Continuous Space & Nondeterministic Actions

Outline

I. Local search in continuous spaces

II. Search with non-deterministic actions

* Figures are from the textbook site (or by the instructor) unless the source is specifically cited.

I. Local Search in Continuous Space

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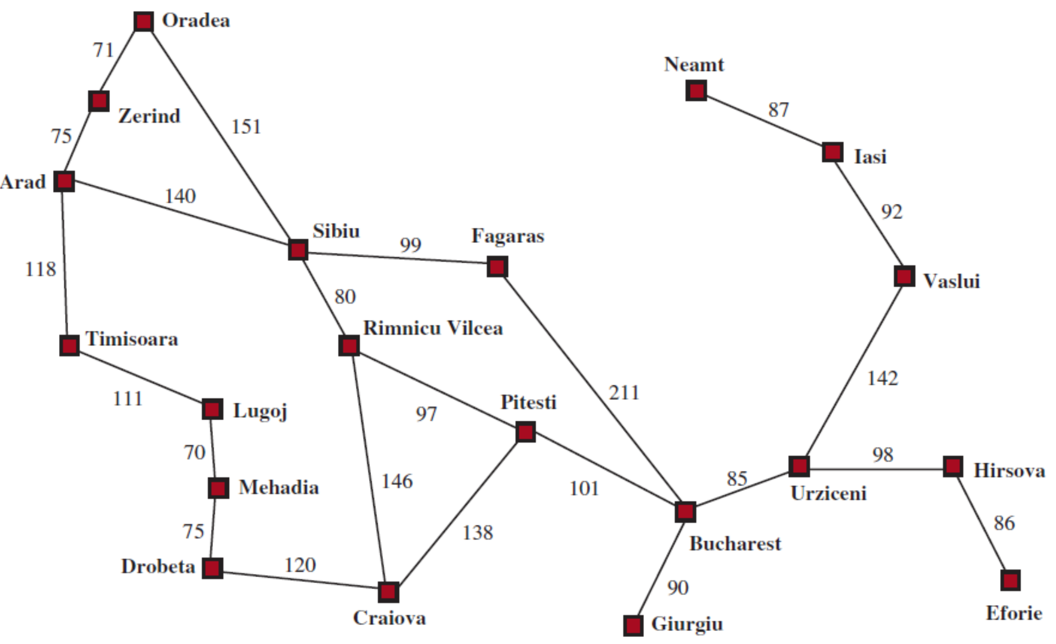

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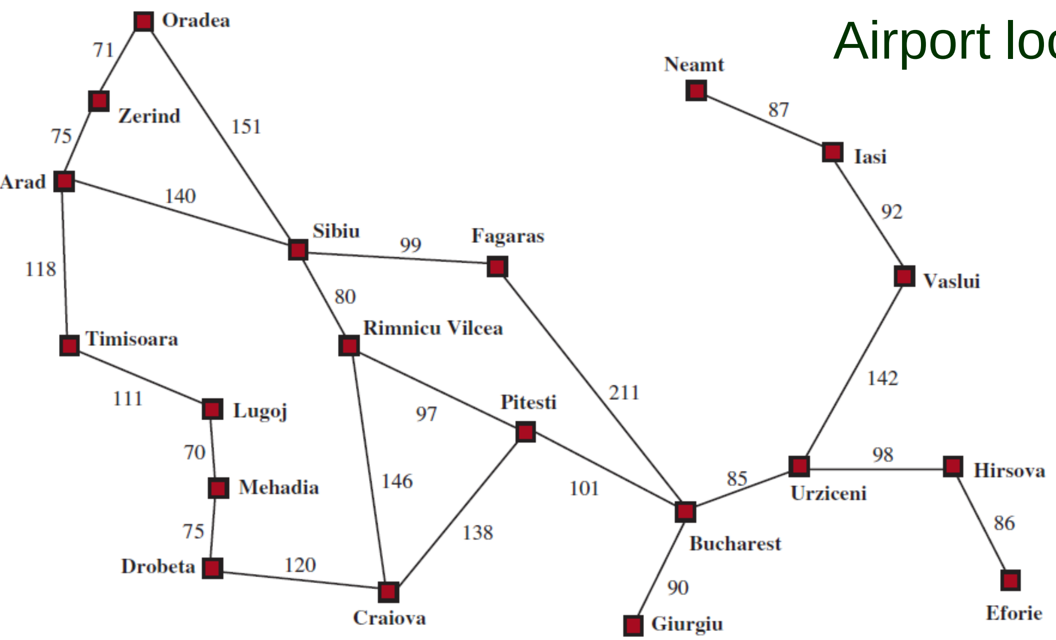

