Minicourse on *information, complexity and organisation in multidimensional symbolic dynamics*

Effect of dynamical constraints on the computational power of multidimensional SFT

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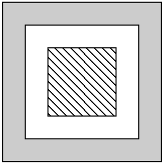
Reminder: the characterization of entropies of multidimensional SFT

Theorem[M.Hochman,T.Meyerovitch]: for all $d \geq 2$, the possible values of entropy for d-dimensional SFT are the non-negative Π_1-computable numbers.

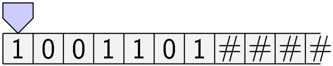
Π_1-computable: exists an algorithm which on input n outputs $r_n \in \mathbb{Q}$ s.t. $r_n \downarrow x$.

Question: what are the possible values for entropy on **irreducible** multidimensional SFT ?



Dynamical constraints and computational power: analogy with the human brain

Classes of dynamical systems *containing* universal computation:



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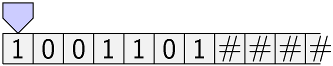

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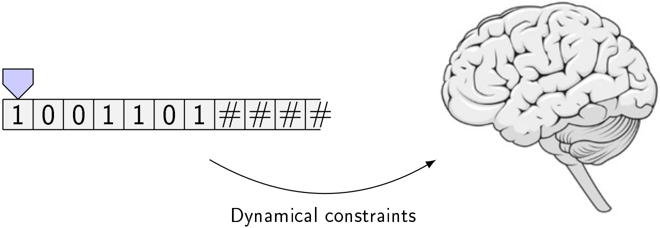

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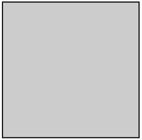

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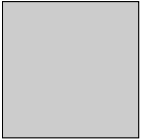

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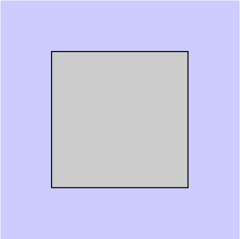

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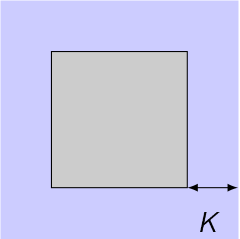

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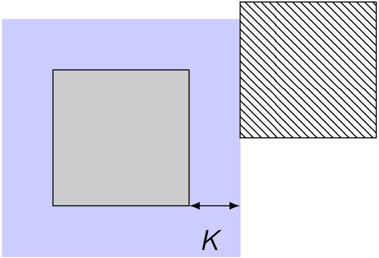

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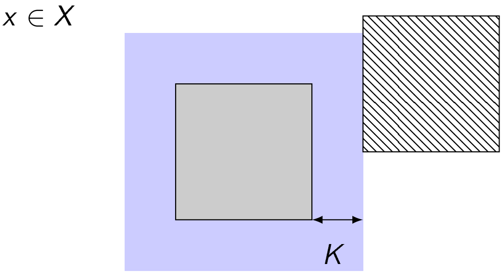

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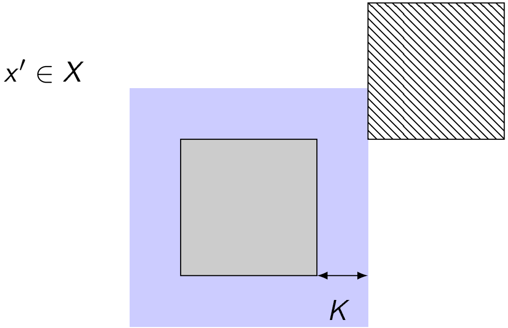
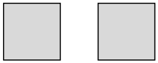


Density of periodic points

Theorem: If X is a 2-dimensional block gluing SFT, its periodic configurations are dense.

Proof: For nearest neighbour subshifts:



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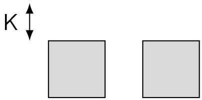

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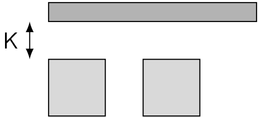

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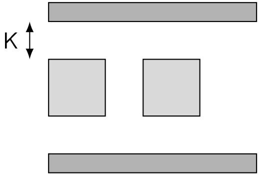

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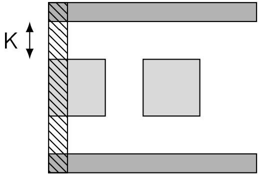

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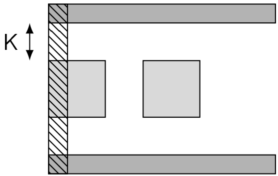

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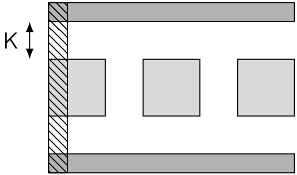

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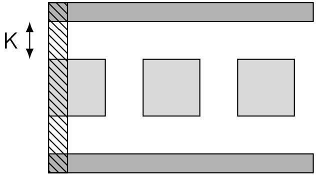

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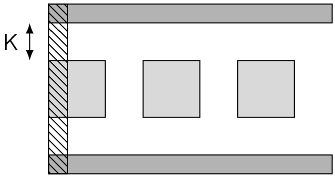

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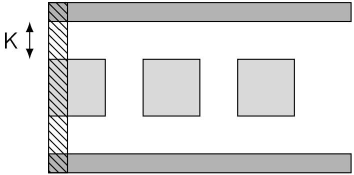

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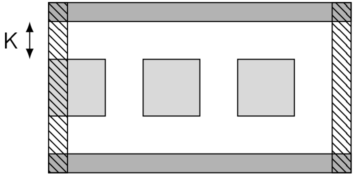

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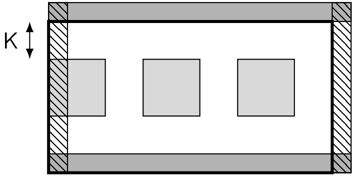

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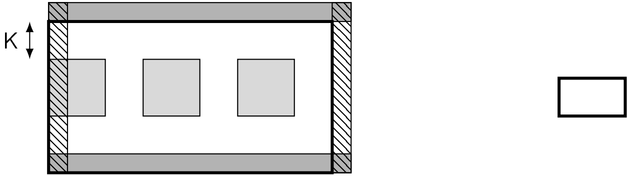

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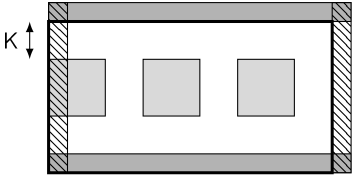

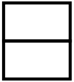

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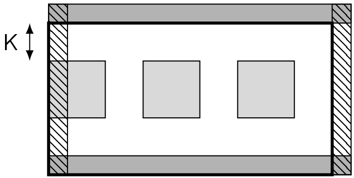

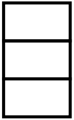

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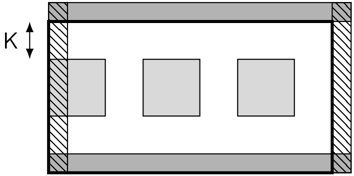

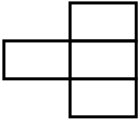

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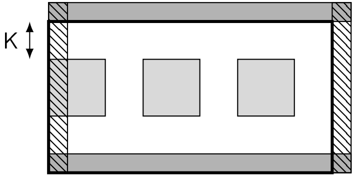

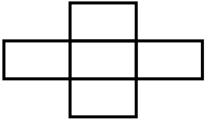

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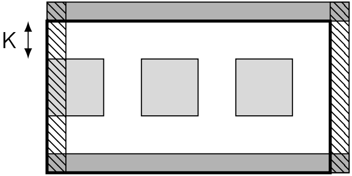

