EE 330 F12

HW 6 solution

Due 9/28/2012

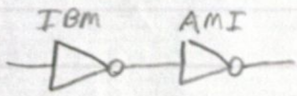
10pts 1. AMI .5$\mu$ CMOS Process

$$\frac{W}{L} = \frac{1}{R \times (\mu C_{ox}) \times (V_{GS} - V_t)} = \frac{1}{1K \times (57.8E{-}6 \cdot 2) \times (3.5 - .79)} = \boxed{3.19}$$

IBM 0.13$\mu$ CMOS Process

$$\frac{W}{L} = \frac{1}{1K \cdot (308E{-}6 \cdot 2) \cdot (1.5 - .41)} = \boxed{1.49}$$

5pts 2.




AMI: Min $\frac{W}{L} = \frac{3}{.6}$

$N_{+active}$ to Poly: 2434 aF/$\mu m^2$

$P_{+active}$ to Poly: 2351 aF/$\mu m^2$

$C_{Load} = 3 \cdot .6\,(2434\,aF + 2351\,aF)$

$= 8.613E{-}15$ F

IBM: Min $W/L = .16/.12$

$$R_{swn} = \frac{1}{W/L \times (\mu C_{ox}) \times (V_{GS} - V_t)} = \frac{1}{.16/.12 \cdot 308E{-}6 \cdot 2 \cdot (1.5 - .41)} = 1{,}117\,\Omega$$

$$R_{swp} = \frac{1}{.16/.12 \cdot 48.8E{-}6 \cdot 2 \cdot (1.5 - .42)} = 7{,}115.21$$

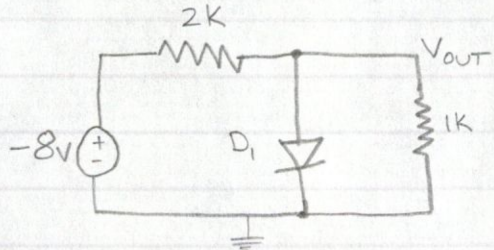
$t_{HL} = R_{swn} \times C_{load} = \boxed{9.621\,ps}$ $\quad t_{LH} = R_{swp} \cdot C_{load} = \boxed{61.3\,ps}$

5pts 3. $$R_{onsc} = \frac{1}{\frac{\theta_2}{\theta_1} \mu C_{ox} (V_{gs} - V_t)^{\alpha/2}} = \frac{1}{1/1 \cdot 308E{-}6 \cdot 2 \cdot (1.5 - .41)^{.23}} = \boxed{1538\,\Omega}$$

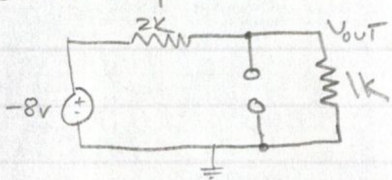
$$R_{on1c} = \frac{1}{\frac{1}{1} \cdot 308E{-}6 \cdot 2 \cdot (1.5 - .41)} = \boxed{1{,}489\,\Omega}$$

4. (a)

10 points




Guess $D_1 \Rightarrow$ OFF

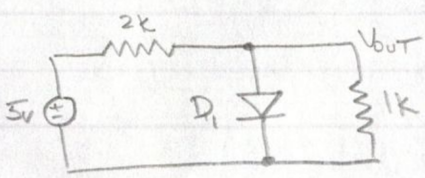



$$V_{OUT} = (-8V)\left(\frac{1K}{1K+2K}\right)$$

$$\boxed{V_{OUT} = -2.667V}$$

Check: $V_{D_1} = V_{OUT} = -2.667 < 0.6$ ✓ $D_1 \Rightarrow$ OFF ← correct Guess

(b)




$$\frac{5 - V_{OUT}}{2k} = I_D + \frac{V_{OUT}}{1K}$$

let $I_s = (50\mu m^2)\left(10^{-15}\frac{A}{\mu m^2}\right)$

$$I_D = I_s\left(e^{\frac{V_D}{V_t}} - 1\right) = (50)(10^{-15})\left(e^{\frac{V_D}{26mV}} - 1\right)$$

