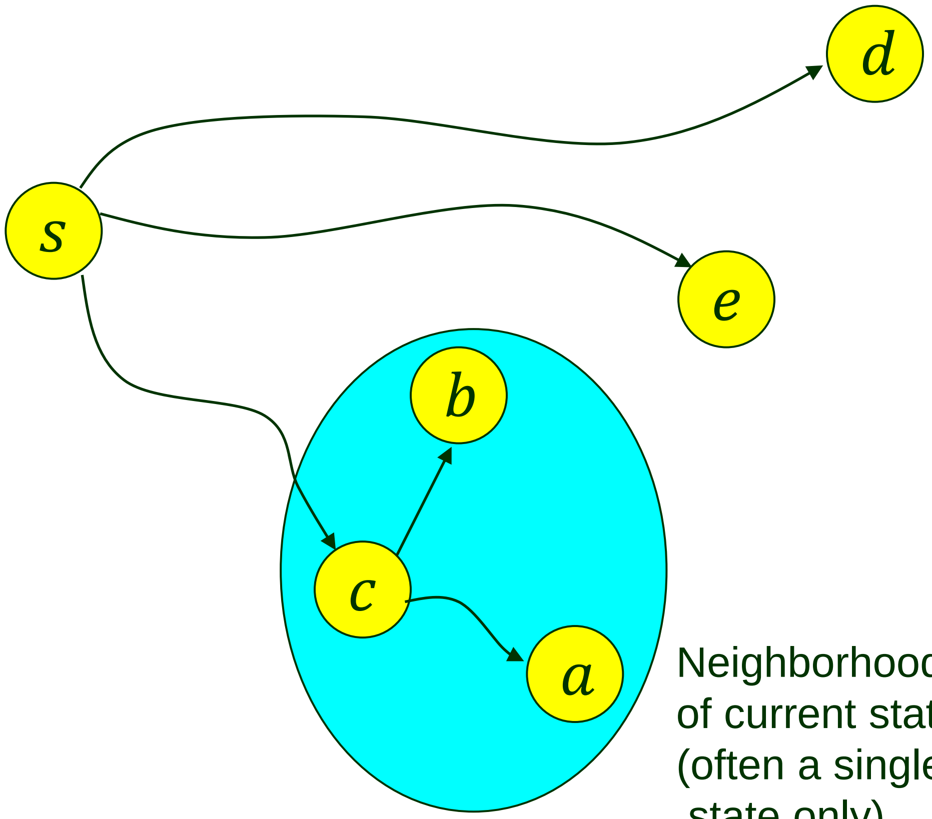
Local Search

Evaluate and modify one or more current states rather than systematically exploring paths from an initial state.



Neighborhood of current states (often a single state only)

* Figures are from the textbook site (or drawn by the instructor) unless the source is specifically cited.

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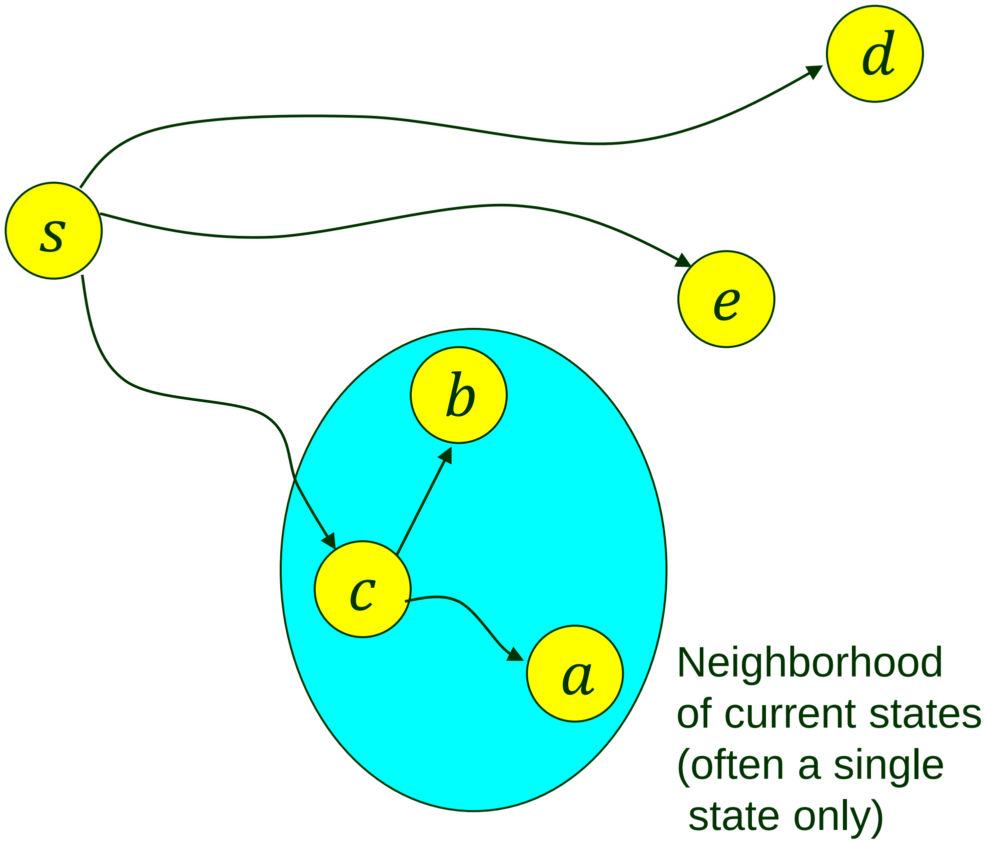
Outline

