

EE 330

Lecture 15

Devices in Semiconductor Processes

- Diodes
- Capacitors
- MOSFETs

Basic Devices and Device Models

- Resistor

 Diode

- Capacitor
- MOSFET
- BJT

Analysis of Nonlinear Circuits

(Circuits with one or more nonlinear devices)

What analysis tools or methods can be used?

KCL ?

Nodal Analysis

KVL?

Mesh Analysis

~~Superposition?~~

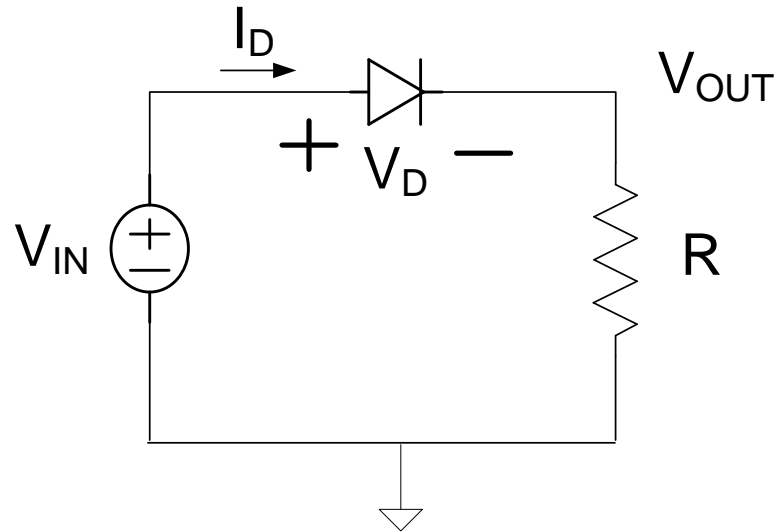
Two-Port Subcircuits

~~Voltage Divider ?~~

~~Current Divider?~~

~~Thevenin and Norton Equivalent Circuits?~~

Consider again the basic rectifier circuit



$$\left. \begin{aligned} V_{IN} &= V_D + I_D R \\ V_{OUT} &= I_D R \\ I_D &= I_S \left(e^{\frac{V_D}{V_t}} - 1 \right) \end{aligned} \right\} \quad V_{OUT} = I_S R \left(e^{\frac{V_{IN} - V_{OUT}}{V_t}} - 1 \right)$$

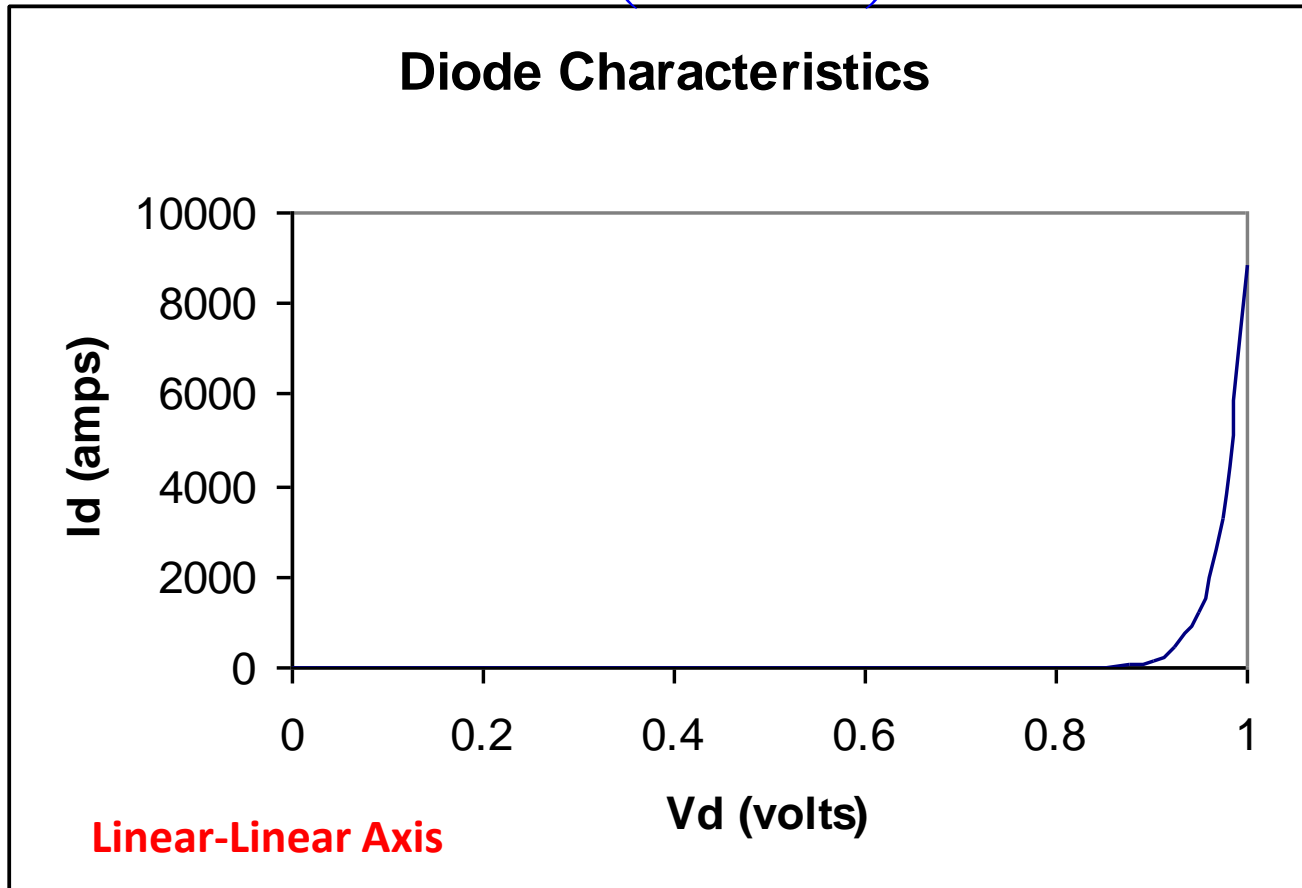
Even the simplest diode circuit does not have a closed-form explicit solution when diode equation is used to model the diode !!

Due to the nonlinear nature of the diode equation

Simplifications are essential if analytical results are to be obtained

Lets study the diode equation a little further

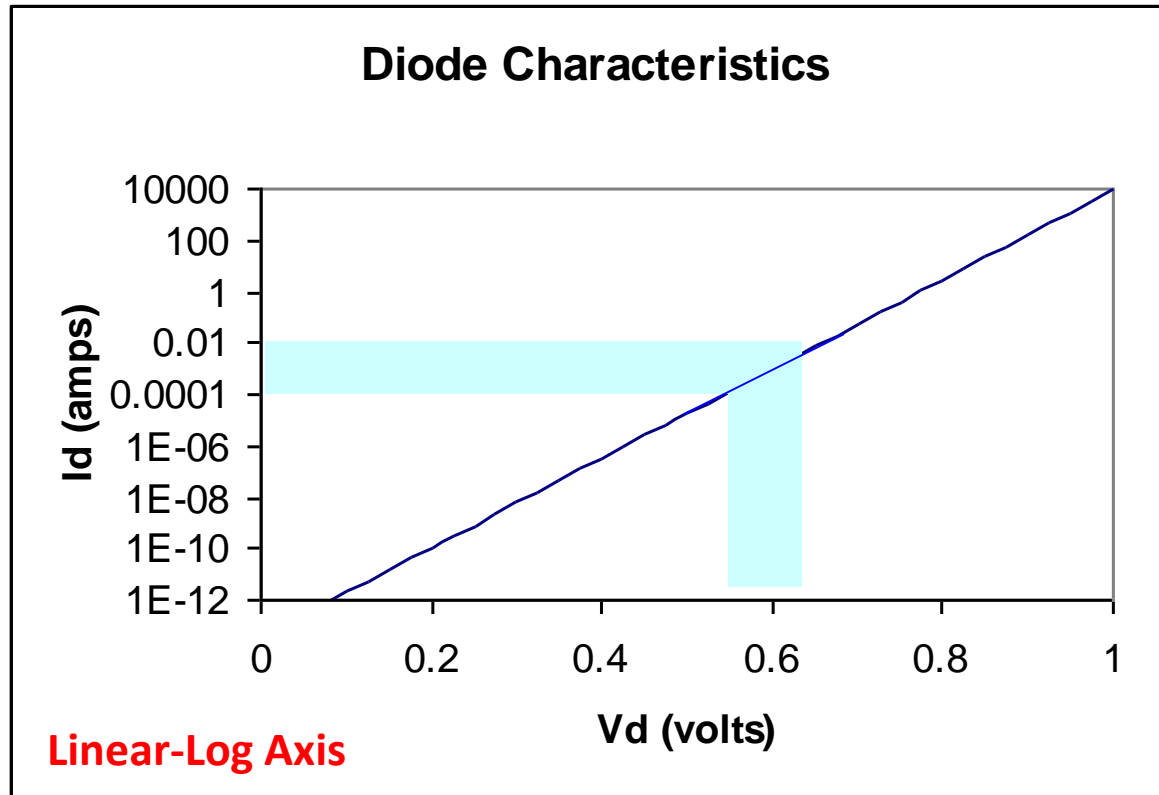
$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Power Dissipation Becomes Destructive if $V_d > 0.85V$ (actually less)

