Bayes Nets: Construction and Conditional Independence

Outline

I. Construction of Bayes nets

II. Conditionally independent relations

III. Efficient representations of conditional distributions

* Figures are either from the textbook site or by the instructor.

* A few slides are based on lecture notes by Dr. Jin Tian.

I. Construction of the Bayesian Network

$$\boldsymbol{P}(X_i \mid X_{i-1}, \dots, X_1) = \boldsymbol{P}(X_i \mid \mathit{Parents}(X_i)) \quad \text{for } i = 2, \dots, n$$

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   c) Write down the conditional probability table (CPT), $\boldsymbol{P}(X_i \mid \mathit{Parents}(X_i))$.

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Chosen order: *MaryCalls*, *JohnCalls*, *Alarm*, *Burglary*, *Earthquake*.

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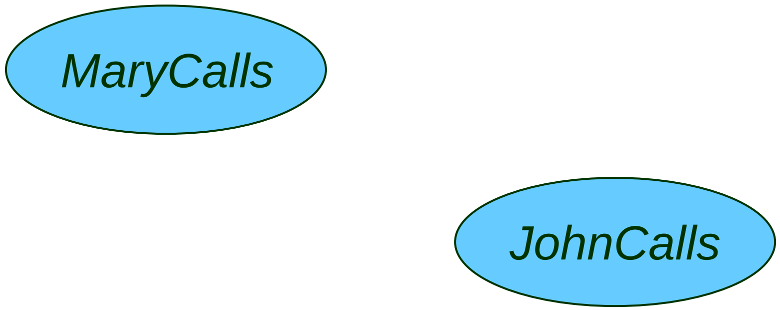

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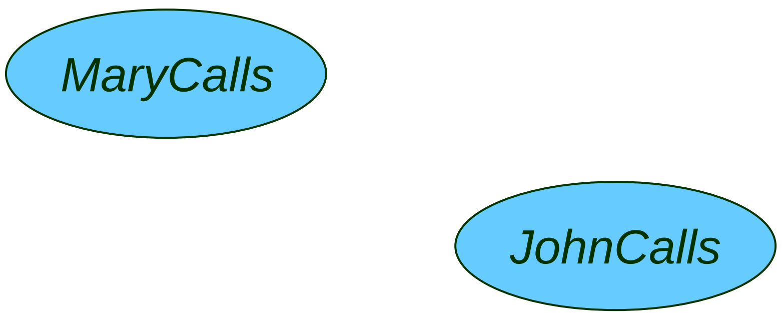

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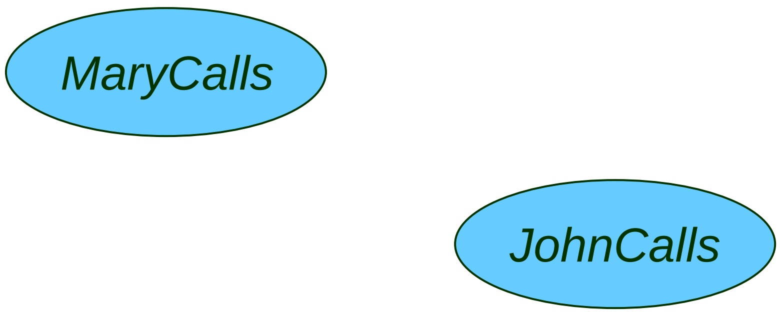

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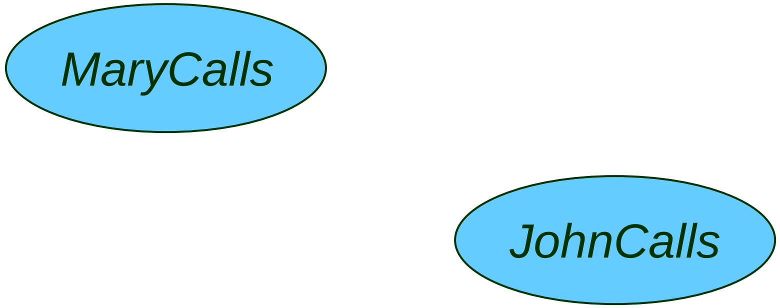

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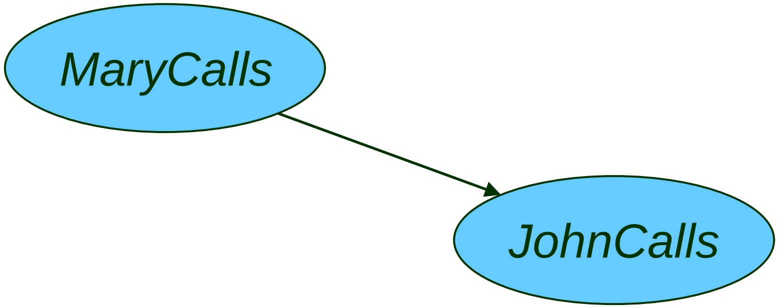

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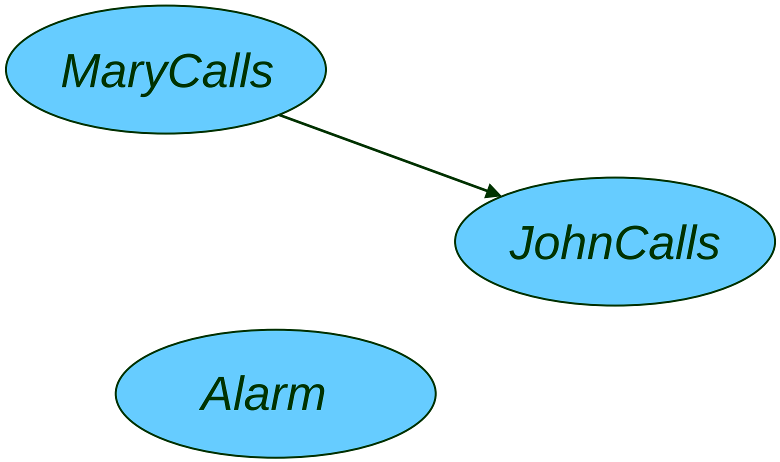

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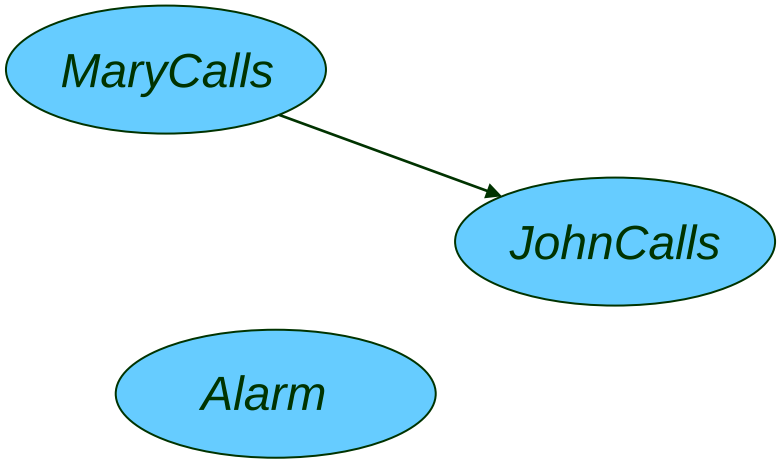

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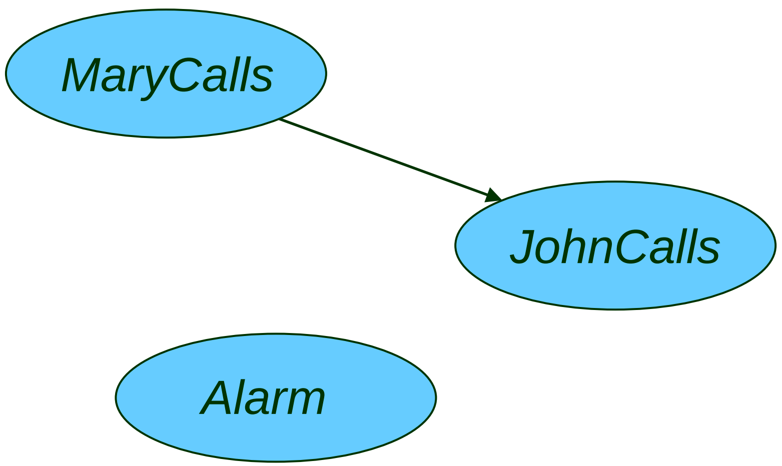

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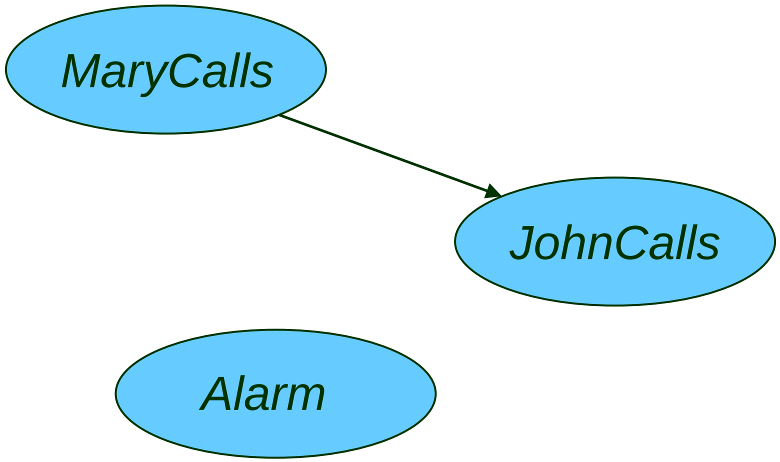

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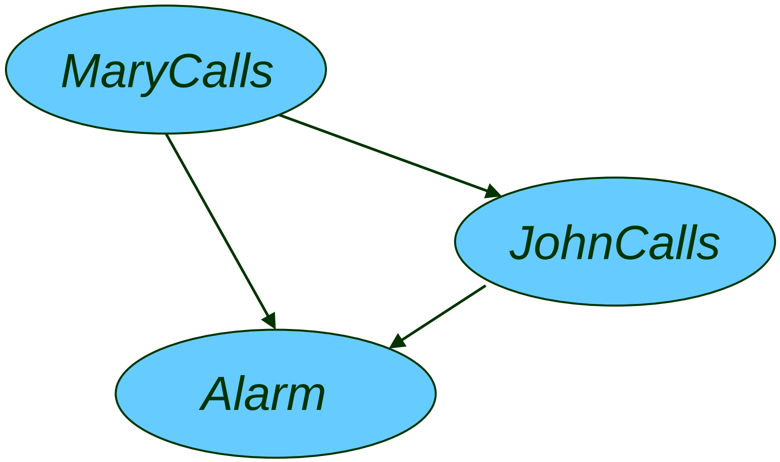

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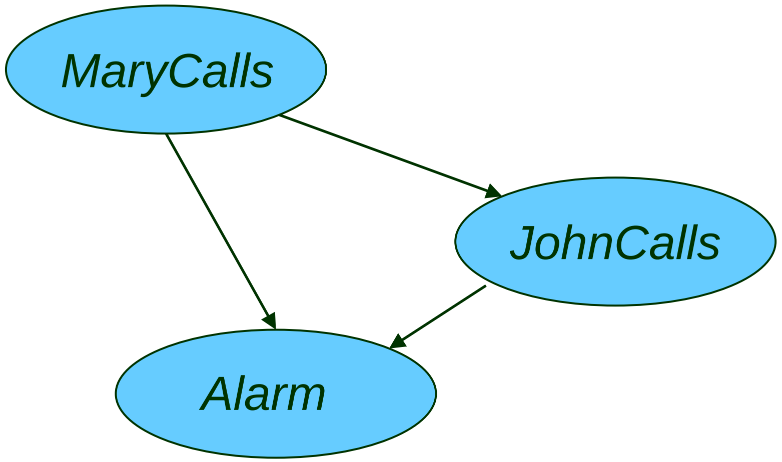

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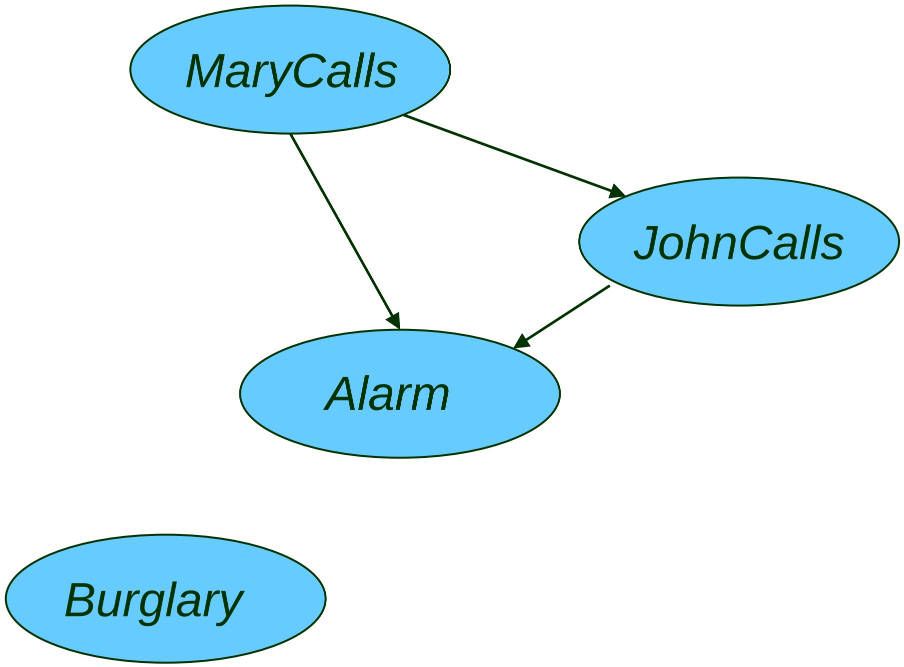

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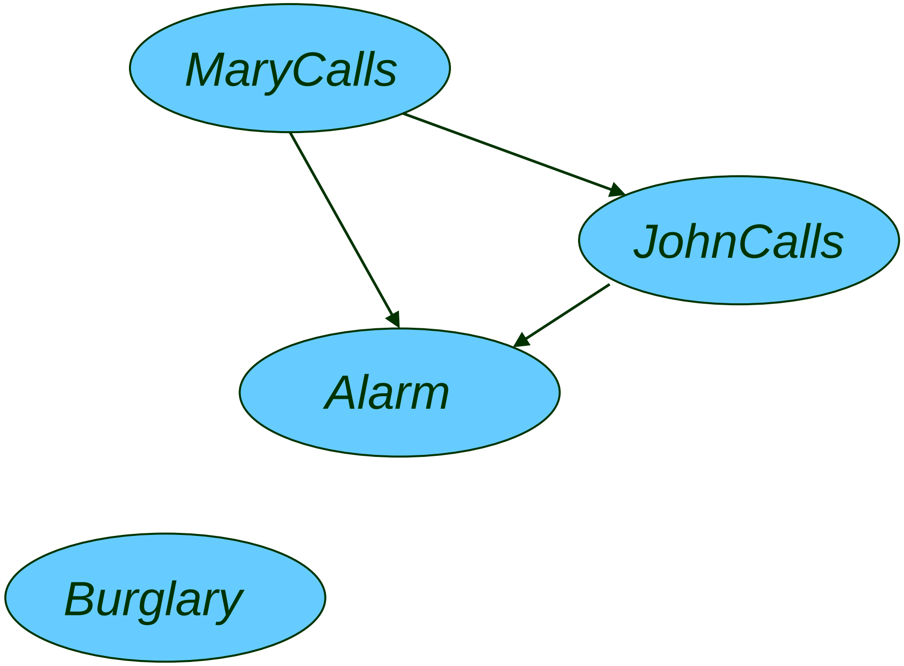