I. Hill climbing

II. Simulated annealing

III. Genetic algorithms



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Advantages of Local Search

- Use of very little memory.
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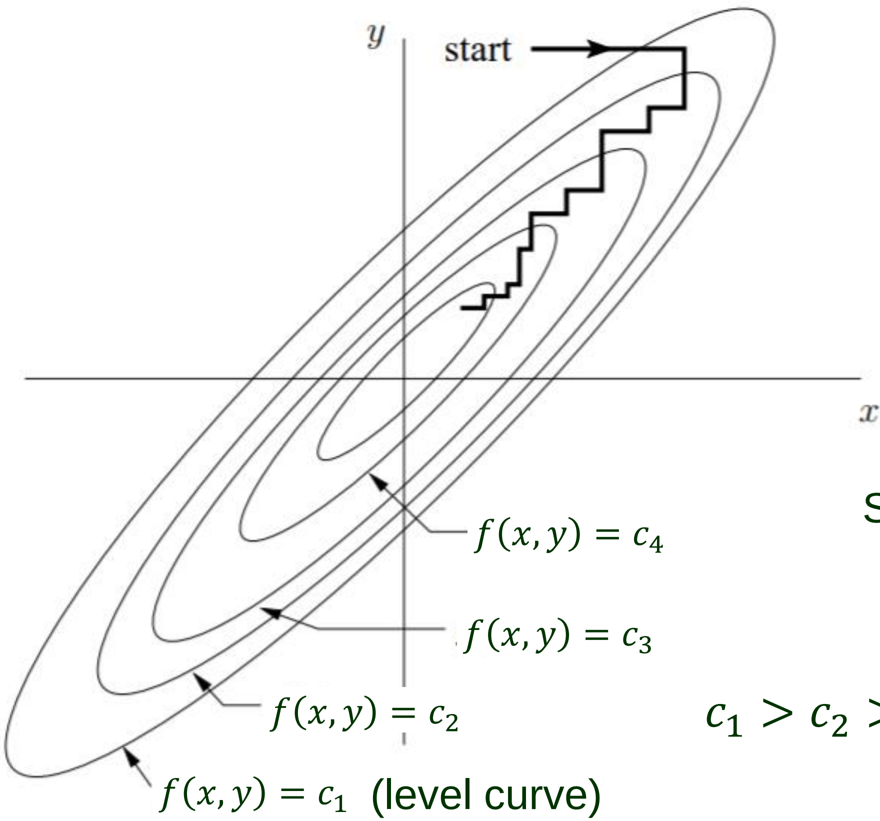
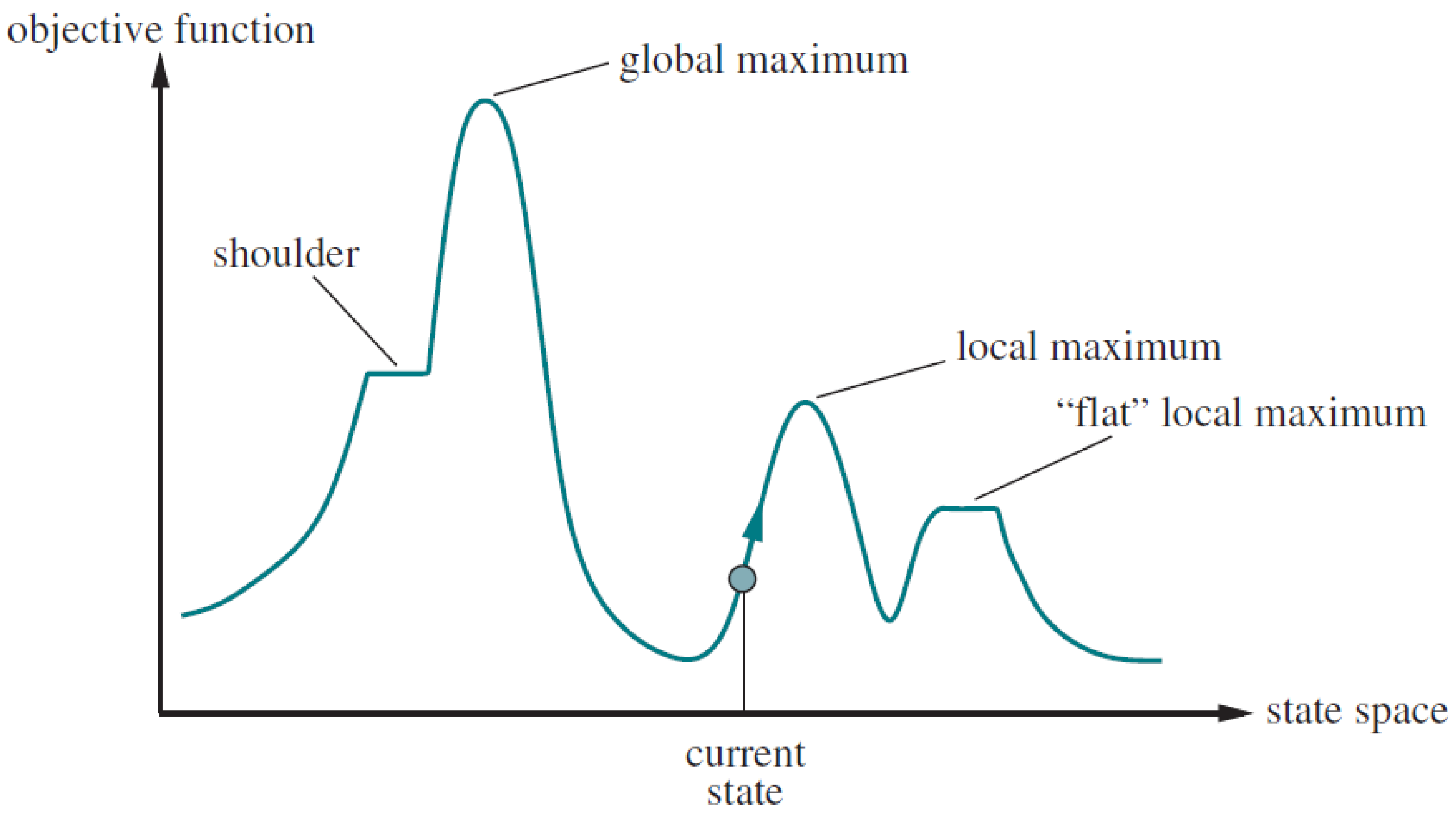


Steepest descent
(along $-\nabla f$)

$c_1 > c_2 > c_3 > c_4$

State Space Landscape



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function HILL-CLIMBING($problem$) **returns** a state that is a local maximum
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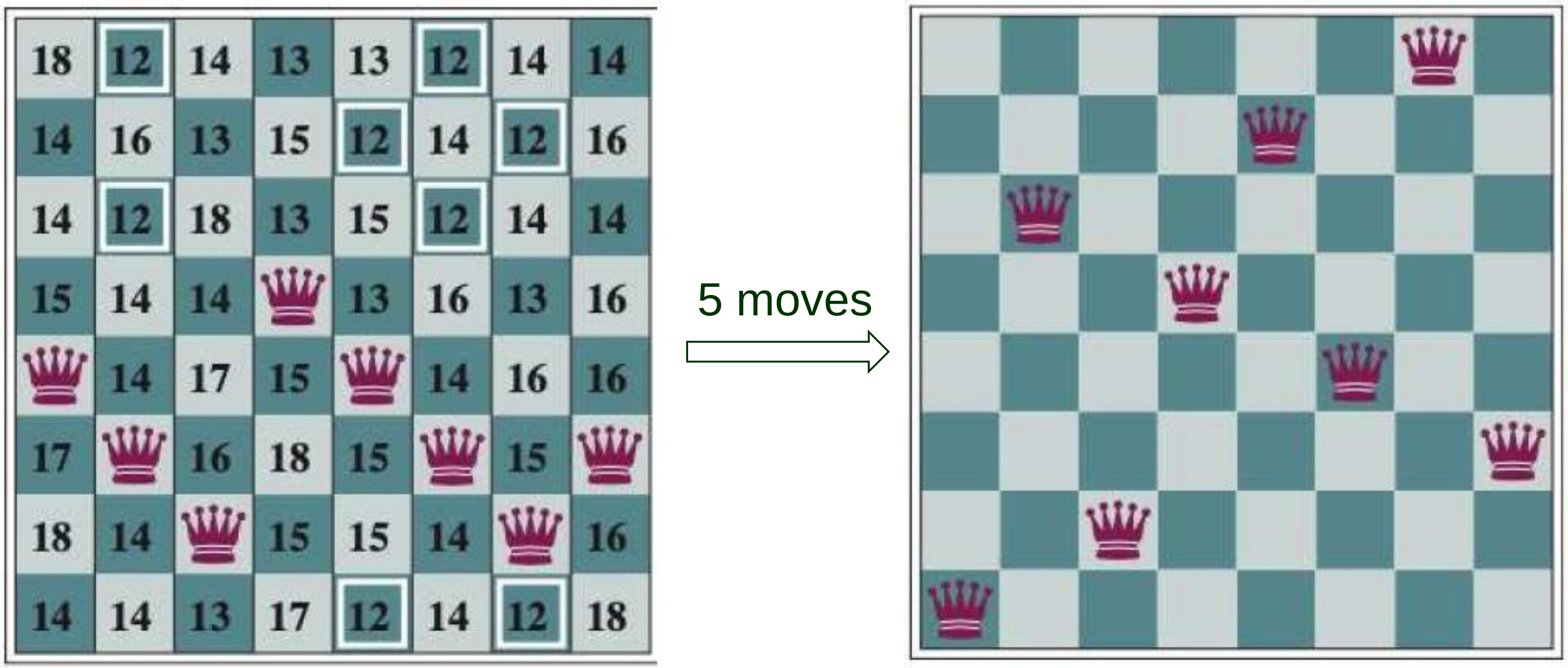
Hill climbing randomly picks one.

Efficiency?