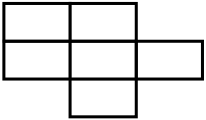

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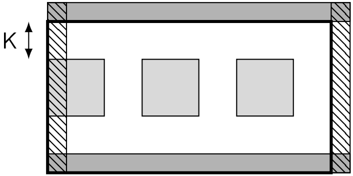

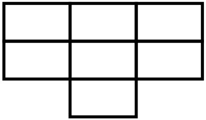

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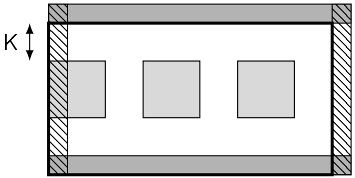

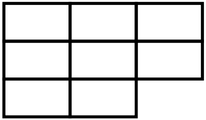

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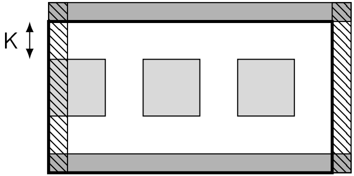

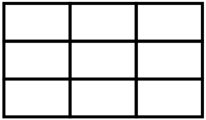

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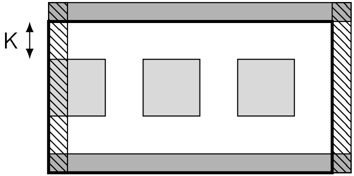

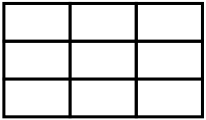

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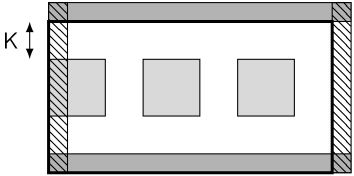

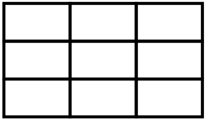

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Question: what happens in dimension 3 ?

Computability of entropy for block gluing SFT

Reminders: x is computable when there is an algorithm which on input n outputs r_n s.t. $|x - r_n| \leq 2^{-n}$.

For X d-dim. subshift:

$$h(X) = \lim_n \frac{\log(N_n(X))}{n^d},$$

where $N_n(X)$ is the number of n-blocks appearing in configurations of X.

Computability of entropy for block gluing SFT

Theorem: For X a 2-dimensional block gluing SFT: $h(X)$ is a computable number.

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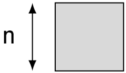

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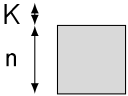

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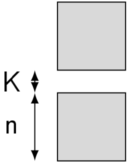

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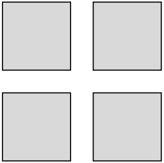

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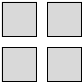

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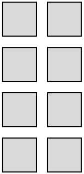

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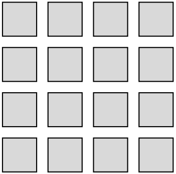

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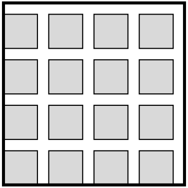

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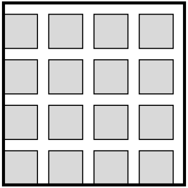

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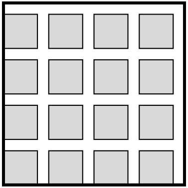

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$$\frac{(mn)^2}{(m(n+K))^2}\frac{\log(N_n(X))}{n^2} \leq \frac{N_{m(n+K)}(X)}{(m(n+K))^2}.$$

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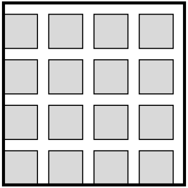

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When $m \to +\infty$:

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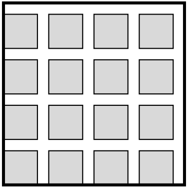
Computability of entropy for block gluing SFT

Theorem: For X a 2-dimensional block gluing SFT: $h(X)$ is a computable number.

Proof:



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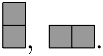
Consider X_0 on $\mathcal{A} = \{\square, \blacksquare\}$, and $\mathcal{F}$ the following patterns:



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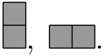

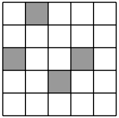

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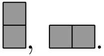

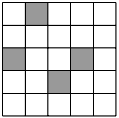

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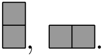

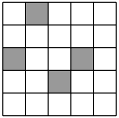

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Question: closed formula for the $h(X_0)$?

The result of R.Pavlov and M.Schraudner

Computability condition $(*)$: x rational or has an infinite continuous fraction expansion $[a_0; a_1; ...]$ such that for (t_n) s.t. $t_0 = 1$, $t_1 = a_1$ and

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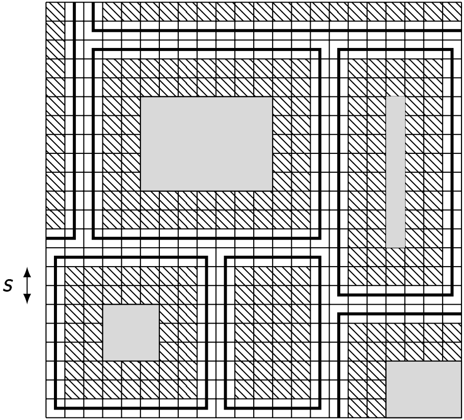
Entropies realizable by block gluing $\mathbb{Z}^d$ shifts of finite type, **R.Pavlov, M.Schraudner**.

Schema of proof: an approach by operators

Let us fix X subshift of finite type.
Denote X_s the following subshift:



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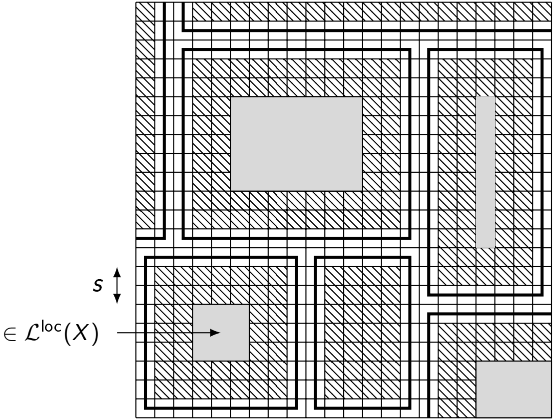

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Definition: X **upgradable**: for all k, l,
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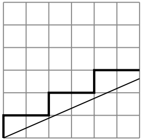

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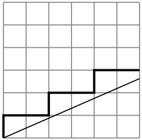

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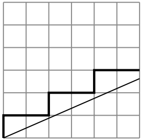

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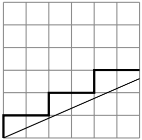

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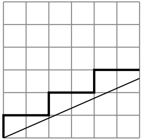

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Stacking copies of this subshift: $\leq (n+1)2^{\alpha n^2 + n}$ (upgradable).

The question of the limit between computability and uncomputability.

1. Two "computability regimes": w/wo block gluing. How to characterize the limit ?

2. What are the exact conditions under which uncomputability phenomena can appear ?

Quantification of block gluing

Consider a function $f : \mathbb{N} \to \mathbb{N}$, non-decreasing and computable.

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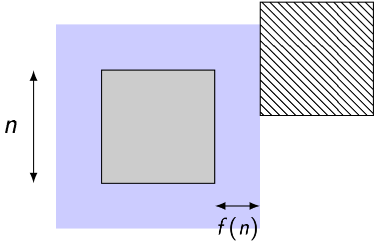

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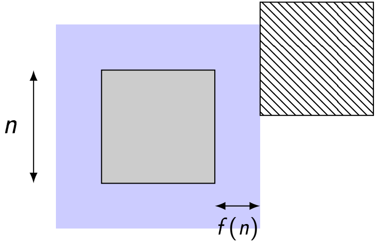
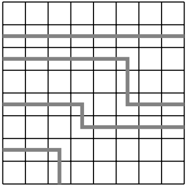


Question: for which functions f the entropy (resp. language) is forced to be computable (resp. decidable) ?

