

Stat 330

Homework 3

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1)

$$\begin{aligned} \text{(a)} \quad P(B|A) + P(B|\bar{A}) &= 1 & \frac{P(B)P(A)}{P(B)} + \frac{P(B)P(\bar{A})}{P(B)} &= 1 \\ \frac{P(B)P(A) + P(B)P(\bar{A})}{P(B)} &= 1 & P(A) + P(\bar{A}) &= 1 \checkmark \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad P(\bar{A}|\bar{B}) &= 1 - P(A \cup B) & &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) & &= (1 - P(A))(1 - P(B)) \\ P(\bar{A}|\bar{B}) &= P(\bar{A})P(\bar{B}) \checkmark \end{aligned}$$

2)

x		-2	-1	0	1	2
(a, b)	$P_X(x)$	0.1	0.3	0.3	0.1	0.2
	$F_X(x)$	0.1	0.4	0.7	0.8	1.0

(c)

- i. $P(X \leq 1) \Rightarrow P(X = 1) + P(X = 2) = 0.1 + 0.2 = 0.3$
- ii. $P(-1 < X \leq 1) \Rightarrow P(X = 0) + P(X = 1) = 0.3 + 0.1 = 0.4$
- iii. $P(X < 0) \Rightarrow P(X = -2) + P(X = -1) = 0.1 + 0.3 = 0.4$

- (d) A test for independance is if $P(A|B) = P(A)$ and $P(B|A) = P(B)$
As the above tests hold, these two events are **independent**.

3)

(a) When drawing from the first urn, the chances are:

Red: $2/6$, White: $4/6$.

If a red is transferred, when drawing from urn 2 the chances are:

Red: $4/5$, **White: $1/5$** .

If a white is transferred, when drawing from urn 2 the chances are:

Red: $3/5$, **White: $2/5$** .

Drawing from urn 1 affects drawing from the second, and must be considered.

Thus, the cumulative chance of selecting white from urn 2 is:

$$\frac{2}{6} * \frac{1}{5} + \frac{4}{6} * \frac{2}{5} = \mathbf{1/3}$$

(b) The two events are not independent, as drawing from the first urn affects the chances of drawing from the second.

Using the values calculated above,

$$P(W2|R1) = \frac{2}{6} * \frac{1}{5} = 1/15$$

$$P(W2|W1) = \frac{4}{6} * \frac{2}{5} = 4/15$$

If the events were independent, the two resulting values would be equivalent.

4)

	$D(.15)$	$\bar{D}(.85)$	$Total$
Pos	.98	.10	?
Neg	?	?	?
$Total$?	?	

$$P(Pos \cap D) = (.98)(.15) = .147$$

$$P(Pos \cap \bar{D}) = (.10)(.85) = .085$$

$$(a) P(Pos) = .147 + .085 = \mathbf{.232}$$

$$(b) P(D|Pos) = .147 / .232 = \mathbf{.634}$$

$$(c) P(\bar{D}|Pos) = .085 / .232 = \mathbf{.366}$$

5) $P(Sys) = P(1st \cap 2nd)$

$$P(1st) = 1 - P(\overline{1st}) = 1 - P(\bar{1} \cap \bar{2} \cap \bar{3}) = 1 - (.3)(.3)(.3) = 0.973$$

$$P(2nd) = 1 - P(\overline{2nd}) = 1 - P(\bar{4} \cap \bar{5}) = 1 - (.3)(.3) = 0.91$$

$$P(1st \cap 2nd) = P(1st) * P(2nd) = 0.973 * 0.91 = \mathbf{0.885}$$

6)

(a) $P(\text{Sys}) = P(A) * P(B) = .95 * .9 = \mathbf{.855}$

(b) $P(\text{Sys}) = 1 - P(\overline{Top} \cap \overline{Bottom}) \Rightarrow$

$P(\text{Top}) = P(A) * P(B) = .95 * .9 = .855$

$P(\text{Bottom}) = P(C) * P(D) = .8 * .7 = .56$

$P(\text{Sys}) = 1 - (1 - .855)(1 - .56) = \mathbf{.9362}$

(c) $P(\text{Sys}) = 1 - P(\overline{A} \cap \overline{B} \cap \overline{C}) = 1 - (1 - .95)(1 - .9)(1 - .8) = \mathbf{.999}$