In search of a measure of organisedness for dynamical systems

Silvere Gangloff (LIP, ENS Lyon)

1er mai 2019

"Symbolic dynamics as an exploration field"

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$$\mathcal{C}[f] = \{\sigma \circ f \circ \sigma^{-1}\}.$$

I. Main theories of consciousness

1. Global neuronal workspace
2. Integrated information theory

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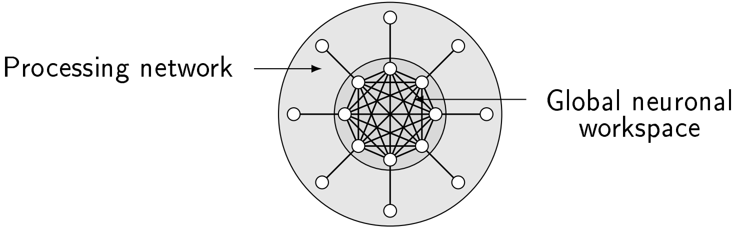

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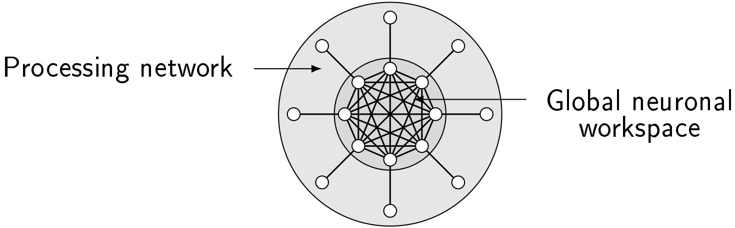

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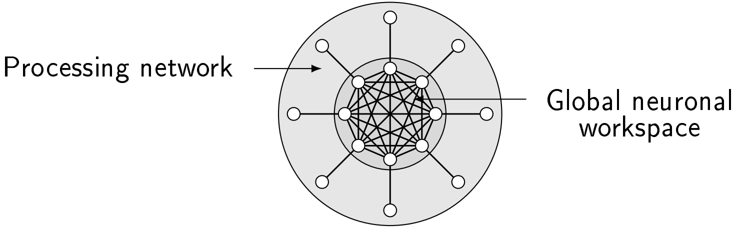

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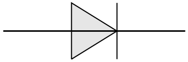
A cognitive theory of consciousness, B.J. Baars, 1988.

2. Integrated information theory : Generation of information : photodiode :



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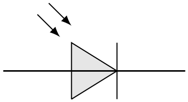

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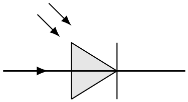

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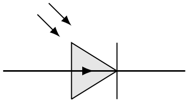

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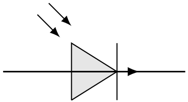

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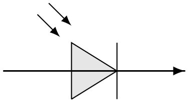

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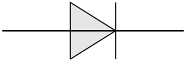
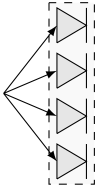


The knowledge of the output generates 1bit of information.

2. Integrated information theory : Set of photodiodes (or camera) :



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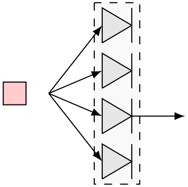

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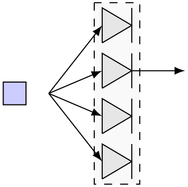

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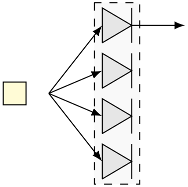

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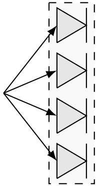

Discriminate more inputs

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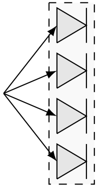
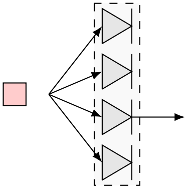


Discriminate more inputs $\Rightarrow$ generates more information.

2. Integrated information theory : Non-integration of information : decomposition into independant smaller mechanisms :



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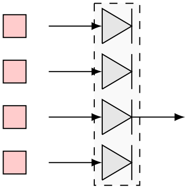

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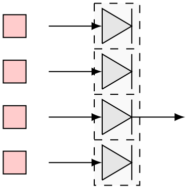

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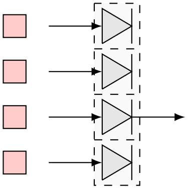

In the human brain : **balance** between **generation** of information and **integration**.