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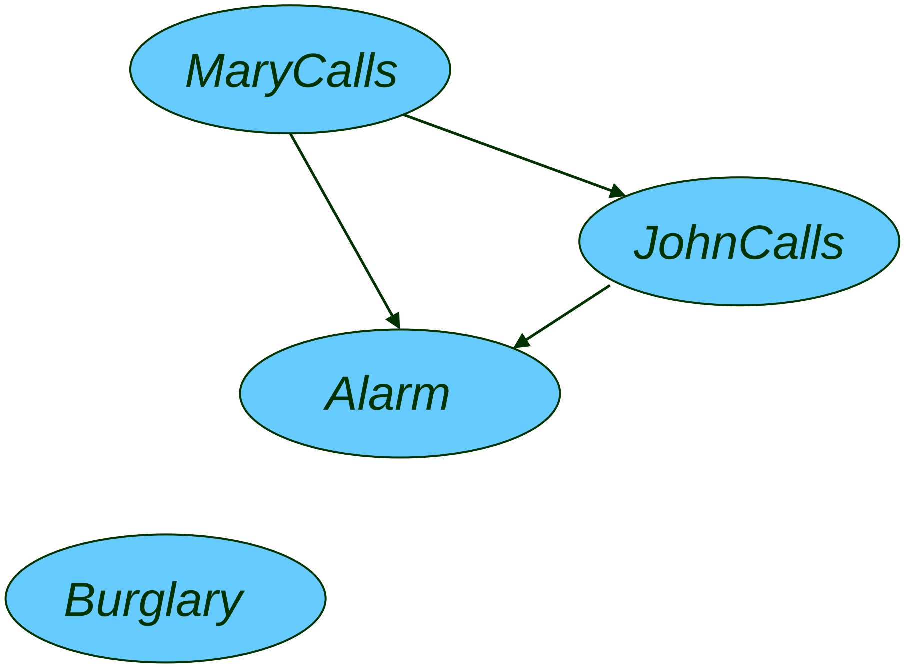

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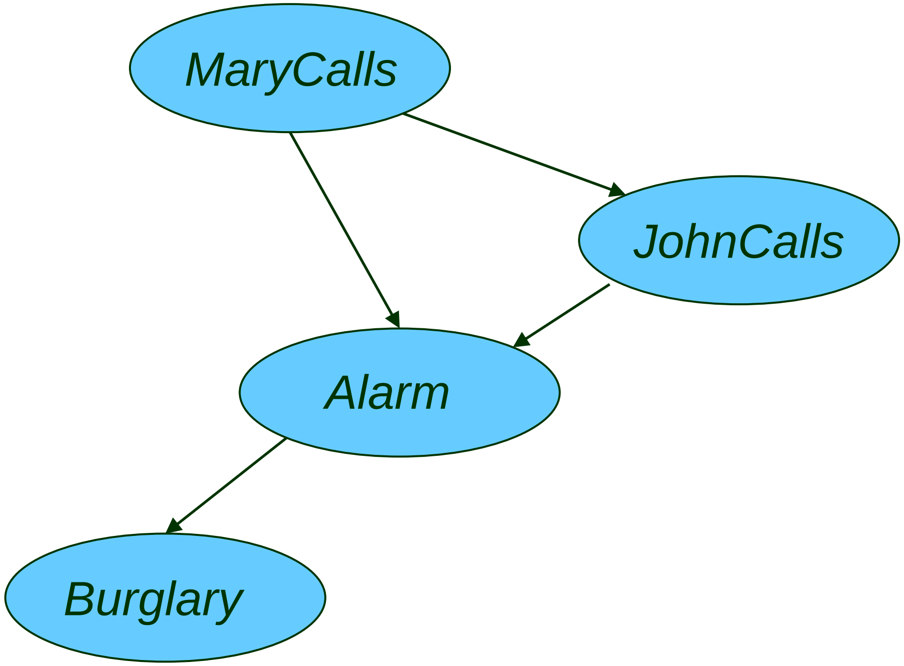

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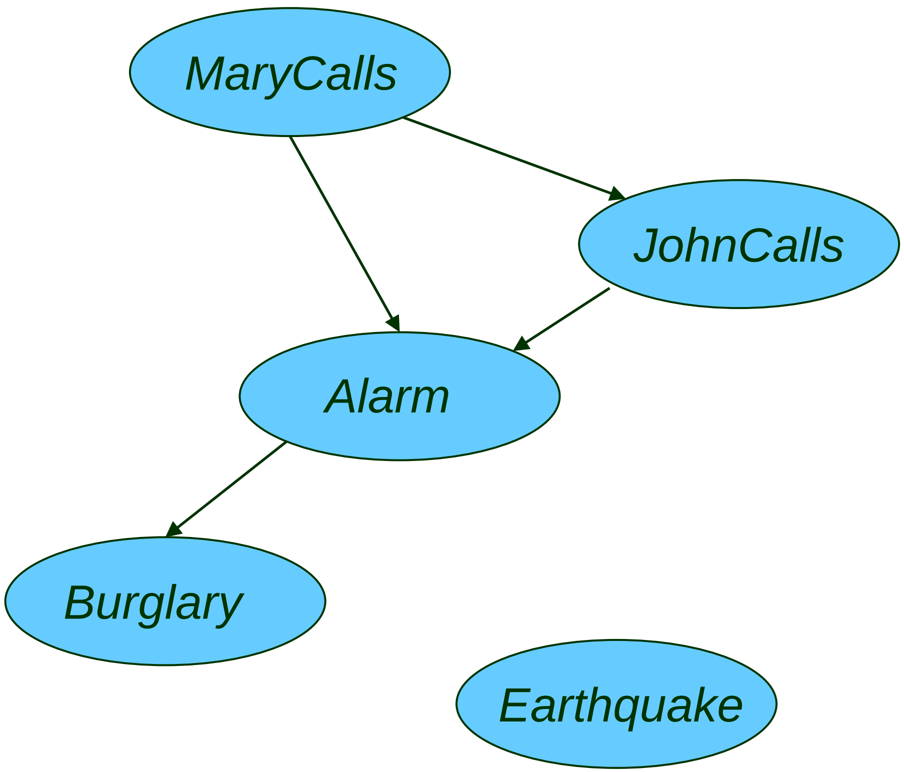

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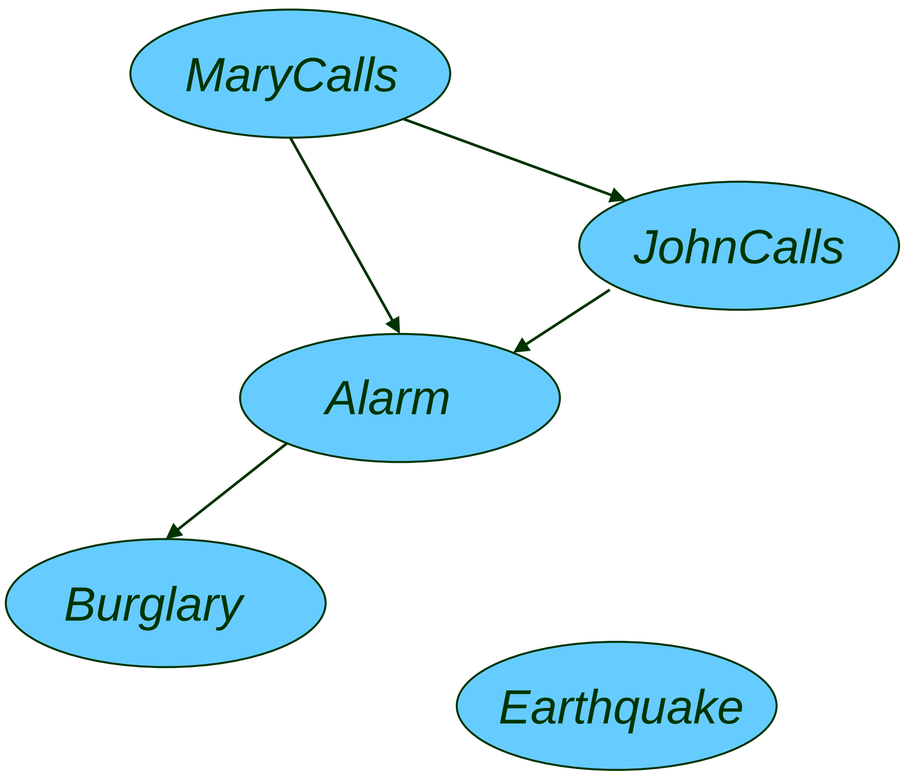

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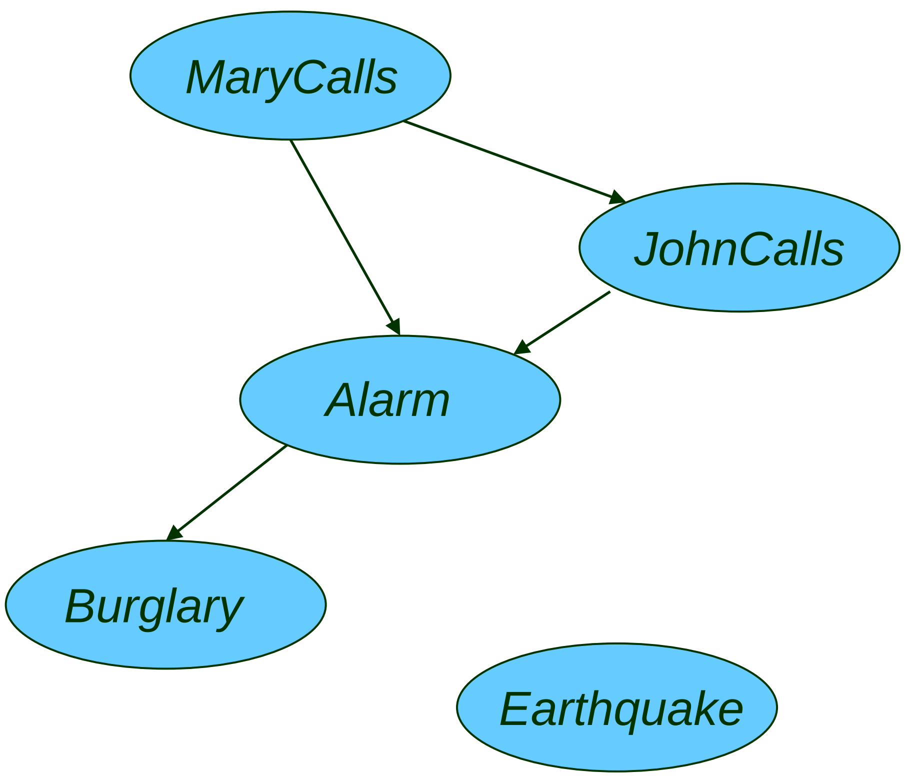

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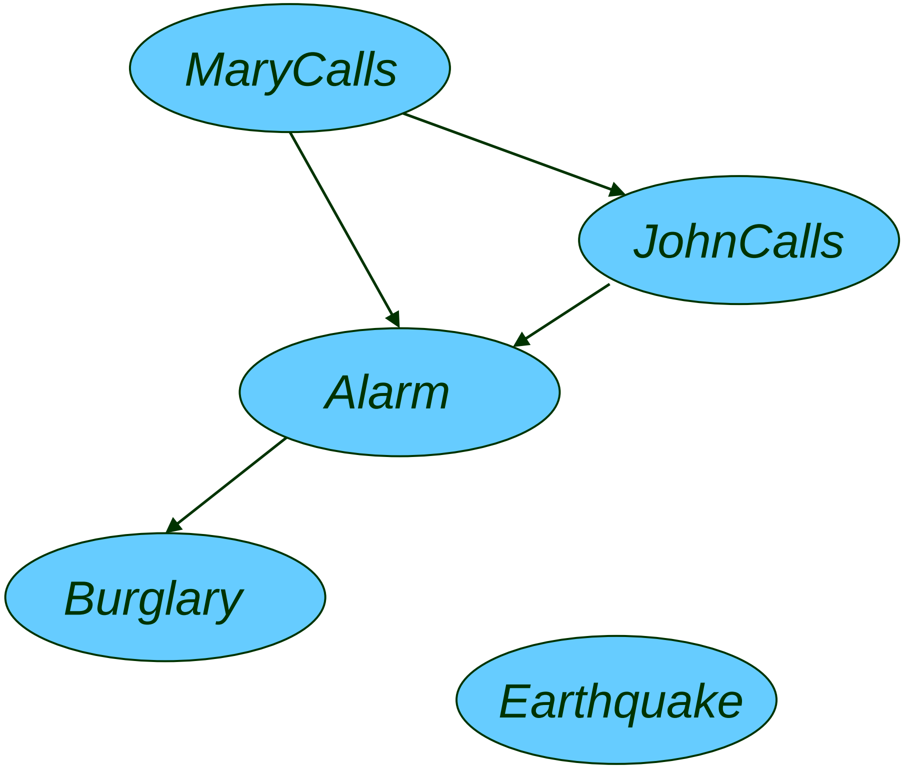

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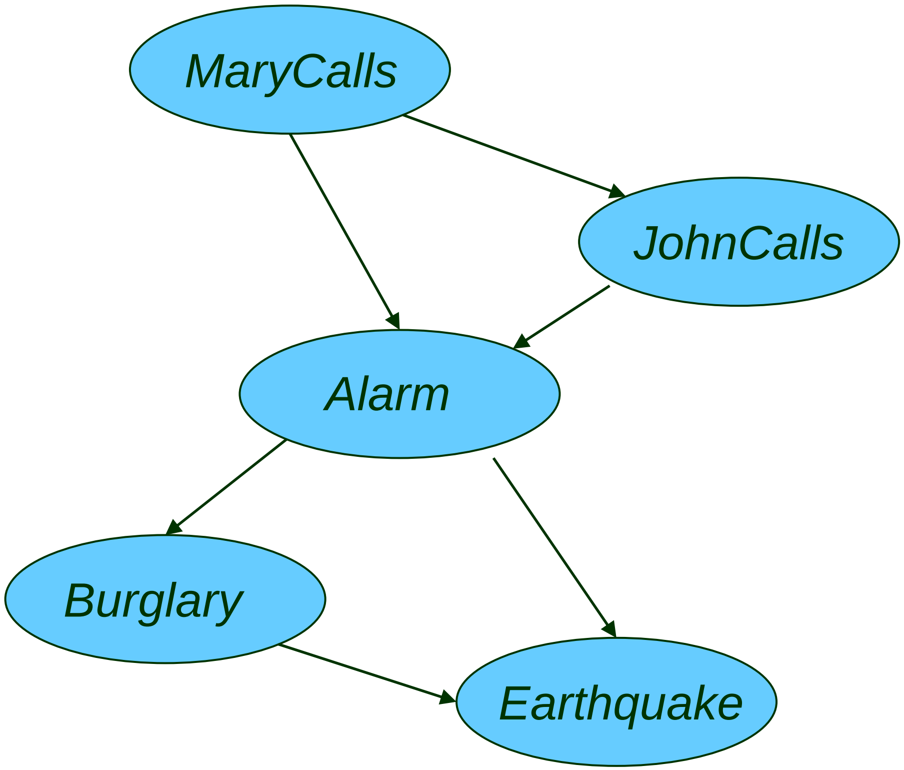

