Problem Set 3
Due: Monday, October 12th

Exercise 6.1

5pts

How many solutions are there for the map-coloring problem in Figure **australia-figure**? How many solutions if four colors are allowed? Two colors?

Exercise 6.6 [nary-csp-exercise]

10pts

Show how a single ternary constraint such as "$A + B = C$" can be turned into three binary constraints by using an auxiliary variable. You may assume finite domains. (*Hint:* Consider a new variable that takes on values that are pairs of other values, and consider constraints such as "X is the first element of the pair Y.") Next, show how constraints with more than three variables can be treated similarly. Finally, show how unary constraints can be eliminated by altering the domains of variables. This completes the demonstration that any CSP can be transformed into a CSP with only binary constraints.

Exercise 6.8

10pts

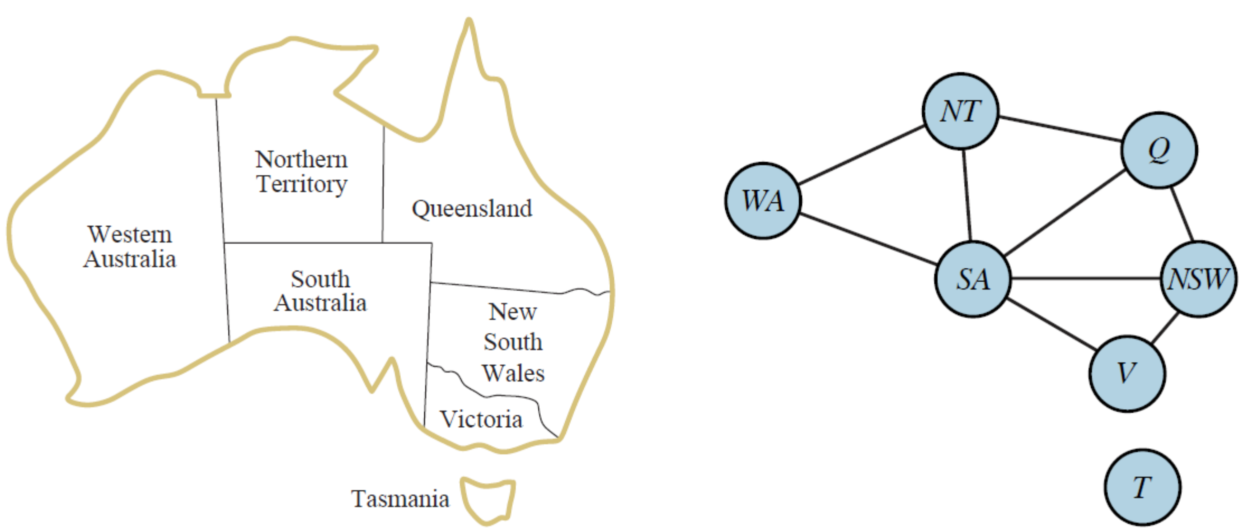
Consider the graph with 8 nodes $A_1, A_2, A_3, A_4, H, T, F_1, F_2$. A_i is connected to A_{i+1} for all i, each A_i is connected to H, H is connected to T, and T is connected to each F_i. Find a 3-coloring of this graph by hand using the following strategy: backtracking with conflict-directed backjumping, the variable order $A_1, H, A_4, F_1, A_2, F_2, A_3, T$, and the value order R, G, B.

Exercise 6.11

10pts

assignment {WA = green, V = red}

Use the AC-3 algorithm to show that arc consistency can detect the inconsistency of the partial assignment $green, Vred$ for the problem shown in Figure **australia-figure**.



australia-figure

Exercise 6.20

4 + 4 + 6 + 6 = 20pts

Consider the problem of tiling a surface (completely and exactly covering it) with n dominoes (2×1 rectangles). The surface is an arbitrary edge-conected (i.e., adjacent along an edge, not just a corner) collection of $2n$ 1×1 squares (e.g., a checkerboard, a checkerboard with some squares missing, a 10×1 row of squares, etc.).

