Stat 330
Exam 3

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1)

(a) E(X) = $\frac{0+30}{20}$ = 15 minutes

(b) F_X(t) = $\frac{t-a}{b-a}$ $\Rightarrow$ $\frac{t-0}{30-0}$ = $\frac{10}{30}$. P(X>10) = 1 - P(X<10) $\Rightarrow$ 1 - $\frac{10}{30}$ = 1-.333 = 0.666

(c) P(15<X<25) = P(X<25) - P(X<15) = $\frac{25}{30}$ - $\frac{15}{30}$ = .833 - .5 = 0.333

(d) .10 = F_X(t) $\Rightarrow$ .10 = $\frac{t}{30}$ $\Rightarrow$ t = 3 minutes

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2)

(a) E(X) = 1/10 seconds/spike, so $\lambda = 10$. Thus, X $\sim$ Exp(10)

(b) E(X) = 1/10, Var(X) = $1/10^2$ = 1/100

(c) F_X(t) = 1-e^{-10t} $\Rightarrow$ P(X$\leq$.07) = 1-$e^{-10*.07}$ = .503

(d) X $\sim$ Gamma(50, 10)

(e) E(T) = 50/10 = 5 seconds

(f) P(T<3), convert gamma to poisson $\rightarrow$ X $\sim$ Pois(10 * 3) = Pois(30)
P(T<3) = P(X$\geq$50) $\Rightarrow$ 1 - P(X$\leq$49) = 1 - 0.99948110 (From appendix) = 5.2 * 10^{-4}

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3)

(a) X $\sim$ N(20, 0.01). P(X<20.1) - P(X<19.9) = P(Z< $\frac{20.1-20}{\sqrt{.01}}$) - P(Z< $\frac{19.9-20}{\sqrt{.01}}$) = P(Z<1) - P(Z<-1) $\Rightarrow$ 0.8413 - 0.1587 (using Z table) = 0.6824

(b) P(Z<$\frac{x-20}{\sqrt{.01}}$) = .1 $\Rightarrow$ P(Z<-1.28) $\rightarrow$ -1.28 = $\frac{x-20}{\sqrt{.01}}$ $\Rightarrow$ x = 19.872

(c) 801/40 = each bottle averages 20.025. P(Z>$\frac{20.025-20}{\sqrt{.01}/\sqrt{40}}$) = 1 - P(X<1.58) = 1 - .9429 = 0.0571

(d) P(Z>$\frac{19.96-20}{\sqrt{.01}/\sqrt{40}}$) = P(X<-2.53) = 0.0057

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4)

(a) P = $\begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix}$

(b) P^3 = $\begin{pmatrix} 0.65 & 0.35 \\ 0.63 & 0.37 \end{pmatrix}$ $\Rightarrow$ P(3rd is A | 1st is B) = 0.63

(c) $[\pi_1\ \pi_2]$ $\begin{pmatrix} 0.75 & 0.25 \\ 0.45 & 0.55 \end{pmatrix}$ $\Rightarrow$ $(.75\pi_1 + .45\pi_2 = \pi_1)$ *and* $(25\pi_1 + .55\pi_2 = \pi_2)$

$\pi_1 = .643, \pi_2 = .357$, thus P(Last is B) = .357

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5)

(a) P = $\begin{pmatrix} 0.3 & 0.7 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

(b) $P_3 = P_0 * P^3 = (0.5, 0.5, 0) * \begin{pmatrix} 0.45 & 0 & 0.55 \\ 0 & 0 & 0 \\ 0.79 & 0 & 0.21 \end{pmatrix} = \begin{pmatrix} 0.225 & 0 & 0.275 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$