cycle. In order to show the effectiveness of the proposed system, we compared our results with our previous results. The echocardiographic images are 190 by 240 pixels in size. 4 different cardiac contours delineated by experts are used for creating the model surfaces.

In order to produce numerical values, the results we have found were compared with the expert delineations because it is not easy to obtain ground truth for echocardiograms. The delineations of experts are also compared with each other to obtain the variation between experts. Our new level set method based on matching 3D surfaces produces better segmentation results compared with the previous results [9] for echocardiographic images. Fig. 3 shows the automatically recovered inner and outer contours and expert detected contours. Table II shows the numerical variations between experts and our system. We can conclude that our system is within the interexpert variation numbers.

Finally, we tested our system with cardiac MR images by extracting inner and outer cardiac walls. The cardiac MR images are not as noisy as echocardiographic images. However, there are low contrast wall sections and unrelated high contrast structures around the heart walls. The cardiac MR images are 256x256 pixels in size. We used the automatically generated contours of Stegmann [10] as model contours. The automatically detected contours by our system are shown in Fig. 4. The visual inspection of our results shows that our system is also very robust on cardiac MR images.

IV. CONCLUSIONS

We proposed a novel segmentation method with level sets based on shape priors. The method proposes using level set evolution to incorporate local image information from the images. It also uses a separate phase to incorporate global or prior information into the final contours. The incorporation of local and global information is done by two repeating separate stages, which brings many advantages to the system. First, the complexity of the main level set functional is not increased

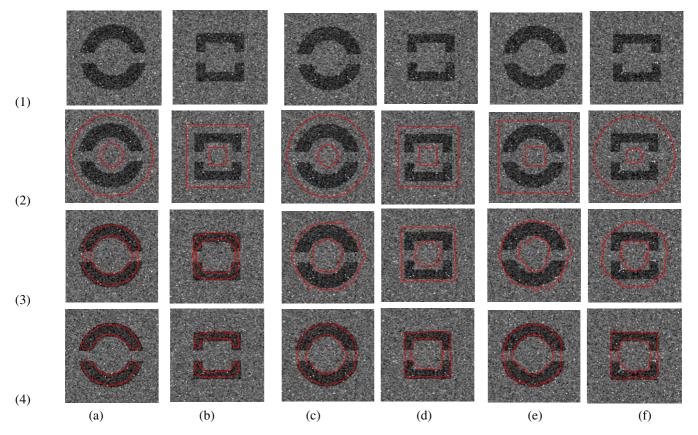


Fig. 2. (1) Synthetic images, (2) Initial contours, (3) Contours while evolving, (4) Finally extracted contours, (a,b) Extraction without prior information, (c,d) Extraction with same shaped initial contours with various shape priors.

