



# *COTI S 342*

Recitation 11/18/2019 –  
11/20/2019

Topic

○ Typelang

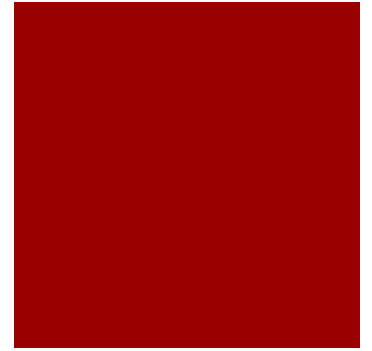
○ Q&A



# Typelang

<i>program</i>	<i>::=</i>	<i>definedecl* exp?</i>	<i>Programs</i>
<i>definedecl</i>	<i>::=</i>	<b>(define</b> <i>identifier</i> : <i>T</i> <i>exp</i> )	<i>Declarations</i>
<i>exp</i>	<i>::=</i>		<i>Expressions</i>
		<i>varexp</i>	<i>Variable expression</i>
		<i>numexp</i>	<i>Number constant</i>
		<i>addexp</i>	<i>Addition</i>
		<i>subexp</i>	<i>Subtraction</i>
		<i>multexp</i>	<i>Multiplication</i>
		<i>divexp</i>	<i>Division</i>
		<i>letexp</i>	<i>Let binding</i>
		<i>lambdaexp</i>	<i>Function creation</i>
		<i>callexp</i>	<i>Function Call</i>
		<i>letrecexp</i>	<i>Letrec</i>
		<i>refexp</i>	<i>Reference</i>
		<i>dereferexp</i>	<i>Dereference</i>
		<i>assignexp</i>	<i>Assignment</i>
		<i>freeexp</i>	<i>Free</i>

# Typelang

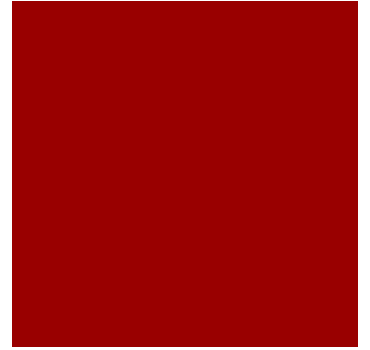


$T ::=$   
| unit  
| num  
| bool  
| (  $T^* \rightarrow T$  )  
| Ref  $T$

*Types*  
*Unit Type*  
*Number Type*  
*Boolean Type*  
*Function Type*  
*Reference Type*

- Base Type
- Recursively-defined types, i.e. their definition makes use of other types: reference, function types

# Typelang



- TypeLang assert that all numeric values (constants) have type `num`

`(Num)`

`n : num`

- TypeLang assert that all Boolean values (constants) have type `bool`

`(Num)`

`n : bool`

# Typelang

○ Atomic: no subexpressions

$(\text{NumExp})$

$(\text{NumExp } n) : \text{num}$



# Typelang

- Conditional assertion: if subexpressions of the addition expression always produce values of type `num`, then the addition expression will produce a value of type `num`.

(ADDEXP)

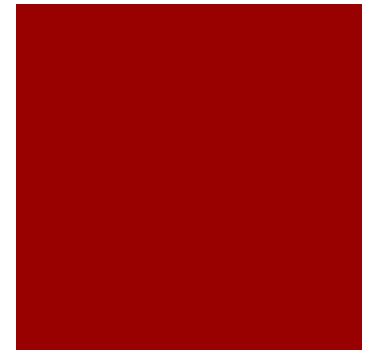
$$tenv \vdash e_i : \text{num}, \forall i \in 1..n$$

---

$$tenv \vdash (\text{AddExp } e_0 \ e_1 \ \dots \ e_n) : \text{num}$$

- if subexpressions  $e_0$  to  $e_n$  have type `num`, then the expression  $(\text{AddExp } e_0 \ , \ e_1 \ , \ \dots \ , \ e_n)$  will have type `num` also

# Typelang



(MULTEXP)

$$\frac{tenv \vdash e_i : \mathbf{num}, \forall i \in 1..n}{tenv \vdash (MultExp\ e_0\ e_1\ \dots\ e_n) : \mathbf{num}}$$

(SUBEXP)

$$\frac{tenv \vdash e_i : \mathbf{num}, \forall i \in 1..n}{tenv \vdash (SubExp\ e_0\ e_1\ \dots\ e_n) : \mathbf{num}}$$

