

EE 330

Lecture 20

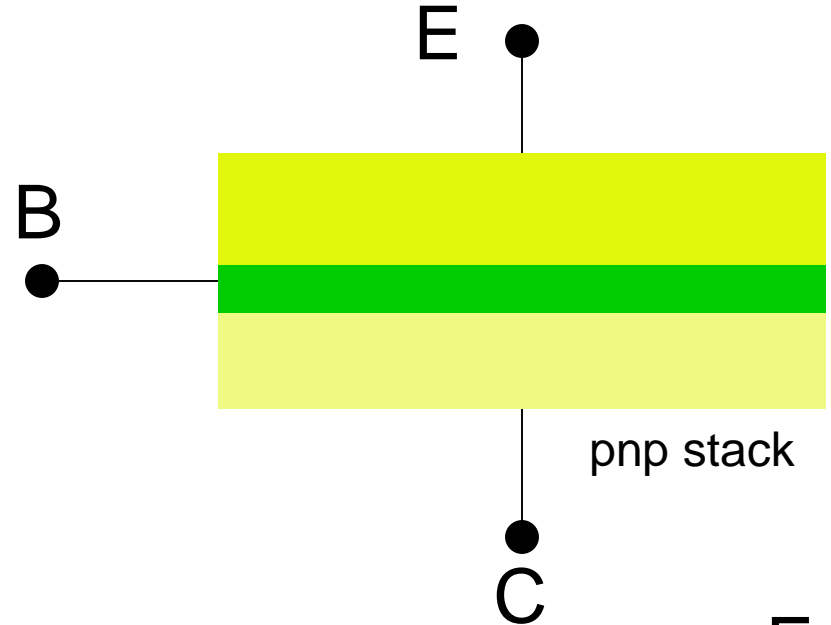
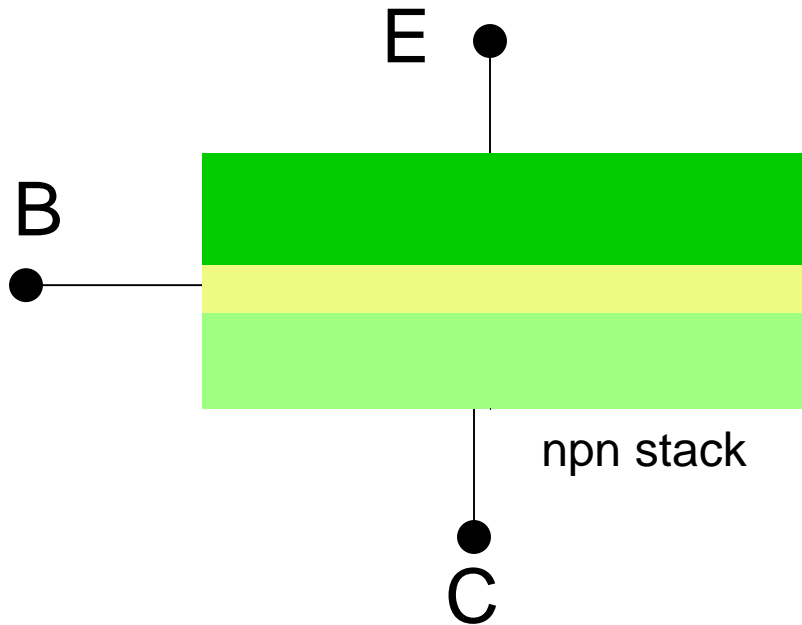
Bipolar Device Modeling

Bipolar Process Flow (if time permits)

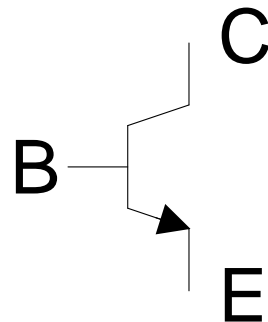
Exam 2 Wed Oct 24

Exam 3 Friday Nov 16

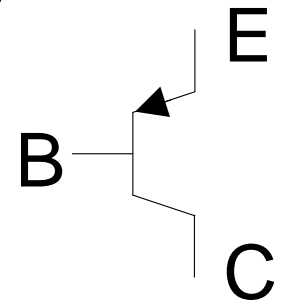
Bipolar Transistors



- Bipolar Devices Show Basic Symmetry
- Electrical Properties not Symmetric
- Designation of C and E critical



npn transistor

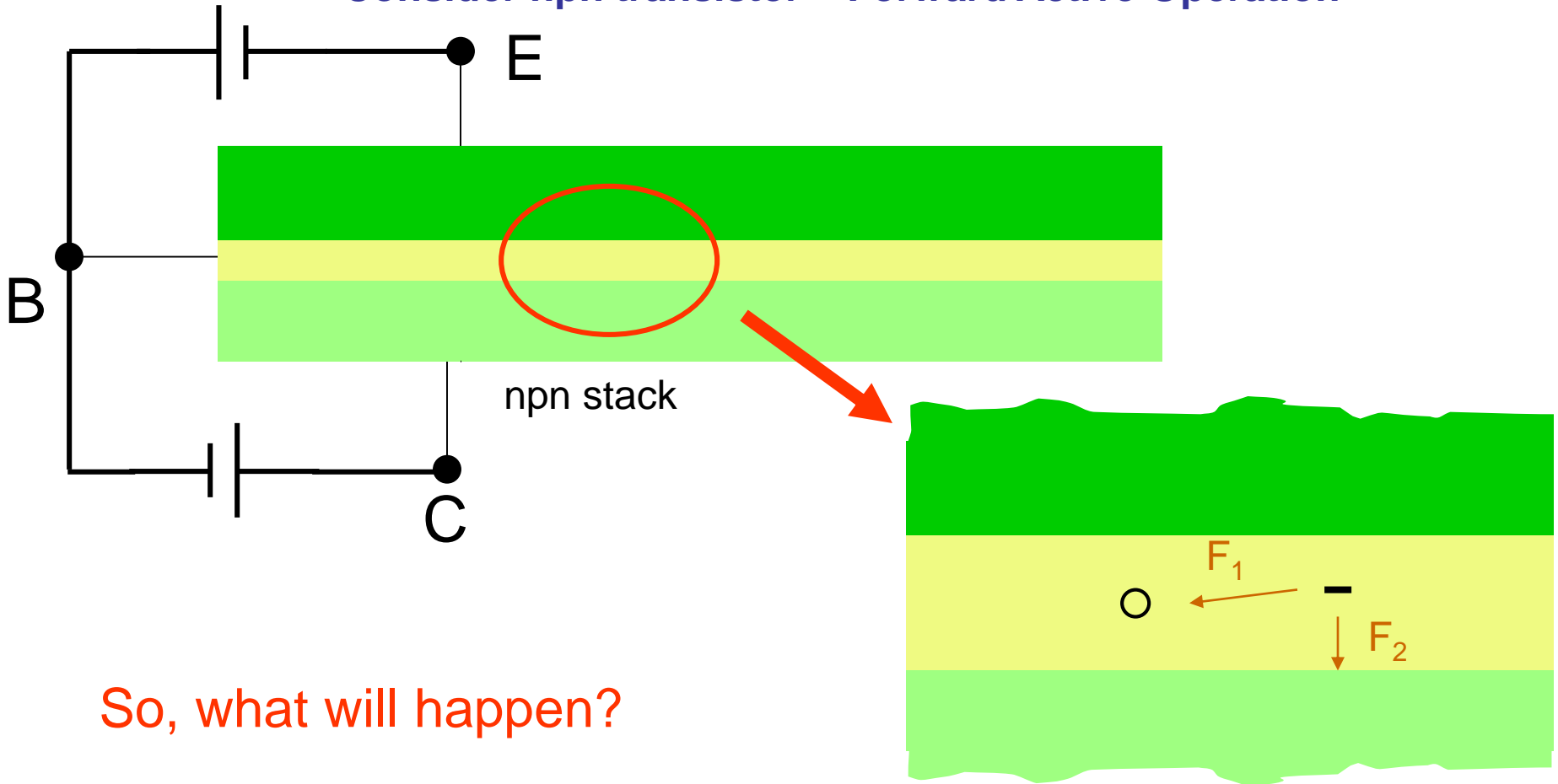


pnp transistor

With proper doping and device sizing these form Bipolar Transistors

Bipolar Operation

Consider npn transistor – Forward Active Operation



So, what will happen?

Some will recombine with holes and contribute to base current and some will be attracted across BC junction and contribute to collector

Size and thickness of base region and relative doping levels will play key role in percent of minority carriers injected into base contributing to collector current

Simple dc model

npn transistor – Forward Active Operation

$$\left. \begin{aligned} I_B &= \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \\ I_C &= J_S A_E e^{\frac{V_{BE}}{V_t}} \end{aligned} \right\}$$
$$V_t = \frac{kT}{q}$$

As with the diode, the parameter J_S is highly temperature dependent

$$J_S = J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right]$$

Typical values for parameters: $J_{SX}=20\text{mA}/\mu^2$, $V_{G0}=1.17\text{V}$, $m=2.3$

The parameter β is also somewhat temperature dependent but much weaker temperature dependence than J_{SX} .

Simple dc model

npn transistor – Forward Active Operation

$$\left. \begin{aligned} I_B &= \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \\ I_C &= \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \\ V_t &= \frac{kT}{q} \end{aligned} \right\} \longrightarrow \left. \begin{aligned} I_B &= \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \\ I_C &= J_S A_E e^{\frac{V_{BE}}{V_t}} \\ V_t &= \frac{kT}{q} \end{aligned} \right\}$$

$k/q = 8.62 \times 10^{-5}$

J_S is termed the saturation current density

Process Parameters : J_S, β

Design Parameters: A_E

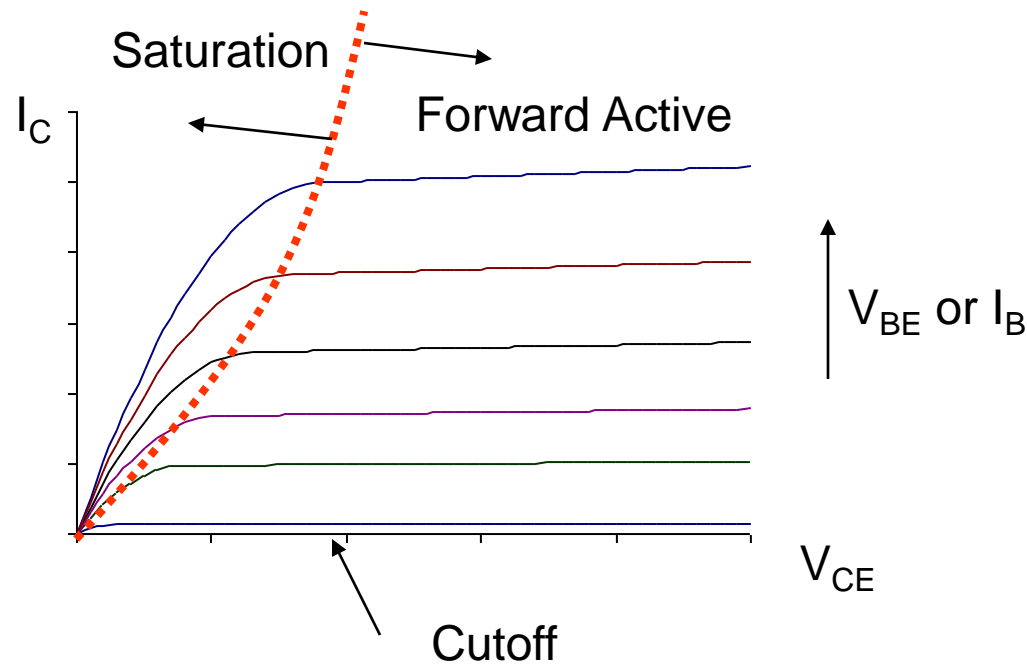
Environmental parameters and physical constants: k, T, q

