ComS 472
Homework 6

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- 13.26 -

1) As we don't know the ratio of taxi colors in Athens, we can't determine how likely the taxi is to be a certain color.

2) Because the color discrimination reliability is given to us...

P(lookBlue | isBlue) = 0.75
P(lookBlue | isGreen) = 0.25
P(lookGreen | isBlue) = 0.25
P(lookGreen | isGreen) = 0.75

Given that 9/10 taxis are Green, P(isBlue) = 0.1 and P(isGreen) = 0.9. Thus

P(isBlue | looksBlue) = 0.75 * 0.1 = 0.075
P(isGreen | looksBlue) = 0.25 * 0.9 = 0.225

$$\text{P(isBlue|lookBlue)}=\frac{P(isBlue|looksBlue)}{P(isBlue|looksBlue)+P(isGreen|looksBlue)}=\frac{0.075}{0.075+0.225}=0.25$$

$$\text{P(isGreen|lookBlue)}=\frac{P(isGreen|looksBlue)}{P(isBlue|looksBlue)+P(isGreen|looksBlue)}=\frac{0.225}{0.075+0.225}=0.75$$

As P(isGreen|lookBlue) > P(isBlue|lookBlue), it is most likely to be Green.

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1) Numerically, because P(B, E) = P(B)P(E), they are independant.

Topologically, because the chance of B and the chance of E are unaffected by anything (no arrows pointing toward them), they are independend from the entire structure, and thus from each other.

2) Independent if P(B, E | A) = P(B | A) * P(E | A)

$$P(A) = \begin{cases} 0.95 * 0.001 * 0.002 = 0.0000019 & B = t \ and \ E = t \\ 0.94 * 0.001 * 0.998 = 0.0009381 & B = t \ and \ E = f \\ 0.29 * 0.999 * 0.002 = 0.0005794 & B = f \ and \ E = t \\ 0.001 * 0.999 * 0.998 = 0.0009970 & B = f \ and \ E = f \end{cases} \Rightarrow$$

P(A) = 0.0000019 + 0.0009381 + 0.0005794 + 0.0009970 = 0.002516

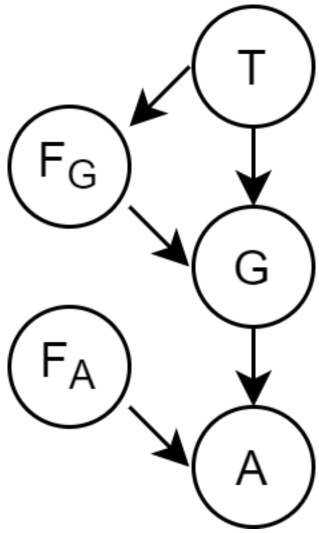
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.0000019+0.0009381}{0.002516} = 0.3736$$

$$P(E|A) = \frac{P(A|E)P(E)}{P(A)} = \frac{0.0000019+0.0005794}{0.002516} = 0.2310$$

P(B|A) * P(E|A) = 0.3736 * 0.2310 = 0.0863

$$P(B,E|A) = \frac{P(A|B,E)P(B,E)}{P(A)} = \frac{0.95*0.001*0.002}{0.0.002516} = 0.0007550$$

As P(B,E|A) $\neq$ P(B|A) * P(E|A), B and E are not independent.



2) The network is not a polytree because there is a set of 3 nodes with connections between them, disallowing the label 'tree'.

3)

	F_G True		**F_G False**	
	T High	**T Normal**	**T High**	**T Normal**
G Normal	(1-y)	y	(1-x)	x
G High	y	(1-y)	x	(1-x)

4)

	G Normal		**G High**	
	F_A True	**F_A False**	**F_A True**	**F_A False**
A On	0	0	0	1
A Off	1	1	1	0

5) As alarm is only influenced by G, I will consider only G.

$$P(T_{High}|G, \neg F_G) = \frac{P(G|T_{High},\neg F_G)P(\neg F_G|T)P(T)}{P(G,\neg F_G)}$$

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1) $P(B|j,m) = \alpha * P(B) * \sum_e P(e) * \sum_a P(a|b,e)P(j|a)P(m|a)$

$$\alpha * P(B) * \sum_e P(e) * \left(0.9 * 0.7 * \begin{pmatrix} 0.95 & 0.94 \\ 0.29 & 0.001 \end{pmatrix} + 0.05 * 0.01 * \begin{pmatrix} 0.05 & 0.06 \\ 0.71 & 0.999 \end{pmatrix}\right)$$

$$\alpha * P(B) * \sum_e P(e) * \left(0.63 * \begin{pmatrix} 0.95 & 0.94 \\ 0.29 & 0.001 \end{pmatrix} + 0.00005 * \begin{pmatrix} 0.05 & 0.06 \\ 0.71 & 0.999 \end{pmatrix}\right)$$

$$\alpha * P(B) * \sum_e P(e) * \left(\begin{pmatrix} 0.5985 & 0.5922 \\ 0.1827 & 0.00063 \end{pmatrix} + \begin{pmatrix} 0.000025 & 0.00003 \\ 0.000355 & 0.0005 \end{pmatrix}\right)$$

$$\alpha * P(B) * \sum_e P(e) * \left(\begin{pmatrix} 0.598525 & 0.59223 \\ 0.183055 & 0.00113 \end{pmatrix}\right)$$

$$\alpha * P(B) * \left(0.002 * \begin{pmatrix} 0.598525 \\ 0.183055 \end{pmatrix} + 0.998 * \begin{pmatrix} 0.59223 \\ 0.00113 \end{pmatrix}\right)$$

$$\alpha * P(B) * \left(\begin{pmatrix} 0.001197 \\ 0.000366 \end{pmatrix} + \begin{pmatrix} 0.591046 \\ 0.001128 \end{pmatrix}\right)$$

$$\alpha * P(B) * \left(\begin{pmatrix} 0.592243 \\ 0.001493 \end{pmatrix}\right)$$

$$\alpha * (0.001 * 0.592243)(0.999 * 0.001493)$$

$$\alpha * (0.000592)(0.001492)$$

$$\langle 0.28428, 0.71616 \rangle$$

2) # Additions = 6
# Multiplications = 16
# Divisions = 2
Total = 24

The algorithm will perform 27 operations.

3) Enumeration requires parsing through 2 complete binary trees for each variable, each one with a depth of n-2. This leaves computation at O(2^n).
Elimination only considers 2 variables at a time, working through each variable one at a time. Thus, the algorithm runs for all n variables at O(n).