AIUB COURSE SOLUTION

PHYSICS -2

ELECTRIC FIELD

NEW MATH SOLUTION

Summer-16

SRABONI BHUIYAN

AIUB COURSE SOLUTION

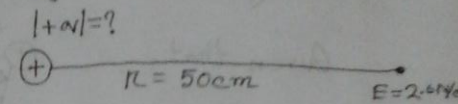
A.C.S

TOGETHER WE CAN ACHIEVES MORE

Electric Field

Electric field due to a point charge :

5E:
What is the magnitude of a point charge whose electric field 50 cm away has the magnitude 2.0 N/C?



Given that,

$r = 50\ cm$

$= \frac{50}{100}$

$= 0.5 m$

$E = 2.0\ N/C$

$\frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2/c^2$

$|+q| = ?$

We know, $E = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}$

$E = K\cdot\frac{q}{r^2}$

$\Rightarrow Er^2 = Kq$

$\Rightarrow Kq = Er^2$

$\Rightarrow q = \frac{Er^2}{K}$

$\Rightarrow q = \frac{2.0\times(0.5)^2}{9\times10^9}\ c$

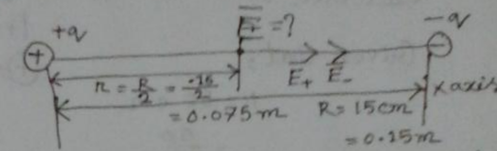
$= 5.5\times10^{-11}\ c$

Answer: $5.5\times10^{-11}\ c$

②

6E: Two particles equal charge magnitudes 2.0×10^{-7} c but opposite signs are held 15 cm apart. What are the magnitude and direction of E at the point midway between the charges?

Given that,



$K=9\times10^{9}\ Nm^2/c^2$

$R = 15\,cm$
$= 0.15\,m$

$r = \frac{R}{2}$
$= \frac{0.15}{2}$
$= 0.075\,m$

$\frac{1}{4\pi\epsilon_0} = 9\times10^{9}\ Nm^2/c^2$

$|q| = 2.0\times10^{-7}\,c$

$E = ?$

We know

$E_+ = \frac{1}{4\pi\epsilon_0}\cdot\frac{|+q|}{r^2}$

$\Rightarrow \overline{E}_+ = K\cdot\frac{q}{r^2}\cdot\hat{i}$

Again,

$E_- = \frac{1}{4\pi\epsilon_0}\cdot\frac{|-q|}{r^2}$

$= K\cdot\frac{q}{r^2}\,\hat{i}$

Now, $\overline{E} = \overline{E}_+ + \overline{E}_-$

$= K\cdot\frac{q}{r^2}\cdot\hat{i} + K\cdot\frac{q}{r^2}\cdot\hat{i}$

$= 2\left(K\cdot\frac{q}{r^2}\right)\hat{i}$

$= 2\times9\times10^{9}\cdot\frac{2.0\times10^{-7}}{(0.075)^2}\,\hat{i}$

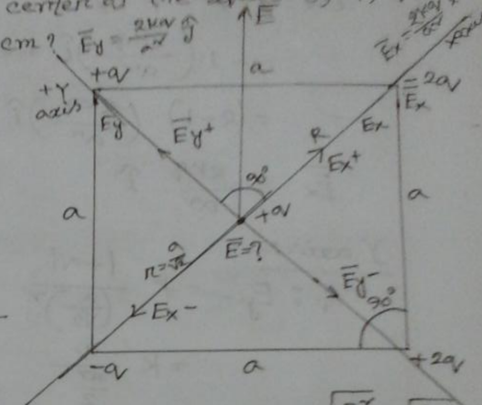
$= 640000\,\hat{i}$

$= 6.4\times10^{5}\,\hat{i}\ N/c$

Answer:/

13P: what is the magnitude and direction of the electric filed at the center of the square of if $q = 1.0\times10^{-8}C$ and $a = 5.0$ cm?