At room temperature, V_t is around 26mV

J_S very small – around .25fA/ μ^2 at room temperature

Simple dc model

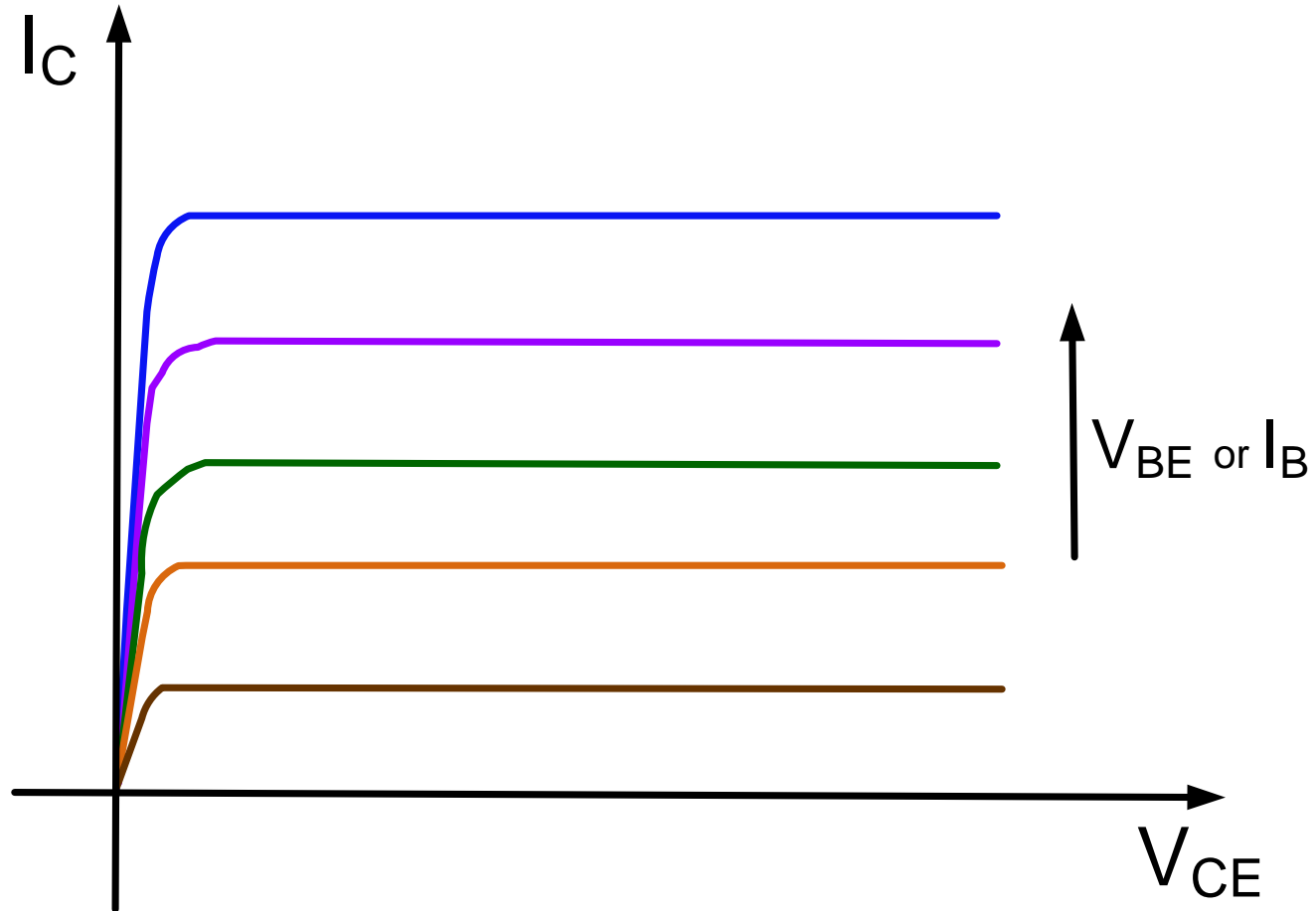
Typical Output Characteristics



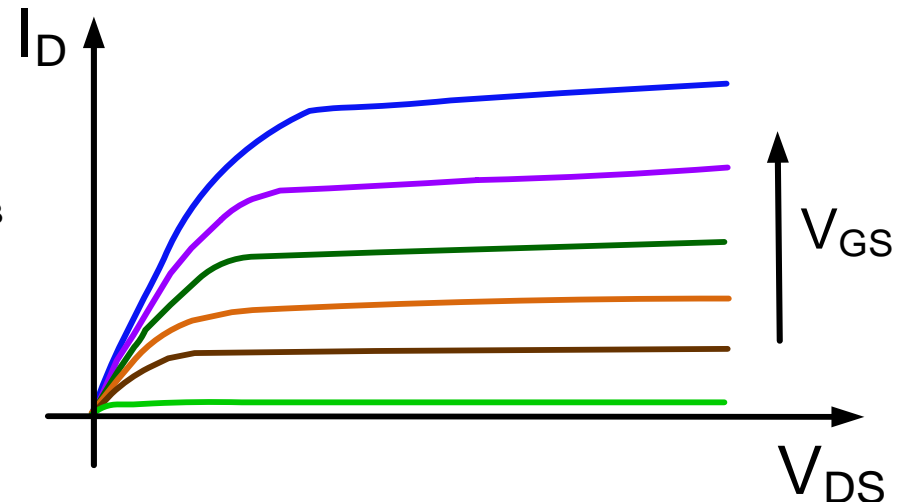
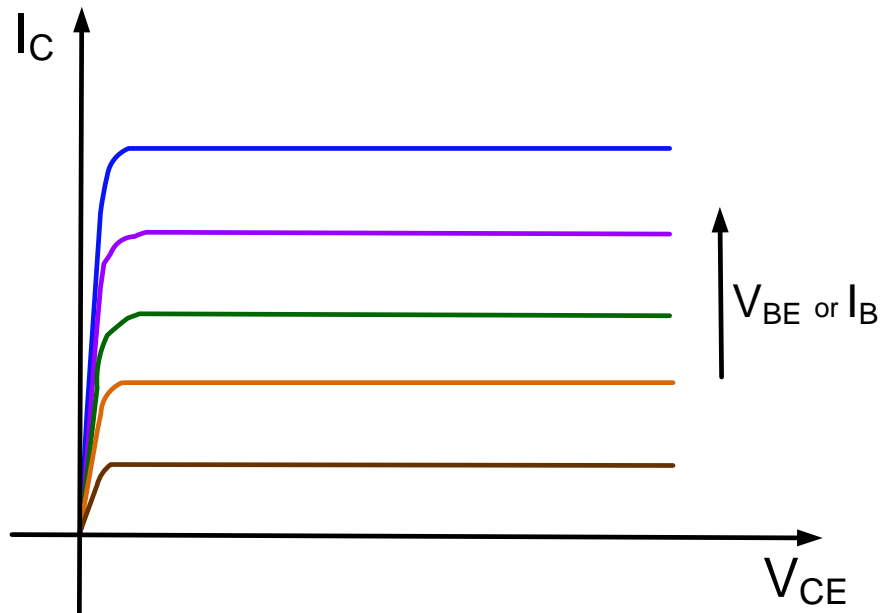
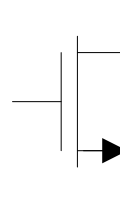
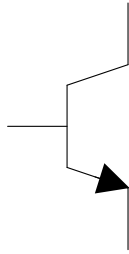
Forward Active region of BJT is analogous to Saturation region of MOSFET

Saturation region of BJT is analogous to Triode region of MOSFET

Better Model of Output Characteristics

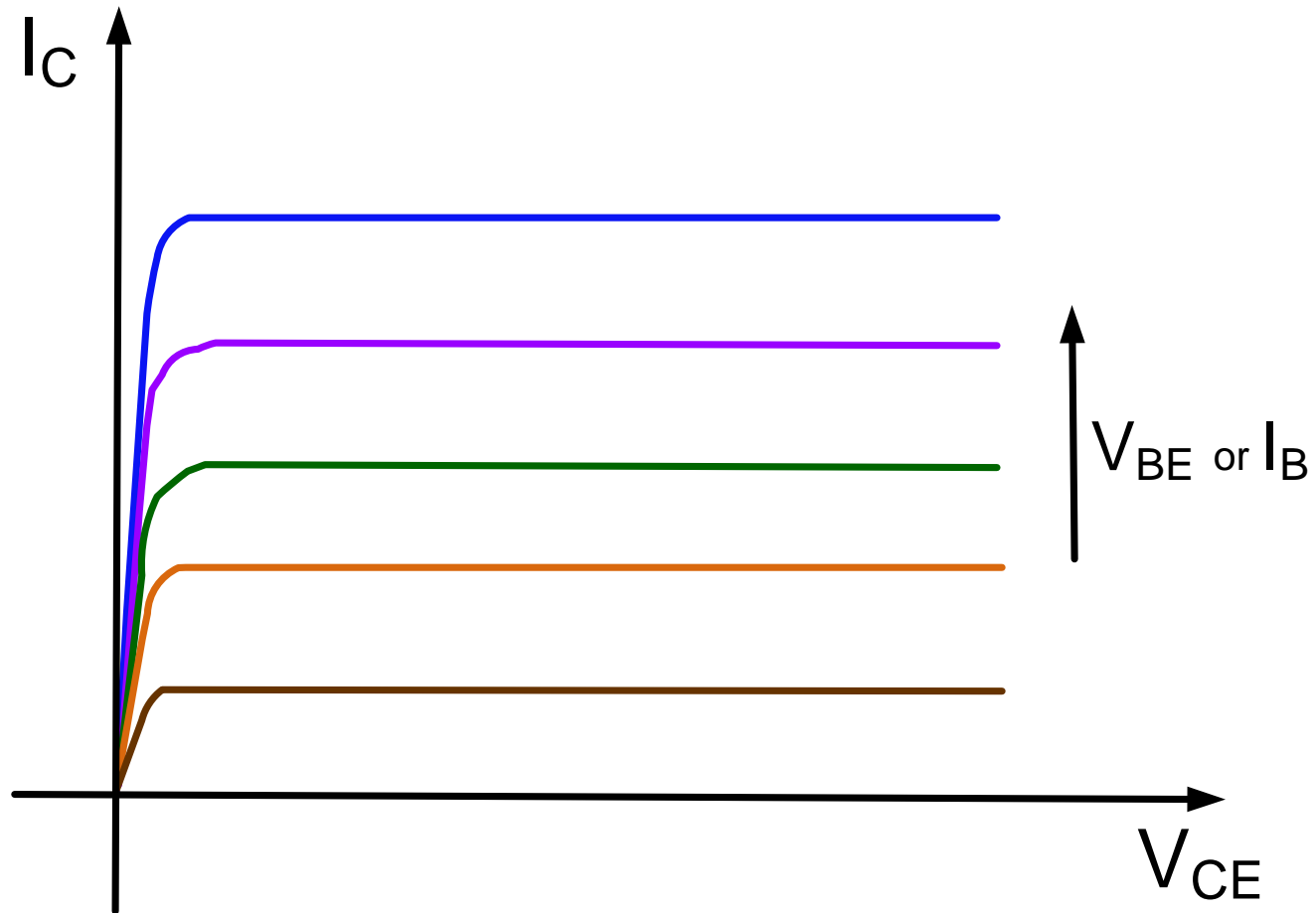


BJT and MOSFET Comparison



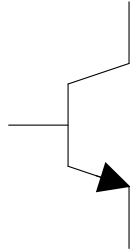
- Same general characteristics
- Spacings a bit different (Exponential vs square law)
- Slope steeper for small V_{CE} compared to V_{DS}

Better Model of Output Characteristics

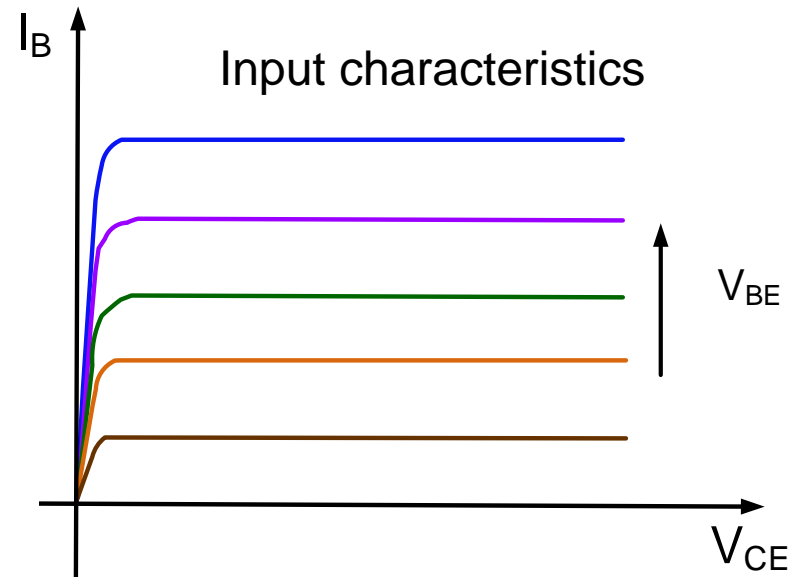
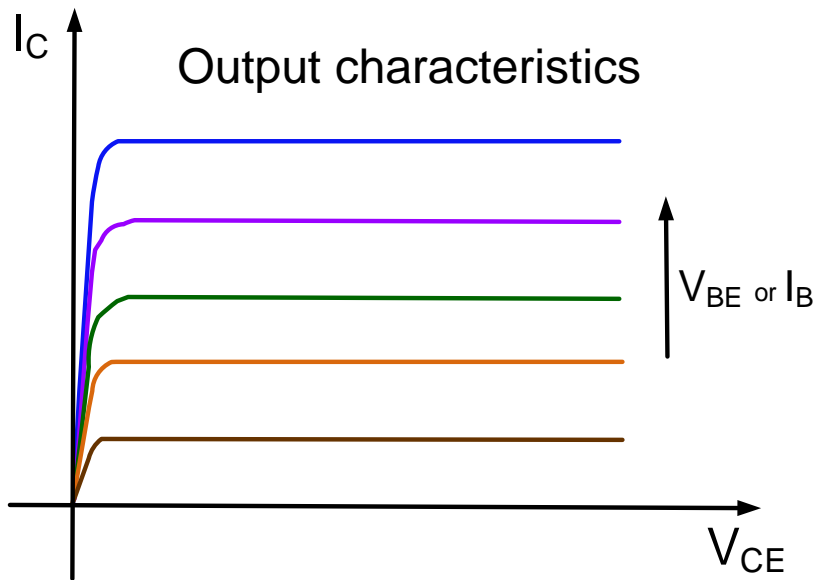