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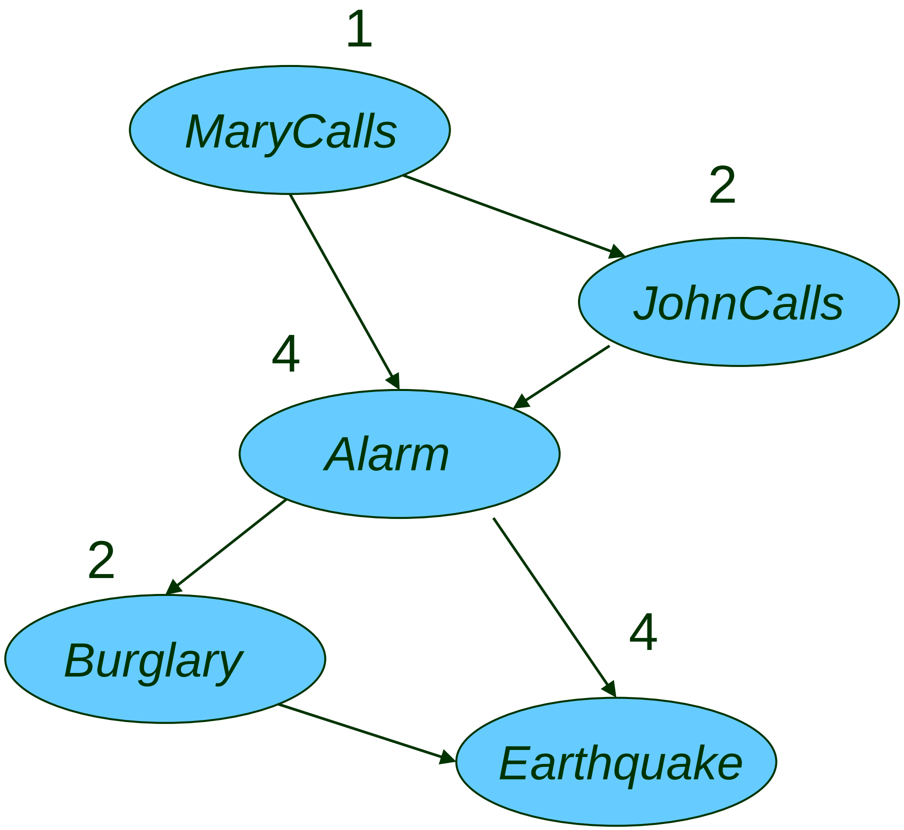

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conditional probabilities

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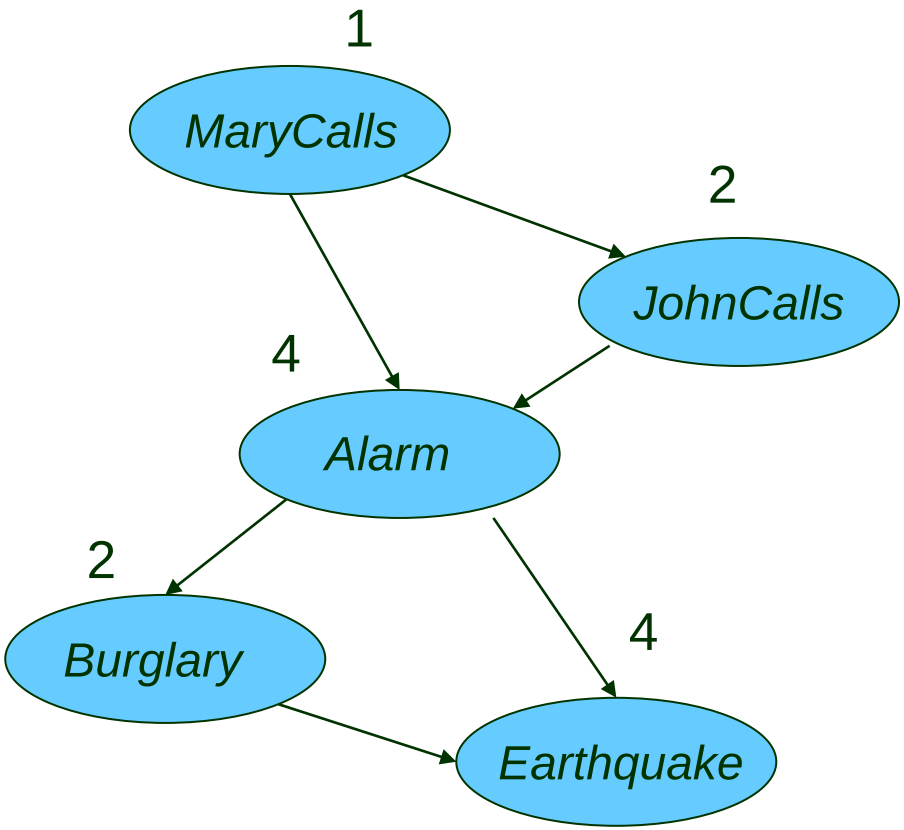

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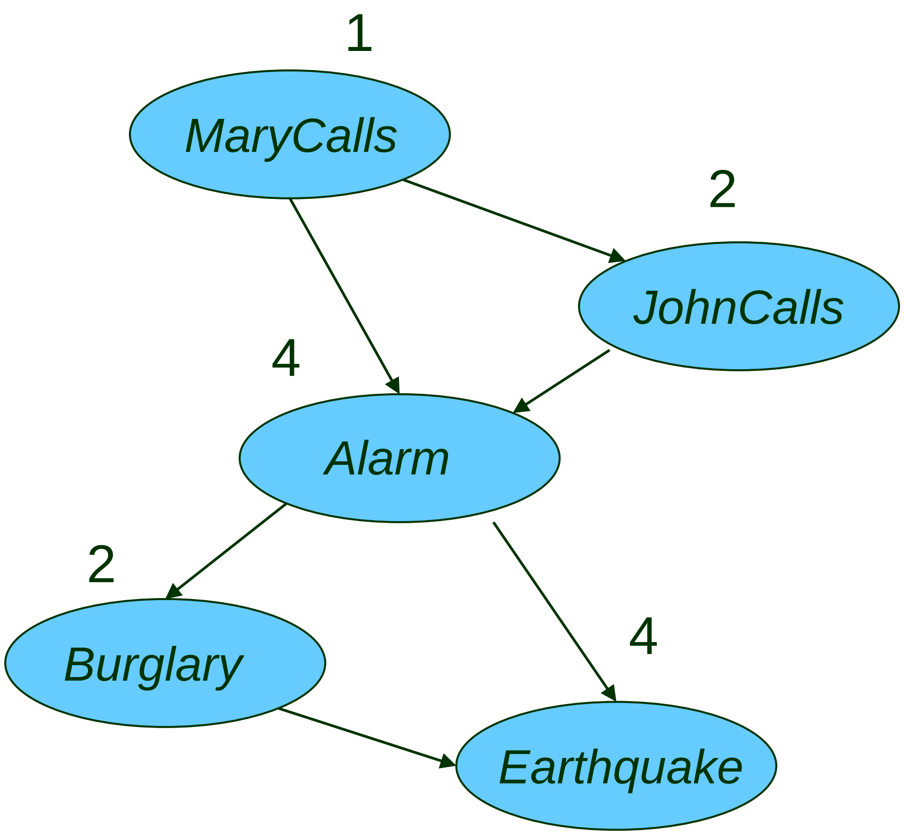

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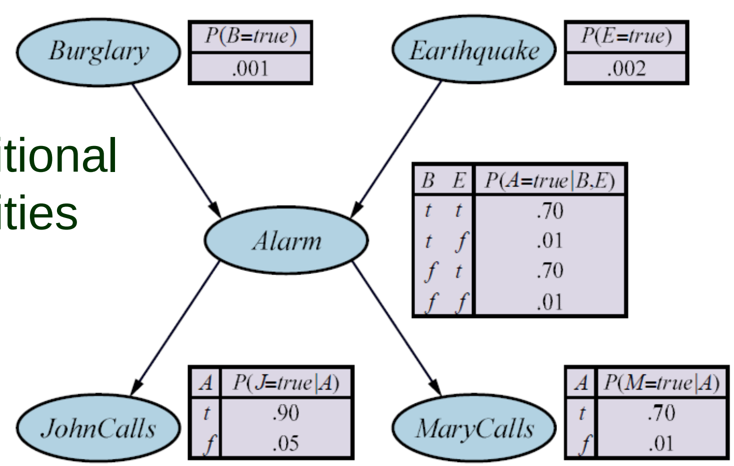

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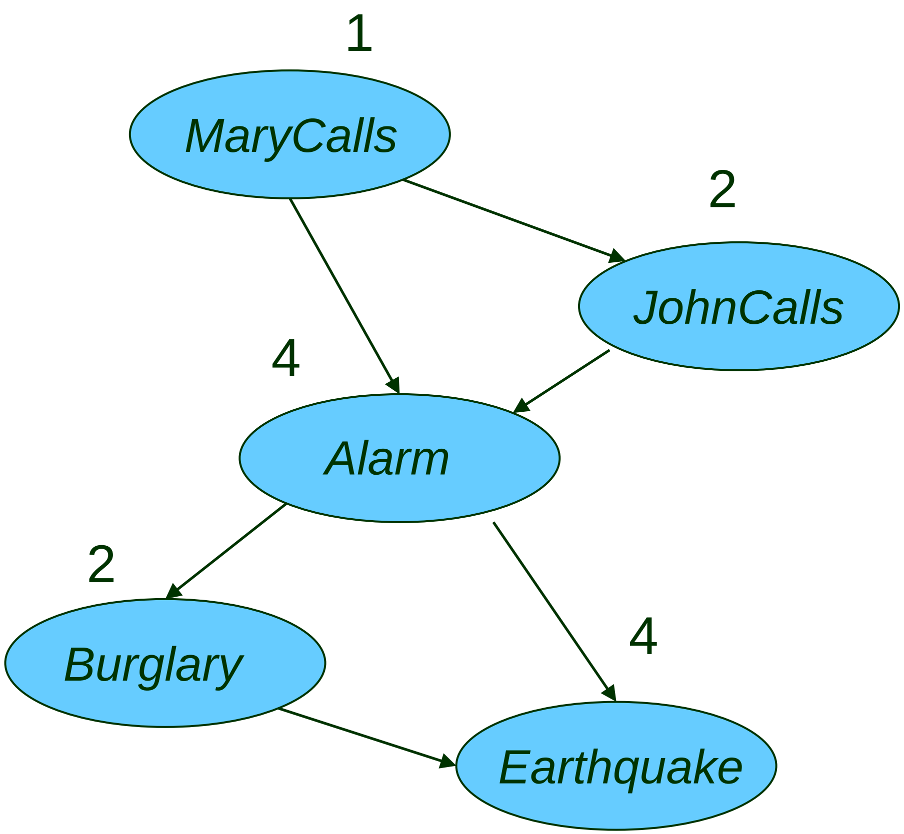

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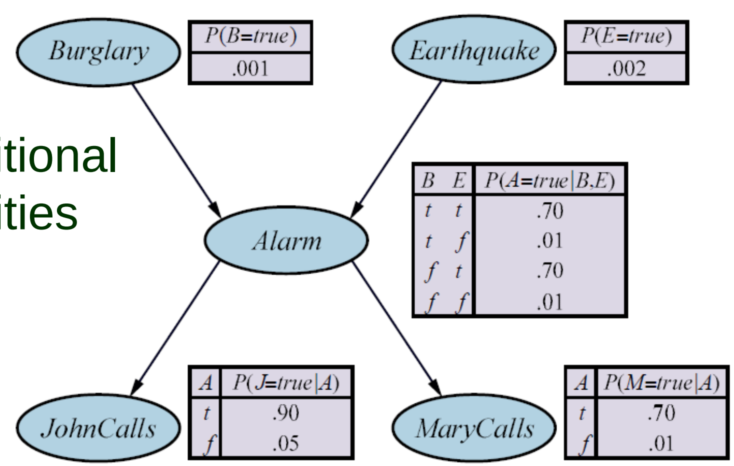

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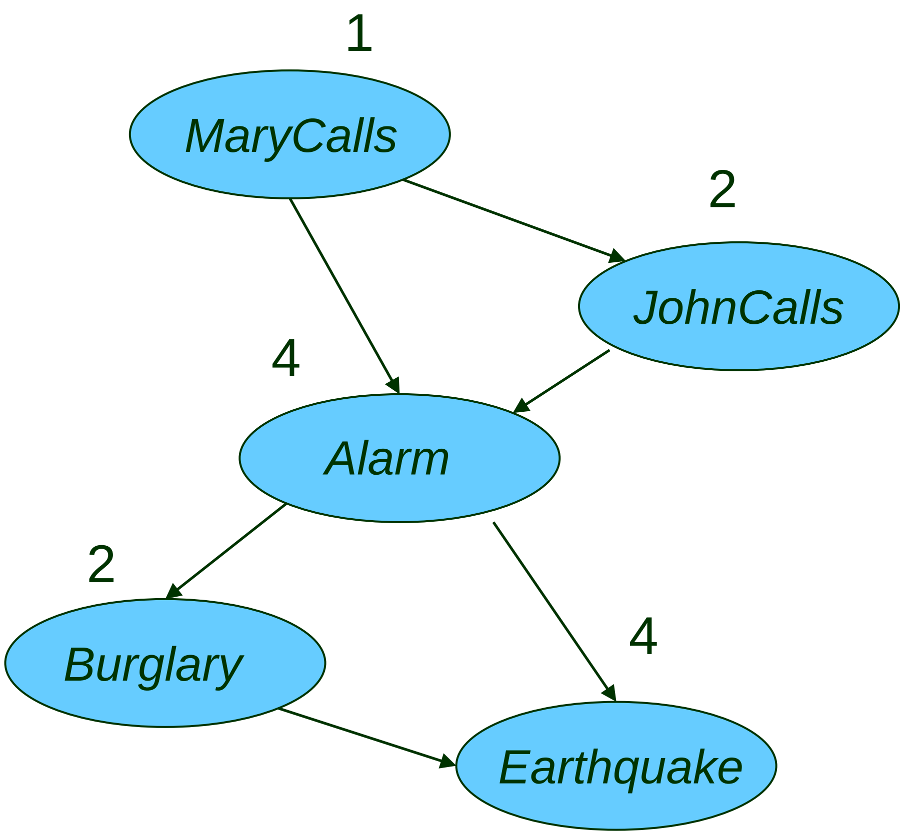

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- ♣ More conditional probabilities than needed.
- ♣ Assessment of unnatural probabilities, e.g., $\boldsymbol{P}(\textit{Earthquake} \mid \textit{Burglary, Alarm})$.
- ♦ Sticking to a causal model results in fewer probabilities that are also easier to come up with.