An example of linear block gluing SFT



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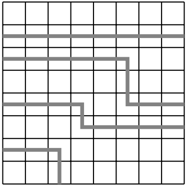

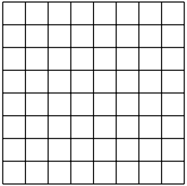

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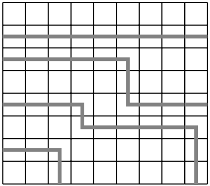

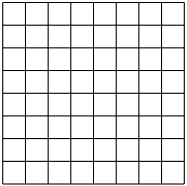

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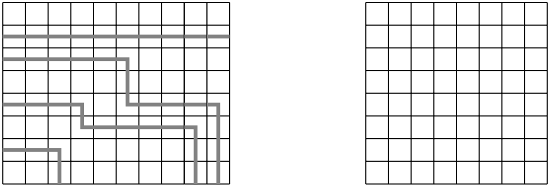

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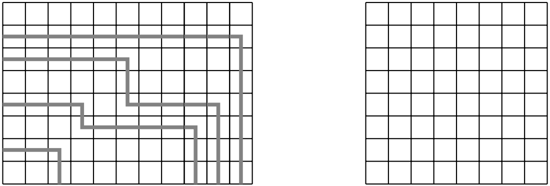

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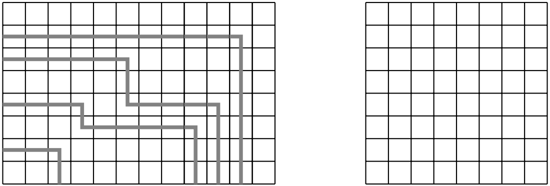

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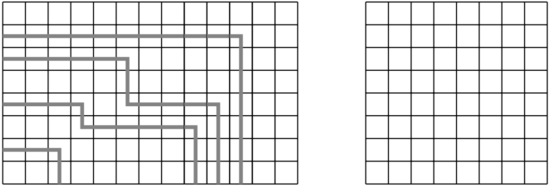

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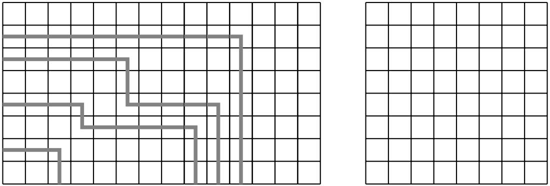

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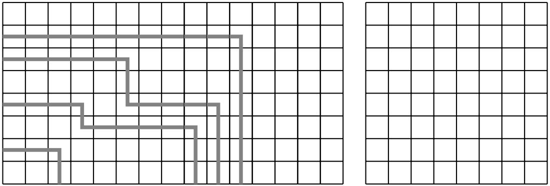

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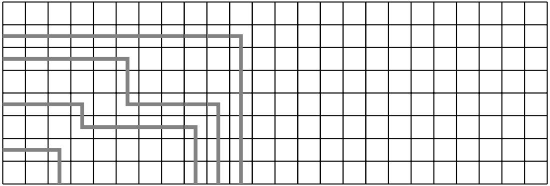

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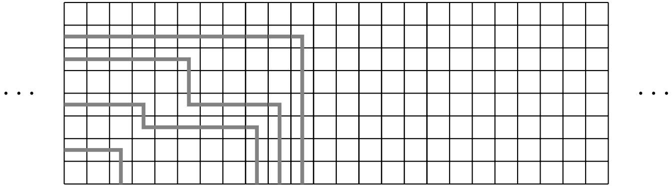

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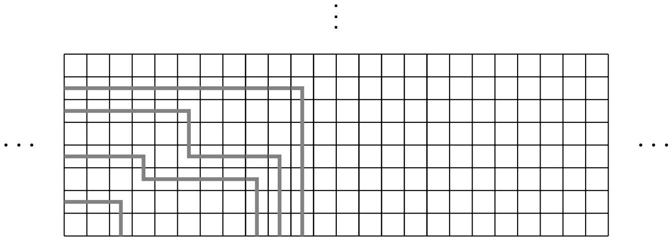

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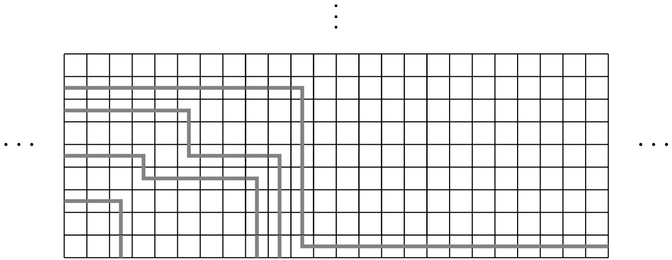

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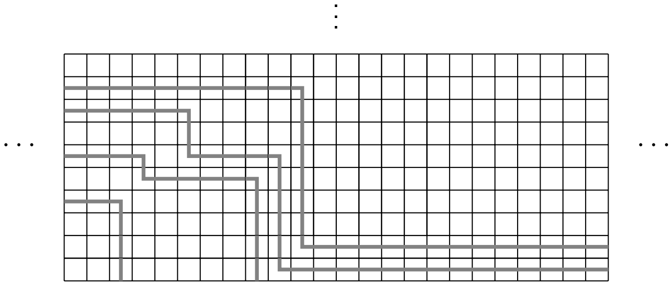

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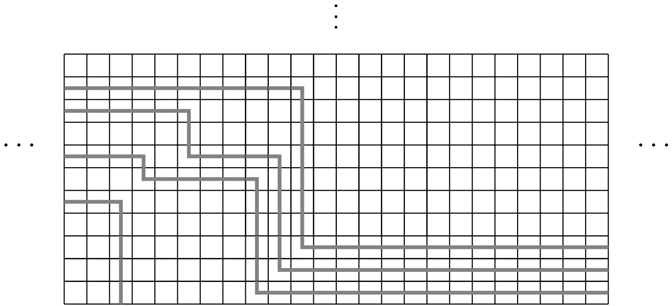

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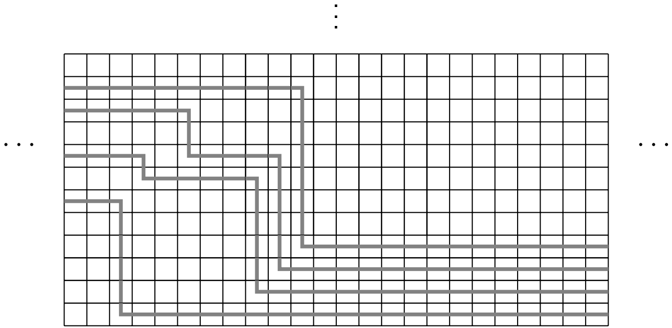

