Inference in First-Order Logic

Outline

I. Knowledge engineering

II. Propositionalization

III. Unification

* Figures are from the textbook site unless a source is specifically cited.

I. Knowledge Engineering Process

- ♣ Identification of questions and facts.

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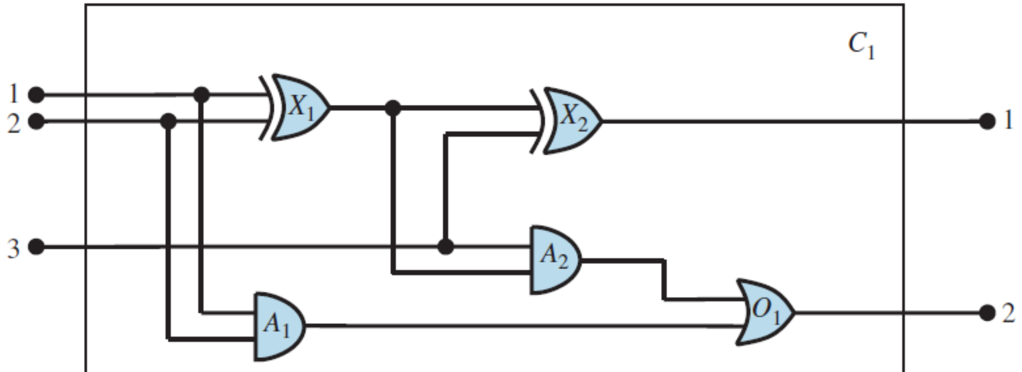
- Debugging and evaluation of the KB.

The Electronic Circuits Domain



A one-bit full adder

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  1 or 2 inputs, and 1 output

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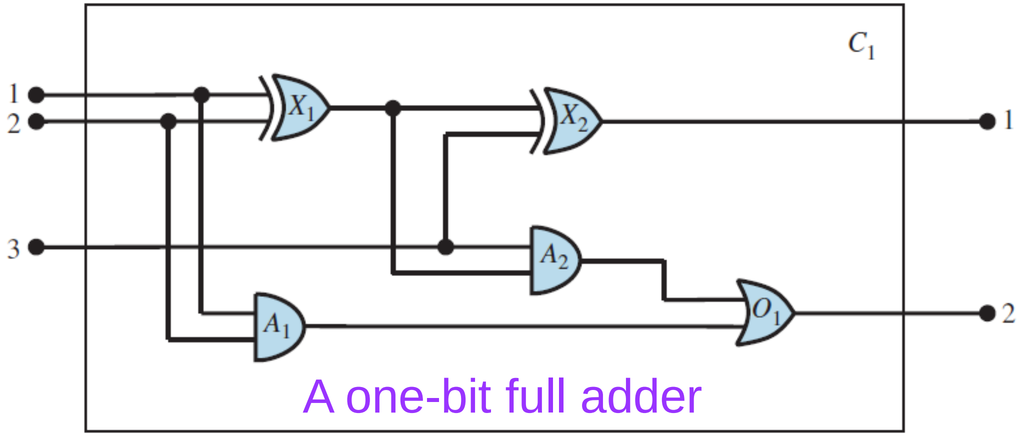

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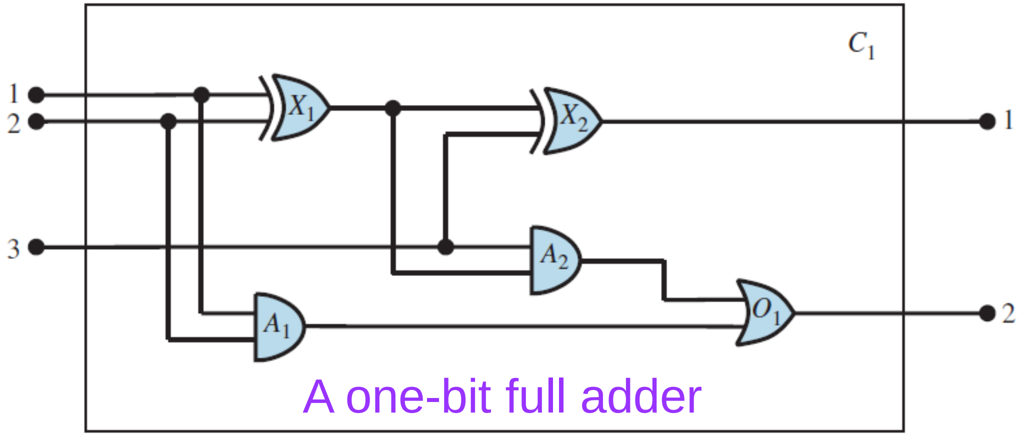