18 12 14 13 13 12 14 14
14 16 13 15 12 14 12 16
14 12 18 13 15 12 14 14
15 14 14 13 16 13 16
14 17 15 14 16 16
17 16 18 15 15
18 14 15 15 14 16
14 14 13 17 12 14 12 18

Efficiency?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5 moves

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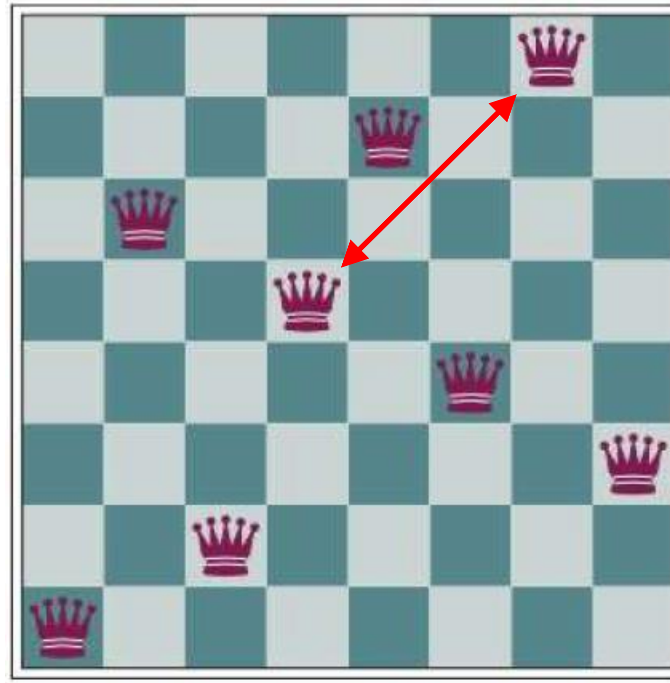
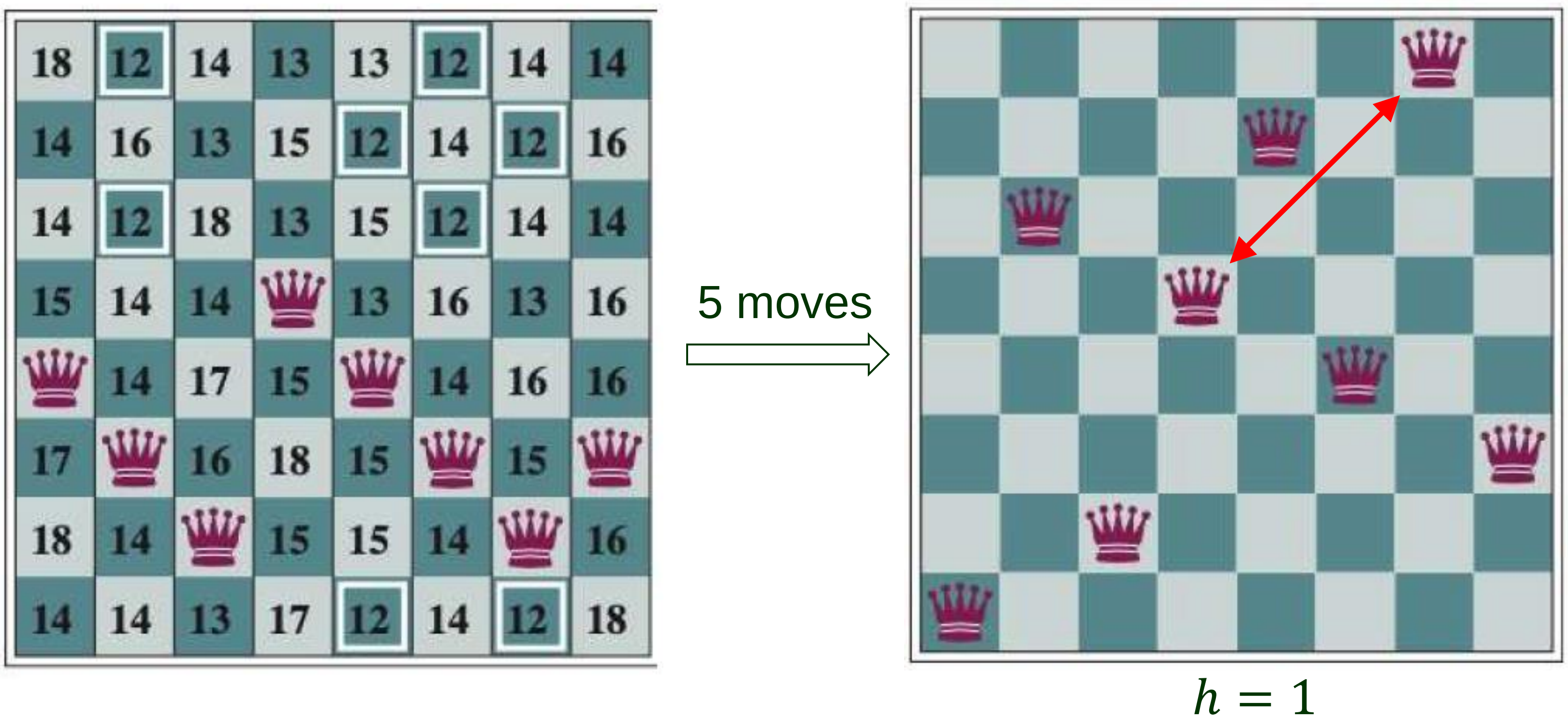


$h = 1$

Efficiency?



- Hill climbing can make rapid progress toward a solution.

Drawback of Hill Climbing (1)

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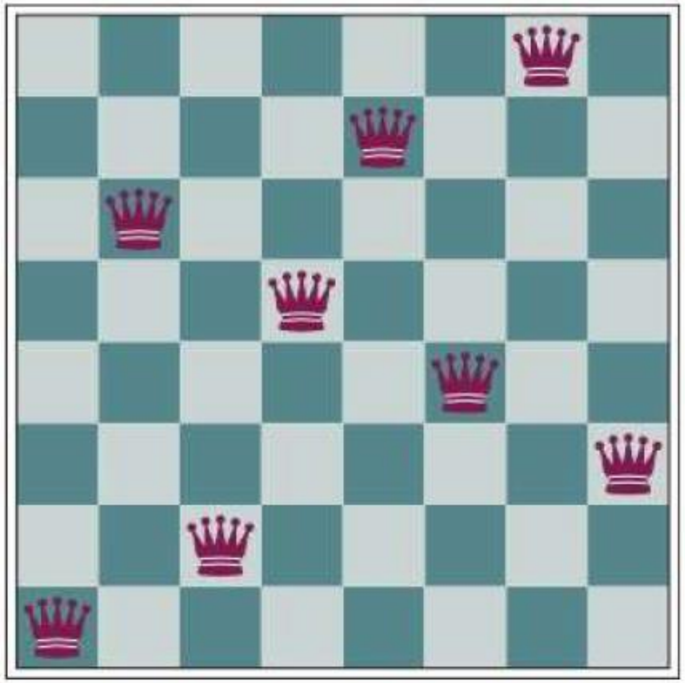

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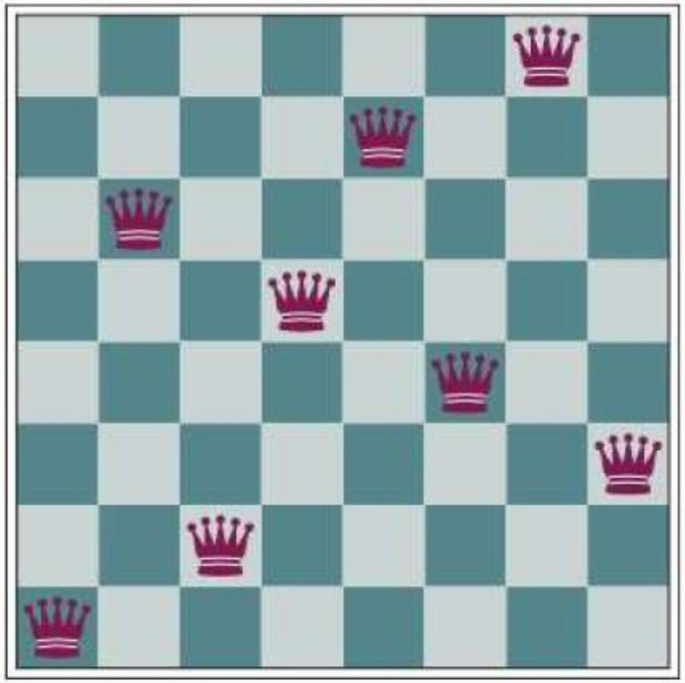

Every move of one queen introduces more conflicts.

