In search of a measure of organisedness for dynamical systems

Silvere Gangloff (LIP, ENS Lyon)

24 avril 2019

"Symbolic dynamics as an exploration field"

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3. **Physical :** what is an observer ?

I. Main theories of consciousness

1. Global neuronal workspace
2. Integrated information theory

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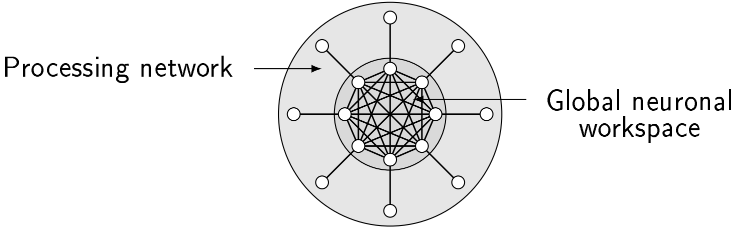

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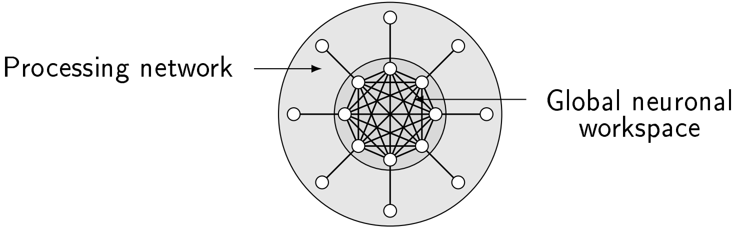

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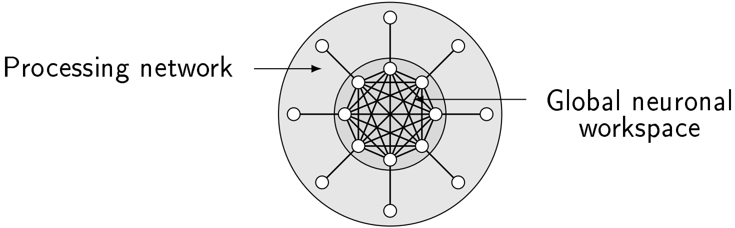

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A cognitive theory of consciousness, B.J. Baars, 1988.

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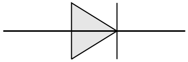

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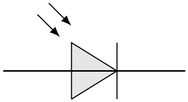

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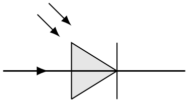

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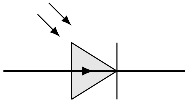

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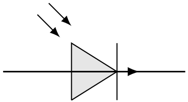

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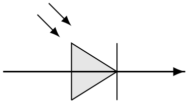

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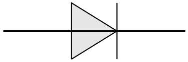
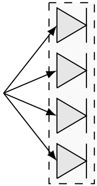


The knowledge of the output generates 1bit of information.

2. Integrated information theory : Set of photodiodes (or camera) :



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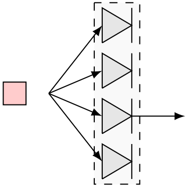

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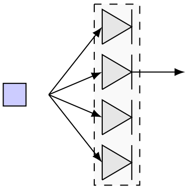

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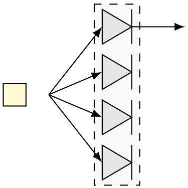

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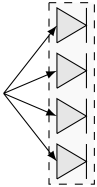

Discriminate more inputs

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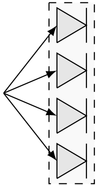
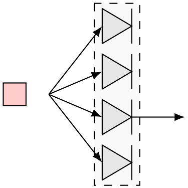


Discriminate more inputs $\Rightarrow$ generates more information.

2. Integrated information theory : Non-integration of information : decomposition into independant smaller mechanisms :



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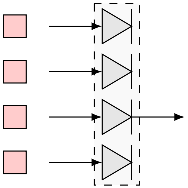

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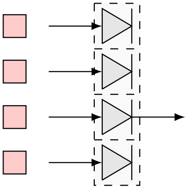

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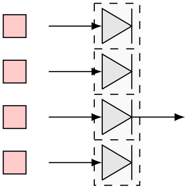

In the human brain : **balance** between **generation** of information and **integration**.