$V_D = V_{OUT}$

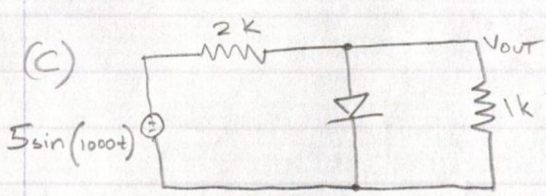
$$-V_{OUT}\left(\frac{1}{2k} + \frac{1}{1K}\right) + \frac{5}{2K} = (50)(10^{-15})\left(e^{\frac{V_{OUT}}{26mV}} - 1\right)$$

$$V_{OUT} = 0.628V$$

$$I_{R_2} = \frac{V_{OUT}}{1K}$$

$$\boxed{I_{R_2} = 0.628mA}$$

(c)

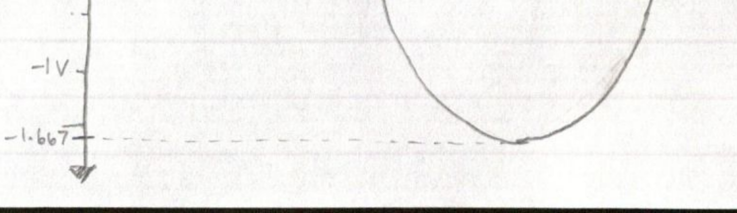



*use 0.6V model for Diode

for $V_{OUT} < 0.6V \Rightarrow$ Reverse bias → Part a

$V_{OUT} > 0.6V \Rightarrow$ forward bias → Part b

for $V_{OUT} = \begin{cases} 0.6V & V_{in} \geq 1.8V \\ \frac{1}{3}V_{in} & V_{in} < 1.8V \end{cases}$




$$t(PA) = \frac{1}{1000}\sin^{-1}\left(0.6 \cdot \frac{3}{5}\right)$$

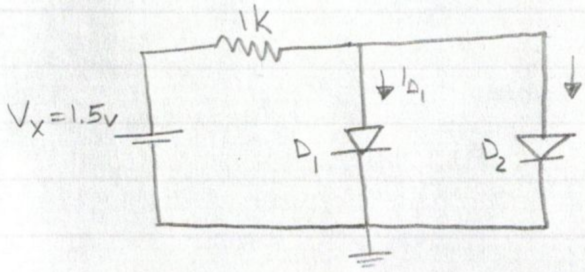
$$\boxed{= 0.368ms}$$

$$t(PB) = \frac{\left(\frac{2\pi}{1000}\right)}{2} - t(PA)$$

$$\boxed{t(PB) = 2.77ms}$$

5

10 points




$A_{D_1} = 100 \mu m^2$

$A_{D_2} = 4 \cdot A_{D_1} = 400 \mu m^2$

$J_s = 5 \frac{fA}{\mu m^2}$

$V_{D_1} = V_{D_2}$

$I_s = I_{D_1} + I_{D_2}$

$$\frac{1.5 - V_{D_1}}{1K} = (A_{D_1})(J_s)\left(e^{\frac{V_{D_1}}{V_T}} - 1\right) + 4 \cdot (A_{D_1})(J_s)\left(e^{\frac{V_{D_1}}{V_T}} - 1\right)$$

$$\frac{1.5 - V_{D_1}}{1K} = 5\left(100 \mu m^2\right)\left(5 \frac{fA}{\mu m^2}\right)\left(e^{\frac{V_{D_1}}{26mV}} - 1\right)$$

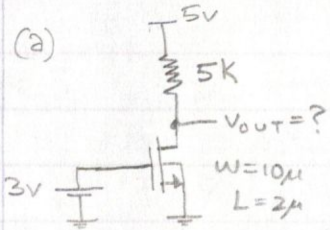
$V_{D_1} = 0.5146\ v$

then $I_{D_1} = (100 \mu m^2)\left(5 \frac{fA}{\mu m^2}\right)\left(e^{\frac{0.5146}{0.026}} - 1\right)$

$$\boxed{I_{D_1} = 0.1971 mA}$$

6 (a)