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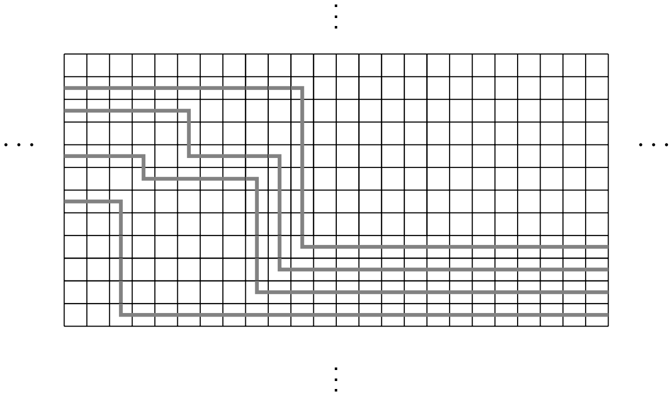
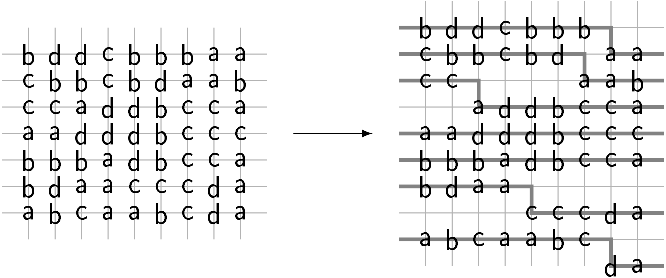


Distortion operator

Transformation on configurations of X subshift on alphabet $\{a, b, c, d\}$:



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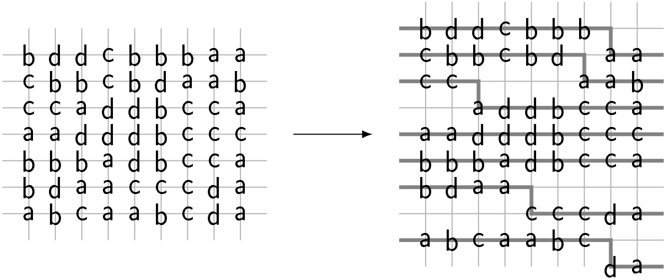

Induces an operator $\mathcal{T}$ on subshifts.

Gluing on sublattices

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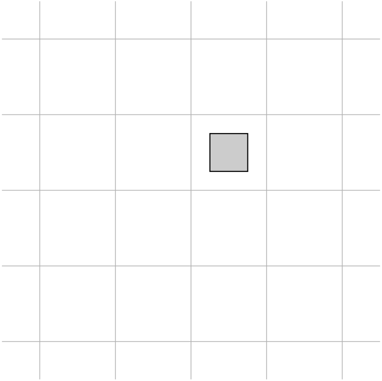

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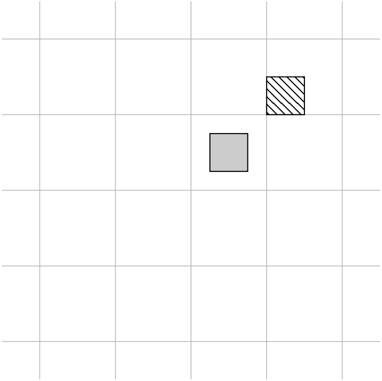

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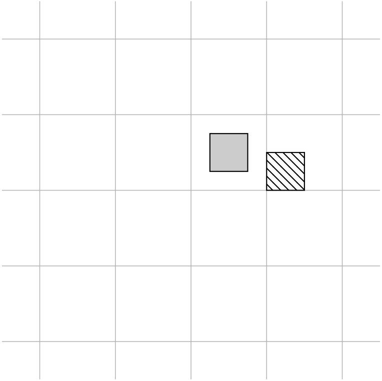

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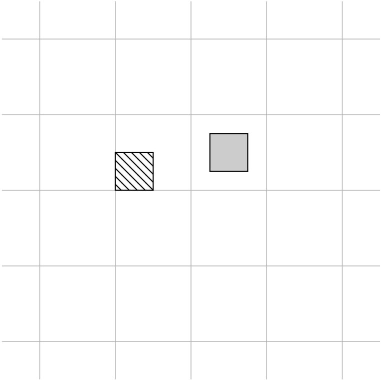

Distortion and block gluing

Theorem: if X has linear block gluing on sublattices, $\mathcal{T}(X)$ has linear block gluing on infinite vertical stripes separated by $O(n)$ columns.

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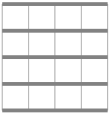

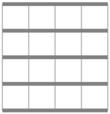

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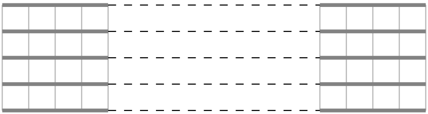

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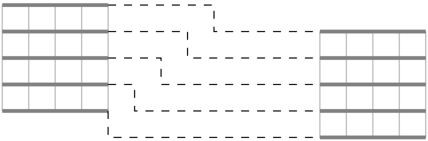

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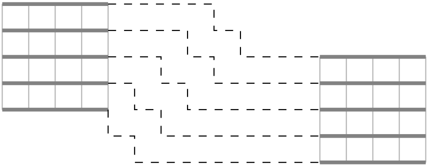

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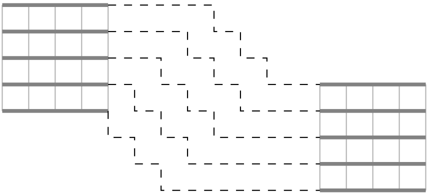

Distortion and aperiodicity

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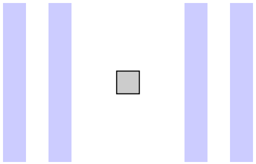

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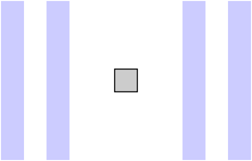
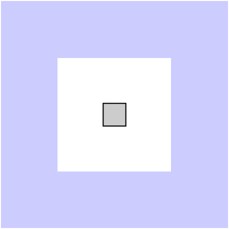


Then a horizontal version of the operator:



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Proof: apply the distortion operator on Robinson tilings.

Also: there exists a bidimensional linear block gluing SFT with undecidable language.

Regime transition for bidimensional SFT: entropy

Theorem[G.,Sablik]: for all $d \geq 2$, the possible values of entropy for linear block gluing d-dimensional SFT are the Π_1-computable non-negative real numbers.

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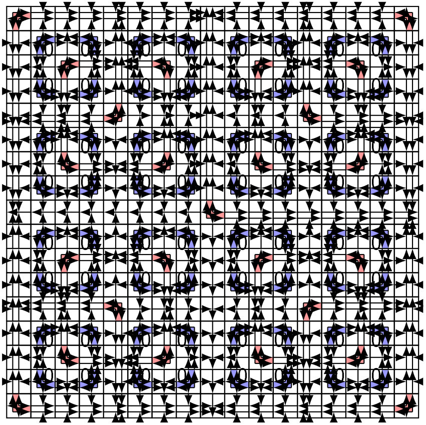
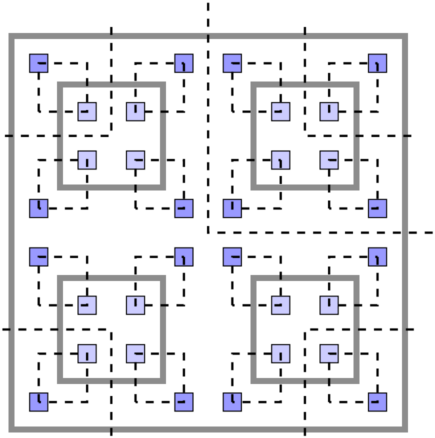
See *Quantified block gluing, aperiodicity and entropy for multidimensional SFT*, **S.Gangloff, M.Sablik**.

