Propositional Logic

Outline

I. Syntax and semantics

II. Truth table enumeration

III. Theorem proving

* Figures are from the [textbook site] unless the source is specifically cited.

I. Syntax of Propositional Logic

An *atomic sentence* is a single proposition symbol.

standing for a proposition that has to be either true or false but not both.

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True : always-true proposition
False: always-false proposition

Complex Sentences

A *complex sentence* is constructed from simpler sentences, using parentheses and *logical connectives* (5 in total).

- $\neg$ (not).
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- $\wedge$ (and).
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- $\vee$ (or).
  - $(W_{1,3} \wedge P_{3,1}) \vee W_{2,2}$ is a *disjunction* whose parts $(W_{1,3} \wedge P_{3,1})$ and $W_{2,2}$ are *disjuncts*.

More Logical Connectives

- $\Rightarrow$ (implies).
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- $\Leftrightarrow$ (if and only if).
  - $W_{1,3} \Leftrightarrow W_{2,2}$ is a *biconditional*.

Grammar of Propositional Logic

John Backus (IBM)

National Medal of Science (1975)
ACM Turing Award (1977)

Peter Naur (U. Copenhagen)

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Backus-Naur form (BNF):

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & (\ \mathit{Sentence}\ ) \\
 & \mid & \neg\ \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \wedge \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \vee \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \Rightarrow \mathit{Sentence} \\
 & \mid & \mathit{Sentence} \Leftrightarrow \mathit{Sentence} \\
\\
\textsc{Operator Precedence} & : & \neg, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{array}
$$

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& \mid & \neg\; Sentence \\
& \mid & Sentence \wedge Sentence \\
& \mid & Sentence \vee Sentence \\
& \mid & Sentence \;\Rightarrow\; Sentence \\
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\end{array}
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$\neg A \vee B \wedge C \Rightarrow D$ is equivalent to

$$((\neg A) \vee (B \wedge C)) \Rightarrow D$$

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Semantics

A model fixes the truth value (*true* or *false*) for every proposition symbols.

$$m_1 = \{P_{1,2} = \textit{false}, P_{2,2} = \textit{false}, P_{3.1} = \textit{true}\}$$
$$m_2 = \{P_{1,2} = \textit{true}, P_{2,2} = \textit{false}, P_{3.1} = \textit{true}\}$$

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The truth value of any sentence can be computed once we know

- how to evaluate the truth of atomic sentences;
- how to compute the truth of sentences formed with each of the five connectives.

Determining the Truth Value

Atomic sentences:

- *true* is true in every model.
- *false* is false in every model.
- The truth value of every other proposition symbol must be specified in a model m.

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Complex sentences in the model m:

- $\neg P$ is true iff P is false.
- $P \land Q$ is true iff P and Q are true.
- $P \lor Q$ is true iff either P or Q is true.
- $P \Rightarrow Q$ is true unless P is true and Q is false.
- $P \Leftrightarrow Q$ is true iff P and Q are both true or both false.

Truth Table

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
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No pit in [1,2].

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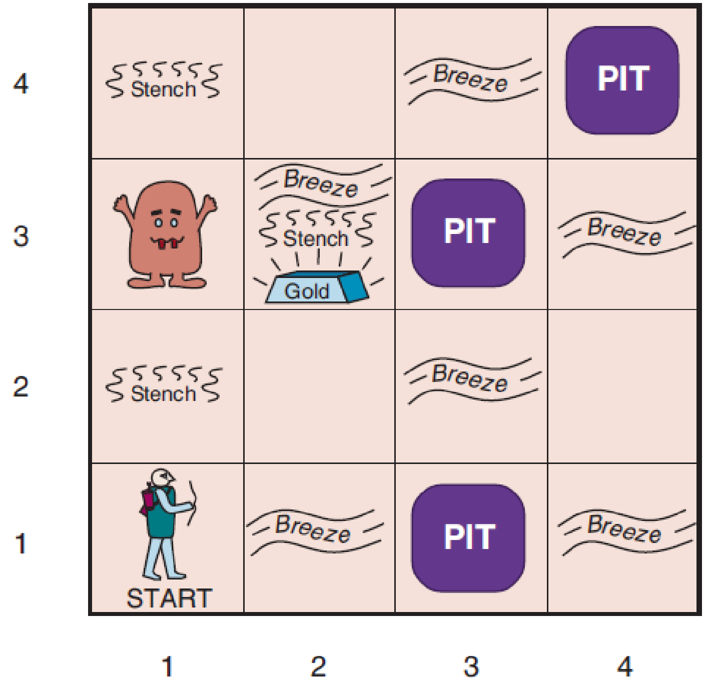
$$= \textit{true}$$

