CprE 310 / Spring 2019

Homework 3 Solutions

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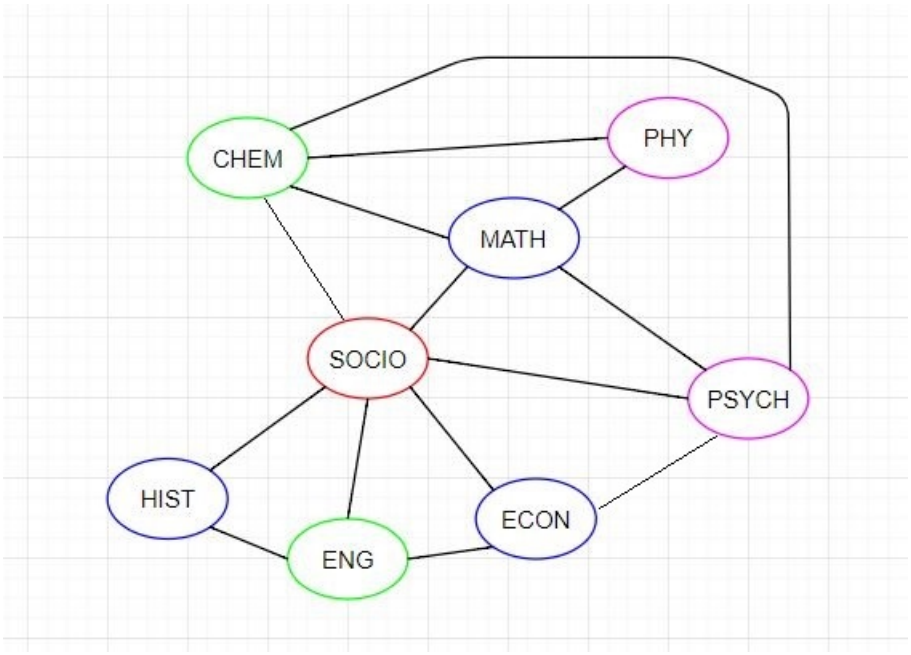
1. **(10 points)** Several departments across campus have wireless access points. However, due to a mistake in the design of the router, *interference* can happen if two access points are within 200 feet of each other and operating on the same frequency. Here is how the departments are situated:

Department	is within 200 feet of
MATH	PHY, PSYCH, CHEM, SOCIO
SOCIO	HIST, ENG, ECON, MATH, CHEM, PSYCH
PHY	MATH, CHEM
PSYCH	MATH, CHEM, SOCIO, ECON
HIST	SOCIO, ENG
ENG	ECON, SOCIO, HIST
ECON	ENG, SOCIO, PSYCH
CHEM	MATH, PSYCH, SOCIO, PHY

   (a) Model the above table using an undirected graph, where nodes denote departments and the "is within 200 feet of" information denotes edges.
   (b) Assign labels (or colors) to the nodes such that no two nodes connected by an edge are assigned the same label/color.
   (c) Play around with possible colorings. Show that the *fewest* number of labels/colors needed to color this graph is 4.
   (d) Conclude by inferring the minimum number of frequencies required for the overall wireless system to function without interference

   Solution



(a) through (c):

(b) 4 is the minimum number of frequencies required for the overall wireless system to function without interference.

2. **(20 points)** *This question involves programming. You are free to use any language – Java, C++, Python, etc – of your choice, as long as you include your code, comments explaining what the code does, and a screenshot of the output of your program.*

Suppose you are the inventor of a social network (call it Fussbook). The attached file, network_fussbook.txt, describes the friendships between individuals on Fussbook. Assume that all relationships are symmetric, i.e., the graph is undirected.

(a) Write a small program that takes a filename as input, opens and parses the file, and outputs the number of nodes and edges in the network.
(b) Suppose you are tasked with finding the "most well-connected" person on Fussbook. In your view, which concept from graph theory (discussed in class) is a plausible measure of well-connectedness?
(c) Write another small program that automatically determines the most well-connected person in Fussbook.

Repeat parts a and c for a rival social network (call it Instaface); a description of friend relationships on this network is given in network_instaface.txt.

Solution

(a) and (c) Expected outputs for Fussbook.txt: number of nodes = 11 number of edges = 10 most well connected person = 11

Expected outputs for Instaface: number of nodes = 17 number of edges = 24 most well connected person = 9

3. **(10 points)** Determine whether or not the following statements are true or false. If you think a statement is true, give your reasoning. If you think it is false, provide a counterexample.

   (a) For all sets A and B, if $B \subseteq \bar{A}$, then $A \cap B = \emptyset$.
   (b) For all sets A, B, C, if $B \subseteq C$ and $A \cap C = \emptyset$, then $A \cap B = \emptyset$.

   Solution

(a) True

Proof by contradiction:

Assume that $A \cap B \neq \emptyset$

Then there exists an element x such that x $\in$ A and x $\in$ B.

Because x $\in$ B and B $\subseteq \bar{A}$, x $\in \bar{A}$, which contradicts to the assumption that x $\in$ A.

(b) True

Proof by contradiction:

Assume that $A \cap B \neq \emptyset$

Then there exists an element x such that x $\in$ A and x $\in$ B.

Because x $\in$ B and B $\subseteq$ C, x $\in$ C.

Because x $\in$ C and $A \cap C = \emptyset$, x $\notin$ A, which contradicts to the assumption that x $\in$ A.

4. **(10 points)** Indicate which of the following relationships are true and which are false, together with a brief explanation in words why you think that is the case:

   (a) $Z^+ \subseteq Q$.
   (b) $Q \subseteq Z$.

   (c) $Q \cap R = Q$.
   (d) $Z^+ \cap R = Z^+$.
   (e) $\emptyset \subset \mathbb{N}$.

   Solution

   a) True

   b) False. $1/2 \in Q$, but $1/2 \notin Z$

   c) True

   d) True

   e) True