With scaled V_{CE} axis, transition in saturation very steep

BJT Model



$$I_B = f_1(V_{BE}, V_{CE})$$
$$I_C = f_2(V_{BE}, V_{CE})$$

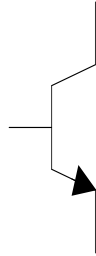


Require two graphical representations though vertical axis scales different by factor of β

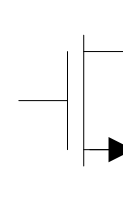
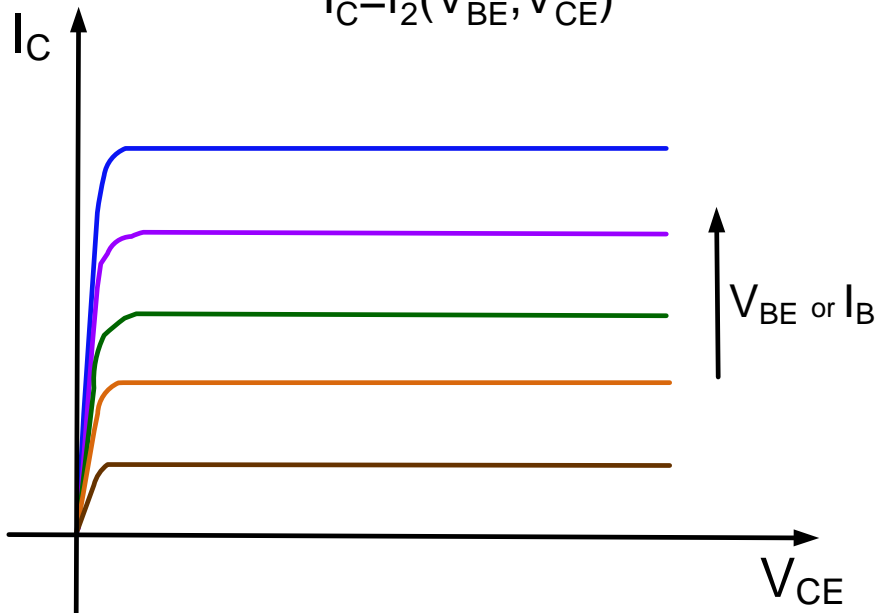
Since $I_B = f(V_{BE})$, can use independent (V_{BE}) or dependent (I_B) variable for 2-D visualization of 3-dimensional I_C function

BJT and MOSFET Comparison

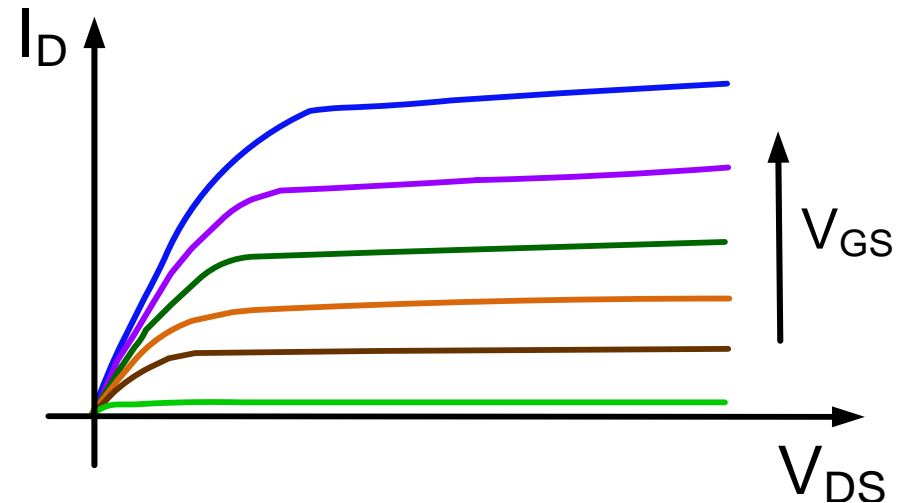
Output Characteristics



$$I_C = f_2(V_{BE}, V_{CE})$$



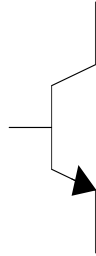
$$I_D = f_{2M}(V_{GS}, V_{DS})$$



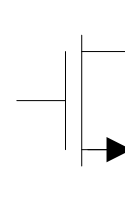
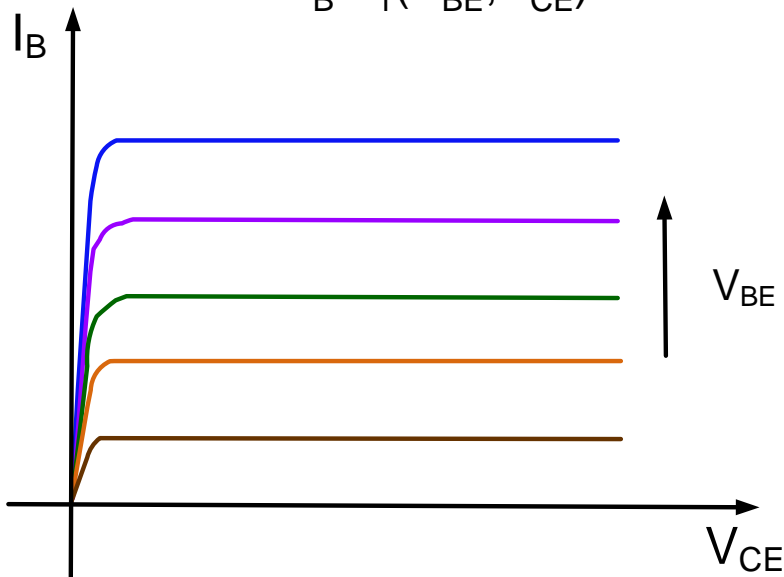
- Same general characteristics
- Spacings a bit different (Exponential vs square law)
- Slope steeper for small V_{CE} compared to small V_{DS}

BJT and MOSFET Comparison

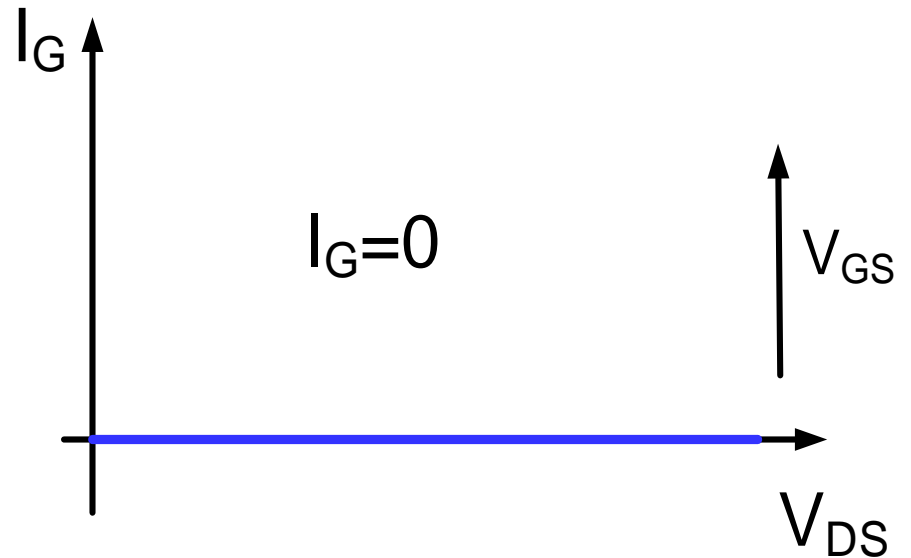
Input Characteristics



$$I_B = f_1(V_{BE}, V_{CE})$$



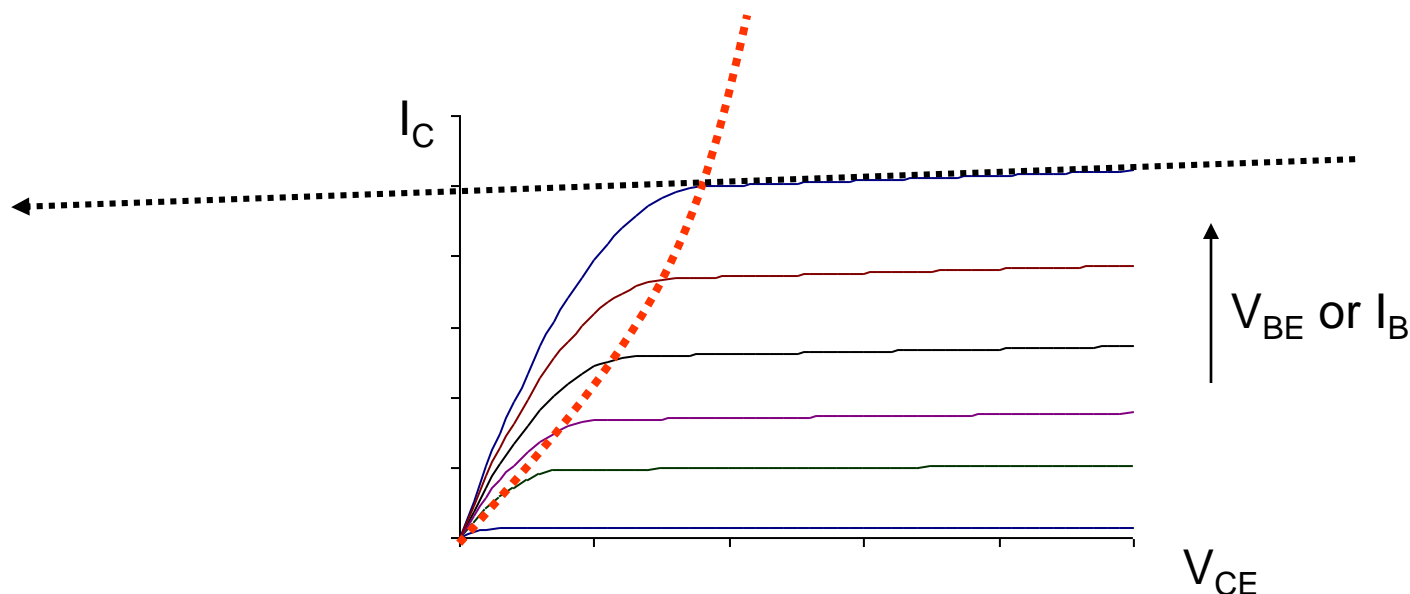
$$I_G = f_{1M}(V_{GS}, V_{DS})$$



Did not need to graphically show input characteristics for MOS transistors since $I_G = 0$

