Using First-Order Logic

Outline

- I. Assertions and queries
- II. Numbers and sets in FOL
- III. The wumpus world in FOL

^{*} Figures are from the <u>textbook site</u> unless a source is specifically cited.

I. Knowledge Engineering

A field of AI dedicated to representing information about the world in a form that can be utilized by a computer to solve complex tasks such as:

- medical diagnosis
- dialog in a natural language
- etc.
- Knowledge representation (logical rules, semantic nets, frames, etc.)
- Automated reasoning (inference engines, theorem provers, etc.)

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A *domain* is some part of the world about which we wish to express some knowledge.

Add sentences, called <u>assertions</u>, to a KB using Tell.

```
Tell(KB, Likes(John, Icecream))

Tell(KB, Father(Zeus, Athena))

Tell(KB, \forall x \exists y \; Brother(x, y) \Rightarrow Sibiling(x, y))
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Query: question asked
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Ask the KB questions using Ask.

Any query is entailed by the KB should be answered affirmatively.

Suppose another KB has the following predicates:

Bird(Swan), Bird(Crane), Bird(Parrot),

Quantified query

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To know what values of x make the sentence true

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 $\{x / \text{Swan}\}, \{x / \text{Crane}\}, \text{ and } \{x / \text{Parrot}\}$

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```
\{x / \text{Swan}\}, \{x / \text{Crane}\}, \text{ and } \{x / \text{Parrot}\}
          substitution or binding list
```

```
// One's mother is the person's female parent. \forall m, c \; Mother(c) = m \Leftrightarrow Female(m) \land Parent(m, c)
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// One's mother is the person's female parent. \forall m, c \; Mother(c) = m \Leftrightarrow Female(m) \land Parent(m, c) // One's husband is the person's male spouse. \forall w, h \; Husband(h, w) \Leftrightarrow Male(h) \land Spouse(h, w) // Parent and child are inverse relations. \forall p, c \; Parent(p, c) \Leftrightarrow Child(c, p)
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// Parent and child are inverse relations.  \forall p, c \; Parent(p, c) \Leftrightarrow Child(c, p) 
// A grand parent is a parent of one's parent  \forall g, c \; GrandParent(g, c) \Leftrightarrow \exists p \; (Parent(g, p) \land Parent(p, c))
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\forall p, c \; Parent(p, c) \Leftrightarrow Child(c, p)
// A grand parent is a parent of one's parent
\forall g, c \; GrandParent(g, c) \Leftrightarrow \exists p \; (Parent(g, p) \land Parent(p, c))
// A sibling is another child of one's parent
\forall x, s \; Sibling(x, s) \Leftrightarrow x \neq s \land \exists p \; (Parent(p, x) \land Parent(p, s))
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Kinship relations are represented by binary predicates.

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Axioms

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These are *definitions* in the form of $\forall x, y \ P(x, y) \Leftrightarrow ...$ and built upon a basic set of predicates *Child*, *Male*, *Female*, etc.

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- ♠ Theorems are logical sentences entailed by axioms.

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```
\forall x \ Person(x) \Rightarrow \cdots // partial specification of \forall x \cdots \Rightarrow Person(x) // properties
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II. Domain of Natural Numbers

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 $\forall n \; NatNum(n) \Rightarrow NatNum(S(n))$

II. Domain of Natural Numbers

Describe the theory of natural numbers using merely:

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- one function symbol, S (successor)
- one predicate, NatNum
- Recursive definition:

NatNum(0)

 $\forall n \; NatNum(n) \Rightarrow NatNum(S(n))$

Axioms to constrain the successor function:

 $\forall n \ 0 \neq S(n)$

 $\forall m, n \mid m \neq n \Rightarrow S(m) \neq S(n)$

 $\forall m \ NatNum(m) \Rightarrow +(0,m) = m$

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$$prefix \ \textit{notation}$$

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Use infix notation for readability.

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 $\forall m \ \textit{NatNum}(m) \Rightarrow 0 + m = m$ $infix \ \textit{notation}$

Use infix notation for readability.

$$\forall m, n \ NatNum(m) \land NatNum(n) \Rightarrow +(S(m), n) = S(+(m, n))$$

$$\forall m \ \textit{NatNum}(m) \Rightarrow +(0,m) = m$$

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Use infix notation for readability.

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$$\forall m, n \ NatNum(m) \land NatNum(n) \Rightarrow +(S(m), n) = S(+(m, n))$$

Write S(n) as n + 1.

$$\forall m, n \; NatNum(m) \land NatNum(n) \Rightarrow (m+1) + n = (m+n) + 1$$

$$\forall m \; NatNum(m) \Rightarrow +(0,m) = m$$

$$prefix \; notation$$

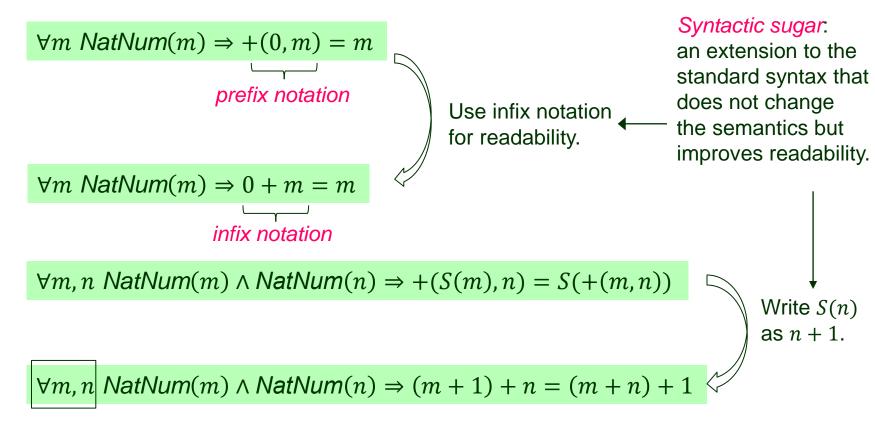
$$\forall m \; NatNum(m) \Rightarrow 0 + m = m$$

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Syntactic sugar: $\forall m \forall n$ according to syntax

Defining Addition

$$\forall m \ NatNum(m) \Rightarrow +(0,m) = m$$

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- Binary functions, ∩ (intersection), ∪ (union), Add
 s₁ ∩ s₂

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Binary functions, ∩ (intersection), ∪ (union), Add

$$s_1 \cap s_2$$
 $s_1 \cup s_2$ $Add(x,s)$
the set resulting from add element x to set s

◆ A set is either an empty set or made by adding something to a set.

$$\forall s \; \mathsf{Set}(s) \Leftrightarrow (s = \{\}) \; \lor \; (\exists x, s_0 \; \; \mathsf{Set}(s_0) \land s = \mathsf{Add}(x, s_0))$$

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The empty set has no elements added to it.

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\neg \exists x, s \; Add(x, s) = \{\} // equivalently, no way to decompose \{\}
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$$\forall x, s \ x \in s \Leftrightarrow \exists y, s_2 \ (s = Add(y, s_2) \land (x = y \lor x \in s_2))$$
// expressed recursively

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must be true at some recursion level

Subset relationship

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$$\forall s_1, s_2 \quad s_1 = s_2 \Leftrightarrow (s_1 \subseteq s_2 \land s_2 \subseteq s_1)$$

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Set equality

$$\forall s_1, s_2 \quad s_1 = s_2 \Leftrightarrow (s_1 \subseteq s_2 \land s_2 \subseteq s_1)$$

Intersection

$$\forall x, s_1, s_2 \quad x \in s_1 \cap s_2 \Leftrightarrow (x \in s_1 \land x \in s_2)$$

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Intersection

$$\forall x, s_1, s_2 \quad x \in s_1 \cap s_2 \Leftrightarrow (x \in s_1 \land x \in s_2)$$

Union

$$\forall x, s_1, s_2 \quad x \in s_1 \cup s_2 \Leftrightarrow (x \in s_1 \vee x \in s_2)$$

First-order logic axioms are much more concise than propositional axioms.

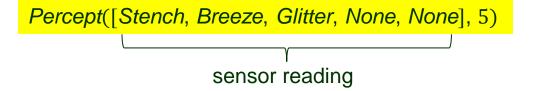
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Predicate for percept

Percept([Stench, Breeze, Glitter, None, None], 5)

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Predicate for percept



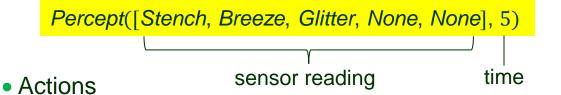
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Querying the KB for best action

AskVars(KB, BestAction(a, 5))

A binding list, e.g., $\{a/Grab\}$, is returned.

Input/Output Rules

No need for the use of fluents (e.g., Forward^t, Bump^{t+1})!

