Midterm Clarification

October 26, 2019

Grammar

- a way to specify the patterns for strings
- Chomsky hierarchy: regular, context-free, context-sensitive, recursive enumerable (the format of their production rule is different)

- Do ambiguous context-free languages exist?
- **Yes!**

$$\{ \ a^{\mathbf{n}} b^{\mathbf{m}} c^{\mathbf{k}} \ | \ n = m \ \ or \ \ m = k \ \}$$

Example strings: $a^2 b^2 c =$ aabbc, abbcc

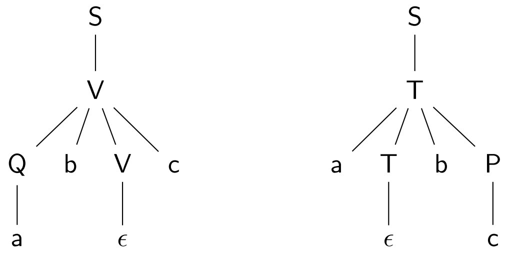
- The original proof is combinatorial, using classical techniques of language theory (pumping lemmas, ...)

- Is the problem difficult?
- **Yes!**
- Some languages seem to resist (discrete) combinatorial approaches
- The problem is **undecidable**: there is no algorithm to check whether a given context-free language is ambiguous.

- $\{ \; a^{\mathbf{n}} b^{\mathbf{m}} c^{\mathbf{k}} \mid n = m \;\; or \;\; m = k \; \}$

$$
\begin{array}{lcl}
\mathsf{S} & \rightarrow & \mathsf{T} \mid \mathsf{V} \\
\mathsf{T} & \rightarrow & \mathsf{aTbP} \mid \epsilon \\
\mathsf{V} & \rightarrow & \mathsf{QbVc} \mid \epsilon \\
\mathsf{P} & \rightarrow & \mathsf{Pc} \mid \epsilon \\
\mathsf{Q} & \rightarrow & \mathsf{Qa} \mid \epsilon
\end{array}
$$

- Two parse tree can be drawn for the string "abc"



- A grammar is **ambiguous** if there exists a word with at least two derivation trees in its generated language.
- A context-free language $\mathcal{L}$ is **ambiguous** (**inherently ambiguous**) if **every** grammar that generates $\mathcal{L}$ is ambiguous.

- $\{a^n \mid n \geq 1\}$ is generated by $S \to SS \mid a$, which is an **ambiguous grammar . . .**
- but $\{a^n \mid n \geq 1\}$ is also generated by the non-ambiguous $S \to Sa \mid a$, and is therefore a **non-ambiguous language**.

- **Main focus:** sufficient conditions that ensure the ambiguity of a context-free language.

Designing Grammars

3. To generate languages with matching, balanced, or related numbers of symbols, write productions which generate strings from the middle

   $\{a^n b^n \mid n \geq 0\}$ // N a's followed by N b's

   $S \rightarrow aSb \mid \varepsilon$

   Example derivation: $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$

   $\{a^n b^{2n} \mid n \geq 0\}$ // N a's followed by 2N b's

   $S \rightarrow aSbb \mid \varepsilon$

   Example derivation: $S \Rightarrow aSbb \Rightarrow aaSbbbb \Rightarrow aabbbb$

Designing Grammars

4. For a language that is the union of other languages, use separate nonterminals for each part of the union and then combine

$\{ a^n(b^m|c^m) \mid m > n \geq 0\}$

Can be rewritten as

$\{ a^n b^m \mid m > n \geq 0\} \cup \{ a^n c^m \mid m > n \geq 0\}$

$S \rightarrow T \mid V$

$T \rightarrow aTb \mid U$

$U \rightarrow Ub \mid b$

$V \rightarrow aVc \mid W$

$W \rightarrow Wc \mid c$