1. Formulate this problem precisely as a CSP where the dominoes are the variables.
2. Formulate this problem precisely as a CSP where the squares are the variables, keeping the state space as small as possible. (*Hint:* does it matter which particular domino goes on a given pair of squares?)
3. Construct a surface consisting of 6 squares such that your CSP formulation from part (b) has a *tree-structured* constraint graph.
4. Describe exactly the set of solvable instances that have a tree-structured constraint graph.

Exercise 7.4

12 × 1 = 12pts

provide a brief reasoning for each answer.

Which of the following are correct?

1. $False \models True$.
2. $True \models False$.
3. $(A \wedge B) \models (A \ \Leftrightarrow \ B)$.
4. $A \ \Leftrightarrow \ B \models A \vee B$.
5. $A \ \Leftrightarrow \ B \models \neg A \vee B$.
6. $(A \wedge B)\!: \ \Rightarrow : C \models (A\!: \ \Rightarrow : C) \vee (B\!: \ \Rightarrow : C)$.
7. $(C \vee (\neg A \wedge \neg B)) \equiv ((A\!: \ \Rightarrow : C) \wedge (B\!: \ \Rightarrow : C))$.
8. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.
9. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.
10. $(A \vee B) \wedge \neg(A\!: \ \Rightarrow : B)$ is satisfiable.
11. $(A \ \Leftrightarrow \ B) \wedge (\neg A \vee B)$ is satisfiable.
12. $(A \ \Leftrightarrow \ B) \ \Leftrightarrow \ C$ has the same number of models as $(A \ \Leftrightarrow \ B)$ for any fixed set of proposition symbols that includes A, B, C.

Exercise 7.6 [deduction-theorem-exercise]

5 × 4 = 20pts

Prove each of the following assertions:

1. α is valid if and only if $True \models \alpha$.
2. For any α, $False \models \alpha$.
3. $\alpha \models \beta$ if and only if the sentence $(\alpha\!: \ \Rightarrow : \beta)$ is valid.
4. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \ \Leftrightarrow \ \beta)$ is valid.
5. $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg \beta)$ is unsatisfiable.

Exercise 7.7

4 + 5 + 4 = 13pts

Prove, or find a counterexample to, each of the following assertions:

1. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \land \beta) \models \gamma$
2. If $(\alpha \land \beta) \models \gamma$ then $\alpha \models \gamma$ or $\beta \models \gamma$ (or both).
3. If $\alpha \models (\beta \lor \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

Exercise 7.15

15pts

the clauses are given in Exercise 7.25

Use resolution to prove the sentence $\neg A \land \neg B$ from the clauses in Exercise **convert-clausal-exercise**.

S1: $A \ \Leftrightarrow \ (B \lor E)$.
S2: E: $\Rightarrow : D$.
S3: $C \land F$: $\Rightarrow : \neg B$.
S4: E: $\Rightarrow : B$.
S5: B: $\Rightarrow : F$.
S6: B: $\Rightarrow : C$

convert-clausal-exercise clauses from Exercise 7.25

Exercise 7.16 [inf-exercise]

5 + 5 + 5 = 15pts

This exercise looks into the relationship between clauses and implication sentences.

1. Show that the clause $(\neg P_1 \lor \cdots \lor \neg P_m \lor Q)$ is logically equivalent to the implication sentence $(P_1 \land \cdots \land P_m) \Rightarrow Q$.
2. Show that every clause (regardless of the number of positive literals) can be written in the form $(P_1 \land \cdots \land P_m) \Rightarrow (Q_1 \lor \cdots \lor Q_n)$, where the Ps and Qs are proposition symbols. A knowledge base consisting of such sentences is in implicative normal form or **Kowalski form** @Kowalski:1979.
3. Write down the full resolution rule for sentences in implicative normal form.