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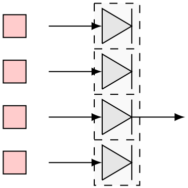

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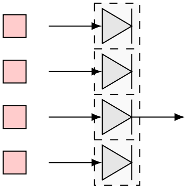

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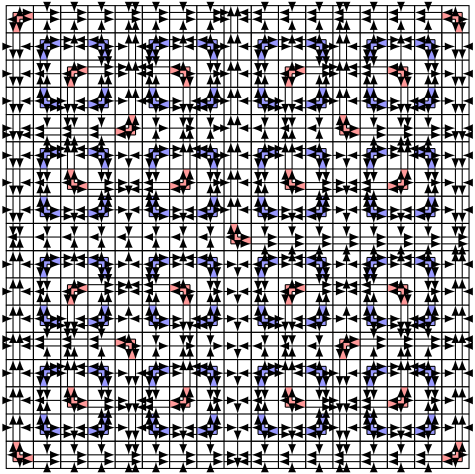
Question : quantity which measures this balance ?

II. Symbolic dynamics and universal computation

1. Hierarchical aperiodic tilings and further constructions
2. Minimal tile sets generating aperiodic tilings

1. Hierarchical aperiodic tilings : Robinson :



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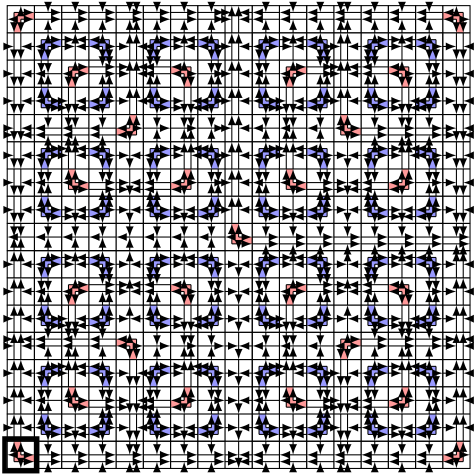

tile

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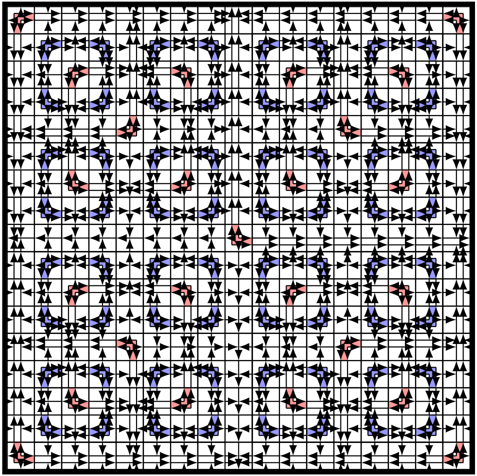
pattern



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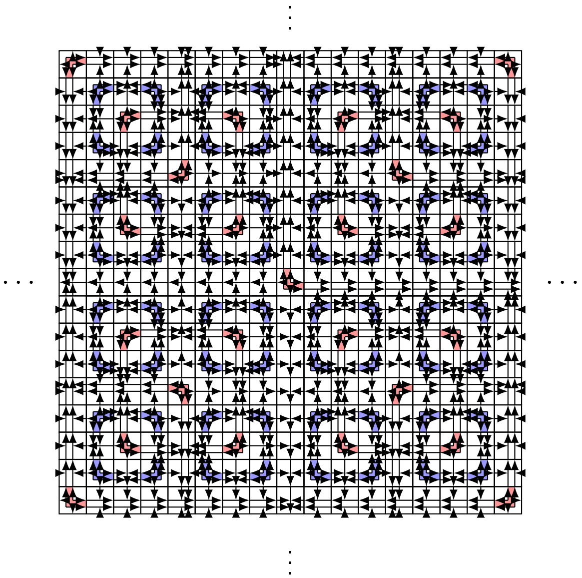
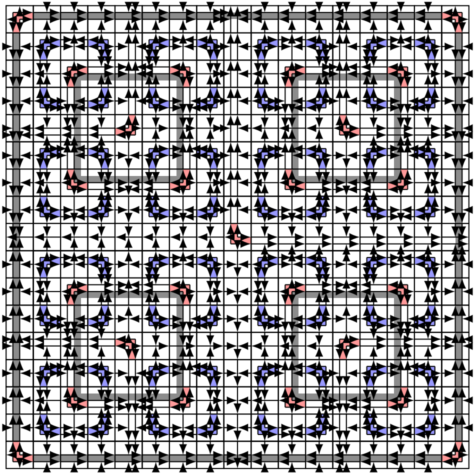
tiling 