Improved simple dc model

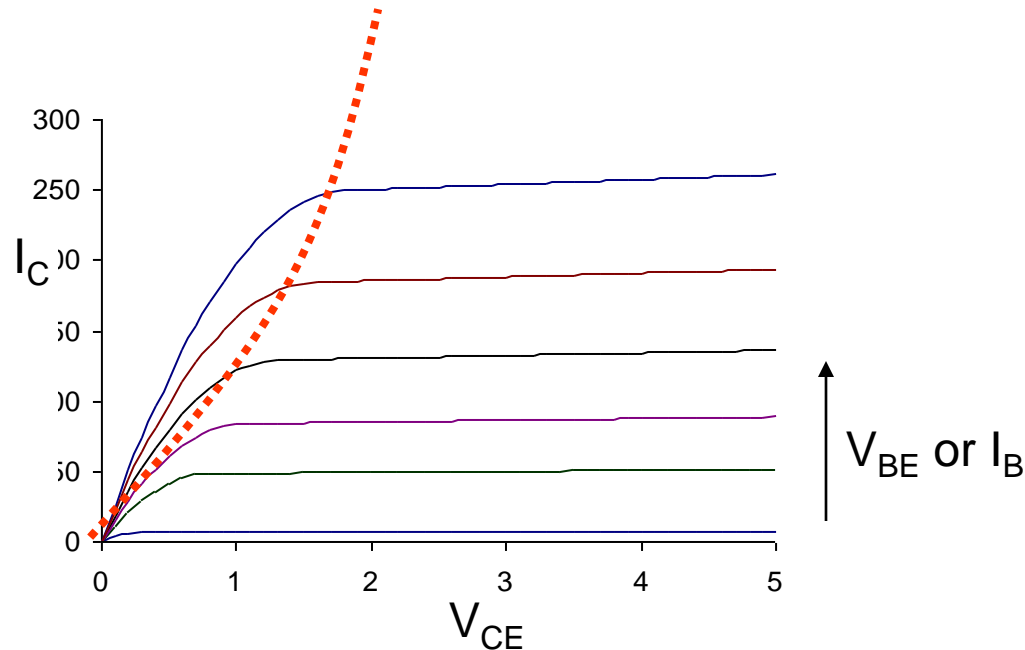
Typical Output Characteristics



- Projections of these tangential lines all intercept the $-V_{CE}$ axis at the same place and this is termed the Early voltage, V_{AF} (actually $-V_{AF}$ is intercept)
- Typical values of V_{AF} are in the 100V to 200V range
- Can multiply expression for I_C in Forward Active Region by term $\left(1 + \frac{V_{CE}}{V_{AF}}\right)$ to account for slope

Improved simple dc model

(graphically showing only output characteristics)

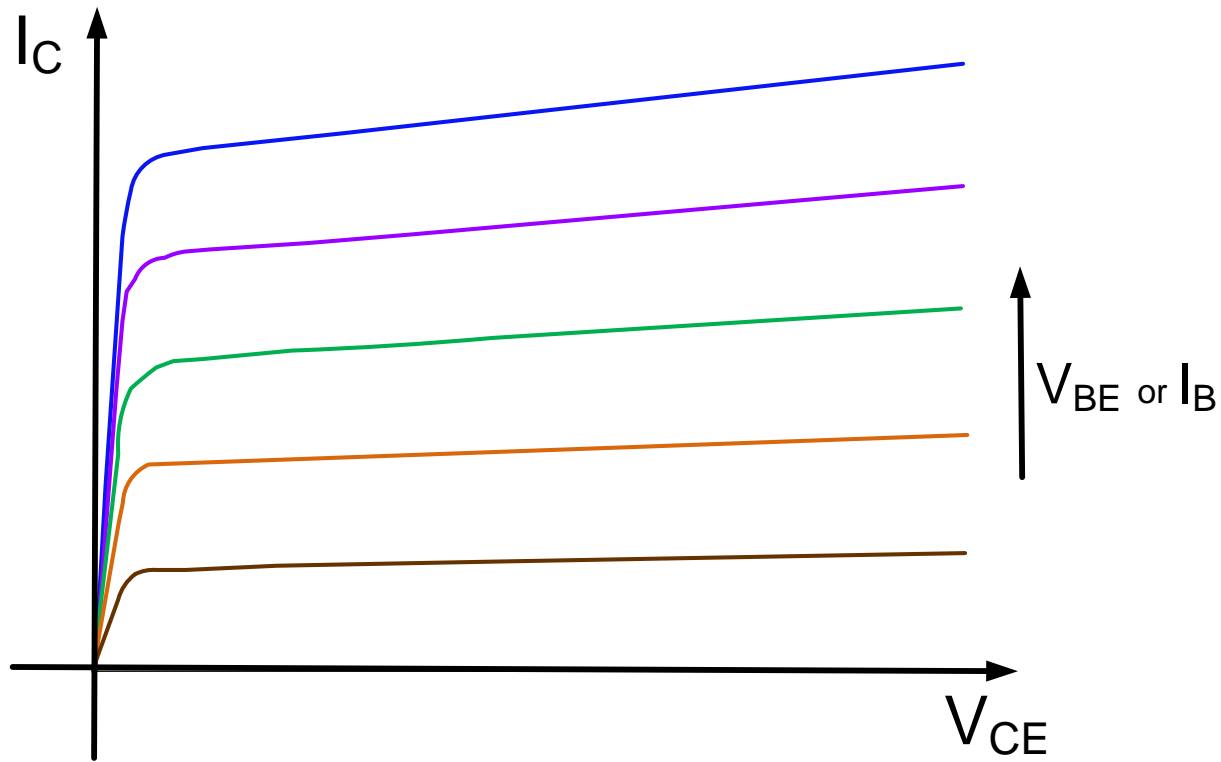


$$\left. \begin{aligned} I_B &= \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \\ I_C &= J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}} \right) \end{aligned} \right\} \text{Valid only in Forward Active Region}$$

Need models in saturation and cutoff regions

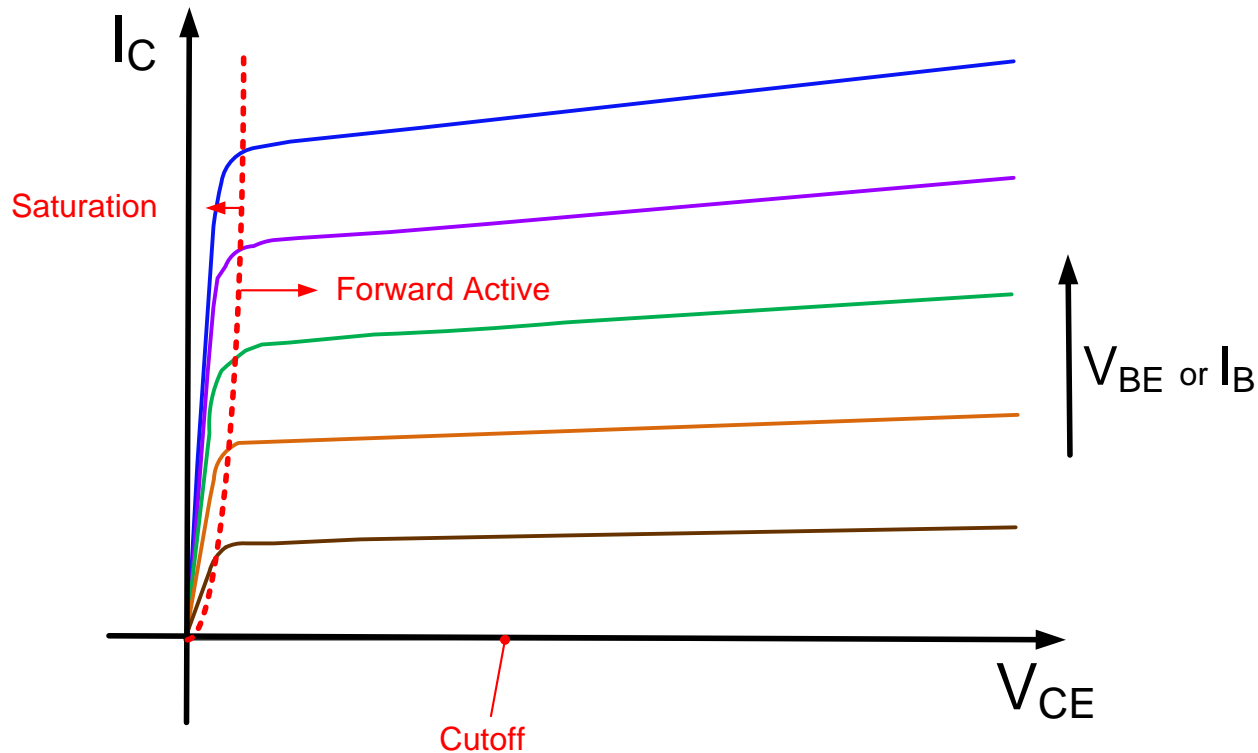
Improved simple BJT dc model

Typical Output Characteristics



Improved simple BJT dc model

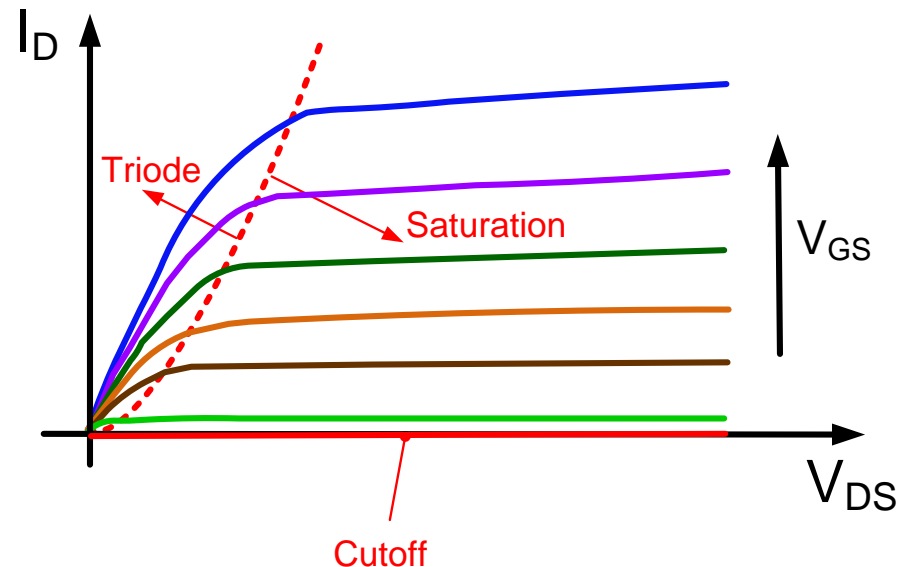
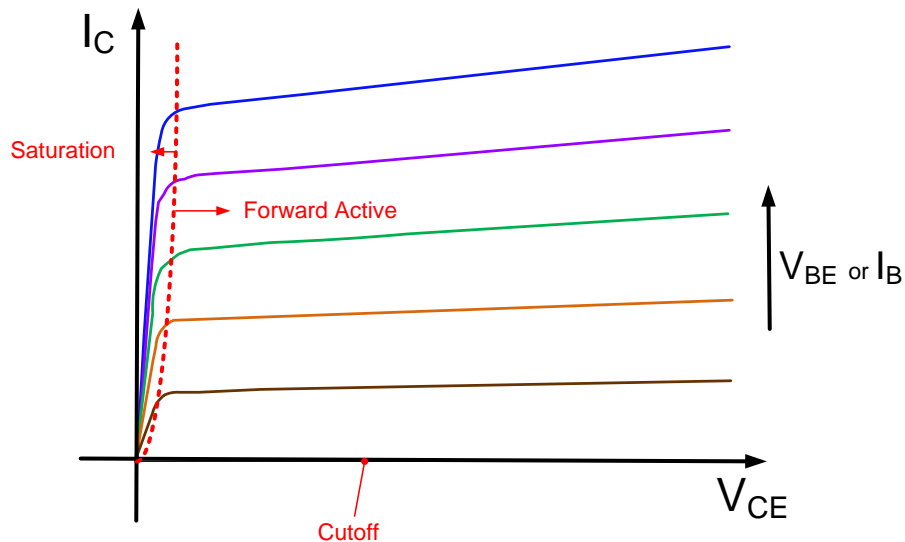
Typical Output Characteristics



