Practice problem solutions

1.) $\Omega = \{ \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{1,6\},$
$\{2,3\}, \{2,4\}, \{2,5\}, \{2,6\},$
$\{3,4\}, \{3,5\}, \{3,6\},$
$\{4,5\}, \{4,6\},$
$\{5,6\} \}$

$|\Omega| = 15$

$A =$ sum to 5 $\Rightarrow A = \{\{1,4\}, \{2,3\}\} \Rightarrow \mathbb{P}(A) = \frac{|A|}{|\Omega|} = \boxed{2/15}$

$B =$ Sum $= 4$ or 5

$B = \{ \{1,3\}, \{1,4\}, \{2,3\} \} \Rightarrow \mathbb{P}(B) = \frac{|B|}{|\Omega|} = \boxed{3/15}$

2.) $M =$ motherboard problems
$H =$ hard drive problems

$\mathbb{P}(M) = .4$, $\mathbb{P}(H) = .3$, $P(M \cap H) = .15$

	H	$\overline{H}$	
M	.15	.25	.4
$\overline{M}$	.15	.45	.6
	.3	.7	

a.) $\mathbb{P}(\overline{M} \cap \overline{H}) = \boxed{.45}$

b.) $\mathbb{P}(M \cup H) = \mathbb{P}(M) + \mathbb{P}(H) - \mathbb{P}(M \cap H) = .4 + .3 - .15 = .55$

3.) Let $T_i =$ Test i discovers error

$\mathbb{P}(T_1) = .2$, $\mathbb{P}(T_2) = .3$, $\mathbb{P}(T_3) = .5$

$\mathbb{P}(\text{At least one test discovers}) = 1 - \mathbb{P}(\text{None discover})$

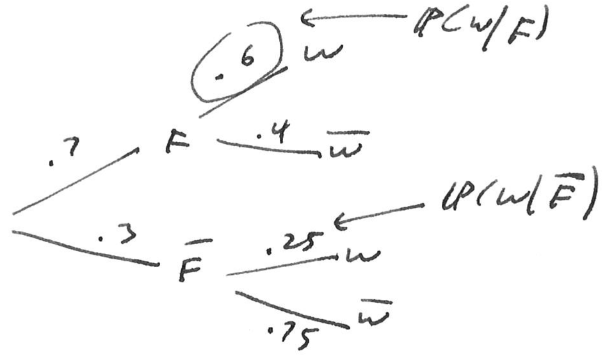
$= 1 - [\mathbb{P}(\overline{T}_1 \cap \overline{T}_2 \cap \overline{T}_3)]$

$= 1 - [\mathbb{P}(\overline{T}_1)\mathbb{P}(\overline{T}_2)\mathbb{P}(\overline{T}_3)]$ (b/c Independent)

$= 1 - (.8)(.7)(.5)$

$= \boxed{.72}$

4.) F = first string QB plays
W = ISU wins



$\mathbb{P}(F) = .7 \qquad \mathbb{P}(W/F) = .6$

$\mathbb{P}(\overline{F}) = .3 \qquad \mathbb{P}(W/\overline{F}) = .25$

$$\begin{aligned}\mathbb{P}(W) &= \mathbb{P}(F \cap W) + \mathbb{P}(\overline{F} \cap W) \\ &= \mathbb{P}(F)\mathbb{P}(W/F) + \mathbb{P}(\overline{F})\mathbb{P}(W/\overline{F}) \\ &= (.7)(.6) + (.3)(.25) \\ &= \boxed{.495}\end{aligned}$$

5.) $|\Omega|$ = # of ways to choose 6 laptops from 10 (unordered w/o Replacement $\Rightarrow$ combinations)

$|\Omega| = \binom{10}{6} = 210$

a.) $\mathbb{P}$(2 defective)?

def	good
5	5

$|A| = \binom{5}{2}\binom{5}{4} = 50 \Rightarrow \mathbb{P}(A) = \frac{|A|}{|\Omega|} = \frac{50}{210}$

b.) $\mathbb{P}$(2 def. | At least 2 def) $= \dfrac{\mathbb{P}(\text{2 def.} \cap \text{At least 2 def.})}{\mathbb{P}(\text{At least 2 def.})}$

$= \dfrac{\mathbb{P}(\text{2 def})}{\mathbb{P}(\text{At least 2 def})}$ (numerator is part a.)

$\mathbb{P}$(At least 2 defective) $= 1 - \mathbb{P}$(less than 2 def)

$= 1 - [\underbrace{\mathbb{P}(\text{0 def})}_{=0 \text{ (why?)}} + \mathbb{P}(\text{1 def})]$

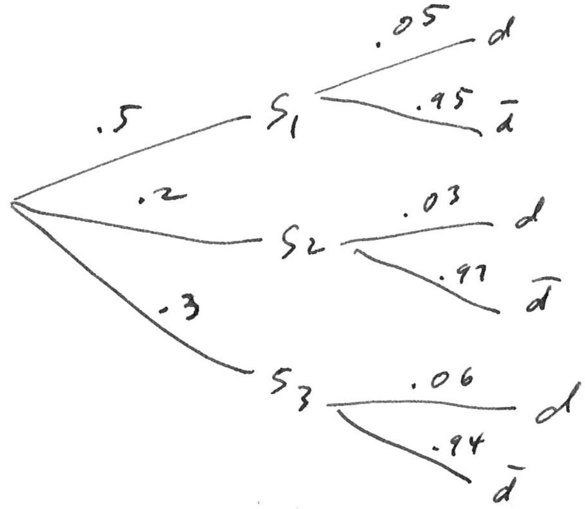
$= 1 - \left[\binom{5}{1}\binom{5}{5}/210\right] = .976$

So, we have

$$\frac{\frac{50}{210}}{.976} = \boxed{.244}$$

6.) $\mathbb{P}(S_1) = .5$, $\mathbb{P}(S_2) = .2$, $\mathbb{P}(S_3) = .3$ let d = defective part

$\mathbb{P}(d|S_1) = .05$, $\mathbb{P}(d|S_2) = .03$, $\mathbb{P}(d|S_3) = .06$



a.) $\mathbb{P}(d) = \mathbb{P}(S_1 \cap d) + \mathbb{P}(S_2 \cap d) + \mathbb{P}(S_3 \cap d)$

$= \mathbb{P}(S_1)\mathbb{P}(d|S_1) + \mathbb{P}(S_2)\mathbb{P}(d|S_2) + \mathbb{P}(S_3)\mathbb{P}(d|S_3)$

$= (.5)(.05) + (.2)(.03) + (.3)(.06)$

$= \boxed{.049}$

b.) $\mathbb{P}(S_1|d) = \dfrac{\mathbb{P}(S_1 \cap d)}{\mathbb{P}(d)} = \dfrac{\mathbb{P}(S_1)\mathbb{P}(d|S_1)}{\mathbb{P}(d)} = \dfrac{(.5)(.05)}{.049}$

$= \boxed{.51}$