

Stat 330

Homework 3

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1)

(a) $\text{Im}(X) = \{0, 1, 2, 3\}$

(b) $P(X = 0) = \{GGG\} = \frac{3}{7} * \frac{2}{6} * \frac{1}{5} = \frac{1}{35}$

$$P(X = 1) = \{RGG\} = \frac{4}{7} * \frac{3}{6} * \frac{2}{5} = \frac{4}{35}$$

$$P(X = 2) = \{RRG\} = \frac{4}{7} * \frac{3}{6} * \frac{3}{5} = \frac{6}{35}$$

$$P(X = 3) = \{RRR\} = \frac{4}{7} * \frac{3}{6} * \frac{2}{5} = \frac{4}{35}$$

(c)

x	0	1	2	3
$P_X(x)$	1/35	4/35	6/35	4/35

2)

x		-2	-1	0	1	2
(a, b)	$P_X(x)$	0.1	0.3	0.3	0.1	0.2
	$F_X(x)$	0.1	0.4	0.7	0.8	1.0

(c)

i. $P(X \leq 1) \Rightarrow P(X = 1) + P(X = 2) = 0.1 + 0.2 = 0.3$

ii. $P(-1 < X \leq 1) \Rightarrow P(X = 0) + P(X = 1) = 0.3 + 0.1 = 0.4$

iii. $P(X < 0) \Rightarrow P(X = -2) + P(X = -1) = 0.1 + 0.3 = 0.4$

(d)

i. $F(1) = 0.8$

ii. $F(0.5) = F(0) = 0.7$

iii. $P(X \geq 0) = 1 - F(-1) = 0.6$

(e) $E(X) = -2(0.1) + -1(0.3) + 0(0.3) + 1(0.1) + 2(0.2) = 0.2$

$E(X^2) = -2^2(0.1) + -1^2(0.3) + 0^2(0.3) + 1^2(0.1) + 2^2(0.2) = 0.6$

Variance = $E(X^2) - E(X)^2 = 0.6 - (0.2)^2 = 0.56$

3)

(a) $\text{Im}(Y) = \{8, 6, 4, 2, 0\}$

(b) $E(X) = 8(0.1) + 6(0.3) + 4(0.3) + 2(0.1) + 0(0.2) = 4$

$E(X^2) = 8^2(0.1) + 6^2(0.3) + 4^2(0.3) + 2^2(0.1) + 0^2(0.2) = 22.4$

Variance = $E(X^2) - E(X)^2 = 22.4 - (4)^2 = 38.4$

4) $\text{Var}(aX) = E([aX]^2) - [E(aX)]^2 = a^2E(X^2) - a^2E(X)^2 = a^2(E(X^2) - E(X)^2) = a^2\text{Var}(X)$

5)

(a) $P(X=2) = \binom{6}{2}(0.05)^2(0.95)^{6-2}$

$15*(0.05)^2(0.95)^4 = .0305$

(b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) \Rightarrow$

$\binom{6}{0}(0.05)^0(0.95)^{6-0} + \binom{6}{1}(0.05)^1(0.95)^{6-1} + \binom{6}{2}(0.05)^2(0.95)^{6-2} =$
 $0.735 + 0.232 + 0.031 = 0.998$

6)

$$(a) P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10) \Rightarrow$$

$$\binom{10}{5}(0.2)^5(0.8)^{10-5} + \binom{10}{6}(0.2)^6(0.8)^{10-6} + \binom{10}{7}(0.2)^7(0.8)^{10-7} +$$

$$\binom{10}{8}(0.2)^8(0.8)^{10-8} + \binom{10}{9}(0.2)^9(0.8)^{10-9} + \binom{10}{10}(0.2)^{10}(0.8)^{10-10} =$$

$$0.02642 + 0.00551 + .00079 + .00007 + \sim 0 + \sim 0 = .033$$

$$(b) P(X \geq 5) = 1 - P(X \leq 4) = (.2)(.8)^{4-1} + (.2)(.8)^{3-1} + (.2)(.8)^{2-1} + (.2)(.8)^{1-1} =$$

$$.1024 + .128 + .16 + .2 = .5904$$