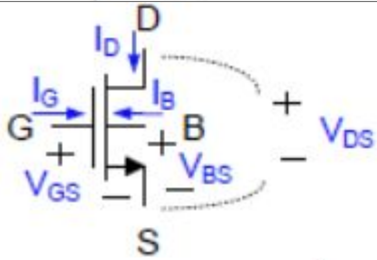
Sqr Law

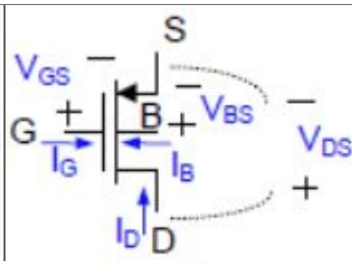
$$I_D = \begin{cases} 0 & V_{GS} \le V_T & \text{Cutoff} \\ \mu C_{OX}\dfrac{W}{L}\left(V_{GS}-V_T-\dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS}-V_T & \text{Triode} \\ \mu C_{OX}\dfrac{W}{2L}\left(V_{GS}-V_T\right)^2 & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS}-V_T & \text{Saturation} \end{cases}$$

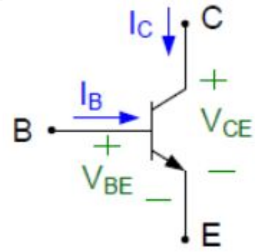
Extended

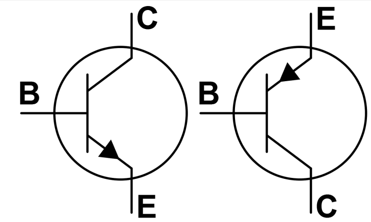
$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{ox}\dfrac{W}{L}\left(V_{GS}-V_T-\dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS}-V_T \\ \mu C_{ox}\dfrac{W}{2L}\left(V_{GS}-V_T\right)^2 \bullet \left(1+\lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS}-V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

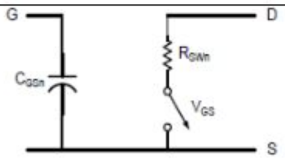








NPN PNP



Determine R_{SW} and C_{GS} for an n-channel MOSFET from square-law model in the 0.5u ON CMOS process if L=1u, W=1u

(Assume μC_{OX}=100µAV^{-2}, C_{OX}=2.5fFu^{-2}, V_{T0}=1V, V_{DD}=3.5V, V_{SS}=0)

When on operating in deep triode

$$I_D = \mu C_{ox}\frac{W}{L}\left(V_{GS}-V_T-\frac{V_{DS}}{2}\right)V_{DS} \cong \mu C_{ox}\frac{W}{L}\left(V_{GS}-V_T\right)V_{DS}$$

$$R_{SQ} = \left.\frac{V_{DS}}{I_D}\right|_{V_{GS}=V_{DD}} = \left.\frac{1}{\mu C_{ox}\frac{W}{L}\left(V_{GS}-V_T\right)}\right|_{V_{GS}=3.5V} = \frac{1}{(E-4)\left(\frac{1}{1}\right)(3.5-1)} = 4K\Omega$$

C_{GS}= $C_{OX}WL$ = (2.5fFµ^{-2})(1µ^2) = 2.5fF

$$I_C = \beta I_B$$

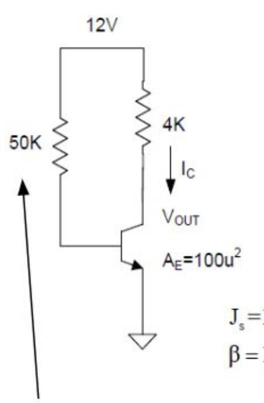
$V_{BE} = 0.6V$ | V_{BE}>0.4V, V_{BC}<0 | **Forward Active**

$$V_t = \frac{kT}{q}$$

V_{BE}=0.7V, V_{CE}=0.2V | I_C<βI_B | **Saturation**

I_C=I_B=0 | V_{BE}<0, V_{BC}<0 | **Cutoff**

Example: Determine I_C and V_{OUT}.



