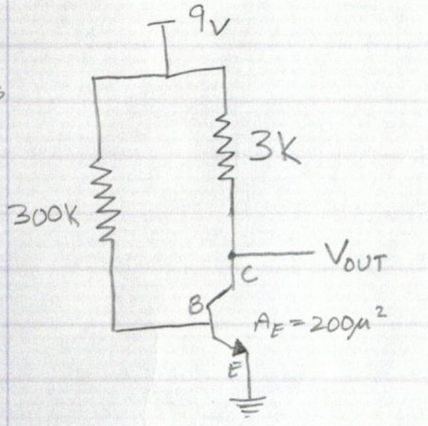
EE330 F12

HW 7 - solution

Due 10/5/2012

**1**

10 points




$\beta = 100 \qquad J_s = 10^{-14}\frac{A}{\mu^2}$

$$I_C = \frac{9 - V_{out}}{3K} = J_S A_E e^{\frac{V_{BE}}{V_t}} = (10^{-14})(200) e^{\frac{V_{BE}}{V_t}} \quad \text{where } V_t = 26mV$$

$$I_B = \frac{I_C}{\beta} \qquad I_B = \frac{9 - V_{be}}{300K} = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} = \frac{(10^{-14})(200)}{100} e^{\frac{V_{BE}}{V_t}}$$

$$\frac{9 - V_{be}}{300K} = \frac{(10^{-14})(200)}{100} e^{\frac{V_{be}}{V_t}}$$

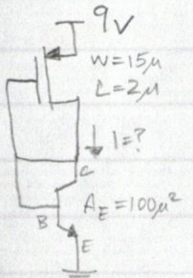
$V_{be} = 0.5477\ [V] \quad > 0.4 \quad$ forward active ✓

$$I_C = (10^{-14})(200) e^{\frac{0.5477}{0.026}} = 2.817mA$$

$$V_{out} = 9 - 3k \cdot I_C \qquad \boxed{V_{out} = 0.5477\ [V]}$$

**2** (a)

10 points




PMOS → saturation

$$I_D = \mu C_{ox}\left(\frac{W}{L}\right)\left(\frac{1}{2}\right)(V_{gs} - V_T)^2 = 33\frac{\mu A}{V^2}\left(\frac{15\mu}{2\mu}\right)\left(\frac{1}{2}\right)(V_{out} - 9 + 1)^2$$

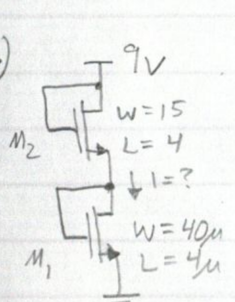
$$I_D = I_C$$

$$I_C = (100\mu^2)\left(10^{-14}\frac{A}{\mu^2}\right) e^{\frac{V_{out}}{26mV}}$$

$$100\mu^2\left(10^{-14}\frac{A}{\mu^2}\right) e^{\frac{V_{out}}{0.026}} = 33\frac{\mu A}{V^2}\left(\frac{15\mu}{2\mu}\right)\left(\frac{1}{2}\right)(V_{out} - 9 + 1)^2$$

$V_{out} = 0.5886 \quad$ then $\quad \boxed{I = 6.797\ mA}$

(b)




$M_1$ + $M_2$ in saturation

$$I_{D_1} = I_{D_2} \qquad 100\frac{\mu A}{V^2}\cdot\left(\frac{40}{4}\right)\left(\frac{1}{2}\right)(V_{out} - 0 - 1)^2 = 100\frac{\mu A}{V^2}\cdot\left(\frac{15}{4}\right)\left(\frac{1}{2}\right)(9 - V_{out} - 1)^2$$

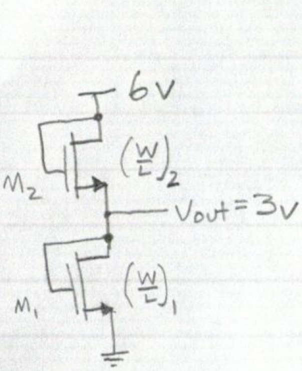
$$V_{out} = 3.65857$$

$$I = 100\frac{\mu A}{V^2}\left(\frac{40}{4}\right)\left(\frac{1}{2}\right)(3.65857 - 1)^2$$

