

## Recitation 11

- Here is a set of additional problems. They range from being very easy to very tough. The best way to learn the material in 310 is to solve problems on your own.
  - Feel free to ask (and answer) questions about this problem set on Piazza.
  - This is an **optional** problem set; do not turn this in for grading.
  - While you don't have to turn this in, be warned that this material **can** appear in a quiz or exam.
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1. Prove by mathematical induction the following properties:
  - a. The sum of the first  $n$  entries of the geometric progression  $1, r, r^2, \dots, r^{n-1}$  (for  $r < 1$ ) is given by  $\frac{1-r^n}{1-r}$ . What is the answer if  $r > 1$  What is the answer if  $r = 1$ ?
  - b. The sum of the first  $n$  entries of the arithmetic progression  $d, 2d, 3d, \dots, nd$  (for  $d > 0$ ) is given by  $dn(n+1)/2$ .
2. Prove that every amount of postage that is at least  $12c$  can be made from some combination of  $4c$  and  $5c$  stamps. (Hint: (i) strong induction. (ii) you need to check multiple base cases.)
3. The game of *Nim* is a two-player game involving a box of matchsticks. Two piles of  $n$  matchsticks each are placed on a table. Players take turns, and in each turn a player removes some (non-zero) number of matchsticks from one of the two piles. The player who removes the last matchstick wins.
  - a. Find another student in your recitation class, and play the game using  $n = 4$  and  $n = 5$ .
  - b. The player who has the second move *always wins*. Figure out the winning strategy.
  - c. Prove that the winning strategy always works using strong induction.