Stat 330
Homework 4

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1)

(a) X= # of drivers until one doesn't make a full stop.
$X \sim$ Geo(1 - .85) = Geo(.15)
P(X<10) = ?

(b) X = # of correct answers out of total answers.
$X \sim$ Bin(20, .6)
P(X$\geq$12) = ?

(c) X = # of customers that arrive between 1:00 pm and 2:00 pm.
$X \sim$ Pois(16)
P(X=14) = ?

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2)

(a) There are 6 possible doubles rolls out of 36 possible rolls. Thus, p = 6/36 = $.16\tilde{6}$

(b) $E(X) = 0(1\text{-}p) + 1(p) = 0(.83\tilde{3}) + 1(.16\tilde{6}) = .16\tilde{6}$
$E(X^2) = 0^2(1\text{-}p) + 1^2(p) = 0^2(.83\tilde{3}) + 1^2(.16\tilde{6}) = .16\tilde{6}$
$Var(X) = E(X^2) - (E(X))^2 = .16\tilde{6} - (.16\tilde{6})^2 = .16\tilde{6} - 0.027\tilde{7} = 0.138\tilde{8}$

(c) $Y \sim$ Bin(5, .167)

(d) E(Y) = np = 5*.167 = 0.835

(e) $P(Y=3) = \binom{5}{3}(0.167)^3(0.835)^{5-3} = .032$

(f)

(g)

3)

(a) X = Bin(

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4)

(a) X $\sim$ Pois(10)

(b) $\dfrac{e^{-10}(10)^8}{8!} =$

(c) X $\sim$ Pois(10/(60/12)) = Pois(2)

(d) $\dfrac{e^{-2}(2)^3}{3!} =$

(e) E(X) = λ = 2

(a, b)

x	-2	-1	0	1	2
$P_X(x)$	0.1	0.3	0.3	0.1	0.2
$F_X(x)$	0.1	0.4	0.7	0.8	1.0

(c)

i. $P(X \leq 1) \Rightarrow P(X = 1) + P(X = 2) = 0.1 + 0.2 = 0.3$

ii. $P(-1 < X \leq 1) \Rightarrow P(X = 0) + P(X = 1) = 0.3 + 0.1 = 0.4$

iii. $P(X < 0) \Rightarrow P(X = -2) + P(X = -1) = 0.1 + 0.3 = 0.4$

(d)

i. $F(1) = 0.8$

ii. $F(0.5) = F(0) = 0.7$

iii. $P(X \geq 0) = 1 - F(-1) = 0.6$

(e) E(X) = -2(0.1) + -1(0.3) + 0(0.3) + 1(0.1) + 2(0.2) = 0.2

E(X^2) = -2^2(0.1) + -1^2(0.3) + 0^2(0.3) + 1^2(0.1) + 2^2(0.2) = 0.6

Variance = E(X^2) - E(X)^2 = 0.6 - (0.2)^2 = 0.56

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3)

(a) Im(Y) = {8, 6, 4, 2, 0}

(b) E(X) = 8(0.1) + 6(0.3) + 4(0.3) + 2(0.1) + 0(0.2) = 4

E(X^2) = 8^2(0.1) + 6^2(0.3) + 4^2(0.3) + 2^2(0.1) + 0^2(0.2) = 22.4

Variance = E(X^2) - E(X)^2 = 22.4 - (4)^2 = 38.4

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4) Var(aX) = E([aX]^2) - [E(aX)]^2 = a^2E(X^2) - a^2E(X)^2 = a^2(E(X^2) - E(X)^2) = a^2Var(X)

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5)

(a) P(X=2) = $\binom{6}{2}(0.05)^2(0.95)^{6-2}$

$15*(0.05)^2(0.95)^4$ = .0305

(b) P(X≤2) = P(X=0) + P(X=1) + P(X=2) ⇒

$\binom{6}{0}(0.05)^0(0.95)^{6-0} + \binom{6}{1}(0.05)^1(0.95)^{6-1} + \binom{6}{2}(0.05)^2(0.95)^{6-2} =$

0.735 + 0.232 + 0.031 = 0.998

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6)

(a) $P(X\geq 5) = P(X{=}5) + P(X{=}6) + P(X{=}7) + P(X{=}8) + P(X{=}9) + P(X{=}10) \Rightarrow$

$$\binom{10}{5}(0.2)^5(0.8)^{10-5} + \binom{10}{6}(0.2)^6(0.8)^{10-6} + \binom{10}{7}(0.2)^7(0.8)^{10-7} +$$
$$\binom{10}{8}(0.2)^8(0.8)^{10-8} + \binom{10}{9}(0.2)^9(0.8)^{10-9} + \binom{10}{10}(0.2)^{10}(0.8)^{10-10} =$$

$0.02642 + 0.00551 + .00079 + .00007 + \sim 0 + \sim 0 = .033$

(b) $P(X\geq 5) = 1 - P(X\leq 4) = (.2)(.8)^{4-1} + (.2)(.8)^{3-1} + (.2)(.8)^{2-1} + (.2)(.8)^{1-1} =$

$.1024 + .128 + .16 + .2 = .5904$