ComS 474
Final Exam

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1 Regular Problems

1) $\begin{pmatrix} 1 & 1/2 & 1/2 \\ 1/3 & 1/2 & 1 \end{pmatrix} * \begin{pmatrix} 0.5 & 1 & 6 \\ 3 & -4 & 2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 & 3 \\ 1 & -2 & 2 \end{pmatrix}$

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2) (a) $\begin{pmatrix} 1 & 1/2 & 1/2 \\ 1/3 & 1/2 & 1 \end{pmatrix} * \begin{pmatrix} 0.5 & 3 \\ 1 & -4 \\ 6 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 6.667 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1/3 \\ 1/2 & 1/2 \\ 1/2 & 1 \end{pmatrix} * \begin{pmatrix} 0.5 & 1 & 6 \\ 3 & -4 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 6.667 \\ 2 & 1 \end{pmatrix}$

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3) $\left(\begin{pmatrix} 1 & 1/3 \\ 1/2 & 1/2 \\ 1/2 & 1 \end{pmatrix} * \begin{pmatrix} 0.5 & 1 & 6 \\ 3 & -4 & 2 \end{pmatrix} \right) + 1 \Rightarrow \begin{pmatrix} 4 & 6.667 \\ 2 & 1 \end{pmatrix} + 1 = \begin{pmatrix} 5 & 7.667 \\ 3 & 2 \end{pmatrix}$

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4) $\hat{y} = \phi(w^T x) = ((1/2) * 2)^2 + ((1/3) * 3)^2 + ((1/4) * 4)^2 + ((1/5) * 5)^2 = 4$

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5) As $\hat{y} = (w^T x)^2$...

(a) $\frac{\partial E}{\partial \hat{y}} = \frac{\partial (y+\hat{y})}{\partial \hat{y}} = \frac{\partial y}{\partial \hat{y}} + \frac{\partial \hat{y}}{\partial \hat{y}} = 0+1 = 1$

(b) $\frac{\partial \hat{y}}{\partial w^T x} = \frac{\partial (w^T x)^2}{\partial w^T x} = \frac{\partial (u)^2}{\partial u} = 2u = 2w^T x =$
$2((1/2)*2) + 2((1/3)*3) + 2((1/4)*4) + 2((1/5)*5) = 8$

(c) $\frac{\partial w^T x}{\partial x_1} = \frac{\partial (w_0 x_0 + w_1 x_1 + w_2 x_2 + w_3 x_3)}{\partial x_1} = w_1$

(d) $\frac{\partial E}{\partial x_1} = \frac{\partial E}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w^T x} \frac{\partial w^T x}{\partial x_1} = 1 * 8 * w_1 = 1 * 8 * 3 = 24$

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6)

(a) $\frac{\partial E}{\partial x} = \begin{pmatrix} \frac{\partial E}{\partial x_0} \\ \frac{\partial E}{\partial x_1} \\ \frac{\partial E}{\partial x_2} \\ \frac{\partial E}{\partial x_3} \end{pmatrix} = \begin{pmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{pmatrix} = w = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$

(b) $\frac{\partial E}{\partial w} = \begin{pmatrix} \frac{\partial E}{\partial w_0} \\ \frac{\partial E}{\partial w_1} \\ \frac{\partial E}{\partial w_2} \\ \frac{\partial E}{\partial w_3} \end{pmatrix} = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = x = \begin{pmatrix} 1/2 \\ 1/3 \\ 1/4 \\ 1/5 \end{pmatrix}$

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2 Bonus Problems

7) $x^1 = \phi\left[\begin{pmatrix}1 & -1 & 0.1\\ 1 & -1 & 0.1\\ 1 & -1 & 0.1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}\right] = max\left(\begin{pmatrix}0.1\\0.1\\0.1\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}\right) = \begin{pmatrix}0.1\\0.1\\0.1\end{pmatrix}$

$x^2 = \phi\left[\begin{pmatrix}0.5 & 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0.5 & 0.5\end{pmatrix}\begin{pmatrix}1\\0.1\\0.1\\0.1\end{pmatrix}\right] = max\left(\begin{pmatrix}0.65\\0.65\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix}\right) = \begin{pmatrix}0.65\\0.65\end{pmatrix}$

$x^3 = \phi\left[\begin{pmatrix}0.25 & 0.25 & 0.25\\ 0.25 & 0.25 & 0.25\end{pmatrix}\begin{pmatrix}1\\0.65\\0.65\end{pmatrix}\right] = max\left(\begin{pmatrix}0.575\\0.575\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix}\right) = \begin{pmatrix}0.575\\0.575\end{pmatrix}$

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8) $\delta^{(3)} = \left(\begin{pmatrix}x_1^{(3)}\\x_2^{(3)}\end{pmatrix} - \begin{pmatrix}y_1\\y_2\end{pmatrix}\right)^2 = \begin{pmatrix}(x_1^{(3)}y_1)^2\\(x_2^{(3)}y_2)^2\end{pmatrix}$

$\left(\begin{pmatrix}0.575\\0.575\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix}\right)^2 = \begin{pmatrix}7.6\\8.6\end{pmatrix}$