Need analytical models in saturation and cutoff regions

Improved simple BJT dc model

Typical Output Characteristics



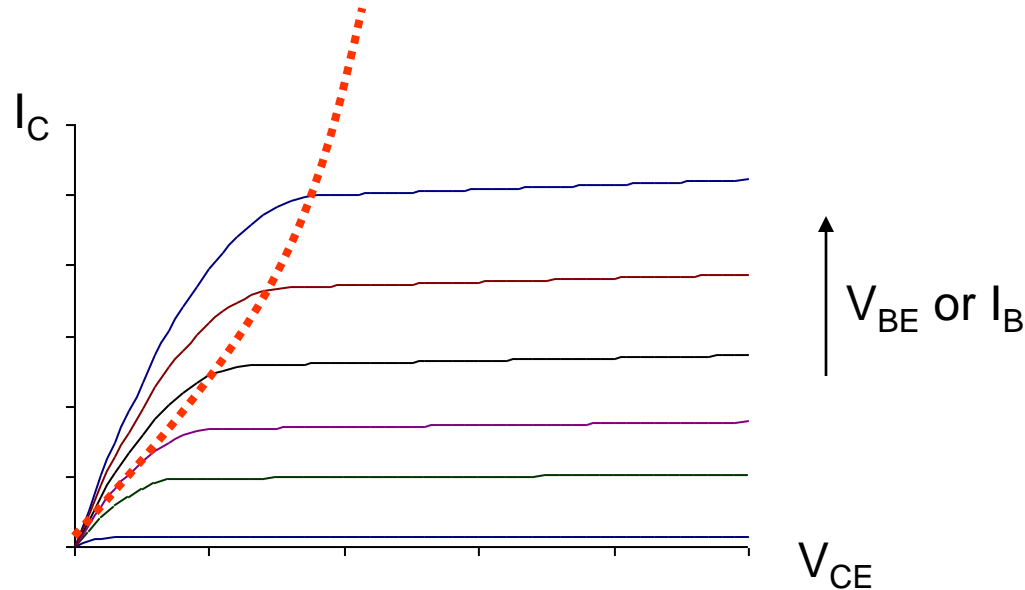
Recall:

Forward Active region of BJT is analogous to Saturation region of MOSFET

Saturation region of BJT is analogous to Triode region of MOSFET

Improved dc model

(graphically showing only output characteristics)



$$V_t = \frac{kT}{q}$$

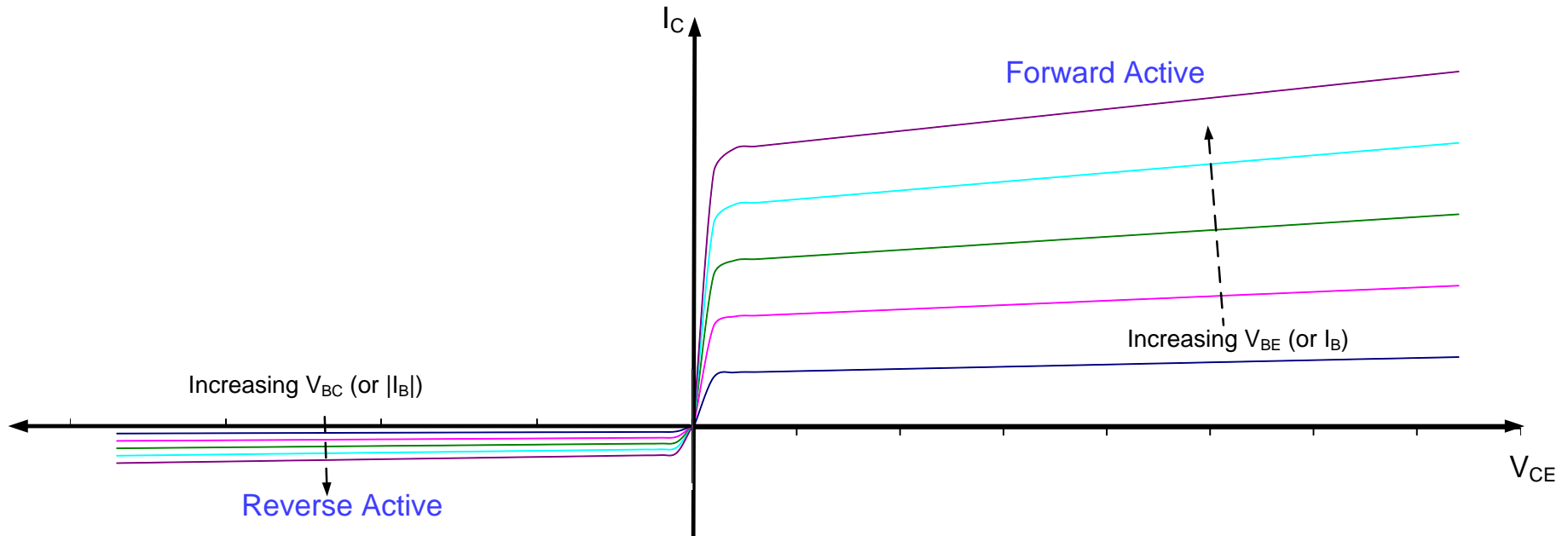
$$I_E = -\frac{J_S A_E}{\alpha_F} \left(e^{\frac{V_{BE}}{V_t}} - 1 \right) + J_S A_E \left(e^{\frac{V_{BC}}{V_t}} - 1 \right)$$

$$I_C = J_S A_E \left(e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_S A_E}{\alpha_R} \left(e^{\frac{V_{BC}}{V_t}} - 1 \right)$$

- Valid in All regions of operation
- V_{AF} effects can be added
- Not mathematically easy to work with
- Note dependent variables changes
- Termed Ebers-Moll model
- Reduces to previous model in FA region
- Little use in Reverse Active Region

Improved dc model

(graphically showing only output characteristics)



$$V_t = \frac{kT}{q}$$

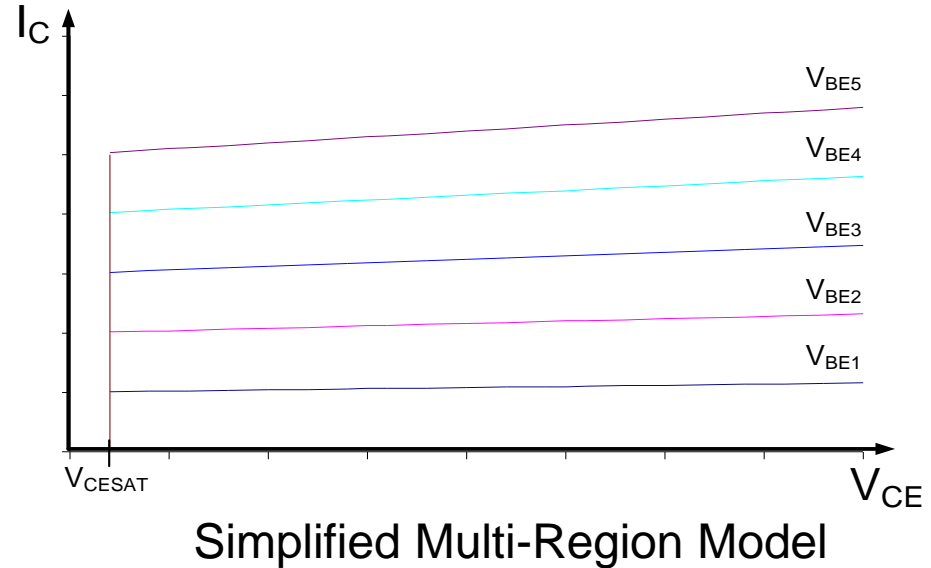
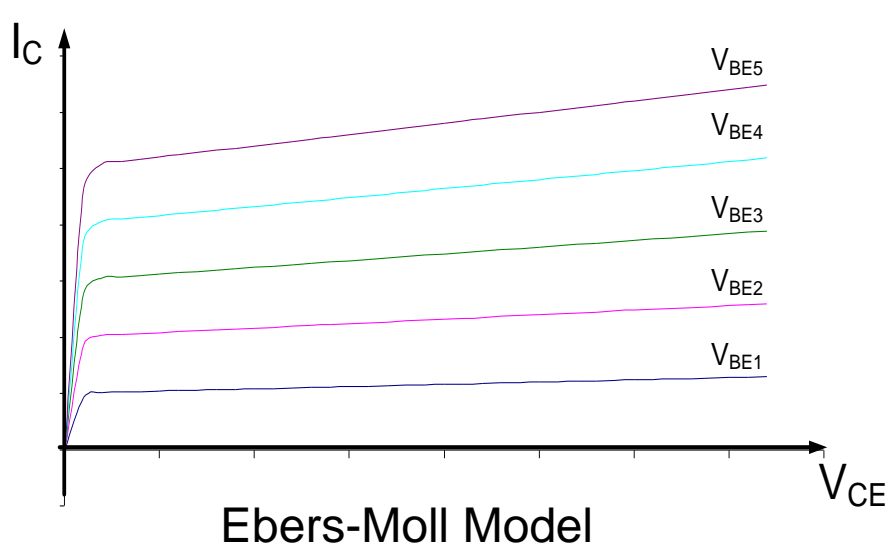
$$I_E = -\frac{J_S A_E}{\alpha_F} \left(e^{\frac{V_{BE}}{V_t}} - 1 \right) + J_S A_E \left(e^{\frac{V_{BC}}{V_t}} - 1 \right)$$

$$I_C = J_S A_E \left(e^{\frac{V_{BE}}{V_t}} - 1 \right) - \frac{J_S A_E}{\alpha_R} \left(e^{\frac{V_{BC}}{V_t}} - 1 \right)$$

- Model using I_E and I_C as dependent variables
- Valid in All regions of operation
- V_{AF} effects can be added
- Not mathematically easy to work with
- Note dependent variables changes
- Termed Ebers-Moll model
- Reduces to previous model in FA region
- Little use in Reverse Active Region

Simplified Multi-Region Model

(graphically showing only output characteristics)

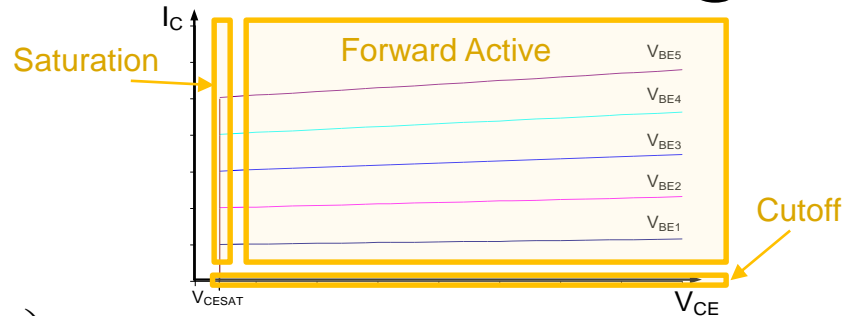


- Observe V_{CE} around 0.2V when saturated
- V_{BE} around 0.6V when saturated
- In most applications, exact V_{CE} and V_{BE} voltage in saturation not critical

Simplified model in saturation:

$$\left. \begin{array}{l} V_{BE}=0.7V \\ V_{CE}=0.2V \end{array} \right\} \text{ Saturation}$$

Simplified Multi-Region Model



$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}} \right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

Forward Active

$$V_{BE} = 0.7V$$

$$V_{CE} = 0.2V$$

Saturation

$$I_C = I_B = 0$$

Cutoff

- This is a piecewise model suitable for analytical calculations
- Can easily extend to reverse active mode but of little use
- Still need conditions for operating in the 3 regions

Simplified Multi-Region Model

“Forward” Regions : $\beta = \beta_F$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}} \right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$V_{BE} = 0.7V$$

$$V_{CE} = 0.2V$$

$$I_C = I_B = 0$$

Conditions

$$V_{BE} > 0.4V \quad V_{BC} < 0$$

$$I_C < \beta I_B$$

$$V_{BE} < 0 \quad V_{BC} < 0$$

Forward Active

Saturation

Cutoff

Process Parameters: $\{J_S, \beta, V_{AF}\}$

$$V_t = \frac{kT}{q}$$

Design Parameters: $\{A_E\}$

- Process parameters highly process dependent
- J_S highly temperature dependent as well, β modestly temperature dependent
- This model is dependent only upon emitter area, independent of base and collector area !
- Currents scale linearly with A_E and not dependent upon shape of emitter
- A small portion of the operating region is missed with this model but seldom operate in the missing region

