ComS 472
Homework 5

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- 9.24 -

1) $L = \exists p \forall q\ S(p,q) \Leftrightarrow \neg S(q,q)$

2) $\exists p \forall q\ S(p,q) \Leftrightarrow \neg S(q,q)$
   $\exists p \forall q\ (S(p,q) \Rightarrow \neg S(q,q)) \wedge (\neg S(q,q) \Rightarrow S(p,q))$
   $\exists p \forall q\ (\neg S(p,q) \vee \neg S(q,q)) \wedge (S(q,q) \vee S(p,q))$
   $\forall q\ (\neg S(P,q) \vee \neg S(q,q)) \wedge (S(q,q) \vee S(P,q))$
   $(\neg S(P,q) \vee \neg S(q,q)) \wedge (S(q,q) \vee S(P,q))$

3) S(p,q) and $(\neg S(p,q) \vee \neg S(q,q)) \wedge (S(q,q) \vee S(p,q))$ becomes $(\neg S(q,q))$
   ¬ S(p,q) and $(\neg S(p,q) \vee \neg S(q,q)) \wedge (S(q,q) \vee S(p,q))$ becomes $(S(q,q))$

- 9.29 -

1) For every natural number x, there is a natural number y that is less than or equal to x.
   There is a natural number y that is less than or equal to every natural number x.

2) Yes
3) Yes
4) No
5) Yes

6) $\forall x \exists y (x \geq y) \Rightarrow \exists y \forall x (x \geq y)$
   $\neg(\forall x \exists y (x \geq y)) \vee (\exists y \forall x (x \geq y))$
   $\forall y \exists x \neg(x \geq y) \vee (\exists y \forall x (x \geq y))$
   $\forall y \neg(A \geq y) \vee (\forall x (x \geq B))$
   These map to {x/A, y/B}

7) $\exists y \forall x (x \geq y) \Rightarrow \forall x \exists y (x \geq y)$
   $\neg(\exists y \forall x (x \geq y)) \vee (\forall x \exists y (x \geq y))$
   $(\forall y \exists x \neg(x \geq y)) \vee (\forall x \exists y (x \geq y))$
   $(\forall y \neg(A(y) \geq y)) \vee (\forall x \exists y (x \geq B(x)))$
   There is no possible mapping, so this is disproven.

- 10.3 -

1) Fly(P1, JFK, SFO) and Fly(P2, SFO, JFK)

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1) $Init(At(Monkey, A) \wedge At(Bananas, B) \wedge At(Box, C) \wedge$
$Height(Monkey, Low) \wedge Height(Bananas, High) \wedge Height(Box, Low) \wedge$
$Monkey(Monkey) \wedge Bananas(Bananas) \wedge Box(Box) \wedge$
$Location(A) \wedge Location(B) \wedge Location(C))$

2) $Action(Go(m, from, to),$
Precond: $At(m, from) \wedge Monkey(m) \wedge Location(from) \wedge Location(to)$
Effect: $\neg At(m, from) \wedge At(m, to))$

$Action(Push(m, obj, from, to),$
Precond: $At(m, from) \wedge At(obj, from) \wedge Monkey(m) \wedge Box(obj) \wedge$
$Location(from) \wedge Location(to)$
Effect: $\neg At(m, from) \wedge \neg At(obj, from) \wedge At(m, to) \wedge At(obj, to))$

$Action(ClimbUp(m, b, loc),$
Precond: $At(m, loc) \wedge At(b, loc) \wedge Monkey(m) \wedge Box(b) \wedge Location(loc) \wedge Height(m, Low)$
Effect: $Height(m, High))$

$Action(ClimbDown(m, b, loc),$
Precond: $At(m, loc) \wedge At(b, loc) \wedge Monkey(m) \wedge Box(b) \wedge Location(loc) \wedge Height(m, High)$
Effect: $Height(m, Low))$

