

Confidence Interval Examples

1. A start-up company purchases commercials during the Super Bowl to improve name recognition among the public. The day after the game, a pollster contacts 200 randomly selected adults and finds that 84 of them recognize the company. Make a 95% confidence interval for the true proportion of adults that recognize the company, and give it's interpretation.

Answer:

Given: $n = 200$, $\hat{p} = \frac{84}{200} = 0.42$, 95% confidence $\rightarrow z_{\alpha/2} = 1.96$

$$\begin{aligned}\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ 0.42 \pm 1.96 \sqrt{\frac{0.42(1-0.42)}{200}} \\ 0.42 \pm 0.0684 \rightarrow (0.3516, 0.4884)\end{aligned}$$

We are 95% confident that the true proportion of adults that recognize the company is between 0.3516 and 0.4884.

2. In a random sample of 100 students from a school district, the mean IQ was found to be 110 with a standard deviation of 15. construct and interpret a 90% confidence interval for the true mean IQ of students at this school district.

Answer:

Given: $n = 100$, $\bar{x} = 110$, $s = 15$, 90% confidence $\rightarrow z_{\alpha/2} = 1.645$

$$\begin{aligned}\bar{x} \pm z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right) \\ 110 \pm 1.645 \left(\frac{15}{\sqrt{100}} \right) \\ 110 \pm 2.4675 \rightarrow (107.53, 112.47)\end{aligned}$$

We are 90% confident that the true mean IQ of students is between 107.53 and 112.47 units.

3. A survey of 66 randomly chosen adults was taken as part of the National Health and Nutrition Examination Survey (NHANES) from 2009-2010. In the survey, 33 adults said they smoke and 33 said they do not smoke. The mean age of the adults that smoke was 48.18 with a standard deviation of 18.07. The mean age of adults that do not smoke was 57.39 with a standard deviation of 15.44. Calculate a 95% confidence interval for the difference in the population means between smokers and non-smokers. Interpret the confidence interval.

Answer:

Group 1 (non-smokers): $n_1 = 33$, $\bar{x}_1 = 57.39$, $s_1 = 15.44$

Group 2 (smokers): $n_2 = 33$, $\bar{x}_2 = 48.18$, $s_2 = 18.07$, 95% confidence $\rightarrow z_{\alpha/2} = 1.96$

$$\begin{aligned}(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \\ (57.39 - 48.18) \pm 1.96 \sqrt{\frac{15.44^2}{33} + \frac{18.07^2}{33}} \\ 9.21 \pm 8.1095 \rightarrow (1.1005, 17.3195)\end{aligned}$$

We are 95% confident that the difference in population mean ages between non-smokers and smokers (non-smoker - smoker) is between 1.1005 and 17.3195 years.

or

We are 95% confident that the population mean age of non-smokers is between 1.1005 and 17.3195 years larger than the population mean age of smokers from 2009-2010.

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4. A student polls two schools ("A" and "B") to see if students in the schools are for or against the new legislation regarding school uniforms. She surveys 600 students from school A and finds that 480 are against uniforms. She surveys 800 students from school B and finds that 650 are against uniforms. Build a 99% confidence interval for the difference in proportion of student against school uniforms between the 2 schools.

Answer:

Group 1 (A) : $n_1 = 600$, $\hat{p}_1 = \frac{480}{600} = 0.80$

Group 2 (B) : $n_2 = 800$, $\hat{p}_2 = \frac{592}{800} = 0.74$, 99% confidence $\rightarrow z_{\alpha/2} = 2.576$

$$\begin{aligned} & (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}} \\ & (0.80 - 0.74) \pm 2.576 \sqrt{\frac{0.80(1 - 0.80)}{600} + \frac{0.74(1 - 0.74)}{800}} \\ & 0.06 \pm 0.058 \rightarrow (0.0020, 0.1180) \end{aligned}$$

We are 99% confident that the difference in proportion of students against school uniforms between school A and B (A-B) is between 0.0020 and 0.1180.

or

We are 99% confident that the proportion of students against school uniforms in school A is between 0.0020 and 0.1180 more than the proportion of students against school uniforms in school B.