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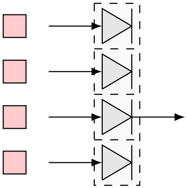

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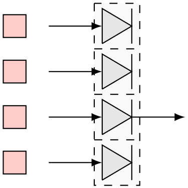

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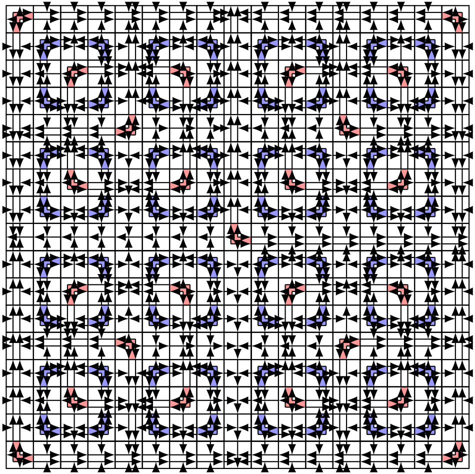
Question : quantity which measures this balance ?

II. Symbolic dynamics and universal computation

1. Hierarchical aperiodic tilings and further constructions
2. Minimal tile sets generating aperiodic tilings

1. Hierarchical aperiodic tilings : Robinson :



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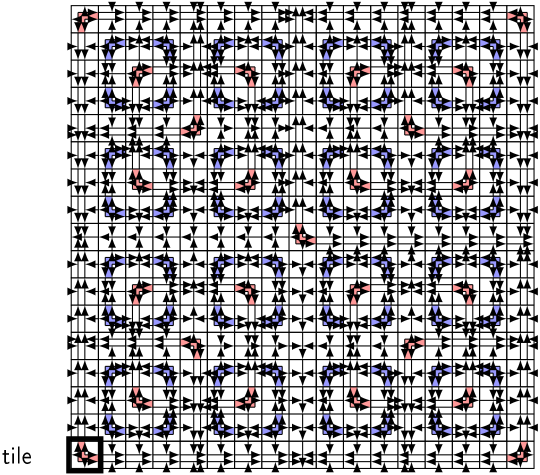

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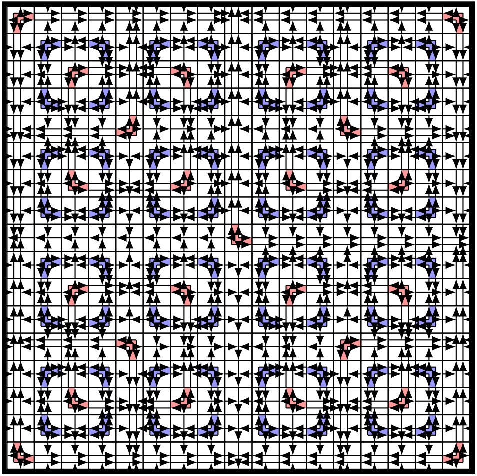
pattern



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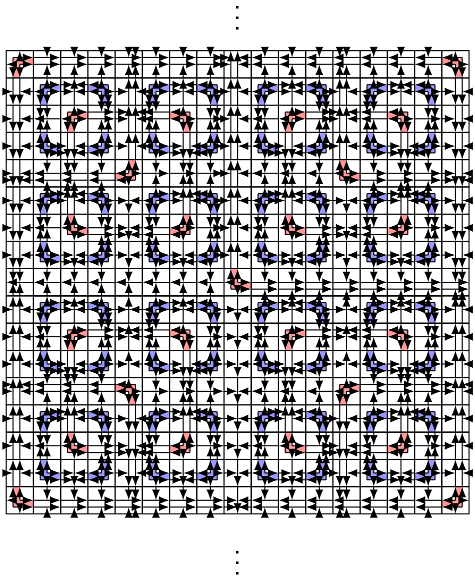
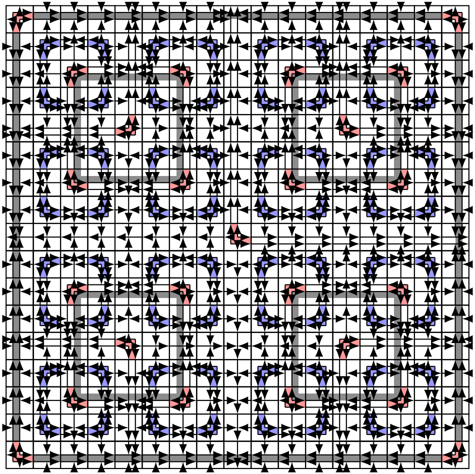
tiling ⋯  ⋯

Computation boards :