10 points




$V_{TN} = 1V$

$\mu_n C_{ox} = 100 \frac{\mu A}{V^2}$

Guess triode

$V_{GS} \geq V_T$ $\quad V_{DS} < V_{GS} - V_T$

$(V_{GS} = 3v) \geq 1$ ✓ $\quad V_{DS} < (3 - 1)$

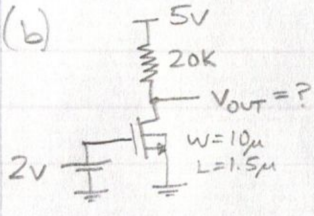
$I_D = \mu_n C_{ox} \cdot \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right) \cdot V_{DS}$

$V_{DS} = V_{OUT}$

$$\frac{5 - V_{DS}}{5K} = \left(100 \frac{\mu A}{V^2}\right)\left(\frac{10}{2}\right)\left(3 - 1 - \frac{V_{DS}}{2}\right) \cdot V_{DS}$$

$\boxed{V_{OUT} = 1.07335\ v}$ triode ✓

(b)




Guess triode

$V_{GS} \geq V_T$ $\quad V_{DS} < V_{GS} - V_T$

$2v \geq 1v$ ✓ $\quad (V_{DS} = V_{OUT}) < (2 - 1)$

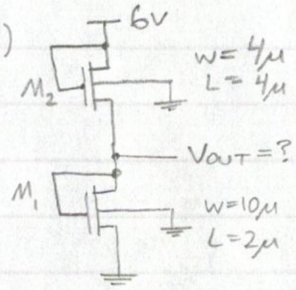
$V_{DS} = V_{OUT}$

$$I_D = \frac{5 - V_{DS}}{20K} = \left(100 \frac{\mu A}{V^2}\right)\left(\frac{10}{1.5}\right)\left(2 - 1 - \frac{V_{DS}}{2}\right) \cdot V_{DS}$$

$V_{DS} = \boxed{V_{OUT} = 0.43811\ v}$ triode ✓

7 (a)

10 points




$V_{TN} = 1 \quad \mu_n C_{ox} = 100 \frac{\mu A}{V^2}$

SATURATION since $V_{DS} = V_{GS}$ & $V_{TN} = 1$

↳ for Both M1 + M2

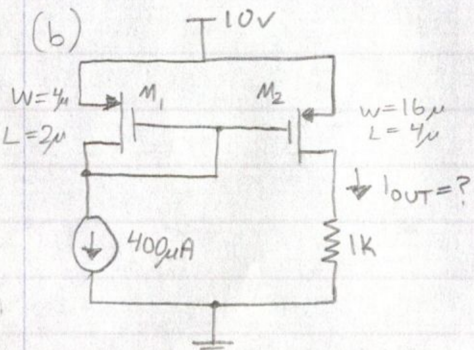
$I_{D_1} = I_{D_2}$

$$\cancel{\mu_n C_{ox}} \frac{W_2}{2 \cdot L_2} \cdot (V_{GS_2} - V_T)^2 = \cancel{\mu_n C_{ox}} \frac{W_1}{2L_1} \cdot (V_{GS_1} - V_T)^2$$

$$\frac{4}{2 \cdot 4} \left( (6 - V_{OUT}) - 1 \right)^2 = \frac{10}{2 \cdot 2} \cdot (V_{OUT} - 1)^2$$

$$\boxed{V_{OUT} = 2.236 \, [V]}$$

(b)




