

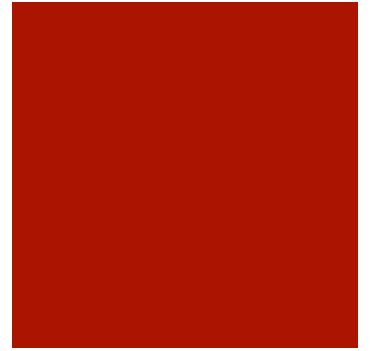


COM S 342

Recitation 09/09/2019 – 09/11/2019

Topic

- Context free grammar:
 - Derivation,
 - Parse tree,
 - Grammar design,
 - Remove ambiguity.



Grammar (review)

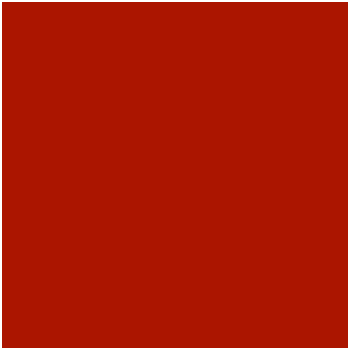


- Specification of legal strings in a programming language.
- Key elements:
 - α Production rule, non-terminal, terminal.
 - What symbols can be expanded?
 - Backus Naur Form (BNF)
 - * means zero or more, + means one or more, | defines alternatives
 - α Syntax derivation: leftmost, rightmost
 - Systematic proof that a string belongs to a language
 - Replace exactly one symbol in a single step

Derivation

- Grammar:

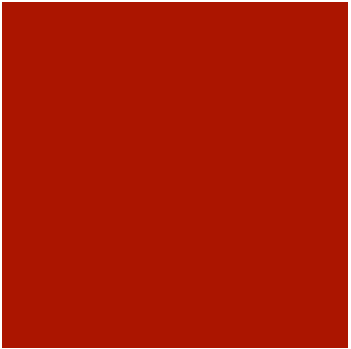
Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Write leftmost and rightmost derivation of the string “babbab”.


$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow AAA \\ A &\rightarrow a \\ A &\rightarrow bA \\ A &\rightarrow Ab \end{aligned}$$

Derivation

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Write leftmost and rightmost derivation of the string “babbab”.


$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow AAA \\ A &\rightarrow a \\ A &\rightarrow bA \\ A &\rightarrow Ab \end{aligned}$$

Leftmost Derivation: At each derivation point, the leftmost non-terminal is expanded.

Derivation

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Write leftmost and rightmost derivation of the string “babbab”.

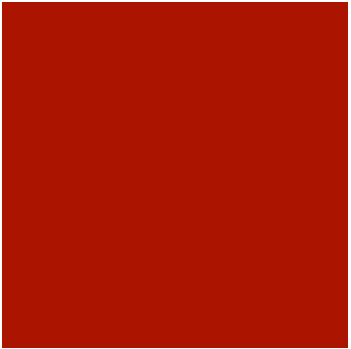
$S \rightarrow AA \rightarrow bAA \rightarrow bAbA \rightarrow babA \rightarrow babbA$
 $\rightarrow babbAb \rightarrow babbab$

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$

Derivation

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Write leftmost and rightmost derivation of the string “babbab”.


$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow AAA \\ A &\rightarrow a \\ A &\rightarrow bA \\ A &\rightarrow Ab \end{aligned}$$

Rightmost Derivation: At each derivation point, the rightmost non-terminal is expanded.

Derivation

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Write leftmost and rightmost derivation of the string “babbab”.

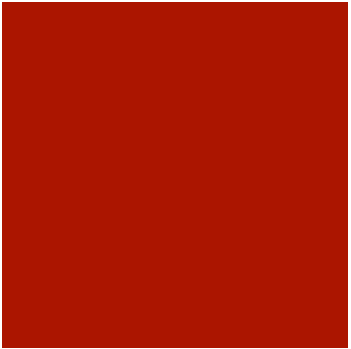
$S \rightarrow AA \rightarrow AAb \rightarrow Aab \rightarrow Abab \rightarrow Abbab$
 $\rightarrow bAbbab \rightarrow babbab$

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$

Parse Tree

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol).