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

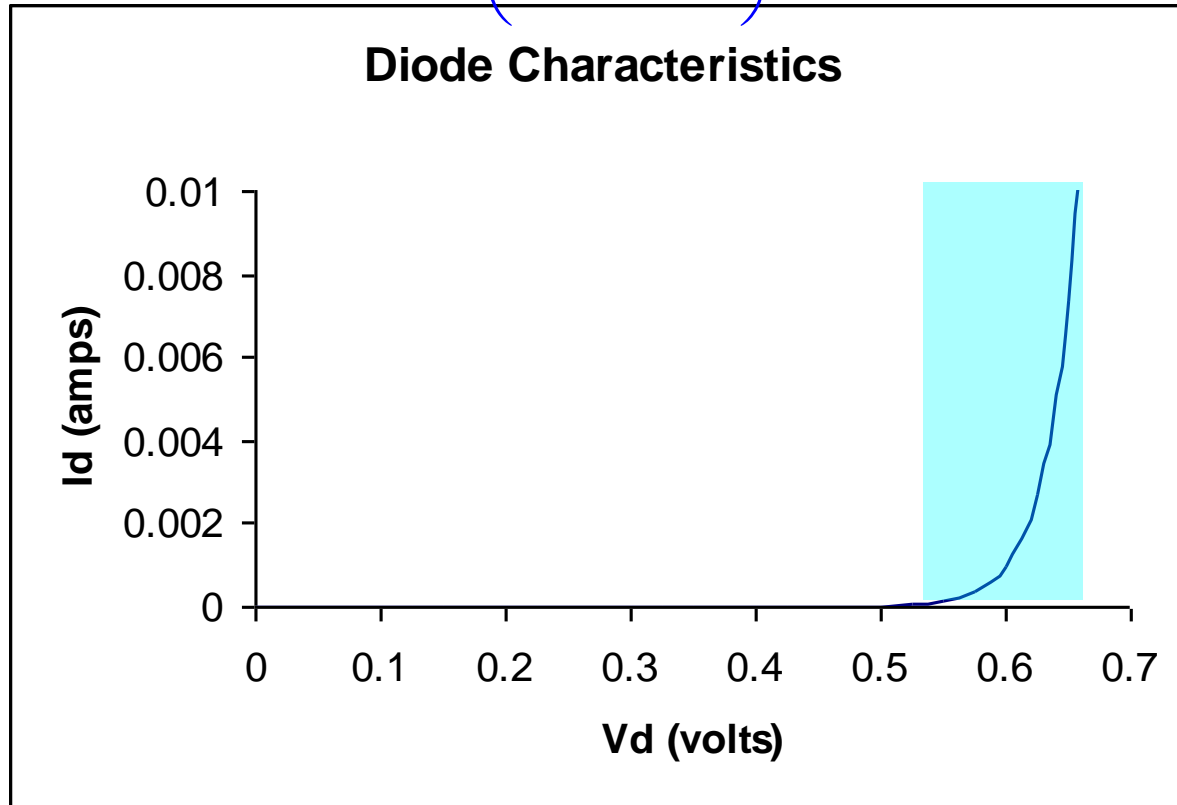


For two decades of current change, V_d is close to 0.6V

This is the most useful conducting current range for many applications

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



For two decades of current change, V_d is close to 0.6V

This is the most useful current range when conducting for many applications

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

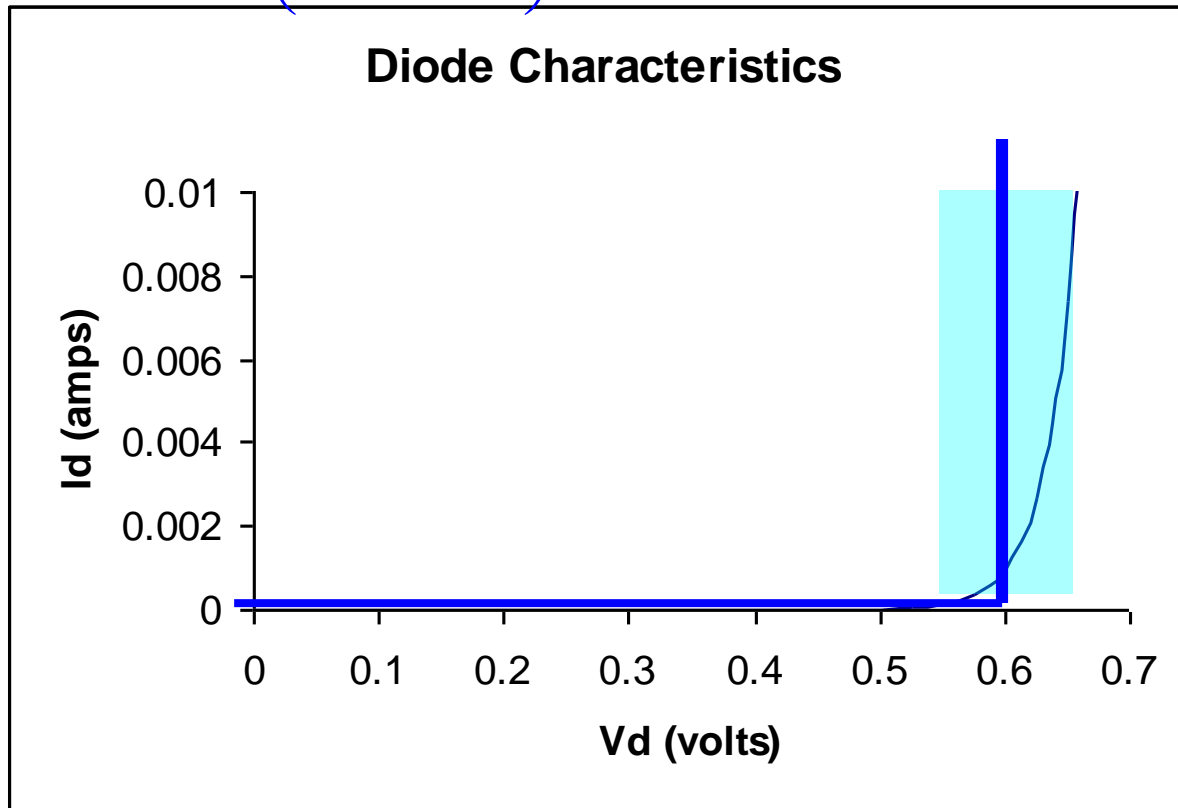


$$I_d = 0$$

$$V_d = 0.6V$$

$$V_d < 0.6V$$

$$I_d > 0$$

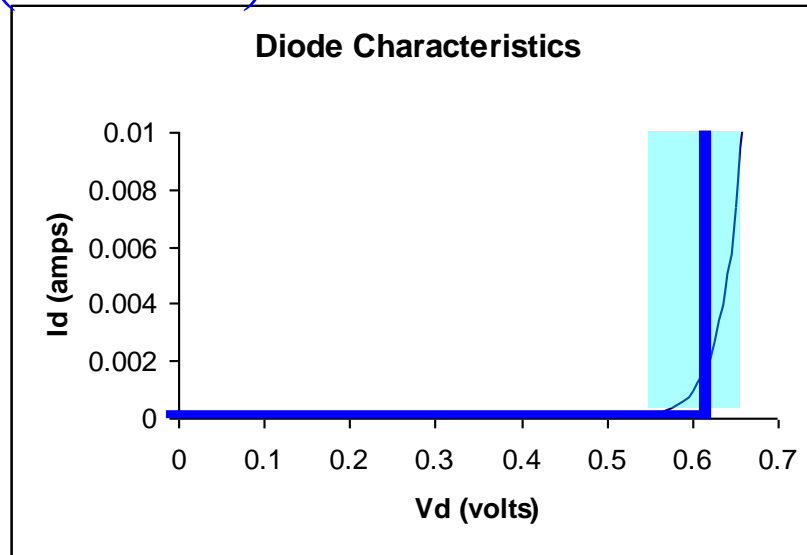


Widely Used Piecewise Linear Model

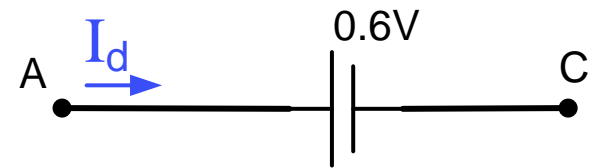
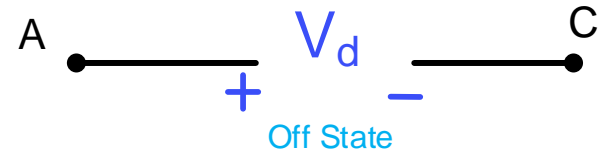
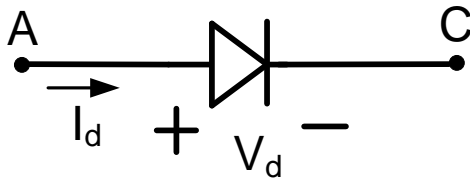
Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