Guess saturation $\quad \mu_p C_{ox} = 33 \frac{\mu A}{V^2} \quad V_{Tp} = -1V$

$$I_{D_1} = 400\mu A = \mu_p C_{ox} \frac{W_1}{2L_1} \cdot (V_{GS_1} - V_{Tp})^2$$

$$400\mu A = \left(33 \frac{\mu A}{V^2}\right)\left(\frac{4}{2 \cdot 2}\right)(V_{GS_1} + 1)^2$$

$$V_{GS_1} = -4.48155$$

$V_{GS_1} = V_{GS_2}$

$$I_{D_2} = 33 \frac{\mu A}{V^2} \left(\frac{16}{2 \cdot 4}\right)(-4.48155 + 1)^2$$

$$\boxed{I_{D_2} = 800\mu A}$$

check saturation

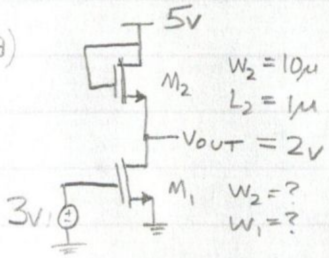
$V_{DS} = (800\mu A \cdot 1K) - 10 = -9.2$

$V_{GS} = -4.48155 \, V$

$(-9.2) < (-4.48 + 1)$ ✓ saturation

8 (a)

15 points




$V_{T_N} = 1V$ $\quad \mu_n C_{ox} = 100 \frac{\mu A}{V^2}$

saturation for $M_1 + M_2$

$$I_{D_1} = I_{D_2}$$

$$\cancel{\mu_n C_{ox}} \frac{W_2}{2 \cdot L_2} (V_{GS_2} - V_T)^2 = \cancel{\mu_n C_{ox}} \frac{W_1}{2 \cdot L_1} (V_{GS_1} - V_T)^2$$

$$\frac{10}{2 \cdot 1} ((5-2)-1)^2 = \frac{W_1}{L_1} \cdot \left(\frac{1}{2}\right) \cdot (3-1)^2$$

$$\boxed{\frac{W_1}{L_1} = 10} \quad \text{pick} \quad \boxed{\begin{array}{l} W_1 = 10\mu \\ L_1 = 1\mu \end{array}}$$

(b) for $V_{OUT} = 0.5V$

$M_2 \Rightarrow$ saturation

$M_1 \Rightarrow$ triode $\qquad 0.5 < 3-1$

$$I_{D_1} = I_{D_2}$$

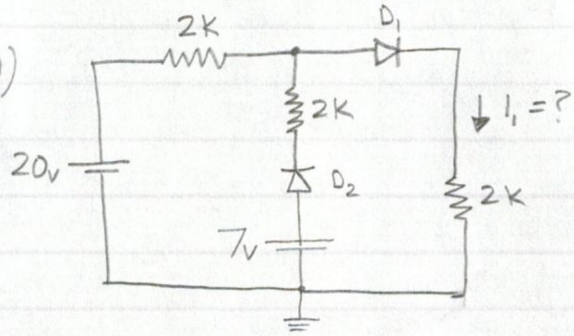
$$\cancel{\mu C_{ox}} \cdot \frac{W_2}{2 \cdot L_2} \cdot (V_{GS_2} - V_T)^2 = \cancel{\mu C_{ox}} \cdot \frac{W_1}{L_1} \left(V_{GS_1} - V_T - \frac{V_{DS_1}}{2}\right) \cdot V_{DS_1}$$

$$\frac{10}{2 \cdot 1} \cdot ((5-0.5)-1)^2 = \frac{W_1}{L_1} \cdot \left(3 - 1 - \frac{0.5}{2}\right) \cdot 0.5$$

$$\boxed{\frac{W_1}{L_1} = 70} \quad \text{pick} \quad \boxed{\begin{array}{l} W_1 = 70\mu \\ L_1 = 1\mu \end{array}}$$

$\boxed{9}$ (a)

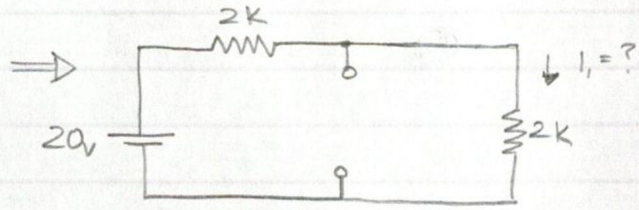
10 points




Guess $D_1 \Rightarrow$ ON ✓ (+ current) (ideal = short)