Proof of the characterization: first layers reminder

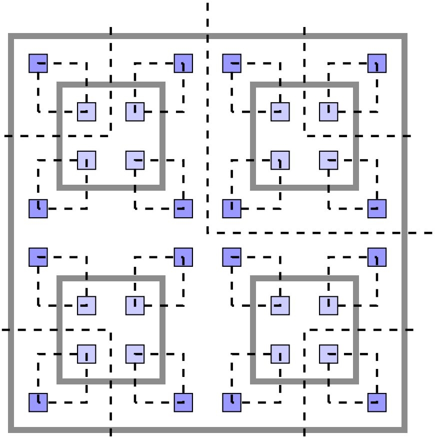
Hierarchical structures and computation areas:





Proof of the characterization: first layers reminder

Implementation of computing machines in hierarchical structures:



a'_0	a'_1	a'_2	$q^h_{a_3}$	a_4	a_5	a_6	a_7
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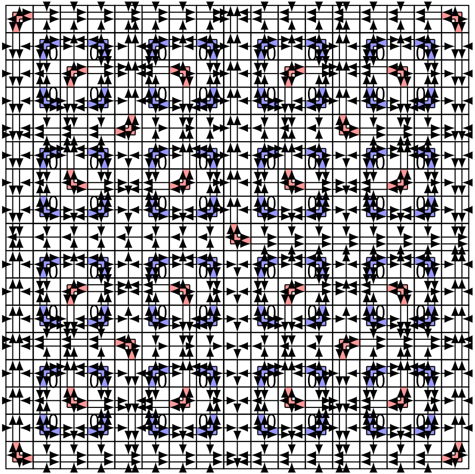
Proof of the characterization: first layers reminder

Control sets:



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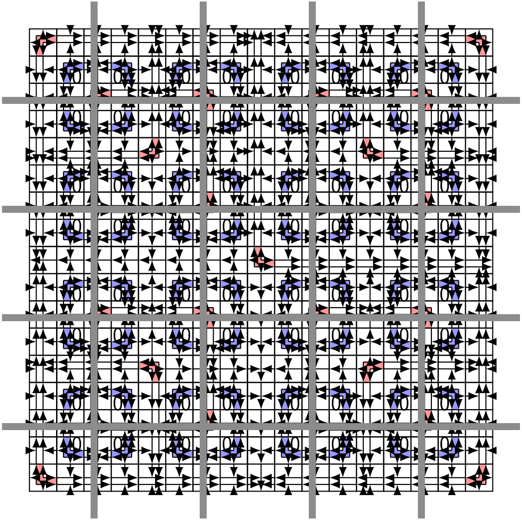

$\Lambda_0,\ \epsilon_0$

Proof of the characterization: first layers reminder

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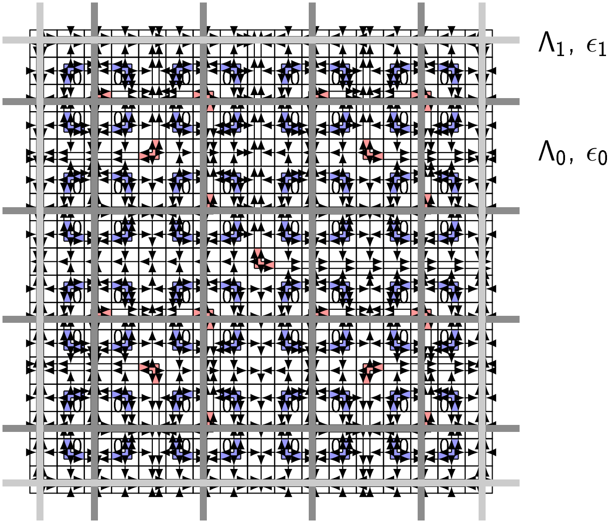
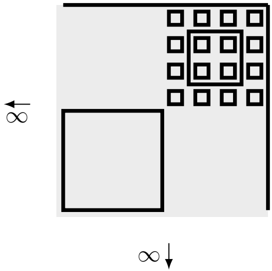


Obstacles to linear block gluing

1. **Degenerated behavior** of the computing machines (infinite computation areas, without initialisation).
2. **Rigidity** of hierarchical structures.



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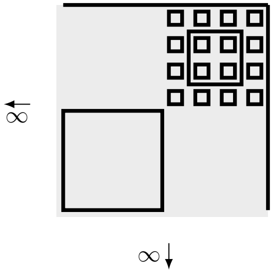

Solutions:

1. **Simulate** degenerated behaviors of the machines everywhere.

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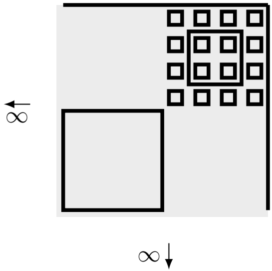
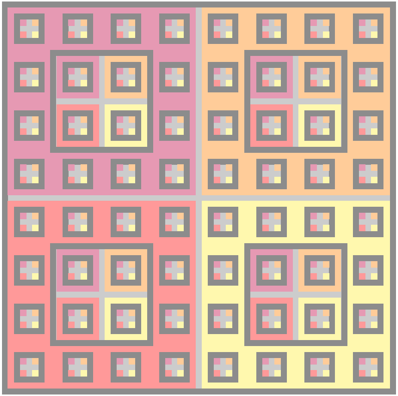


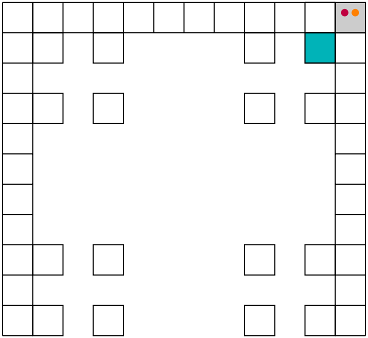
Solutions:

1. **Simulate** degenerated behaviors of the machines everywhere.
2. Use a **distortion operator** to render the structure more *flexible*.

Simulating degenerated behavior: functional subdivision



Simulating degenerated behavior: error signals



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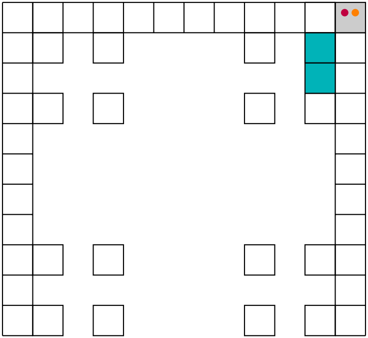

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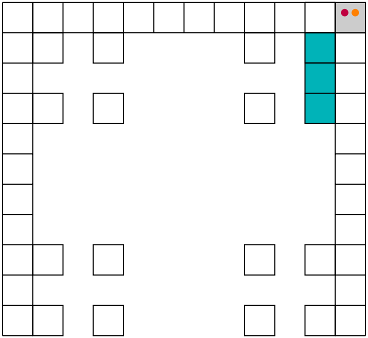

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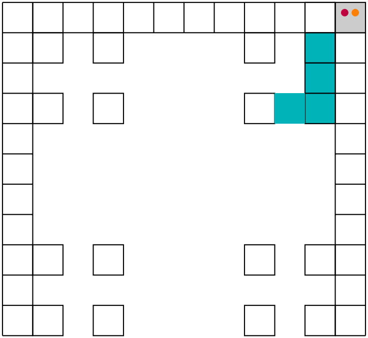

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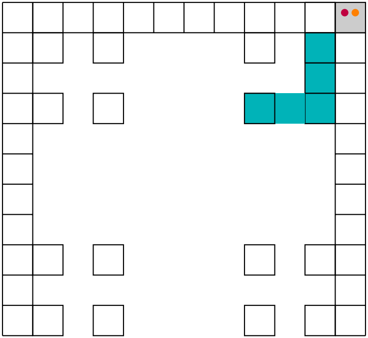

