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Education and jobs

▷ **Current position:** 2018-2019: Post-doctoral researcher at LIP, ENS Lyon.

▷ 2015-2018 : **PhD**, in Computer science and Mathematics, at University of Aix-Marseille. **Title**: Computability of the growth-type invariants of subshifts of finite type under dynamical restrictions. The manuscript can be found here.

▷ 2011-2015 : **ENS Paris**. Master in fundamental mathematics, University Pierre et Marie Curie. Agrégation de mathématiques.

Publications

Most of the articles are very long, and thus still under submission process.

In international journals:

▷ Effect of quantified irreducibility on the computability of subshift entropy, with B. Hellouin.

Submitted:

▷ First author: Block gluing intensity of bidimensional SFT - computability of the entropy and periodic points

▷ First author:A characterization of entropy dimensions of tridimensional subshifts of finite type

▷ First author:Simulation of minimal effective dynamical systems on the Cantor sets by minimal tridimensional subshifts of finite type

▷ Only author: A proof that square ice entropy is $\frac{3}{2}\log_2(4/3)$.

Scientific activities

▷ Sept - Nov 2015 and Nov-Dec 2018 : Invited researcher at Santiago, Chile, CMM.

▷ Short selection of talks:

1. Computability, Randomness and applications, CIRM, Marseille, June 2016.
2. Combinatorics, Automata and Number Theory, CIRM, Marseille, December 2016.
3. Seminar of LIP team, ENS Lyon, February 2017.
4. Seminar Systemas dynamicos, December 2018, Santiago Chile.

5. Colloquium University of Denver, February 2019.

6. Ernest seminar, Marseille, March 2019

▷ March 2018: Organization of a workshop, around Algorithmic questions in dynamical systems, Toulouse.

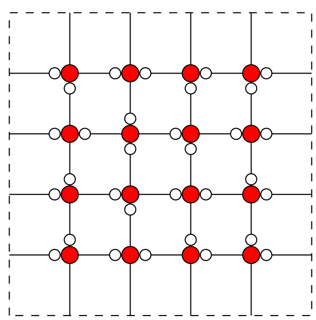
▷ 2018-2019: Organization of a workgroup, around computation of multidimensional SFT.

Scientific orientation

Subshifts are sets of colorings of the infinite grid $\mathbb{Z}^k$ with colors in a finite set $\mathcal{A}$. These objects appeared in dynamical systems theory as symbolic codings of dynamical systems, according to a topological partition for instance, but also in statistical mechanics where they are called **lattice models**, and as computation models in information theories.

A **pattern** p on shape $\mathbb{U} \subset \mathbb{Z}^k$ is an element of $\mathcal{A}^{\mathbb{U}}$ and it appears in an element (called configuration) of a subshift X when the restriction of this configuration to some $\mathbf{v} + \mathbb{U}$ is p. A subshift X is **decidable** when there is an algorithm that decides if a pattern on alphabet $\mathcal{A}$ appears in a configuration of X or not. It is said to be **of finite type** when it is defined by forbidding a finite set of patterns on shape $\mathbb{U}$ finite.

A widely studied **example** of subshift of finite type is the **square ice**. An example of representation of a configuration in this subshift is as follows:



The elements of the alphabet corresponds to local configurations of molecules, and forbidden patterns to interactions between them.

For a subshift X, we denote $N_n(X)$ the number of patterns on $[\![1, n]\!]^k$ that appear in a configuration of X. The **entropy of** X is

$$h(X) = \lim_n \frac{\log_2(N_n(X))}{n^k}.$$

This invariant appeared in dynamical systems to be a tool in the project of classification of systems. This quantity corresponds to the thermodynamics entropy, and to a quantity of information in information theories.

We say that a real number x is **computable** when there is an algorithm that on input n outputs a rational number q_n such that $|x - q_n| \leq 2^{-n}$. It is said to be **right recursively computable** when there is an algorithm which on input n outputs q_n such that $\inf_n q_n = x$.

