```
> restart: with(plots):
> mode := (x,t) -> xi^(t) * exp(I*k*x);
```

$$mode := (x, t) \mapsto \xi^{t}\, e^{I k x} \tag{1}$$

```
> up := mode(0,1); hr := mode(0,0); dn := mode(0,-1);
```

$$up := \xi$$

$$hr := 1$$

$$dn := \frac{1}{\xi} \tag{2}$$

```
> lt := mode(-1,0); rt := mode(1,0);
```

$$lt := e^{-I k}$$

$$rt := e^{I k} \tag{3}$$

```
> (lt + rt - 2*hr)/dx^2 = (up + dn - 2*hr)/dt^2; dt := alpha*dx;
```

$$\frac{e^{-I k} + e^{I k} - 2}{dx^{2}} = \frac{\xi + \frac{1}{\xi} - 2}{dt^{2}}$$

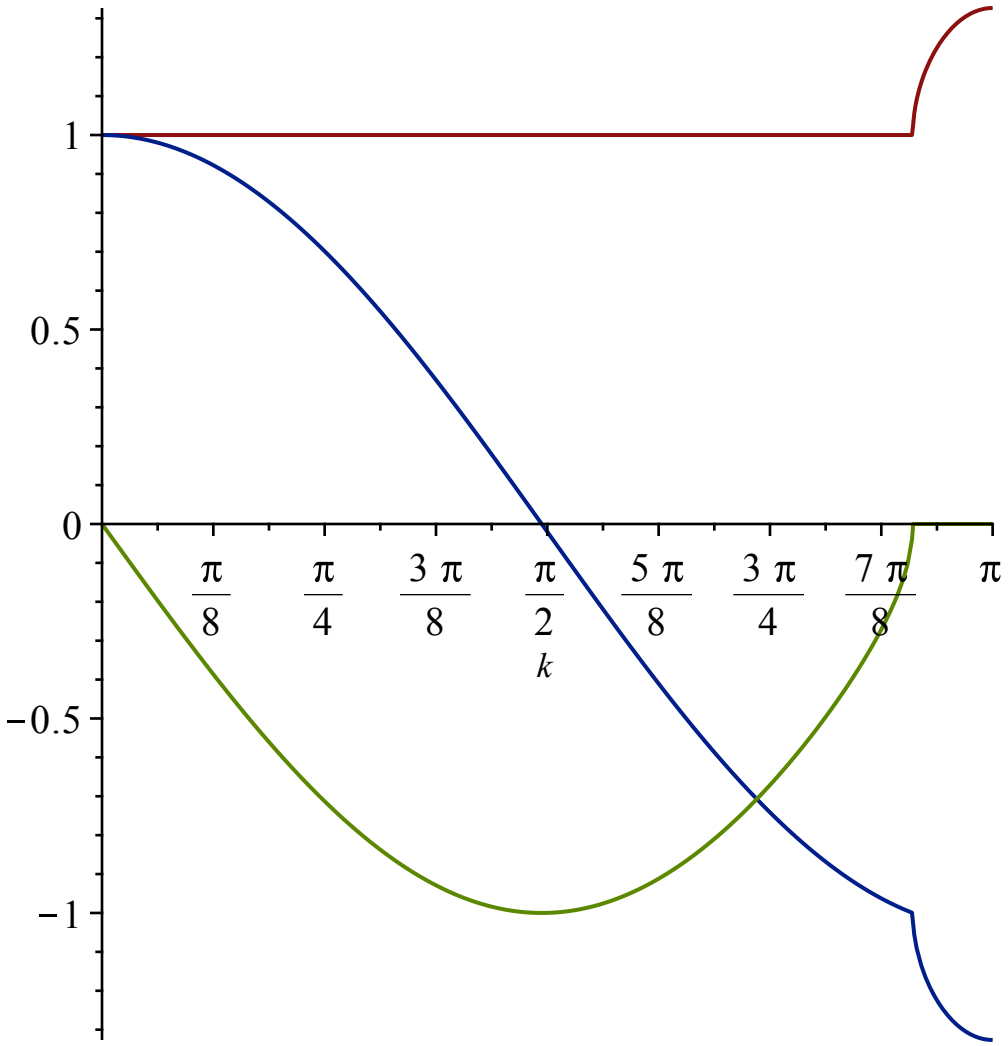
$$dt := \alpha\, dx \tag{4}$$

```
> X:=solve(%%,xi):
> xi[1] := simplify(X[1]); xi[2] :=simplify(X[2]);
```