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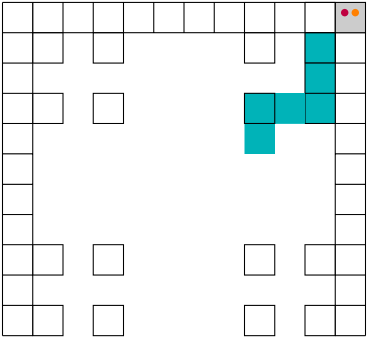

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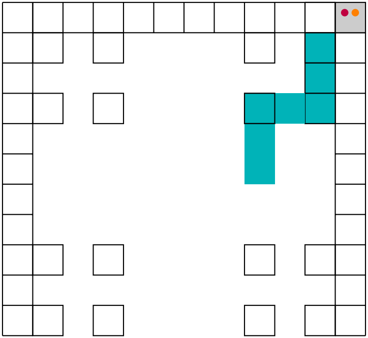

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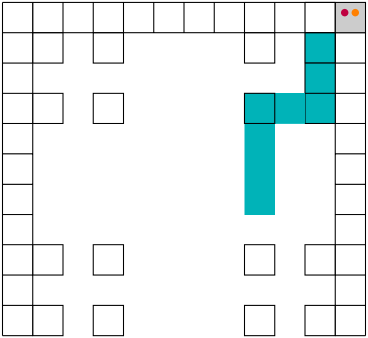

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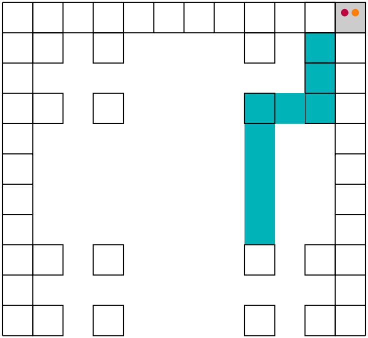

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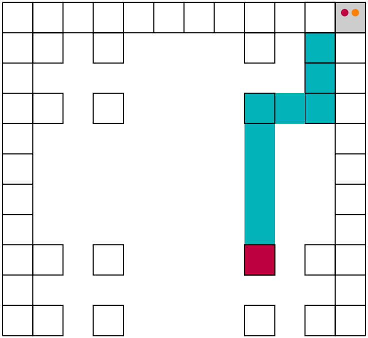

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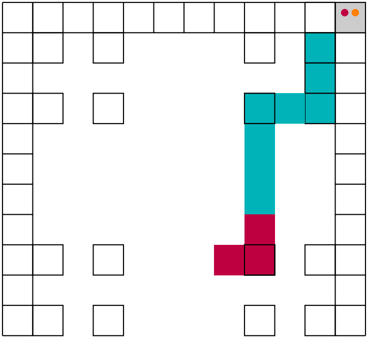

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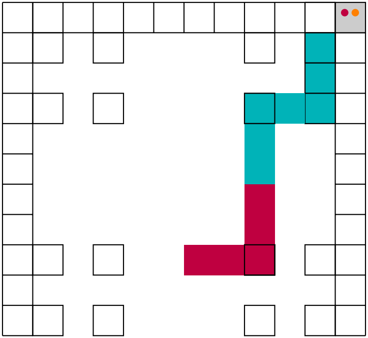

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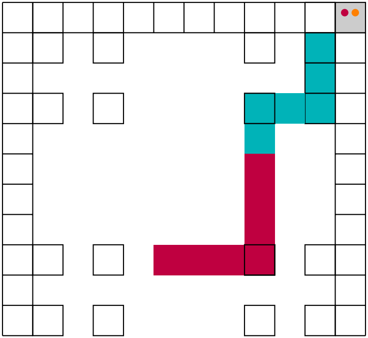

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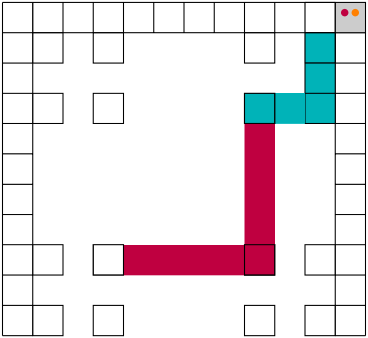

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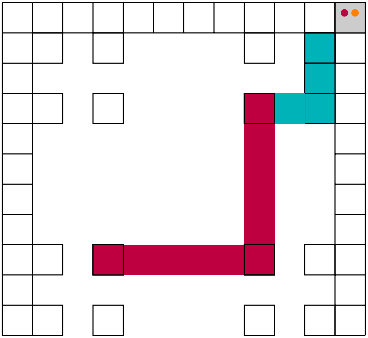

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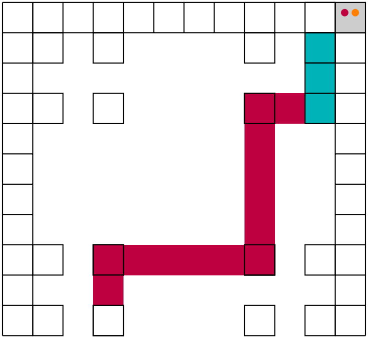

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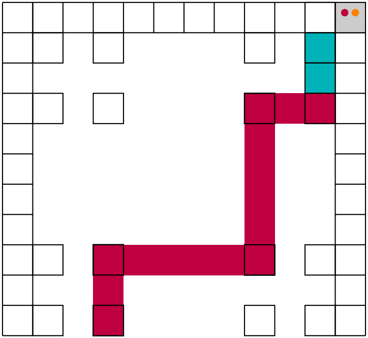

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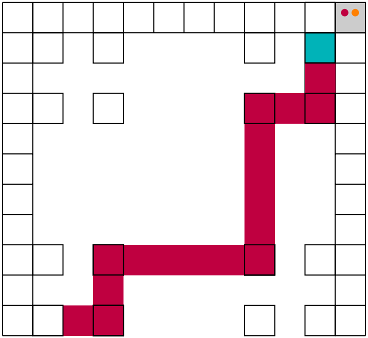

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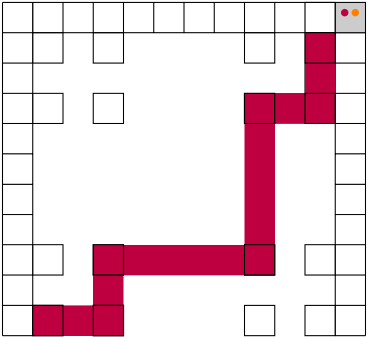

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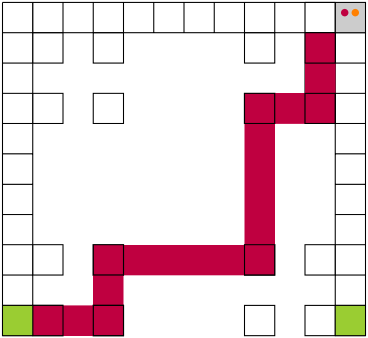

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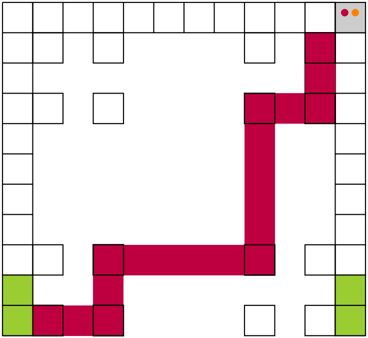

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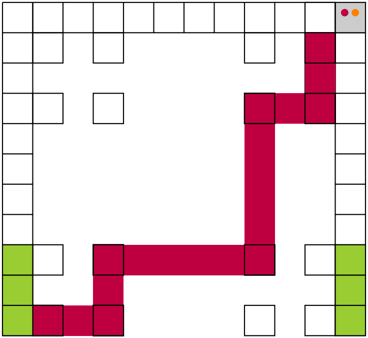

