Tutoring Lesson Notes: Lesson 3 (29/4)

Let us finish off the topic of Momentum by looking at a question:

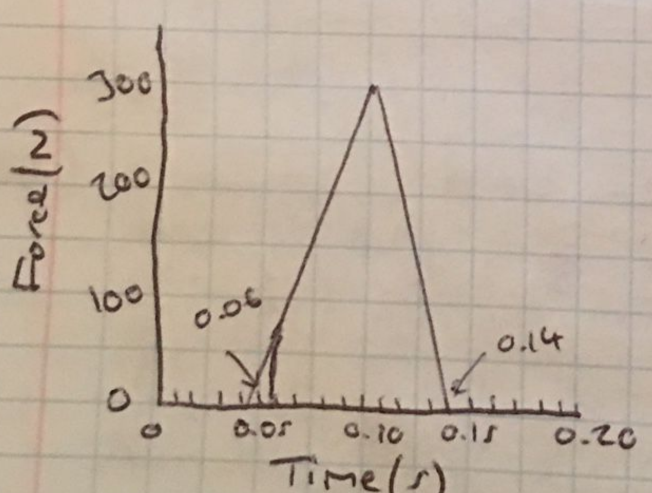
Q89-91 (Practice Test 2)

A ball of mass 0.15kg is travelling in a straight horizontal path at speed 30 ms^{-1}. The ball has a perfectly elastic collision with a body moving directly towards it and rebounds in the opposite direction. The force that acts on the ball while in contact with the body is plotted below.

No net external force acts on the ball and body during the collision.

Under these conditions there is a change in momentum:

$$\Delta p = F\Delta t$$



89) After rebounding from the body, what is the magnitude of the momentum of the ball?

Answer: 7.5 Ns

Looking at the question, the initial momentum is

$p_0 = mv = 0.15 \cdot 30 = 4.5 \text{ kg m s}^{-1}$

Now let's calculate $\Delta p = F \Delta t$

$\Delta t = 0.14 - 0.06 = 0.08$

What force do we use though? We can't use F = 300N because that would mean F = 300 for the whole time period. In fact, we want the ~~average~~ average force

$F_{Avg} = \frac{1}{2} \cdot 300 = 150N$

$\Rightarrow \Delta p = \frac{1}{2} \cdot 300 \cdot 0.08 = 12 \text{ kg m s}^{-1}$ (note this is the AREA of the graph

Thus $p_1 = \Delta p - p_0 = 7.5 \text{ kg m s}^{-1}$
$(= 7.5 \text{ Ns})$

If you're confused by the unit change, note that as $F = ma$, $1N = 1 \text{ kg m s}^{-2}$

$\Rightarrow$ ~~1 kg ms~~ $1 \text{ Ns} = 1 (\text{kg m s}^{-2}) \text{s} = 1 \text{ kg m s}^{-1}$

90) As a consequence of the elastic collision, the kinetic energy of the ball:

Answer: Increases by 120J

$E_{k,initial} = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 0.15 \cdot 30^2 = 67.5J$

$V_{final} = \frac{p}{m} = \frac{7.5}{0.15} = 50 \text{ m s}^{-1}$

$\Rightarrow E_{k,final} = \frac{1}{2} \cdot 0.15 \cdot 50^2 = 187.5J$

increase of 120J

(If flurtered, just remember that the momentum is increasing, so the velocity is increasing, so the kinetic energy is increasing. For the multiple choice options given, this logic would have given you the answer straight away).

91) Consider the combined momentum and kinetic energy of the ball and body.

Is the momentum conserved?
Is the kinetic energy conserved?

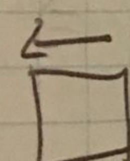
Answer: Yes and Yes

Momentum always conserved in the absence of external forces

Collision is elastic: by definition kinetic energy is conserved.

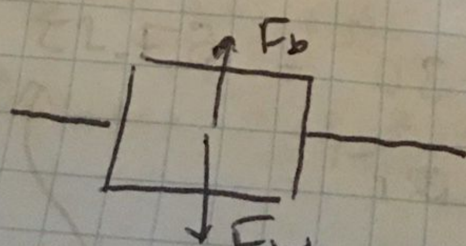
Now let's study fluids:

Fluids - deform under shear stress
- liquid, gas



Archimedes' Principle:

Buoyant force on a body = weight of the fluid submerged

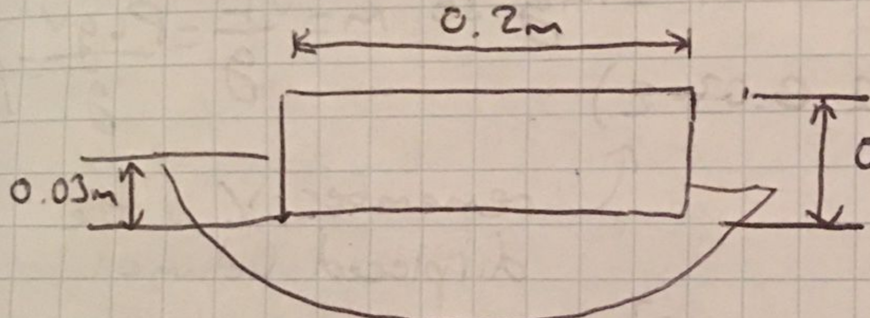


$$\boxed{F_b = \rho g V}$$

it is a common mistake to use the ~~...~~ density of the object

F_b = Buoyancy force
ρ = density of the fluid
$g = 9.8\, ms^{-2}$ (~ 10)
V = volume of the displaced fluid

Q29-32 (Practice Test 2)



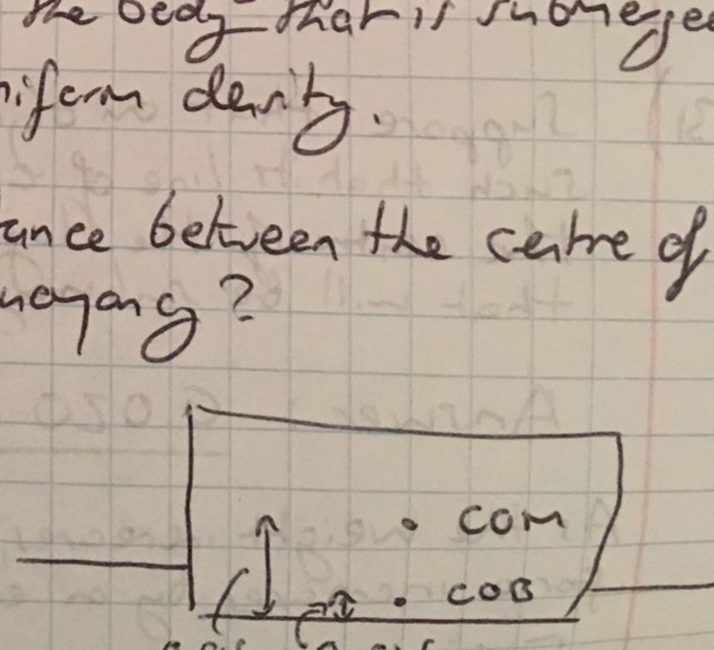
$\rho_{water} = 1000 kg$

log dimensions are $0.2 \times 0.1 \times 2\, m$

The point of application of the buoyancy force is the centre of mass of the part of the body that is submerged. Assume that the log is of uniform density.

29) What is the vertical distance between the centre of mass and the centre of buoyancy?

Answer: <u>0.035m</u>



$COB = \frac{0.03}{2} = 0.015m$ from bottom

$COM = \frac{0.1}{2} = 0.05m$ from bottom

$0.05 - 0.015 = \underline{0.035m}$

30) What is the density of the log?

Answer: $\underline{300 \text{ kg m}^{-3}}$

density $= \frac{\text{mass}}{\text{volume}}$ $\left(\rho = \frac{m}{V} \right)$

$V_{log} = 0.2 \cdot 0.1 \cdot 2 = 0.04 \text{ m}^3$

The mass is just $m = \rho_b V$

(From $F = \rho_b g V$, F is a buoyancy force equal to weight in magnitude, and $F = mg$ so $m = \frac{F}{g} = \frac{\rho_b g V}{g} = \rho_b V$

$m = 1000 \cdot (0.2 \cdot 0.03 \cdot 2)$

$= 12 \text{ kg}$

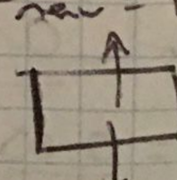
remember V is the displaced volume!

$\rho = \frac{12}{0.04} = \underline{300 \text{ kg m}^{-3}}$

31) Suppose that an object of 20kg is placed on the log such that its line of action of its weight passes through the centre of the log. What is the additional volume that will be submerged?

Answer: $\underline{0.020 \text{ m}^3}$

$F_{new} = F + (20 \cdot 10)$



$W_{new} = W + (20 \cdot 10)$

As the weight increases, the buoyancy force increases by an equal amount.

$F_{new} = 200 + 120 = 320 \text{N}$

From $F_{new} = \rho g V_{new}$ where $V_{new} = 0.2 \cdot 2 \cdot x$

$x = \frac{320}{1000 \cdot 10 \cdot 2 \cdot 0.2} = 0.08 \text{m}$

$\Rightarrow (0.05 - 0.03) \cdot 0.2 \cdot 2 = 0.02 \text{m}^3$

32) Suppose an object is placed on the log such that the line of action of its weight passes through the centre of mass of the log.
If the object is to completely submerge the log, what is the smallest mass that the object can have?

