Stat 330 (Spring 2020): Homework 9

Due: April 8, 2020

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. Every day, Eric takes the same street from his home to the university. There are 4 street lights along his way, and Eric has noticed the following Markov dependence. If he sees a green light at an intersection, then 60% of time the next light is also green, and 40% of time the next light is red. However, if he sees a red light, then 75% of time the next light is also red, and 25% of time the next light is green. Let 1 = "green light" and 2 = "red light" with the state space $\{1, 2\}$.
   (a) Construct the 1-step transition probability matrix for the street lights.
   (b) If the first light is red, what is the probability that the third light is red?
   (c) Eric's classmate Jacob has *many* street lights between his home and the university. If the *first* street light is red, what is the probability that the *last* street light is red? (Use the steady-state distribution.)

2. * We want to model the daily movement of a particular stock (say Amazon, ticker = AMZN) using a homogenous markov-chain. Suppose at the close of the market each day, the stock can end up higher or lower than the previous day's close. Assume that if the stock closes higher on a day, the probability that it closes higher the next day is 0.58. If the stock closes lower on a day, the probability that it closes higher the next day is 0.46.
   (a) What is the 1-step transition matrix? (Let 1 = higher, 2 = lower)
   (b) On Monday, the stock closed higher. What is the probability that it will close higher on Thursday (three days later)

3. * A Markov chain has 3 possible states: A, B, and C. Every hour, it makes a transition to a *different* state. From state A, transitions to states B and C are equally likely. From state B, transitions to states A and C are equally likely. From state C, it always makes a transition to state A.
   (a) Write down the transition probability matrix.
   (b) If the initial distribution for states A, B, and C is $P_0 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, find the distribution of state after 2 transitions, i.e., the distribution of X_2.
   (c) Show that this is a regular Markov Chain.
   (d) Find the steady-state distribution of states.

4. Every second in a hockey game, we recorded the possession status of a hockey puck where the possibilities are that team A possesses the puck, team B possesses the puck, or nobody possesses the puck (this is called a loose puck). Then a Markov chain model of the possession status of the puck is

$$P = \begin{array}{c} \\ A \\ B \\ L \end{array} \begin{array}{c} \begin{array}{ccc} A & B & L \end{array} \\ \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0.5 & 0.4 & 0.1 \end{pmatrix} \end{array}$$

   (a) What is the probability that team A retains possession of the puck in 1 second?
   (b) What is the probability that team B losses the puck to team A in 1 second?
   (c) Which team is better at picking up loose pucks? Why?
   (d) What is the probability that a loose puck is **stays** loose for 2 seconds, i.e. it was loose at 1 second and again at 2 seconds?
   (e) What is the probability that if a puck is loose now, that it will be loose after 2 seconds? (This probability is different than the previous.)

(f) Find the steady-state distribution of this Markov chain. Use a computer program that can do matrix multiplication to make things easier.

(g) At the end of the game, what is the expected proportion of time that team A will possess the puck? (Note: A hockey game has 3 20-minute periods for a total of 3600 seconds in the game).