```
\forall t, s, g, w, c Percept([s, Breeze, g, w, c], t) \Rightarrow Breeze(t)

\forall t, s, g, w, c Percept([s, None, g, w, c], t) \Rightarrow \neg Breeze(t)

\forall t, s, b, w, c Percept([s, b, Glitter, w, c], t) \Rightarrow Glitter(t)

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\vdots
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\forall t, s, b, w, c \ \ Percept([s, b, None, w, c], t) \Rightarrow \neg Glitter(t)
\vdots
\forall t \ \ Glitter(t) \Rightarrow BestAction(Grab, t) \ \ // simple "reflex" behavior
```

Adjacency of two squares

```
\forall x, y, a, b \;  Adjacent([x, y], [a, b]) \Leftrightarrow \\ (x = a \land (y = b - 1 \lor y = b + 1)) \lor (y = b \land (x = a - 1 \lor x = a + 1))
```

// if using proportional logic, we would have to name every square, say, Square_{i,j}, // for $1 \le i,j \le 4$, and would need one such fact for 120 different pairs of squares!

Adjacency of two squares

```
square at row x and column y
\begin{vmatrix} h & Adjacent([x, y], [a, h]) \Leftrightarrow \\ \end{pmatrix}
```

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\forall x, y, a, b \ \textit{Adjacent}([x, y], [a, b]) \Leftrightarrow \\ (x = a \land (y = b - 1 \lor y = b + 1)) \lor (y = b \land (x = a - 1 \lor x = a + 1))
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• Unary predicate Pit (no reason to distinguish among pits).

 $Pit([x, y]) \equiv true$ if and only if the square [x, y] contains a pit.

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 $Pit([x, y]) \equiv true$ if and only if the square [x, y] contains a pit.

Constants Agent, Wumpus

Adjacency of two squares

square at row x and column y

```
\forall x, y, a, b \; Adjacent([x, y], [a, b]) \Leftrightarrow
       (x = a \land (y = b - 1 \lor y = b + 1)) \lor (y = b \land (x = a - 1 \lor x = a + 1))
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```
\forall t \; At(Wumpus, [1,3], t) // fixed location for the wumpus
```

```
\forall x, s_1, s_2, t At(x, s_1, t) \land At(x, s_2, t) \Rightarrow s_1 = s_2 // only one location at a time
```

• Percepts of breeze, stench, etc.

 $\forall s, t \; At(Agent, s, t) \land Breeze(t) \Rightarrow Breezy(s)$

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Diagnostic (inferring cause from effect)

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\forall s \; Breezy(s) \Leftrightarrow \exists r \; Adjacent(r,s) \land Pit(r)
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