

ComS 342  
Recitation 2, 10:00 Tuesday  
Homework 6

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1a)

$$\begin{aligned} & (( (\lambda(x)(x\ x)) (\lambda(y)(y\ x)) )\ z) \\ & (( (x\ x)\ [x \rightarrow \lambda(y)(y\ x)] )\ z) \end{aligned}$$

$$\begin{aligned} & (( (\lambda(y)(y\ x)\ \lambda(y)(y\ x)) )\ z) \\ & (( (y\ x)\ [y \rightarrow \lambda(y)(y\ x)] )\ z) \end{aligned}$$

$$\begin{aligned} & (( (\lambda(y)(y\ x)\ x) )\ z) \\ & (( (y\ x)\ [y \rightarrow x] )\ z) \end{aligned}$$

$$((x\ x)\ z)$$

1b)

$$\begin{aligned} & (( (\lambda(a)(\ \lambda(b)(a\ b)) )\ ((\lambda(c)\ c)\ x) )\ y) \\ & (( (\lambda(a)(\ \lambda(b)(a\ b)) )\ ((c)\ [c \rightarrow x]) )\ y) \end{aligned}$$

$$\begin{aligned} & (( (\lambda(a)(\ \lambda(b)(a\ b)) )\ x )\ y) \\ & (( (\lambda(b)(a\ b))\ [a \rightarrow x] )\ y) \end{aligned}$$

$$\begin{aligned} & ((\ \lambda(b)(x\ b))\ y) \\ & ((x\ b)\ [b \rightarrow y]) \\ & (x\ y) \end{aligned}$$

1c)

$$(( (\lambda(x)(x\ x)) (\lambda(y)\ y) ) (\lambda(y)\ y) )$$
$$(( (x\ x) [x \rightarrow \lambda(y)\ y] ) (\lambda(y)\ y) )$$
$$(( (\lambda(y)\ y) (\lambda(y)\ y) ) (\lambda(y)\ y) )$$
$$(( y [y \rightarrow (\lambda(y)\ y) ] ) (\lambda(y)\ y) )$$
$$( (\lambda(y)\ y) (\lambda(y)\ y) )$$
$$( y [y \rightarrow (\lambda(y)\ y) ] )$$
$$(\lambda(y)\ y)$$

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2a)

$$( (\lambda(x)\ p) ( (\lambda(y)(y\ y)) (\lambda(z)(z\ z)) ) )$$
$$( (p) [x \rightarrow ( (\lambda(y)(y\ y)) (\lambda(z)(z\ z)) ) ] )$$
$$(p)$$

2b)

$$( (\lambda(x)\ p) ( (\lambda(y)(y\ y)) (\lambda(z)(z\ z)) ) )$$
$$( (\lambda(x)\ p) ( (y\ y) [y \rightarrow (\lambda(z)(z\ z)) ] ) )$$
$$( (\lambda(x)\ p) ( ( (\lambda(z)(z\ z)) (\lambda(z)(z\ z)) ) ) )$$
$$( (\lambda(x)\ p) ( ( (z\ z) [z \rightarrow (\lambda(z)(z\ z)) ] ) ) )$$

...

$$( (\lambda(x)\ p) ( ( (\lambda(z)(z\ z)) (\lambda(z)(z\ z)) ) ) )$$
$$( (\lambda(x)\ p) ( ( (z\ z) [z \rightarrow (\lambda(z)(z\ z)) ] ) ) )$$

...

$$( (\lambda(x)\ p) ( (\lambda(z)(z\ z)) (\lambda(z)(z\ z)) ) )$$
$$( (p) [x \rightarrow ( (\lambda(z)(z\ z)) (\lambda(z)(z\ z)) ) ] )$$
$$(p)$$

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3) and =  $(\lambda(a)(\lambda(b)\ \text{ite } a\ b\ \text{false}))$

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4a)

```
((λ(z)((two f ) z)) (succ zero))  
((λ(z)((two f ) z)) 1)  
(((two f ) z) [z → 1])  
((two f ) 1)  
(f(f(1)))
```

This function applies f to the successor of 0 (1) two times.

4b) zero = λ(f)(λ(x)x), one = λ(f)(λ(x) f(x)), two = λ(f)(λ(x) f(f(x)))

i. (g zero)

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((λ(n)((n unknown) false)) zero)  
(zero unknown) false  
(((λ(f)(λ(x)x)) unknown) false)  
(false)
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ii. (g one)

```
((λ(n)((n unknown) false)) one)  
(one unknown) false  
(((λ(f)(λ(x) f(x) )) unknown) false)  
(unknown (false))  
((λ(x)(λ(y)(λ(z) y))) (false))  
(λ(y)(λ(z) y))  
(true)
```

iii. (g two)

```
((λ(n)((n unknown) false)) two)  
(two unknown) false  
(((λ(f)(λ(x) f(f(x)) )) unknown) false)  
(unknown (unknown (false)))  
(unknown ((λ(x)(λ(y)(λ(z) y))) (false)))  
(unknown (λ(y)(λ(z) y)))  
(unknown (true))  
((λ(x)(λ(y)(λ(z) y))) (true))  
((λ(y)(λ(z) y)))  
(true)
```

iv. (n != 0)

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5) (a) (g one)  
 $((\lambda(a)(\lambda(b)(\lambda(c)((a\ b)\ ((a\ b)\ c))))\ one)$   
 $((\lambda(b)(\lambda(c)((one\ b)\ ((one\ b)\ c))))$   
 $((\lambda(b)(\lambda(c)((one\ b)\ (b(c))))))$   
 $(\lambda(b)\ (\lambda(c)\ (b(b(c)))\ )\ ))$   
 (two)

(b) (g two)  
 $((\lambda(a)(\lambda(b)(\lambda(c)((a\ b)\ ((a\ b)\ c))))\ two)$   
 $((\lambda(b)(\lambda(c)((two\ b)\ ((two\ b)\ c))))$   
 $((\lambda(b)(\lambda(c)((two\ b)\ (b(b(c)))\ ))))$   
 $(\lambda(b)(\lambda(c)\ (b(b\ (b(b(c)))\ ))\ ))$   
 (four)

(c) (a \* 2)