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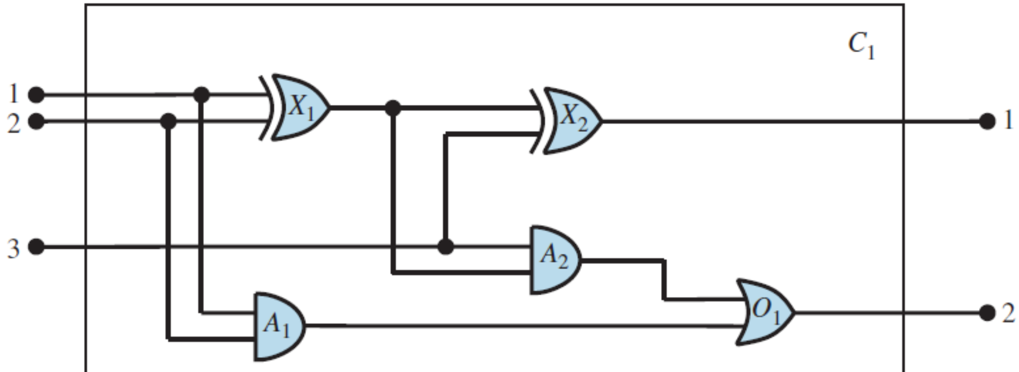

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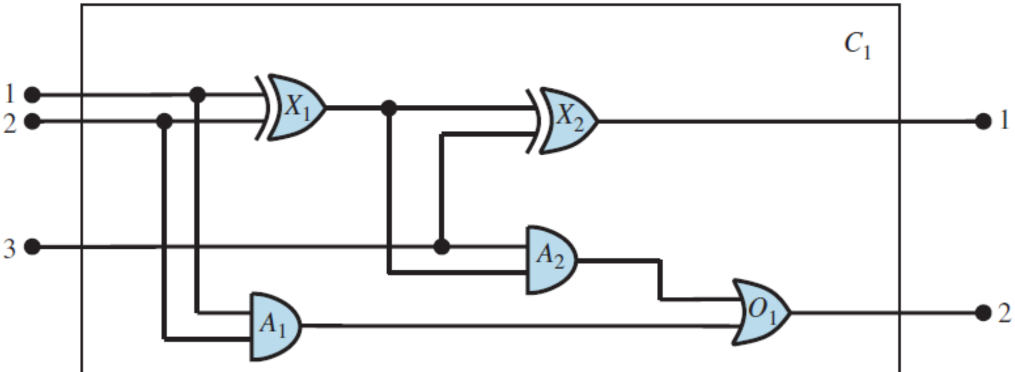

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  - *Type* (X_1): type of gate X_1 (which is *XOR*)
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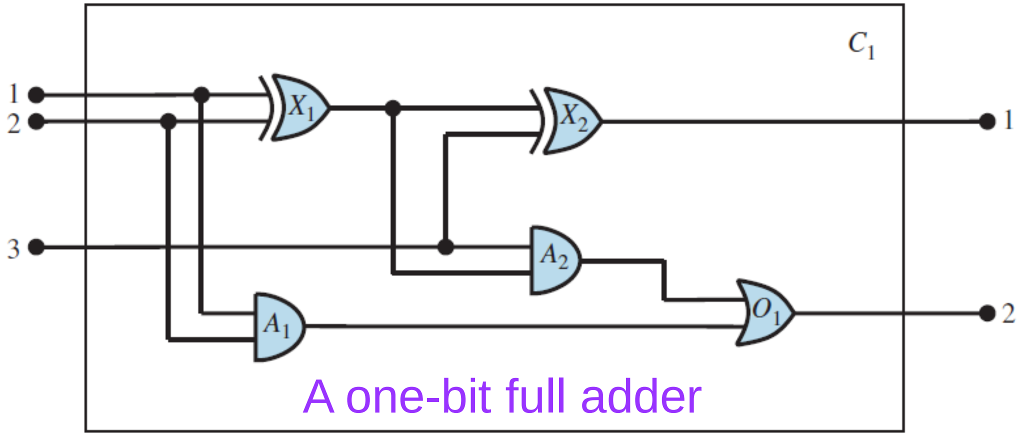