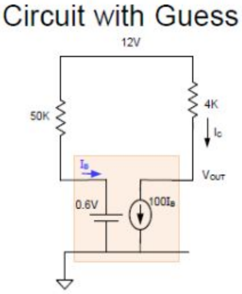
$J_s = 10^{-16} A/\mu^2$

$\beta = 100$

500k → 50k

Solution:

1. Guess Forward Active Region
2. Solve Circuit with Guess



$$I_B = \frac{(12-0.6)}{50K}$$

$$I_C = \beta I_B = 100\frac{(12-0.6)}{50K} = 22.8mA$$

$$V_{OUT} = 12 - I_C \bullet 4K = -79.2V$$

3. Verify FA Region V_{BE}>0.4V and V_{BC}<0

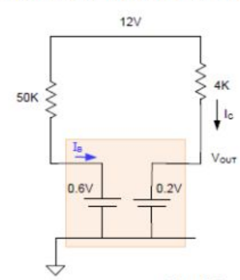
$$V_{BE} = 0.6V > 0.4V$$

$$V_{BC} = 0.6V - -79.2V = +79.8V > 0$$

Verify Fails so solution is not valid

Solution:

4. Guess Saturation
5. Solve Circuit with Guess



$$I_B = \frac{(12-0.6)}{50K} = 228\mu A$$

$$I_C = \frac{(12-0.2)}{4K} = 2.95mA$$

$$V_{OUT} = 0.2V$$

6. Verify SAT Region I_C<βI_B

$$\beta I_B = 100 \bullet 228\mu A = 22.8mA$$

$$I_C = 2.95mA$$

$$I_C = 2.95mA < \beta I_B = 22.8mA$$

Verify Passes so solution is valid

$$I_C = 2.95mA \qquad V_{OUT} = 0.2V$$

$I_G = 0$

$I_B = 0$

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX}\dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX}\dfrac{W}{2L}(V_{GS} - V_T)^2 \bullet (1+\lambda V_{DS}) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

$$y_{11} = \left.\frac{\partial I_G}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad y_{12} = \left.\frac{\partial I_G}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad y_{13} = \left.\frac{\partial I_G}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$y_{21} = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = g_m \qquad y_{22} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = g_o \qquad y_{23} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = g_{mb}$$

$$y_{31} = \left.\frac{\partial I_B}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad y_{32} = \left.\frac{\partial I_B}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad y_{33} = \left.\frac{\partial I_B}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

Small-Signal 4-terminal Model Extension

$$I_D = \mu C_{OX}\frac{W}{2L}(V_{GS} - V_T)^2 \bullet (1+\lambda V_{DS})$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Definition:

$V_{EB} = V_{GS} - V_T$

$V_{EBQ} = V_{GSQ} - V_{TQ}$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX}\frac{W}{\cancel{2}L}\cancel{2}(V_{GS} - V_T)^1 \bullet (1+\lambda V_{DS})\Big|_{\vec{V}=\vec{V}_Q} \cong \mu C_{OX}\frac{W}{L}V_{EBQ}$$

Same as 3-term

$$g_o = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX}\frac{W}{2L}2(V_{GS} - V_T)^2 \bullet \lambda\Big|_{\vec{V}=\vec{V}_Q} \cong \lambda I_{DQ}$$

Same as 3-term

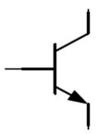
$$g_{mb} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX}\frac{W}{\cancel{2}L}\cancel{2}(V_{GS} - V_T)^1 \bullet \left(-\frac{\partial V_T}{\partial V_{BS}}\right) \bullet (1+\lambda V_{DS})\Big|_{\vec{V}=\vec{V}_Q}$$

$$g_{mb} = \left.\frac{\partial I_D}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} \cong \mu C_{OX}\frac{W}{L}V_{EBQ} \bullet \left.\frac{\partial V_T}{\partial V_{BS}}\right|_{\vec{V}=\vec{V}_Q} = \left(\mu C_{OX}\frac{W}{L}V_{EBQ}\right)(-\cancel{1})\gamma\frac{1}{2}(\phi - V_{BS})^{-\frac{1}{2}}\Big|_{\vec{V}=\vec{V}_Q}(-\cancel{1})$$