$\overline{E}_y = \frac{2kq}{a^2}\hat{j}$



X-axis:

$-q$: $E = k\frac{|-q|}{r^2}$

$E_{x^-} = k\frac{q}{\left(\frac{a}{\sqrt{2}}\right)^2}$

$= k\frac{q}{\frac{a^2}{2}}$

$\overline{E}_{x^-} = \frac{2kq}{a^2}(-\hat{i})$

$\overline{E}_{x^-} = -\frac{2kq}{a^2}\hat{i}$

$\sqrt{R^2} = \sqrt{a^2+a^2}$

$\Rightarrow R = \sqrt{2a^2}$

$= a\sqrt{2}$

$\frac{R}{2} = \frac{a\sqrt{2}}{2}$

$r = \frac{a\cancel{\sqrt{2}}}{\sqrt{2}\cancel{\sqrt{2}}}$

$= \frac{a}{\sqrt{2}}$

$+2q$: $E_{x^+} = k\frac{|-2q|}{r^2}$

$= k\frac{2q}{\left(\frac{a}{\sqrt{2}}\right)^2}$

$= k\frac{2q}{\frac{a^2}{2}}$

$\overline{E}_{x^+} = 2\left(\frac{2kq}{a^2}\right)\hat{i}$

$= 2\left(\frac{2kq}{a^2}\right)\hat{i}$

$\overline{E}_{x^+} = 2\left(\frac{2kq}{a^2}\right)\hat{i}$

(1)

$$\bar{E}_x = \bar{E}_{x-} + \bar{E}_{x+}$$

$$= -1\left(\frac{2kq}{a^2}\right)\hat{i} + 2\left(\frac{2kq}{a^2}\right)\hat{i}$$

$$= (2-1)\left(\frac{2kq}{a^2}\right)\hat{i}$$

$$\bar{E}_x = \frac{2kq}{a^2}\hat{i}$$

<u>Y axis:</u>

$+q$: $E_{y-} = k\dfrac{|+q|}{\left(\frac{a}{\sqrt{2}}\right)^2}$

$$= k\frac{q}{\frac{a^2}{2}}$$

$$\bar{E}_{y-} = \frac{2kq}{a^2}(-\hat{j})$$

$$= -\frac{2kq}{a^2}\hat{j}$$

$+2q$: $E_{y+} = k\dfrac{|+2q|}{\left(\frac{a}{\sqrt{2}}\right)^2}$

$$= k\frac{2q}{\frac{a^2}{2}}$$

$$\bar{E}_{y+} = 2\left(\frac{2kq}{a^2}\right)\hat{j}$$

$$\bar{E}_y = \bar{E}_{y-} + \bar{E}_{y+}$$

$$= -1\left(\frac{2kq}{a^2}\right)\hat{j} + 2\left(\frac{2kq}{a^2}\right)\hat{j}$$

$$= (2-1)\left(\frac{2kq}{a^2}\right)\hat{j}$$

$$\bar{E}_y = \frac{2kq}{a^2}\hat{j}$$

$$E^2 = E_x^2 + E_y^2 + 2E_xE_y\cos 90^\circ$$

$$\sqrt{E^2} = \sqrt{E_x^2 + E_y^2} \qquad \left[E_x = E_y = \frac{2kq}{a^2}\right]$$

$$E = \sqrt{E_x^2 + E_y^2}$$

$$= \sqrt{E_x^2 + E_x^2}$$

$$= \sqrt{2E_x^2}$$

$$= \frac{2kq}{a^2}\sqrt{2} \qquad = \frac{2 \times 9\times10^9 \times 1.0\times10^{-8}}{(0.05)^2} \times \sqrt{2}$$

Given that,

$$= 1.02 \times 10^5 \text{ N/C}$$

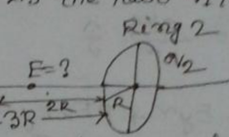
Answer

$a = 5.0\text{cm} = 0.05\text{m}$

$q = 1.0\times10^{-8}\text{ C}$

6

18E: Shows two parallel nonconducting rings arranged with their central axes along a common line. Ring 1 has uniform charge q_1 and radius R; ring 2 has uniform charge q_2 and the same radius R. The rings are separated by a distance 3R. The net electric field at point P on the common line, at distance R from ring 1, is zero, what is the ratio q_1/q_2?



