

HW4

1) Prove  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \rightarrow$ 

$$x \in A \cup (B \cap C) \Rightarrow x \in A \text{ or } x \in (B \cap C) \Rightarrow$$

$$x \in A \text{ or } (x \in B \text{ and } x \in C) \Rightarrow x \in (A \text{ or } B) \text{ and } x \in (A \text{ or } C) \Rightarrow$$

$$x \in (A \cup B) \text{ and } x \in (A \cup C) \Rightarrow x \in (A \cup B) \cap (A \cup C)$$

2) One-to-One? No,  $F(-2, -2) = F(2, 2) = 4$ Onto? Yes,  $F(1, 0) = 0$ 

3)

a) If  $F$  is a surjection,  $y \in Y$  and  $x \in X$ , that  $F(x) = y$ As  $g$  is also a surjection, For every  $z \in Z$  there exists a  $y \in Y$  that $g(y) = z$ . As there are any possible  $y$ , there are any possible  $z$ .b) We have found that  $h$  is surjective, looking for injective -As  $g$  &  $F$  are injective,  $g(F(a)) = g(F(a')) \Rightarrow$ 

$$F(a) = F(a') \Rightarrow a = a'$$

4) a) The graph cannot have cycles of size 1, as a player cannot beat themselves. Cycles of size 2 are also impossible, as a player cannot beat and be beaten by the same player.

b) i. Never asymmetric, neither cycles of size 1 nor 2 are allowed

ii. Never reflexive, a player cannot beat themselves

iii. Always irreflexive, no loops of size 1

iv. Never transitive, if player 2 beats player 3, and player 1 beats player 2, it does not guarantee that player 1 beats player 3



- 5) a) - Reflexive:  $w_1$  and  $w_2$  have the same first letter;  $he(o) = he(o)$  ✓  
- Symmetric: The order does not matter,  $w_1(o) = w_2(o) \equiv w_2(o) = w_1(o)$   
 $he(o) = he(o) \equiv he(o) = he(o)$  ✓  
- Transitive:  $he(o) = he(o)$   $he(o) = his(o)$   $he(o) = his(o)$  ✓

As all three conditions are met, the relation is equivalent.

b)  $[The] = [he] = [he] \in [television] = [tuned] = [to] \mid [above] = [a]$   
 $[color] = [channel]$