$$g_{mb} \cong g_m\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}$$

Large and Small Signal Model Summary

Large Signal Model



$$I_C = \beta I_B\left(1+\frac{V_{CE}}{V_{AF}}\right) \qquad V_{BE}>0.4V$$

$$I_B = \frac{J_S A_E}{\beta}e^{\frac{V_{BE}}{V_t}} \qquad V_{BC}<0$$

Forward Active

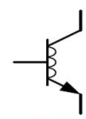
$V_{BE}=0.7V$ $\quad I_C<\beta I_B$

$V_{CE}=0.2V$

$I_C=I_B=0$ $\quad V_{BE}<0$

$V_{BC}<0$

Small Signal Model



Forward Active

$$i_b = g_\pi v_{be}$$

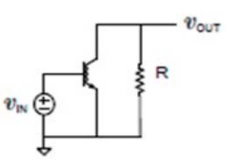
$$i_c = g_m v_{be} + g_0 v_{ce}$$

where

$$g_m = \frac{I_{CQ}}{V_t}$$

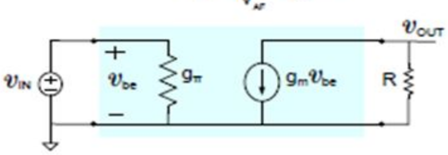
$$g_\pi = \frac{I_{CQ}}{\beta V_t}$$

$$g_o \cong \frac{I_{CQ}}{V_{AF}}$$



Neglect V_{AF} effects (i.e. $V_{AF}=\infty$) to be consistent with earlier analysis

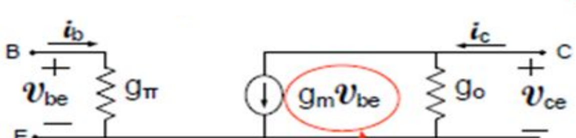
$$g_o = \frac{I_{CQ}}{V_{AF}} \underset{V_{AF}=\infty}{=} 0$$



$$\left.\begin{aligned} v_{OUT} &= -g_m R v_{BE} \\ v_{IN} &= v_{BE} \end{aligned}\right\} \qquad A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R$$

$$g_m = \frac{I_{CQ}}{V_t}$$

$$A_V = -\frac{I_{CQ}R}{V_t}$$



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Observe :

$$g_\pi v_{be} = i_b$$

$$g_m v_{be} = i_b\frac{g_m}{g_\pi}$$

$$\frac{g_m}{g_\pi} = \frac{\left[\frac{I_Q}{V_t}\right]}{\left[\frac{I_Q}{\beta V_t}\right]} = \beta$$

$$g_m v_{be} = \beta i_b$$

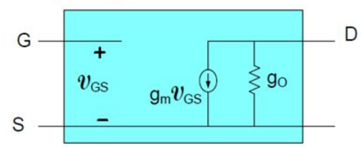
Can replace the voltage dependent current source with a current dependent current source

Review from last lecture

Small-Signal Model of MOSFET

$$g_m = \mu C_{OX}\frac{W}{L}(V_{GSQ} - V_T)$$

$$g_o \cong \lambda I_{DQ}$$

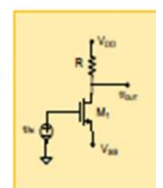


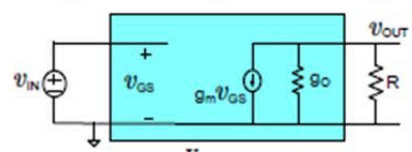
Alternate g_m

$$g_m = \sqrt{2\mu C_{OX}\frac{W}{L}} \bullet \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$

Small-signal analysis example

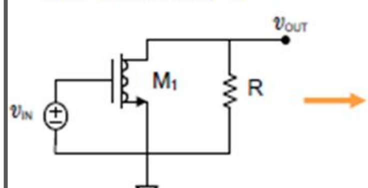




$$A_v = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_o + 1/R}$$

This gain is expressed in terms of small-signal model parameters

For $\lambda=0$, $g_O = \lambda I_{DQ} = 0$



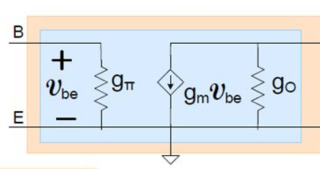
$$A_v = \frac{v_{OUT}}{v_{IN}} = -g_m R$$