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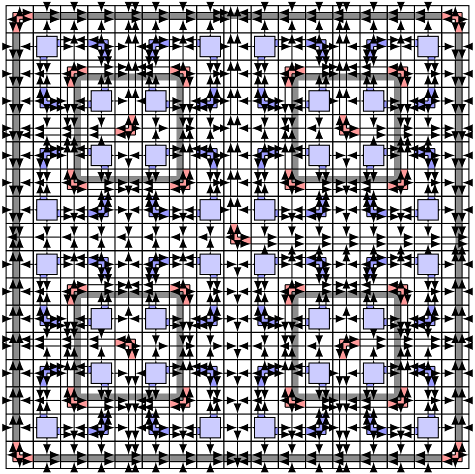

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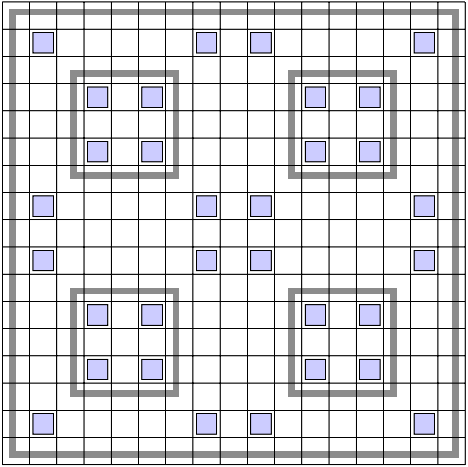
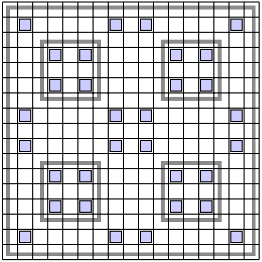


Implementing Turing computing machines :



a'_0	a'_1	a'_2	${}^{q_h}a_3$	a_4	a_5	a_6	a_7
a'_0	a'_1	a'_2	${}^{q_h}a_3$	a_4	a_5	a_6	a_7
a'_0	a'_1	${}^{q^2}a_2$	a_3	a_4	a_5	a_6	a_7
a'_0	${}^{q^1}a_1$	a_2	a_3	a_4	a_5	a_6	a_7
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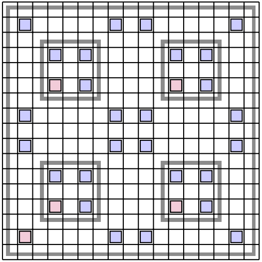

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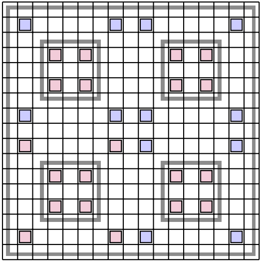

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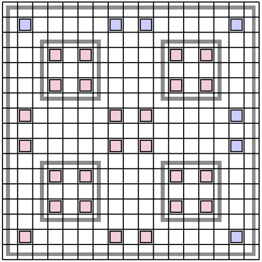

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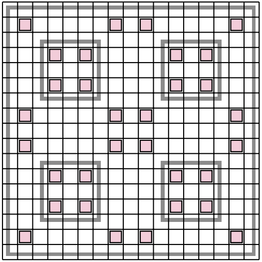

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Original problem : Undecidability of tiling problem, H. Wang (formulation), R. Berger (solution), R. Robinson (simplification).

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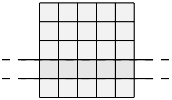

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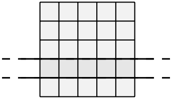
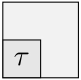
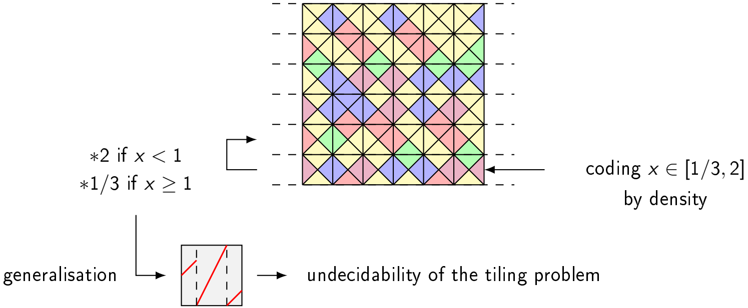


3. B. Durand, A. Romaschchenko, A. Shen : **fixed-point** tilings constructions.



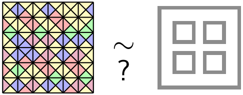
$\tau' \equiv \tau$

Small aperiodic tile sets : Kari-Culik (13) :

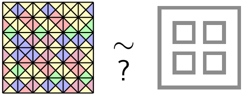


Minimal : Jeandel-Rao (11).

Question :



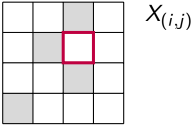
No : entropy of Kari-Culik > 0, and 0 for Robinson [*Aperiodic tilings and entropy*, B. Durand, G. Gamard, A. Grandjean].