$$\begin{aligned} &\longrightarrow I_d = 0 & V_d < 0.6V \\ &V_d = 0.6V & I_d > 0 \end{aligned}$$

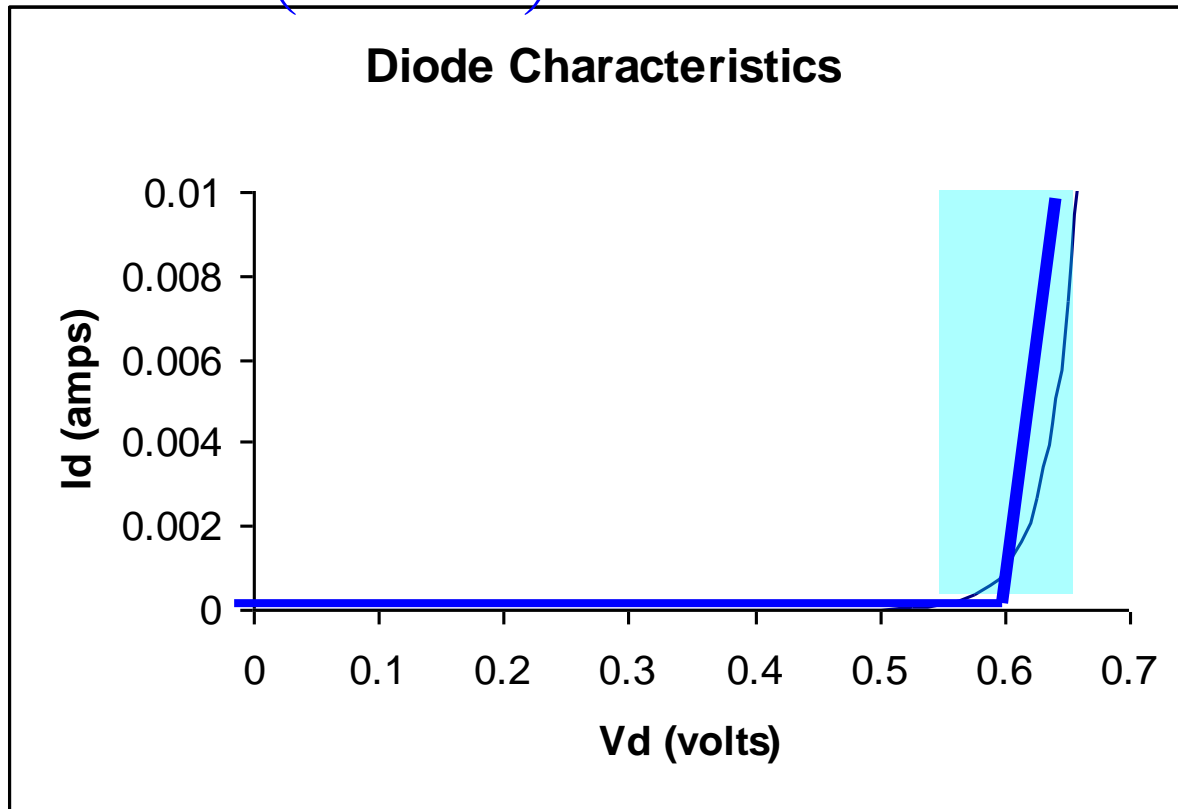


Equivalent Circuit



Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Better model in “ON” state though often not needed

Includes Diode “ON” resistance

Lets study the diode equation a little further

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

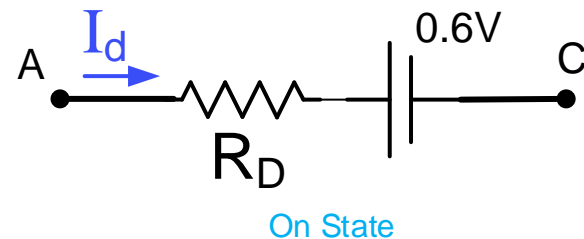
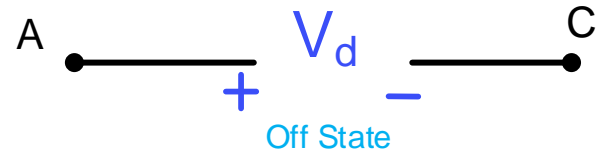
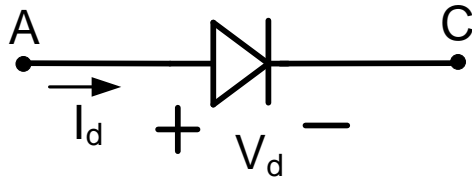
Piecewise Linear Model with Diode Resistance

$$I_d = 0 \quad \text{if } V_d < 0.6V$$

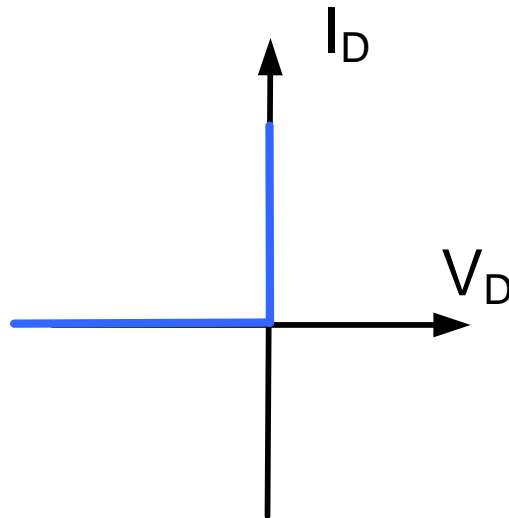
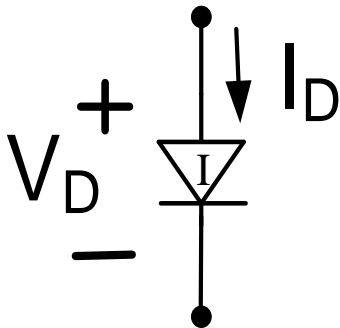
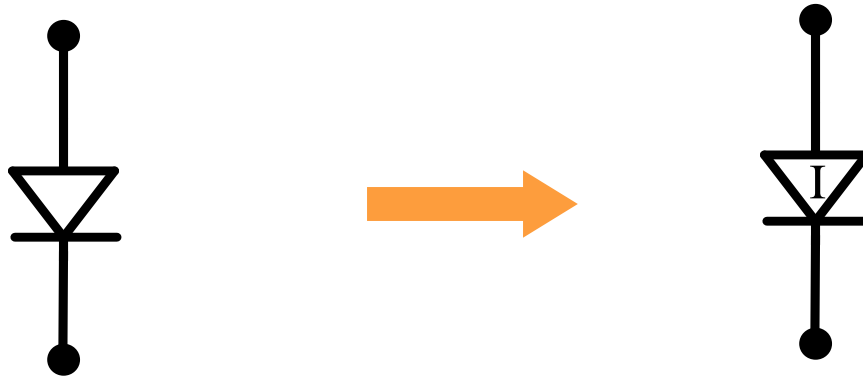
$$V_d = 0.6V + I_d R_D \quad \text{if } I_d > 0$$