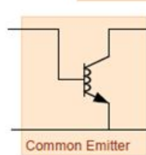
but

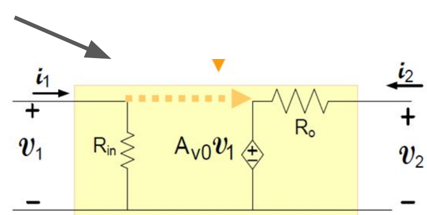
$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} \qquad V_{GSQ} = -V_{SS}$$

thus

$$A_v = \frac{2I_{DQ}R}{[V_{SS} + V_T]}$$





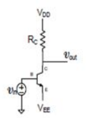


By Thevenin : Norton Transformations

$$R_{in} = \frac{1}{g_\pi} \qquad A_{V0} = -\frac{g_m}{g_0} \qquad R_0 = \frac{1}{g_0} \qquad A_{VR} = 0$$

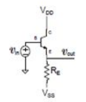
Basic Amplifier Characteristics Summary

CE/CS



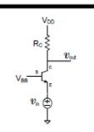
- Large inverting gain
- Moderate input impedance
- Moderate (or high) output impedance
- Widely used as the basic high gain inverting amplifier

CC/CD



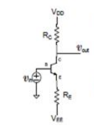
- Gain very close to +1 (little less)
- High input impedance for BJT (high for MOS)
- Low output impedance
- Widely used as a buffer

CB/CG



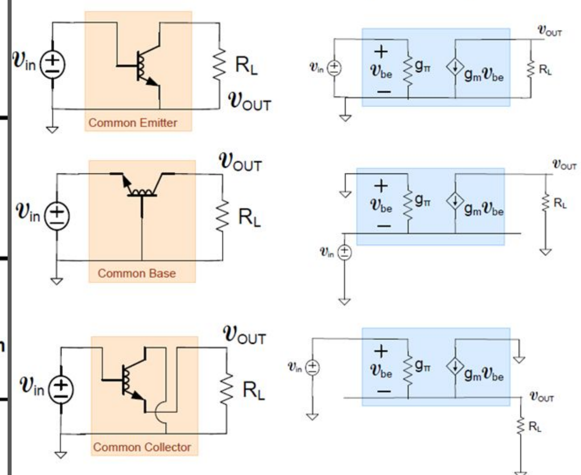
- Large noninverting gain
- Low input impedance
- Moderate (or high) output impedance
- Used more as current amplifier or, in conjunction with CD/CS to form two-stage cascode

CEwRE/ CSwRS



- Reasonably accurate but somewhat small gain (resistor ratio)
- High input impedance
- Moderate output impedance
- Used when more accurate gain is required

The three basic amplifier types for both MOS and bipolar processes



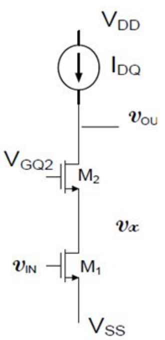
$$\left.\begin{aligned} v_{OUT} &= -g_m R_L v_{be} \\ v_{IN} &= v_{be} \end{aligned}\right\} \qquad A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R_L$$

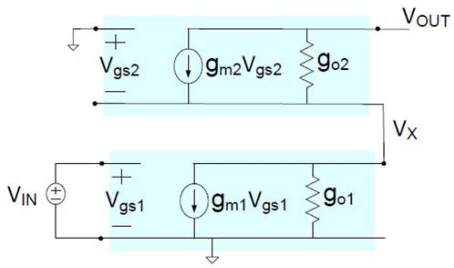
$$\left.\begin{aligned} v_{OUT} &= -g_m R_L v_{be} \\ v_{IN} &= -v_{be} \end{aligned}\right\} \qquad A_V = \frac{v_{OUT}}{v_{IN}} = g_m R_L$$

$$\left.\begin{aligned} v_{OUT} &= (g_m + g_\pi) v_{be} R_L \\ v_{IN} &= v_{be} + (g_m + g_\pi) v_{be} R_L \end{aligned}\right\} \qquad A_V = \frac{v_{OUT}}{v_{IN}} = \frac{(g_m+g_\pi)R_L}{1+(g_m+g_\pi)R_L} \cong 1$$