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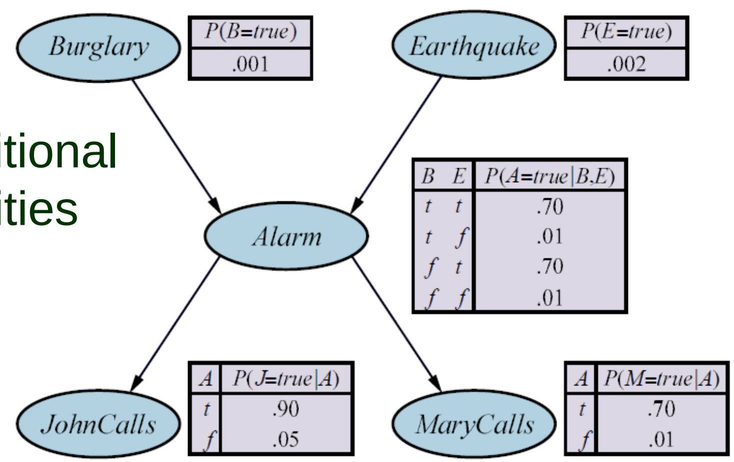
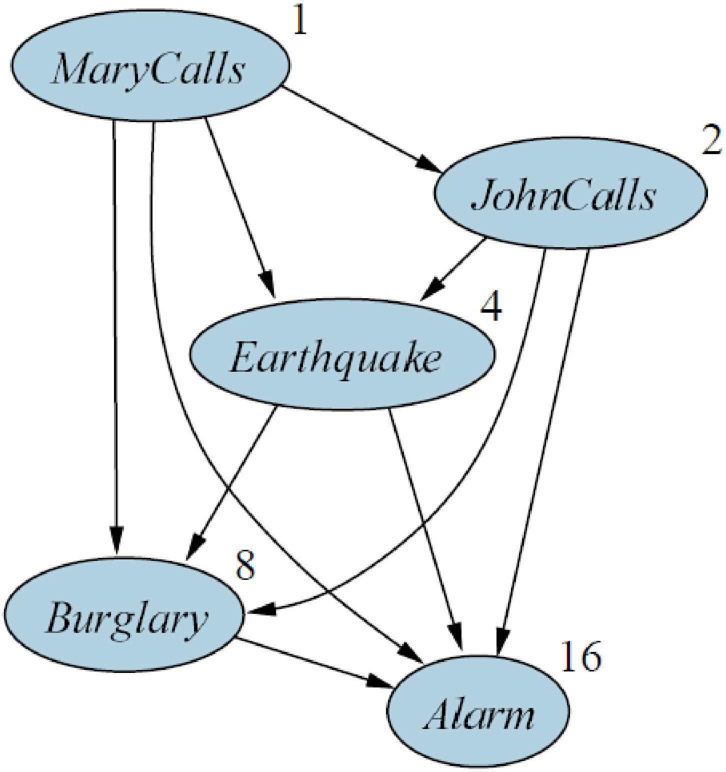


Bad Node Ordering

MaryCalls, *JohnCalls*, *Earthquake, Burglary*, *Alarm*.



$1 + 2 + 4 + 8 + 16 = 31$ distinct probabilities (exactly the same as the full joint distribution)!

Roles of Casualty

- Deciding conditional independence is hard in noncausal directions.
(Causal models and conditional independence seem hardwired for humans!)

- Assessing conditional probabilities is hard in noncausal directions.

- The interpretation of directed acyclic graphs as carriers of independence assumptions does not necessarily imply causation

- The ubiquity of DAG models in statistical and AI applications stems (often unwittingly) primarily from their causal interpretation.

- In practice, DAG models are rarely used in any variable ordering other than those which respect the direction of time and causation.

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♦ To avoid a fully connected network, leave out links that represent slight dependencies.

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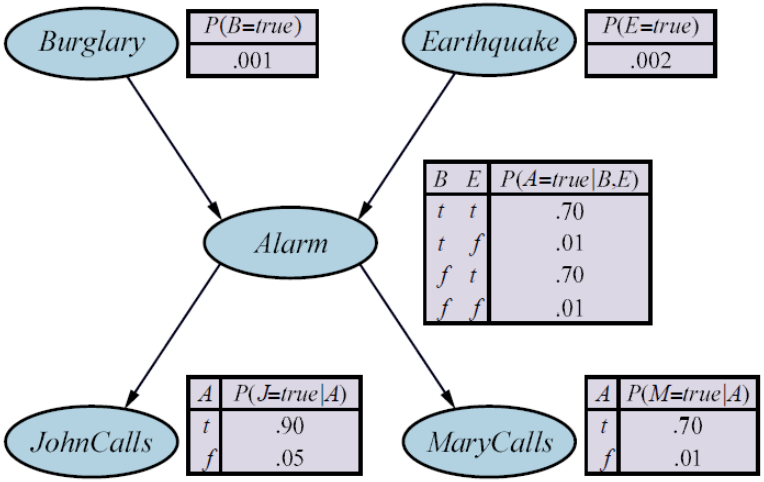
Every variable is conditionally independent of its non-descendants, given the values of its parents.



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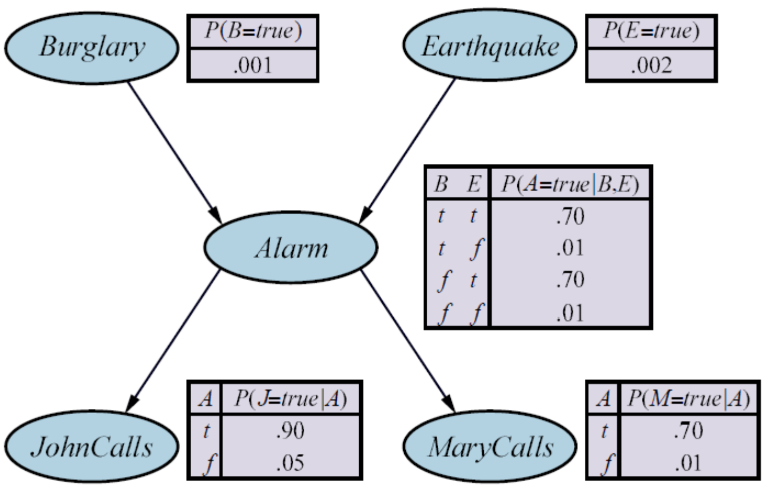

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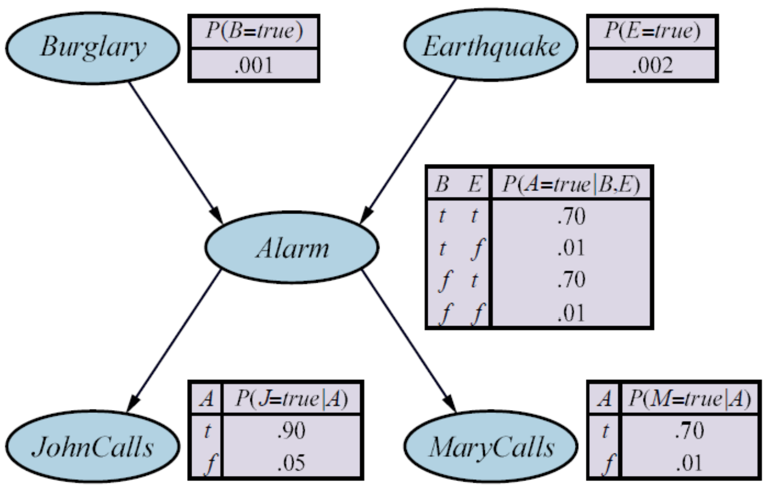

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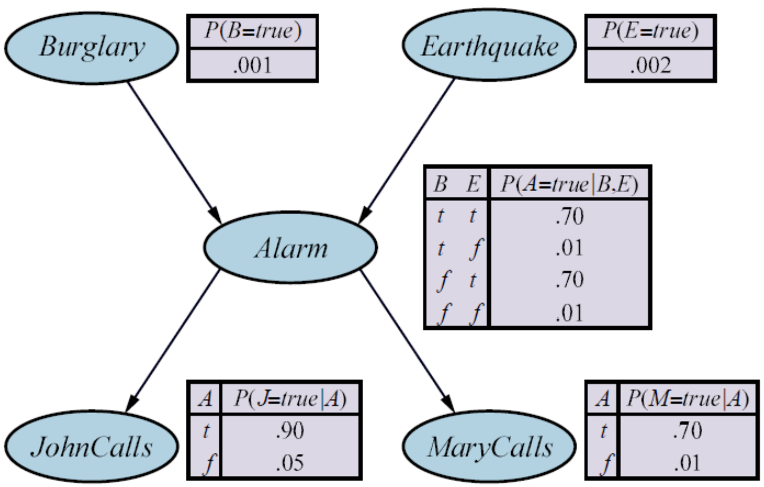
// network parameter interpretation

$$P(x_i \mid Parents(X_i)) = \theta_i(x_i \mid parents(X_i))$$

II. Non-Descendants Property

Every variable is conditionally independent of its non-descendants, given the values of its parents.



P(B=true)
.001

P(E=true)
.002

B	E	P(A=true | B,E)
t	t	.70
t	f	.01
f	t	.70
f	f	.01

A	P(J=true | A)
t	.90
f	.05

A	P(M=true | A)
t	.70
f	.01

Given the value of *Alarm*, *JohnCalls* is independent of *Burglary*, *Earthquake*, and *Marycalls*.

// network parameter interpretation

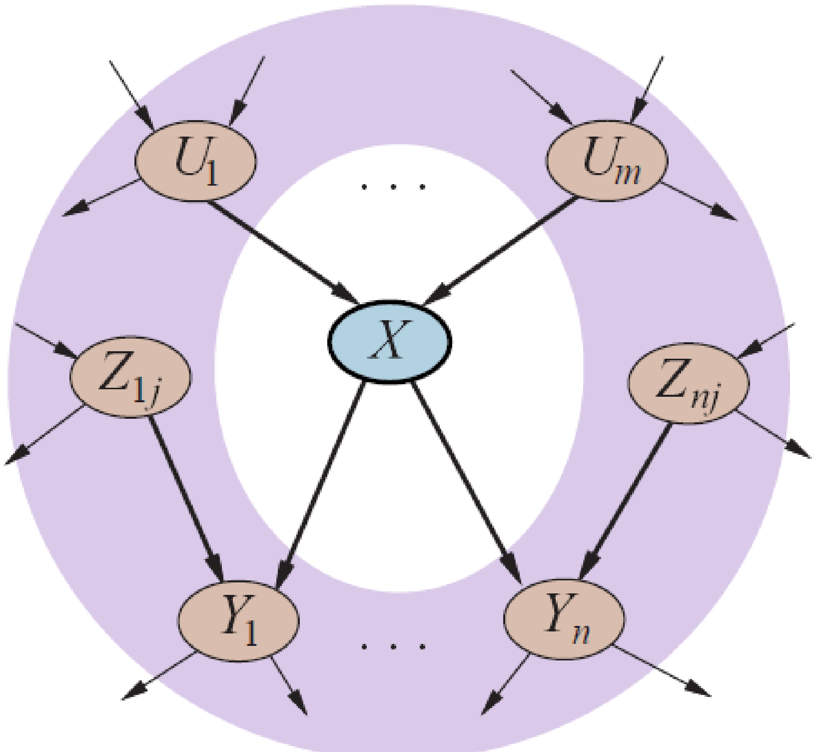
$$\underbrace{\text{Non-descendants property} \quad P(x_i \mid \mathit{Parents}(X_i)) = \theta_i(x_i \mid \mathit{parents}(X_i))}$$

$$\Downarrow$$

The full joint distribution $P(x_1, \dots, x_n) = \prod_{i=1}^{n} P(x_i \mid \mathit{Parents}(X_i))$

Markov Blanket

The *Markov blanket* of a node consists of its parents, children, and children's parents.



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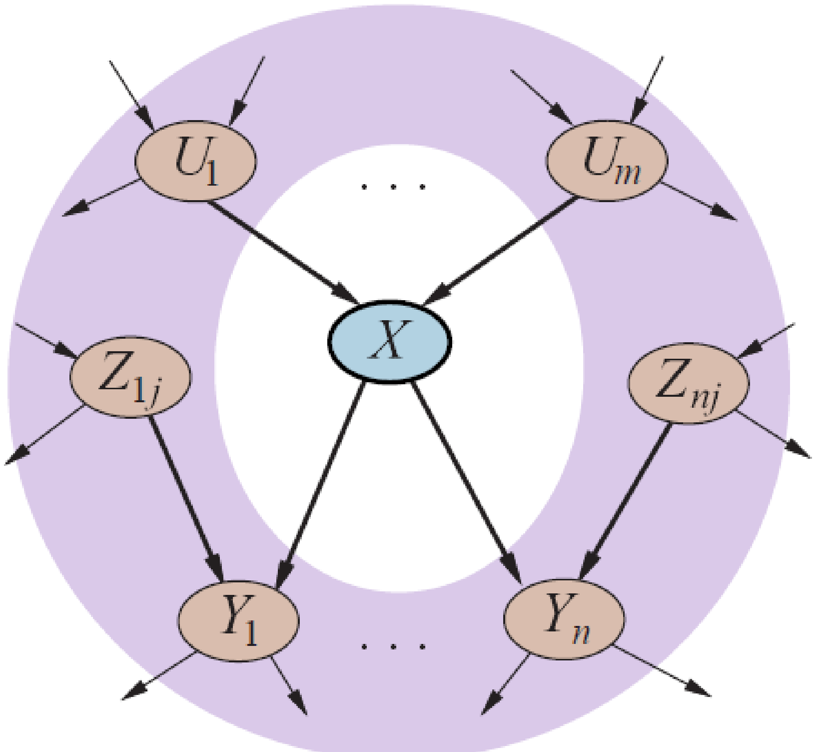

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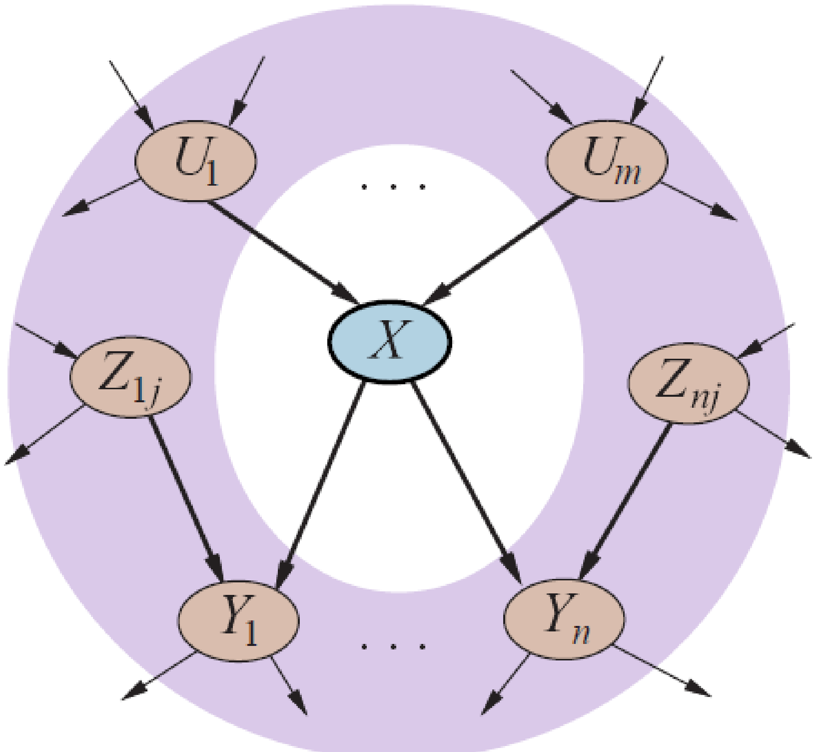

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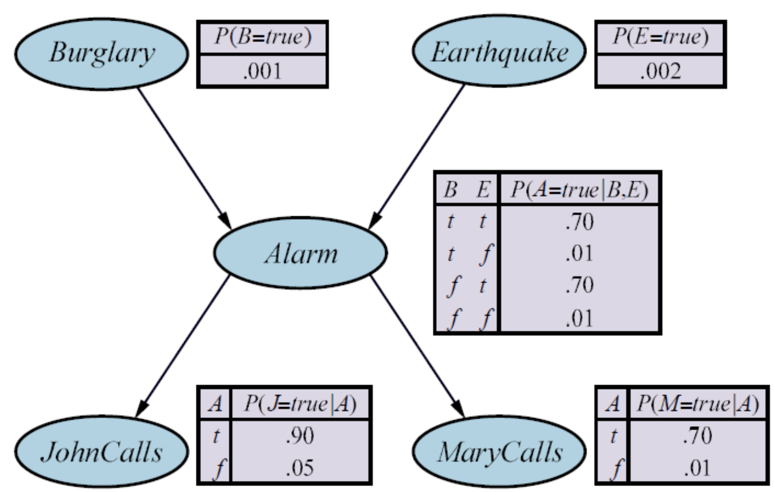

