ComS 474
Midterm 1

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I affirm that the work on this exam is my own and I will not use any people to help me nor will I share any part of this exam or my work with others without permission of the instructor.

1) Supervised, unsupervised, and Reinforcement learning.

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2) $w^T x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} * (1, 1, 1) = 6 > 1$, so $\hat{y} = 1$

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3) $(w^T x - y)^2 = (6 - (-1))^2 = 49$

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4) As ŷ can only be ± 1, $\sum(\hat{y} - y)^2$ can detect whether the classifier is right or wrong, but loses information on **how** right or wrong it is.

If a classifier predicts a sample's score to be 57 when it really is -1, $(\hat{y} - y)^2$ makes it appear just as wrong as a sample predicted as 2 when it really is -1.

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5) There are 2 samples with b > 5, and 4 samples with b <= 5. Then...

$Pr(class = +1|b > 5) = 1/2 = 0.5$
$Pr(class = -1|b > 5) = 1/2 = 0.5$
$Pr(class = +1|b <= 5) = 2/4 = 0.5$
$Pr(class = -1|b <= 5) = 2/4 = 0.5$

Using $G(condition) = 1 - (Pr(class = +1|condition))^2 - (Pr(class = -1|condition))^2$ and the above values...

$G(b > 5) = 1 - (0.5)^2 - (0.5)^2 = 0.5$
$G(b \leq 5) = 1 - (0.5)^2 - (0.5)^2 = 0.5$

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6) There are 6 total samples, 2 of which are > 5 and 4 of which are <= 5. Thus...
$Pr(b > 5) = 2/6 =$ **0.333**, and $Pr(b <= 5) = 4/6 =$ **0.667**

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7) Expectation $= Pr(b > 5) * G(b > 5) + Pr(b <= 5) * G(b <= 5) \quad \Rightarrow$

$(0.333)(0.5) + (0.667)(0.5) = 0.5$

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8) Any sample with $\lambda > 0$ is considered to be a support vector. Thus, as 3 $\lambda > 0$, 3 of the samples were chosen to be support vectors (samples 1, 3, and 4).

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9) $\begin{pmatrix} w1 \\ w2 \\ w3 \end{pmatrix} = \lambda_1 * y_1 * \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix} + \lambda_2 * y_2 * \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix} + \lambda_3 * y_3 * \begin{pmatrix} a_3 \\ b_3 \\ c_3 \end{pmatrix} + \lambda_4 * y_4 * \begin{pmatrix} a_4 \\ b_4 \\ c_4 \end{pmatrix} \Rightarrow$

$6.13 * (1) * \begin{pmatrix} 0.5 \\ 0.25 \\ 0.125 \end{pmatrix} + 0 * (1) * \begin{pmatrix} 0.4 \\ 0.15 \\ 0.225 \end{pmatrix} + 4.08 * (-1) * \begin{pmatrix} .3 \\ .75 \\ .325 \end{pmatrix} + 2.05 * (-1) * \begin{pmatrix} 0.2 \\ 0.65 \\ .425 \end{pmatrix} \Rightarrow$

$$\text{w} = \begin{pmatrix} 1.431 \\ -2.86 \\ -1.431 \end{pmatrix}$$

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10)

(1) $\begin{pmatrix} 1.431 \\ -2.86 \\ -1.431 \end{pmatrix} * \begin{pmatrix} 0.5 \\ 0.25 \\ 0.125 \end{pmatrix} + 1.18 = -0.1784 + 1.18 = 1.0016$

(2) $\begin{pmatrix} 1.431 \\ -2.86 \\ -1.431 \end{pmatrix} * \begin{pmatrix} 0.4 \\ 0.15 \\ 0.225 \end{pmatrix} + 1.18 = -0.1786 + 1.18 = 1.0014$

(3) $\begin{pmatrix} 1.431 \\ -2.86 \\ -1.431 \end{pmatrix} * \begin{pmatrix} 0.3 \\ 0.75 \\ 0.325 \end{pmatrix} + 1.18 = -2.1808 + 1.18 = -1.0008$

(4) $\begin{pmatrix} 1.431 \\ -2.86 \\ -1.431 \end{pmatrix} * \begin{pmatrix} 0.2 \\ 0.65 \\ 0.425 \end{pmatrix} + 1.18 = -2.181 + 1.18 = -1.001$

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11) A point is inside the margin when $|wx + w_b| < d$, where d = 1:

(1) $|1.0016| > 1$, so this point falls outside of the margin.
(2) $|1.0014| > 1$, so this point falls outside of the margin.
(3) $|-1.0008| > 1$, so this point falls outside of the margin.
(4) $|-1.001| > 1$, so this point falls outside of the margin.

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12) The value of $w^T x_i + w_b$ for a support vector should equal ± 1, but none of the above values were exactly ± 1.