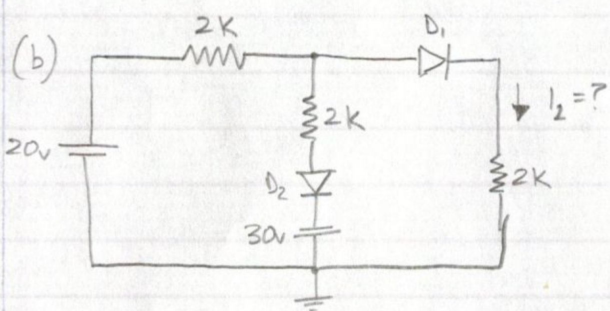
$D_2 \Rightarrow$ OFF ✓ (− voltage) (ideal = open)

$\Rightarrow$




$$I_1 = \frac{20\text{v}}{4\text{K}\Omega}$$ $$\boxed{I_1 = 5\text{mA}}$$

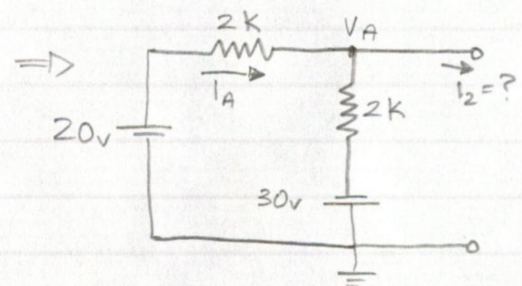
(b)




Guess $D_1 \Rightarrow$ OFF (−voltage) ideal = open

$D_2 \Rightarrow$ ON (+current) ideal = short

$\Rightarrow$




$$-20 + 2\text{k} \cdot I_A + 2\text{k} \cdot I_A - 30 = 0 \qquad I_A = \frac{50\text{v}}{4\text{K}\Omega} = 12.5\text{mA}$$

$$V_A = 20 - 2\text{k} \cdot I_A = -5\text{v} \qquad V_A = -5\text{v}$$

since the current through $D_2$ is positive (forward bias)
and the voltage accross $D_1$ is negative (reverse bias)
the current $\boxed{I_2 = 0\ \text{A}}$

10pts 10. $I = \frac{1}{2} \mu_n C_{ox} \cdot \frac{W}{L} (V_{GS} - V_{tn})^2 = \frac{1}{2} \mu_p C_{ox} \cdot \frac{W}{L} (V_{GS} - V_{tp})^2$

assuming saturation

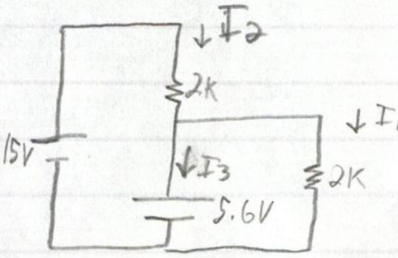
$I = \frac{1}{2} \cdot 100E\text{-}6 \cdot \frac{1}{1} (V_{out} - 1)^2 = \frac{1}{2} \cdot 33E\text{-}6 \cdot \frac{1}{1} (-2 + 1)^2$

