Stat 330 (Spring 2020): Homework 4

Due: February 19, 2020

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. A box contains seven marbles. Four of them are red and three of them are green. You reach in and choose three at random without replacement. Define a random variable X as: X = the number of red marbles selected.
   (a) What are the possible values X can take on? (i.e. give $Im(X)$)
   (b) Find $\mathbb{P}(X = x)$ for all x in $Im(X)$.
   (c) Make a table for the probability distribution of X as shown in lecture. (Leave probabilities as fractions)

2. * Let X be a random variable with image $Im(X) = \{-2, -1, 0, 1, 2\}$.
   (a) Fill in the blank in the table below to make it a valid probability mass function:

x	-2	-1	0	1	2
$p_X(x)$	0.1	0.3	0.3	0.1	

   (b) Add the cumulative distribution function, $F_X(x)$ to the table.
   (c) Using $p_X(x)$, determine the probabilities that...
      i. X is at least 1.
      ii. X is greater than -1 and at most 1
      iii. X is a negative value
   (d) Using $F_X(x)$, find...
      i. $F_X(1)$
      ii. $F_X(.5)$
      iii. $\mathbb{P}(X \geq 0)$ (*rewrite this first in terms of* $F_X(x)$)
   (e) Find the expected value and variance of X.

3. * Let Y be a random variable with $Y = 4 - 2X$ where X was defined in the previous problem.
   (a) Determine the image of Y.
   (b) Using the rules for computing expected values and variances of a linear function of a random variable, find the expected value and variance of Y, using the corresponding values of X.

4. Let X be a random variable and a be a constant. Using the "short-cut" definition of variance, prove that $Var(aX) = a^2 Var(X)$.

5. A quality control engineer tests the quality of produced computers in a shipment of 6 computers. Suppose that 5% of computers have defects, and defects occur independently of each other.
   (a) Find the probability of exactly 2 defective computers in the shipment.
   (b) Find the probability of at most 2 defective computers in the shipment.

6. An internet search engine looks for a certain keyword in a sequence of independent web sites. It is believed that 20% of the sites contain this keyword.
   (a) Compute the probability that at least 5 of the first 10 sites contain the given keyword.
   (b) Compute the probability that the search engine had to visit at least 5 sites in order to find the first occurrence of a keyword.