ComS 342
Recitation 2, 10:00 Tuesday
Homework 6

Sean Gordon

October 29, 2019

1a)
(((λ(x)(x x)) (λ(y)(y x))) z)
(((x x) [x → λ(y)(y x)]) z)

(((λ(y)(y x) λ(y)(y x))) z)
(((y x) [y → λ(y)(y x)]) z)

(((λ(y)(y x) x)) z)
(((y x) [y → x]) z)

((x x) z)

1b)
(((λ(a)(λ(b)(a b))) ((λ(c) c) x)) y)
(((λ(a)(λ(b)(a b))) ((c) [c → x])) y)

(((λ(a)(λ(b)(a b))) x) y)
(((λ(b)(a b)) [a → x]) y)

((λ(b)(x b)) y)
((x b) [b → y])
(x y)

1c)
(((λ(x)(x x)) (λ(y) y)) (λ(y) y))
(((x x) [x → λ(y) y]) (λ(y) y))

(((λ(y) y) (λ(y) y)) (λ(y) y))
((y [y → (λ(y) y)]) (λ(y) y))

((λ(y) y) (λ(y) y))
(y [y → (λ(y) y)])

(λ(y) y)

2a)
((λ(x) p) ((λ(y)(y y)) (λ(z)(z z))))
((p) [x → ((λ(y)(y y)) (λ(z)(z z)))])
(p)

2b)
((λ(x) p) ((λ(y)(y y)) (λ(z)(z z))))
((λ(x) p) ((y y) [y → (λ(z)(z z))]))

((λ(x) p) (((λ(z)(z z)) (λ(z)(z z)))))
((λ(x) p) (((z z) [z → (λ(z)(z z))])))
...
((λ(x) p) (((λ(z)(z z)) (λ(z)(z z)))))
((λ(x) p) (((z z) [z → (λ(z)(z z))])))
...

((λ(x) p) ((λ(z)(z z)) (λ(z)(z z))))
((p) [x → ((λ(z)(z z)) (λ(z)(z z)))])
(p)

---

3) and = (λ(a)(λ(b) ite a b false))

4a)
((λ(z)((two f) z)) (succ zero))
((λ(z)((two f) z)) 1)
(((two f) z) [z → 1])
((two f) 1)
(f(f(1)))
This function applies f to the successor of 0 (1) two times.

4b) zero = λ(f)(λ(x)x), one = λ(f)(λ(x) f(x)), two = λ(f)(λ(x) f(f(x)))
i. (g zero)
((λ(n)((n unknown) false)) zero)
((zero unknown) false)
(((λ(f)(λ(x)x)) unknown) false)
(false)

ii. (g one)
((λ(n)((n unknown) false)) one)
((one unknown) false)
(((λ(f)(λ(x) f(x))) unknown) false)
(unknown (false))
((λ(x)(λ(y)(λ(z) y))) (false))
(λ(y)(λ(z) y))
(true)

iii. (g two)
((λ(n)((n unknown) false)) two)
((two unknown) false)
(((λ(f)(λ(x) f(f(x)))) unknown) false)
(unknown (unknown (false)))
(unknown ((λ(x)(λ(y)(λ(z) y))) (false)))
(unknown (λ(y)(λ(z) y)))
(unknown (true))
((λ(x)(λ(y)(λ(z) y))) (true))
((λ(y)(λ(z) y)))
(true)

iv. (n != 0)

---

5) (a) (g one)
((λ(a)(λ(b)(λ(c)((a b) ((a b) c))))) one)
((λ(b)(λ(c)((one b) ((one b) c)))))
((λ(b)(λ(c)((one b) (b(c))))))
(λ(b) (λ(c) (b(b(c)))))
(two)

(b) (g two)
((λ(a)(λ(b)(λ(c)((a b) ((a b) c))))) two)
((λ(b)(λ(c)((two b) ((two b) c))))))
((λ(b)(λ(c)((two b) (b(b(c)))))))
(λ(b)(λ(c) (b(b (b(b(c)))))))
(four)

(c) (a * 2)