Stat 330
Homework 6

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1)

(a)

$X \backslash Y$	0	1	2	$p_x(x)$
0	0.3	0.1	0.1	0.5
1	0.2	0.1	0	0.3
2	0.1	0.1	0	0.2
$p_y(y)$	0.6	0.3	0.1	1

(b) E(X) = (0)(0.5) + (1)(0.3) + (2)(0.2) = 0.7
E(Y) = (0)(0.6) + (1)(0.3) + (2)(0.1) = 0.5

$E(X^2) = (0)^2(0.5) + (1)^2(0.3) + (2)^2(0.2) = 1.1$
$E(Y^2) = (0)^2(0.5) + (1)^2(0.3) + (2)^2(0.2) = 0.7$

$Var(X) = E(X^2) - |E(X)|^2 = 1.1 - 0.7^2 = 0.61$
$Var(Y) = E(Y^2) - |E(Y)|^2 = 0.7 - 0.5^2 = 0.45$

(c) E(XY) = (0)(0)(0.3) + (1)(0)(0.1) + (2)(0)(0.1) +
(0)(1)(0.2) + (1)(1)(0.1) + (2)(1)(0) +
(0)(2)(0.1) + (1)(2)(0.1) + (2)(2)(0) = 0.3

Cov(X, Y) = E(XY) - E(X)E(Y) = 0.3 - (0.7)(0.5) = -0.05

$$\text{Corr(X, Y)} = \frac{Cov(X,Y)}{\sqrt{Var(X)*Var(Y)}} = \frac{-0.05}{\sqrt{0.61*0.45}} = \text{-0.095}$$

(d) Covariance != 0, and $p_{x,y}(1,0)$!= $p_x(1)$ * $p_y(0)$.
Therefore the two days are not independent.

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2)

(a) P(X=Y) = $p_{x,y}(0,0)$ + $p_{x,y}(1,1)$ + $p_{x,y}(2,2)$ = 0.3 + 0.1 + 0 = 0.4

(b) P(X<Y) = $p_{x,y}(0,1)$ + $p_{x,y}(0,2)$ + $p_{x,y}(1,2)$ = 0.1 + 0.1 + 0 = 0.2

(c) P(X>Y) = $p_{x,y}(1,0)$ + $p_{x,y}(2,0)$ + $p_{x,y}(2,1)$ = 0.2 + 0.1 + 0.1 = 0.4

(d) $p_{x,y}(0,0) = 0.3$

(e) $p_{x,y}(1,2) = 0$

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3)

(a, b)

X \ Y	0	1	2	3	$p_x(x)$
0	0.125	0.25	0.125	0	.5
1	0	0.125	0.25	0.125	.5
$p_y(y)$	0.125	0.375	0.375	0.125	1

(c) P(Y=1 | X=1) = $\frac{P(X{=}1,Y{=}1)}{P(X{=}1)} \Rightarrow \frac{0.125}{0.5} = 0.25$

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4)

(a)

X \ Y	2	3	4	$p_x(x)$
1	0.083	0.167	0	0.25
2	0.167	0	0.333	0.5
3	0.083	0.167	0	0.25
$p_y(y)$	0.333	0.333	0.333	1

(b) Two variables are independent if, for all values of X and Y:
P(x | y) = P(x)
P(x ∩ y) = P(x)*P(y)
The variables are dependent, as P(x ∩ y) for (2, 3) = 0, but P(X=2)*P(Y=3) = .167
This violates the second rule of independence.

(c)

A \ B	2	3	4	$p_x(x)$
1	0.083	0.083	0.083	0.25
2	0.167	0.167	0.167	0.5
3	0.083	0.083	0.083	0.25
$p_y(y)$	0.333	0.333	0.333	1

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5)

(a) $\int_0^2$cx = 1 $\quad\Rightarrow\quad (\frac{cx^2}{2})\Big|_0^2 \quad\Rightarrow\quad$ 0+2c = 1 $\quad\Rightarrow\quad$ c = 1/2

(b) For $0 \leq x \leq 2$, $F_X(x) = \int$cx = $\int$(1/2)x = $\frac{x^2}{4}$
For other, $F_X(x) = 0$

Thus, $F_X(x) = \begin{cases} x^2/4 & 0 \leq x \leq 2 \\ 0 & otherwise \end{cases}$

(c) P($0.5 \leq X \leq 1.5$) = $\int_{0.5}^{1.5}$cx = $(\frac{x^2}{4})\Big|_{0.5}^{1.5} = \frac{1}{2}$

(d) P($1 \leq X \leq 2$) = $F_X(2) - F_X(1) = \frac{2^2}{4}$ - $\frac{1^2}{4}$ = 1 - .25 = .75

(e) .75 = $F_X(x) \Rightarrow \frac{x^2}{4}$ = .75 $\Rightarrow$ x = 1.732

(f) E(X) = $\int_0^2$x*f(x) $\Rightarrow \int_0^2 \frac{x^2}{2} \Rightarrow (\frac{x^3}{6})\Big|_0^2 \Rightarrow \frac{4}{3}$ = 1.333

(g) Var(X) = $\int_0^2$(x - E(X))^2*f(x) $\Rightarrow \int_0^2$ (x - 1.333)^2*$\frac{x}{2} = \frac{2}{9}$ = 0.2222

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6)

(a) For $0 \leq x \leq 1$, $F_X(x) = \int_0^x$x = $\frac{x^2}{2}$
For $1 < x \leq 1.5$, $F_X(x) = \int_0^1$x + $\int_1^x$1 = $(\frac{x^2}{2})\Big|_0^1$ + (x)$\Big|_1^x$ = $\frac{1}{2}$ + x - 1 = x - $\frac{1}{2}$

Thus, $F_X(x) = \begin{cases} x^2/2 & 0 \leq x \leq 1 \\ x - 1/2 & 1 \leq x \leq 1.5 \\ 0 & otherwise \end{cases}$

(b) P($0.5 \leq X \leq 1.2$) = $F_X(1.2) - F_X(0.5)$ = (1.2 - 1/2) - (0.125) = 0.575

(c) E(X) = $\int_0^1$x*(x) + $\int_1^{1.5}$x*(1) = (1/3) + (5/8) = 0.958