$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow AAA \\ A &\rightarrow a \\ A &\rightarrow bA \\ A &\rightarrow Ab \end{aligned}$$

A parse tree results from the derivation sequence.

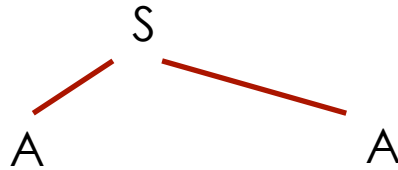
- Each node in the tree is a terminal or non-terminal in the production rule.
- Each edge in the tree from a non-terminal results from the application of production rule on the non-terminal.
- Application of production rule always result in new nodes in the tree.
- A terminal is a leaf node

Parse Tree

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Draw a parse tree for the string “babbab”.

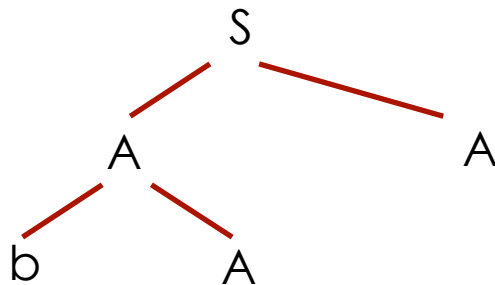
$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$



Parse Tree

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Draw a parse tree for the string “babbab”.



$$S \rightarrow AA$$

$$A \rightarrow AAA$$

$$A \rightarrow a$$

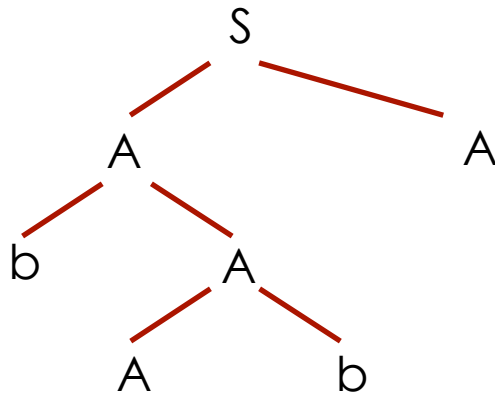
$$A \rightarrow bA$$

$$A \rightarrow Ab$$

1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 26

- Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Draw a parse tree for the string “babbab”.

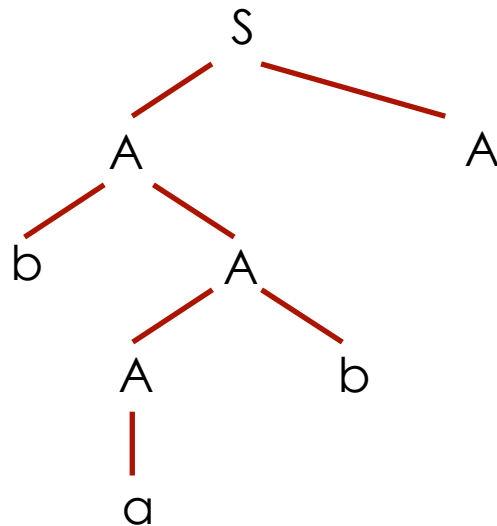
$$S \rightarrow AA$$
$$A \rightarrow AAA$$
$$A \rightarrow a$$
$$A \rightarrow bA$$
$$A \rightarrow Ab$$


Parse Tree

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Draw a parse tree for the string “babbab”.