(DIVEXP)

$$\frac{tenv \vdash e_i : \mathbf{num}, \forall i \in 1..n}{tenv \vdash (DivExp\ e_0\ e_1\ \dots\ e_n) : \mathbf{num}}$$



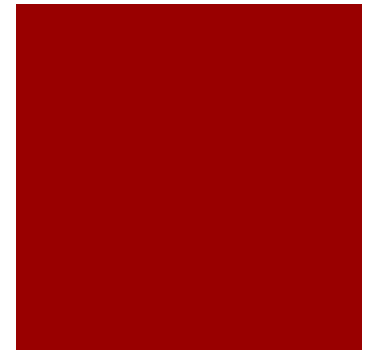
# Typelang

$$\text{get}(\text{tenv}, v') = \begin{cases} \text{Error} & \text{tenv} = (\text{EmptyEnv}) \\ t & \text{tenv} = (\text{ExtendEnv } v \ t \ \text{tenv}') \\ & \text{and } v = v' \\ \text{get}(\text{tenv}', v') & \text{Otherwise.} \end{cases}$$

(VAREXP)

$$\frac{\text{get}(\text{tenv}, \text{var}) = t}{\text{tenv} \vdash (\text{VarExp } \text{var}) : t}$$

# Typelang



$letexp ::= (\text{let } ((\text{identifier} : T \text{ exp})^+) \text{ exp})$

*Let expression*

(LETEXP)

$tenv \vdash e_i : t_i, \forall i \in 0..n$

$tenv_n = (\text{ExtendEnv } var_n \ t_n \ tenv_{n-1}) \ \dots$

$tenv_0 = (\text{ExtendEnv } var_0 \ t_0 \ tenv)$

$tenv_n \vdash e_{body} : t$

---

$tenv \vdash (\text{LetExp } var_0, \dots, var_n, t_0, \dots, t_n, e_0, \dots, e_n, e_{body}) : t$

# Typelang



$\text{lambdaexp} ::= (\text{lambda } (\{ \text{identifier} : T \}^*) \text{exp})$  Lambda

```
(lambda
  (
    x : num    //Argument 1
    y : num    //Argument 2
    z : num    //Argument 3
  )
  (+ x (+ y z))
)
```

- Declares a function with three arguments x, y and z
- Type for this function is,
- $\text{num num num} \rightarrow \text{num}$
- Return type is num as well
- Type checks!

```
(
  (lambda
    (
      x : num    //Argument 1
      y : num    //Argument 2
      z : num    //Argument 3
    )
    (+ x (+ y z))
  )
  1 2 3
)
```

- Declares the same function and also calls it by passing integer parameters 1, 2 and 3 for arguments x, y and z
- Type checks!

# Typelang

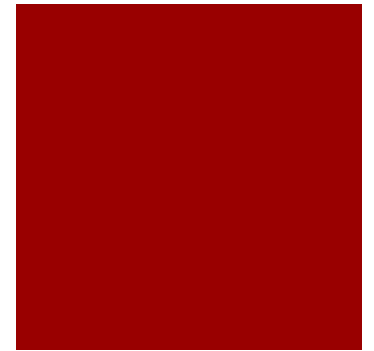
(LAMBDAEXP)

$$\frac{\begin{array}{l} \text{tenv}_n = (\text{ExtendEnv } \text{var}_n \ t_n \ \text{tenv}_{n-1}) \dots \\ \text{tenv}_0 = (\text{ExtendEnv } \text{var}_0 \ t_0 \ \text{tenv}) \quad \text{tenv}_n \vdash e_{\text{body}} : t \end{array}}{\text{tenv} \vdash (\text{LambdaExp } \text{var}_0 \ \dots \ \text{var}_n, t_0 \ \dots \ t_n, e_{\text{body}}) : (t_0 \ \dots \ t_n \multimap t)}$$

(CALLEXP)

$$\frac{\text{tenv} \vdash e_{\text{op}} : (t_0 \ \dots \ t_n \multimap t) \quad \text{tenv} \vdash e_i : t_i, \forall i \in 0..n}{\text{tenv} \vdash (\text{CallExp } e_{\text{op}} \ e_0 \ \dots \ e_n) : t}$$

# Typelang



$T ::=$   
| unit  
| num  
| bool  
| (  $T^* \rightarrow T$  )  
| Ref  $T$

*Types*  
*Unit Type*  
*Number Type*  
*Boolean Type*  
*Function Type*  
*Reference Type*

*type*  $::=$  ...  
| String  
| (  $T, T$  )  
| List  $\langle T \rangle$

*Types*  
*String Type*  
*Pair Type*  
*List Type*

Q&A