Computation boards :



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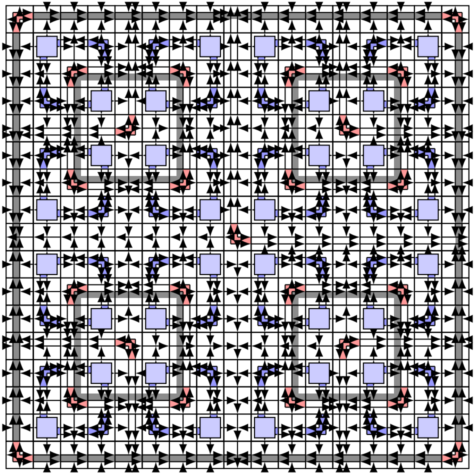

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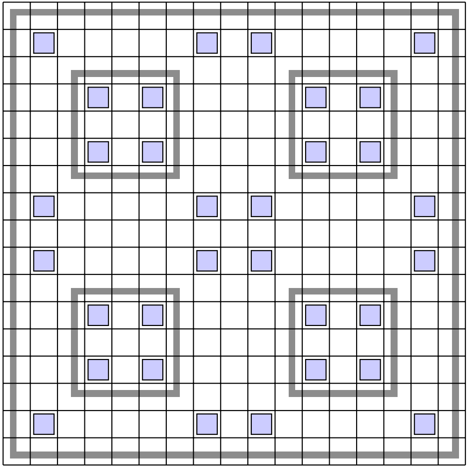
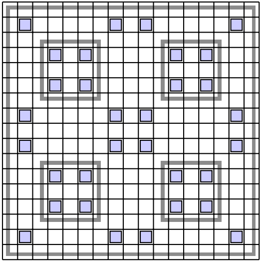


Implementing Turing computing machines :



a'_0	a'_1	a'_2	q_h a_3	a_4	a_5	a_6	a_7
a'_0	a'_1	a'_2	q_h a_3	a_4	a_5	a_6	a_7
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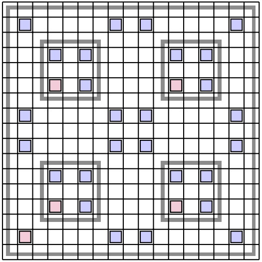

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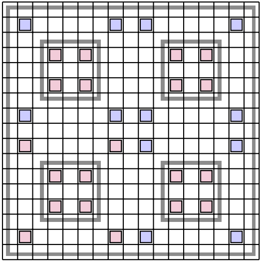

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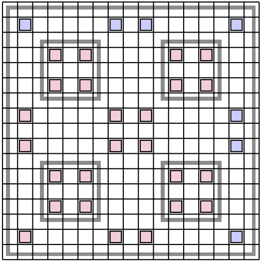

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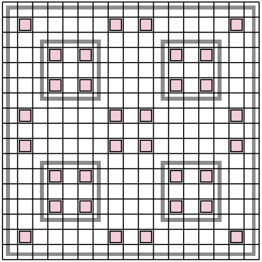

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Original problem : Undecidability of tiling problem, H. Wang (formulation), R. Berger (solution), R. Robinson (simplification).