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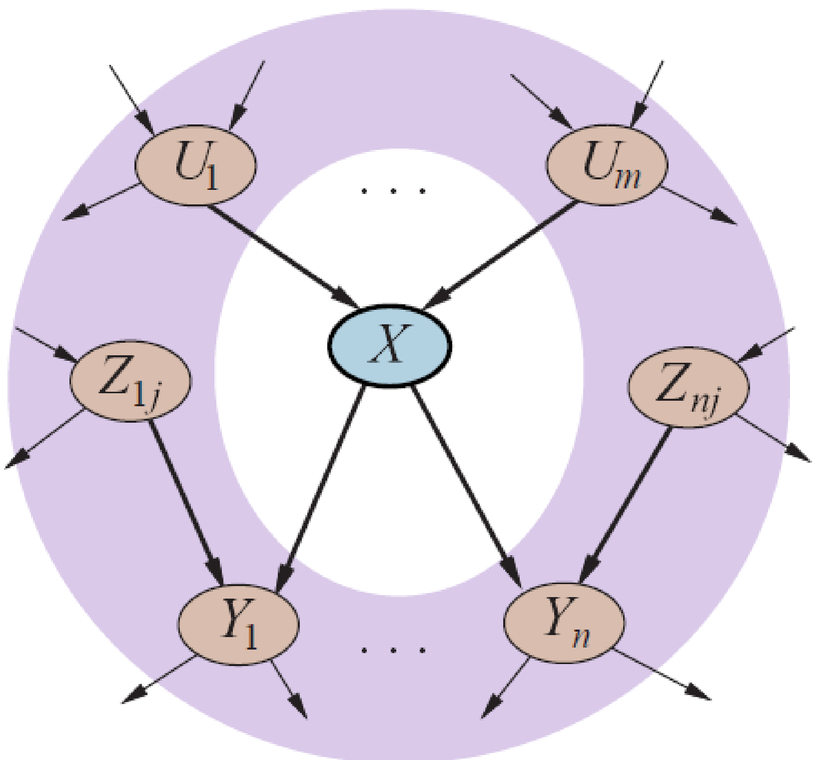

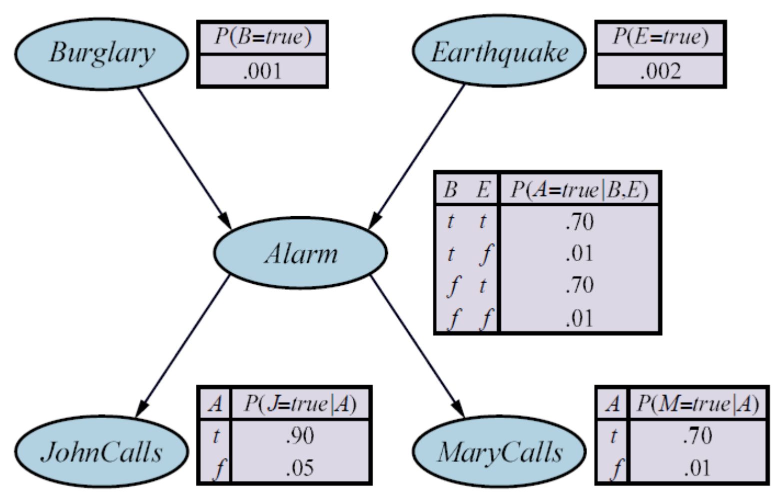

Given *Alarm* and *Earthquake*, *Burglary* is independent of *JohnCalls* and *Marycalls*.

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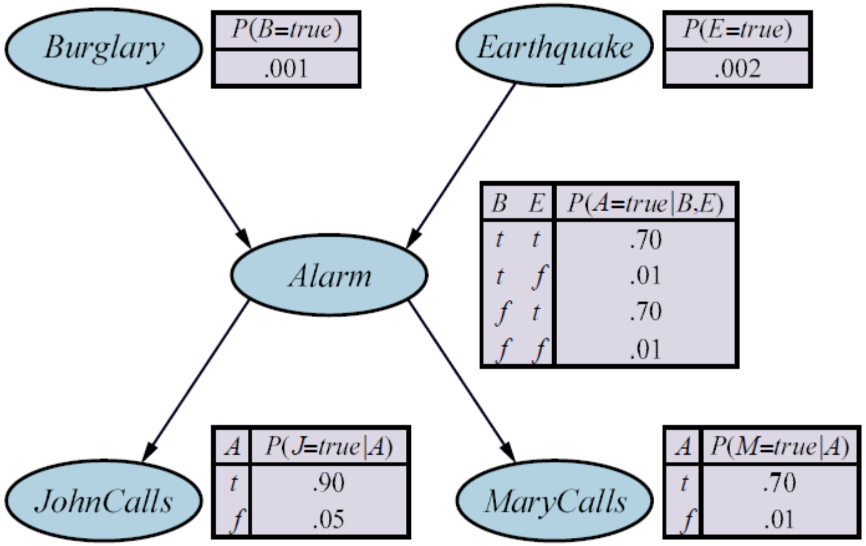
Examples



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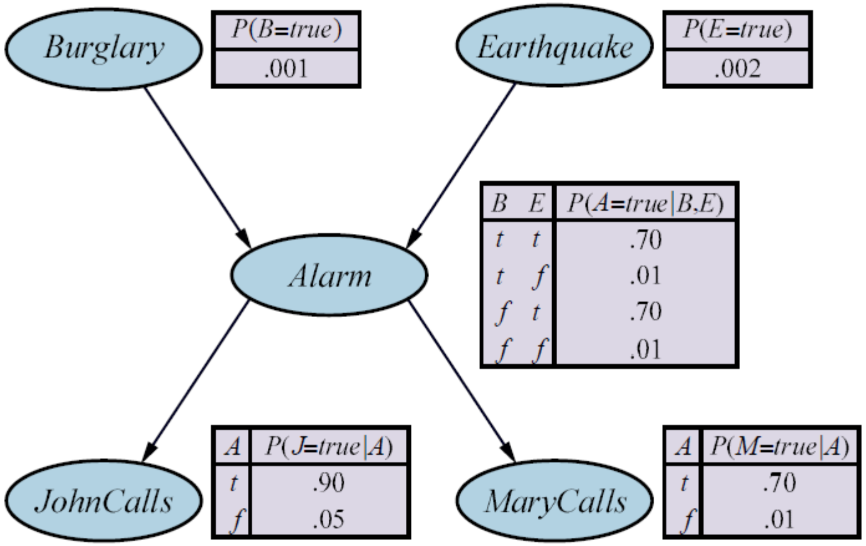

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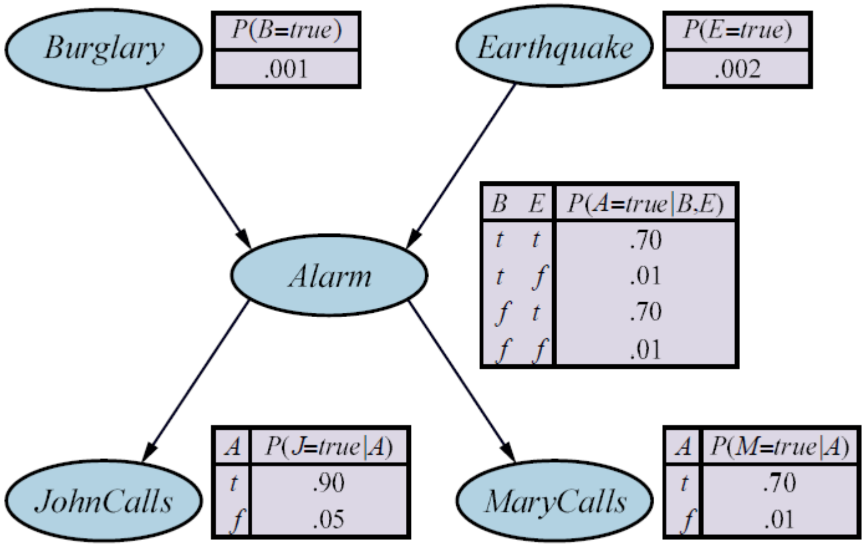

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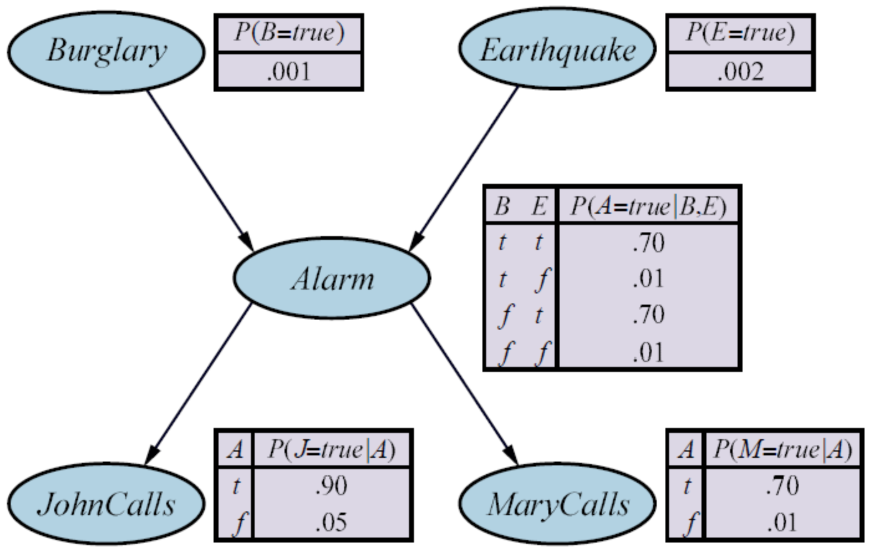

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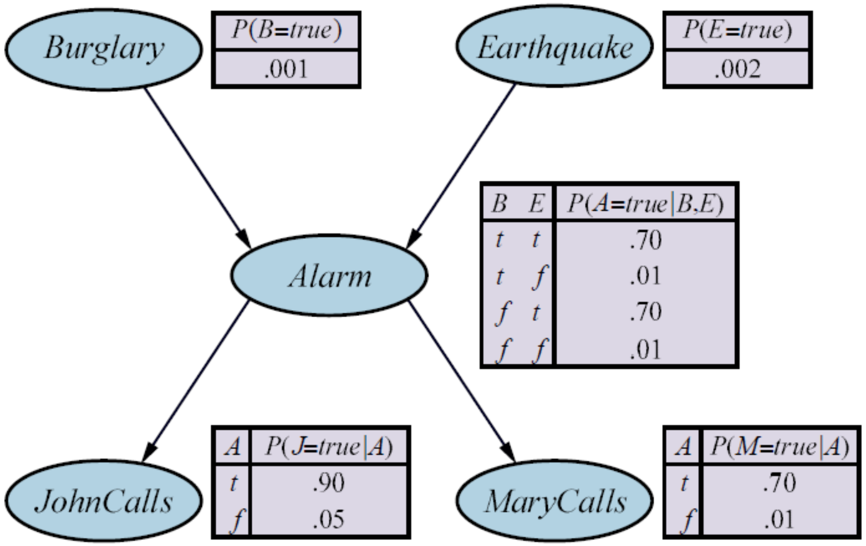

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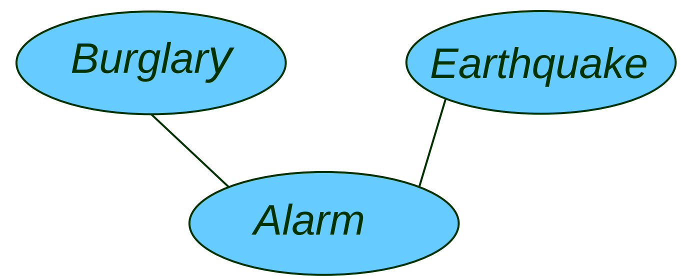

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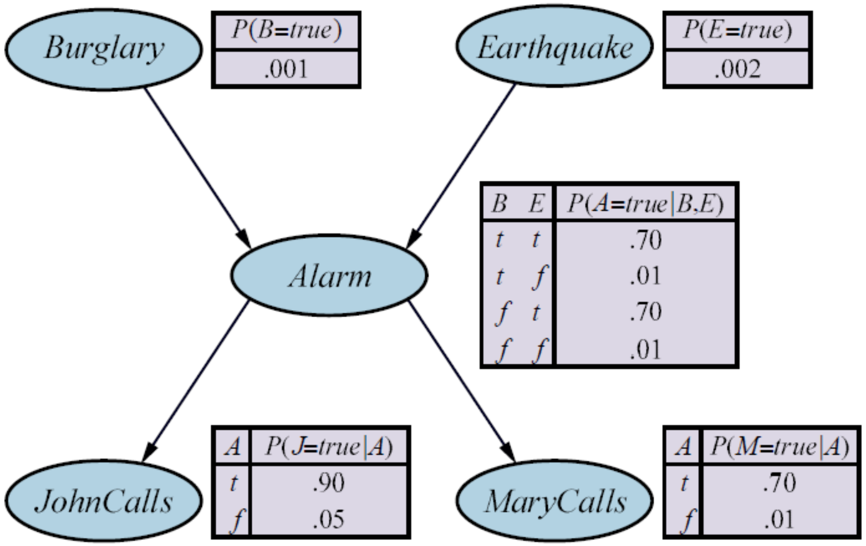

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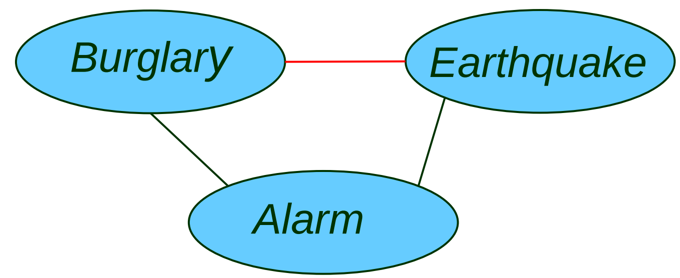

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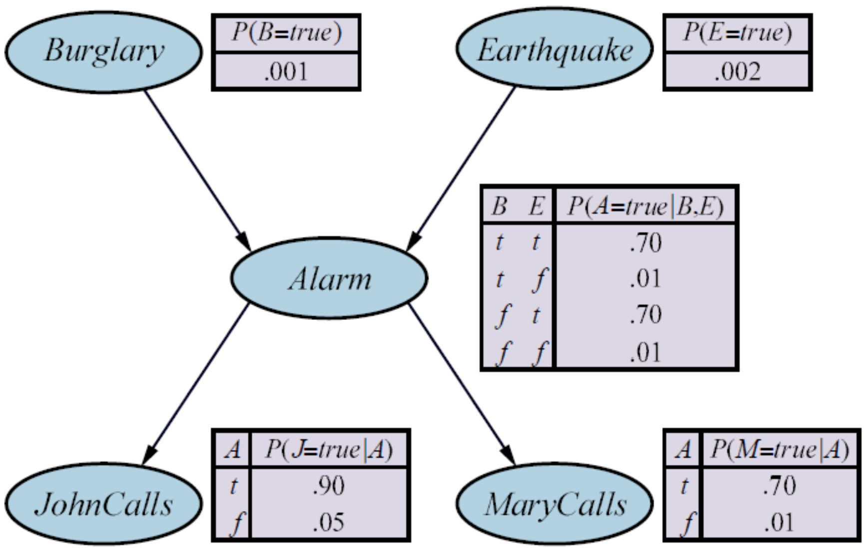