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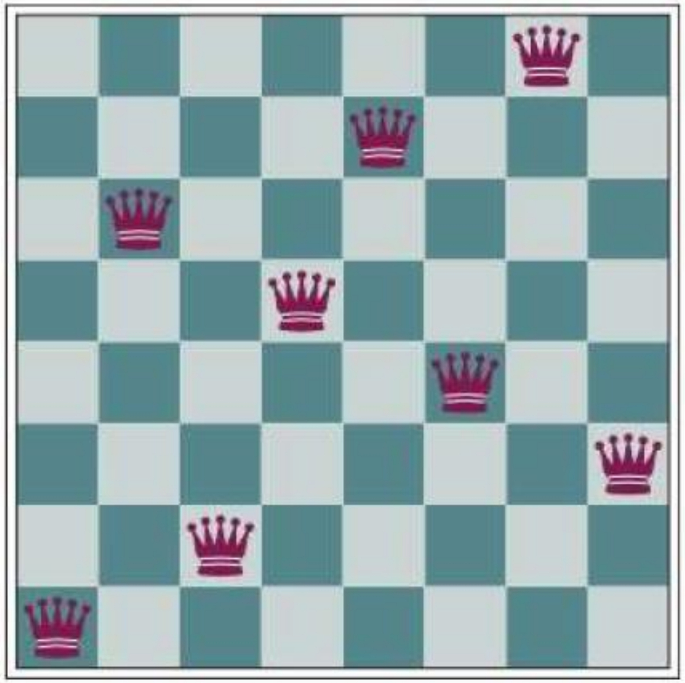
  Hill climbing in the vicinity of a local maximum will be drawn toward it and then get stuck there.



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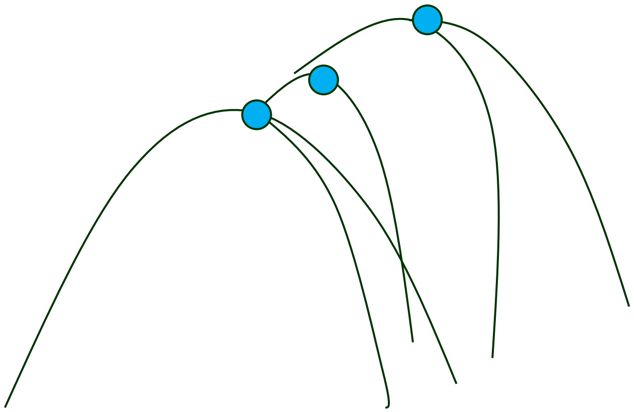
Drawback of Hill Climbing (2)

♠ Ridge: A sequence of local maxima difficult to navigate.



At each local maximum, all available actions are downhill.

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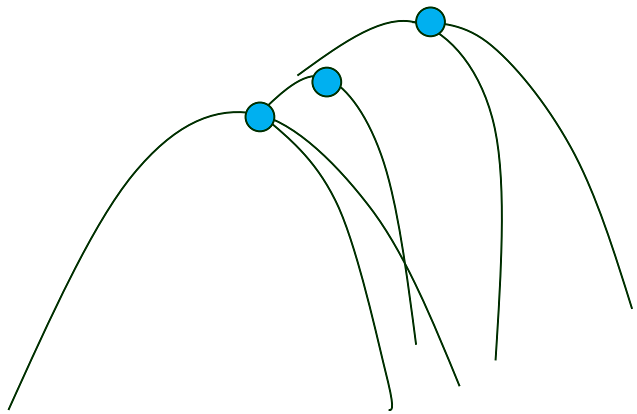

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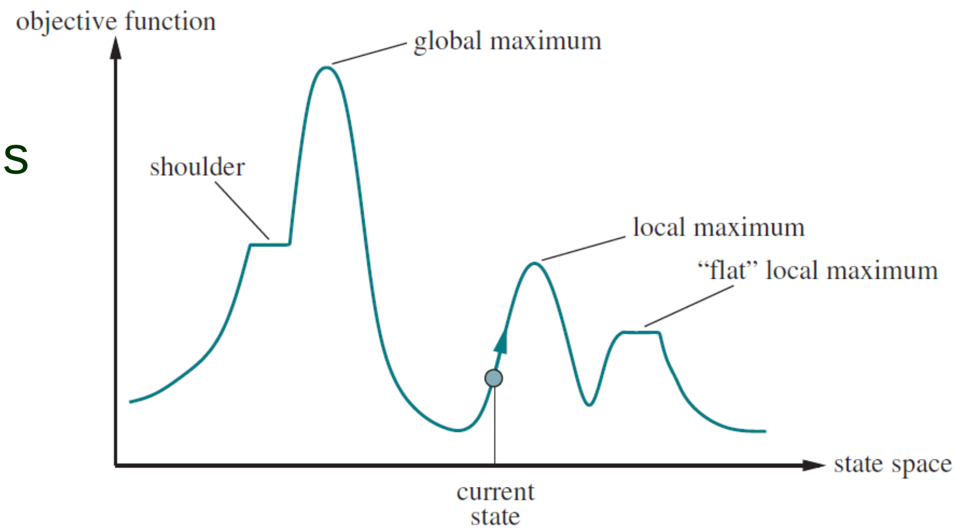

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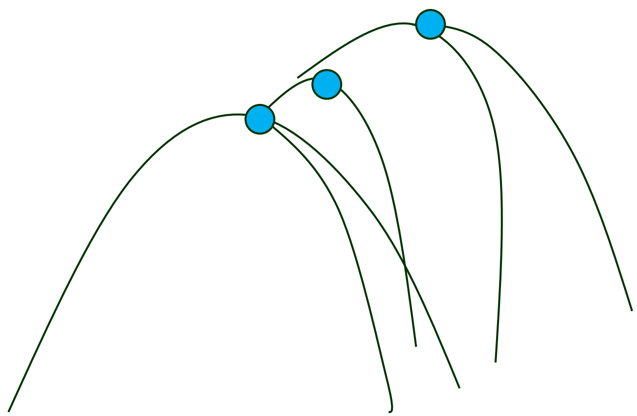

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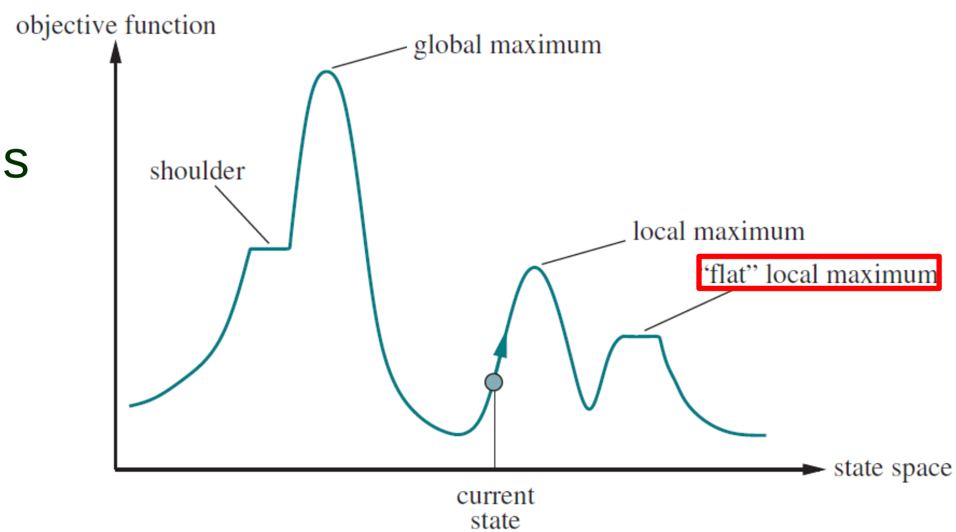

