ComS 311
Recitation 3, 2:10 Monday
Homework 1

Sean Gordon

September 12, 2019

1a) $8n^2 + 35n + 46 \leq 8n^2 + n^2$
$9n^2 \rightarrow \quad c = 9, N = 0$

b) $log(2^{2^{n+1}}) \leq log(c) + log(2^{2^n})$
$2^{n+1} \leq log(c) + 2^n$
$2^{n+1} - 2^n \leq log(c)$
Incorrect, inequality fails
as $n \rightarrow \infty$

c) $n^3(5 + n^{.5}) \leq n^3$
$5n^3 + n^{3.5} \leq c * n^3$
$5 + n^{.5} \leq c$
Incorrect, inequality fails
as $n \rightarrow \infty$

d) If a is the smallest it can be (0)
$2^n \leq 2^n$, which checks out.
However, if a=1
$2^{n+1} \leq 2^n \Rightarrow \ 2^n * 2 \leq 2^n \Rightarrow$
$2 \leq 0$, which is incorrect.

e) If $f \in O(g) \Rightarrow f \leq g$, then $fh \in O(gh) \Rightarrow fh \leq gh$
$h * (f) \leq h * (g) \Rightarrow \frac{h}{h} * (f) \leq g \Rightarrow f \leq g$

2a) $i \rightarrow n, j \rightarrow n$
$C1(n) * C2(n) \Rightarrow O(n^2)$

b) As $i \rightarrow n, j \rightarrow n * n$
$C1(n) * C2(n^2) \Rightarrow O(n^3)$

c) pow(2,n) has a runtime of n
$i \rightarrow 2^n, and \ j \rightarrow 2^n$
However, i scales by 2 each loop, so that $i \rightarrow n$. Then,
$Cpow(n) + C1(n) * C2(2^n) \Rightarrow$
$O(n * 2^n)$

d) While i starts == n, it is /2 each pass, becoming log(n).
The inner loop is O(n), so
$C1(log(n)) * C2(n) \Rightarrow$
$O(nlog(n))$

3) This algorithm is incorrect, shown by the counterexample below:

[redacted]

4a) Provided prime numbers:
$prime1_9 = 100,000,007;$
$prime2_9 = 100,000,037;$

Runtime of gcd: 638 ms
Runtime of fastgcd: 0 ms

b) Provided prime numbers:
$prime1_{10} = 1,000,000,123;$
$prime2_{10} = 1,000,000,181;$

Runtime of gcd: 6527 ms
Runtime of fastgcd: 0 ms

---

5) Given array A and value t

```
n = A.length
x = 1

for i in range [0, n-1]
    sum += A[i] * x
    x = x*t
```

If the size of the given array is n this algorithm runs in O(n) time, as there is only 1 loop from $0 \rightarrow n$.