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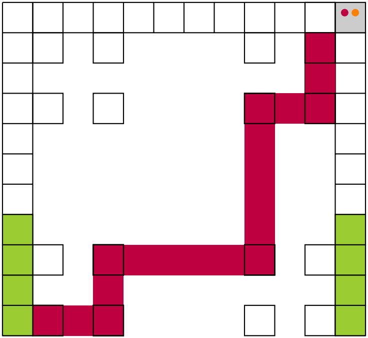

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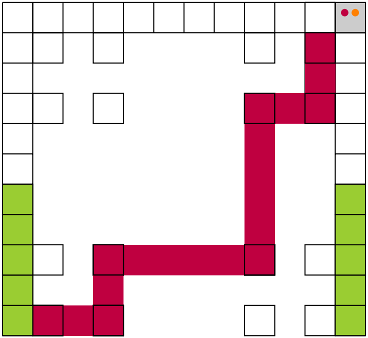

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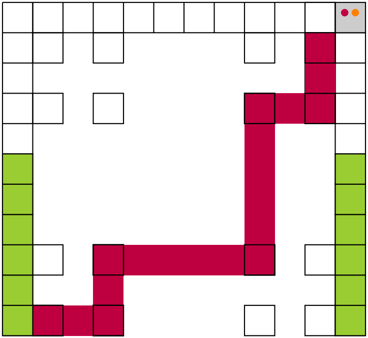

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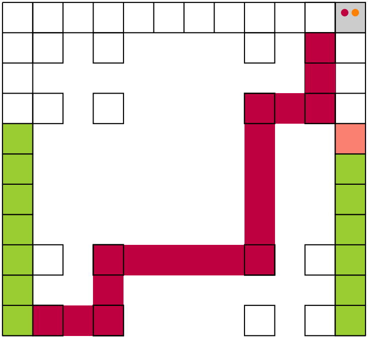

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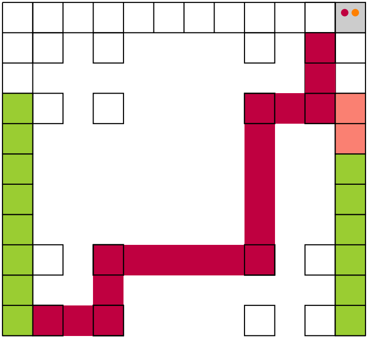

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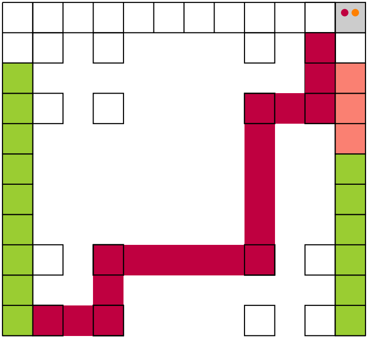

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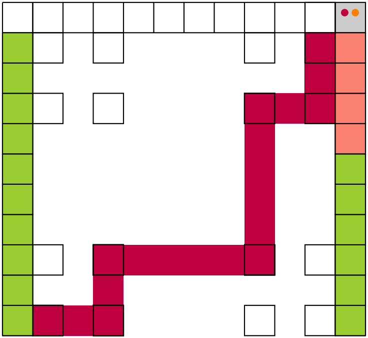

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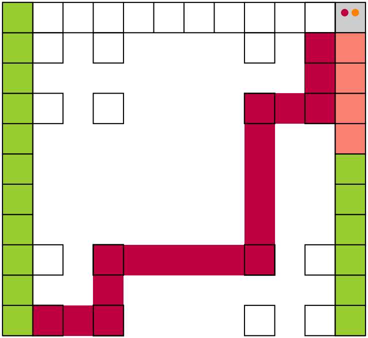

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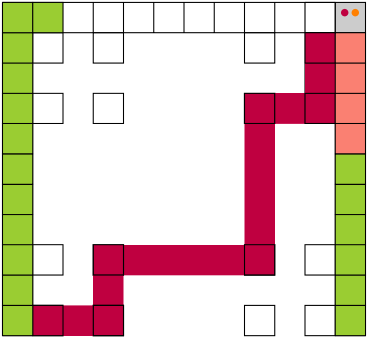

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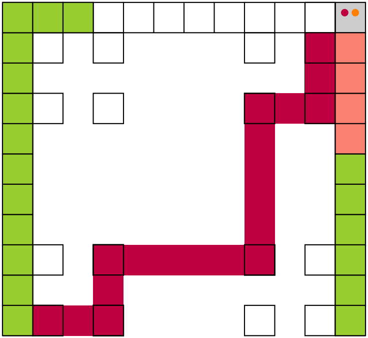

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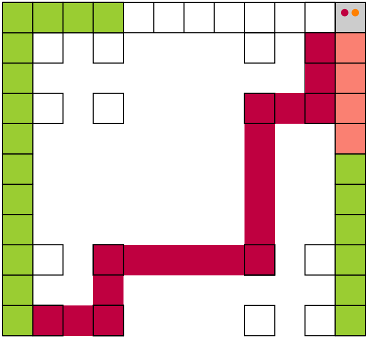

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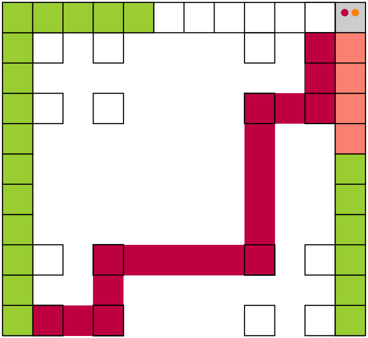

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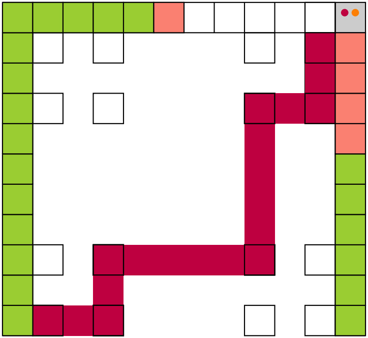

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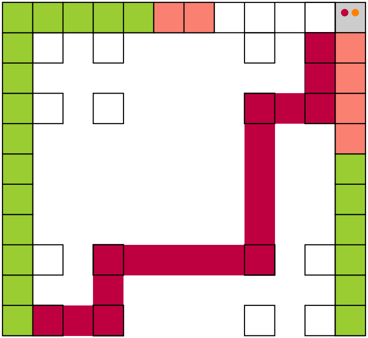

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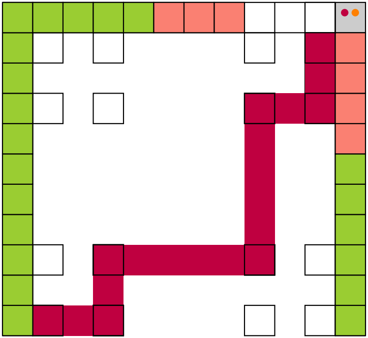

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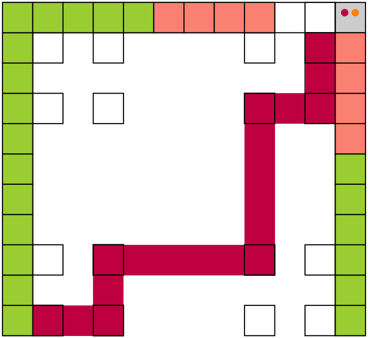