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$

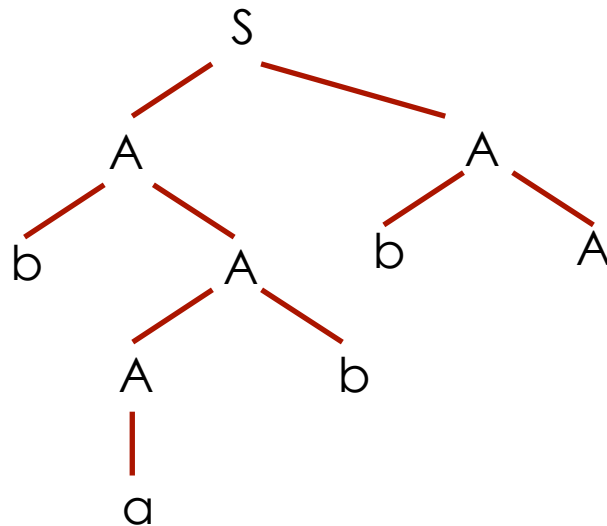


Parse Tree

■ Grammar:

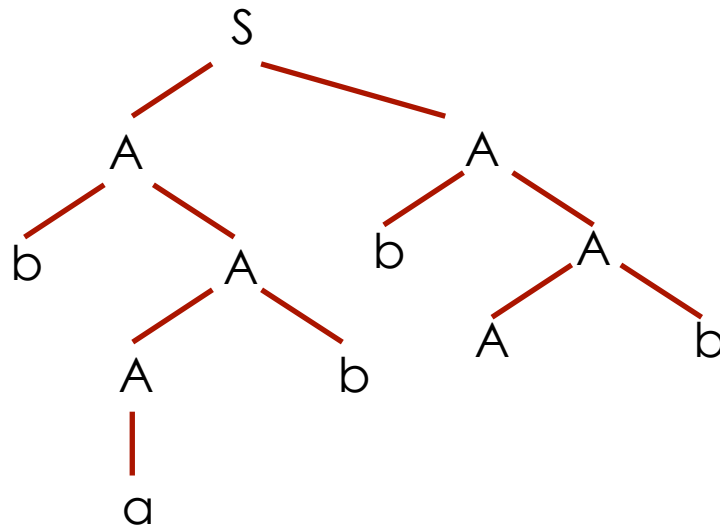
Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Draw a parse tree for the string “babbab”.

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$



1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 26

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Draw a parse tree for the string “babbab”.

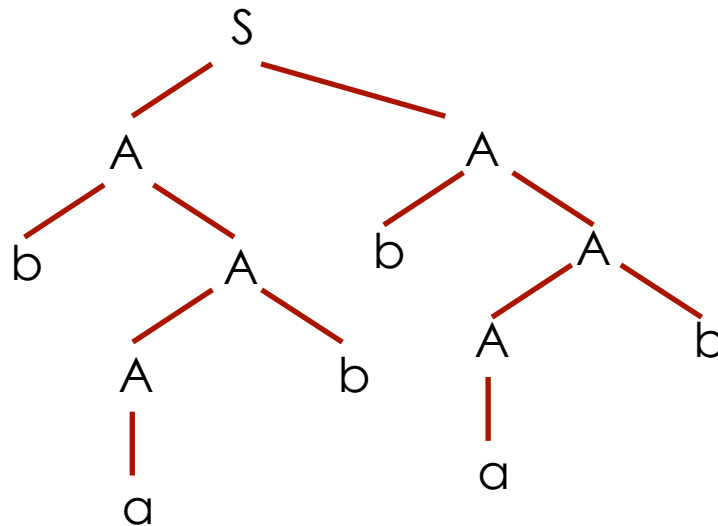
$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow AAA \\ A &\rightarrow a \\ A &\rightarrow bA \\ A &\rightarrow Ab \end{aligned}$$


Parse Tree

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol). Draw a parse tree for the string “babbab”.

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$



Ambiguity

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol).

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$

Prove or disprove the grammar is ambiguous.

Ambiguity

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol).

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$

Prove or disprove the grammar is ambiguous.

