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Homework #1 (Due: Jan. 19)

Total 200 points

Please write down your **name** on your homework.
Please submit your homework online through Canvas by **Friday 10:00pm**.
Late homework will be penalized.
Important: Your submission must be **in .pdf or .jpg format**.

Class 2

1. Suppose you want to represent unsigned integers in binary. Indicate at least how many bits are required to represent each of the following sets of integers:
   (a) (5 points) The integers from 0 to 128 inclusively. $\boxed{\text{8 bits}}$
   (b) (10 points) The integers from 0 to n-1 inclusively. $\boxed{\log_2(n)}$

   145
   64 32 16 8 4 2 1

2. Indicate how large a value can be represented by each of the unsigned binary quantities:
   (a) (5 points) A 7-bit quantity. $\boxed{127}$
   (b) (5 points) A 16-bit quantity. $\boxed{65535}$

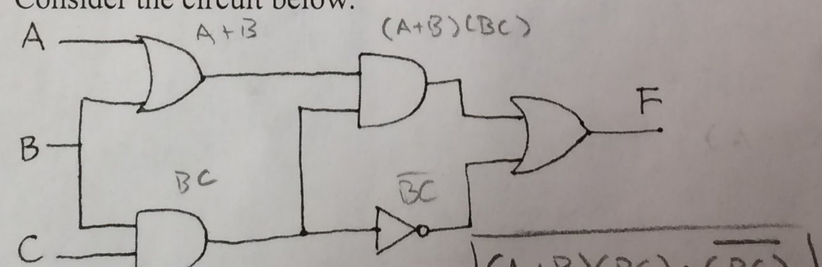
   2√1 R1
   2√3 R1
   2√7 R 0
   2√14 R 0
   2√28 R 1
   2√57

3. (5 points) Convert the decimal number 57 into binary. $\boxed{111001}$

4. (5 points) Convert the binary number 101101 into decimal. $\boxed{45}$

   32 16 8 4 2 1

5. Consider the circuit below.



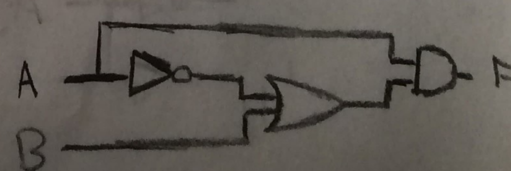
$\boxed{(A+B)(BC)\cdot(\overline{BC})}$

   (a) (15 points) Please write the logic expression for the circuit.
   (b) (20 points) Please write the truth table for the circuit.

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

6. Let the logic function f(A, B) = A . (A' + B).
   (a) (15 points) Please draw the circuit diagram for f(A, B).
   (b) (10 points) Please write the truth table for f(A, B).

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1



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(c) (5 points) By looking at the truth table in (b), what observation can you make about f(A, B)?

It should be simplified to "AB"

Class 3

For all exercises using Boolean algebra in the following, please only use axioms / theorems / properties 1-17. Please explicitly show **all** the steps except those using commutative properties (10a and 10b) and associative properties (11a and 11b). For each step, please state the axioms/theorems/properties (i.e., 1a, 1b, ...,9, 12a, 12b, ..., 17b) that you used.

1. Prove the equality (a + b).(a'.c' + a) = a + b.c'
   (a) (15 points) by truth table.
   (b) (10 points) by Boolean algebra.
   (c) (15 points) by proving its dual theorem using Boolean algebra. (In other words, first construct the dual theorem of the given equality. Then prove that the dual theorem is correct.)

2. Consider the logic function f(a,b,c) = abc + ab'c + a'bc + a'b'c + a'b'c'.
   (a) (15 points) Draw the logic circuit for the function f given above.
   (b) (3 points) Let the cost of a logic circuit be the total number of gates plus the total number of inputs to all gates in the circuit. (See page 49-50 of textbook for examples.) What is the cost of the circuit in part (a)?
   (c) (10 points) Simplify f by Boolean algebra as much as possible.
   (d) (5 points) Draw the logic circuit for the simplified version of f in part (c).
   (e) (2 points) What is the cost of the circuit in part (d)?

3. (25 points) Show that a'.b'.c + a'.b.c' + a'.b.c + a.b'.c' + a.b'.c + a.b.c' + a.b.c = a + b + c using Boolean algebra.

1a)

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

1b) $(a+b)\cdot(a'c'+a)$

$aa'c' + a + a'bc' + ab$

$0 + a + a'bc' + ab$

$a + b(a'c'+a)$

$a + bc' \cdot 1$

$\boxed{a + bc'}$

1c) $(a+b)\cdot(a'c'+a) \Rightarrow a + bc'$

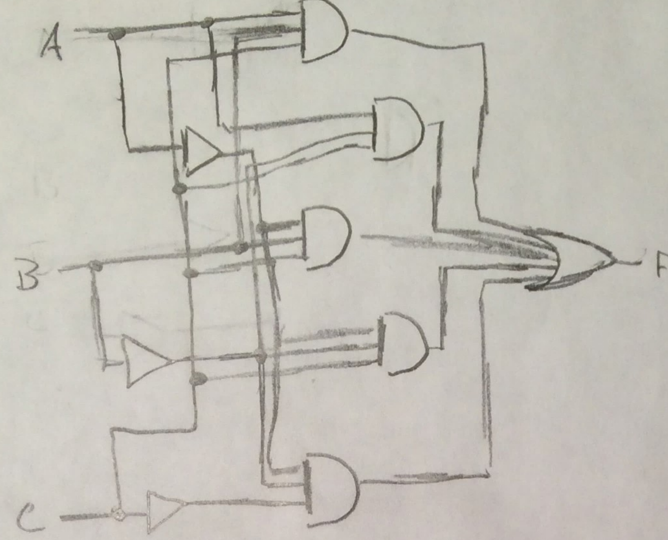
$(ab) + (a'+c'a) = ab + c'$

$(ab) + (c') = ab + c'$

$\boxed{ab + c'} = ab + c'$

✓

2) a]



b] Cost:

8 gates, 23 inputs

$\boxed{31 \text{ cost}}$

c] $abc + ab'c + a'bc + a'b'c + a'b'c'$

$a(bc+b'c) + a'(bc+b'c) + a'b'c'$

$(a+a')(bc+b'c) + a'b'c'$

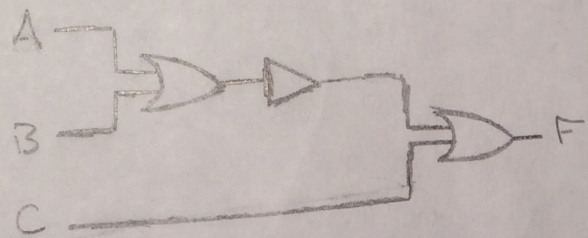
$1 \cdot c + a'b'c'$

$c + a'b'c'$

$c + \overline{(abc)}$

$\boxed{c + \overline{(a+b)}}$

d]



e] Cost:

3 gates, 5 inputs

$\boxed{8 \text{ cost}}$

3) $a'b'c + a'bc' + a'bc + ab'c' + ab'c + abc' + abc = a + b + c$

$a'(b'c + bc' + bc) + a(b'c' + b'c + bc' + bc)$

$a'(b'c + bc' + bc) + a$

$c'(b + c) + a$

$a + (b + c)$

$\boxed{a + b + c}$ ✓