Simplified Multi-Region Model

Alternate equivalent model

$$I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}} \right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

$$V_{BE} = 0.7V$$

$$V_{CE} = 0.2V$$

$$I_C = I_B = 0$$

Conditions

$$V_{BE} > 0.4V$$

$$V_{BC} < 0$$

Forward Active

$$I_C < \beta I_B$$

Saturation

$$V_{BE} < 0$$

$$V_{BC} < 0$$

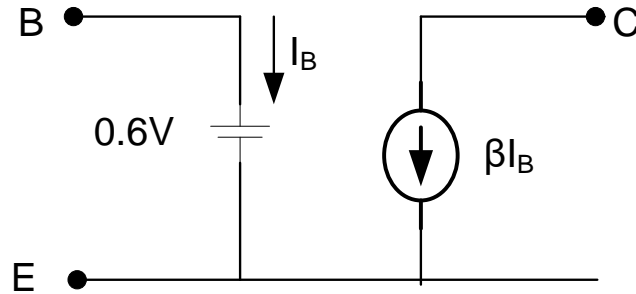
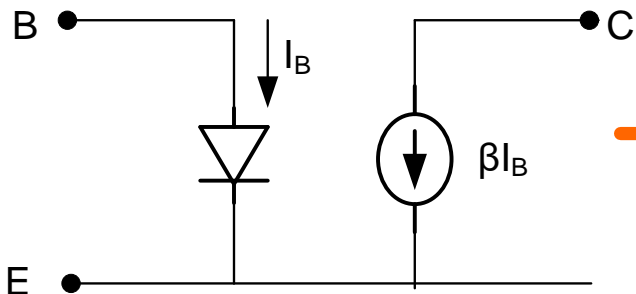
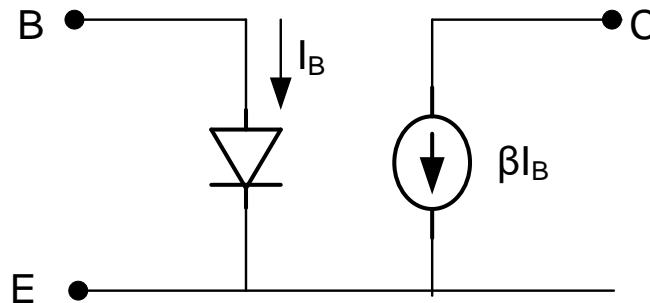
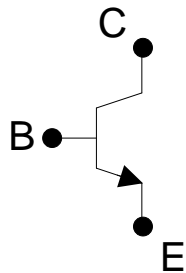
Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region

Further Simplified Multi-Region dc Model

(by neglecting V_{AF})

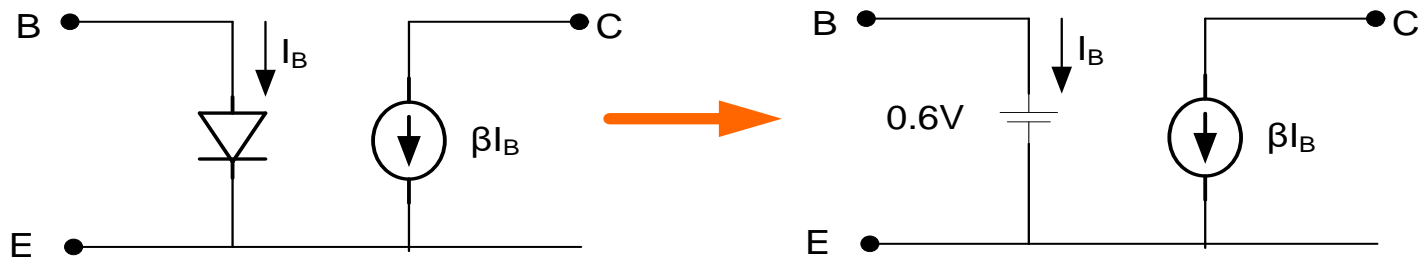
Forward Active



Adequate when it makes little difference whether $V_{BE}=0.6V$ or $V_{BE}=0.7V$

Simplified Multi-Region dc Model

Forward Active



Mathematically

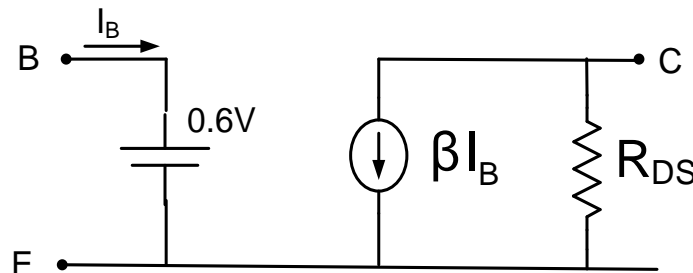
$$V_{BE} = 0.6V$$

$$I_C = \beta I_B$$

Or, if want to show slope in I_C - V_{CE} characteristics

$$V_{BE} = 0.6V$$

$$I_C = \beta I_B (1 + V_{CE}/V_{AF})$$



$$R_{DS} = \frac{V_{AF}}{\beta I_{BQ}}$$

R_{DS} highly nonlinear

Further Simplified Multi-Region dc Model

Equivalent Further Simplified Multi-Region Model

$$I_C = \beta I_B$$

$$V_{BE} = 0.6V$$

$$V_{BE} > 0.4V$$

$$V_{BC} < 0$$

Forward Active

$$V_t = \frac{kT}{q}$$

$$V_{BE} = 0.7V$$

$$V_{CE} = 0.2V$$

$$I_C < \beta I_B$$

Saturation

$$I_C = I_B = 0$$

$$V_{BE} < 0$$

$$V_{BC} < 0$$

Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region

Conditions for Regions of Operation in Multi-Region Model

$$V_{BE} > 0.4V$$

Forward Active

$$V_{BC} < 0$$

$$I_C < \beta I_B$$

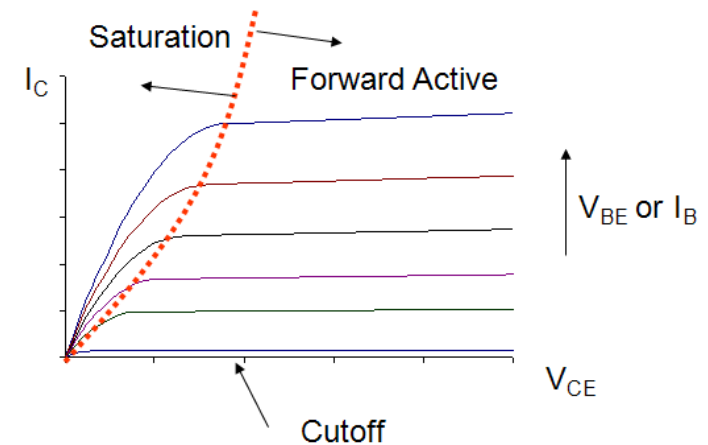
Saturation

$$V_{BE} < 0$$

$$V_{BC} < 0$$

Cutoff

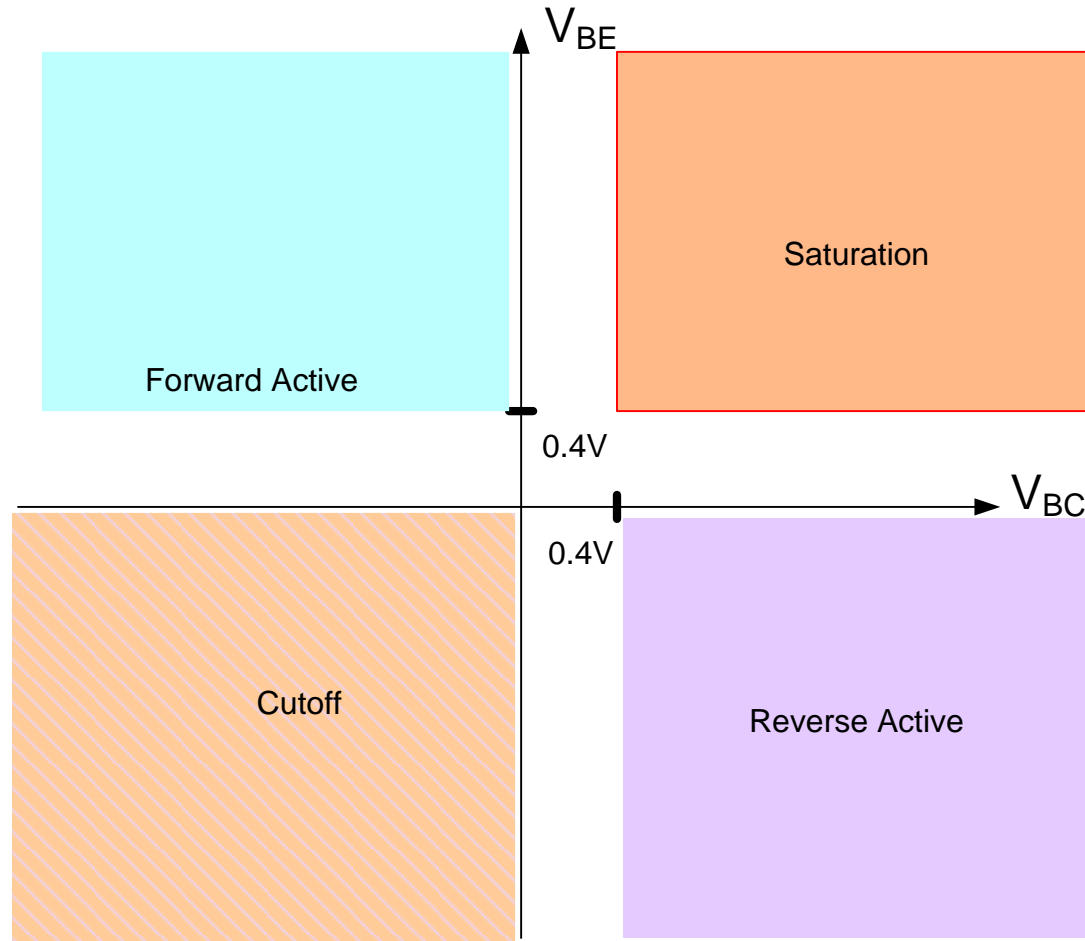
Note: One condition is on dependent variables !



Observe that in saturation, $I_C < \beta I_B$

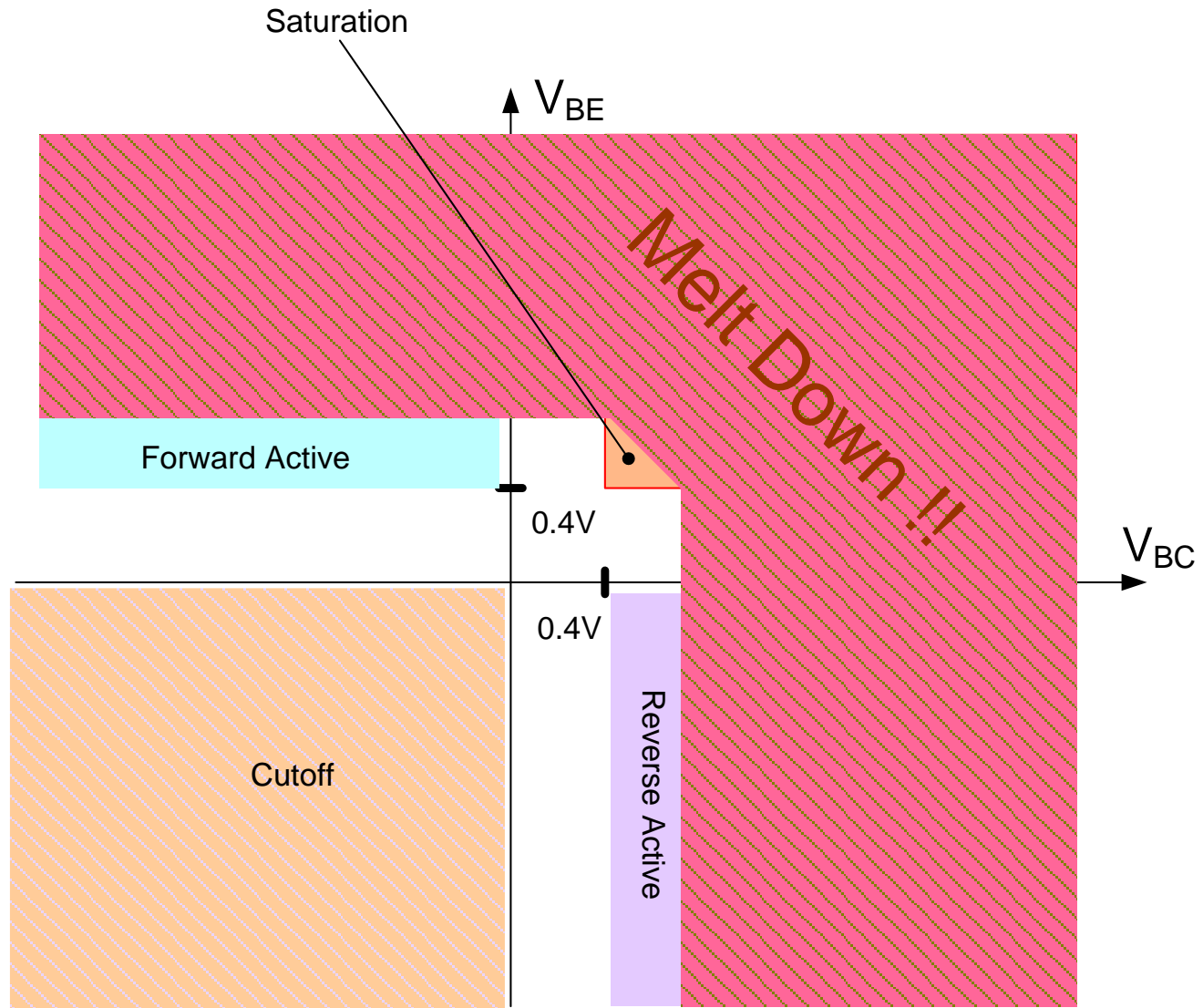
Can't condition on independent variables in saturation because they are fixed in the model