(R_D is rather small: often in the 20Ω to 100Ω range):

Equivalent Circuit

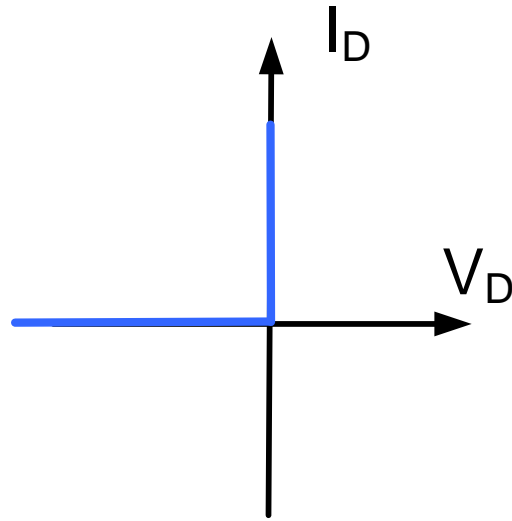


The Ideal Diode



$$\begin{aligned} I_D &= 0 & \text{if } V_D &\leq 0 \\ V_D &= 0 & \text{if } I_D &> 0 \end{aligned}$$

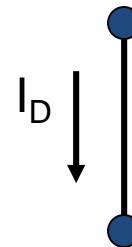
The Ideal Diode



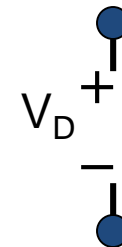
$$\begin{aligned} I_D &= 0 & \text{if } V_D &\leq 0 & \text{"OFF"} \\ V_D &= 0 & \text{if } I_D &> 0 & \text{"ON"} \end{aligned}$$



"ON"



"OFF"

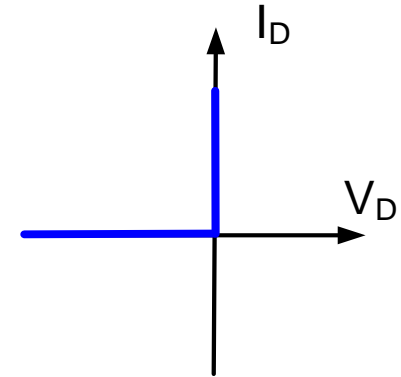
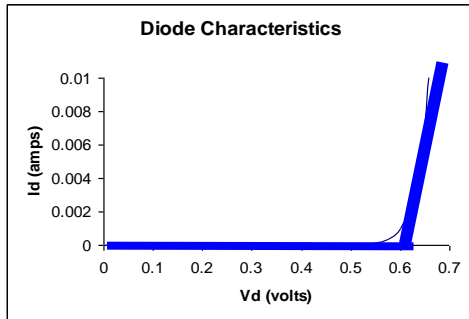
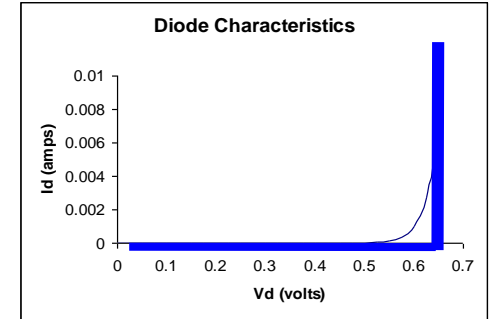
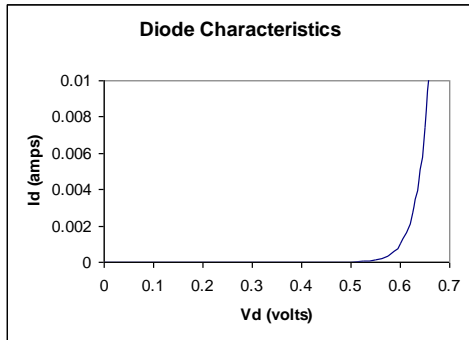


Valid for

$$I_D > 0$$

$$V_D \leq 0$$

Diode Models



Which model should be used?

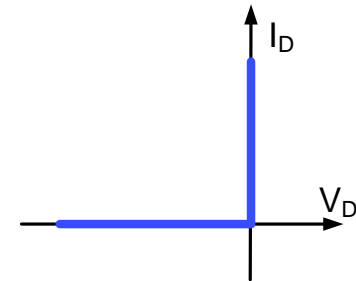
The simplest model that will give acceptable results in the analysis of a circuit

Diode Model Summary

Piecewise Linear Models

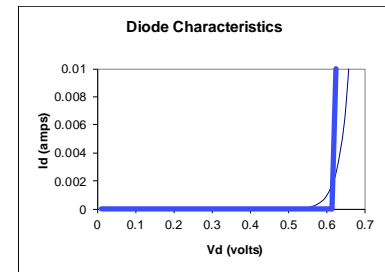
$$I_d = 0 \quad \text{if } V_d < 0$$

$$V_d = 0 \quad \text{if } I_d > 0$$



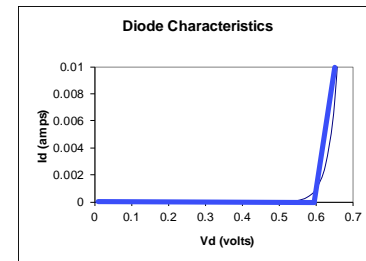
$$I_d = 0 \quad \text{if } V_d < 0.6V$$

$$V_d = 0.6V \quad \text{if } I_d > 0$$



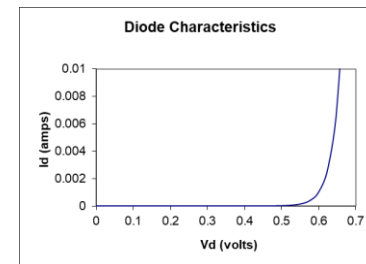
$$I_d = 0 \quad \text{if } V_d < 0.6$$

$$V_d = 0.6 + I_d R_d \quad \text{if } I_d > 0$$



Diode Equation

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Diode Model Summary

Piecewise Linear Models

$$I_d = 0 \quad \text{if } V_d < 0$$

$$V_d = 0 \quad \text{if } I_d > 0$$

$$I_d = 0 \quad \text{if } V_d < 0.6V$$

$$V_d = 0.6V \quad \text{if } I_d > 0$$

$$I_d = 0 \quad \text{if } V_d < 0.6$$

$$V_d = 0.6 + I_d R_d \quad \text{if } I_d > 0$$

Diode Equation

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

When is the ideal model adequate?

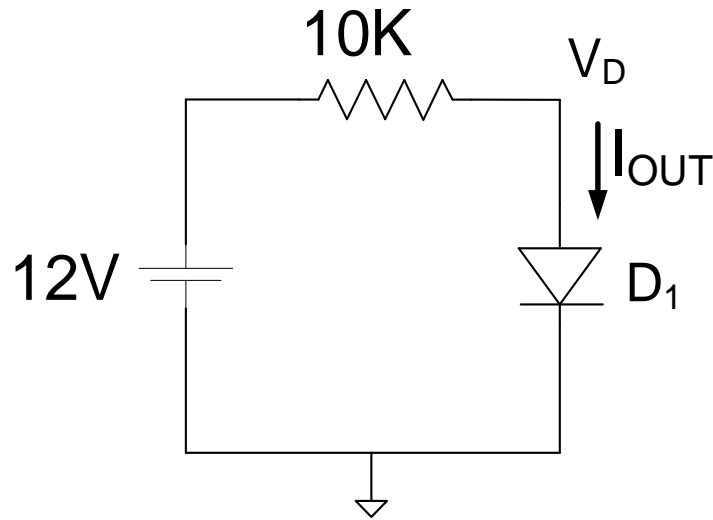
When it doesn't make much difference whether $V_d = 0V$ or $V_d = 0.6V$

When is the second piecewise-linear model adequate?