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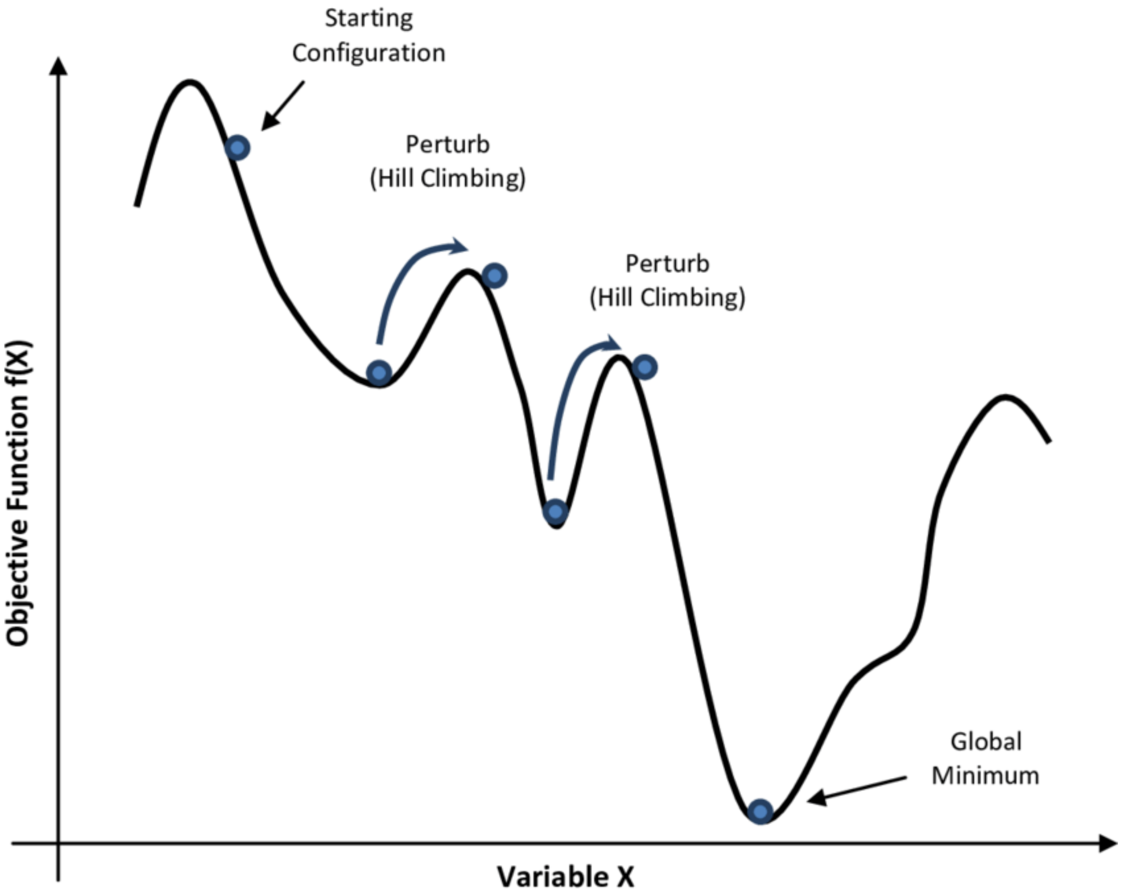

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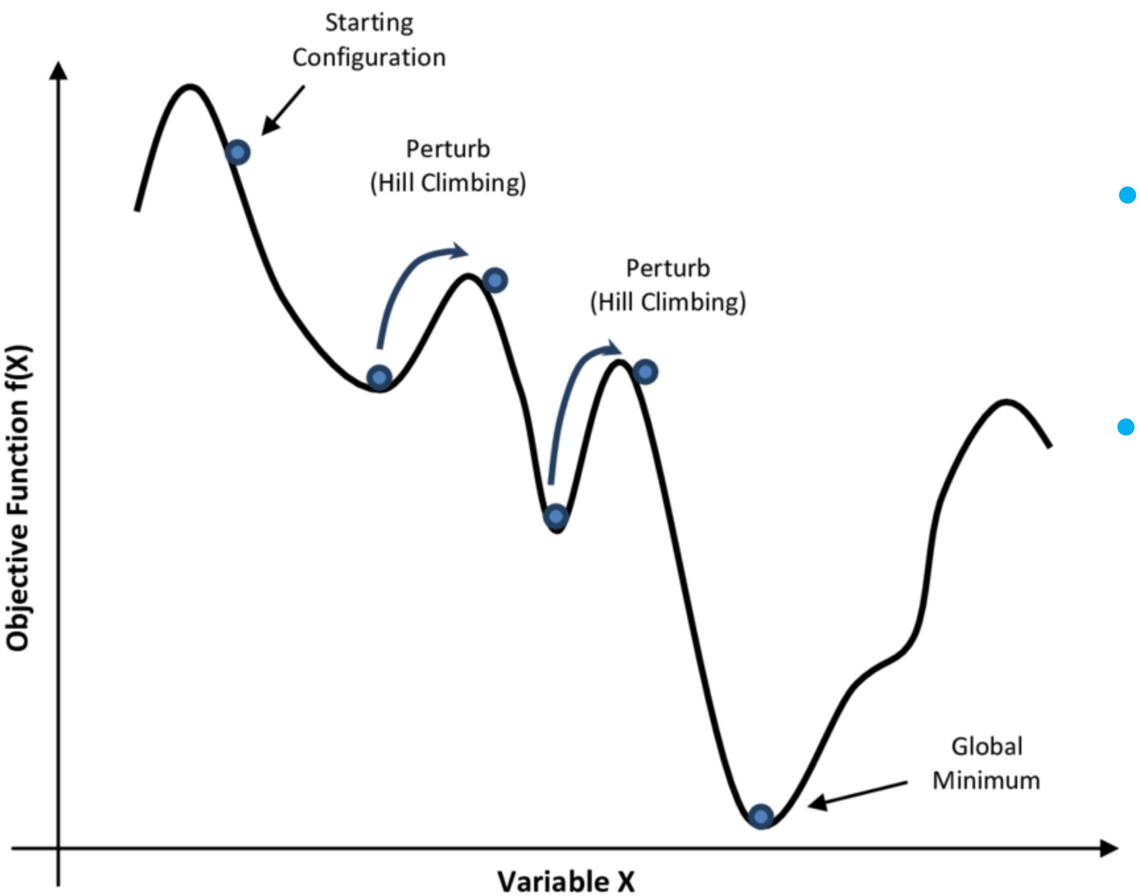

- Start by shaking hard (i.e., at high temperature).
- Gradually reduce the intensity of shaking (i.e., lower the temperature).

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Simulated Annealing Algorithm

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
    current ← *problem*.INITIAL
    for t = 1 **to** ∞ **do**
temperature → $T \leftarrow schedule(t)$
        if T = 0 **then return** *current* // solution
        next ← a randomly selected successor of *current*
badness → $\Delta E \leftarrow$ VALUE(*current*) – VALUE(*next*)
        if $\Delta E > 0$ **then** *current* ← *next*
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- Escape local minima by allowing bad moves.

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- Applied to VLSL layout problems, factory scheduling, aircraft trajectory planning, NP-hard optimization problems such as the traveling salesman problem, and large-scale stochastic optimization tasks.

Local Beam Search

Keep track of k states rather than one.

1. Start with k randomly generated states.
2. Generate all their successors.
3. Stop if any successor is a goal.
4. Otherwise, keep the k best successors and go back to step 2.

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Solution: stochastic beam search which chooses successors with probabilities proportional to their values.

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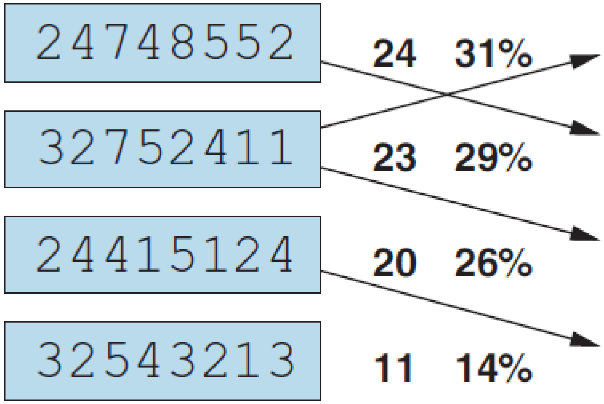
4. Go back to step 2 and repeat until *sufficiently fit* states are discovered (in which case the best one is chosen as a solution).