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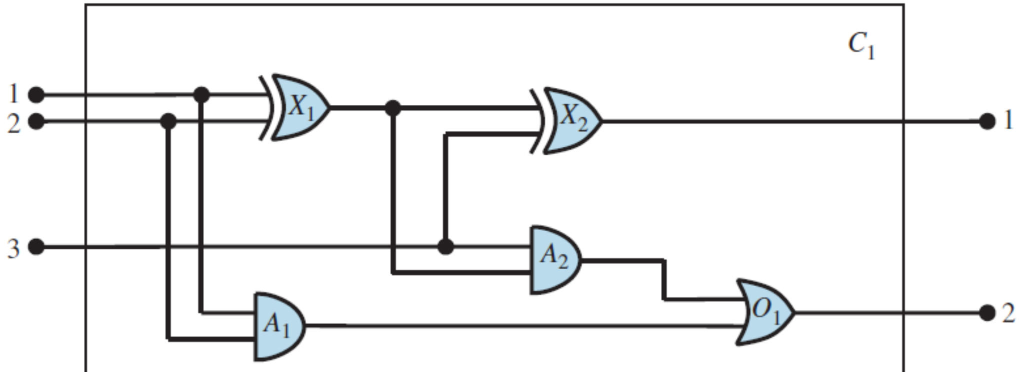

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♦ Connectivity predicate: *Connected*$\big($*Out*$(1, X_1)$, *In*$(1, X_2)\big)$

♦ Signal function: *Signal*(t) has value 1 or 0 at time t.

Encoding General Domain Knowledge

// Two connected terminals have the same signal.

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// An AND gate outputs 0 if and only if its input is 0.

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// An OR gate outputs 1 if and only if any of its input is 1.

$\forall g \;\; \mathit{Gate}(g) \land \mathit{Type}(g) = \mathit{OR} \Rightarrow (\mathit{Signal}(\mathit{Out}(1, g)) = 1 \Leftrightarrow \exists n \;\; \mathit{Signal}(\mathit{In}(n, g)) = 1)$

cont'd

// An XOR gate outputs 1 if and only if its inputs are different.

$$\forall g \quad Gate(g) \wedge Type(g) = XOR \Rightarrow (Signal(Out(1,g)) = 1 \Leftrightarrow Signal(In(1,g)) \neq Signal(In(2,g)))$$

// An NOT gate's output is different from its input.

$$\forall g \quad Gate(g) \wedge Type(g) = NOT \Rightarrow (Signal(Out(1,g)) = 1 \Leftrightarrow Signal(Out(1,g)) \neq Signal(In(1,g)))$$

// All the gates (except for NOT) have two inputs and one output.

$$\forall g \quad Gate(g) \wedge Type(g) = NOT \Rightarrow Arity(g,1,1)$$

$$\forall g,k \quad Gate(g) \wedge k{=}Type(g) \wedge (k = AND \vee k = OR \vee k = XOR) \Rightarrow Arity(g,2,1)$$

// A circuit has terminals exactly up to its input and output arity.

$$\begin{gathered}\forall c,i,j \quad Circuit(c) \wedge Arity(c,i,j) \Rightarrow \\ \forall n \ (n \leq i \Rightarrow Terminal(In(n,c)) \wedge (n > i \Rightarrow In(n,c) = Nothing)) \wedge \\ \forall n \ (n \leq j \Rightarrow Terminal(Out(n,c)) \wedge (n > j \Rightarrow Out(n,c) = Nothing))\end{gathered}$$

// Gates and terminals are all distinct.