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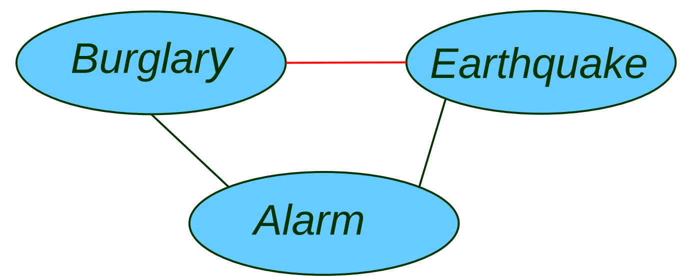

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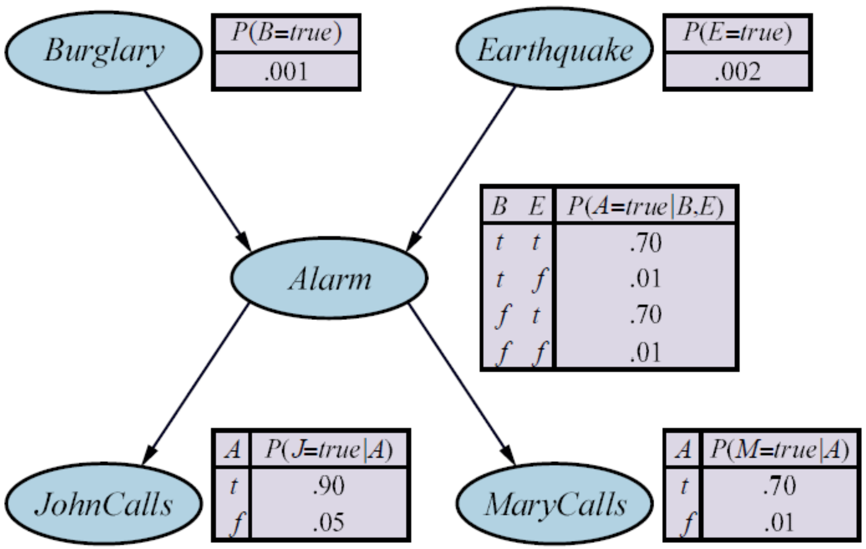

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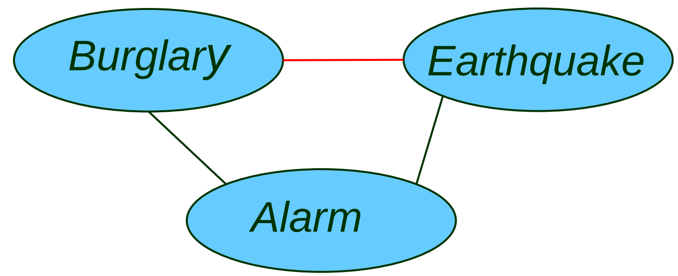

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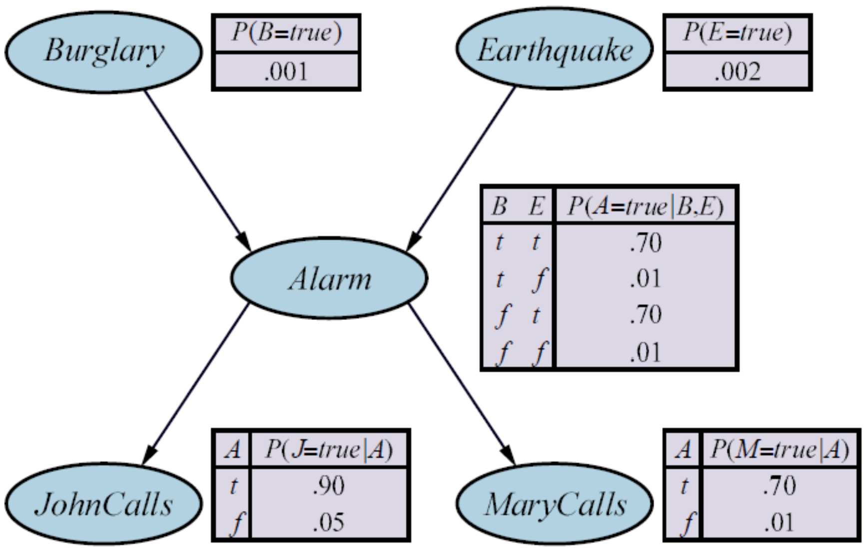

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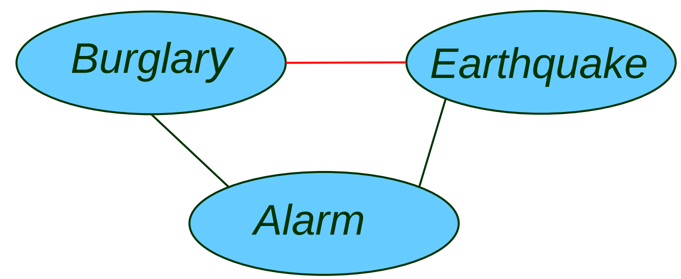

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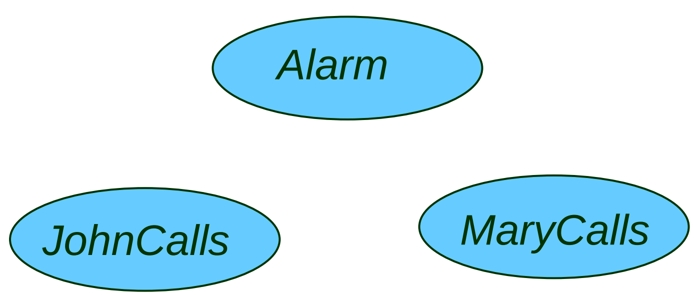

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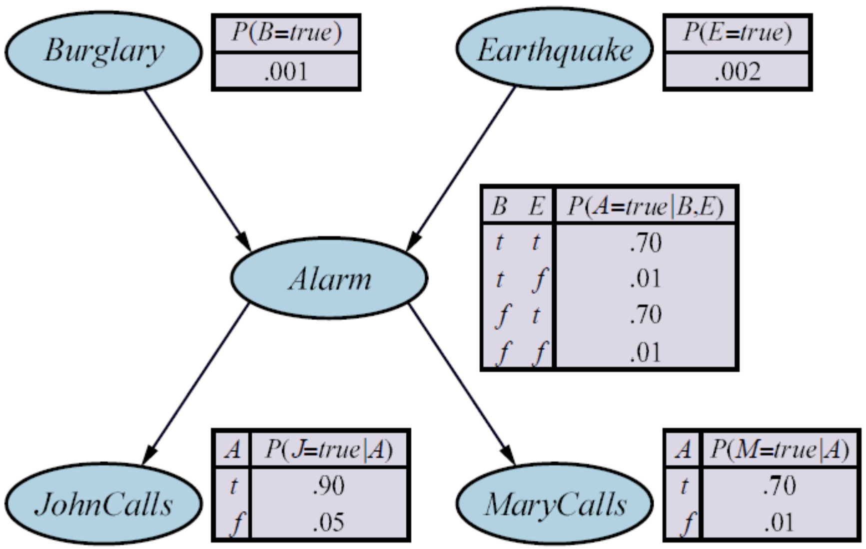

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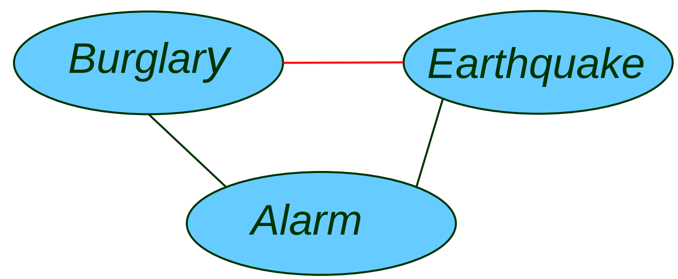

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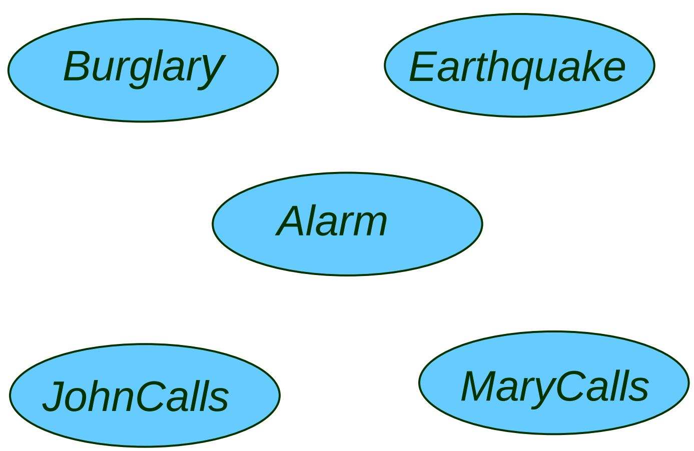

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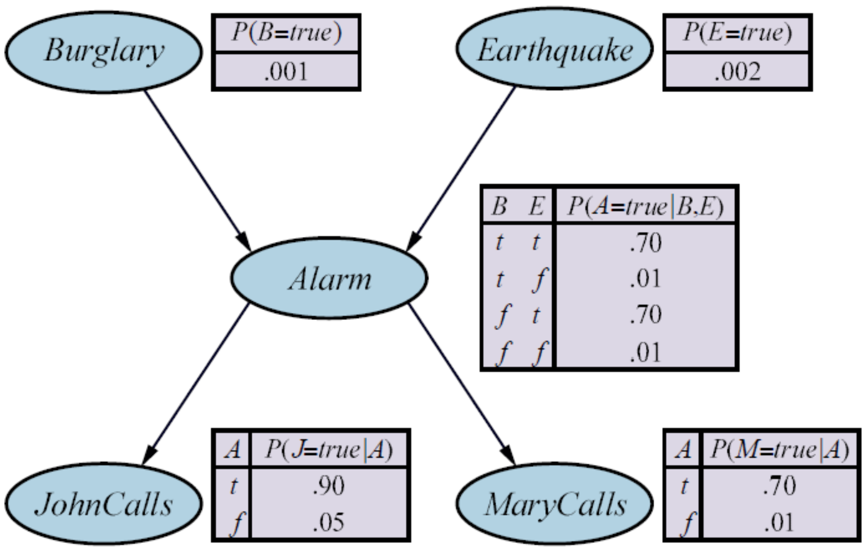

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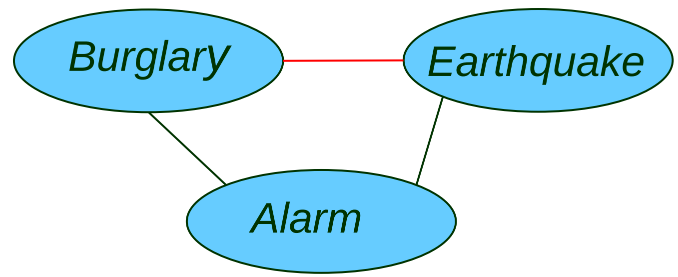

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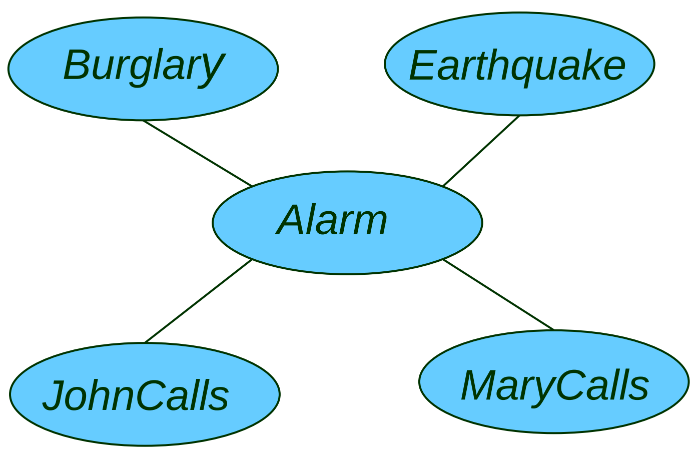

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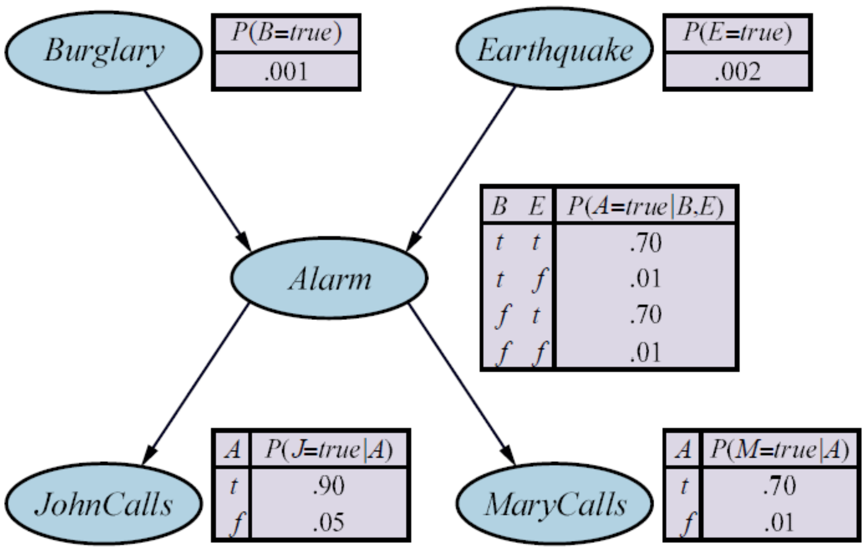

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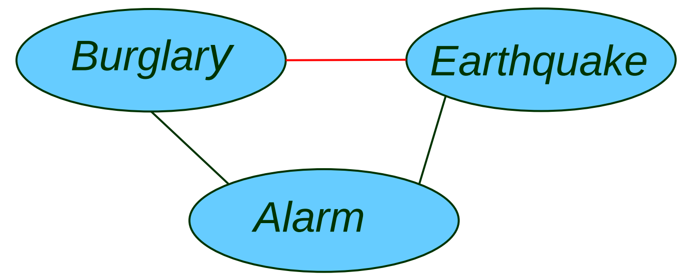

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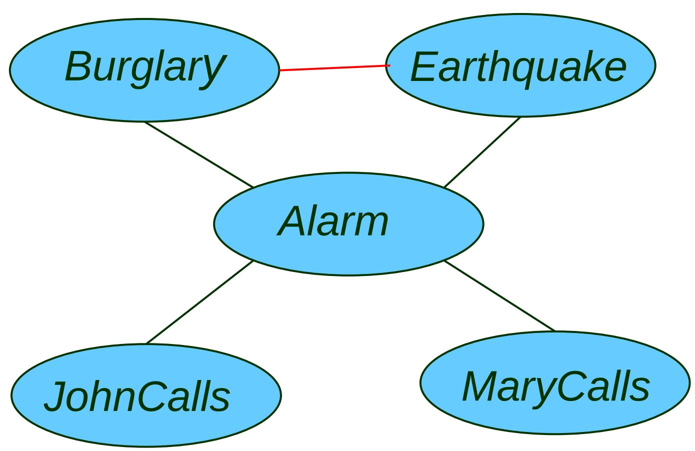