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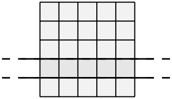

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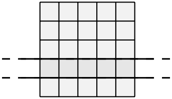
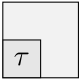
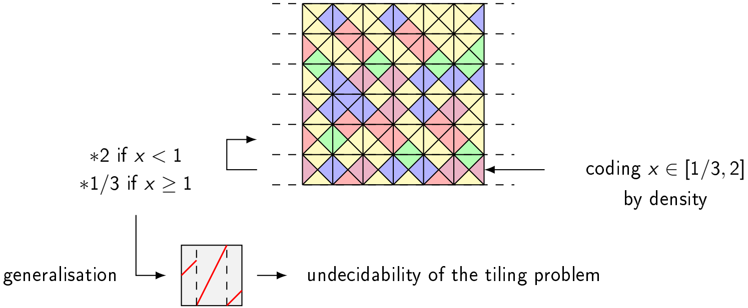


3. B. Durand, A. Romaschchenko, A. Shen : **fixed-point** tilings constructions.



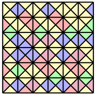
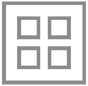
$\tau' \equiv \tau$

Small aperiodic tile sets : Kari-Culik (13) :

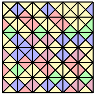
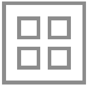


Minimal : Jeandel-Rao (11).

Question :

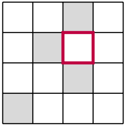
 $\underset{?}{\sim}$ 

No : entropy of Kari-Culik > 0, and 0 for Robinson [*Aperiodic tilings and entropy*, B. Durand, G. Gamard, A. Grandjean].

 $\underset{?}{\sim}$  + random symbols

III. A starting point : neural complexity and intricacies

Neural complexity of a finite family of random variables :



$X_{(i,j)}$

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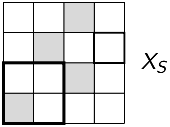

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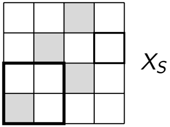

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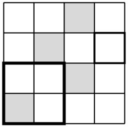

X_S

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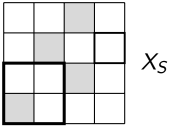
A measure for brain complexity: relating functional segregation and integration in the nervous system, G. Tononi, O.Sporns, G.M.Edelman,1994.

Neural complexity of a finite family of random variables :



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A measure for brain complexity: relating functional segregation and integration in the nervous system, G. Tononi, O.Sporns, G.M.Edelman,1994.

Synchronous or independant : $\mathcal{N}(X) = 0$.

Intricacies : [Buzzi, Zambotti 2009]

$$\mathcal{I}(X,c) = \frac{1}{|I|+1} \sum_{S} c_S * MI(X_S, X_{S^c}).$$

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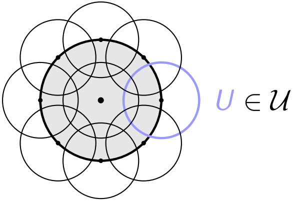
(Number of) maxima of intricacy ?

Definition for dynamical systems : [Petersen, Wilson 2016]
(X, f) dynamical system :

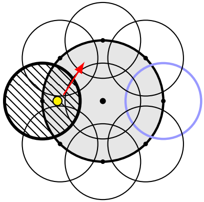
$$S = \{s_1, ..., s_k\} \subset \{1, ..., n\}$$



$$\mathcal{I}(X, f, \mathcal{U}, c) = \lim_n \sum_{S \subset \{1,...,n\}} c_S^n \log_2 \left(\frac{|\mathcal{U}_S| \cdot |\mathcal{U}_{S^c}|}{|\mathcal{U}_{\{1,...,n\}}|} \right).$$

Definition for dynamical systems : [Petersen, Wilson 2016]
(X, f) dynamical system :

$t = s_1$ $\qquad$ $S = \{s_1, ..., s_k\} \subset \{1, ..., n\}$



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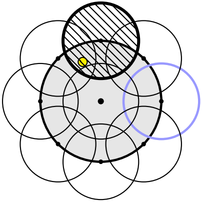
$t = s_2$ $\qquad$ $S = \{s_1, ..., s_k\} \subset \{1, ..., n\}$



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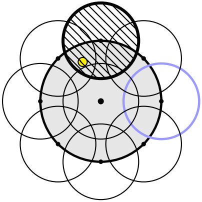


