

Recitation 8

- Here is a set of additional problems. They range from being very easy to very tough. The best way to learn the material in 310 is to solve problems on your own.
- Feel free to ask (and answer) questions about this problem set on Piazza.
- This is an **optional** problem set; do not turn this in for grading.
- While you don't have to turn this in, be warned that this material **can** appear in a quiz or exam.

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1. In a congregation of 97 people, a pastor instructs everyone to stand up and shake hands with exactly 3 other people (the pastor doesn't participate in this activity). Use graph theory to explain why this cannot be done.
 2. You are tasked with painting the centerlines of the streets in downtown. The map of downtown consists of blocks in a regular 3x3 grid, as shown in the graph below. (Each edge represents a street.)

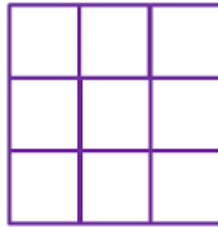


Figure 1: 3x3 grid of streets

- (a) Is it possible to paint all the centerlines without traversing a street in the above map more than once? Assume that all streets are two-way streets.
 - (b) Justify your answer using graph theory.
3. A complete graph with n nodes (denoted by the symbol K_n) is a simple, undirected graph with precisely one edge joining every pair of distinct nodes. Use the First Degree theorem to deduce the number of edges in this graph in terms of n .
 4. A simple graph is called *cubic* if *every* node has degree 3.
 - (a) Draw examples of cubic graphs with $n = 4, 6, 8$ nodes.
 - (b) Argue why you cannot draw cubic graphs with an odd number of nodes.
 5. Define the *distance* between two nodes in a graph as the number of edges along the shortest path between the nodes. Then, the *diameter* of a connected graph is the largest distance between any pair of nodes in the graph.

- (a) What is the biggest possible diameter for any connected graph with n nodes? Draw (or describe in words) a graph with this maximum diameter.
- (b) What is the smallest possible diameter for any connected graph with n nodes? Draw (or describe in words) a graph with this minimum diameter.