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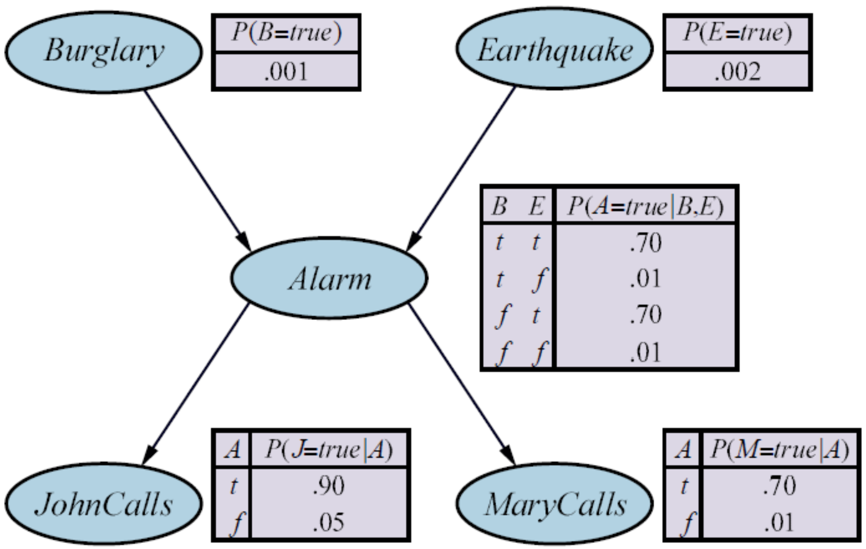

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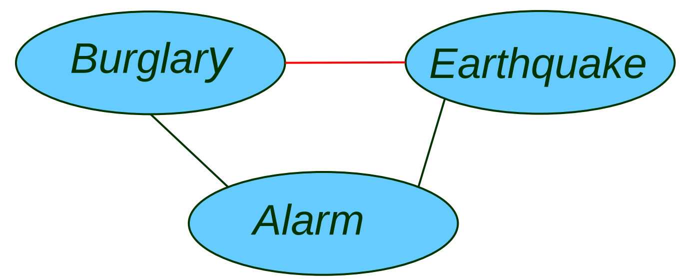

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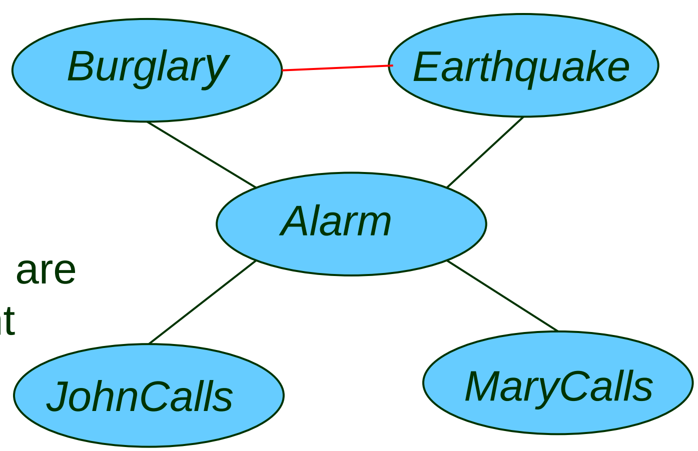

JohnCalls and *MayCalls* are conditionally independent given *Alarm*.

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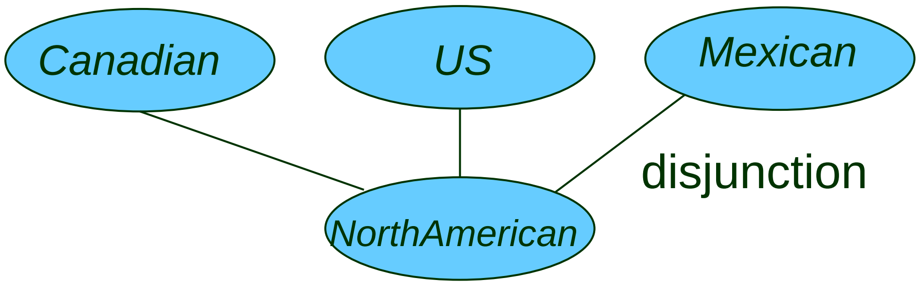

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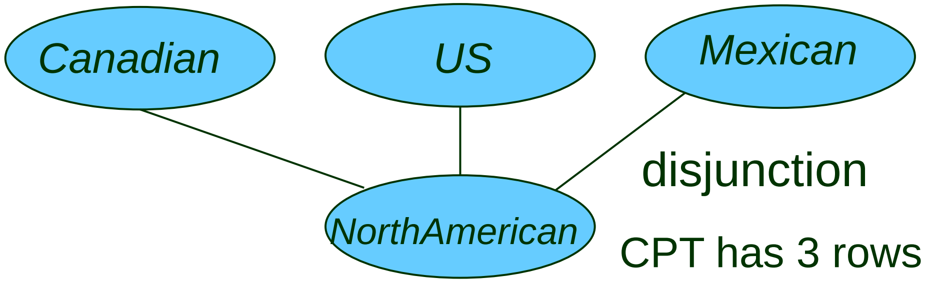

disjunction

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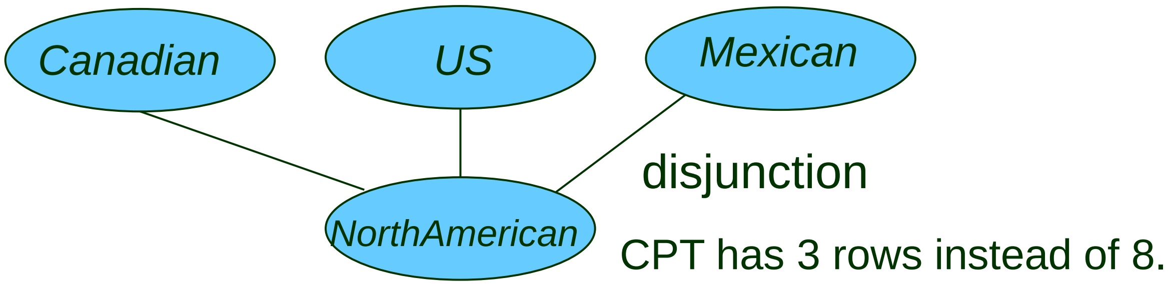

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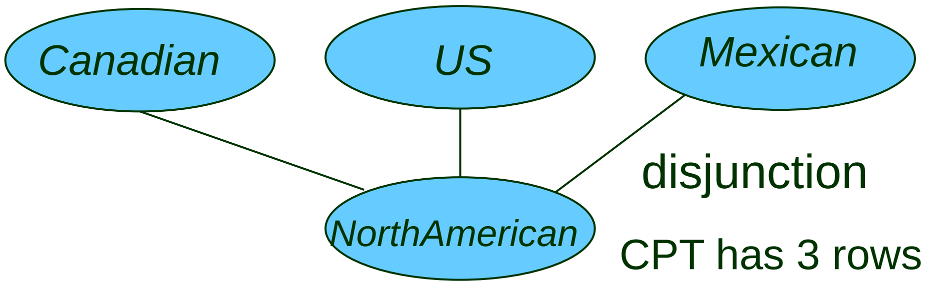

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$$\boldsymbol{P}(\textit{Damage} \mid \textit{Ruggedness}, \textit{Accident}) = $$
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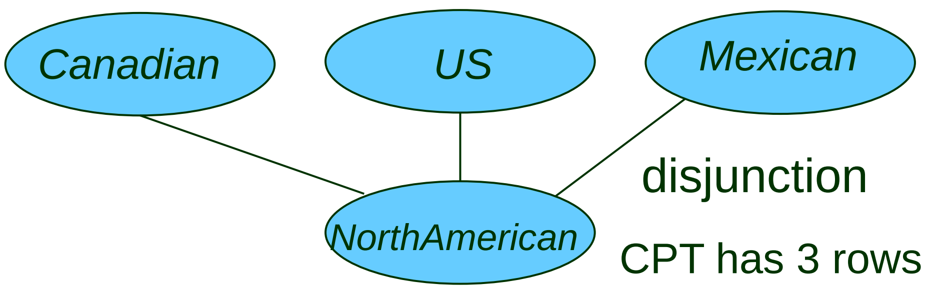

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f	f	f	0.0	1.0
f	f	t	0.9	**0.1**
f	t	f	0.8	**0.2**
f	t	t	0.98	$0.02 = 0.2 \times 0.1$
t	f	f	0.4	**0.6**
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- Whatever inhibits cold from causing a fever is independent of whatever inhibits flu from causing a fever.
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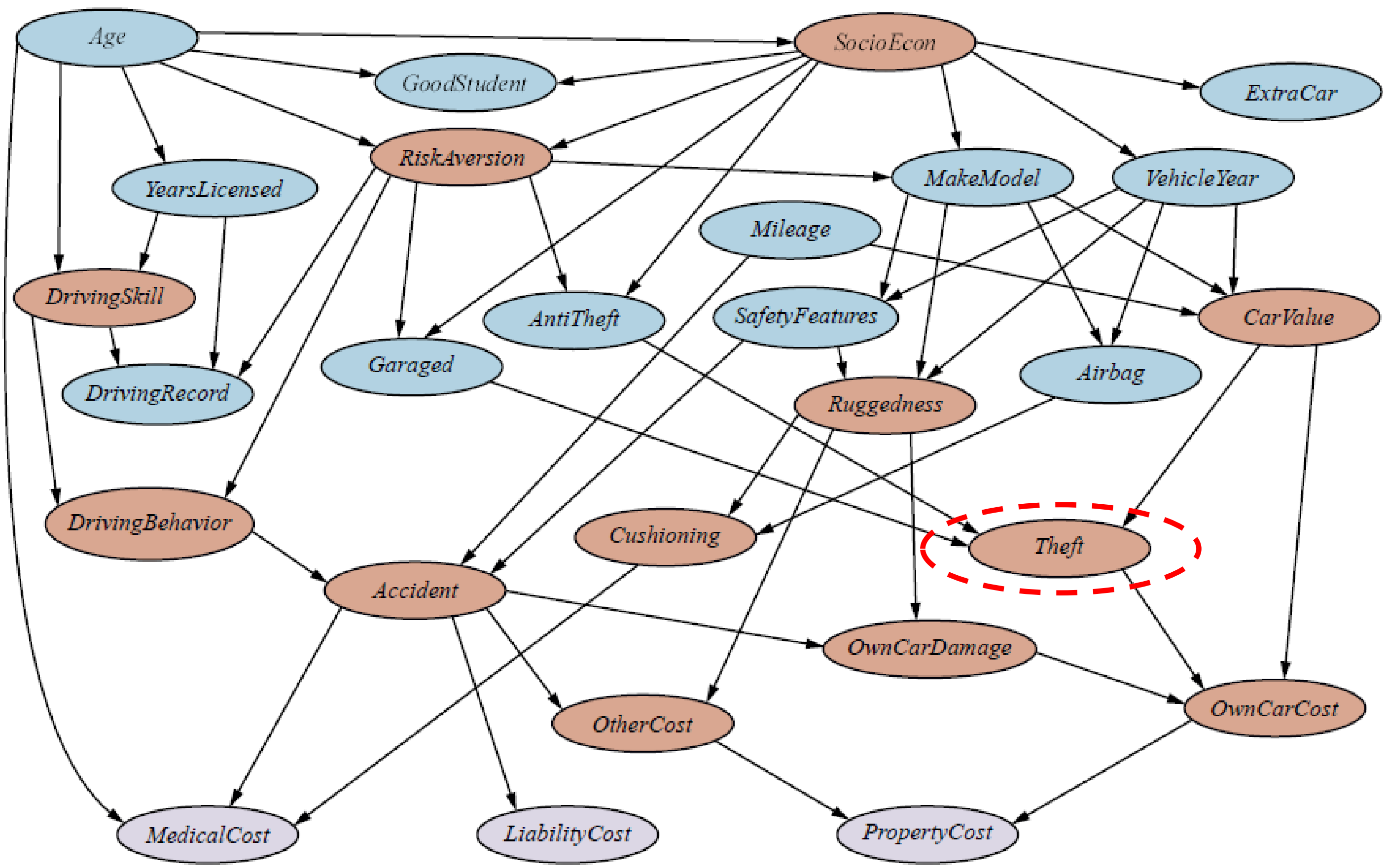
$$P(x_i \mid \mathit{parents}(X_i)) = 1 - \prod_{\{j : X_j = \mathit{true}\}} q_j$$

Case Study: Car Insurance

A car insurance company processes a car insurance application to decide on the annual premium based on the anticipated claims it will pay out for the applicant.



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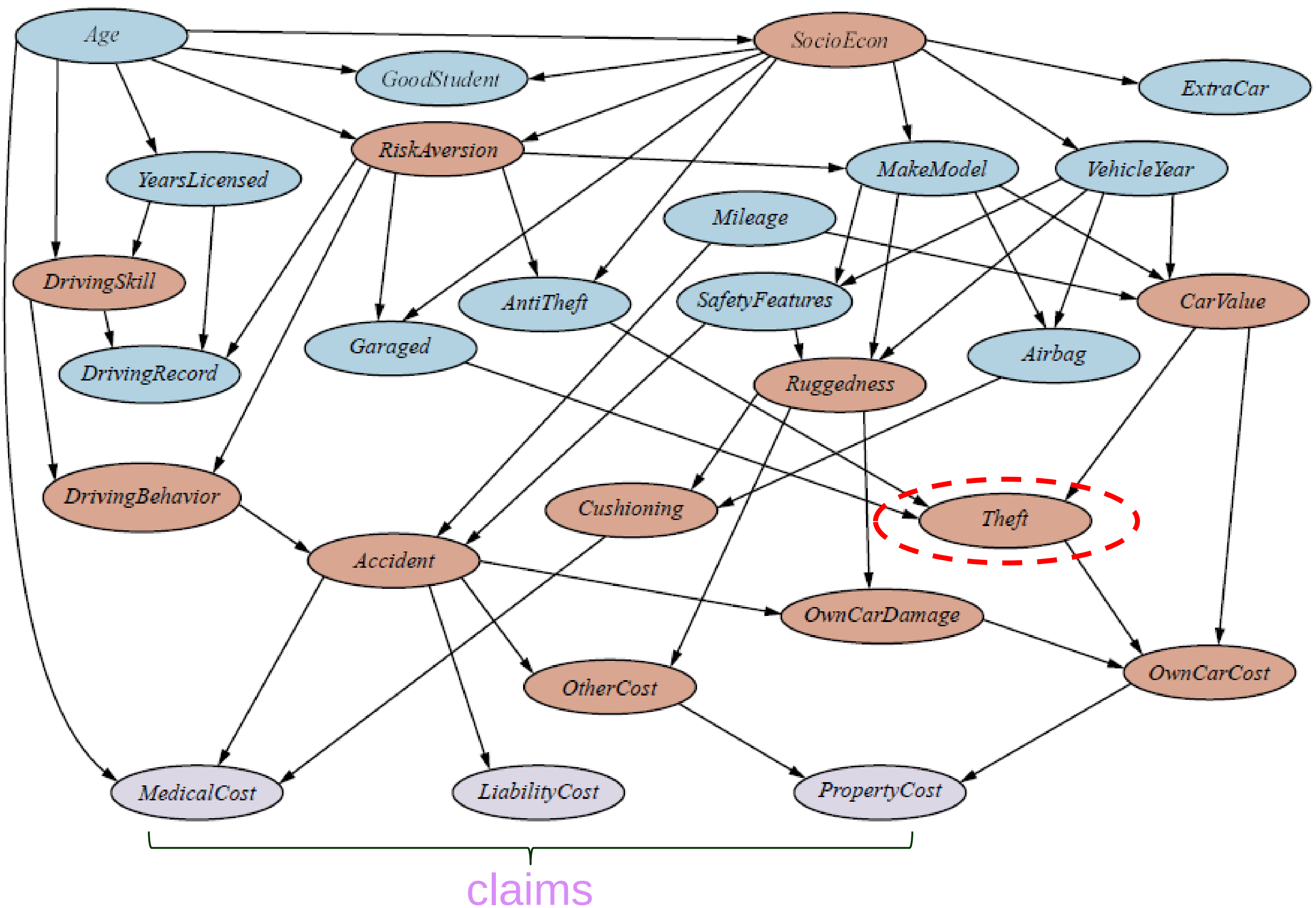