$$\boxed{I = 3.534\ mA}$$

**3**

10 points




$M_1$ + $M_2$ ⇒ Saturation

$$I_{D_1} = I_{D_2}$$

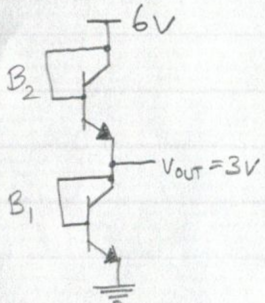
$$\cancel{\mu C_{ox}}\left(\frac{W}{L}\right)_1 \cancel{\left(\frac{1}{2}\right)}(3-1)^2 = \cancel{\mu C_{ox}}\left(\frac{W}{L}\right)_2 (6-3-1)^2 \cancel{\left(\frac{1}{2}\right)}$$

$$\boxed{\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2}$$

let $\frac{W}{L} = 5$ then $\boxed{\begin{array}{c} W_1 = W_2 = 15\mu \\ L_1 = L_2 = 3\mu \end{array}}$

**3'**

10 points extra credit




$$I_{B_2} = \frac{J_s \cdot A_{E_1} e^{\frac{V_{be}}{V_t}}}{\beta}$$

$$I_{E_2} = I_{B_2} + I_{C_2}$$

$$I_{E_2} = I_{E_1} = I_{B_1} + I_{C_1}$$

$$I_{B_1} = \frac{J_s A_{E_2} e^{\frac{V_{be}}{V_t}}}{\beta}$$

$$J_s A_{E_1} e^{\frac{V_{be_1}}{V_t}} = J_s A_{E_2} e^{\frac{V_{be_2}}{V_t}}$$

$V_{be_1} = 3V$

$V_{be_2} = 3V$

then $A_{E_1} = A_{E_2}$

$$\boxed{\begin{array}{l} \text{let } \beta = 100 \\ A_{E_1} = A_{E_2} = 100\mu^2 \\ J_s = 10^{-14} \frac{A}{\mu^2} \end{array}}$$

**4**

10 points

$C = 500fF = \dfrac{C_{jo} \cdot A}{\left(1 - \frac{V_D}{\phi_B}\right)^n}$ $\qquad n = 0.5$

$V_B = 0V$ $\qquad \phi_B = 0.6$

$500fF = \dfrac{C_{jo} \cdot A}{\left(1 - \frac{0}{0.6}\right)^{0.5}}$ $\qquad 500fF = C_{jo} \cdot A$

$C = \dfrac{500fF}{\left(1 - \frac{-3}{0.6}\right)^{0.5}}$ $\qquad \boxed{C = 2.0412 \times 10^{-13}\,[F] \quad V_D = -3V}$

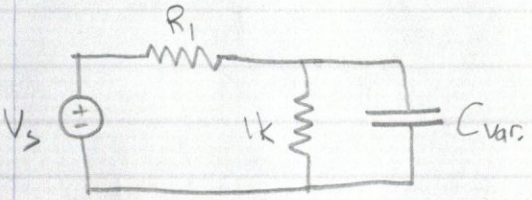
$C = \dfrac{500fF}{\left(1 - \frac{250mV}{0.6}\right)^{0.5}}$ $\qquad \boxed{C = 6.5465 \times 10^{-13}\,[F] \quad V_D = 250mV}$

**5**

10 points

2pF @ 0V

3pF @ 4V



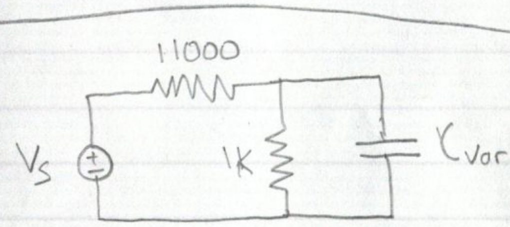

$C = \dfrac{C_{jo} \cdot A}{\left(1 - \frac{V_D}{\phi_B}\right)^n}$ $\qquad n = 0.5$, $\phi_B = 0.6$

@ 0V, $V_s = 0$ $\qquad 2pF = \dfrac{C_{jo} \cdot A}{\left(1 - \frac{0}{0.6}\right)^{0.5}}$ $\qquad C_{jo} \cdot A = 2pF$

@ 4V, $V_s = 4V$ $\qquad 3pF = \dfrac{2pF}{\left(1 - \frac{V_D}{0.6}\right)^{0.5}}$ $\qquad V_D = 0.333\,[V]$