Knowledge Base for the Wumpus World

Proposition symbols

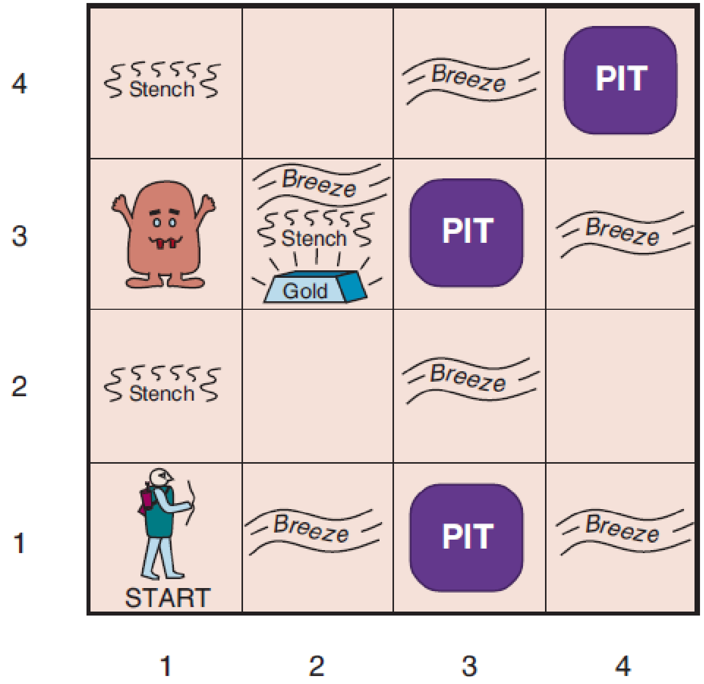
- $P_{x,y}$ is true if there is a pit in $[x, y]$.
- $W_{x,y}$ is true if there is a wumpus in $[x, y]$, dead or alive.
- $B_{x,y}$ is true if the agent perceives a breeze in $[x, y]$.
- $S_{x,y}$ is true if the agent perceives a stench in $[x, y]$.



Knowledge Base (cont'd)

- General knowledge (partial – only for relevant squares):
  - There is no pit in [1,1].

    R_1: $\neg P_{1,1}$



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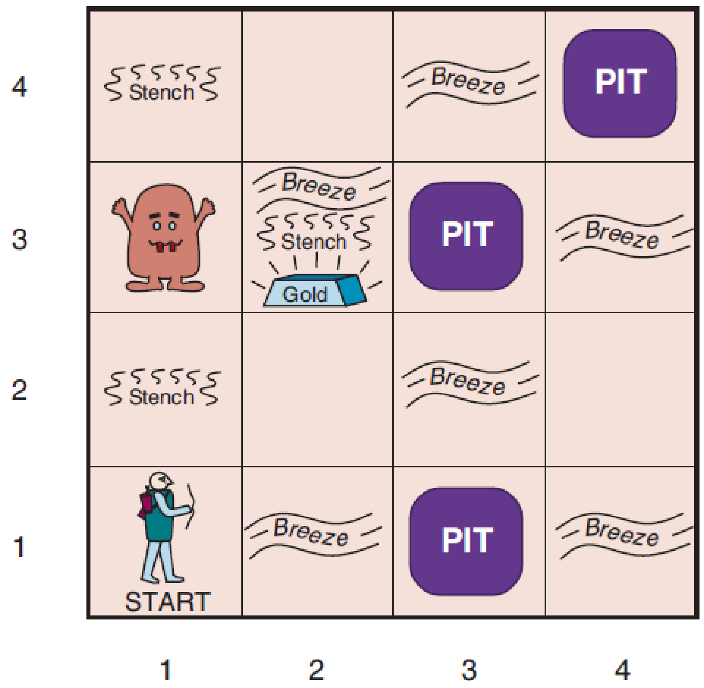
  - A square is breezy if and only if a neighboring square has a pit.