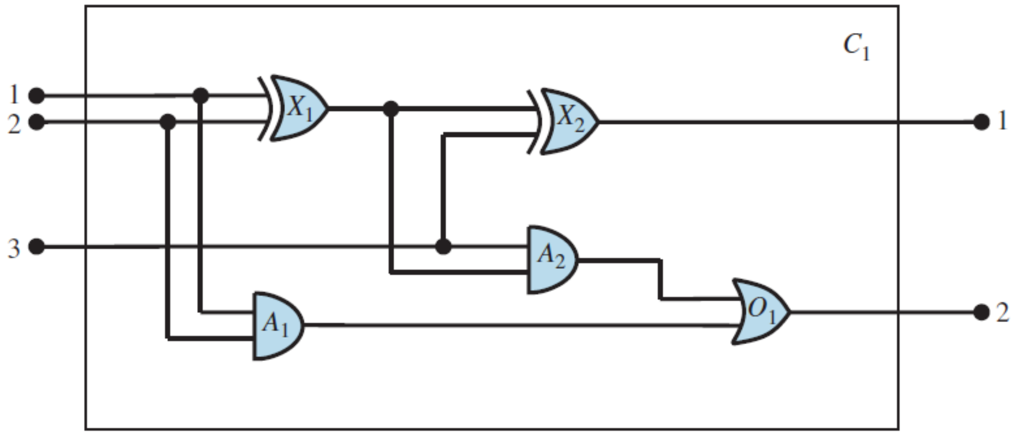
$$\forall g,t,\cancel{s} \quad Gate(g) \wedge Terminal(t) \cancel{\wedge Signal(s)} \Rightarrow g \neq t \cancel{\wedge g \neq s \wedge t \neq s}$$

function not predicate

// Gates are circuits.

$$\forall g \quad Gate(g) \Rightarrow Circuit(g)$$

Encoding a Problem Instance



- Circuit and component gates:

  $Circuit(C_1) \wedge Arity(C_1, 3, 2)$

  $Gate(X_1) \wedge Type(X_1) = XOR$

  $Gate(X_2) \wedge Type(X_2) = XOR$

  $Gate(A_1) \wedge Type(A_1) = AND$

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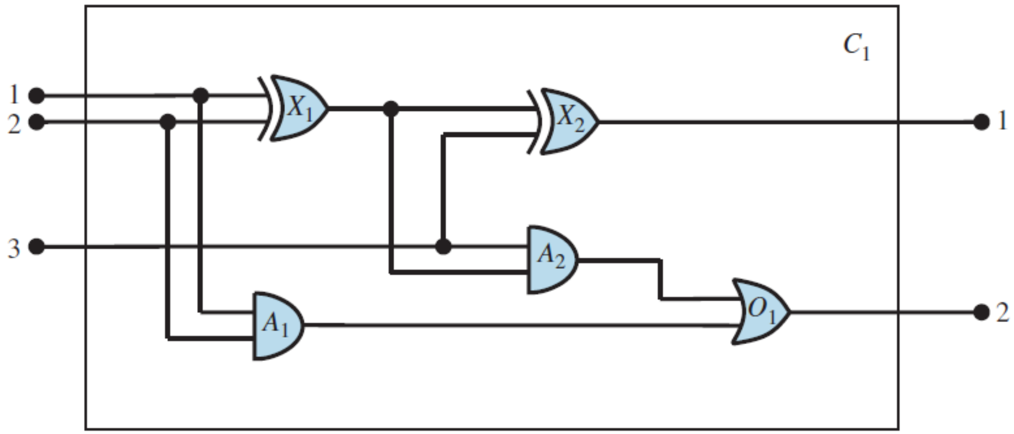

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  $Gate(A_1) \land Type(A_1) = AND$

  $Gate(A_2) \land Type(A_2) = AND$

  $Gate(O_1) \land Type(O_1) = OR$

- Connections between the circuit and component gates:

  $Connected(Out(1, X_1), In(1, X_2))$

  $Connected(Out(1, X_1), In(2, A_2))$

  $Connected(Out(1, A_2), In(1, O_1))$

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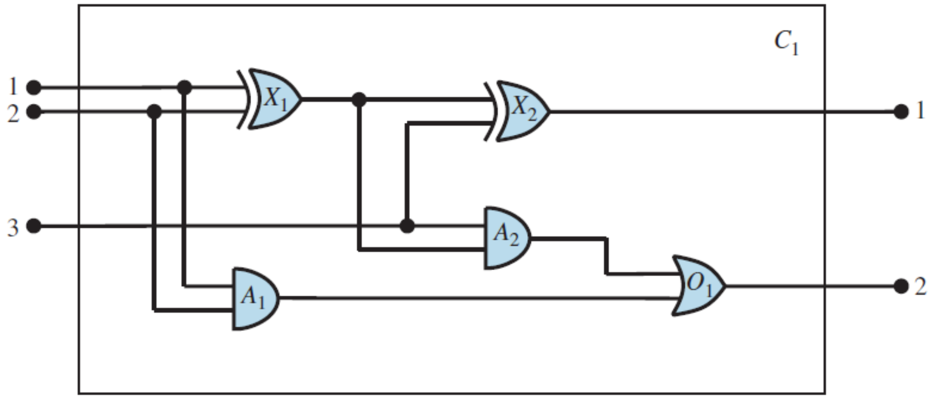
Queries