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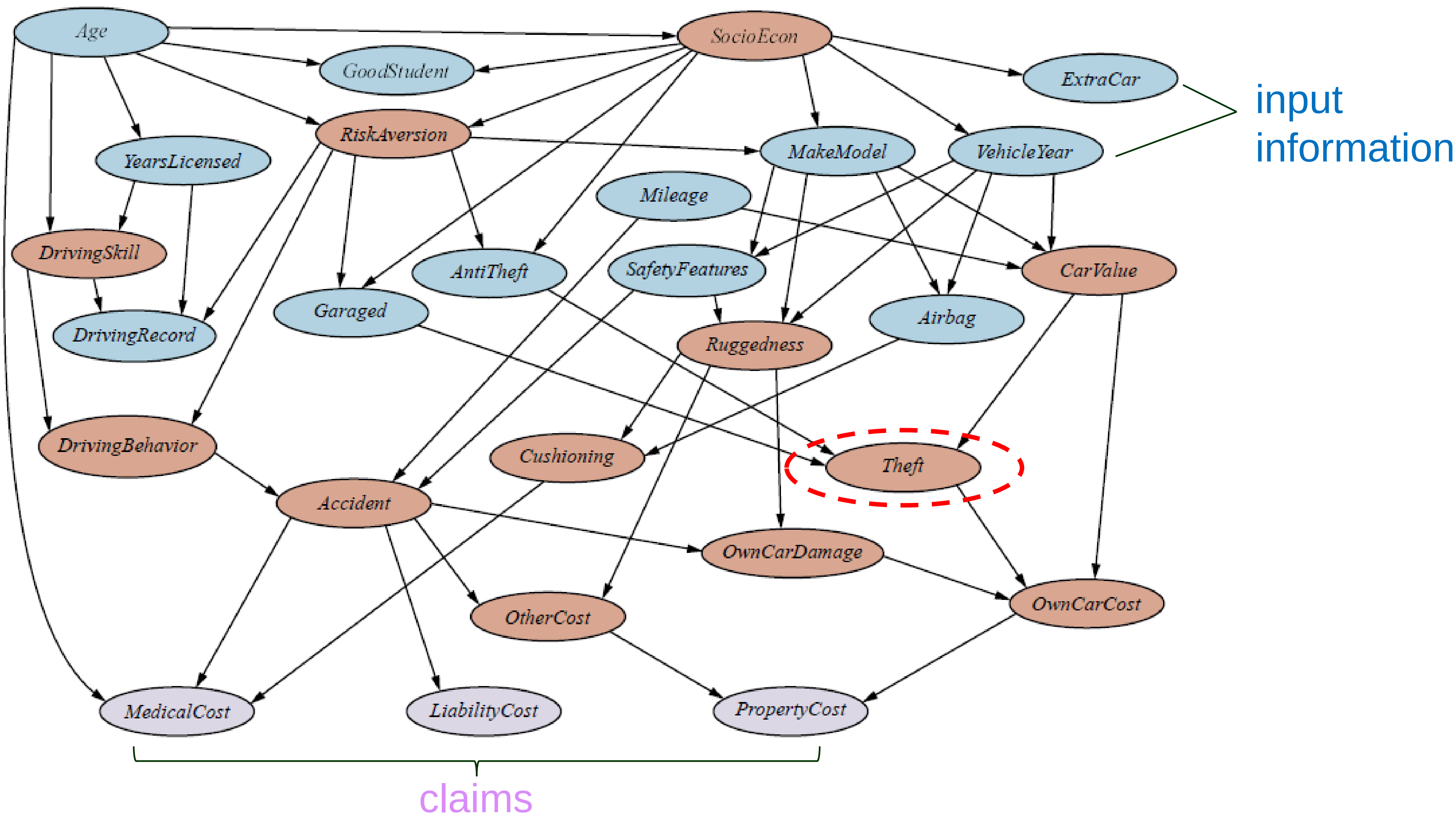

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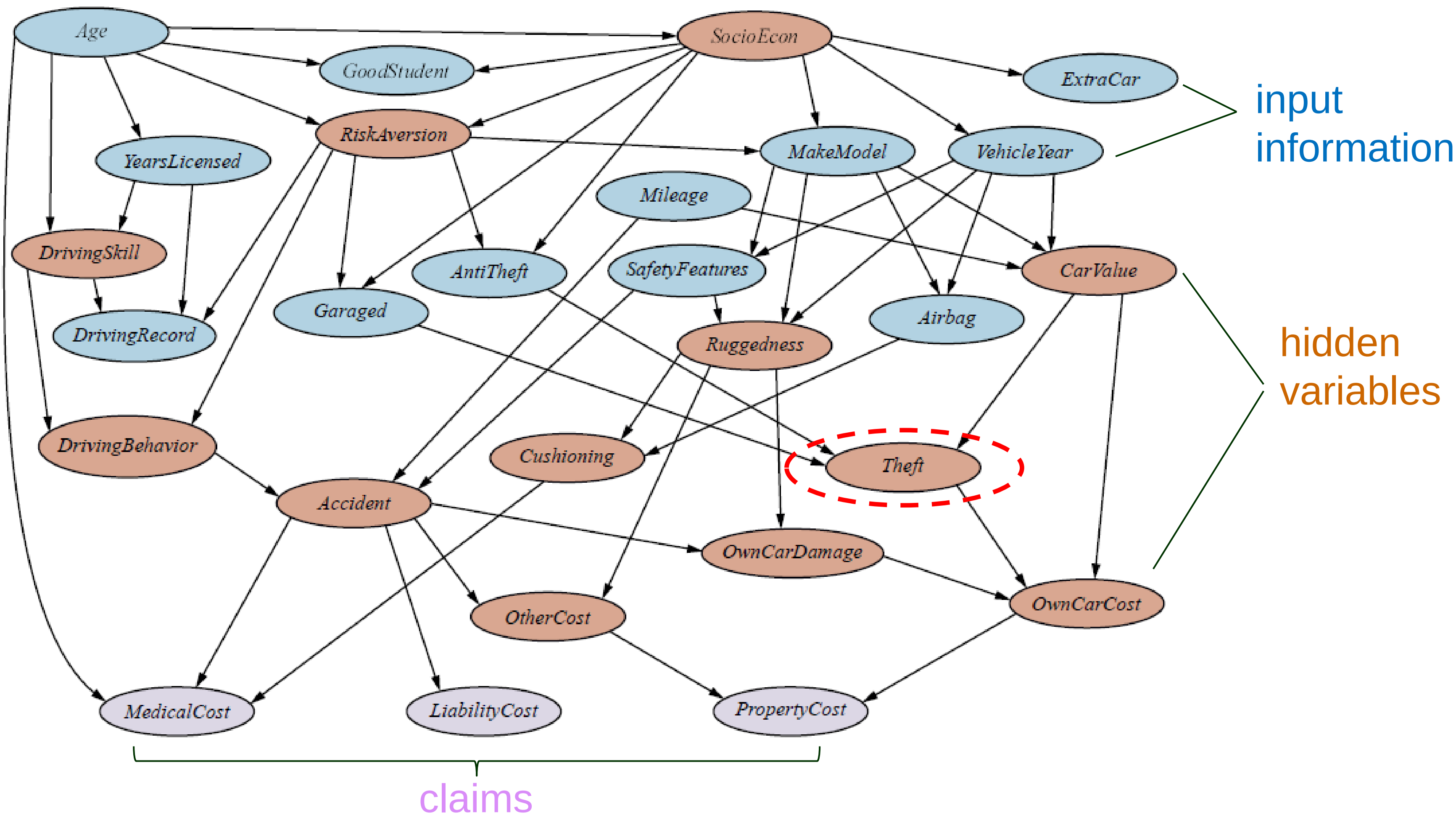

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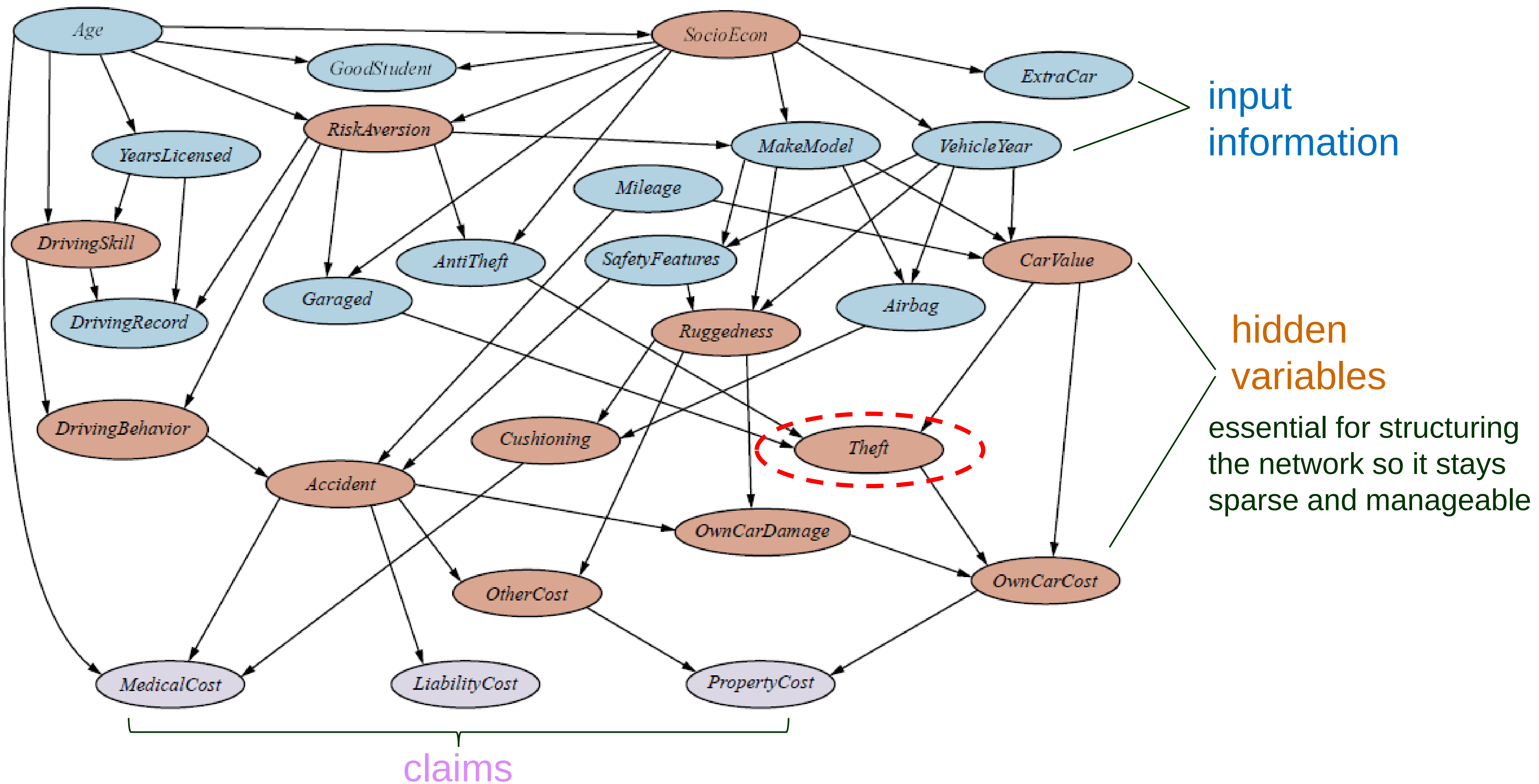

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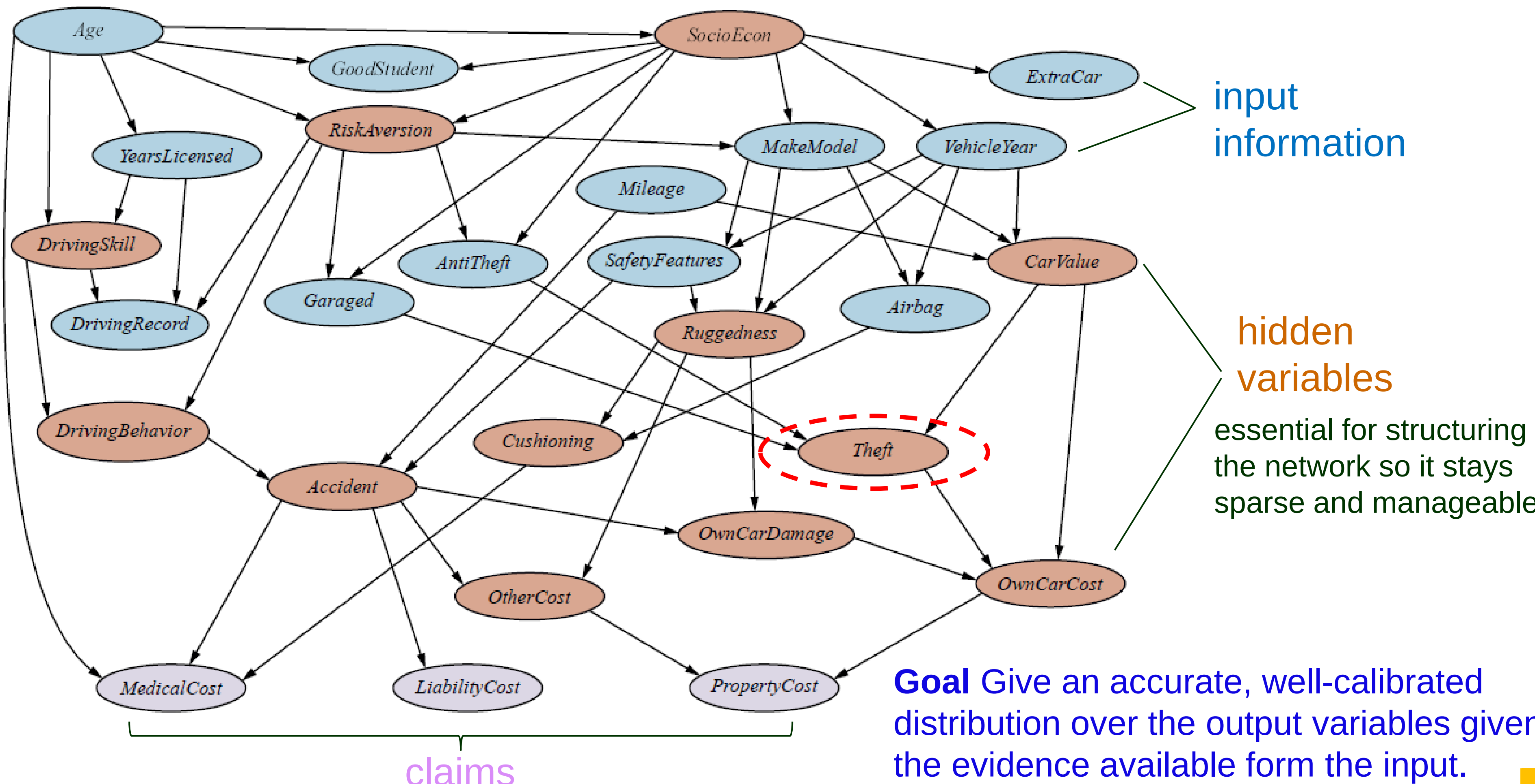

Goal Give an accurate, well-calibrated distribution over the output variables given the evidence available form the input.