Spring 2020 **Statistics 330 - Section 1** **Exam I**

(- points)

Please write your first and last name here:

Name ______________________________

Instructions:

- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- The questions are roughly in the order in which the material is presented in class, so they are not necessarily ordered easiest to hardest.
- If you get stuck on one, it may be a good idea to move on and come back to that question at the end.
- You may use your prepared notes (1 page, both sides) and a calculator only.

1. Suppose you are playing a game involving a color wheel with a spinner. The color wheel has 4 equally spaced colors (Yellow, Blue, Red, Green). Spin the wheel 2 times, and record the color you get each time. (For the computation of probabilities, assume each outcome in your sample space is equally likely.)

   Let Y = yellow, B=blue, R = red, G = green

   (a) Give the sample space for this game. (5 points)

   Answer:

   $$\begin{aligned}\Omega = \{&YY, YB, YR, YG,\\ &BY, BB, BR, BG,\\ &RY, RB, RR, RG,\\ &GY, GB, GR, GG\}\end{aligned}$$

   (b) You earn 5 points if you obtain the same color for both spins. List the outcomes for this event. (5 points)

   Answer:

   $$A = \{YY, BB, RR, GG\}$$

   (c) Calculate the probability of obtaining the same color for both spins. (5 points)

   Answer:

   $$\mathbb{P}(A) = \frac{|A|}{|\Omega|} = \frac{4}{16} = \frac{1}{4}$$

   (d) You lose 3 points if you land a red color in any of your spins. Calculate the probability of this event. (5 points)

   Answer:

   $$B = \{YR, BR, RY, RB, RR, RG, GR\}$$
   $$\mathbb{P}(B) = \frac{|B|}{|\Omega|} = \frac{7}{16}$$

2. Two events A and B are such that $\mathbb{P}(A) = .45$, $\mathbb{P}(B) = .25$, and $\mathbb{P}(A \cup B) = .6$. Find the probability that

   (a) Both A and B occur. (*Then you can make a picture*) (3 points)
   Answer: $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

   $\Rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cup B) = .25 + .45 - .6 = .10$

   (b) Either A occurs or B occurs, but not both. (3 points)
   Answer: This is $\mathbb{P}(A \cap \overline{B}) + \mathbb{P}(\overline{A} \cap B)$

   Which is $[\mathbb{P}(A) - \mathbb{P}(A \cap B)] + [\mathbb{P}(B) - \mathbb{P}(A \cap B)] = (.45 - .1) + (.25 - .1) = .50$

   Or if you have a table made, you can pick out $\mathbb{P}(A \cap \overline{B})$ and $\mathbb{P}(\overline{A} \cap B)$

   (c) A occurs given B occurred. (3 points)
   Answer: $\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} = \frac{.10}{.25} = .40$

   (d) Are events A and B independent? Justify your answer. (3 points)
   Answer: $\mathbb{P}(A \cap B) = \mathbb{P}(A) \times \mathbb{P}(B)$?

   $.10 \neq (.45)(.25) = .1125 \Rightarrow A$ and B are not independent.

3. Harry Potter's closet contains 12 brooms. 7 brooms are *Comet 260*s, 4 brooms are *Nimbus 2000*s, and 1 broom is a *Firebolt*. Harry, Ron, George and Fred want to sneak out in the middle of the night for a game of Quidditch. They are afraid to turn on the light in case they get caught. Harry reaches into the closet and pulls 4 brooms out at once without looking.

   (a) What is the probability that all 4 chosen brooms are *Comet 260*s? (5 points)

   Answer:

$$\frac{\binom{7}{4} \cdot \binom{4}{0} \cdot \binom{1}{0}}{\binom{12}{4}} = \frac{35}{495}$$

   (b) What is the probability that Harry pulled out 1 *Comet 260*, 2 *Nimbus 2000*s, and 1 *Firebolt* broom? (5 points)

   Answer:

$$\frac{\binom{7}{1} \cdot \binom{4}{2} \cdot \binom{1}{1}}{\binom{12}{4}} = \frac{42}{495}$$

   (c) What is the probability that at least 1 of the 4 chosen brooms is a *Comet 260*? (5 points)

   Answer:

$$\begin{aligned}
P(\text{at least one C}) &= 1 - P(\text{no } C) \\
&= 1 - \Big[P(\text{0C, 4N, 0F}) + P(\text{0C, 3N, 1F})\Big] \\
&= 1 - \left[\frac{\binom{7}{0} \cdot \binom{4}{4} \cdot \binom{1}{0}}{\binom{12}{4}} + \frac{\binom{7}{0} \cdot \binom{4}{3} \cdot \binom{1}{1}}{\binom{12}{4}}\right] \\
&= 1 - \left[\frac{1}{495} + \frac{4}{495}\right] \\
&= 1 - \frac{5}{495} \\
&= 0.9899
\end{aligned}$$

4. A communications channel transmits the digits 0 and 1. However, due to static, the digit that is transmitted is incorrectly received with a certain probability. To reduce the chance of error, the digit that is sent is replicated three times and that string is sent across the channel. Each digit can then flip in transmission. The receiver will decode the message being sent after receiving the three digit string using a "majority wins" decision. The process looks like this: (*just for illustration*)

$$0 \to 000 \to 010 \to 0$$

Suppose that a 0 is sent 60% of the time in the channel. If a 0 is sent, a 0 is received with probability .91. On the other hand, if a 1 is sent, a 0 is received with probability .27. Let A be the event a 0 is sent and B be the event a 0 is received.