When it doesn't make much difference whether $V_d = 0.6V$ or $V_d = 0.7V$

Example:

Determine I_{OUT} for the following circuit



Solution:

If the diode equation model is used will obtain:

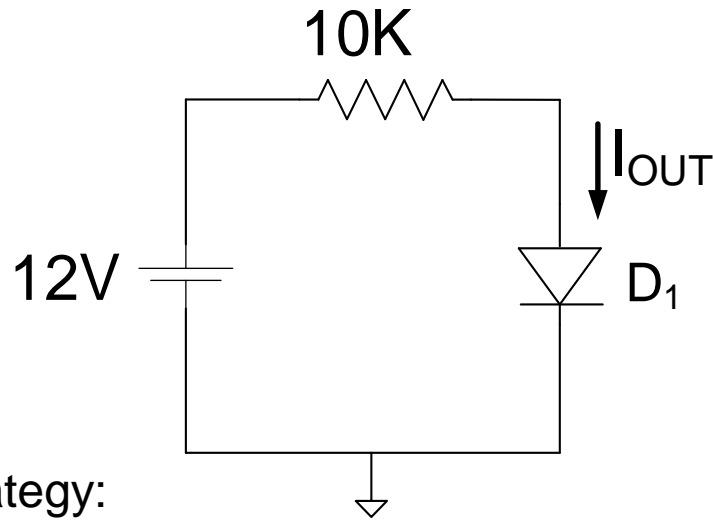
$$\left. \begin{aligned} 12 &= I_{OUT} \cdot 10K + V_D \\ I_{OUT} &= I_S \left(e^{\frac{V_D}{V_t}} - 1 \right) \end{aligned} \right\} \Rightarrow I_{OUT} = I_S \left(e^{\frac{-I_{OUT} \cdot 10K}{V_t}} e^{\frac{12}{V_t}} - 1 \right)$$

As in previous example, a closed-form explicit expression for I_{OUT} does not exist

**Will now establish alternate approach for solving this
(and other) nonlinear circuits !**

Example:

Determine I_{OUT} for the following circuit



Alternate Solution Strategy:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2 (verify the “if” condition in model)

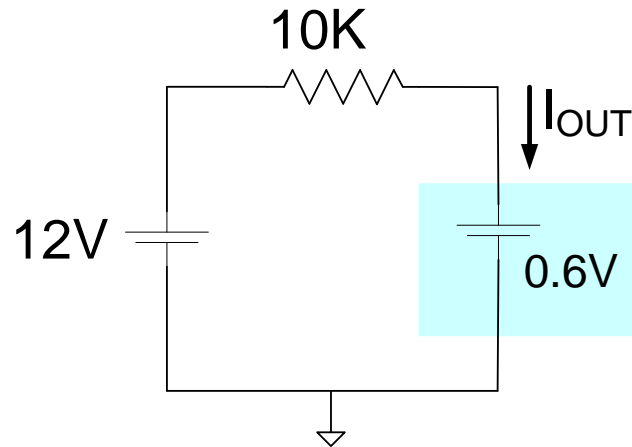
Select
Model

5. Assume PWL with $V_D=0.7V$
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6 (verify the “if” condition in model)
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Validate
Model

Solution:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)



3. Analyze circuit with model

$$I_{OUT} = \frac{12V - 0.6V}{10K} = 1.14mA$$

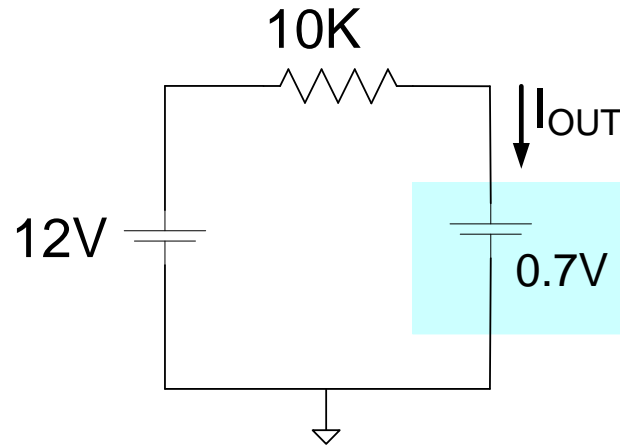
4. Validate state of guess in step 2

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 1.14mA > 0$$

Solution:

5. Assume PWL model with $V_D=0.7V$, $R_D=0$
6. Guess state of diode (ON)



7. Analyze circuit with model

$$I_{OUT} = \frac{12V - 0.7V}{10K} = 1.13mA$$

8. Validate state of guess in step 6

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 1.13mA > 0$$

Solution:

9. Show difference between results using these two models is small

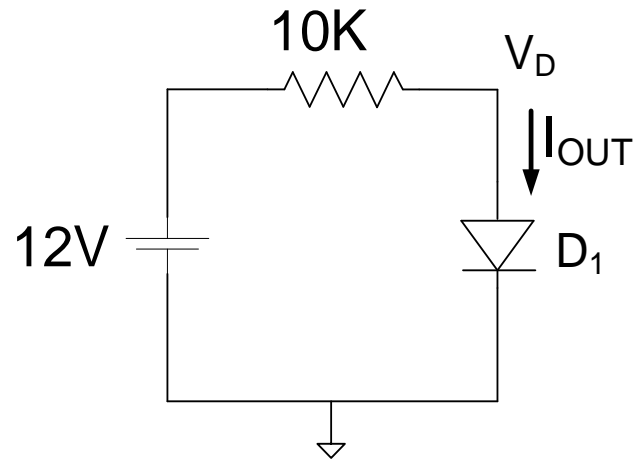
$$I_{\text{OUT}}=1.14\text{mA and } I_{\text{OUT}}=1.13 \text{ mA} \quad \text{are close}$$

Thus, can conclude

$$I_{\text{OUT}} \cong 1.14\text{mA}$$

Example:

Determine I_{OUT} for the following circuit



How do the two solutions compare?

With diode equation model :

$$I_{OUT} = I_S \left(e^{\frac{-I_{OUT} \cdot 10K}{V_t}} e^{\frac{12}{V_t}} - 1 \right)$$

With PWL model:

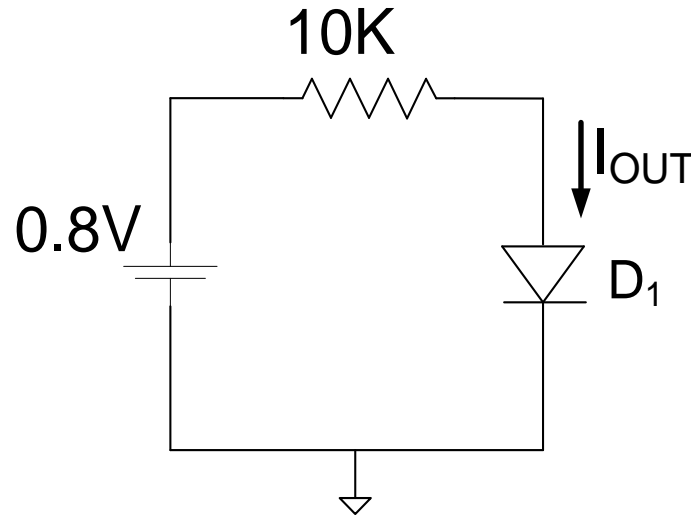
$$I_{OUT} \cong 1.14\text{mA}$$

What was the major reason the PWL model simplified the analysis?