Genetic Algorithm on 8-Queen



(a) Initial Population

(b) Fitness Function

Genetic Algorithm on 8-Queen

Row number of the
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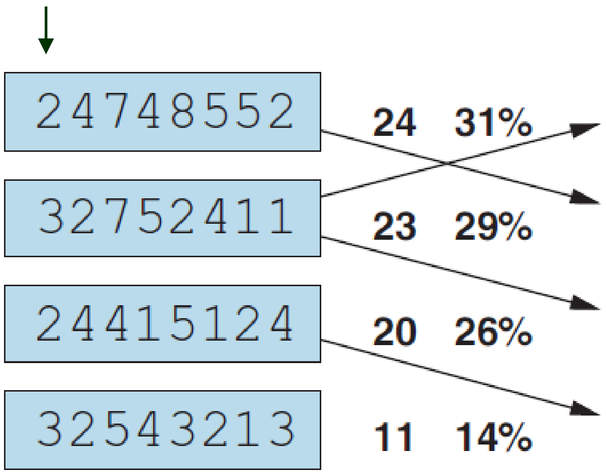

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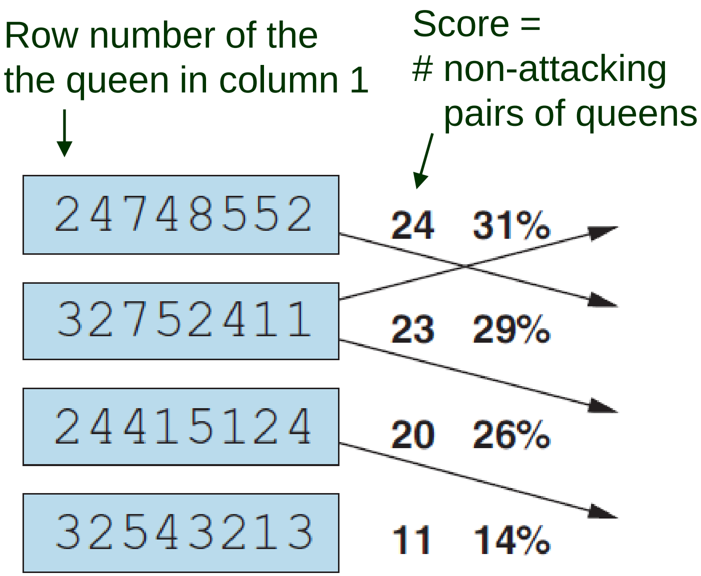

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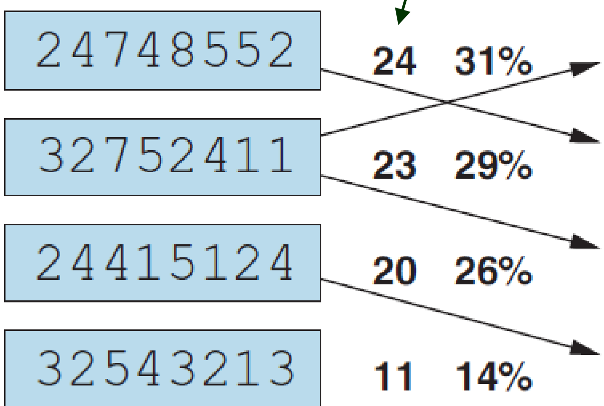
Score =
non-attacking
pairs of queens

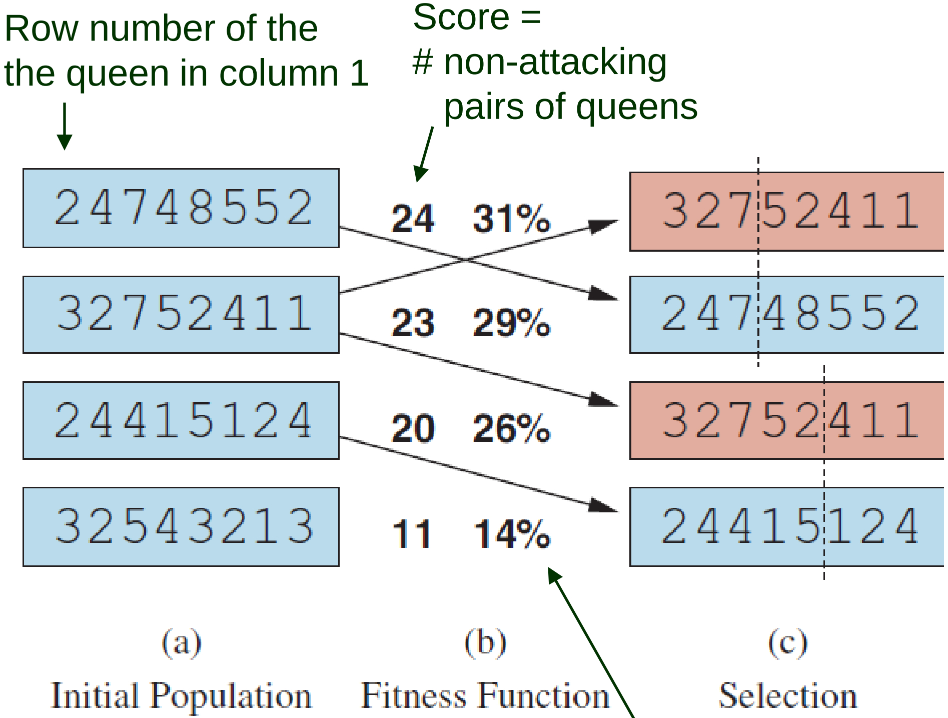


(a) Initial Population

(b) Fitness Function

$$14\% = \frac{11}{24 + 23 + 20 + 11}$$

Genetic Algorithm on 8-Queen



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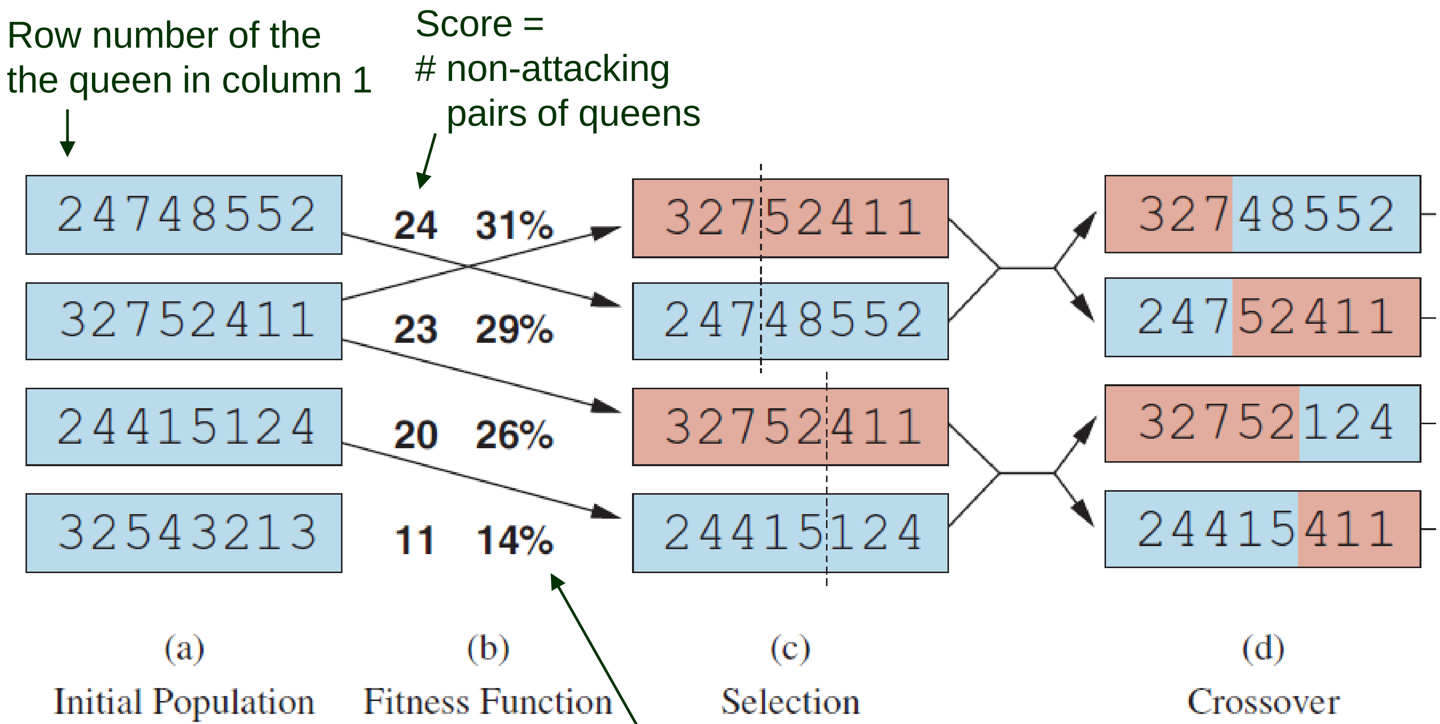
Genetic Algorithm on 8-Queen