State of the art

1. **Intertwinement of dynamics and computability**

   The intertwinement between dynamical systems theory and computability theory, and more widely computation theories, has been developped during the last decades. A particularly expressive result in this direction was the characterization of entropies of multidimensional subshifts of finite type, by M. Hochman and T. Meyerovitch [HM10], with a simple recursion-theoretic criterion: they are exactly the non-negative right recursively computable real numbers. In particular, entropy is algorithmically uncomputable on this class of systems, meaning that there is no algorithm that allows to approximate this number up to arbitrary precision.

2. **Towards an understanding between the computable and uncomputable worlds**

   More generally, results of this field typically state the uncomputability of a dynamical quantity over a large enough class of systems, or the computability on more restricted classes, often defined by dynamical restrictions. However, a gap always remains between computable and uncomputable regimes. The general **long-term aim of this project** is to explore various aspects of the frontier between computable and uncomputable worlds in dynamics.

I. An exact computational threshold phenomenon

There is no definite way to approach the frontier. A particular one is through a quantified version of mixing property, that we introduced: a subshift X is **f-mixing** when for any couple of patterns on $[\![1, n]\!]^k$, whenever put on the grid with relative distance $\geq f(n)$, they can be completed in a configuration of X. This is a generalisation of a notion introduced in [PS15].

1. **Saillant result [GH18] : Exhibition of a sharp computational threshold phenomenon**

   When the gap function f verifies
   $$\sum_n \frac{f(2^n)}{2^n} < +\infty,$$
   the entropy of a f-mixing decidable subshift is computable. When this sum is infinite, the possible entropies are the non-negative right recursively computable numbers.

2. **New perspectives**

   To our knowledge, this is the first time that one is able to characterise a threshold (no gap) between a computability regime and a (general) uncomputability one. We hope that this type of result could be found for other types of dynamical systems.

3. **Research direction: what happens on the frontier?**

   (a) **Potential benefits of interactions** with other theories about thresholds phenomena. For instance: percolation theory.

   (b) **Example of exploratory question**

   Can we make more precise the statement about computability of entropy when one get closer and closer to the frontier? [Example: computability speed].

   (c) **A more precise question**

   Characterisation of the entropies of f-mixing subshifts with $\sum f(2^n)/2^n < +\infty$?

4. **Research direction: other types of dynamical systems?**

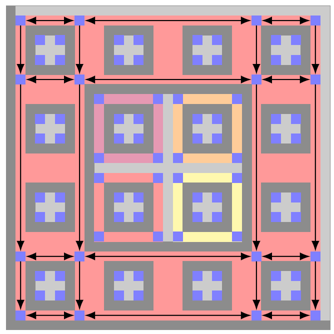
   (a) **Formalisation work done**

   Development of computability notions for topological dynamical systems. General computability properties of entropy, and some computability restriction under dynamical constraints.

(b) **Specific systems: on the interval** A characterization of entropies with computability criterion, and characterization for systems under constraints. Similar quantified constraints as f-mixing ?

(c) **Potential interactions with complex dynamics:** example of work done in this direction: computability and uncomputability of Julia sets.

II. Extension of the *uncomputable* world in statistical physics:



We got interested with M. Sablik in what happens of subshifts of finite type, which are more pertinent for physicists. Amongst the results that we obtained is the following:

1. **Saillant result [GS17a]**

   The characterization of M. Hochman and T. Meyerovitch is still true under $O(n)$-mixing for multidimensional SFT (lattice models in statistical physics).

   (a) **Interpretation**
   This condition is verified by all known models in statistical physics. This work is interpreted as a **localisation of computable statistical physics world** and an extension of the knowledge on its uncomputable part.

   (b) **Acquired skills**
   Ability to **manage highly complex constructions**, constructive methods of dynamical systems involving Turing machines. These constructions are complex in the sense that any modification of local arguments involve complete change of the whole construction.
   Formalisation of complex material.

