Fraction of Undetectable Error Bursts of Length L (when L – 1 > n – k)

Step 1.

According to the definition of an error burst, the error polynomial for an error burst of length L looks like:

$$e(x) = (x^{(L-1)} + ??? + 1) * x^i, \text{ where i is the starting position of the error burst.}$$

Step 2.

There are a total of (L-2) terms of "?", and each of them has 2 choices on the coefficient: 0 or 1. Therefore, the total number of different error bursts of length L is: $2^{(L-2)} * (n-L+1)$, where (n-L+1) is the number of different starting positions.

Step 3.

According to the definition of an undetectable error for a CRC code with a generator polynomial of g(x), if an error burst of length L is undetectable, we must have:

$$e(x) = (x^{(L-1)} + ??? + 1) * x^i = g(x) * c(x) * x^i, \text{ where c(x) is some polynomial.}$$

Let's denote $(x^{(L-1)} + ??? + 1)$ as d(x). Then, we must have:

$$(x^{(L-1)} + ??? + 1) = d(x) = g(x) * c(x).$$

Step 4.

Finding the total number of different d(x)'s that have the format of $(x^{(L-1)} + ??? + 1)$, and is a multiple of g(x), is equivalent to finding the total number of different c(x)'s that satisfy the above equation in Step 3.

As $g(x) = x^{(n-k)} + \ldots + 1$, c(x) must have the following format:

$$c(x) = x^{(L-1)-(n-k)} + ??? + 1.$$

There are a total of (L-1)-(n-k)-1 terms of "?", and each of them has 2 choices on the coefficient: 0 or 1. Therefore, the total number of different c(x)'s is: $2^{(L-1)-(n-k)-1}$.

Step 5.

Hence, the total number of undetectable error bursts of length L is:

$$2^{(L-1)-(n-k)-1} * (n-L+1), \text{ where (n-L+1) is the number of different starting positions.}$$

Step 6.

Now, we can calculate the fraction of undetectable error bursts of length L as follows:

$$FUE = (2^{(L-1)-(n-k)-1} * (n-L+1)) / (2^{(L-2)} * (n-L+1)) = 1/2^{(n-k)}.$$