Regions of Operation in Independent Parameter Domain used In multi-region models

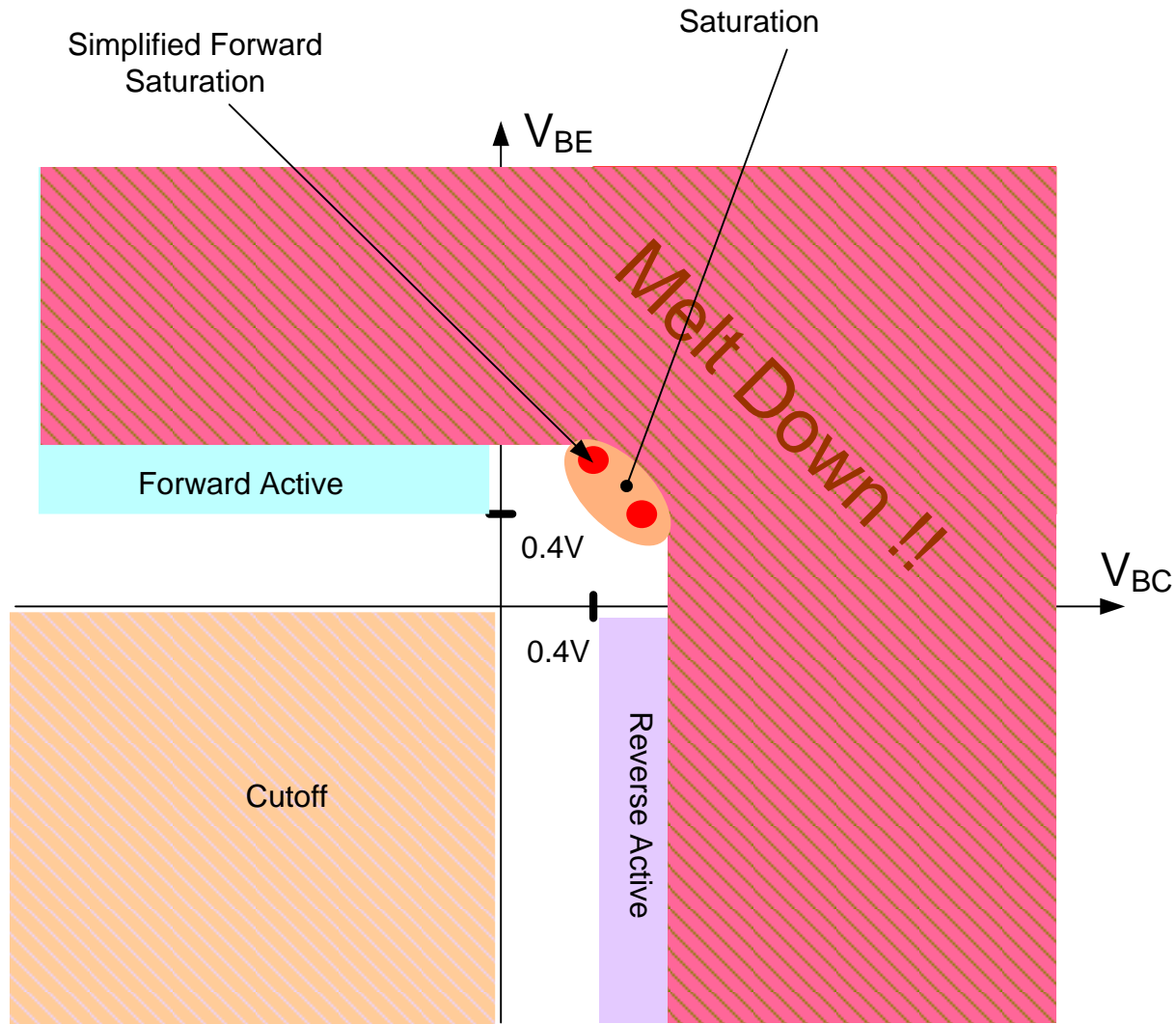


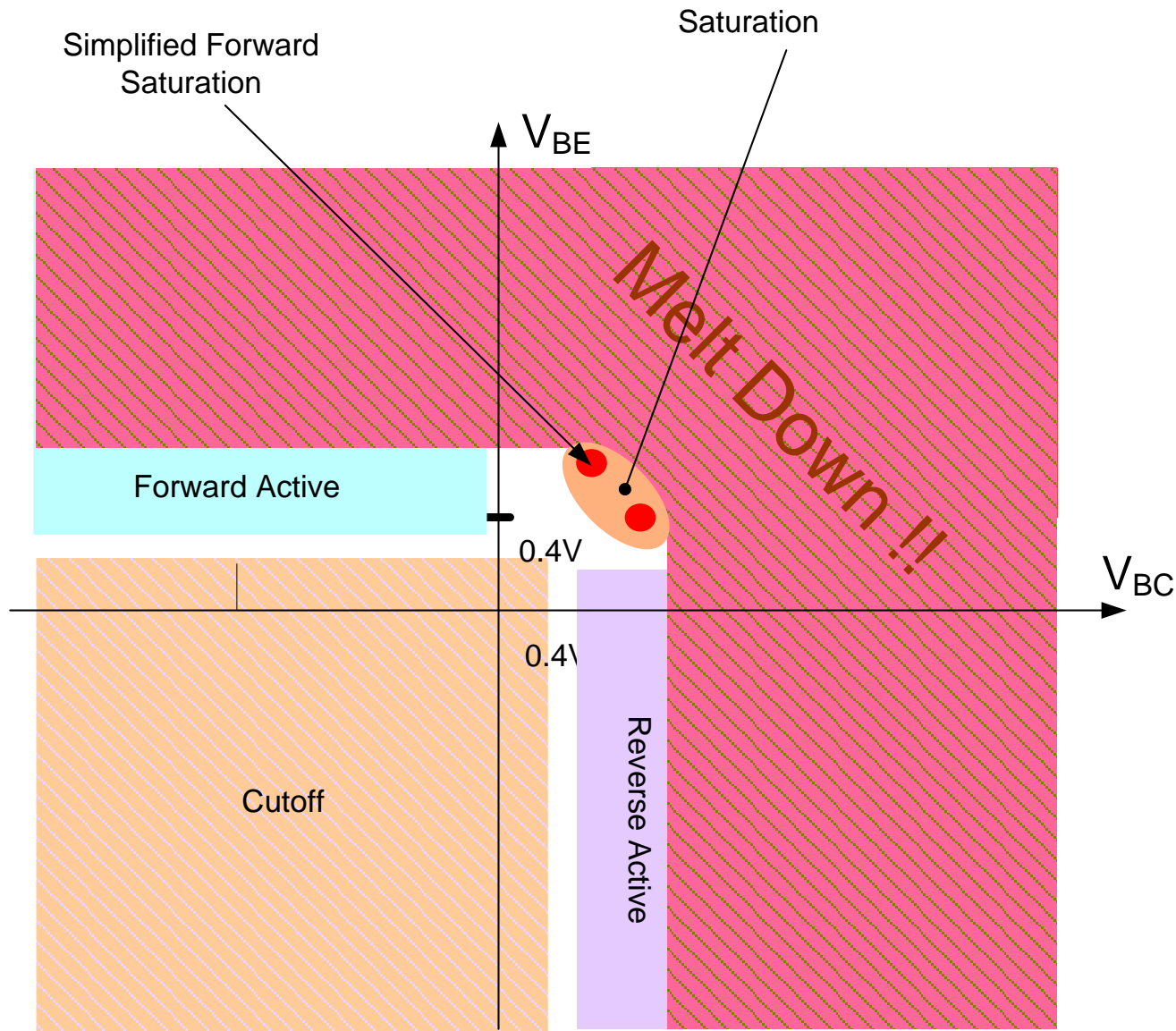
- Seldom operate in regions excluded in this picture
- Limited use in Reverse Active Mode

Excessive Power Dissipation if either junction has large forward bias



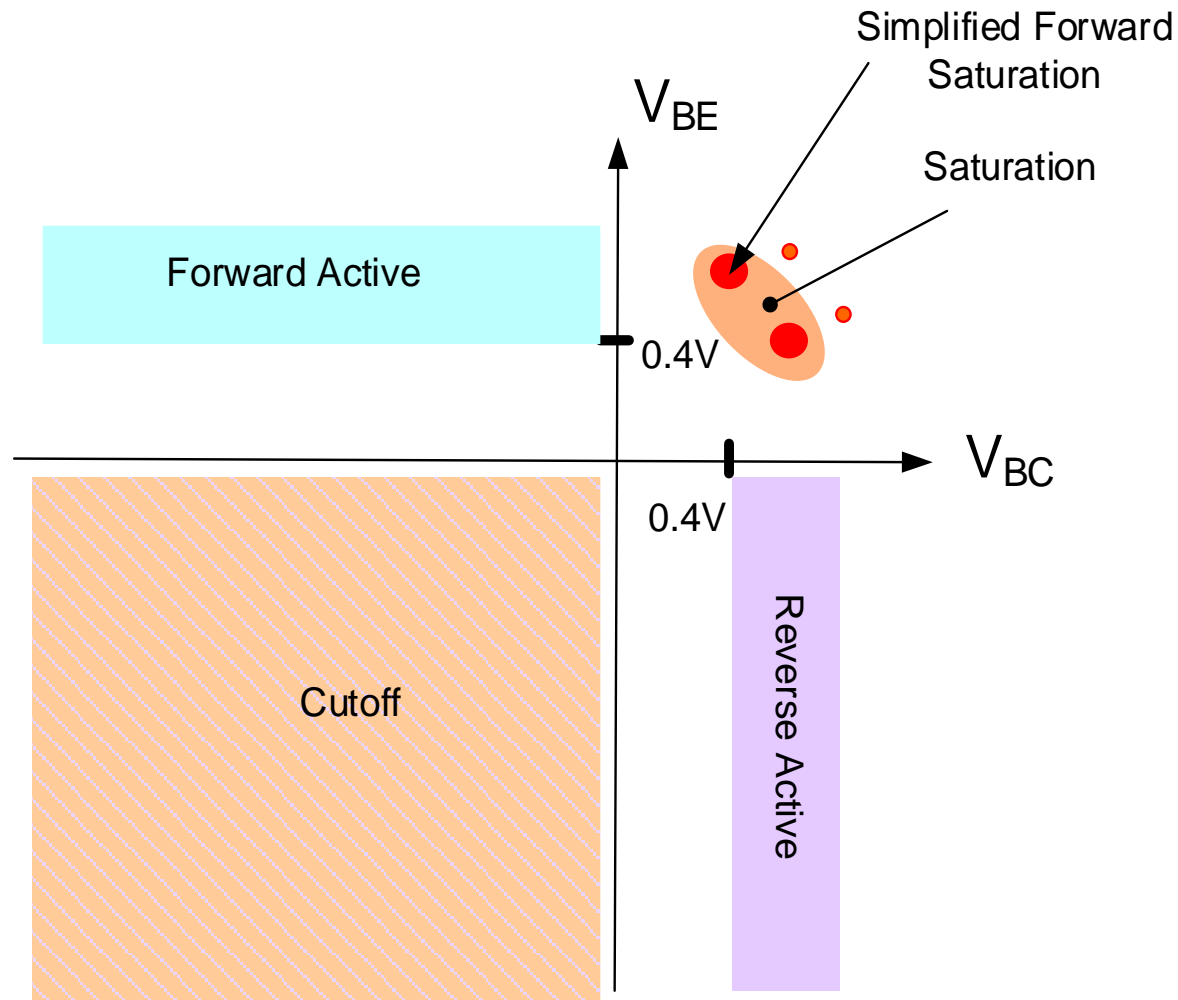
Safe regions of operation



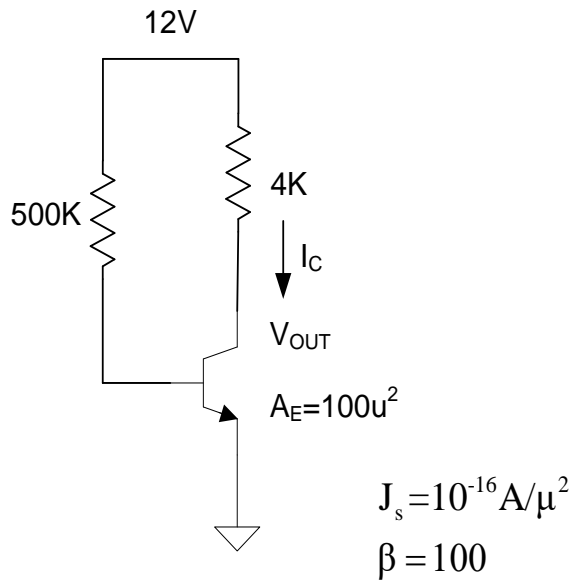


Actually cutoff, forward active, and reverse active regions can be extended modestly as shown and multi-region models still are quite good