   (c) **Developement of new tools**
   Multiple refinements of embedding Turing computations constructions that led to new tools for uncomputability results under dynamical constraints.

   (d) **Application and refinements of these tools**
   We complexified further the methods to prove a characterization of entropy dimensions and dynamical systems than can be simulated by minimal multidimensional SFT [GS17b],[GS18].

2. **Research directions**

   (a) **Minimal multidimensional SFT**
   These systems are not very well understood, and our work is the first to provide construction tools to explore this class of systems. We expect more constructions to come, related to other aspects of multidimensional dynamics.

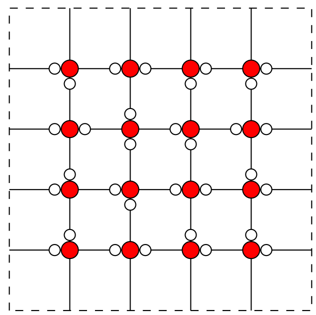
(b) **Short-term research direction**

Comparison with other uncomputability results obtained independantly for the spectral gap [CPGW], which was proven interesting for physicists [K17]. Possible simplification using our tools?

(c) **Long-term direction**

Uncomputability of other physically pertinent quantities and influence of physically pertinent constraints on their computability?

III. Extension of the *computable* world in statistical physics



After my doctoral thesis, N. Aubrun and M. Rao pointed out that some computations methods were developped by physicists to compute exactly entropy for some lattice models. I got naturally interested in them and obtained the following:

1. **Saillant result [G19]**

   Proof of the exact value of square ice entropy.

2. **Acquired skills**

   Tradional **tools of statistical physics** (partition functions, transfer matrices, etc). **Dealing with non-rigorous arguments**, and strong **bibliography abilities** (form of the literature in physics,etc).

3. **Research directions**

   (a) **Purely algebraic proof**

   The proof presented has, besides algebraic arguments, a heavy analytical part, that we would like to avoid. We expect that some generalisation of R. Baxter method of commuting matrices would lead to a simplification of this part.

   (b) **More proofs of physicists computations**

   Still a lot of work to be done in this direction, on eight-vertex model and hard hexagons. R. Baxter himself admits the lack of rigour of these developments. Highly probable profit of understanding of computation methods for square ice.

   (c) **Extension to models outside the scope of physicists**

   Examples of such models involve Kari-Culik (known to have positive entropy) or Jeandel-Rao tilings, that appeared in interactions between mathematics and computer science, subshifts of square ice, that appeared in our work on uncomputability results, and many other multidimensional SFT.

IV. Approaching the frontier: a specific notion of complexity

This axis is still in developement, but I have a good knowledge of bibliography, and some ideas of developments, in which I am highly interested.

1. **Context: a notion of complexity for organised systems**

   G. Tononi et al. published in 1994 [TSE] a seminal paper that defines a quantity called neural complexity, based on information theory, that aims at measuring the balance in a system between independance of its parts and their integration into a whole that have *organised sytems* such as the brain. This quantity has then started to be studied by mathematicians, who generalized it into the notion of intricacies, defined for general dynamical systems (see [BZ12] and [PW18]).

2. **Research directions**

   (a) **Understanding existing quantites:** Concepts far from understood by the community, still a lot of work. Examples of tasks:

   - computing the neural complexity or intricacies for simple systems (would benefit skills acquired in part III).
   - and understanding their maxima and minima.

   (b) **A formalisation work to be done**

   This question is interesting for me since the notion of *organisation* of dynamical systems matches intuitively with the constructions of multidimensional SFT under constraints (thus approaching the frontier) that I have done with M. Sablik. In particular, they exhibit complex information processing phenomena, with natural analogies with biological systems: functional segregation, multiple exchanges of informations between functional parts, inhibition of information transport, etc. An apparently possible work would be to provide formal definitions for these phenomena.

