

Method of Moments Estimation

Suppose $X_1 \stackrel{iid}{\sim} \text{Exp}(\lambda)$ for $i = 1, \dots, n$.

1. Find the method of moments estimator for λ
2. If we observe data $x = 2, 4, 7, 10$, give the value for the method of moments estimate.

Answer:

1. Methods of Moments

Since there's only one unknown parameter, we will just use the 1st moment. The 1st (population) moment is $\mu_1 = E(X)$

$$\mu_1 = E(X) = \frac{1}{\lambda} \quad (\text{known for exponential distribution})$$

The 1st sample moment is always $m_1 = \frac{\sum_{i=1}^n x_i}{n} = \bar{x}$

Set the 1st (population) moment equal to 1st sample moment, and solve for λ .

$$\begin{aligned}\mu_1 = E(X) &\stackrel{set}{=} \bar{X} = m_1 \\ \implies \frac{1}{\lambda} &= \bar{x} \\ \implies \hat{\lambda}_{MoM} &= \frac{1}{\bar{x}}\end{aligned}$$

The method of moments estimator for λ is $\hat{\lambda}_{MoM} = \frac{1}{\bar{X}}$

2. Based on our data, $\bar{x} = \frac{2+4+7+10}{4} = 5.75$. Plugging this into our estimator, The method of moments estimate for λ is $\hat{\lambda}_{MoM} = \frac{1}{\bar{x}} = \frac{1}{5.75} = 0.1739$