(a) Below is a tree diagram of the two step process (send a digit, receive a digit). Write down the six probabilities that would go on the tree. (6 points)

$\mathbb{P}(A) =$

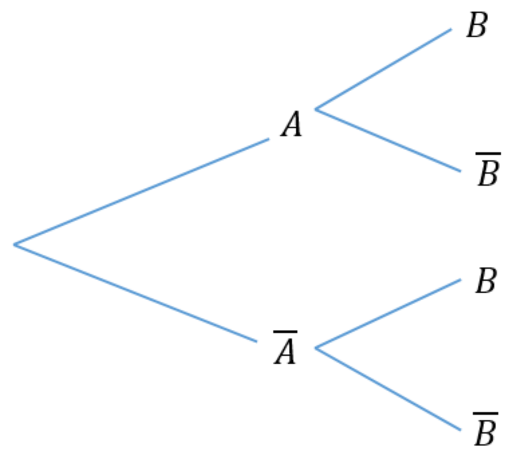
$\mathbb{P}(\overline{A}) =$

$\mathbb{P}(B|A) =$

$\mathbb{P}(\overline{B}|A) =$

$\mathbb{P}(B|\overline{A}) =$

$\mathbb{P}(\overline{B}|\overline{A}) =$



Answer: From the problem we have: $\mathbb{P}(A) = .6, \mathbb{P}(\overline{A}) = .4, \mathbb{P}(B|A) = .91, \mathbb{P}(\overline{B}|A) = .09, \mathbb{P}(B|\overline{A}) = .27, \mathbb{P}(\overline{B}|\overline{A}) = .73$

(b) In a random transmission, what is the probability that a 0 is received? (6 points)

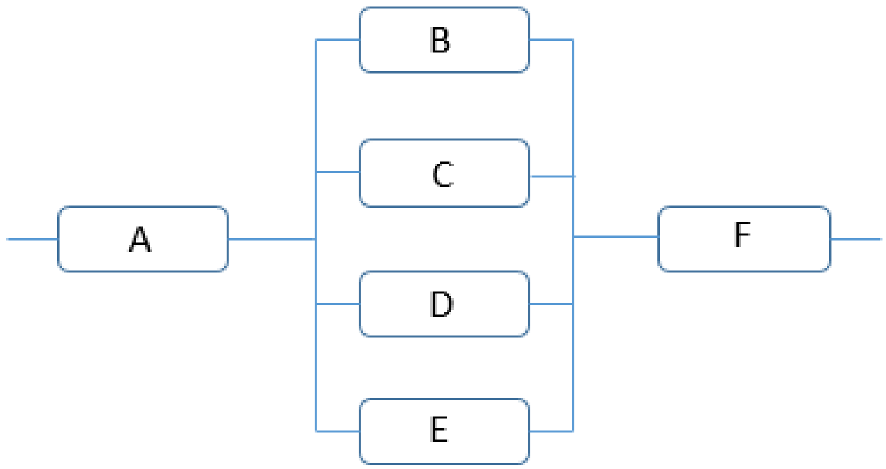
Answer:

$$\begin{aligned}\mathbb{P}(B) &= \mathbb{P}(A \cap B) + \mathbb{P}(\overline{A} \cap B)\\ &= \mathbb{P}(A)\mathbb{P}(B|A) + \mathbb{P}(\overline{A})\mathbb{P}(B|\overline{A})\\ &= (.6)(.91) + (.4)(.27)\\ &= .654\end{aligned}$$

(c) A digit is sent and 0 is received. What is the probability that a 0 was originally sent? (6 points)

Answer: Using Baye's rule we have: $\mathbb{P}(A|B) = \frac{\mathbb{P}(A)\mathbb{P}(B|A)}{\mathbb{P}(B)} = \frac{(.6)(.91)}{.654} = .835$

5. In the following system, the probability of the individual components A, B, C, D, E, and F working are 0.8, 0.9, 0.9, 0.9, 0.9, and 0.8 respectively. Compute the system's reliability. (10 points)



Answer: Combine B, C, D, and E, which has reliability $1 - [1 - (0.9)]^4 = 0.99$. Combine B to E. Now we have a series system as show below.

Thus the reliability is now:

$P(\text{A works and G works and F works}) = (0.8)(0.99)(0.8) = 0.634$

Scratch Paper