Recitation 9 Solution

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1. Graph theory lets us analyze the structure of tournaments in sports.

   a. Suppose 64 basketball teams are selected to compete in March Madness. Estimate the number of games that are played in the tournament using a graph-theoretic approach.

   b. Suppose, now, that instead of the usual elimination-style tournament (involving a round-of-64, round-of-32, Sweet 16, Elite 8, etc), the NCAA decides to change its rules. It is now a "Last Team Standing" tournament; the top seed is given the Championship Belt, and teams *in random order* are allowed to challenge the Belt Holder for a chance to win the Belt. Once a team wins the Belt, they are crowned the Champion, and the previous Belt Holder is eliminated. How many games are played in this tournament? (Again, use graph theory.)

   Solution

   63 in both cases (both are trees).

2. As you have (undoubtedly) realized by now, The real learning in CPRE 310 happens during recitations. Suppose it happened that 8 recitation sections were needed, with two or three TAs per section. The assignment of TA to recitation sections is as follows:

   R1: Maverick, Goose, Iceman

   R2: Maverick, Stinger, Viper

   R3: Goose, Merlin

   R4: Slider, Stinger, Cougar

   R5: Slider, Jester, Viper

   R6: Jester, Merlin

   R7: Jester, Stinger

   R8: Goose, Merlin, Viper

Two recitations can not be held in the same time slot if they share a common TA. The problem is to determine the minimum number of time slots required to complete all the recitations.

a. Model the above table using an undirected graph, where nodes denote recitation sections and the "shares TA" information are modeled by edges. Draw this graph, and clearly mark the nodes.

b. Assign colors to the nodes such that no two nodes connected by an edge are assigned the same color.

c. Use your answer above to find the minimum number of slots required to schedule the recitations.

Solution

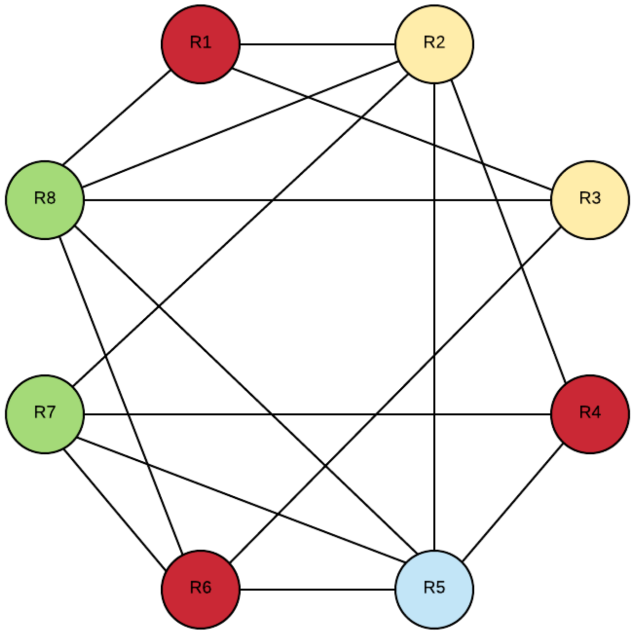

Figure 1: Answer for Problem 3

3. Recall that the complete graph K_n with n nodes is defined as the graph in which every pair of nodes are connected via an undirected edge.

a. Let e_n be the sequence denoting the number of edges in K_n. Evaluate the sequence $e_1, e_2, e_3, e_4, e_5, \ldots$.

b. Guess a recurrence relation for the number of edges, $e(n)$, in this graph. (Hint: how do you construct K_n given K_{n-1}?)

c. In the previous recitation, we used the first degree theorem to find a closed form expression for e_n. Recall it (or re-derive it).

d. Verify that your recurrence relation in part b is correct by plugging in the derived expressions for e_n and e_{n-1}.

Solution

a. $e_1 = 0$, $e_2 = 1$, $e_3 = 3$, $e_4 = 6$, $e_5 = 10$.

b. $e_n = e_{n-1} + n - 1$.

c. $e_n = \frac{n(n-1)}{2}$

d.

$$\begin{aligned}
e_n &= e_{n-1} + n - 1 \\
&= \frac{(n-1)(n-2)}{2} + n - 1 \\
&= \frac{(n-1)(n-2)}{2} + \frac{2(n-1)}{2} \\
&= \frac{(n-1)(n-2+2)}{2} \\
&= \frac{n(n-1)}{2}
\end{aligned}$$