$U \in \mathcal{U}$

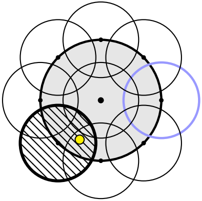
U_{s_1} $\quad$ $U_{s_2} U_{s_3}$

$$\mathcal{I}(X, f, \mathcal{U}, c) = \lim_n \sum_{S \subset \{1,...,n\}} c_S^n \log_2 \left(\frac{|\mathcal{U}_S| \cdot |\mathcal{U}_{S^c}|}{|\mathcal{U}_{\{1,...,n\}}|} \right).$$

Definition for dynamical systems : [Petersen, Wilson 2016]
(X, f) dynamical system :

$t = s_4$

$S = \{s_1, ..., s_k\} \subset \{1, ..., n\}$



U_{s_1} $U_{s_2} U_{s_3}$ U_{s_4}

$$\mathcal{I}(X, f, \mathcal{U}, c) = \lim_n \sum_{S \subset \{1,...,n\}} c_S^n \log_2 \left(\frac{|\mathcal{U}_S| \cdot |\mathcal{U}_{S^c}|}{|\mathcal{U}_{\{1,...,n\}}|} \right).$$

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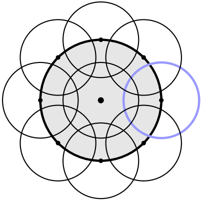

$U \in \mathcal{U}$

$U_{s_1} \quad U_{s_2} U_{s_3} \quad U_{s_4} \quad \in \mathcal{U}_S$

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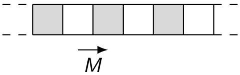
One dimensional tiling sets : [Petersen, Wilson 2016]

X 

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

When $M^2 > 0$:

$$\mathcal{I}(X) = \sum_{k=1}^{+\infty} \frac{\log_2(N_k(X))}{k} - h(X),$$

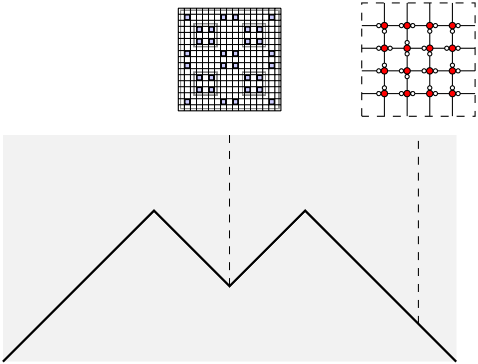
$$h(X) = \lim_k \frac{\log_2(N_k(X))}{k},$$

where $N_k(X)$: number of size k observable patterns.

IV. Symbolic dynamics as an exploration field

1. Strategy
2. A pool of existing formalisms
3. Organisedness and dynamical constraints

1. Strategy : search a quantity with tractable maxima such that :



tot. synchr. tot. indep.

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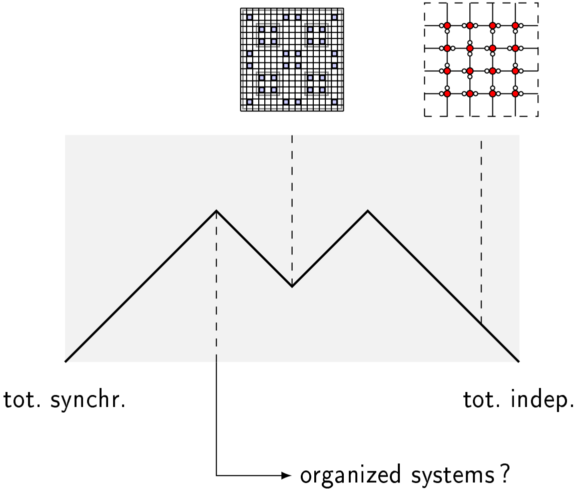

2. Other formalisms :

Kolmogorov complexity of sequences :

$$00000101 \longrightarrow \boxed{\text{Interpreter}} \longrightarrow 0100101010$$

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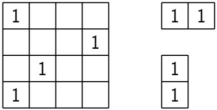
Kolmogorov complexity of sequences :

Minimal number of forbidden patterns :

Ex : hard core model :



forbidden patterns

tiling

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00000101 ⟶ Interpreter ⟶ 0100101010

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Ex : hard core model :

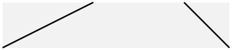
1			
			1
	1		
1			

tiling

1	1

1
1

forbidden patterns



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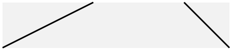
tiling

1	1

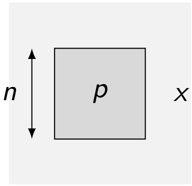
1
1

forbidden patterns



In practive, maxima difficult to apprehend.