+ random symbols

III. A starting point : neural complexity and intricacies

Neural complexity of a finite family of random variables :



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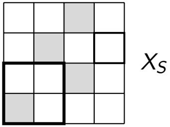

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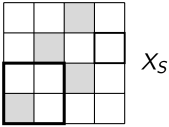

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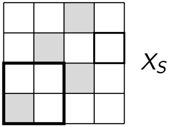

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A measure for brain complexity: relating functional segregation and integration in the nervous system, G. Tononi, O.Sporns, G.M.Edelman,1994.

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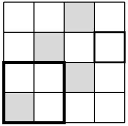

X_S

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Synchronous or independant : $\mathcal{N}(X) = 0$.

Intricacies : [Buzzi, Zambotti 2009]

$$\mathcal{I}(X, c) = \frac{1}{|I| + 1} \sum_{S} c_S * MI(X_S, X_{S^c}).$$

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1. $c_S \geq 0$,

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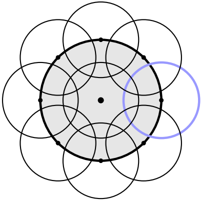
(Number of) maxima of intricacy ?

Definition for dynamical systems : [Petersen, Wilson 2016]
(X, f) dynamical system :

$$S = \{s_1, ..., s_k\} \subset \{1, ..., n\}$$



$$\mathcal{I}(X, f, \mathcal{U}, c) = \lim_n \sum_{S \subset \{1,...,n\}} c_S^n \log_2 \left(\frac{|\mathcal{U}_S| \cdot |\mathcal{U}_{S^c}|}{|\mathcal{U}_{\{1,...,n\}}|} \right).$$

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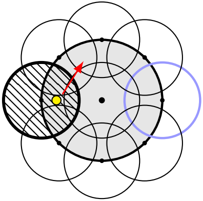

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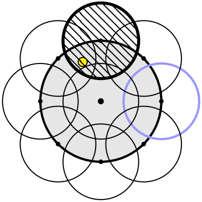

$U \in \mathcal{U}$

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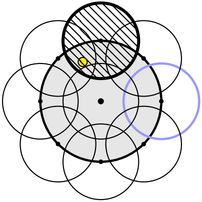

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$t = s_4$

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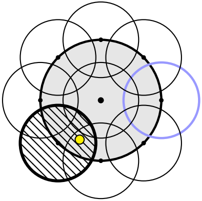

U_{s_1} $U_{s_2} U_{s_3}$ U_{s_4}

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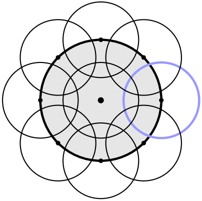

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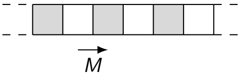
One dimensional tiling sets : [Petersen, Wilson 2016]

X 

$$M = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \square & \blacksquare \\ \end{array}$$

$$M = \begin{array}{cc} & \begin{array}{cc} \square & \blacksquare \end{array} \\ & \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right) \end{array} \begin{array}{c} \\ \square \\ \blacksquare \end{array}$$

When $M^2 > 0$:

$$\mathcal{I}(X) = \sum_{k=1}^{+\infty} \frac{\log_2(N_k(X))}{k} - h(X),$$

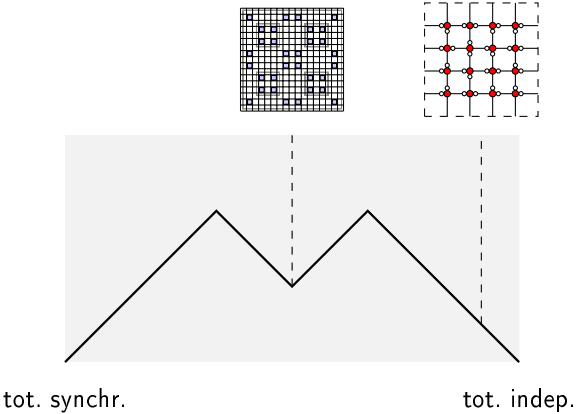
$$h(X) = \lim_k \frac{\log_2(N_k(X))}{k},$$

where $N_k(X)$: number of size k observable patterns.