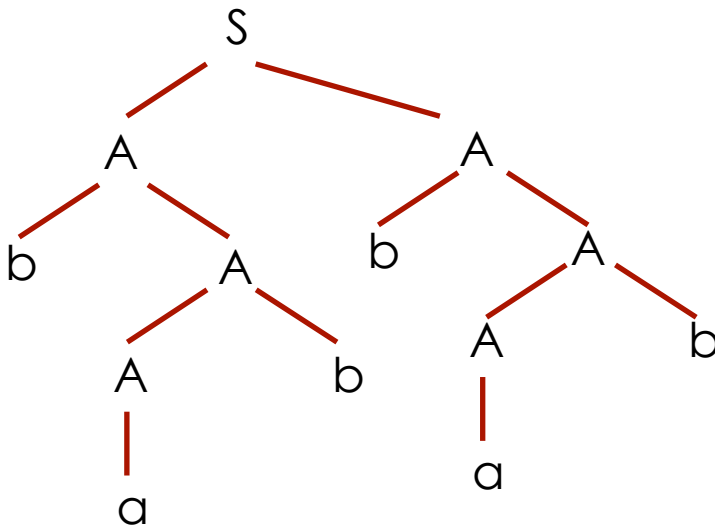
- A grammar is *ambiguous* if there exists at least two distinct parse trees for the derivation of the same string.

Ambiguity

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol).

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$



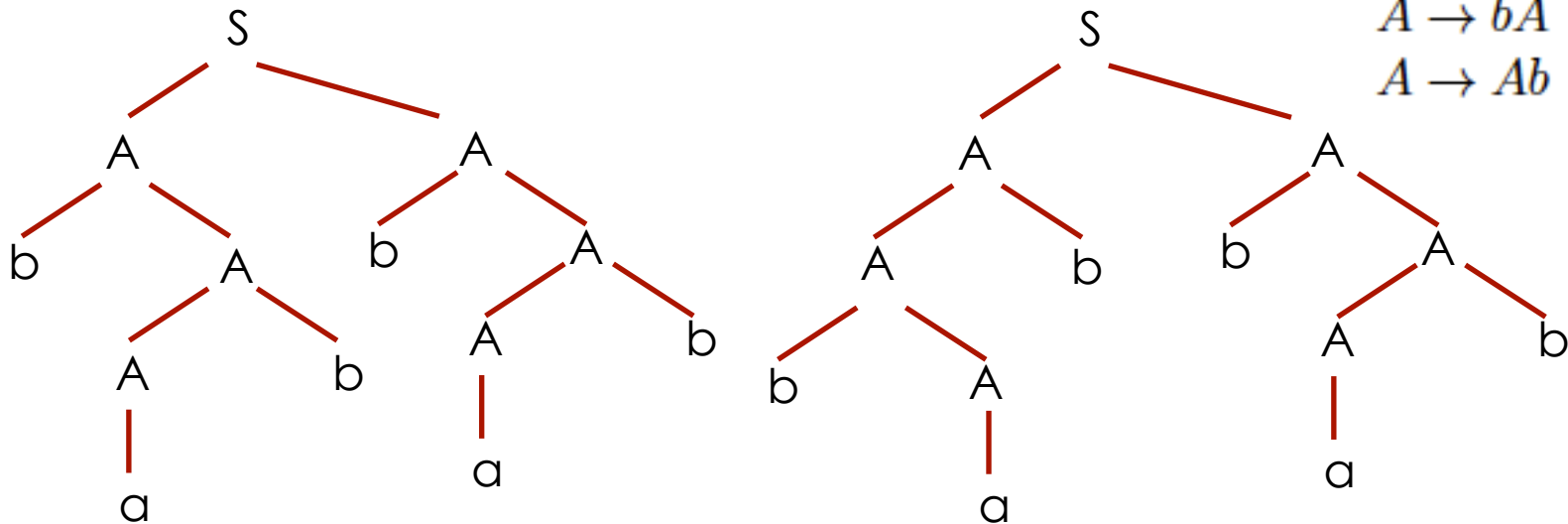
?

Ambiguity

■ Grammar:

Consider the following production rules of a grammar G (all lower-case letters are terminals and S is the start symbol).