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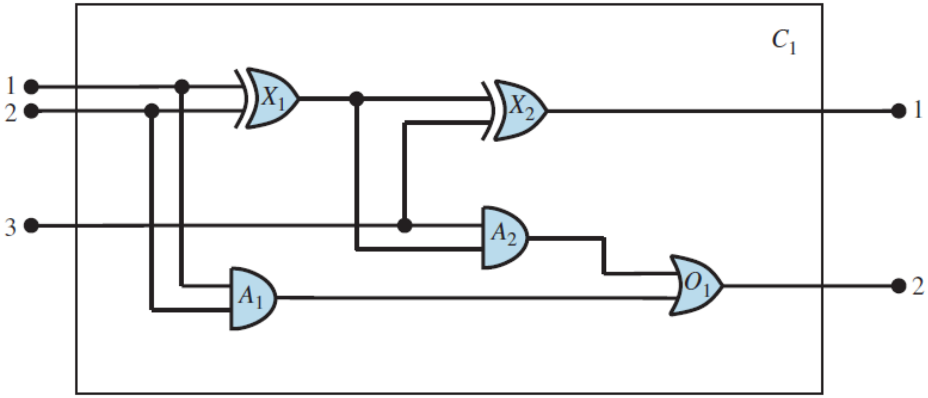

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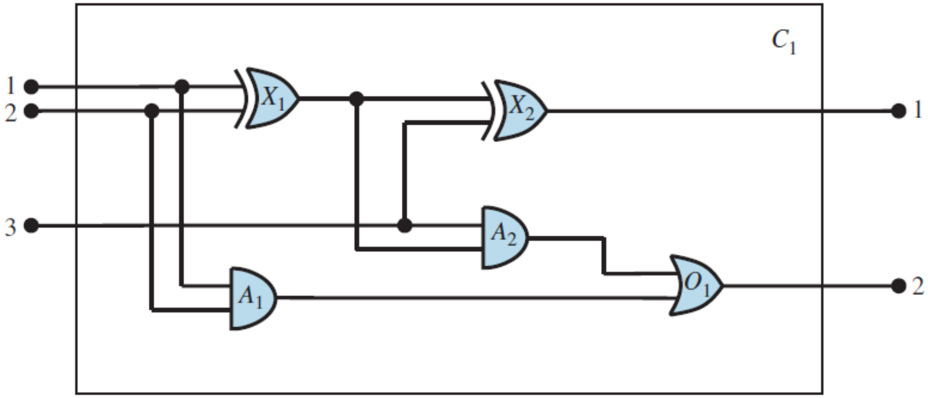

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ASKVARS will give three substitutions as answers.

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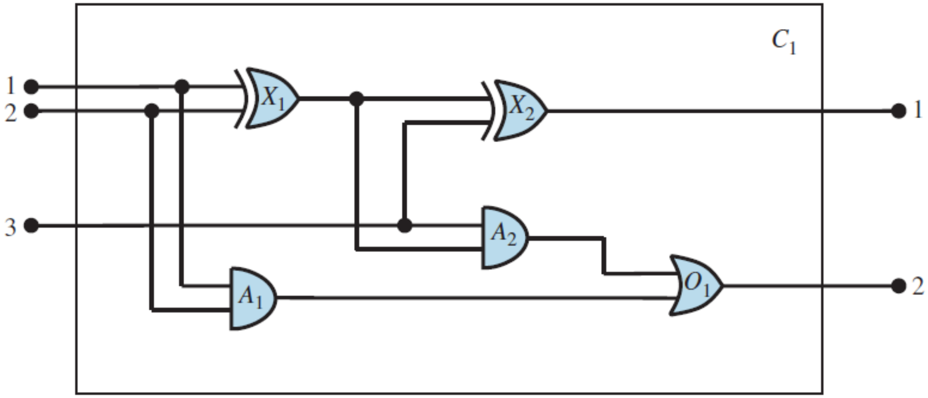

