Sean Gordon
Sgordon4

CprE 310 HW7

1)

a. As order doesn't matter, this is equivalent to $\binom{15}{7}$ = 6435 ways

b.

i. $\binom{9}{4}\binom{6}{3}$ = 2520 ways

ii. As there are 6 possible CprE group members, each group of 7 by default must include an SE student, meaning this is simply equal to $\binom{15}{7}$ = 6435 ways

iii. If the group of seven caps at 4 CprE members, it excludes cases $\binom{9}{2}\binom{6}{5}$ and $\binom{9}{1}\binom{6}{6}$, meaning $\binom{15}{7} - \binom{9}{2}\binom{6}{5} - \binom{9}{1}\binom{6}{6}$ = 6210

2) Using the 'stars and bars' method, we want to partition 100 beans into 4 different sets(colors). As there are no restrictions, I assume that means not every color needs to be used. The result is $\binom{100+(4-1)}{(4-1)}$ = 176851 different combinations

3) The first rook has the ability to be placed anywhere on the board, so in any of the 8*8 = 64 slots. The second rook can now only be placed in any files not occupied by that rook, so in 7*7 = 49 slots. This leaves the total ways both can be placed at (64 * 49) = 3136. However, with this method each slot is counted twice because the rooks are presumed non-identical, but swapping them would make no difference so the result must be divided by 2: 3136 / 2 = 1568 possible ways

4)

a. As for each one of the *p* beads we can independently chose any of the α colors, this can be represented as α*α*α*…*α *p* times, leaving us with a total of α^p possible sequences.

b. There are only α sequences with a full sequence of one color, so the number of sequences with at least 2 colors is found by $\alpha^p - \alpha$

c. For one bracelet to have two indistinct sequences, it must have a sequence of beads that repeats, which can only occur if the sequence has an even number of beads or it is made entirely of one color. The former, when we remember that per-bracelet α > 2, can only happen with non-prime numbers (and 2, but for

p = 2, both beads must be of differing colors), and the latter cannot happen *because* $\alpha > 2$. Therefore, as long as p is prime and $\alpha > 2$, there cannot be any repeating sets within a sequence. As there are no repeating sets allowed, each rotation of the bracelet split at the same point must be unique. For a bracelet of size p, there are p possible states(rotations). Therefore there are p distinct sequences that can create a bracelet of size p.

d. As there are $\alpha^p - \alpha$ unique p-length sequences when p is prime and $\alpha > 2$, because every p unique sequences can be partitioned into 'necklaces' it is guaranteed that $\alpha^p - \alpha$ is divisible by p.