   (c) **Other possible definitions**

   We expect that, based on a better understanding of these phenomena in our constructions and their formal counterparts, we would be able to develop a quantity that charges this type of systems, intuitively analogical to biological ones, and would bring new ideas to this research area.

Teaching experience

▷ 2012-2014: **Writer** for teaching books (HK editions).

▷ 2012-2015: Oral interrogations for preparatory school students (Lycée LLG and Bellevue).

▷ 2015-2019: **Tutorials** (mathematics) at the university (first and second year students). **Writted** corrections for the tutorials 2017-2018.

Teaching principles

This part is aimed to describe some principles that came out of my teaching experience at the university - mainly tutorials. I extracted them from my mistakes while teaching to students but also from communicating to other researchers. I see this analysis as a way to make research and teaching interact and enlighten the fact that teaching is a sort of research in itself. In some way, understanding how to

understand and perceive an external flux of information can be helpful in order to guide oneself in the spectrum of research possibilities. On the other hand, the difficulty of communication in research helps see us how difficult is communication in general and how we can improve our way to teach.
These principles are rather simple and can be shortly described, on one hand, as the observation that teaching has always to be adapted more and more to the students (their perception of the class, their mind set, level, etc.), and that there are moments in teaching for which the interaction between the teacher and the students have to be differenciated.

On tutorials

There are two things I want to say about tutorials. First, I believe that the tutorial time in itself is a moment of informal exchange, between a specialist in mathematics who is used to mathematical conventions, language, interests of a mathematician, mathematical approach of a problem (non exhaustive list), and students who are not used to these. This is important since they are the key to an understanding of mathematics. Of course, they also need to get familiar with the object of the course. A way to achieve both is to interact in a direct way with students. This allows an adaptation to the person (his or her difficulties or expectations, according to the his/her level), which is crucial for communicating the interest in mathematics, and at the same time a direction of their attention directly to the object, in a way that is adapted to them. That is the reason why, when I have choice, I prefer to forget about lecturing in order to have more time for interacting directly with students.

Teaching documents

While I focus on direct interaction during the tutorials, students still need a written model that they can follow, memorize and reproduce (after adapting it) during the exams. For this purpose, I write and propose them a full model of writing for any problem that was considered during the tutorial, and leave to demand the corrections of other exercises. It is the moment for me to remember the main difficulties that were encountered during the tutorial and write them as remarks, so that it helps most of them. I include drawings, as much as possible, and combine informal explainations before writing the solutions with very formal and detailed solution, which is a model of what is expected for the exam, and even more.

Correction of exams

The way we correct the exams have to depend on the group of students and also on the object of the class itself: on the difficulties that were encountered, on the matching between the level expected and the actual level of most of the students, etc. In order for me to adapt to these constraints, I first read a significant proportion of the copies in order to understand better the spectrum of capacities the students have in the group, what they are able to do easily and what they don't, as well as what was not done by any of them. With an adapted level of expectation, students are not discouraged and can feel some progress, and actualise it. According to this process, the second step is to establish a detailed program to attributed grades.

Some teaching proposition

Besides mathematics classes, I would be interested in teaching some class around scientific writting methods, in which I could use the strong writting skills I acquired during my doctoral thesis, including the organisation of the text, general tools, such as LaTeX, but also more specific ones, such as TikZ (for illustrations).

Programming

Python, $\LaTeX$, HTML.

Languages

French, English (fluent) Spanish, Korean, Chinese (school).

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[GH18] S.Gangloff and B.Hellouin Effect of quantified irreducibility on the computability of subshift entropy *Discrete and continuous dynamical systems*, 2018.

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[CHK92] L.P. Hurd and J. Kari and K. Culik. The topological entropy of cellular automata is uncomputable. *Ergodic theory and dynamical systems*, 12:2551–2065, 1992.

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