    $R_2\!: B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

    $R_3\!: B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$



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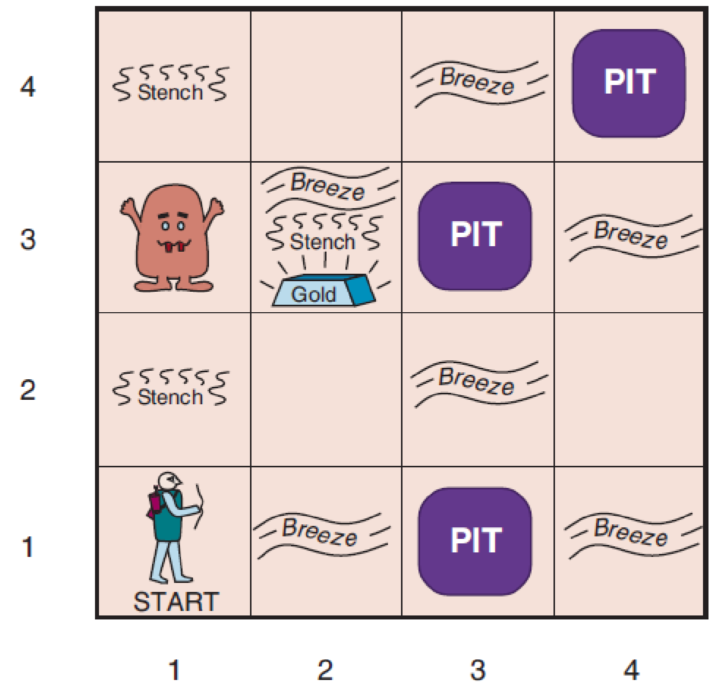
    R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

- Percepts for the first two squares:

    R_4: $\neg B_{1,1}$

    R_5: $B_{2,1}$



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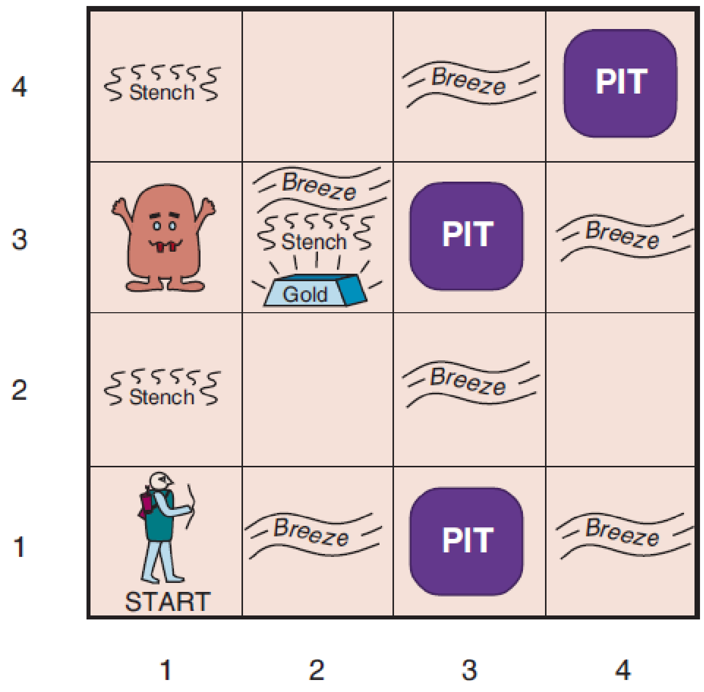

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$\Downarrow$

$2^7 = 128$ possible models!

Truth Table Enumeration

128 rows

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Truth Table Enumeration

KB is true in only 3 models.

128 rows

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

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$\neg P_{1,2}$ is true in all 3.

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$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

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$\Rightarrow$ $KB \vDash \neg P_{1,2}$

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false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

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$\Rightarrow$ $KB \models \neg P_{1,2}$

$P_{2,2}$ is true in only 2 of 3.

128 rows

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false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Truth Table Enumeration

KB is true in only 3 models.
$\neg P_{1,2}$ is true in all 3.
$\Rightarrow$ $KB \vDash \neg P_{1,2}$

$P_{2,2}$ is true in only 2 of 3. $\Rightarrow$ No inference of $KB \vDash P_{2,2}$

128 rows

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Bad News

Suppose KB and α have n symbols. $\Longrightarrow$ 2^n models!

