Stat 330 (Spring 2020): Homework 6 Due: March 4, 2020

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, student ID, the course number, and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. * Consider the following joint distribution for the weather in two consecutive days. Let X and Y be the random variables for the weather in the first and the second days, with the weather coded as 0 for sunny, 1 for cloudy, and 2 for rainy.

$X \backslash Y$	0	1	2
0	0.3	0.1	0.1
1	0.2	0.1	0
2	0.1	0.1	0

   (a) Find the marginal probability mass functions for X and Y.
   (b) Calculate the expectation and variance for X and Y.
   (c) Calculate the covariance and correlation between X and Y. Are they correlated?
   (d) Are the weather in two consecutive days independent?

2. * Using the joint distribution table given in problem 1, calculate the following probabilities:
   (a) $\mathbb{P}(X = Y)$
   (b) $\mathbb{P}(X < Y)$
   (c) $\mathbb{P}(X > Y)$
   (d) Probability that the weather is sunny on two consecutive days.
   (e) Probability that the weather is cloudy on the first day, and rainy on the second day.

3. Suppose a fair coin is tossed 3 times. Let X = the number of heads on the last toss, and let Y = the total number of heads in the 3 tosses.
   (a) Write down the joint PMF for X and Y in table form.
   (b) Give $p_X(x)$ and $p_Y(y)$ in table form.
   (c) Find $\mathbb{P}(Y = 1|X = 1)$.
   (d) Are X and Y independent? Explain your answer.

4. Suppose X and Y are two random variables and their joint pmf is given by this table:

$X \backslash Y$	2	3	4
1	1/12	1/6	0
2	1/6	0	1/3
3	1/12	1/6	0

   (a) Find the marginal probability mass functions for X and Y.
   (b) Show that X and Y are dependent.
   (c) Give a joint probability table (like we have above for X and Y) for random variables U and V that have the same marginal distributions as X and Y respectively *but are* independent.

5. * Suppose a continuous random variable X has the following probability density function

$$f_X(x) = \begin{cases} cx & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of c that makes $f_X(x)$ a valid probability density function. (Recall a property that a PDF must have)

(b) Give the CDF, $F_X(x)$.

(c) Find $\mathbb{P}(0.5 \leq X \leq 1.5)$ using $f_X(x)$.

(d) Find $\mathbb{P}(1 \leq X \leq 2)$ using $F_X(x)$.

(e) Find the value of x such that the probability of being less than x is .75

(f) Find $\mathbb{E}(X)$.

(g) Find $Var(X)$.

6. A supplier of kerosene has a 150-gallon tank that is filled at the beginning of each week. His weekly demand shows a relative frequency behavior that increases steadily up to 100 gallons and then levels off between 100 and 150 gallons. If X denotes weekly demand in hundreds of gallons, the relative frequency of demand can be modeled by

$$f_X(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 1 & 1 < x \leq 1.5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $F_X(x)$. (Remember to cover all cases)

(b) Find $\mathbb{P}(0.5 \leq X \leq 1.2)$.

(c) Find $\mathbb{E}(X)$.