$V_c = V_s \dfrac{1k}{R_1 + 1k}$ $\qquad 0.333 = 4V \cdot \dfrac{1k}{R_1 + 1k}$

$R_1 = 11k\Omega$




$C_{jo} \cdot A = 2pF$

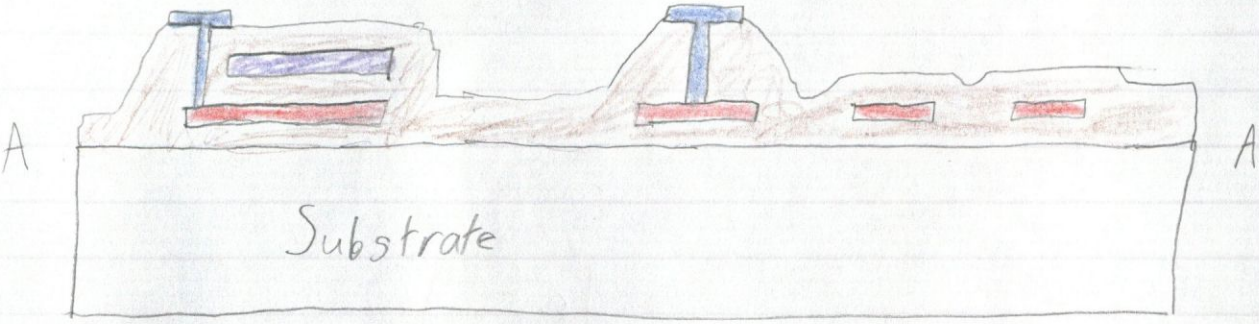
$\phi_B = 0.6 \qquad n = 0.5$

$V_s = 0 \qquad C = 2pF$

$V_s = 4 \qquad C = 3pF$

6

10 points

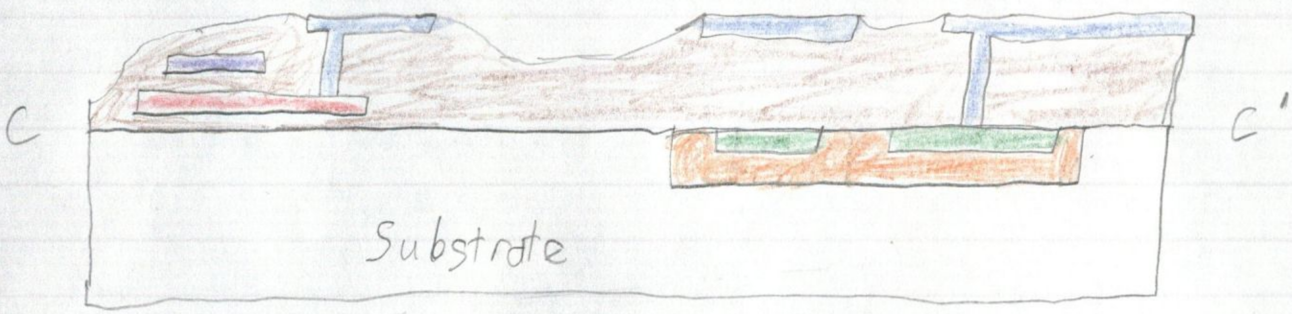



metal Poly Poly 2 ~~Oxide~~ Oxide

Active N-well

6'

10 points extra credit

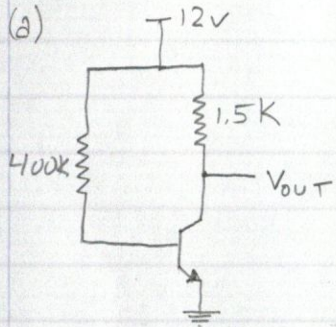


$J_s = 50\,\frac{fA}{\mu^2}$ $A_E = 200\mu^2$ $A_B = 500\mu^2$ $\beta = 100$ $V_t = 26mV$

for 7, 9, 10

$\boxed{7}$ (a)

10 points




$$I_B = \frac{12 - V_{be}}{400K} \qquad I_c = \frac{12 - V_{OUT}}{1.5K} = \left(50\frac{fA}{\mu^2}\right)(200\mu^2)\cdot e^{\frac{V_{be}}{V_t}}$$

then $$\frac{12 - V_{be}}{400K} = \left(\frac{50fA \cdot 200}{100}\right) e^{\frac{V_{be}}{0.026}}$$

