

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. * The price of a particular tablet among dealers nationwide is assumed to have a Normal distribution with mean $\mu = \$600$ and variance $\sigma^2 = 36$.
 - (a) What is the probability that the tablet, chosen randomly from a dealer, will cost less than \$595?
 - (b) What is the probability that the tablet, chosen randomly from a dealer, will cost more than \$603?
 - (c) What is the probability that the tablet, chosen randomly from a dealer, will cost between \$595 and \$603?
 - (d) The manufacturer doesn't want dealers to markup these tablets above the 90th percentile of the price distribution. Approximately, what is the 90th percentile of the distribution of the price of these tablets?
 - (e) What is the probability that the **average** price of 30 tablets, chosen randomly from a dealer, will be greater than \$603?
2. Consider two brands of batteries for calculators, Xmax and Y-cell, manufactured **independently** by two companies. Assume that the lifetimes of each battery has a Normal distribution. The Xmax batteries has a mean of 1000 hours with a standard deviation of 100 hours, and Y-cell batteries has a mean of 1200 hours with a standard deviation of 200 hours. *For each of the following questions, you must state the random variable you are using and the distribution assumption you make.*
 - (a) What is the probability that an Xmax battery will last at most 800 hours? What is this probability for a Y-cell battery?
 - (b) Assume, two batteries are randomly selected, one Xmax and one Y-cell. What can you say about the difference D in their lifetimes? **It turns out that D is also Normally distributed.** Find the expected value and variance of the random variable D .
 - (c) You notice that, on average, Y-cell batteries will last longer than Xmax batteries. But what is the exact probability that the Y-cell batteries will last longer than Xmax batteries?
3. An iid sample of n observations is drawn from a population with mean equal to 50 and standard deviation equal to 5. Let \bar{X} be the sample mean.
 - (a) Given $n = 25$. Approximate $P(49 < \bar{X} < 51)$.
 - (b) Given $n = 100$. Approximate $P(49 < \bar{X} < 51)$.
 - (c) Based on your answer to (a) and (b), what happens to the probability that the sample mean is between 49 and 51 as your sample size n increases?
 - (d) What is the smallest sample size so that the probability that $|\bar{X} - 50| > 1$ is at most 0.05?
4. The average height of professional basketball players is around 6 feet 7 inches, and the standard deviation is 3.89 inches. Assuming Normal distribution of heights within this group,
 - (a) What percent of professional basketball players are taller than 7 feet?
 - (b) If your favorite player is within the tallest 20% of all players, what can his height be?
5. Suppose the installation time in hours for a software on a laptop has probability density function $f(x) = \frac{4}{3}(1 - x^3)$, $0 \leq x \leq 1$.
 - (a) Find the probability that the software takes between 0.3 and 0.5 hours to be installed on your laptop.
 - (b) Let X_1, \dots, X_{30} be the installation times of the software on 30 different laptops. Assume the installation times are independent. Find the probability that the *average* installation time is between 0.3 and 0.5 hours. Cite the theorem you use.

- (c) Instead of taking a sample of 30 laptops as in the previous question, you take a sample of 60 laptops. Find the probability that the *average* installation time is between 0.3 and 0.5 hours. Cite the theorem you use.
6. * Installation of some software package requires downloading 100 files. On average, it takes 15 seconds to download one file, with a variance of 25 sec^2 . What is the (approximate) probability that the software is installed in less than 23 minutes? (Use the Central Limit Theorem)