Piecewise Linear Model

Example:

Determine I_{OUT} for the following circuit



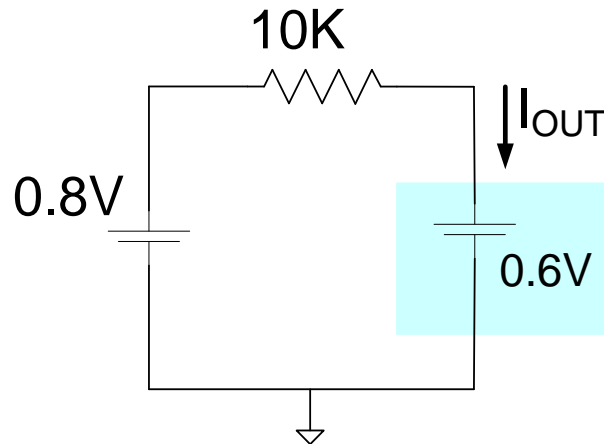
Solution:

Strategy:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2
5. Assume PWL with $V_D=0.7V$
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Solution:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)



3. Analyze circuit with model

$$I_{OUT} = \frac{0.8 - 0.6V}{10K} = 20\mu A$$

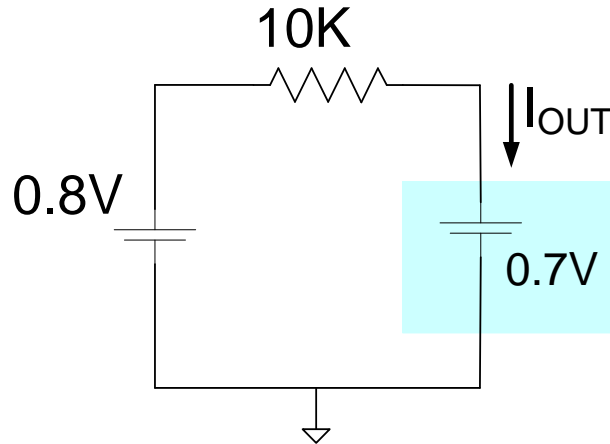
4. Validate state of guess in step 2

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 20\mu A > 0$$

Solution:

5. Assume PWL model with $V_D=0.7V$, $R_D=0$
6. Guess state of diode (ON)



7. Analyze circuit with model

$$I_{OUT} = \frac{0.8V - 0.7V}{10K} = 10\mu A$$

8. Validate state of guess in step 6

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 10\mu A > 0$$

Solution:

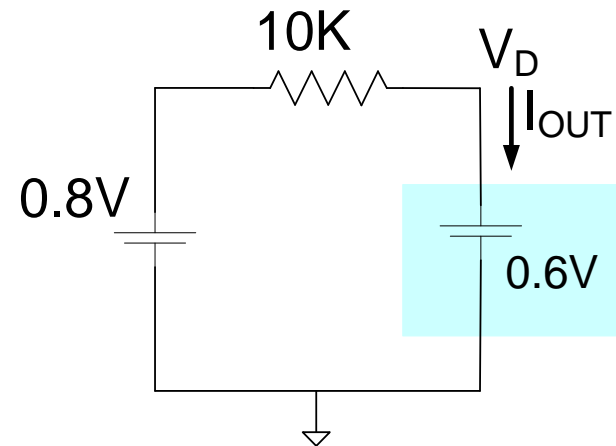
9. Show difference between results using these two models is small

$$I_{OUT}=10\mu A \text{ and } I_{OUT}=20\mu A \quad \text{are not close}$$

10. If difference is not small, must use a different model

Thus must use diode equation to model the device

$$\left. \begin{aligned} I_{OUT} &= \frac{0.8 - V_D}{10K} \\ I_{OUT} &= I_S e^{\frac{V_D}{V_t}} \end{aligned} \right\}$$



Solve simultaneously, assume $V_t=25\text{mV}$, $I_S=1\text{fA}$

Solving these two equations by iteration, obtain $V_D=0.6148\text{V}$ and $I_{OUT}=18.60\mu A$

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

Observations:

- Analysis generally simplified dramatically (particularly if piecewise model is linear)
- Approach applicable to wide variety of nonlinear devices
- Closed-form solutions give insight into performance of circuit
- Usually much faster than solving the nonlinear circuit directly
- Wrong guesses in the state of the device do not compromise solution (verification will fail)
- Helps to guess right the first time
- Detailed model is often not necessary with most nonlinear devices
- For practical circuits, the simplified approach usually applies

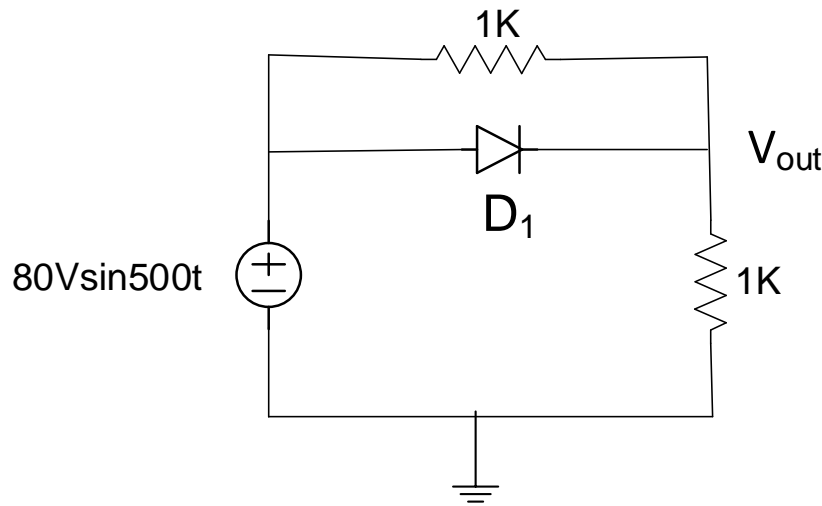
Key Concept For Analyzing Circuits with Nonlinear Devices

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

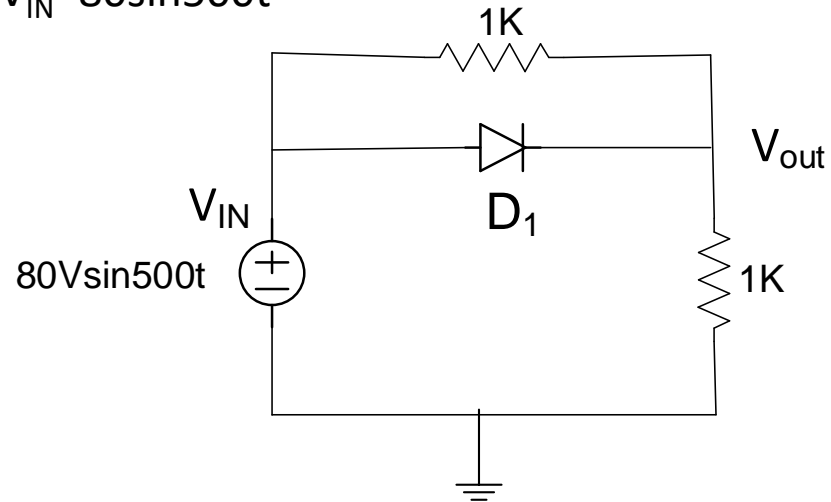
1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

What about nonlinear circuits (using piecewise models) with time-varying inputs?

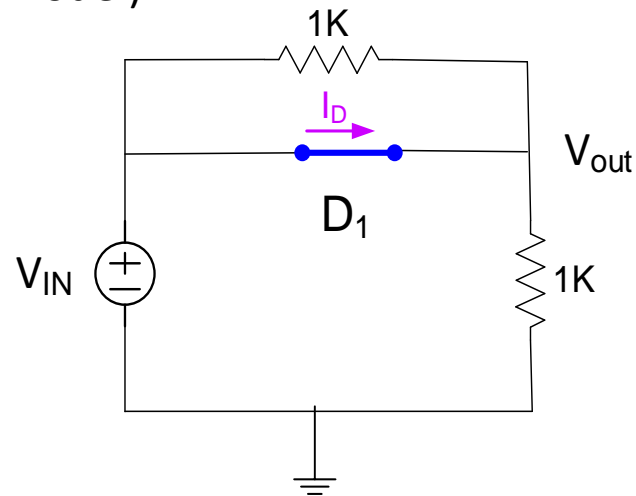


Same process except state verification (step 3) may include a range where solution is valid

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Guess D_1 ON (will use ideal diode model)