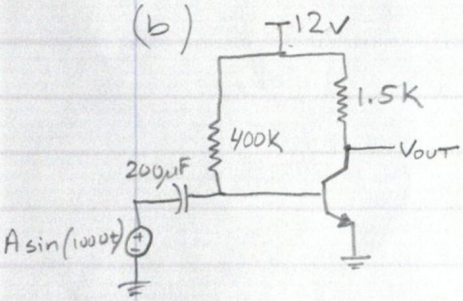
$$V_{be} = 0.50638\ [v]$$

then $I_C = 2.8734\ mA$

and $V_{out} = 12 - 1.5k \cdot I_c$

$$\boxed{V_{OUT} = 7.690\ [v]}$$

(b)




DC equivalent is the same as in part (a)

so $V_{be} = 0.50638$ w/ $A\sin(1000t)$

$V_{be} = 0.50638 \pm 0.01V$

$V_{be_1} = 0.49638\ V$

then $I_{c_1} = \left(50\frac{fA}{\mu^2}\right)(200\mu^2)e^{\frac{0.49638}{0.026}} = 1.956\ mA$

and $V_{OUT} = 9.066\ [V]$

$V_{be_2} = 0.51638$

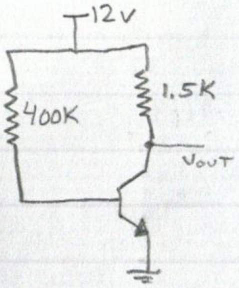
then $I_{c_2} = \left(50\frac{fA}{\mu^2}\right)(200\mu^2)\cdot e^{\frac{0.51638}{0.026}} = 4.22\ mA$

and $V_{OUT} = 5.6683$

> with a sin() input with Amplitude 0.01 [V]
> the output Voltage $V_{OUT}$ will range from [5.6683, 9.066] [V]

8

10 points




$J_S = 10^{-14} \frac{A}{\mu^2}$

$A_E = 100\mu^2$

$V_t = 26mV$

$$I_B = \frac{12 - V_{be}}{400K} = \frac{J_S A_E}{\beta} e^{\frac{V_{be}}{V_t}}$$

for $\beta = 85$ $$\frac{12 - V_{be}}{400K} = \frac{(10^{-14})(100)}{85} e^{\frac{V_{be}}{0.026}}$$

$$V_{be} = 0.56189\ [V]$$

$$I_{C_1} = (10^{-14})(100) e^{\frac{0.56189}{0.026}} = 2.43\ mA$$

then $V_{out} = 12 - 1.5K \cdot I_C$ $\underline{V_{OUT_1} = 8.354\ [V]}$

for $\beta_2 = 125$ $$\frac{12 - V_{be}}{400K} = \frac{(10^{-14})(100)}{125} e^{\frac{V_{be}}{0.026}}$$

$$V_{be} = 0.5719$$

$$I_{C_2} = (10^{-14})(100) \cdot e^{\frac{0.5719}{0.026}} \quad I_{C_2} = 3.57\ mA$$

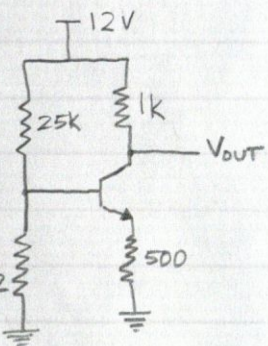
then $V_{OUT} = 12 - 1.5K \cdot I_C$ $\underline{V_{OUT_2} = 6.643\ [V]}$

$$\Delta V_{OUT} = 1.7109\ [V]$$

> as $\beta$ changes from 85 to 125 the output voltage $V_{OUT}$ changes from 8.354[V] to 6.643[V] a change of $\Delta V_{OUT} = 1.7109$ [V]

9

10 points




* $J_s$ $A_E$ $B$ inherited from problem 7 $J_S = \frac{50fA}{\mu^2}$ $A_E = 200\mu^2$ $\beta = 100$ $V_t = 26mV$

$$V_{OUT} = 7.690\ [V]$$

$$I_C = \frac{12 - 7.690}{1K} = \left(50\frac{fA}{\mu^2}\right)(200) e^{\frac{V_{be}}{0.026}} \quad V_{BE} = 0.5169\ [V]$$

$$I_B = \frac{I_C}{\beta} \qquad \frac{12 - V_B}{25K} = I_B + \frac{V_B}{R_2} \qquad I_B = \frac{(50fA)(200)}{100} e^{\frac{0.5169}{0.026}}$$

$$I_B = 43.1\mu A$$

$$I_E = I_C + I_B = 101(I_B) = 4.353mA$$

$$I_E = \frac{V_B - V_{BE}}{500} \qquad 4.353mA = \frac{V_B - V_{BE}}{500} \qquad V_B = 2.69344\ [V]$$