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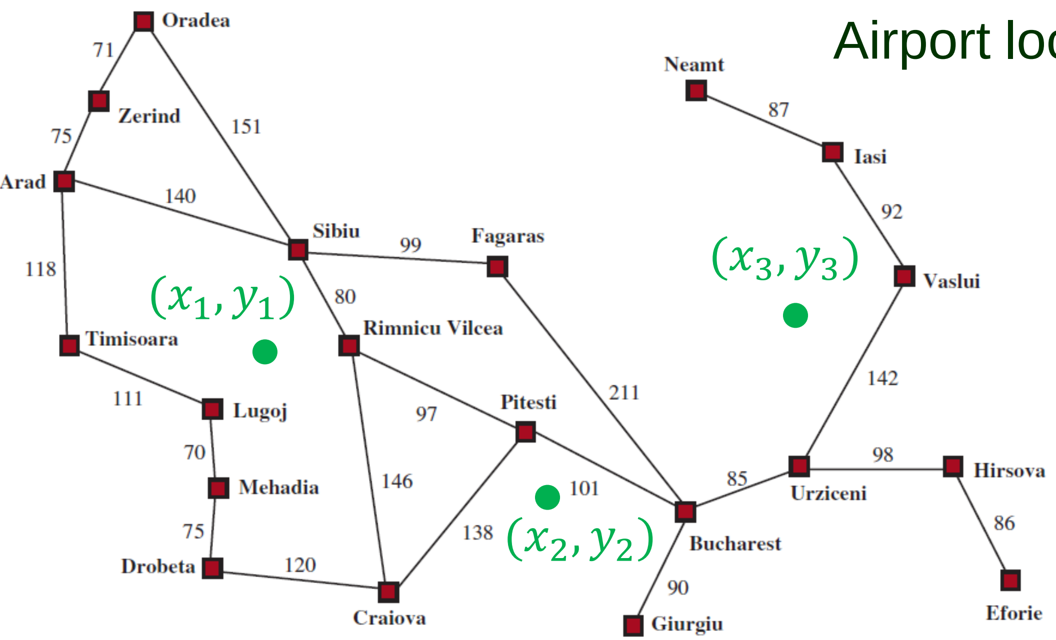

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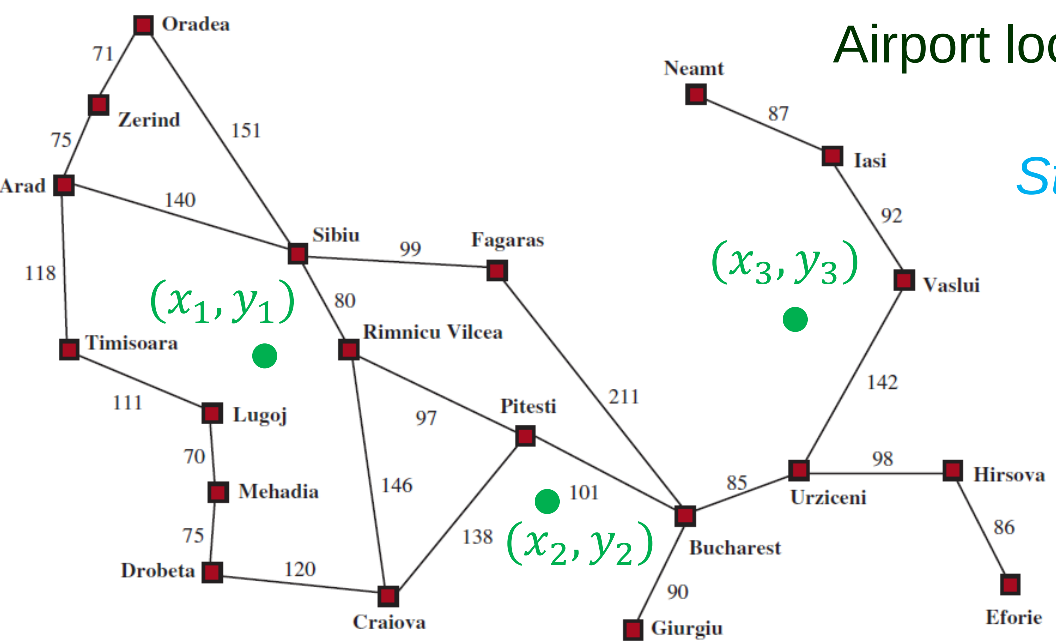