- Significantly different gain characteristics for the three basic amplifiers
- There are other significant differences too (R_{IN}, R_{OUT}, …) as well

Example: Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda \neq 0$





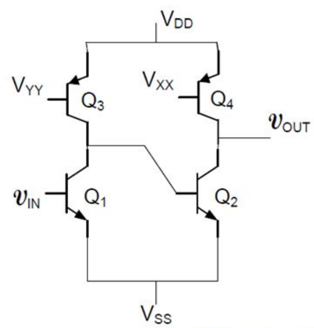
By KCL

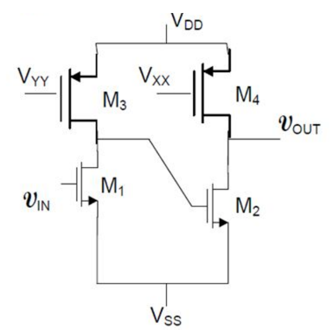
$$\left.\begin{aligned} g_{m1} v_{IN} + g_{o1} v_X &= g_{m2} v_{GS2} + g_{o2}(v_{OUT} - v_X) \\ v_{GS2} &= -v_X \\ (v_{OUT} - v_X) g_{o2} + g_{m2} v_{GS2} &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} g_{m1} v_{IN} + (g_{m2} + g_{o1} + g_{o2}) v_X &= g_{o2} v_{OUT} \\ v_{OUT} g_{o2} &= (g_{m2} + g_{o2}) v_X \end{aligned}\right\}$$

thus:

$$A_v = \frac{v_{OUT}}{v_{IN}} = -\frac{g_{m1}g_{m2} + g_{m1}g_{o2}}{g_{o1}g_{o2}} \cong -\frac{g_{m1}}{g_{o1}}\frac{g_{m2}}{g_{o2}}$$

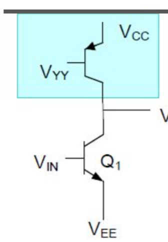


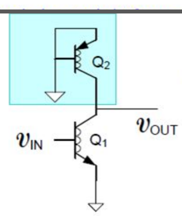


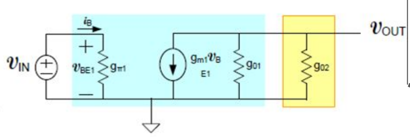
Two-stage CE:CE or CS:CS Cascade

$$A_{VCB} \cong \left[\frac{-g_{m1}}{g_{01}+g_{03}+g_{\pi2}}\right]\left[\frac{-g_{m2}}{g_{02}+g_{04}}\right] \cong \frac{g_{m1}g_{m2}}{2g_{\pi2}g_{02}} = \beta\frac{g_{m1}}{2g_{02}}$$

$$A_{VCM} = \left[\frac{-g_{m1}}{g_{01}+g_{03}}\right]\left[\frac{-g_{m2}}{g_{02}+g_{04}}\right] = \frac{g_{m1}g_{m2}}{4g_{01}g_{02}}$$