$S \rightarrow AA$
 $A \rightarrow AAA$
 $A \rightarrow a$
 $A \rightarrow bA$
 $A \rightarrow Ab$



Ambiguous!

Ambiguity



Remove ambiguity:

- Add delimiters (e.g., parenthesis; begin and end in if statements)
- Add operator precedence and associativity

Operator Precedence:

$$S \rightarrow S @ S \mid S \# S \mid b$$

Example:

$$b @ b \# b @ b$$

Ambiguity



Remove ambiguity:

- Operator Precedence:
- If more than one operator is present in the expression, the precedence order decides the order in which the operators should be applied.
 - Add non-terminals for each precedence level. Push the higher levels towards the bottom of the parse-tree (stratification of tree)

$$S \rightarrow S @ S \mid S \# S \mid b$$

Example:

b @ b # b @ b

Ambiguity

Remove ambiguity:

Operator Precedence:

- If more than one operator is present in the expression, the precedence order decides the order in which the operators should be applied.
- Add non-terminals for each precedence level. Push the higher levels towards the bottom of the parse-tree (stratification of tree)

$$S \rightarrow S @ S \mid S \# S \mid b$$

Example:

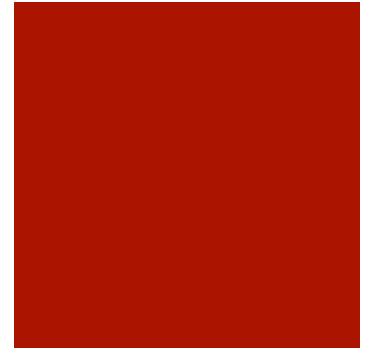
b @ b # b @ b

$$\begin{aligned} S &\rightarrow S @ S \mid A \\ A &\rightarrow A \# A \mid b \end{aligned}$$

Example:

b @ b # b @ b

Ambiguity



Remove ambiguity:

Associativity:

If the same operator appears more than once in the same expression, then associativity rule decides the order in which the operators should be applied.

Ambiguity

Remove ambiguity:

Associativity:

$$\begin{array}{lcl} S & \longrightarrow & S + S \mid S - S \mid T \\ T & \longrightarrow & T * T \mid T / T \mid \text{part} \end{array}$$

Example string: $3 + 4 + 2$.

The grammar allows two different derivation trees for the string $3 + 4 + 2$, one corresponding to the structure $(3 + 4) + 2$ and one corresponding to the structure $3 + (4 + 2)$.

Ambiguity

Remove ambiguity:

Associativity:

$$\begin{array}{lcl} S & \longrightarrow & S + S \mid S - S \mid T \\ T & \longrightarrow & T * T \mid T / T \mid \text{part} \end{array}$$

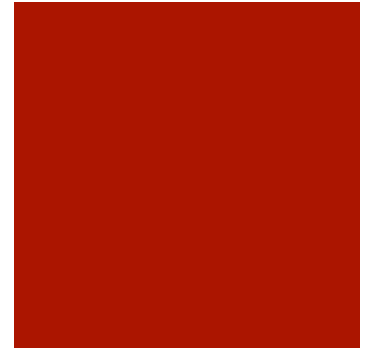
For the op in question,

We can impose left-associativity (resp. right-associativity) by using a left-recursive (resp. right-recursive) production

$$\begin{array}{lcl} S & \longrightarrow & S + T \mid S - S \mid T \\ T & \longrightarrow & T * T \mid T / T \mid \text{part} \end{array}$$

Designing a Grammar

Design a grammar that allows $(a|b)^*abb$



Designing a Grammar

Design a grammar that allows $(a|b)^*abb$

Example accepted strings:

abb

babb

baabb

ababb



Designing a Grammar

Design a grammar that allows $(a|b)^*abb$

$$S \rightarrow aS \mid bS \mid aA_1$$

$$A_1 \rightarrow bA_2$$

$$A_2 \rightarrow bA_3$$

$$A_3 \rightarrow \varepsilon$$