$\delta^{(2)} = \begin{pmatrix}0(1-0)\\3.8(1-3.8)\\3.8(1-3.8)\end{pmatrix} \circ \left(\begin{pmatrix}1 & 1\\1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}7.6\\8.6\end{pmatrix}\right) = \begin{pmatrix}0\\-172.368\\-172.368\end{pmatrix}$

$\delta^{(1)} = \begin{pmatrix}0(1-0)\\0.3(1-0.3)\\0.3(1-0.3)\\0.3(1-0.3)\end{pmatrix} \circ \left(\begin{pmatrix}2 & 2\\2 & 2\\2 & 2\\2 & 2\end{pmatrix}\begin{pmatrix}-172.368\\-172.368\end{pmatrix}\right) = \begin{pmatrix}0\\-144.789\\-144.789\\-144.789\end{pmatrix}$

9) $W^{(0)} : 3 \times 3,\ W^{(1)} : 4 \times 2,\ W^{(2)} : 3 \times 2$

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10) $x^1 = \phi \left[\begin{pmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right] = \begin{pmatrix} 0.3 \\ 0.3 \\ 0.3 \end{pmatrix}$

$x^2 = \phi \left[\begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0.3 \\ 0.3 \\ 0.3 \end{pmatrix} \right] = \begin{pmatrix} 3.8 \\ 3.8 \end{pmatrix}$

$x^3 = \phi \left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3.8 \\ 3.8 \end{pmatrix} \right] = \begin{pmatrix} 8.6 \\ 8.6 \end{pmatrix}$

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11) $\delta^{(3)} = \begin{pmatrix} 8.6 \\ 8.6 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7.6 \\ 8.6 \end{pmatrix}$

$\delta^{(2)} = \begin{pmatrix} 0(1-0) \\ 3.8(1-3.8) \\ 3.8(1-3.8) \end{pmatrix} \circ \left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 7.6 \\ 8.6 \end{pmatrix} \right) = \begin{pmatrix} 0 \\ -172.368 \\ -172.368 \end{pmatrix}$

$\delta^{(1)} = \begin{pmatrix} 0(1-0) \\ 0.3(1-0.3) \\ 0.3(1-0.3) \\ 0.3(1-0.3) \end{pmatrix} \circ \left(\begin{pmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} -172.368 \\ -172.368 \end{pmatrix} \right) = \begin{pmatrix} 0 \\ -144.789 \\ -144.789 \\ -144.789 \end{pmatrix}$

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12) $\nabla^{(2)} = x^{(2)}(\delta^{(3)})^T = \begin{pmatrix} 1 \\ 3.8 \\ 3.8 \end{pmatrix} [7.6, 8.6] = \begin{pmatrix} 7.6 & 8.6 \\ 28.88 & 32.68 \\ 28.88 & 32.68 \end{pmatrix}$

$$\nabla^{(1)} = x^{(1)}(\delta^{(2)})^T = \begin{pmatrix} 1 \\ 0.3 \\ 0.3 \\ 0.3 \end{pmatrix} [-172.368, -172.368] = \begin{pmatrix} -172.368 & -172.368 \\ -51.71 & -51.71 \\ -51.71 & -51.71 \\ -51.71 & -51.71 \end{pmatrix}$$

$$\nabla^{(0)} = x^{(0)}(\delta^{(1)})^T = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} [-144.789, -144.789, -144.789] = \begin{pmatrix} -144.789 & -144.789 & -144.789 \\ -144.789 & -144.789 & -144.789 \\ -144.789 & -144.789 & -144.789 \end{pmatrix}$$

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13) $\mathbb{W}^2 \leftarrow \mathbb{W}^2 - \rho\nabla^{(2)} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 7.6 & 8.6 \\ 28.88 & 32.68 \\ 28.88 & 32.68 \end{pmatrix} = \begin{pmatrix} -6.6 & -7.6 \\ -27.88 & -31.68 \\ -27.88 & -31.68 \end{pmatrix}$

$$\mathbb{W}^1 \leftarrow \mathbb{W}^1 - \rho\nabla^{(1)} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{pmatrix} - \begin{pmatrix} -172.368 & -172.368 \\ -51.71 & -51.71 \\ -51.71 & -51.71 \\ -51.71 & -51.71 \end{pmatrix} = \begin{pmatrix} 174.368 & 174.368 \\ 53.71 & 53.71 \\ 53.71 & 53.71 \\ 53.71 & 53.71 \end{pmatrix}$$

$$\mathbb{W}^0 \leftarrow \mathbb{W}^0 - \rho\nabla^{(0)} = \begin{pmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{pmatrix} - \begin{pmatrix} -144.789 & -144.789 & -144.789 \\ -144.789 & -144.789 & -144.789 \\ -144.789 & -144.789 & -144.789 \end{pmatrix} =$$

$$\begin{pmatrix} 144.889 & 144.889 & 144.889 \\ 144.889 & 144.889 & 144.889 \\ 144.889 & 144.889 & 144.889 \end{pmatrix}$$