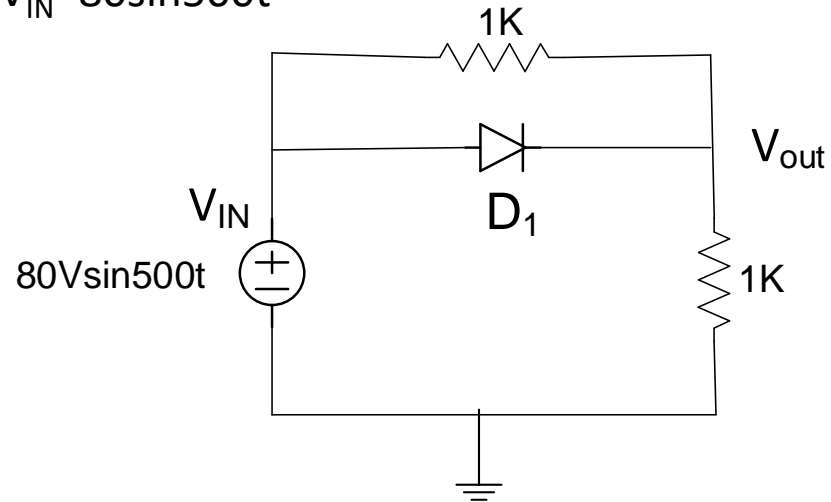


$$V_{OUT}=V_{IN}=80\sin(500t)$$

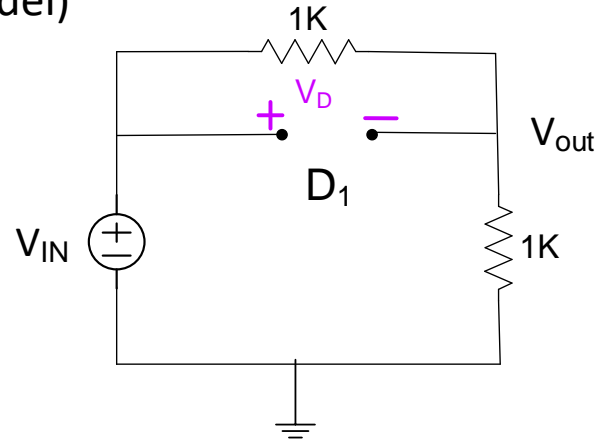
Valid for $I_D > 0$ $I_D = \frac{V_{IN}}{1K}$

Thus valid for $V_{IN} > 0$

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Guess D_1 OFF (will use ideal diode model)

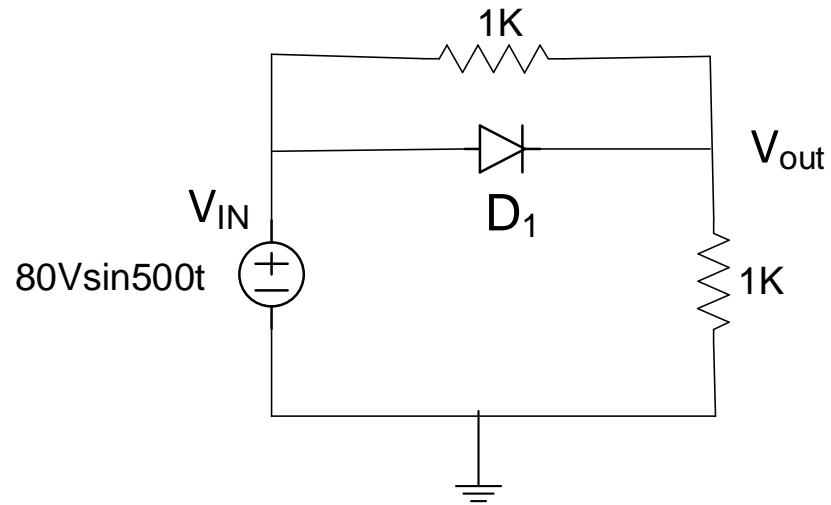


$$V_{OUT}=V_{IN}/2=40\sin(500t)$$

$$\text{Valid for } V_D < 0 \quad V_D = \frac{V_{IN}}{2}$$

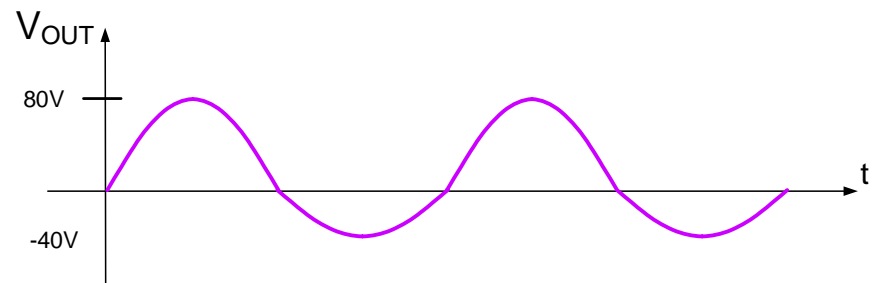
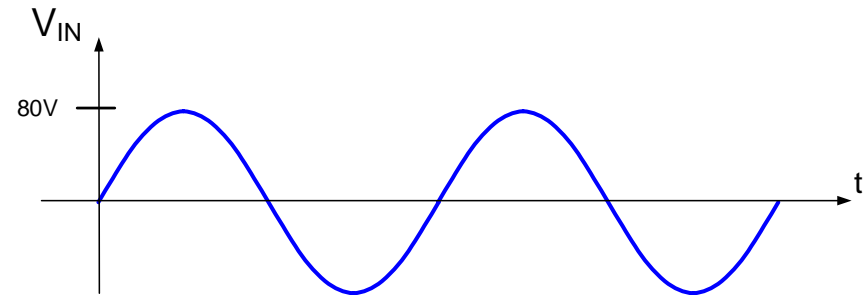
Thus valid for $V_{IN} < 0$

Example: Determine V_{OUT} for $V_{IN}=80\sin 500t$



Thus overall solution

$$V_{OUT} = \begin{cases} 80 \sin 500t & \text{for } V_{IN} > 0 \\ 40 \sin 500t & \text{for } V_{IN} < 0 \end{cases}$$

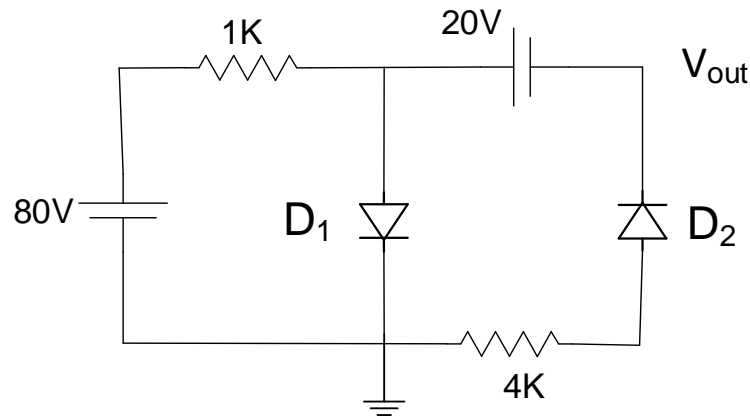


Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

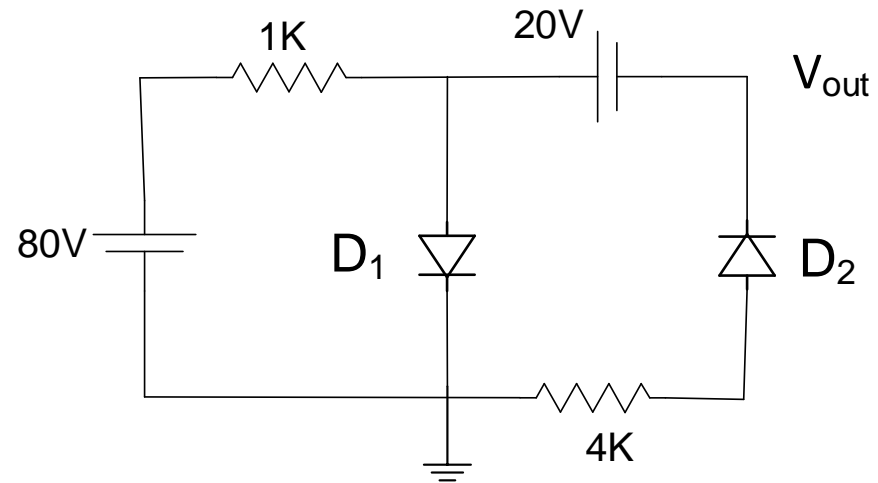
1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

What about circuits (using piecewise models) with multiple nonlinear devices?