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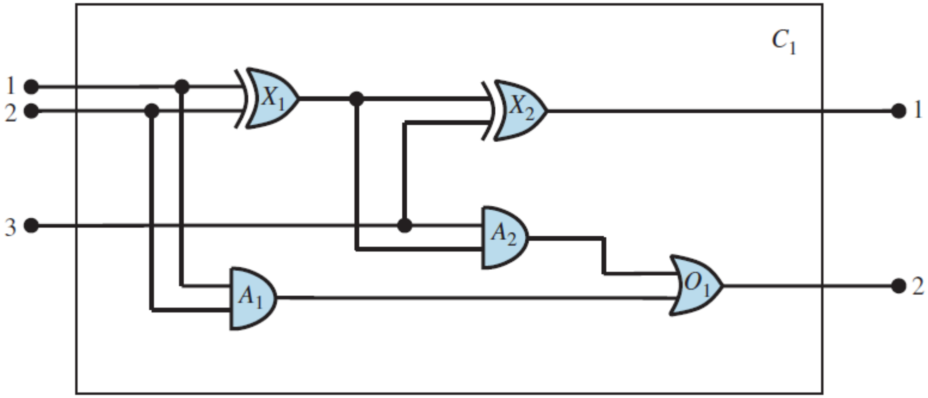

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$WarmBlooded(Ostrich) \wedge HaveWings(Ostrich)$

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$$\forall x\ \overbrace{\mathit{Bird}(x) \Rightarrow \mathit{WarmBlooded}(x) \wedge \mathit{HaveWings}(x)}^{\alpha}$$

Universal and Existential Instantiations

A *ground term* in FOL is a term without variables.

Substitute a ground term for a universally quantified variable.

$$\frac{\overset{\text{sentence}}{\forall v\ \alpha}}{\textsc{Subst}(\underbrace{\{v/g\}}_{\text{substitution } \theta\text{, e.g., } \theta = \{x/\mathit{Socrates}\}}, \alpha)}$$

E.g., $\theta = \{x/\mathit{Ostrich}\}$

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$\textsc{Subst}(\theta, \alpha) \quad \equiv \quad \mathit{Bird}(\mathit{Ostrich}) \Rightarrow \mathit{WarmBlooded}(\mathit{Ostrich}) \wedge \mathit{HaveWings}(\mathit{Ostrich})$

Existential Instantiation

Substitute a single new constant symbol for an existentially quantified variable.

$$\frac{\exists v\ \alpha}{\textsc{Subst}(\{v/g\}, \alpha)}$$

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we can infer

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as long as *LiamsMom* does **not** appear elsewhere in the KB.

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Skolem constant

Propositionalization

$$\forall x \forall y \quad Ancestor(x, y) \rightarrow Parent(x, y) \vee \exists z (Ancestor(x, z) \wedge Ancestor(z, y))$$

KB:

$Ancestor(John, David)$

$Parent(John, David)$

$Parent(David, Lisa)$

Propositionalization

$$\forall x \forall y \quad Ancestor(x, y) \rightarrow Parent(x, y) \vee \exists z (Ancestor(x, z) \wedge Ancestor(z, y))$$

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$$Ancestor(John, David)$$
$$\rightarrow Parent(John, David) \vee (Ancestor(John, JohnDavidAnc) \wedge Ancestor(JohnDavidAnc, David))$$

Propositionalization

$$\forall x \forall y \quad \mathit{Ancestor}(x, y) \rightarrow \mathit{Parent}(x, y) \vee \exists z (\mathit{Ancestor}(x, z) \wedge \mathit{Ancestor}(z, y))$$

KB:

$\mathit{Ancestor}(\mathit{John}, \mathit{David})$

$\mathit{Parent}(\mathit{John}, \mathit{David})$

$\mathit{Parent}(\mathit{David}, \mathit{Lisa})$

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$$\mathit{Ancestor}(\mathit{David}, \mathit{John}) \rightarrow \mathit{Parent}(\mathit{John}, \mathit{David}) \vee (\mathit{Ancestor}(\mathit{David}, \mathit{DavidJohnAnc}) \wedge \mathit{Ancestor}(\mathit{DavidJohnAnc}, \mathit{David}))$$

$$\mathit{Ancestor}(\mathit{John}, \mathit{Lisa}) \rightarrow \mathit{Parent}(\mathit{John}, \mathit{Lisa}) \vee (\mathit{Ancestor}(\mathit{John}, \mathit{JohnLisaAnc}) \wedge \mathit{Ancestor}(\mathit{JohnLisaAnc}, \mathit{Lisa}))$$

$$\vdots$$

Skolemization

A more standard way to eliminate an existential quantifier is to introduce a new function symbol, which is, however, not applicable in generating a PL sentence.