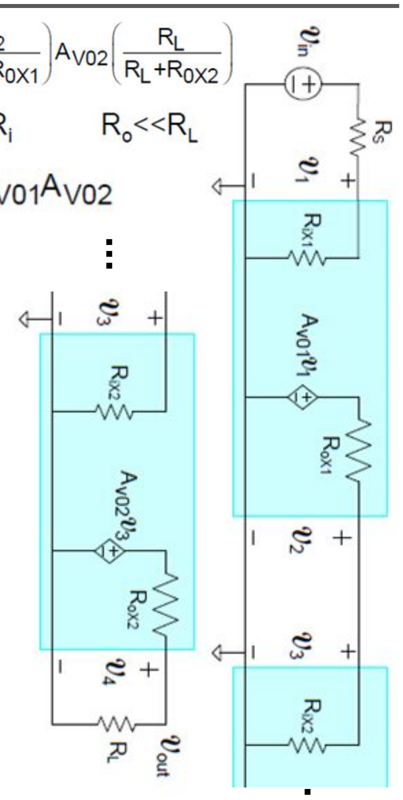


$$A_V = \frac{-g_{m1}}{g_{01}+g_{02}} \cong \frac{-g_{m1}}{2g_{01}}$$

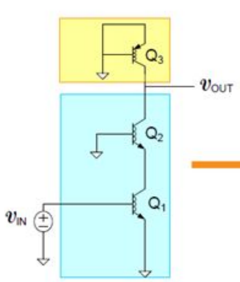
$$A_V = \frac{v_{out}}{v_{in}} = \left(\frac{R_{iX1}}{R_{iX1}+R_S}\right)A_{V01}\left(\frac{R_{iX2}}{R_{iX2}+R_{0X1}}\right)A_{V02}\left(\frac{R_L}{R_L+R_{0X2}}\right)$$

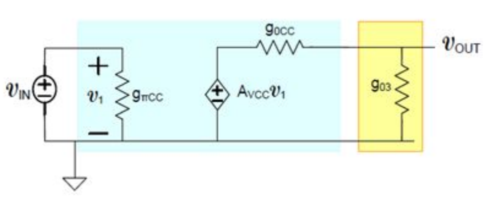
If $R_o \ll R_i$ $\qquad R_S \ll R_i$ $\qquad R_o \ll R_L$

$$A_V \cong A_{V01}A_{V02}$$



High-gain amplifier comparisons





It thus follows that

$$A_V = A_{VCC}\left[\frac{g_{0CC}}{g_{03}+g_{0CC}}\right]$$

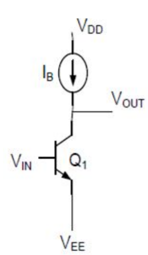
But $g_{0CC} \simeq g_{03}/\beta$

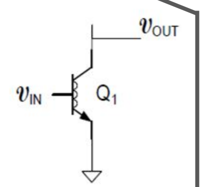
$$A_V \cong A_{VCC}\left[\frac{g_{0CC}}{g_{03}}\right] \cong \frac{A_{VCC}}{\beta}$$

$$A_V \cong A_{VCC}\left[\frac{g_{0CC}}{g_{03}}\right] \cong \frac{A_{VCC}}{\beta}$$

But recall

$$A_{VCC} \cong -\left[\frac{g_{m1}}{g_{01}}\right]\beta$$





$$A_V = \frac{-g_m}{g_0}$$

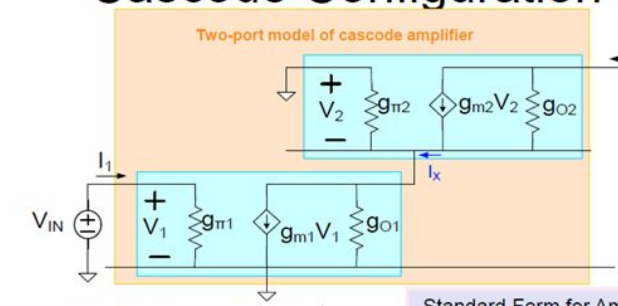
Thus

$$A_V \cong -\left[\frac{g_{m1}}{g_{01}}\right]$$

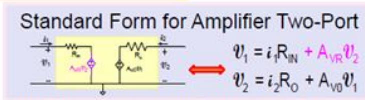
$$A_V \cong -\left[\frac{I_{CQ}/V_t}{I_{CQ}/V_{AF}}\right] = -\left[\frac{V_{AF}}{V_t}\right] \cong -8000$$

Cascode Configuration

Discuss



$$\left.\begin{array}{l}(V_X+V_2)g_{02}+V_2g_{m2}=I_X \\ V_1\, g_{m1}-V_2(g_{01}+g_{\pi 2})=I_X\end{array}\right\}$$



Observing V_1=V_{IN} and eliminating V_2 between these two equations, we obtain

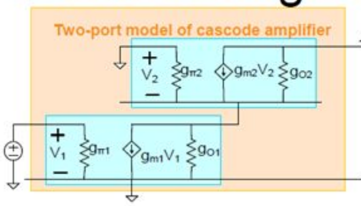
$$V_{IN}=I_1\bullet\frac{1}{g_{\pi 1}}$$

and

$$V_X=I_X\bullet\left[\frac{g_{01}+g_{02}+g_{\pi 2}+g_{m2}}{g_{02}(g_{01}+g_{\pi 2})}\right]-V_{IN}\bullet\left[\frac{g_{m1}(g_{02}+g_{m2})}{g_{02}(g_{\pi 2}+g_{01})}\right]$$