Number of extender sets : X tiling set



$x \in \mathcal{E}_X(p)$

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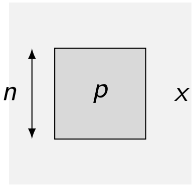

$x \in \mathcal{E}_X(p)$

Set of extender sets of X : $\mathcal{E}_n(X) = \{\mathcal{E}_X(p) : p \in \mathcal{L}_n(X)\}$.

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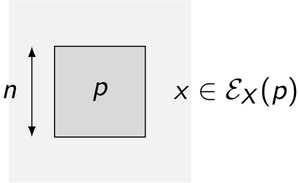

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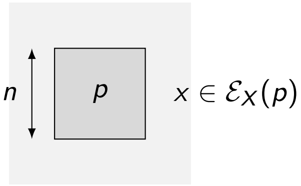

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Extender sets $\equiv$ measure of how a pattern constrains a tiling.

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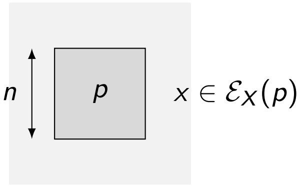

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Extender sets $\equiv$ measure of how a pattern constrains a tiling.
Principle of organisation : a lot of patterns exert a non-trivial constraint.

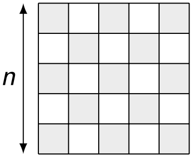
Pattern constructibility : minimal sets of constructibility for p :

n

1	0	1	0	1
0	1	0	1	0
1	0	1	0	1
0	1	0	1	0
1	0	1	0	1

p

Pattern constructibility : minimal sets of constructibility for p :



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n

1		1		1
	1		1	
1		1		1
	1		1	
1		1		1

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	1		1	
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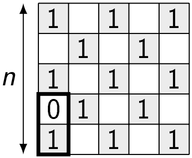

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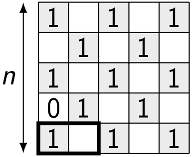

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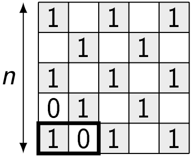

Pattern constructibility : minimal sets of constructibility for p :

	1	0	1	0	1
	0	1	0	1	0
n	1	0	1	0	1
	0	1	0	1	0
	1	0	1	0	1

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n

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0	1	0	1	0
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0	1	0	1	0
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Pattern constructibility : minimal sets of constructibility for p :

$$n \updownarrow \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 & 1 \\ \hline \end{array}$$

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Related complexity notion :

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n

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0	1	0	1	0
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Expressive power of multidimensional tiling sets under constraints : [G.,Sablik]

Minimality : every pattern that appears in a configuration appears in all the configurations.

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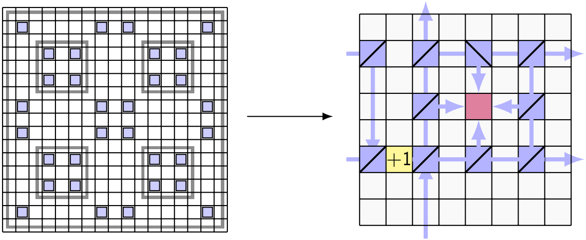

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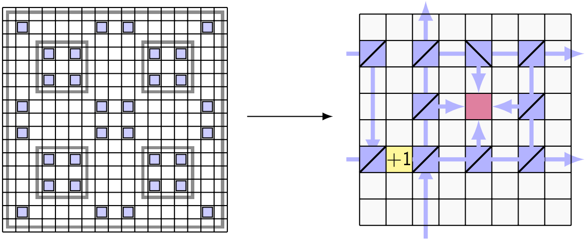

Analysis of this kind of phenomenon in relation with organisedness ?

THE END