Answer: <u>28kg</u>

$$F = \rho g V = 1000 \cdot 10 \cdot (0.2 \cdot 0.1 \cdot 2) = 400N$$

$$\frac{400 - 120}{10} = \underline{28kg}$$

Bernoulli's ~~R~~ Law:

At any point in a fluid:

$$\boxed{P + \rho g h + \frac{1}{2} \rho v = \text{constant}}$$

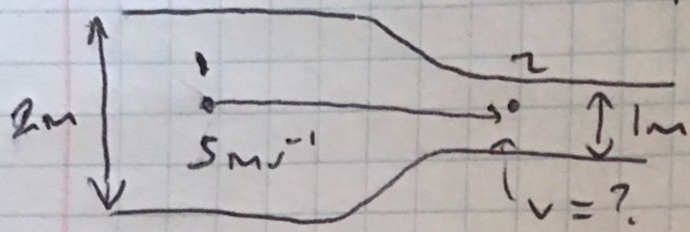
P = Pressure
ρ = density
v = velocity

Assumptions include:
- Fluid is incompressible
- Flow is non-turbulent

Also note that for a liquid that is moving, the flow rate $= Av$, where A is the cross-sectional area of the tube.

The flow rate is constant

$$f = A_1 v_1 = A_2 v_2$$



$$F = A_1 v_1 = \pi\left(\frac{2}{2}\right)^2 \cdot 5 = 5\pi \sim \underline{15\,m^3 s^{-1}}$$

$$15 = \pi\left(\frac{1}{2}\right)^2 \cdot v_2$$

$$\Rightarrow v_2 = \underline{20\,m s^{-1}}$$

Also, by Bernoulli's Law:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2$$

Qs 66–68:

66) A horizontal tube with diameter 2d branches into two horizontal tubes, each with diameter d. The speed of liquid flow in the first tube is v. What is the speed of flow in each smaller tube?

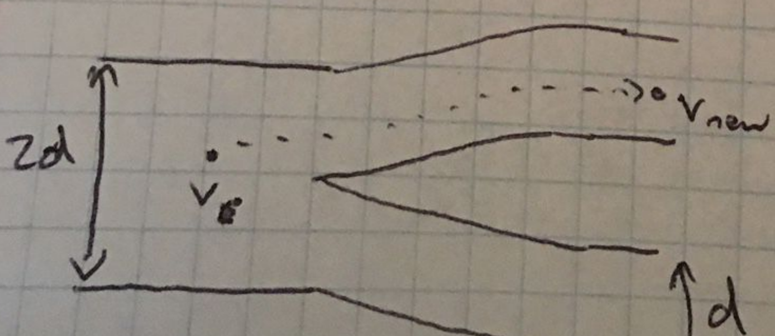
Answer: $\underline{2v}$

$$f = A_1 v_1 = 2 A_2 v_2$$

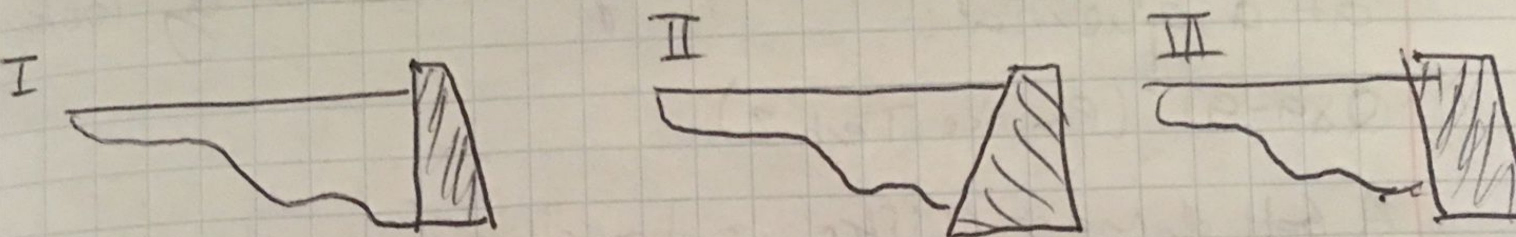
$$\pi d^2 v = 2\left(\pi\left(\frac{d}{2}\right)^2\right) v_{new}$$

$$\Rightarrow \pi d^2 v = 2\left(\frac{\pi d^2}{4}\right) v_{new}$$

$$\Rightarrow \underline{v_{new} = 2v}$$



67) For which dam will the pressure at the base of the dam wall be the greatest?



Answer: Same for all three

$P = P_{atm} + \rho g y$. As y (depth) is equal for all three, $P_I = P_{II} = P_{III}$.

68) For which design will the total force of water be the least?

Force = Pressure × Area

I has the smallest surface area, so

Answer: I