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$$\exists y \; P(y, x_1, \ldots., x_n)$$ // y depends on $x_1, \ldots., x_n$

$\Downarrow$ eliminate y by introducing function f

$$P(f(x_1, \ldots., x_n), x_1, \ldots., x_n)$$

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- That $P(y, x_1, \ldots, x_n) = $ *true* implicitly defines y as a function of $x_1, \ldots, x_n$ (analogous to the implicit function theorem in multivariate calculus).

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$\exists y$ *Mother*$(y,$ *Liam*$)$

$\exists y$ *Mother*$(y,$ *Sophia*$)$

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$\exists y \ \ Mother(y, Liam)$

$\exists y \ \ Mother(y, Sophia)$

$\xrightarrow{\text{new unary function } mom()}$

$Mother(mom(Liam), Liam)$

$Mother(mom(Sophia), Sophia)$

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$\exists y \;\; \textit{Mother}(y, \textit{Liam})$

$\exists y \;\; \textit{Mother}(y, \textit{Sophia})$

$\xrightarrow{\text{new unary function } \textit{mom}()}$

$\textit{Mother}(\textit{mom}(\textit{Liam}), \textit{Liam})$

$\textit{Mother}(\textit{mom}(\textit{Sophia}), \textit{Sophia})$

- Advantage: one function instead of two new constants to denote the moms of Liam and Sophia.

Generalized Modus Ponens

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q), \quad p_1', p_2', \ldots, p_n'$$

Suppose there exists a substitution θ such that

$$\text{SUBST}(\theta, p_i) = \text{SUBST}(\theta, p_i') \qquad \text{for } 1 \leq i \leq n$$

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KB:

$\mathit{Gate}(X_1)$, $\mathit{Terminal}(\mathit{In}(1, C_1))$ $\qquad$ $\mathit{Gate}(g) \wedge \mathit{Terminal}(t) \Rightarrow g \neq t$

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KB:

$Gate(X_1)$, $Terminal(In(1, C_1))$ $\qquad Gate(g) \wedge Terminal(t) \Rightarrow g \neq t$

$\textsc{Subst}(\theta, q)$ $\qquad \theta = \{g/X_1, t/(In(1, C_1))\}$

q is $g \neq t$

Generalized Modus Ponens

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$\text{SUBST}(\theta, q)$ $\Downarrow$ $\theta = \{g/X_1, t/(\mathit{In}(1, C_1))\}$

q is $g \neq t$

$$X_1 \neq \mathit{In}(1, C_1)$$

III. Unification

- The process of finding substitutions that make different logical expressions look identical.

- Carried out by the algorithm UNIFY.

$\text{UNIFY}(p, q) = \theta$ where $\text{SUBST}(\theta, p) = \text{SUBST}(\theta, q)$

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Query: $AskVars(Knows(John, x))$ // what does John know?

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sentences

Query: *AskVars*(*Knows*(*John*, x)) // what does John know?

Answers: all the sentences in the KB found to unify with *Knows*(*John*, x).

UNIFY(*Knows*(*John*, x), *Knows*(*John*, *Jane*)) = {x/*Jane*}

UNIFY(*Knows*(*John*, x), *Knows*(y, *Bill*)) = {x/*Bill*, y/*John*}

UNIFY(*Knows*(*John*, x), *Knows*(y, *Mother*(y))) = {y/*John*, x/*Mother*(*John*)}

Unification (cont'd)

- Conflicting substitutions

$\text{U}\text{NIFY}(\mathit{Knows}(\mathit{John}, x), \mathit{Knows}(x, \mathit{Elizabeth})) = \mathit{failure}$

Unification (cont'd)

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$\{x/\textit{John}\}$

Unification (cont'd)

♠ Conflicting substitutions

UNIFY(*Knows*(*John*, x), *Knows*(x, *Elizabeth*)) = *failure*

$\{x/\mathit{John}\}$ $\{x/\mathit{Elizabeth}\}$

Unification (cont'd)

- Conflicting substitutions

UNIFY(*Knows*(*John*, x), *Knows*(x, *Elizabeth*)) = *failure*

$\{x/John\}$ $\{x/Elizabeth\}$

x cannot take on the values *John* and *Elizabeth* at the same time!

