CprE 310 / Spring 2019

Recitation 4 Solutions

- Feel free to solve this collaboratively during recitation, and ask (and answer) questions about this problem set on Piazza.
- This is an **optional** problem set; do not turn this in for grading.
- While you don't have to turn this in, be warned that this material **can** appear in a quiz or exam.

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1. Identify where the conceptual bug(s) in the following "proof" is/are, and give a brief explanation:

- **Claim**: If a and b are real numbers and $a = b$, then $a = 0$.
- **Proof**: The proof proceeds as follows.
  - Suppose that $a = b$;
  - therefore, $a^2 = ab$;
  - therefore, $a^2 - b^2 = ab - b^2$;
  - therefore, $(a + b)(a - b) = b(a - b)$;
  - therefore, $a + b = b$;
- therefore, $a = 0$.

Solution

Fourth line to fifth line. We cannot divide $(a - b)$ on both sides since $a - b = 0$.

2. Prove, via contradiction, that $sqrt2$ is irrational.

Solution

Suppose that $\sqrt{2}$ is rational such that $\sqrt{2} = \frac{n}{d}$ in the lowest term while both n and d are integers and $d \neq 0$.

By squaring both sides and multipy cross terms, we get:

$$n^2 = 2d^2$$

Since the right hand side is a multiple of 2, the quantity n^2 is even and consequently n is also even. Wrting $n = 2q$ and substituting to the above equations, we get:

$$2q^2 = d^2$$

This shows that d^2 is also a multiple of 2.

Since both n and d are even, they share a common factor of 2 which contradicts our assumptions. Therefore, $\sqrt{2}$ is an irrational number.

3. The following table lists driving distances between some California cities (in miles):

- Barstow to Fresno: 245 miles
- Eureka to Fresno: 450 miles
- Barstow to LA: 115 miles
- Eureka to LA: 645 miles
- Fresno to LA: 220 miles
- San Diego to Barstow: 175 miles
- San Diego to LA: 125 miles

Draw the above information in the form of a graph. Clearly label the nodes, define what you mean by an edge, and draw all relevant edges. Assign "weights" to the edges according to the driving distance information provided.

Using the graph, guess the route for a round trip starting from San Diego and visiting all other cities using as few miles as possible (no proof needed).

Solution

The shortest round trip of visiting every cities starting from San Diago: "San Diago - LA - Eureka - Fresno - Barstow - San Diago". (Reverse path also works.)

4. Let $A = \emptyset$, $B = \{\emptyset\}$, $C = \{\emptyset, \{\emptyset\}\}$.

a. Which of these three sets has/have $\emptyset$ as an *element*?
b. Which of these three sets has/have $\emptyset$ as a *subset*?

Solution

a. B, C

b. A, B, C