**Spring 2020** **Statistics 330 - Section 2** **Exam III**
**(75 points)**

Please write your first and last name here:

**First Name** ______________________________ **Last Name** ______________________________

**Instructions:**

- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- The questions are roughly in the order in which the material is presented in class, so they are not necessarily ordered easiest to hardest.
- If you get stuck on one, it may be a good idea to move on and come back to that question at the end.
- You may use your prepared notes, a calculator, and tables only.

1. You arrive at a bus stop at 10:00, knowing that the bus will arrive at some time uniformly distributed between 10:00 and 10:30. Let $X$ = time you wait for the bus to arrive (in minutes). Thus, $X \sim \text{Unif}(0, 30)$

   (a) How many minutes do you expect to wait? (3 points)

   (b) What is the probability you that you have to wait longer than 10 minutes? (3 points)

   (c) If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes? (4 points)

   (d) Suppose you go grab a coffee right when you get to the stop at 10:00. What time do you need to be back at the stop, such that there is only a 10% chance you miss the bus? (3 points)

2. A *spike train*, commonly used to study neural activity, is a sequence of recorded times at which a neuron fires an action potential (spike). The time in between consecutive spikes is called the *interspike interval (ISI)*. Answer the questions below for an experiment in which the firing rate for a neuron is 10 per second.

   (a) Let $X$ represent a single *interspike interval (ISI)* having an exponential distribution. State the distribution of $X$ and give its parameter value(s). (2 points)

   (b) Give the expected value and variance for an *interspike interval (ISI)*. (4 points)

   (c) What is the probability that an ISI is less than 0.07 seconds? (4 points)

   (d) In the experiment, the *spike train* is composed of 50 ISIs. State the distribution for the total time for the *spike train* and give its parameter value(s). (4 points)

   (e) Give the expected value for the total time for the *spike train*. (2 points)

   (f) What is the probability that the *spike train* takes less than 3 sec? (4 points)

3. The amount of water filled in a particular brand of water bottle follows a normal distribution with a mean of 20 fl oz and variance of 0.01 fl oz$^2$.

   (a) What is the probability that the water bottle contains between 19.9 and 20.1 fluid oz of water? (4 points)

   (b) 10% of water bottles have less that what volume? (4 points)

   (c) A large case contains 40 individual water bottles. What is the probability that its *total* volume is more than 801 fl. oz? (4 points)

   (d) For the same case in the part (c), what is the probability that its *average* volume is less than 19.96 fl. oz? (5 points)

4. An Uber driver only provides service in city A and city B dropping off passengers and immediately picking up a new one at the same spot. He finds the following Markov dependence. For each trip, if the driver is in city A, the probability that he has to drive passengers to city B is 0.25. If he is in city B, the probability that he has to drive passengers to city A is 0.45.

   (a) What is the 1-step transition matrix? (Let 1 = City A and 2 = City B) (5 points)

   (b) Suppose he is in city B, what is the probability he will be in city A after two trips? (5 points)

   (c) After *many* trips between the two cities, what is the probability he will be in city B? (6 points)

5. A Markov chain with the states $\{1, 2, 3\}$ has the transition probability matrix

$$P = \begin{pmatrix} 0.3 & \phantom{0.3} & 0 \\ 0 & 0 & \phantom{0.3} \\ 1 & & \end{pmatrix}$$

(a) Fill in the blanks in the transition matrix above. (3 points)

(b) Assume that the Markov chain has the initial distribution, $P_0 = (0.5,\ 0.5,\ 0)$. Find the distribution of state after 2 transitions. (6 points)