Recitation 10

- Here is a set of additional problems. They range from being very easy to very tough. The best way to learn the material in 310 is to solve problems on your own.
- Feel free to ask (and answer) questions about this problem set on Piazza.
- This is an **optional** problem set; do not turn this in for grading.
- While you don't have to turn this in, be warned that this material **can** appear in a quiz or exam.

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1. Recall that the complete graph over n nodes is defined as the graph in which every pair of nodes are connected via an undirected edge.

   a. Find a recurrence relation for the number of edges, $e(n)$, in this graph.

   b. Previously, we used the first degree theorem to find a closed form expression for $e(n)$. This time, prove it via induction using your answer from part a.

2. A *Koch snowflake* is created by starting with an equilateral triangle with sides one unit in length. Then, on each side of the triangle, a new equilateral triangle is created on the middle third of that side. This process is repeated continuously, as shown in Figure 1 below.

   Prove that the number of sides (colored in black) for the n^{th} Koch snowflake is given by $3 \cdot 4^n$.

3. Let $f : \mathbb{N} \to \mathbb{N}$ such that $f(0) = 1$, $f(1) = 2$, and $f(a+b) = f(a)f(b)$ for all $a, b \in \mathbb{N}$. Prove via induction that
$$f(n) = 2^n.$$

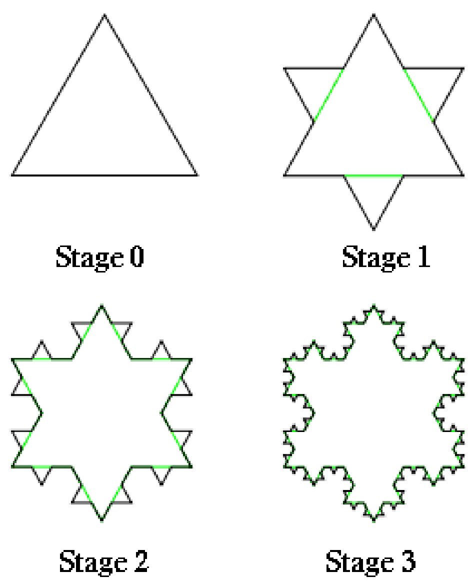

Figure 1: Koch snowflake