$V_{out} = 1.57\ V$

Saturation confirmed

# Extra Credit

+10pts 11. $I_1 = ?$ Assume Forward Bias



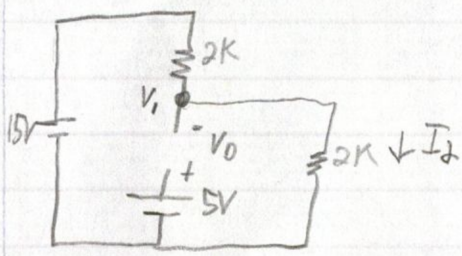

$I_1 = 5.6/2K = \boxed{2.8mA}$

$I_2 = (15-5)/2K = 5mA$

$I_3 = I_2 - I_1 = 2.2mA$

Since $I_3$ is positive, it confirms the diode is in forward bias

$I_2 = ?$ Assume Reverse Bias




$I_2 = 15/4K = 3.75mA$

$V_1 = 15/2 = 7.5V$

$V_D = 5 - 7.5 = -2.5 < 0$ ∴ confirms reverse bias

+10pts 12. $I = [J_{sx} \cdot A\,(T^m e^{-V_{G0}/V_t})] \cdot e^{V_D/V_t}$

$J_{sx} = .5$ $V_{g0} = 1.17$ $m = 2.3$ $A = 100$ $V_D = .575V$

$V_t = K \cdot T/q = 1.38E{-23} \cdot T/1.602E{-19}$

a) $T = -20°C = 253.15K$ $V_t = .0218V$

$I = 2.36E{-5}A$ $V_{out} = I \cdot 10K + .575 = \boxed{.811V}$

b) $T = 40°C = 313.15K$ $V_t = .0270V$

$I = 7.390E{-3}A$ This current would create a voltage greater than the bias voltage ∴ $\boxed{V_{out} = 20V}$

c) $T = 120°C = 393.15K$ $V_t = .0339V$

$I = 1.106A$ This current again would create a voltage greater than the bias voltage ∴ $\boxed{V_{out} = 20V}$