Unification (cont'd)

- Conflicting substitutions

UNIFY(*Knows*(*John*, x), *Knows*(x, *Elizabeth*)) = *failure*

{x/*John*}   {x/*Elizabeth*}

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- Multiple unifiers

UNIFY(*Knows*(*John*, x), *Knows*(y, z))

could return {y/*John*, x/z}

or {y/*John*, x/*John*, z/*John*}

Unification (cont'd)

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UNIFY(*Knows*(*John*, x), *Knows*(y, z))

could return $\{y/John,\ x/z\}$ $\Longrightarrow$ *Knows*(*John*, z)

or $\{y/John,\ x/John,\ z/John\}$

Unification (cont'd)

- Conflicting substitutions

$\textsc{Unify}(\mathit{Knows}(\mathit{John}, x), \mathit{Knows}(x, \mathit{Elizabeth})) = \mathit{failure}$

$\{x/\mathit{John}\}$ $\{x/\mathit{Elizabeth}\}$

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$\textsc{Unify}(\mathit{Knows}(\mathit{John}, x), \mathit{Knows}(y, z))$

could return $\{y/\mathit{John}, x/z\}$ $\Longrightarrow$ $\mathit{Knows}(\mathit{John}, z)$

or $\{y/\mathit{John}, x/\mathit{John}, z/\mathit{John}\}$ $\Longrightarrow$ $\mathit{Knows}(\mathit{John}, \mathit{John})$

Unification (cont'd)

- Conflicting substitutions

$\text{UNIFY}(Knows(John, x), Knows(x, Elizabeth))$ = *failure*

$\{x/John\}$ $\{x/Elizabeth\}$

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$\text{UNIFY}(Knows(John, x), Knows(y, z))$

could return $\{y/John, x/z\}$ $\Longrightarrow$ $Knows(John, z)$

more general unifier for fewer restriction on variable values

or $\{y/John, x/John, z/John\}$ $\Longrightarrow$ $Knows(John, John)$

Unification (cont'd)

♠ Conflicting substitutions

UNIFY(*Knows*(*John*, x), *Knows*(x, *Elizabeth*)) = *failure*

{x/*John*}   {x/*Elizabeth*}

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UNIFY(*Knows*(*John*, x), *Knows*(y, z))

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less general unifier

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less general unifier

Unification Algorithm

function UNIFY(x, y, θ=*empty*) **returns** a substitution to make x and y identical, or *failure*
  if θ = *failure* **then return** *failure*
  else if $x = y$ **then return** θ
  else if VARIABLE?(x) **then return** UNIFY-VAR(x, y, θ)
  else if VARIABLE?(y) **then return** UNIFY-VAR(y, x, θ)
  else if COMPOUND?(x) **and** COMPOUND?(y) **then**
    return UNIFY(ARGS(x), $\boxed{\text{ARGS}(y)}$, UNIFY($\boxed{\text{OP}(x)}$, OP(y), θ))
  else if LIST?(x) **and** LIST?(y) **then**
    return UNIFY(REST(x), REST(y), UNIFY(FIRST(x), FIRST(y), θ))
  else return *failure*

function symbol of x

argument list of y

function UNIFY-VAR(var, x, θ) **returns** a substitution
  if $\{var/val\} \in \theta$ for some val **then return** UNIFY(val, x, θ)
  else if $\{x/val\} \in \theta$ for some val **then return** UNIFY(var, val, θ)
  else if OCCUR-CHECK?(var, x) **then return** *failure*
  else return add $\{var/x\}$ to θ

Recursively explore two expressions x and y "side by side" to build up a unifier.

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  else if OCCUR-CHECK?(var, x) **then return** *failure* // check whether the variable *var* appears
  else return add $\{var/x\}$ to θ // inside the complex term x. match fails if so
// because no unifier can be constructed.

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