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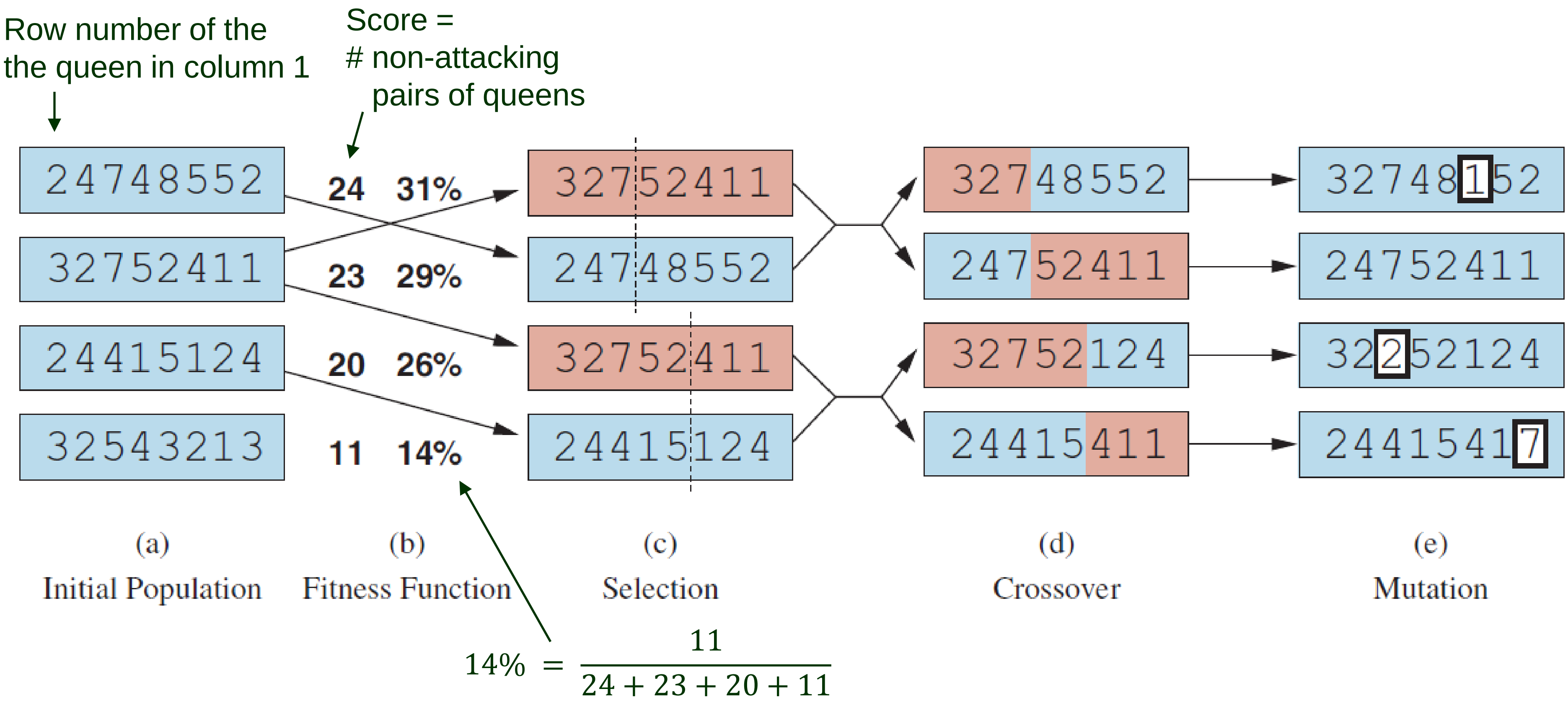
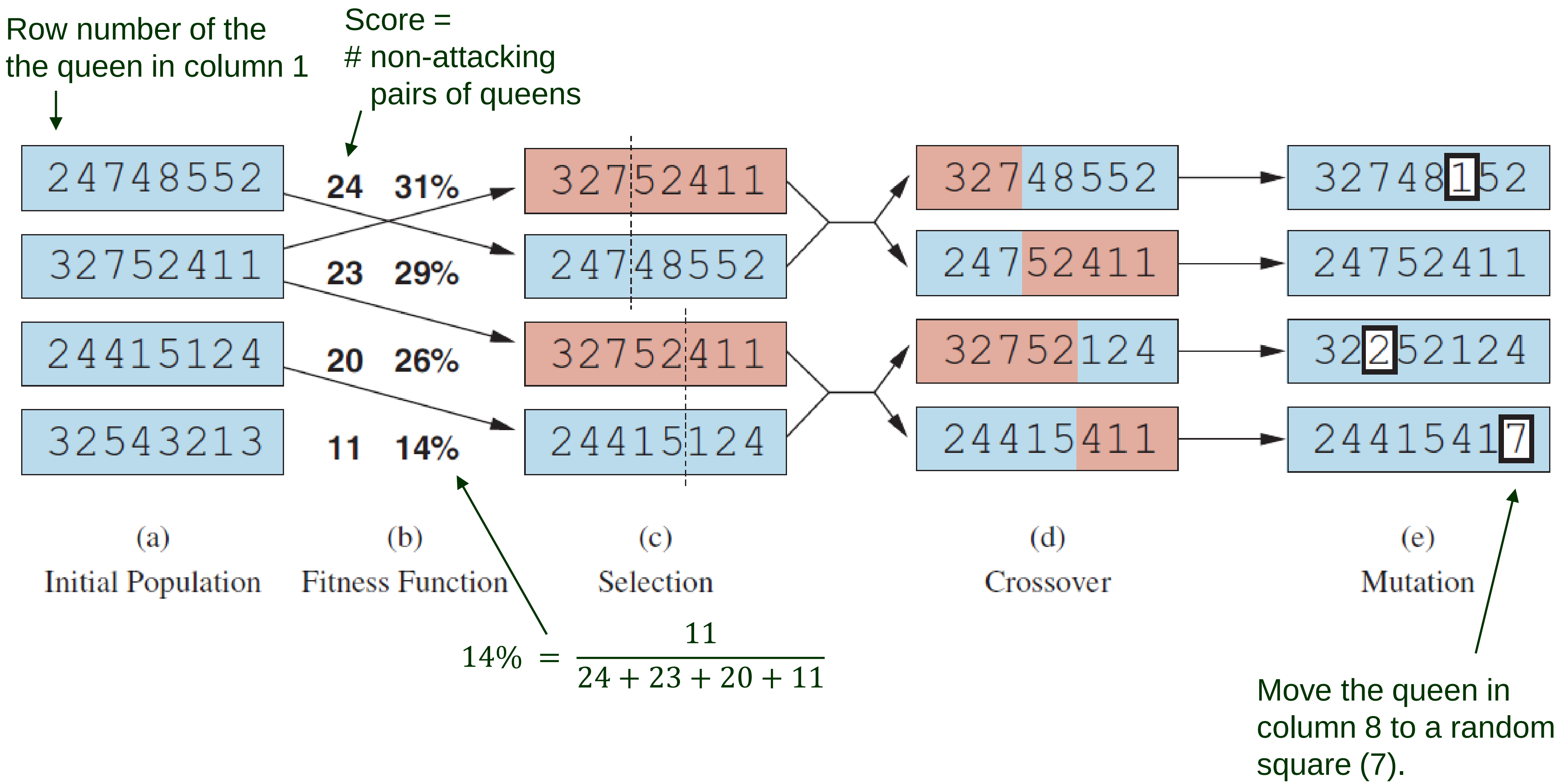
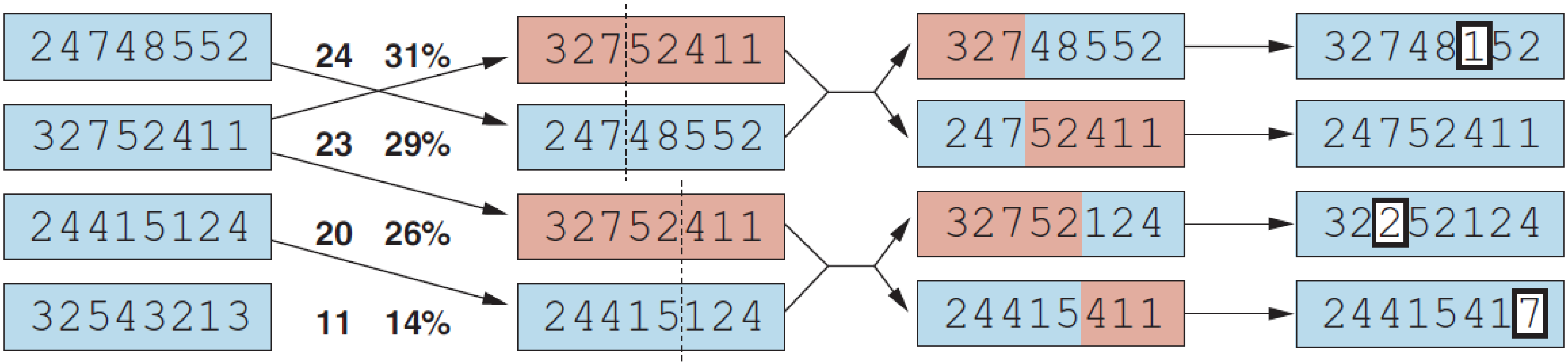
Genetic Algorithm on 8-Queen