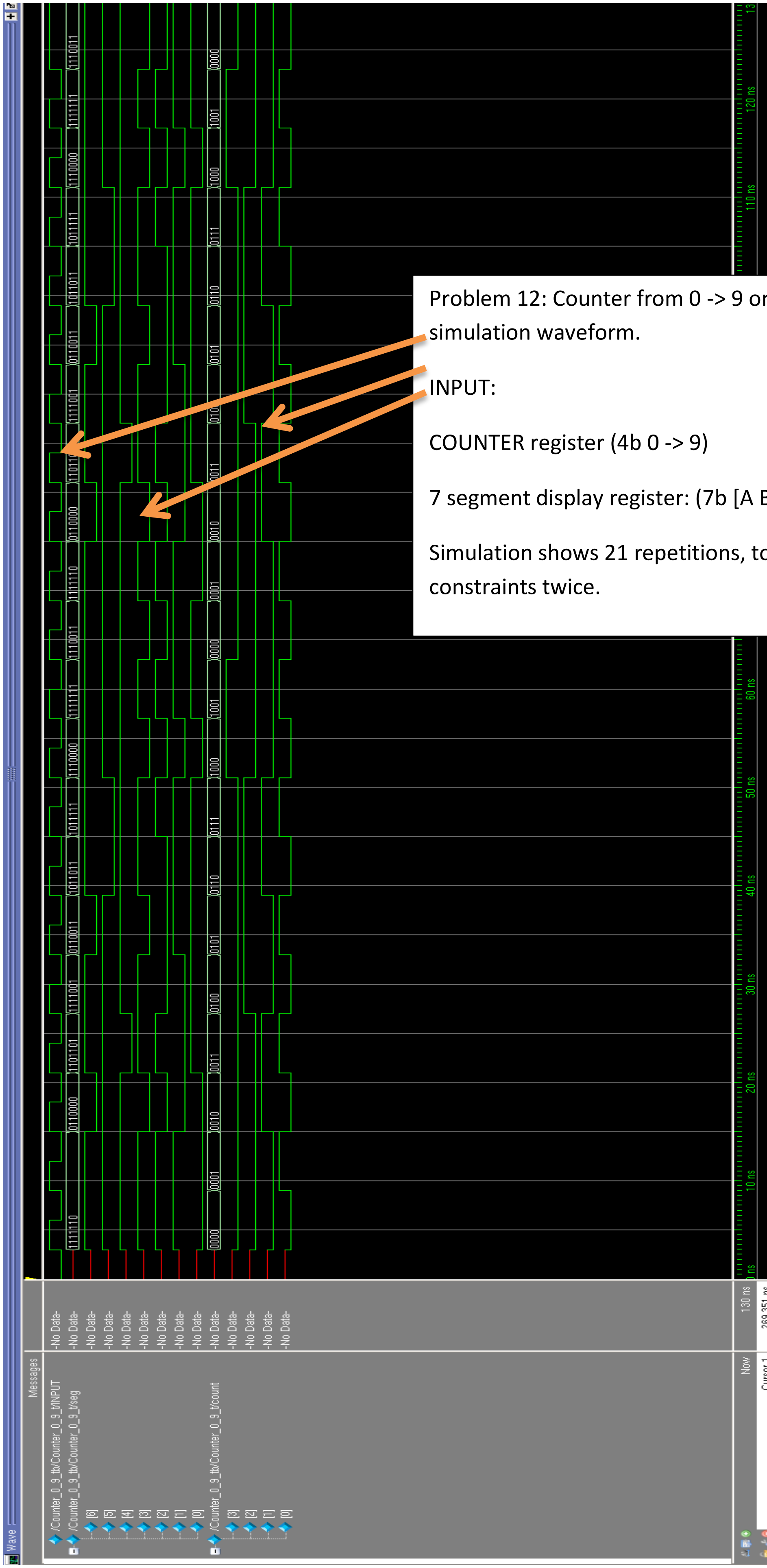
Extra Credit – Problem 13 (20 Points)

```
module Counter_0_9(INPUT, count, seg);
  input INPUT;
  output reg [6:0]seg;//7 segment display register [A B C D E F G]
  output reg [3:0]count;

  always @(posedge(INPUT))//always on rising edge of the clock (INPUT)
  begin
    case (count)//increase the counter
      4'b0000: count<=4'b0001;//0 -> 1
      4'b0001: count<=4'b0010;//1 -> 2
      4'b0010: count<=4'b0011;//2 -> 3
      4'b0011: count<=4'b0100;//3 -> 4
      4'b0100: count<=4'b0101;//4 -> 5
      4'b0101: count<=4'b0110;//5 -> 6
      4'b0110: count<=4'b0111;//6 -> 7
      4'b0111: count<=4'b1000;//7 -> 8
      4'b1000: count<=4'b1001;//8 -> 9
      4'b1001: count<=4'b0000;//9 -> 0
      default: count<=4'b0000; //default value of 0
    endcase
    case(count)//create the 7_seg output
      4'b0000: seg <= 7'b1111110;//display 0
      4'b0001: seg <= 7'b0110000;//display 1
      4'b0010: seg <= 7'b1101101;//display 2
      4'b0011: seg <= 7'b1111001;//display 3
      4'b0100: seg <= 7'b0110011;//display 4
      4'b0101: seg <= 7'b1011011;//display 5
      4'b0110: seg <= 7'b1011111;//display 6
      4'b0111: seg <= 7'b1110000;//display 7
      4'b1000: seg <= 7'b1111111;//display 8
      4'b1001: seg <= 7'b1110011;//display 9
      default: seg <= 7'b1111110;//defaults to a display of 0
    endcase
  end
endmodule
```

**Figure 1: Problem 13: 7 segement counter from 0 -> 9: code**

```
`timescale 1ns / 1ps

module Counter_0_9_tb();
 reg clk_t;//INPUT
 wire [6:0]out_t;//7 segment display register
 wire [3:0]count_t;//counter register

  Counter_0_9 Counter_0_9_t(clk_t, count_t, out_t);

  initial
  begin
    repeat(21) begin
      clk_t<=0;
      #1 $display("Counter = %b", count_t);
      #2 $display("7 seg   = %b", out_t);

      clk_t<=1;
      #1 $display("Counter = %b", count_t);
      #2 $display("7 seg   = %b", out_t);
    end
  end
endmodule
```

**Figure 2: 7 segment counter from 0 to 9 testbench code**



Problem 12: Counter from 0 -> 9 on a 7 segment display simulation waveform.

INPUT:

COUNTER register (4b 0 -> 9)

7 segment display register: (7b [A B C D E F G] of segment)

Simulation shows 21 repetitions, to show the 0->9->0 constraints twice.