The propositional entailment problem of showing $KB \vDash \alpha$ by truth table enumeration requires

$\Theta(2^n n)$ time

$O(n)$ space (not bad)

The problem is co NP-complete (likely not easier than NP-complete).

III. Logical Equivalence

Theorem proving: Apply rules of inference directly to the sentences in KB to construct a proof of a sentence without consulting models.

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$$(\alpha \equiv \beta)$$

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$\alpha \equiv \beta$ if and only if $\alpha \vDash \beta$ and $\beta \vDash \alpha$.

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$\alpha \equiv \beta$ if and only if $\alpha \vDash \beta$ and $\beta \vDash \alpha$.

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) && \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) && \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) && \text{associativity of } \lor \\
\neg(\neg \alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \beta \Rightarrow \neg \alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg \alpha \lor \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg \alpha \lor \neg \beta) && \text{De Morgan} \\
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(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

Validity

A sentence is *valid* if it is true in all models.

$$P \vee \neg P$$

Valid sentences are *tautologies*.

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$\alpha\alpha\Rightarrow\beta\beta$) is valid.
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if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.

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αα⇒ββ) is valid.
αα⇒ββ) is valid.
if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.
if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.
We can decide if $\alpha \vDash \beta$ by checking that $\alpha \Rightarrow \beta$ is a tautology.

Satisfiability

A sentence is *satisfiable* if it is true in, or satisfied by, some model.

R_1	R_2	R_3	R_4	R_5	KB
true	*true*	*true*	*true*	*false*	*false*
true	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*false*	*true*	*true*	*false*
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*true*	*false*	*true*	*false*

...

$(R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5)$ is satisfiable.

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true	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*false*	*true*	*true*	*false*
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*true*	*false*	*true*	*false*

...

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The SAT problem: Determining the satisfiability of sentences in propositional logic.

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true	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*false*	*true*	*true*	*false*
true	*true*	*true*	*true*	*true*	*true*
true	*true*	*true*	*true*	*true*	*true*
true	*true*	*true*	*true*	*true*	*true*
true	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*true*	*false*	*true*	*false*

...

$(R_1 \land R_2 \land R_3 \land R_4 \land R_5)$ is satisfiable.

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first NP-complete problem

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true	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*false*	*true*	*true*	*false*
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*true*	*false*	*true*	*false*

...

$(R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5)$ is satisfiable.

The SAT problem: Determining the satisfiability of sentences in propositional logic.

first NP-complete problem

$\alpha \vDash \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable.

Satisfiability

A sentence is *satisfiable* if it is true in, or satisfied by, some model.

R_1	R_2	R_3	R_4	R_5	KB
true	*true*	*true*	*true*	*false*	*false*
true	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*false*	*true*	*true*	*false*
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*true*	*true*	*true*	*true*	<u>*true*</u>
true	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*true*	*false*	*true*	*false*

...

$(R_1 \wedge R_2 \wedge R_3 \wedge R_4 \wedge R_5)$ is satisfiable.

The SAT problem: Determining the satisfiability of sentences in propositional logic.

first NP-complete problem

Proof by contradiction

$\alpha \vDash \beta$ if and only if $\boxed{\text{the sentence } (\alpha \wedge \neg\beta) \text{ is unsatisfiable.}}$

Inference Rules

Modus Ponens

$$\frac{\alpha \Rightarrow \beta, \qquad \alpha}{\beta}$$

$\alpha \qquad \Rightarrow \qquad \beta$

If **today is Tuesday**, then **John will go to campus**.

Today is Tuesday.

Therefore, **John will go to campus**.

Inference Rules

Modus Ponens

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If **today is Tuesday**, then **John will go to campus**.

Today is Tuesday.

Therefore, **John will go to campus**.

And-elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

$\alpha \qquad \wedge \qquad \beta$

A star is a sphere of gas, and **it is held together by its own gravity.**

A star is a sphere of gas.