Cascode Configuration

Discuss



$$V_X=I_X\bullet\left[\frac{g_{01}+g_{02}+g_{\pi 2}+g_{m2}}{g_{02}(g_{01}+g_{\pi 2})}\right]-V_{IN}\bullet\left[\frac{g_{m1}(g_{02}+g_{m2})}{g_{02}(g_{\pi 2}+g_{01})}\right]$$

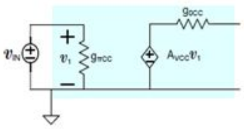
$$V_{IN}=I_1\bullet\frac{1}{g_{\pi 1}}$$

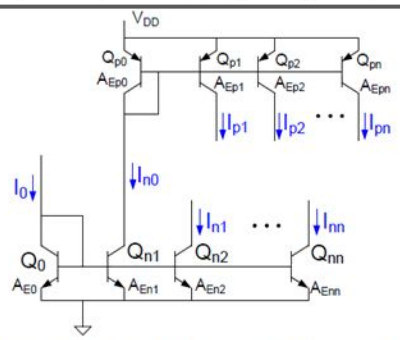
It thus follows for the npn bipolar structure that :

$$A_{VCC}=-\left[\frac{g_{m1}(g_{02}+g_{m2})}{g_{02}(g_{\pi 2}+g_{01})}\right]\cong-\left[\frac{g_{m1}g_{m2}}{g_{02}g_{\pi 2}}\right]$$

$$g_{0CC}=\left[\frac{g_{02}(g_{01}+g_{\pi 2})}{g_{01}+g_{02}+g_{\pi 2}+g_{m2}}\right]\cong\left[\frac{g_{02}g_{\pi 2}}{g_{m2}}\right]$$

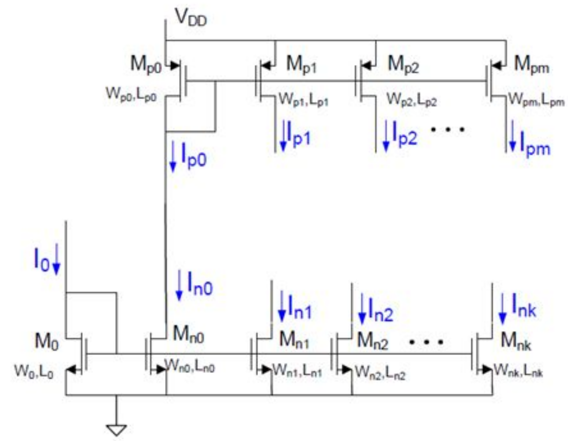
$$g_{\pi CC}=g_{\pi 1}$$





Multiple-Output Bipolar Current Source and Sink

$$I_{nk}=\left[\frac{A_{Enk}}{A_{E0}}\right]I_0 \qquad I_{pk}=\left[\frac{A_{En1}}{A_{E0}}\right]\left[\frac{A_{Epk}}{A_{Ep0}}\right]I_0$$