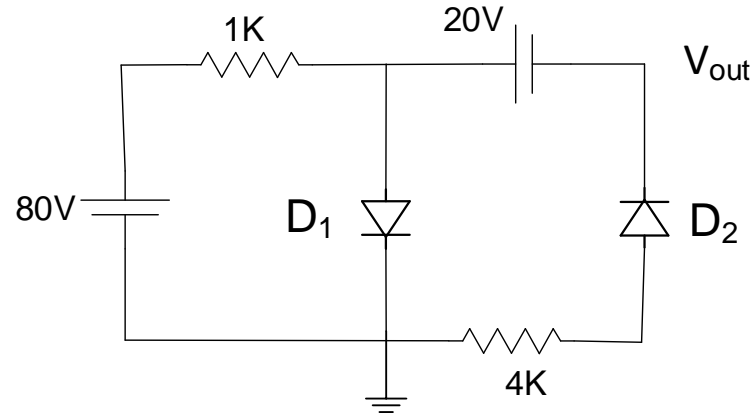


Guess state for each device (multiple combinations possible)

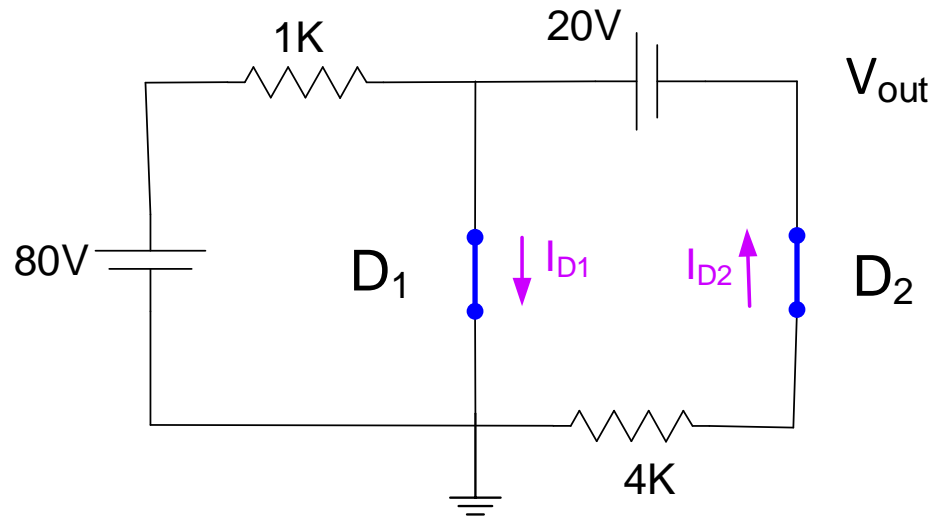
Example: Obtain V_{OUT}



Example: Obtain V_{OUT}



Guess D_1 and D_2 on



$$V_{OUT} = -20V$$

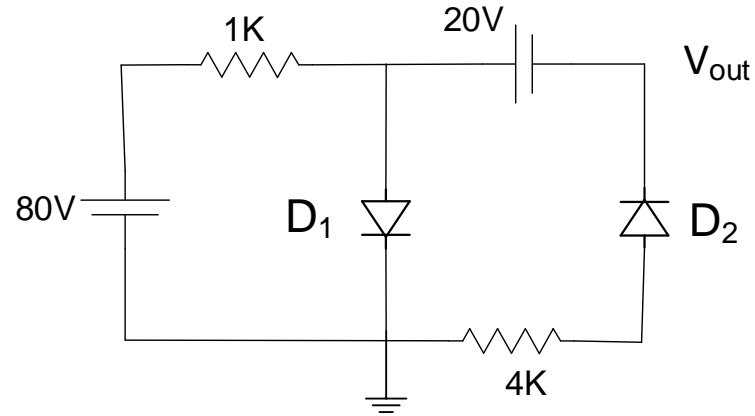
Valid for $I_{D1} > 0$ and $I_{D2} > 0$

$$I_{D2} = \frac{20V}{4K} = 5mA > 0$$

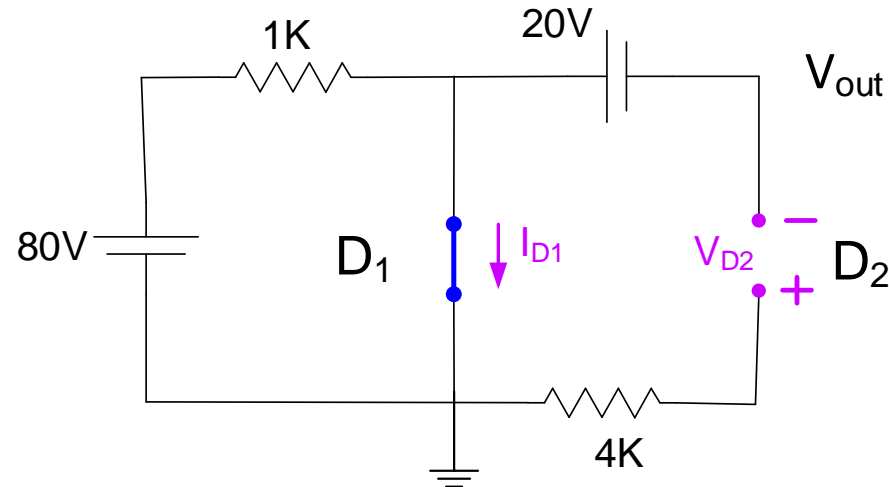
$$I_{D1} = \frac{80V}{1K} + I_{D2} = 85mA > 0$$

Since validates, solution is valid

Example: Obtain V_{OUT}



If we had guessed wrong
Guess D_1 and D_2 off



$$V_{OUT} = -20V$$

Valid for $I_{D1} > 0$ and $V_{D2} < 0$

$$I_{D2} = \frac{20V}{4K} = 5mA > 0$$

$$V_{D2} = +20$$

$$I_{D1} = \frac{80V}{1K} + I_{D2} = 85mA > 0$$

Since fails to validate, solution is not valid so guess is wrong !

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Single Nonlinear Device

Process:

1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

Multiple Nonlinear Devices

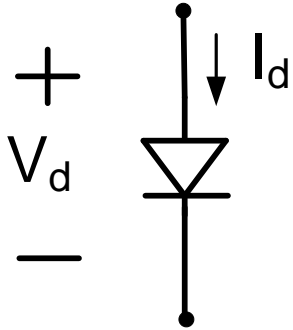
Process:

1. Guess state of each device (may be multiple combinations)
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify models (if necessary)

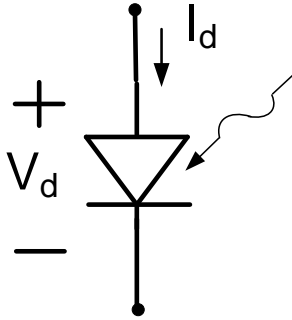
Analytical solutions of circuits with multiple nonlinear devices are often impossible to obtain if detailed non-piecewise nonlinear models are used

Types of Diodes

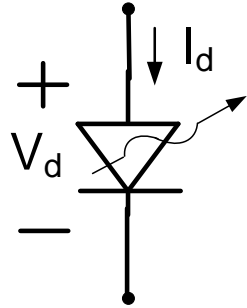
pn junction diodes



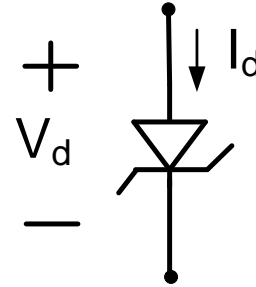
Signal or Rectifier



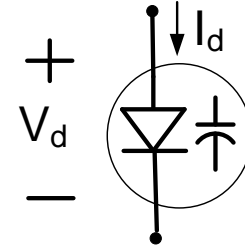
Pin or Photo



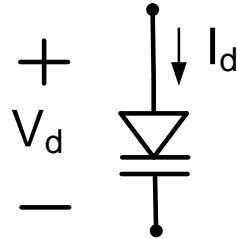
Light Emitting LED
Laser Diode



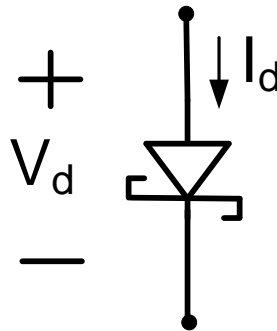
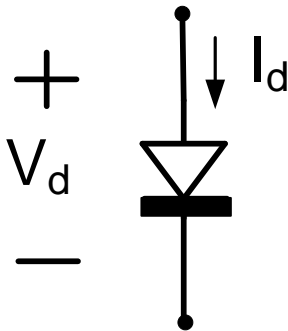
Zener



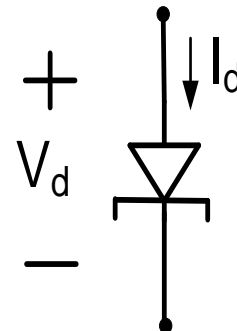
Varactor or Varicap



Metal-semiconductor junction diodes



Schottky Barrier



End of Lecture 15