Other Inference Rules

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) \quad \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) \quad \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) \quad \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) \quad \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
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(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
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\end{aligned}
$$

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$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
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(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

$$\neg(\alpha \vee \beta)$$

Other Inference Rules

$$
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(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

$$\frac{\neg(\alpha \vee \beta)}{\neg \alpha \wedge \neg \beta} \quad \text{De Morgan}$$

Other Inference Rules

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
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(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

$$\frac{\neg(\alpha \vee \beta)}{\neg\alpha \wedge \neg\beta} \quad \text{De Morgan}$$

$$\frac{\neg\alpha \wedge \neg\beta}{\neg\beta} \quad \text{and-elimination}$$

Applying Rules to the Wumpus World

KB:

$R_1: \neg P_{1,1}$

$R_2: B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

$R_3: B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

$R_4: \neg B_{1,1}$

$R_5: B_{2,1}$

Applying Rules to the Wumpus World

KB:

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof for $\neg P_{1,2}$

Applying Rules to the Wumpus World

KB:

$R_1: \neg P_{1,1}$

$R_2: B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$R_3: B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

$R_4: \neg B_{1,1}$

$R_5: B_{2,1}$

Proof for $\neg P_{1,2}$

$$R_2: B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$$

Applying Rules to the Wumpus World

KB:

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof for $\neg P_{1,2}$

$$\frac{R_2\text{: } B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}{R_6\text{: } \left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)}$$

biconditional elimination

Applying Rules to the Wumpus World

KB:

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof for $\neg P_{1,2}$

$$\frac{R_2:\ B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}{R_6:\ \left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)}$$

biconditional elimination

$$\frac{R_6:\ \left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)}{R_7:\ (P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}}$$

and-elimination

Applying Rules to the Wumpus World

KB:

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof for $\neg P_{1,2}$

$$\frac{R_2\colon\ B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}{R_6\colon\ \left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)}$$

biconditional elimination

$$\frac{}{R_7\colon\ (P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}}$$

and-elimination

$$\frac{}{R_8\colon\ \neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})}$$

logical equivalence

Applying Rules to the Wumpus World

KB:

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof for $\neg P_{1,2}$

$$\frac{R_2:\ B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}{R_6:\ \left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)}$$ biconditional elimination

and-elimination

$$R_7:\ (P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}$$

logical equivalence

R_4: $\neg B_{1,1}$

$$R_8:\ \neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})$$

Applying Rules to the Wumpus World

KB:

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof for $\neg P_{1,2}$

$$\frac{R_2:\ B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}{R_6:\ \left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)}$$ biconditional elimination

$$\frac{R_6}{R_7:\ (P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}}$$ and-elimination

$$\frac{R_7}{R_8:\ \neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})}$$ logical equivalence

$$\frac{R_4:\ \neg B_{1,1} \qquad R_8:\ \neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})}{R_9:\ \neg(P_{1,2} \vee P_{2,1})}$$ modus ponens

Applying Rules to the Wumpus World

KB:

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

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R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof for $\neg P_{1,2}$

$$\frac{R_2:\ B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}{R_6:\ \left(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})\right) \wedge \left((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}\right)}$$ biconditional elimination

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$$\frac{R_9:\ \neg(P_{1,2} \vee P_{2,1})}{R_{10}:\ \neg P_{1,2} \wedge \neg P_{2,1}}$$ De Morgan’s rule

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$$\frac{R_6}{R_7:\ (P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}} \quad \text{and-elimination}$$

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$$\frac{R_{10}}{R_{11}:\ \neg P_{1,2}} \quad \text{and-elimination}$$

Applying Rules to the Wumpus World

KB:

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

R_4: $\neg B_{1,1}$

R_5: $B_{2,1}$

Proof for $\neg P_{1,2}$

$$\frac{R_2:\ B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})}{R_6:\ \left(B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})\right) \land \left((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1}\right)}$$ biconditional elimination

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Searching for Proofs

- INITIAL STATE: the initial KB.
- ACTIONS: Apply any inference rule to any sentence that matches its top half.
- RESULT: Add the sentence in the bottom half of the inference rules.
- GOAL: A state containing the sentence to be proved.

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- ♠ The truth-table algorithm runs in time $\Theta(2^n n)$ – always the worst case.
- ♣ Search takes time $O(2^n n)$ – often not the worst case.

Monotonicity

The set of entailed sentences can only *increase* as new information is added to the KB.

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