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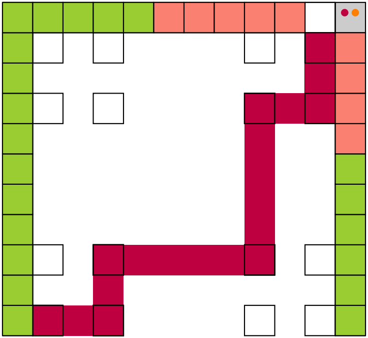

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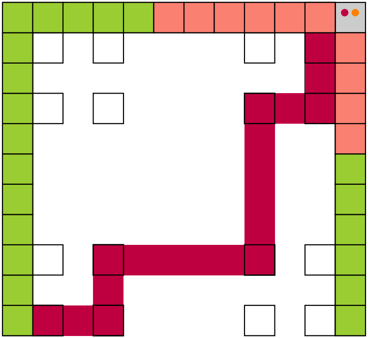
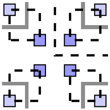


Linear block gluing: completing patterns

Completing a pattern p:



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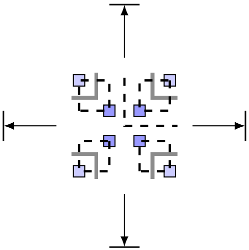

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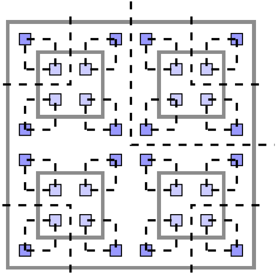
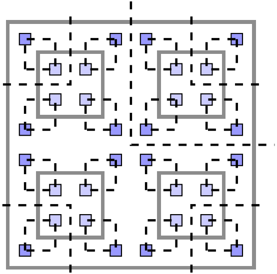


Two cases:

Linear block gluing: completing patterns

Completing a pattern p:



Two cases:

1. Encoding part $\in p$: complete machine layer according to it.

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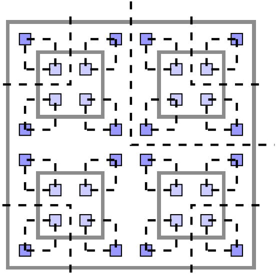

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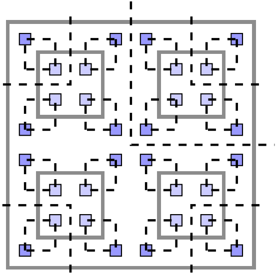

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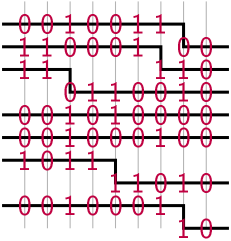
As a consequence, the construction has gluing property on sublattices.

Distortion operator and random bits

Adding random bits $\rightarrow$ operator $\mathcal{T}'$:



With this we have $h(\mathcal{T}'(X)) = h(X) + 1$.

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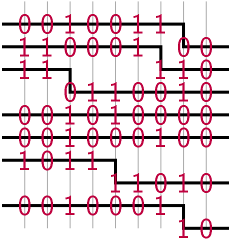

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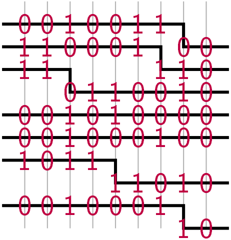

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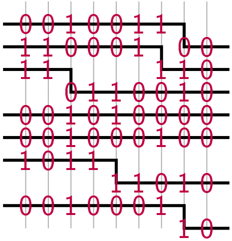

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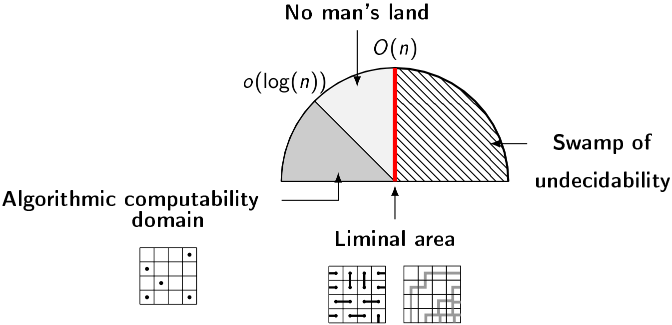
we have:

$$h(\mathcal{T}_r'(X)) = h(X) + \frac{\log(1+r)}{r}$$

Abstract of the results:



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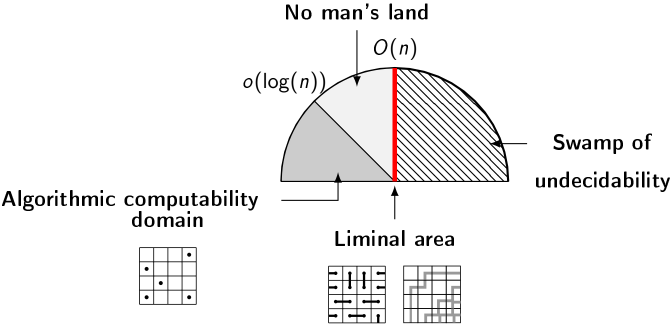

Question: how to discriminate subshift of finite type with computable entropy/non-computable entropy on the liminal area ?

The question of intermediate gap functions

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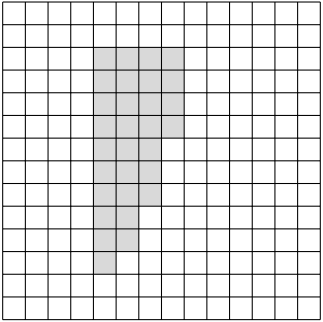

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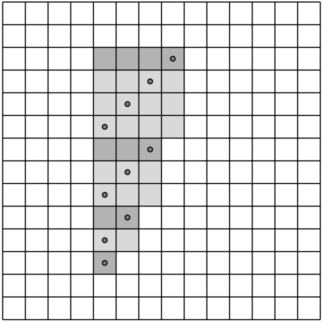

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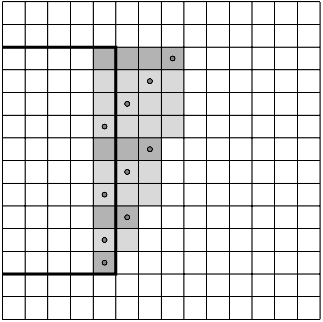

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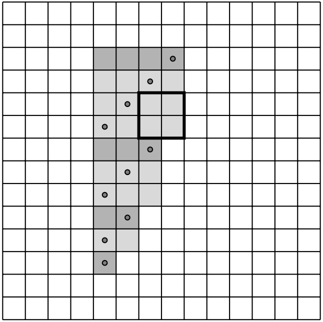

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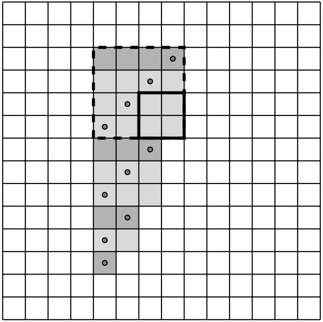

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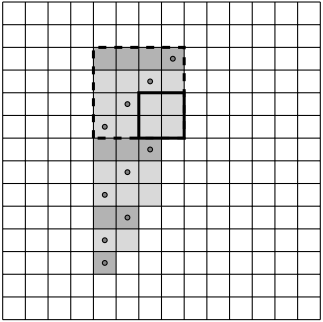

Problems: it is actually linear block gluing.

Universal computation in minimal multidimensional SFT

Reminder: entropy dimension (when exists):

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3. Using Fermat numbers $2^{2^n} + 1$ as periods (co-primes, encoding respecting minimality).

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Reverse approach: given a system, how to identify *functional parts*.

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Formalising these phenomena $\rightarrow$ development of a formalism for:

1. limits on information transport imposed by space-time.
2. how dynamical constraints prevent enforcing universal computation *in* configurations.