Stat 330
Homework 9

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1)

(a) $P = \begin{pmatrix} 0.6 & 0.4 \\ 0.75 & 0.25 \end{pmatrix}$

(b) $P^{(3)} = \begin{pmatrix} 0.65 & 0.35 \\ 0.65 & 0.35 \end{pmatrix} \Rightarrow$ P(3rd Red | 1st Red) = .35

(c) $[\pi_1\ \pi_2] \begin{bmatrix} 0.65 & 0.35 \\ 0.65 & 0.35 \end{bmatrix} \Rightarrow (.6\pi_1 + .75\pi_2 = \pi_1)\ and\ (.4\pi_1 + .25\pi_2 = \pi_2)$

$\pi_1 = .625, \pi_2 = .348$, thus P(Last is Red) = .348

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2)

(a) $P = \begin{pmatrix} 0.58 & 0.42 \\ 0.46 & 0.54 \end{pmatrix}$

(b) $P^{(4)} = \begin{pmatrix} 0.52 & 0.48 \\ 0.52 & 0.48 \end{pmatrix} \Rightarrow$ P(4th is Higher | 1st is Higher) = .52

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3)

(a) $P = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix}$

(b) (Initial) * $P^{(3)}$ = (.333, .333, .333) * $\begin{pmatrix} 0.25 & 0.38 & 0.38 \\ 0.38 & 0.25 & 0.38 \\ 0.75 & 0 & 0.25 \end{pmatrix} \Rightarrow$ (0.46, 0.21, .34)

(c) If this chain is regular, there is a power of P that has only positive, non-zero entries:

$$P^{(4)} = \begin{pmatrix} 0.56 & 0.13 & 0.31 \\ 0.5 & 0.19 & 0.31 \\ 0.25 & 0.38 & 0.38 \end{pmatrix} \checkmark$$

(d) $(\pi_1 * \pi_2 * \pi_3) * \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow \pi_1 = 0.444, \pi_2 = 0.222, \pi_3 = 0.333$

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4)

(a) 0.8

(b) 0.1

(c) Team B, because if a puck is loose the team with the largest probability to pick it up is team B with 0.3

(d) P(Loose | Loose) = 0.1 $\Rightarrow$ P(Loose for 2 seconds) = 0.1 * 0.1 = 0.01

(e) $P^{(3)} = \begin{pmatrix} 0.64 & 0.23 & 0.14 \\ 0.39 & 0.41 & 0.2 \\ 0.52 & 0.32 & 0.17 \end{pmatrix} \Rightarrow 0.17$

(f) $\pi_1 = 0.545, \pi_2 = 0.295, \pi_3 = 0.159$

(g) 3600 seconds * π_1 = 3600 * .545 = 1962 $\Rightarrow$ 32.7 min