

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, student ID, the course number, and the section on every sheet. Problems marked with \* will be graded and one additional randomly chosen problem will be graded.

1. \* Suppose that your bus arrives at your bus stop uniformly between 9:05am and 9:10am. Let  $X$  = time you wait for the bus. Thus we have that  $X \sim \text{Unif}(5, 10)$ .
  - (a) Give the PDF and CDF for  $X$ .
  - (b) What is the expected time that the bus will arrive?
  - (c) Suppose you slept in a little and can make it to the bus stop at 9:07. What is the probability that you will have missed the bus?
2. \* A web page is accessed at an average of 20 times an hour. Assume that waiting time until the next hit has an exponential distribution.
  - (a.) Determine the rate parameter  $\lambda$  of the distribution of the time until the first hit?
  - (b.) What is the expected time between hits?
  - (c.) What is the distribution of the time until the second hit? (Give the name of the distribution and the value(s) of parameter(s).)
  - (d.) What is the probability that the next hit is within 20 minutes?
  - (e.) Describe the distribution of the total waiting time for 5 hits? (Give the name of the distribution and the value(s) of parameter(s).)
  - (f.) What is the expected total waiting time for 5 hits on the web page?
  - (g.) What is the probability that there will be less than 5 hits in the first hour? (Hint: Consider Poisson distribution instead.)
3. The amount of time a postal clerk spends with his customer can be modeled using an exponential distribution. On average, the clerk spends 5 minutes with a customer. Let  $X$  = the amount of time (in minutes) a postal clerk spends with his customer.
  - (a) Give the distribution of  $X$  and the value for its parameter.
  - (b) Give the probability density function (PDF) and the cumulative distribution function (CDF) of  $X$ .
  - (c) What is the probability that the clerk spends less than 5 minutes with a customer?
  - (d) If the clerk hasn't finished assisting the customer in 2 minutes, what is the probability that he spends less than 5 minutes with the customer?
4. On the average, a computer experiences breakdowns every 5 months. The time until the first breakdown and the times between any two consecutive breakdowns are independent Exponential random variables. After the third breakdown, a computer requires a special maintenance.
  - (a) Compute the probability that a special maintenance is required within the next 9 months.
  - (b) Given that a special maintenance was not required during the first 12 months, what is the probability that it will not be required within the next 4 months?
5. Suppose a phone call on average lasts 5 minutes at a phone booth.
  - (a) If a person arrives at a public telephone booth just before you, calculate the probability that you have to wait more than 15 minutes to make your call. (Hint: Use exponential distribution to model waiting time)
  - (b) You get tired of waiting, and decide to do a little shopping and come back. When you come back there are now 2 people ahead of you. What is the probability that that you have to wait more than 20 minutes?