Sufficient regions of operation for most applications



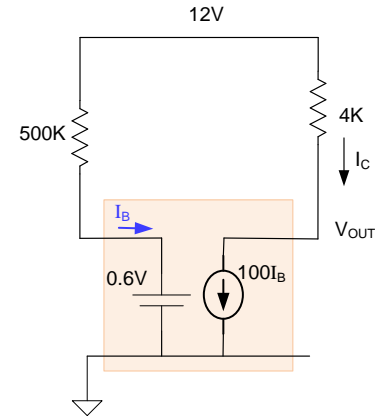
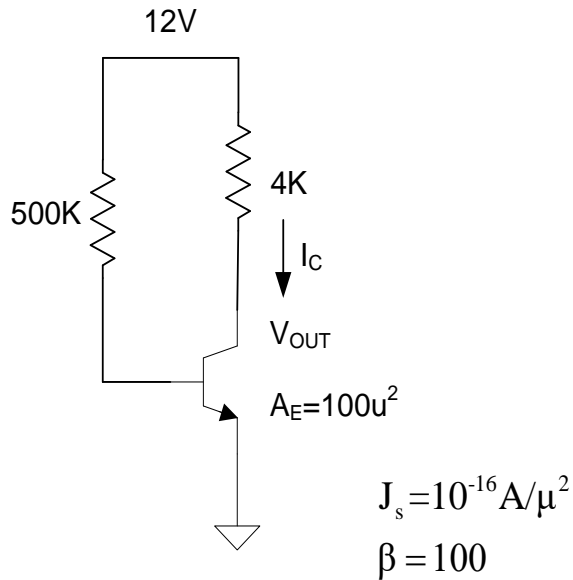
Example: Determine I_C and V_{OUT}



Example: Determine I_C and V_{OUT}

Solution:

1. Guess Forward Active Region
2. Solve Circuit with Guess



$$I_B = \frac{(12 - 0.6)}{500K}$$

$$I_C = \beta I_B = 100 \frac{(12 - 0.6)}{500K} = 2.28mA$$

$$V_{OUT} = 12 - I_C \cdot 4K = 2.88V$$

3. Verify FA Region

$$V_{BE} = 0.6V > 0.4V$$

$$V_{BC} = 0.6V - 2.88V = -2.28V < 0$$

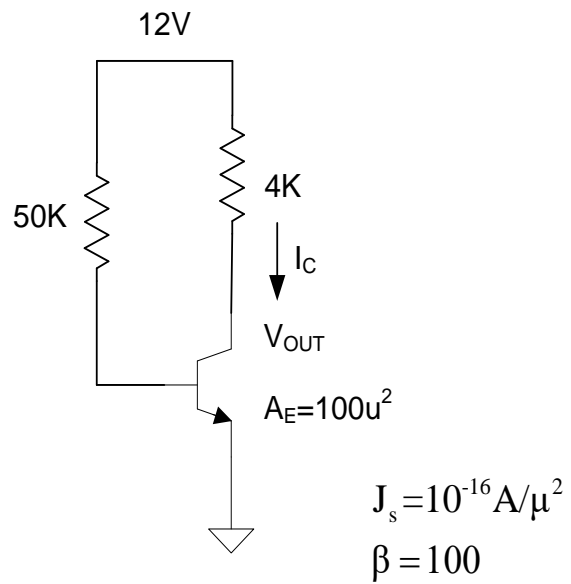
$$V_{BE} > 0.4V \text{ and } V_{BC} < 0$$

Verify Passes so solution is valid

$$I_C = 2.28mA$$

$$V_{OUT} = 2.88V$$

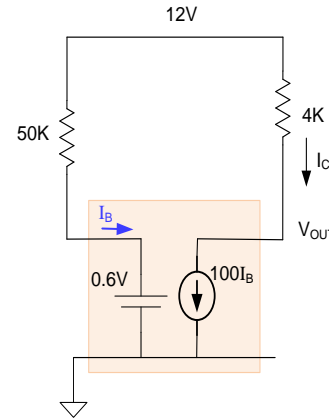
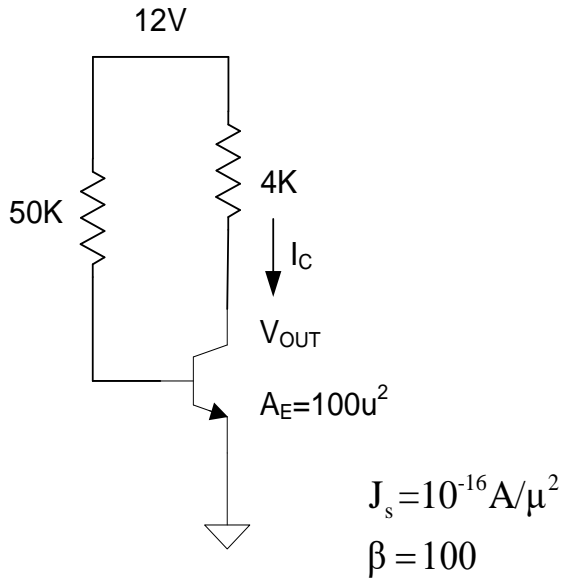
Example: Determine I_C and V_{OUT} ,



Example: Determine I_C and V_{OUT} .

Solution:

1. Guess Forward Active Region
2. Solve Circuit with Guess



$$I_B = \frac{(12 - 0.6)}{50K}$$

$$I_C = \beta I_B = 100 \frac{(12 - 0.6)}{50K} = 22.8 \text{ mA}$$

$$V_{OUT} = 12 - I_C \cdot 4K = -79.2 \text{ V}$$

3. Verify FA Region $V_{BE} > 0.4 \text{ V}$ and $V_{BC} < 0$

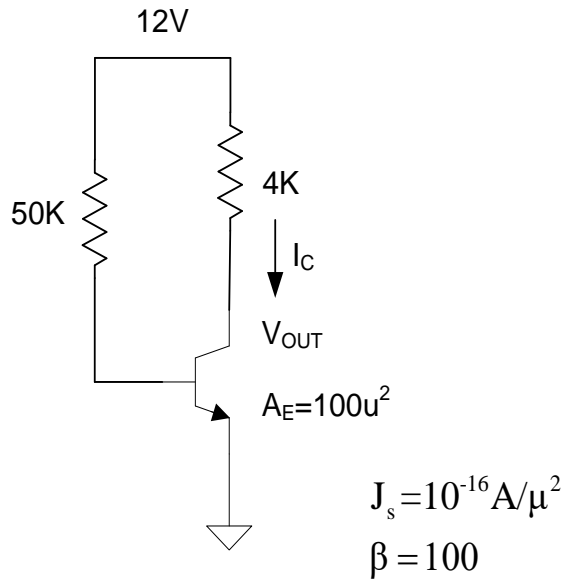
$$V_{BE} = 0.6 \text{ V} > 0.4 \text{ V}$$

$$V_{BC} = 0.6 \text{ V} - (-79.2 \text{ V}) = +79.8 \text{ V} > 0$$

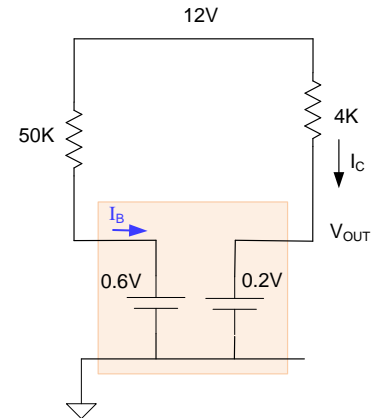
Verify Fails so solution is not valid

Example: Determine I_C and V_{OUT}

Solution:



4. Guess Saturation
5. Solve Circuit with Guess



$$I_B = \frac{(12 - 0.6)}{50K} = 228 \mu A$$

$$I_C = \frac{(12 - 0.2)}{4K} = 2.95 \text{ mA}$$

$$V_{OUT} = 0.2 \text{ V}$$

6. Verify SAT Region

$$I_C < \beta I_B$$

$$\beta I_B = 100 \cdot 228 \mu A = 22.8 \text{ mA}$$

$$I_C = 2.95 \text{ mA}$$

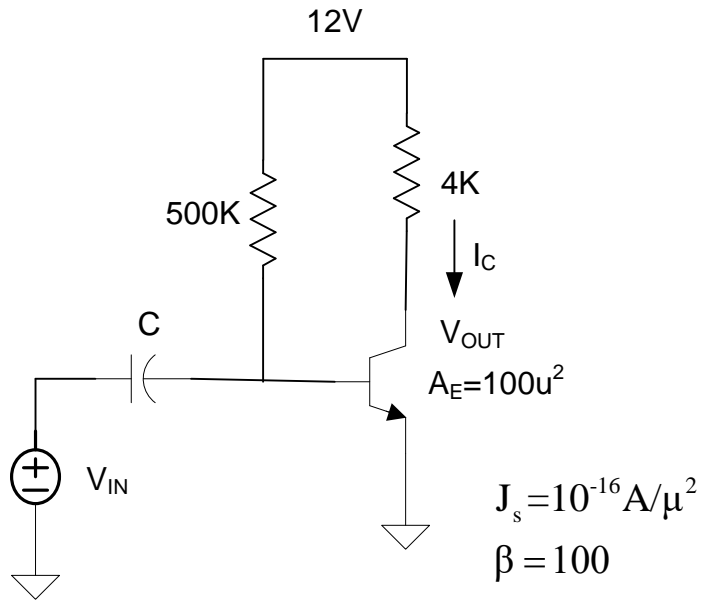
$$I_C = 2.95 \text{ mA} < \beta I_B = 22.8 \text{ mA}$$

Verify Passes so solution is valid

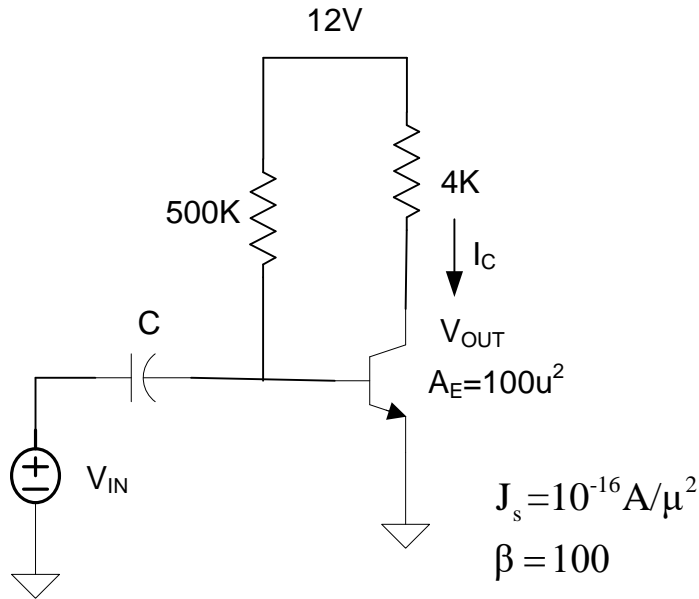
$$I_C = 2.95 \text{ mA}$$

$$V_{OUT} = 0.2 \text{ V}$$

Example: Determine I_C and V_{OUT} . Assume C is large and V_{IN} is very small.



Example: Determine I_C and V_{OUT} . Assume C is large and V_{IN} is very small.



Solution:

Assume $V_{IN} = 0$, then no current flows through C so circuit is identical to circuit of previous-previous example so

$$I_C = 2.28 \text{ mA} \quad V_{OUT} = 2.88 \text{ V}$$

Note: Since V_{IN} is coupled directly to base, will amplify V_{IN} if it is a small time varying signal and gain will be very large

End of Lecture 20