$Action(Grasp(m, b, loc),$
Precond: $At(m, loc) \wedge At(b, loc) \wedge Monkey(m) \wedge Bananas(b) \wedge Location(loc) \wedge$
$Height(m, height) \wedge Height(b, height)$
Effect: $\neg At(b, loc) \wedge Grasped(m, b))$

$Action(Ungrasp(m, b, loc, height),$
Precond: $At(m, loc) \wedge At(b, loc) \wedge Monkey(m) \wedge Bananas(b) \wedge Location(loc) \wedge Height(m, height)$
Effect: $At(b, loc) \wedge Height(b, height) \wedge \neg Grasped(m, b))$

3) $Goal(Grasped(Monkey, Bananas) \wedge At(Box, InitLocation))$
This can't be solved using a classical planning system, as they do not keep track of state. Without knowledge of the initial location state of the box, we cannot return it.

4) $Action(Push(m, obj, from, to),$
Precond: $At(m, from) \wedge At(obj, from) \wedge Monkey(m) \wedge Box(obj) \wedge$
$Location(from) \wedge Location(to) \wedge \neg Weight(obj, Heavy)$
Effect: $\neg At(m, from) \wedge \neg At(obj, from) \wedge At(m, to) \wedge At(obj, to))$

- 13.3 -

1) $P(a|b,c) = P(b|a,c)$ becomes $\frac{P(a,b,c)}{P(b,c)} = \frac{P(b,a,c)}{P(a,c)}$ becomes $\frac{P(a,b,c)P(a,c)}{P(a,b,c)P(b,c)} = \frac{P(b,a,c)P(b,c)}{P(a,b,c)P(a,c)}$ becomes $P(a,c) = P(b,c)$ becomes $\frac{P(a,c)}{P(c)} = \frac{P(b,c)}{P(c)}$ becomes $P(a|c) = P(b|c)$ ✓

2) $P(a|b,c) = P(a)$ states that a is independent of b and c, but says nothing on the relationship between b and c. A counterexample would be:
a=P(H) when flipping coin 1, b=P(H) when flipping coin 2, and b=c

3) $P(a|b) = P(a)$ states that a is independent of b, but says nothing on the relationship between a and c. A counterexample would be:
a=P(H) when flipping coin 1, b=P(H) when flipping coin 2, and a=c

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- 13.8 -

1) P(Toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

2) P(Cavity) = 0.108 + 0.012 + 0.072+ 0.008 = 0.2

3) P(Toothache | Cavity) = P(Toothache ∧ cavity)/P(Cavity) =
(0.108 + 0.012) / (0.108 + 0.012 + 0.072 + 0.008) = 0.6

4) P(Cavity | Toothache ∨ Catch) = $\frac{P(Cavity \land P(Toothache \lor Catch)}{P(Toothache \lor Catch)} =$
$\frac{0.108+0.012+0.072}{0.108+0.012+0.072+0.016+0.064+0.144} = \frac{0.192}{0.416} = 0.462$

- 13.16 -

A: $P(V|A) = \frac{P(A|V)P(V)}{P(A|V)P(V)+P(A|\neg V)P(\neg V)} = \frac{(.95)(.01)}{(.95)(.01)+(.05)(.99)} = 0.088$

B: $P(V|B) = \frac{P(B|V)P(V)}{P(B|V)P(V)+P(B|\neg V)P(\neg V)} = \frac{(.9)(.01)}{(.9)(.01)+(.05)(.99)} = 0.15$

Based on these results, test B is more effective at recognizing the virus.

- 13.18 -

1) The fact that the disease is rare is good news because as the number of people who actually have the disease is much lower than the , the higher the false positive on the test.

2) $\frac{P(T|D)P(D)}{P(T|D)P(D)+P(T|\neg D)P(\neg D)} = \frac{(.99)(.0001)}{(.99)(.0001)+(.01)(.9999)} = 0.0098$