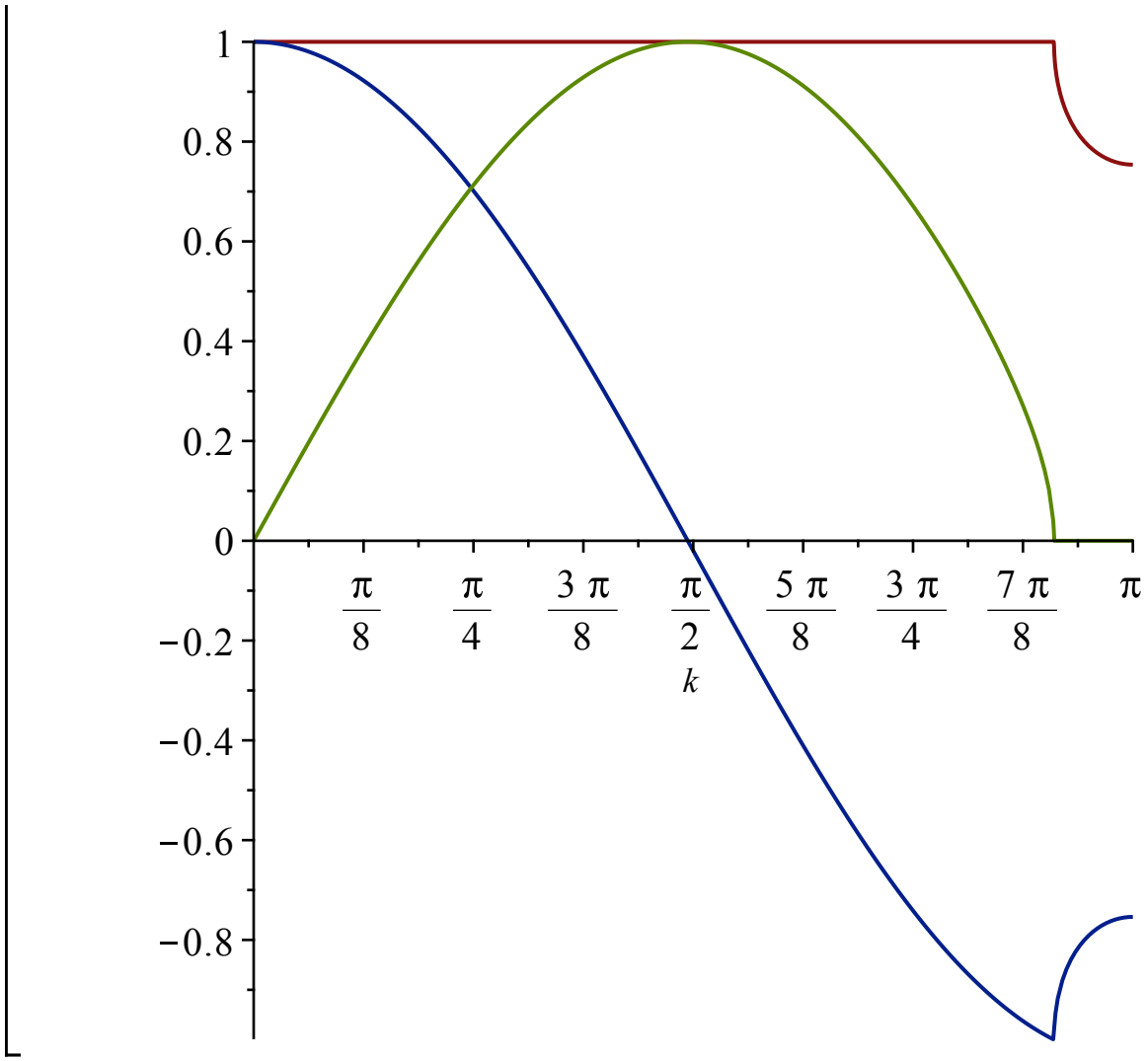
$$\xi_1 := -\alpha^{2} + 1 + \sqrt{\alpha^{2}\left(\cos(k) - 1\right)\left(\cos(k)\,\alpha^{2} - \alpha^{2} + 2\right)} + \cos(k)\,\alpha^{2}$$

$$\xi_2 := -\alpha^{2} + 1 - \sqrt{\alpha^{2}\left(\cos(k) - 1\right)\left(\cos(k)\,\alpha^{2} - \alpha^{2} + 2\right)} + \cos(k)\,\alpha^{2} \tag{5}$$

```
> eval(xi[2],alpha=1.01): plot([abs(%), Re(%), Im(%)], k = 0..Pi);
```




```
> eval(xi[1],alpha=1.01): plot([abs(%), Re(%), Im(%)], k = 0..Pi);
```


1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
π/8
π/4
3 π/8
π/2
5 π/8
3 π/4
7 π/8
π
k