E field due to the q_1 charge of Ring 1 at the point is R.

$$E_1 = \frac{q_1 R}{4\pi\epsilon_0 \cdot (R^2+R^2)^{\frac{3}{2}}}$$

E field due to the q_2 charge of Ring 2 at the point is R,

$$E_2 = \frac{q_2 2R}{4\pi\epsilon_0 (R^2+4R^2)^{\frac{3}{2}}}$$

$$= \frac{q_2 R}{2\pi\epsilon_0 \cdot (R^2+4R^2)^{\frac{3}{2}}}$$

According to the problem,

$$E_1 = E_2$$

$$\Rightarrow \frac{q_1 R}{4\pi\epsilon_0 (R^2+R^2)^{\frac{3}{2}}} = \frac{q_2 R}{2\pi\epsilon_0 (R^2+4R^2)^{\frac{3}{2}}}$$

(7)

$$\Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{2(R^2+R^2)^{\frac{3}{2}}}{(R^2+4R^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{2(2R^2)^{\frac{3}{2}}}{(5R^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{\alpha_1}{\alpha_2} = \_\_\_\_\_\_$$

$$\Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{5.66\,R^3}{11.18\,R^3}$$

$$\therefore \frac{\alpha_1}{\alpha_2} = 0.51$$ Answer:/

23P: A nonconducting rod of length L has charge-q uniformly distributed along its length.

(a) What is the linear charge density of the rod?

(b) What is the electric field at point P, a distance a from the end of the rod?

(c) If P were very far from the rod compared to L, the rod would look like a point charge. Show that your answer to ?

(d) Reduces to the electric field of a point charge for a>>L.

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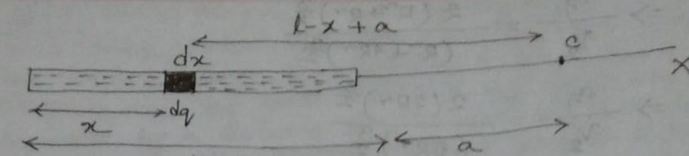


Uniform linear charge density,

$$\lambda = \frac{q}{l} = \frac{dq}{dx}$$

$\therefore dq = \lambda dx$

The electric field due to the charge dq,

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{(l+a-x)^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(l+a-x)^2}$$

$$E = \int_0^l dE$$

$$= \frac{\lambda}{4\pi\epsilon_0} \int_{x=0}^{x=l} \frac{dx}{(l+a-x)^2}$$

Let, $l+a-x = u$

$\Rightarrow -dx = du$

$$\therefore \int \frac{dx}{(l+a-x)^2} = -\int \frac{du}{u^2}$$

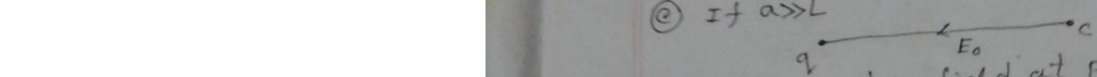
$$\frac{1}{u} = \frac{1}{l+a-x}$$

$$\therefore E = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{l+a-x}\right]_0^l$$

$$E = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{l+a-l} - \frac{1}{l+a-0}\right]$$

$$E = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{l+a}\right]$$

(c) If $a \gg L$



The net electric field at point C.

$$E_0 = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2}$$

Electric field due to an electric dipole

P: Consider two charged particles of magnitude q but opposite sign, separated by a distance d. If $z >> d$, find the electric field E due to the dipole at point P, a distance z from the midpoint of the dipole and on the dipole axis. Hence write E in terms of the electric dipole moment p.

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{r_+^2} \hat{k}$$

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0} \frac{q}{r_-^2} (-\hat{k})$$

$$\therefore \vec{E} = \vec{E}_+ + \vec{E}_-$$

$$= \frac{q}{4\pi\epsilon_0} \cdot \left(\frac{1}{r_+^2} - \frac{1}{r_-^2} \right) \hat{k}$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(z - \frac{d}{2})^2} - \frac{1}{(z + \frac{d}{2})^2} \right] \hat{k}$$

$$\bar{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(z - \frac{d}{2})^2} - \frac{1}{(z + \frac{d}{2})^2} \right] \hat{k}$$

If $z >> d$

Binomeal theoream:

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \cdots$$

$$\therefore \bar{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\{z(1 - \frac{d}{2z})\}^2} - \frac{1}{\{z(1 + \frac{d}{2z})\}^2} \right] \hat{k} \quad \left[\because \frac{z}{z} = \frac{d}{d} \right.$$

$$\bar{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{z^2\left(1-\frac{d}{2z}\right)^2} - \frac{1}{z^2\left(1+\frac{d}{2z}\right)^2}\right]\hat{k}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{z^2}\left[\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right]\hat{k}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]\hat{k}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]\hat{k}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 z^2}\left[\left\{1+\left(-\frac{d}{2z}\right)\right\}^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]\hat{k}$$

$$\bar{E} = \frac{q}{4\pi\epsilon_0 z^2}\left[\left\{1+(-2)\left(-\frac{d}{2z}\right)+\cdots\right\} - \left\{1+(-2)\left(\frac{d}{2z}\right)+\cdots\right\}\right]\hat{k}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1+\frac{d}{z}\right) - \left(1-\frac{d}{z}\right)\right]\hat{k}$$

$$\bar{E} = \frac{q}{4\pi\epsilon_0 z^2}\left(\cancel{1}+\frac{d}{z}-\cancel{1}+\frac{d}{z}\right)\hat{k}$$

$$\vec{E} = \frac{q}{\cancel{4}\pi\epsilon_0 z^2}\left(\cancel{2}\ \frac{d}{z}\right)\hat{k}$$

2

$$\vec{E} = \frac{qd}{2\pi\epsilon_0 z^3}\hat{k}$$

$\therefore$ Electric dipole moment

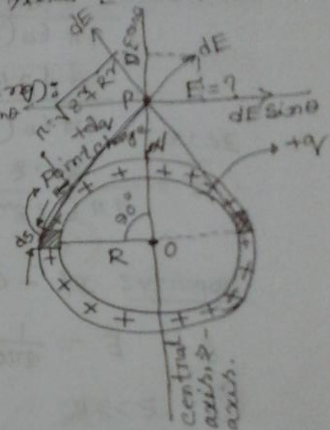
$P = \text{charge} \times \text{distance}$

$\Rightarrow \vec{P} = q \times d\cdot\hat{k} \quad \therefore \vec{E} = \frac{1}{2\pi\epsilon_0 z^3}$

(12)

Electric field due to a line of charge:

Consider a thin ring of radius R with a uniform positive linear charge density λ around its circumference. What is the electric field E at point P.



E due to a line of charge (ring):

$$dE = \frac{1}{4\pi\epsilon_0} \frac{|+dq|}{r^2}$$

Linear charge density, $\lambda = \frac{\text{charge}}{\text{Length}}$

$$\lambda = \frac{dq}{ds}$$

$$dq = \lambda ds$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(\sqrt{z^2+R^2})^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2+R^2)}$$

$$E = \int_0^{2\pi R} dE \cos\theta$$

$$E = \int_0^{2\pi R} \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{z^2+R^2} \frac{z}{\sqrt{z^2+R^2}}$$

z = fixed, variable = S

$$= \int_0^{2\pi R} \frac{\lambda z}{4\pi\epsilon_0} \frac{ds}{(z^2+R^2)^{\frac{3}{2}}}$$

$1 + \frac{1}{2} = \frac{3}{2}$

$$= \frac{\lambda z}{4\pi\epsilon_0 (z^2+R^2)^{\frac{3}{2}}} \int_0^{2\pi R} ds$$

$$= \frac{\lambda z}{4\pi\epsilon_0 (z^2+R^2)^{\frac{3}{2}}} (2\pi R)$$

$$q = \lambda(2\pi R)$$

$$= \frac{z\{\lambda(2\pi R)\}}{4\pi\epsilon_0 (z^2+R^2)^{\frac{3}{2}}}$$

If $+q$,

$$\therefore \bar{E} = \frac{qz}{4\pi\epsilon_0 (z^2+R^2)^{\frac{3}{2}}} \hat{k}$$

Property: If $-q$

① $$\bar{E} = \frac{1}{4\pi\epsilon_0} \frac{qz}{(z^2+R^2)^{\frac{3}{2}}} (-\hat{k})$$

② $z >> R$

$$E = \frac{1}{4\pi\epsilon_0} \frac{qz}{(z^2+0)^{\frac{3}{2}}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{1}{z^{2\times\frac{3}{2}}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{z^2}$$

③ E at the center of the ring.

$z=0$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q(0)}{(0^2+R^2)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0} \frac{q(0)}{R^3}$$

$E=0$