Genetic Algorithm on 8-Queen

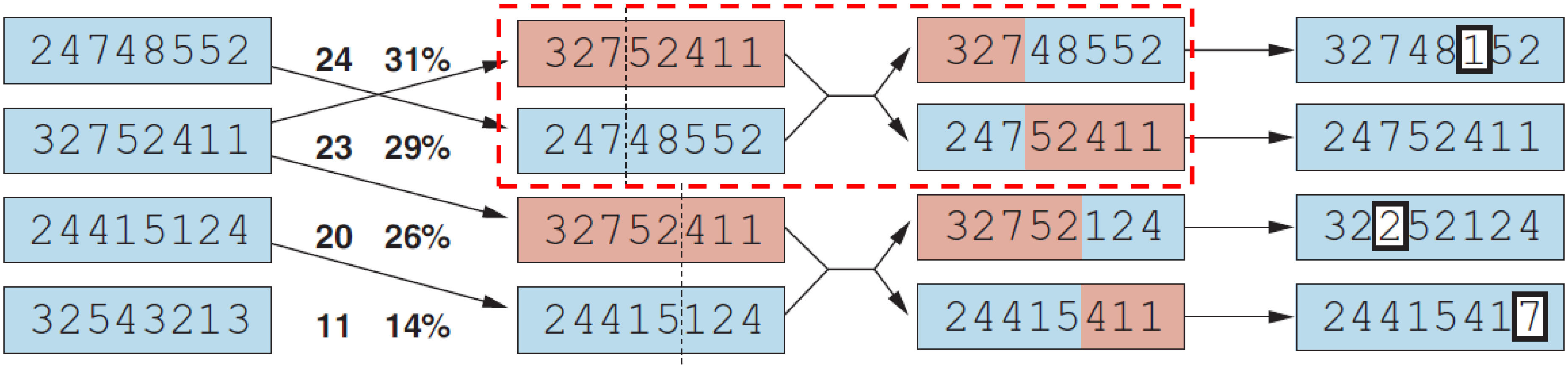


Crossover



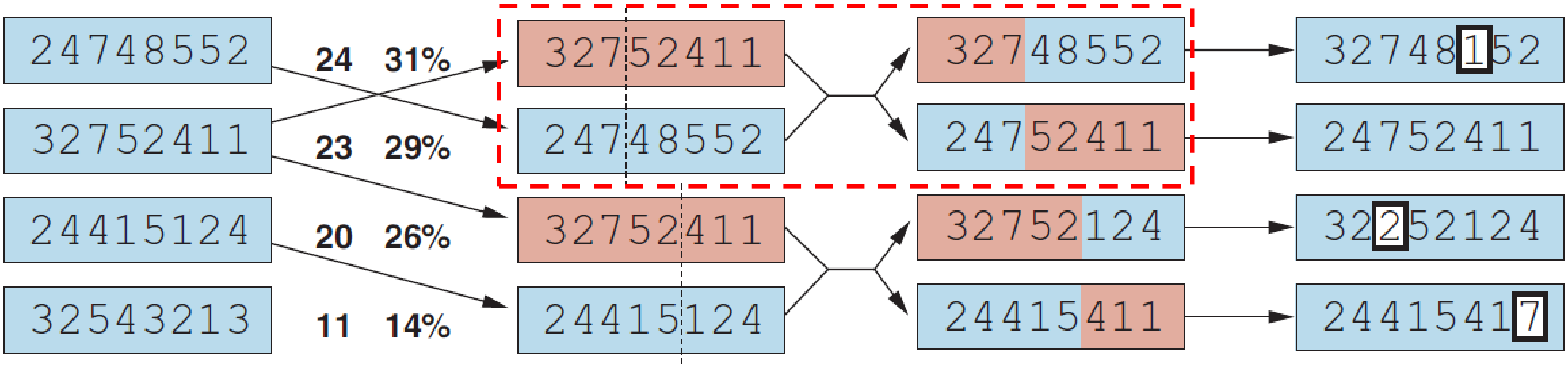
(d)
Crossover

Crossover



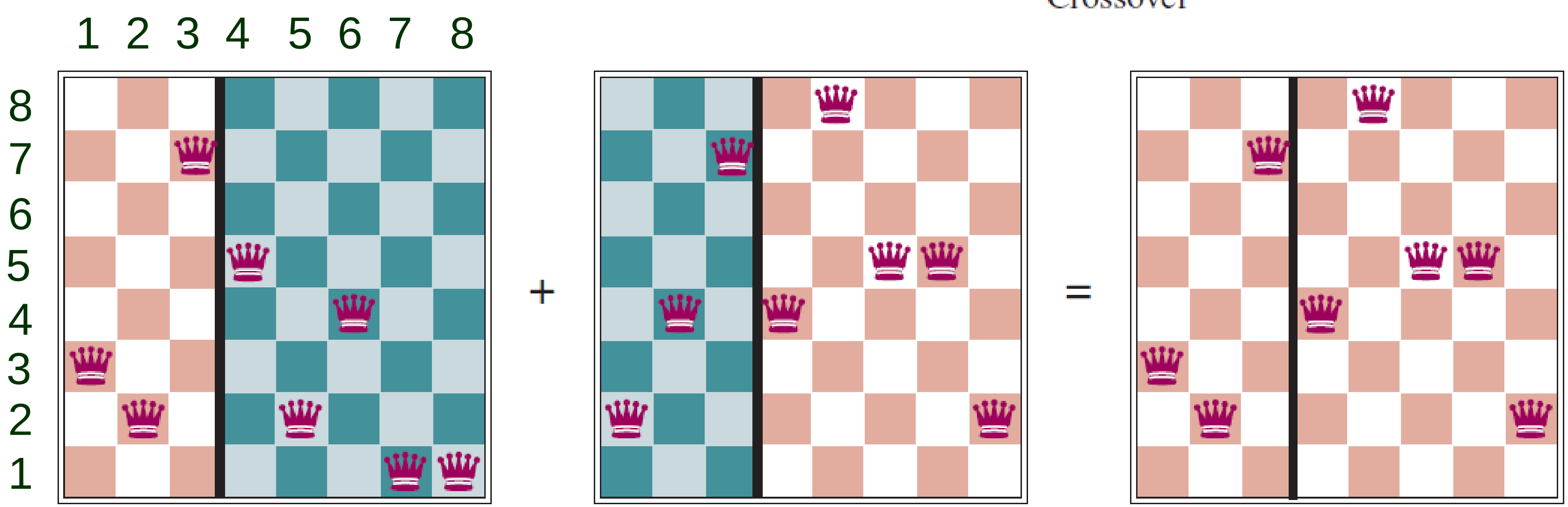
(d)
Crossover

Crossover



(d)

Crossover



Genetic Algorithm (Pseudocode)

```
function GENETIC-ALGORITHM(population, fitness) returns an individual
  repeat
     weights ← WEIGHTED-BY(population, fitness)
     population2 ← empty list
     for i = 1 to SIZE(population) do
        parent1, parent2 ← WEIGHTED-RANDOM-CHOICES(population, weights, 2)
        child ← REPRODUCE(parent1, parent2)
        if (small random probability) then child ← MUTATE(child)
        add child to population2
     population ← population2
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to fitness

function REPRODUCE(parent1, parent2) returns an individual
  n ← LENGTH(parent1)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(parent1, 1, c), SUBSTRING(parent2, c + 1, n))
```

Applications of GA

- Complex structured problems

  Circuit layout, job-shop scheduling

- Evolving the architecture of deep neural networks
- Finding bugs of hardware
- Molecular structure optimization
- Image processing.
- Learning robots, etc.

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