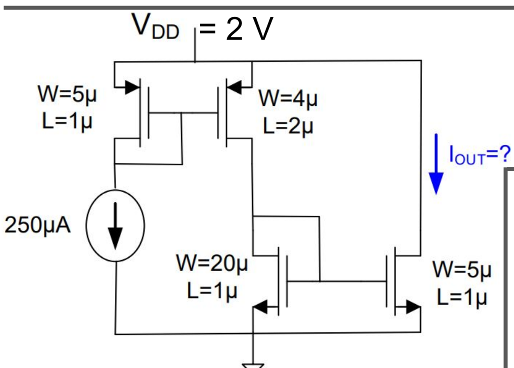


$$I_{pj}=\left[\frac{W_{pj}}{L_{pj}}\bullet\frac{L_{p0}}{W_{p0}}\right]MI_0$$

$$M=\left[\frac{W_{n0}}{L_{n0j}}\bullet\frac{L_0}{W_0}\right]$$

$$I_{nj}=\left[\frac{W_{nj}}{L_{nj}}\bullet\frac{L_0}{W_0}\right]I_0$$

m and k may be different
Often M=1



This is two current mirrors,

$$I_{out}=\frac{(W2/L2)}{(W1/L1)}*\frac{(W4/L4)}{(W3/L3)}*I_{in}=\frac{(4/2)}{(5/1)}*\frac{(5/1)}{(20/1)}250\mu A=\frac{2}{5}*\frac{5}{20}*250\mu$$

$$I_{out}=25\mu A$$

Problem 2 Assume you have available a supply voltage of V_{DD}=3V and a 10mA sourcing current (one end connected to V_{DD}).

a) Design a current mirror that provides two outputs, a sinking current of 100uA and a sourcing current of 10mA using MOS transistors.

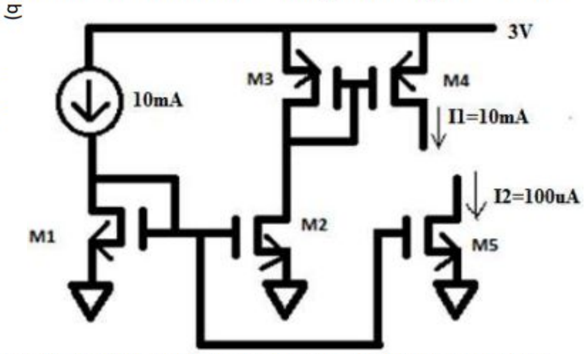
b) What is the maximum voltage for the 10mA current source for your design if it is to work as a current mirror.

b)

$$I_1=\frac{\mu_p C_{ox} W_4}{2L_4}(|V_{GS}|-|V_T|)^2=10mA$$

$$\frac{33*10^{-6}*2}{2*1}(|V_{GS}|-|V_T|)^2=10mA$$

$$|V_{GS}|-|V_T|=\sqrt{\frac{2*1*10mA}{33*10^{-6}*2}}=17.41V$$



a) With this we have to choose the sizes of M_1 and M_5 to determine the 100μA sinking current and M_1, M_2, M_3, and M_4 to create the 10mA sourcing current

As we can choose whatever sizes we want, I am going to use $W_1 = L_1 = 1\mu$, so $\frac{W_1}{L_1}$ = 1.

From there I can set M_5, I want $\frac{W_5}{L_5}=\frac{100\mu A}{10mA}$ = 0.01.

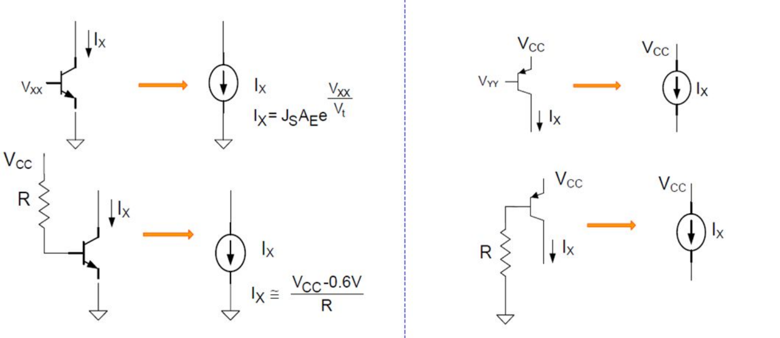
So I will use $W_5 = 1\mu$ and $L_5 = 100\mu$.

We then want to convert 10mA to 10mA, or times 1.This can be easily done in a single mirror, so I will use,

$$M_2 = L_2 = 1\mu, \qquad M_3 = L_3 = 1\mu, \qquad M_4 = 1\mu, L_4 = 1\mu$$

Basic Current Sources and Sinks

Basic Bipolar Current Sinks **Basic Bipolar Current Sources**



- Very practical methods for biasing the BJTs (or MOSFETs) can be used
- Current Mirrors often used for generating sourcing and sinking currents
- Can think of biasing transistors with V_{XX} and V_{YY} in these current sources