 $\label{table I} \mbox{Average pixel errors for synthetic images}$

1			Noise Levels								
-	2	3	4	5	6	7	8	9	10		
0	0	0.01	0	0	0.01	0.01	0.01	0.02	0.02		
0	0	0	1	0	1	1	1	2	2		
0	0	0	0	0.01	0.01	0.01	0.01	0.02	0.02		
0	5	0	0	0	0	5	10	10	20		
).63	0.81	1.37	1.39	1.47	1.18	1.77	2.25	2.06	2.70		
1.20	1.09	1.76	1.53	1.56	1.76	1.68	1.71	1.63	2.95		
1.01	1.17	0.82	0.95	1.00	0.81	1.28	2.70	2.56	3.15		
1.50	1.61	1.63	1.65	1.59	1.67	2.24	2.33	2.39	2.40		
	.01	0 0 0 0 5 0 5 0.63 0.81 1.20 1.09	0 0 0 0 0 0 0 5 0 0.63 0.81 1.37 1.20 1.09 1.76	0 0 0 0 1 0 0 0 0 0 0 5 0 0 0.63 0.81 1.37 1.39 1.20 1.09 1.76 1.53	0 0 0 1 0 0 0 0 0 0.01 0 5 0 0 0 0.63 0.81 1.37 1.39 1.47 1.20 1.09 1.76 1.53 1.56 1.01 1.17 0.82 0.95 1.00	0 0 0 1 0 1 0 0 0 0 0.01 0.01 0 5 0 0 0 0 0.63 0.81 1.37 1.39 1.47 1.18 1.20 1.09 1.76 1.53 1.56 1.76 1.01 1.17 0.82 0.95 1.00 0.81	0 0 0 1 0 1 1 0 0 0 0 0.01 0.01 0.01 0 5 0 0 0 0 5 0.63 0.81 1.37 1.39 1.47 1.18 1.77 1.20 1.09 1.76 1.53 1.56 1.76 1.68 1.01 1.17 0.82 0.95 1.00 0.81 1.28	0 0 0 1 0 1 1 1 0 0 0 0 0.01 0.01 0.01 0.01 0 5 0 0 0 0 5 10 0.63 0.81 1.37 1.39 1.47 1.18 1.77 2.25 1.20 1.09 1.76 1.53 1.56 1.76 1.68 1.71 1.01 1.17 0.82 0.95 1.00 0.81 1.28 2.70	0 0 0 1 0 1 1 1 2 0 0 0 0 0.01 0.01 0.01 0.01 0.02 0 5 0 0 0 0 5 10 10 0.63 0.81 1.37 1.39 1.47 1.18 1.77 2.25 2.06 1.20 1.09 1.76 1.53 1.56 1.76 1.68 1.71 1.63 1.01 1.17 0.82 0.95 1.00 0.81 1.28 2.70 2.56		

 $\label{eq:table_in_table} \text{TABLE II} \\ \text{Average pixel errors for Fig. 3}$

Endocardium Errors					Epicardium Errors						
	Expert2	Expert3	Expert4	Old system[9]	New system	Expert2	Expert3	Expert4	Old system[9]	New system	
Expert 1	2.08	3.63	1.82	4.07	2,07	2.57	1.92	6.18	3.81	2.89	
Expert 2		2.73	1.47	3.89	2.13		3.03	6.17	4.31	3.65	
Expert 3			2.27	3.51	2.88			6.75	4.49	2.62	
Expert 4				3.32	2.37				4.65	6.07	

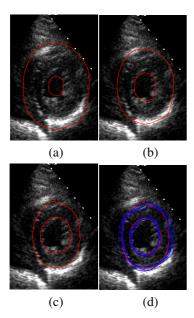


Fig. 3. (a) The initial contours c_1 and c_2 , (b) Evolving contours, (c) Automatically extracted contours by our system, (d) The blue contours are manually detected contours by 4 different experts and red contours are automatically detected contours.

with the additional shape prior terms. This makes our system applicable with any level set formulation and it increases its flexibility. Second, the separate incorporation of prior or global information keeps the computational complexity of the overall system at very low level due to our simpler level set formulation. We observed that enforcing global or prior information in the final contours increases the overall computational time by only 5%. Finally, the system uses a novel technique for shape prior incorporation based on 3D level set surfaces which are created by the best matching surface technique from expert detected contours. Since the best matching surfaces created from expert contours are directly used as model knowledge, there is no explicit training phase to create a model knowledge.

We tested our system with real cardiac images and synthetic images. The experiments with synthetic images showed the robustness of the method with various shape models under various noise levels and missing parts in the objects. The results for the cardiac images showed that the segmentation results are promising and the system is applicable in the real world. We plan to employ the proposed system with different 3D surface matching techniques for other medical imaging applications.

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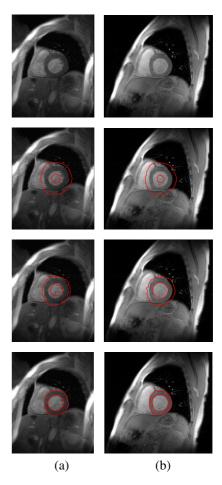


Fig. 4. Column (a) shows one MR image, column (b) shows another. The top row shows the original images. The second row shows the initial contours. The contours during evolution are shown in third row. The bottom row shows the final contours extracted by our system.

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