Stat 330
Homework 3

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1)

(a) $P(B|A) + P(B|\bar{A}) = 1$ $\qquad$ $\dfrac{P(B)P(A)}{P(B)} + \dfrac{P(B)P(\bar{A})}{P(B)} = 1$

$\frac{P(B)P(A)+P(B)P(\bar{A})}{P(B)} = 1$ $\qquad$ $P(A) + P(\bar{A}) = 1$ ✓

(b) $P(\bar{A}|\bar{B}) = 1 - P(A \cup B)$ $\qquad$ $= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - P(A) - P(B) + P(A)P(B)$ $\qquad$ $= (1 - P(A))(1 - P(B))$

$P(\bar{A}|\bar{B}) = P(\bar{A})P(\bar{B})$ ✓

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2)

(a) P(A) = .4, P(B) = .7, P(A ∩ B) = **.28**
(b) P(A|B) = P(A ∩ B) / P(B) = .28 / .7 = **.4**
(c) P(B|A) = P(A ∩ B) / P(A) = .28 / .4 = **.7**
(d) A test for independance is if P(A|B) = P(A) and P(B|A) = P(B)
As the above tests hold, these two events are **independent**.

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3)

(a) When drawing from the first urn, the chances are:
Red: 2/6, White: 4/6.

If a red is transferred, when drawing from urn 2 the chances are:
Red: 4/5, **White: 1/5**.
If a white is transferred, when drawing from urn 2 the chances are:
Red: 3/5, **White: 2/5**.

Drawing from urn 1 affects drawing from the second, and must be considered.
Thus, the cumulative chance of selecting white from urn 2 is:
$\frac{2}{6} * \frac{1}{5} + \frac{4}{6} * \frac{2}{5} = \mathbf{1/3}$

(b) The two events are not independent, as drawing from the first urn affects the chances of drawing from the second.
Using the values calculated above,
P(W2|R1) $= \frac{2}{6} * \frac{1}{5} = 1/15$

P(W2|W1) $= \frac{4}{6} * \frac{2}{5} = 4/15$

If the events were independent, the two resulting values would be equivalent.

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4)

	$D(.15)$	$\overline{D}(.85)$	$Total$
Pos	.98	.10	?
Neg	?	?	?
$Total$	?	?	

P(Pos $\cap$ D) = (.98)(.15) = .147
P(Pos $\cap$ $\overline{D}$) = (.10)(.85) = .085

(a) P(Pos) = .147 + .085 = **.232**
(b) P(D|Pos) = .147 / .232 = **.634**
(c) P($\overline{D}$|Pos) = .085 / .232 = **.366**

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5) P(Sys) = P(1st $\cap$ 2nd)
P(1st) = 1 - P($\overline{1st}$) = 1 - P($\overline{1} \cap \overline{2} \cap \overline{3}$) = 1 - (.3)(.3)(.3) = 0.973
P(2nd) = 1 - P($\overline{2nd}$) = 1 - P($\overline{4} \cap \overline{5}$) = 1 - (.3)(.3) = 0.91
P(1st $\cap$ 2nd) = P(1st) * P(2nd) = 0.973 * 0.91 = **0.885**

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6)

(a) P(Sys) = P(A) * P(B) = .95 * .9 = **.855**

(b) P(Sys) = 1 - P($\overline{Top} \cap \overline{Bottom}$) $\Rightarrow$
P(Top) = P(A) * P(B) = .95 * .9 = .855
P(Bottom) = P(C) * P(D) = .8 * .7 = .56
P(Sys) = 1 - (1 - .855)(1 - .56) = **.9362**

(c) P(Sys) = 1 - P($\overline{A} \cap \overline{B} \cap \overline{C}$) = 1 - (1 - .95)(1 - .9)(1 - .8) = **.999**