IV. Symbolic dynamics as an exploration field

1. Strategy
2. A pool of existing formalisms
3. Organisedness and dynamical constraints

1. Strategy : search a quantity with tractable maxima such that :



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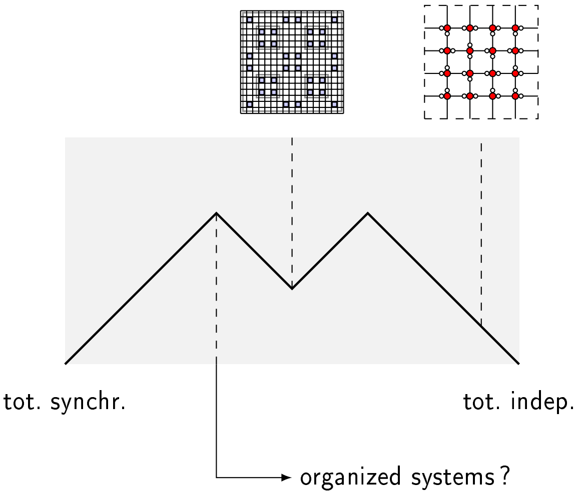

2. Other formalisms :

Kolmogorov complexity of sequences :

$$00000101 \longrightarrow \boxed{\text{Interpreter}} \longrightarrow 0100101010$$

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Minimal number of forbidden patterns :

Ex : hard core model :

1			
			1
	1		
1			

tiling

1	1

1
1

forbidden patterns

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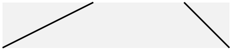

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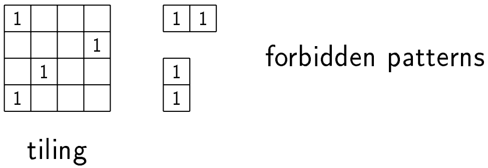
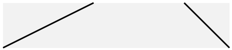
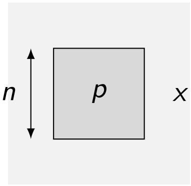




In practive, maxima difficult to apprehend.

Number of extender sets : X tiling set



$x \in \mathcal{E}_X(p)$

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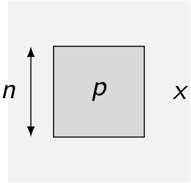

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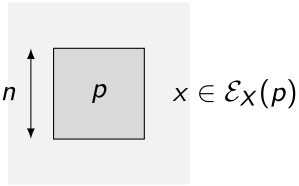

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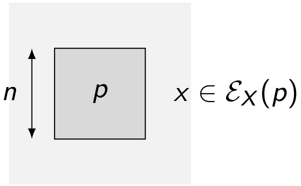

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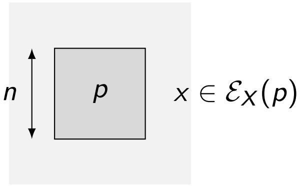

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Extender sets $\equiv$ measure of how a pattern constrains a tiling.
Principle of organisation : a lot of patterns exert a non-trivial constraint.

Pattern constructibility : minimal sets of constructibility for p :

$$n \updownarrow \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 & 1 \\ \hline \end{array} \quad \text{p}$$

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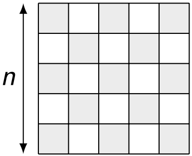

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	1		1		1
		1		1	
n	1		1		1
		1		1	
	1		1		1

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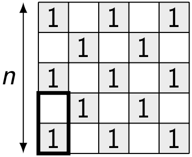

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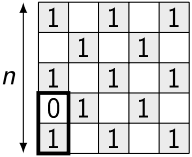

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1		1		1
	1		1	
1		1		1
0	1		1	
1		1		1

n

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n

1		1		1
	1		1	
1		1		1
0	1		1	
1	0	1		1

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n

1	0	1	0	1
0	1	0	1	0
1	0	1	0	1
0	1	0	1	0
1	0	1	0	1

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n

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n

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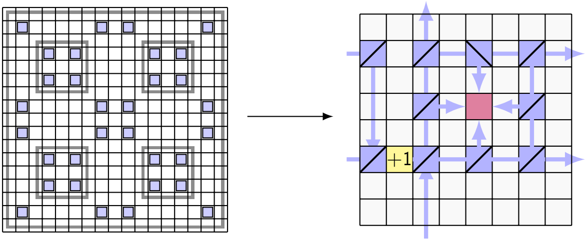

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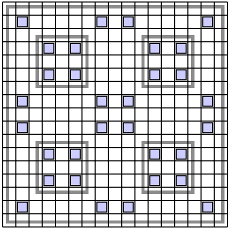

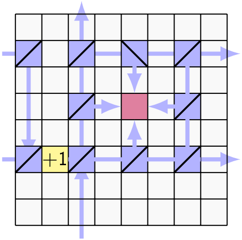

Analysis of this kind of phenomenon in relation with organisedness ?

THE END