Exercise 13.26

(Adapted from Pearl [-@Pearl:1988].) Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable.

1. Is it possible to calculate the most likely color for the taxi? (*Hint:* distinguish carefully between the proposition that the taxi *is* blue and the proposition that it *appears* blue.)

2. What if you know that 9 out of 10 Athenian taxis are green?

Exercise 14.5

Consider the Bayesian network in Figure **burglary-figure**.

1. If no evidence is observed, are **Burglary** and **Earthquake** independent? Prove this from the numerical semantics and from the topological semantics.

2. If we observe **Alarmtrue**, are **Burglary** and **Earthquake** independent? Justify your answer by calculating whether the probabilities involved satisfy the definition of conditional independence.

Exercise 14.13

In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables $\mathbf{A}$ (alarm sounds), $\mathbf{F_A}$ (alarm is faulty), and $\mathbf{F_G}$ (gauge is faulty) and the multivalued nodes $\mathbf{G}$ (gauge reading) and $\mathbf{T}$ (actual core temperature).

1. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.

2. Is your network a polytree? Why or why not?

3. Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is $\mathbf{x}$ when it is working, but $\mathbf{y}$ when it is faulty. Give the conditional probability table associated with $\mathbf{G}$.
4. Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with $\mathbf{A}$.

5. Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is too high, in terms of the various conditional probabilities in the network.

Exercise 14.18 [VE-exercise]

Consider the variable elimination algorithm in Figure **elimination-ask-algorithm** (page **elimination-ask-algorithm**).

1. Section **exact-inference-section** applies variable elimination to the query $\mathbf{P(Burglary \mid JohnCalls = true, MaryCalls = true)}$. Perform the calculations indicated and check that the answer is correct.

2. Count the number of arithmetic operations performed, and compare it with the number performed by the enumeration algorithm.

3. Suppose a network has the form of a *chain*: a sequence of Boolean variables $\mathbf{X_1,\ldots,X_n}$ where $\mathbf{Parents(X_i) X_{i-1}}$ for $\mathbf{i 2,\ldots,n}$. What is the complexity of computing $\mathbf{P(X_1 X_n true)}$ using enumeration? Using variable elimination?

4. Prove that the complexity of running variable elimination on a polytree network is linear in the size of the tree for any variable ordering consistent with the network structure.