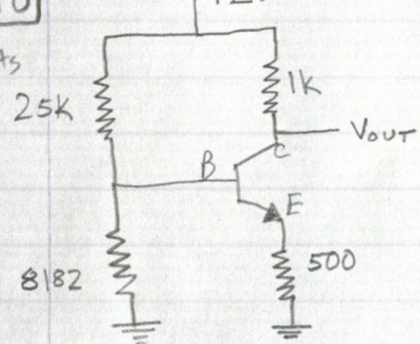
$$\frac{12 - V_B}{25K} - I_B = \frac{V_B}{R_2}$$

$$\frac{12 - 2.69344}{25K} - 43.1\mu A = \frac{2.69344}{R_2} \qquad \boxed{R_2 = 8182.71\Omega}$$

Note: $R_2$ may be different depending on the $J_S$ value used and $A_E$ value used

$\boxed{10}$

10 points




$J_S = 50 \frac{fA}{\mu^2}$ $A_E = 200\mu^2$ ← inherited from problem 9 + 7

$\beta = 85, 125$

$I_E = I_C + I_B$ $I_E = \frac{V_B - V_{BE}}{500}$

$\frac{12 - V_B}{25K} - I_B = \frac{V_B}{8182}$ $I_C = \beta I_B$

$I_C = \frac{12 - V_{out}}{1K} = (50f) \cdot (200) e^{V_{BE}/.026}$ $V_E = \beta I_B \cdot 500$

$\frac{12 - V_B}{25K} - \frac{(50f \cdot 200)}{\beta} \cdot e^{\left(V_B - \beta\left[\frac{12 - V_B}{25K} - \frac{V_B}{8182}\right] \cdot 500\right)/.026} = \frac{V_B}{8182}$

Wolfram Alpha is a good equation solver

when $\beta = 85$ $V_B = 2.74$
when $\beta = 125$ $V_B = 2.80$

$\beta = 85$ $I_C = \beta I_B = 85[(12 - 2.74)/25K - 2.74/8182] = 3.02\,mA$
$V_{out} = 12 - 3.02mA \cdot 1K = 8.98V$

$\beta = 125$ $I_C = 125[(12 - 2.80)/25K - 2.80/8182] = 3.22\,mA$
$V_{out} = 12 - 3.22\,mA \cdot 1K = 8.78\,V$

$\boxed{\Delta V_{out} = 200mV}$

EE330 - F12
HW 7 – solution
Problem 11 – Extra Credit
+ 15 points

Verilog Code

5 points

```
module bin_thermo_3_8_decode(bin, thermo);
  input [2:0]bin;
  output reg [7:0]thermo;

  always @ (bin)
  case(bin)
    0'b000: thermo <= 0'b00000001;
    0'b001: thermo <= 0'b00000010;
    0'b010: thermo <= 0'b00000100;
    0'b011: thermo <= 0'b00001000;
    0'b100: thermo <= 0'b00010000;
    0'b101: thermo <= 0'b00100000;
    0'b110: thermo <= 0'b01000000;
    0'b111: thermo <= 0'b10000000;
  endcase
endmodule
```

Verilog Test Bench simulation code

5 points

```
`timescale 1ns / 1ps

module bin_thermo_3_8_decode_tb();
  reg [2:0]bin_t;
  wire[7:0]thermo_t;

  bin_thermo_3_8_decode bin_thermo_3_8_decode_t(bin_t, thermo_t);

  initial
  begin
    repeat(21) begin
      bin_t<=0'b000;
      #1;

      bin_t<=0'b001;
      #1;

      bin_t<=0'b010;
      #1;

      bin_t<=0'b011;
      #1;

      bin_t<=0'b100;
      #1;

      bin_t<=0'b101;
      #1;

      bin_t<=0'b110;
      #1;

      bin_t<=0'b111;
      #1;
    end
  end
endmodule
```

EE330 - F12
HW 7 – solution
Problem 11 – Extra Credit
+ 15 points

Test Bench waveform

5 points