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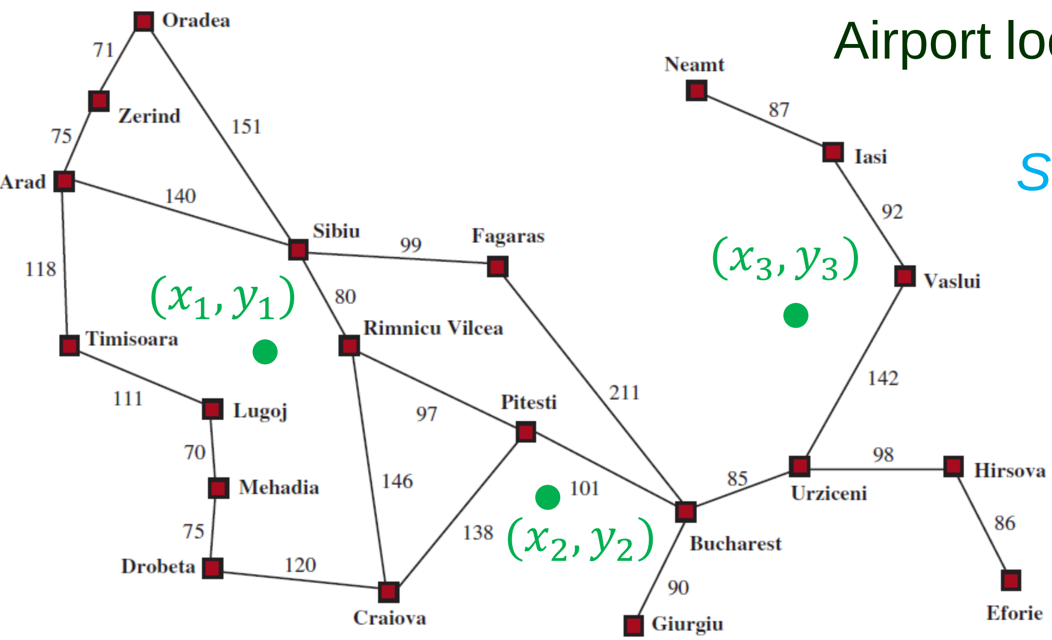

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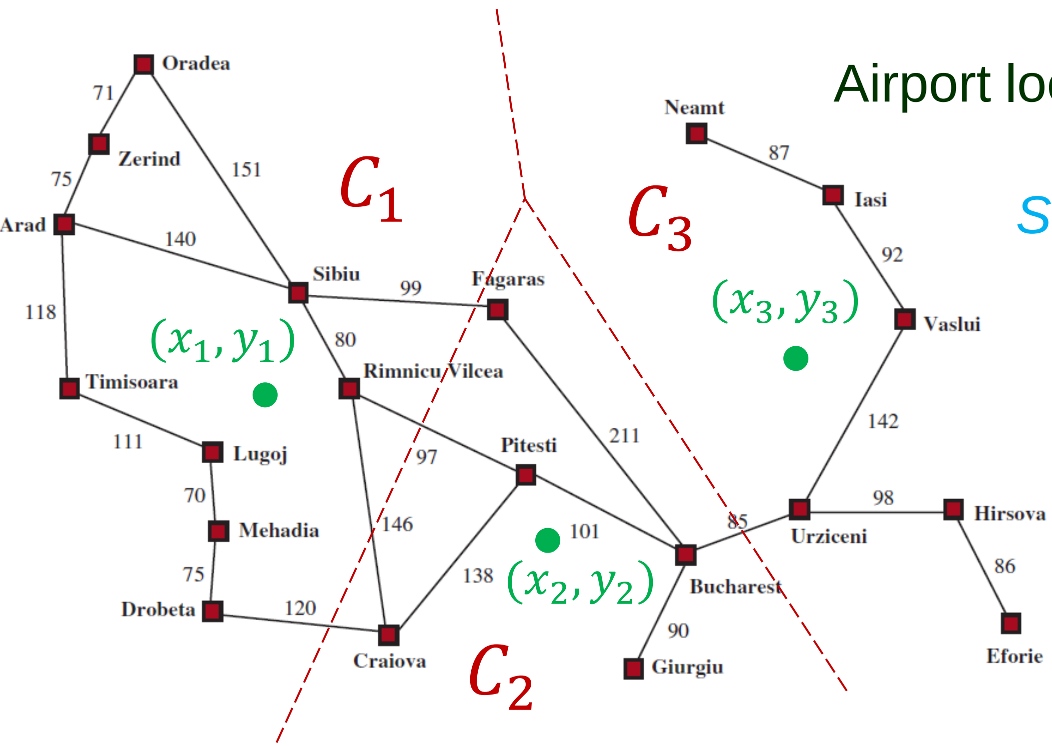

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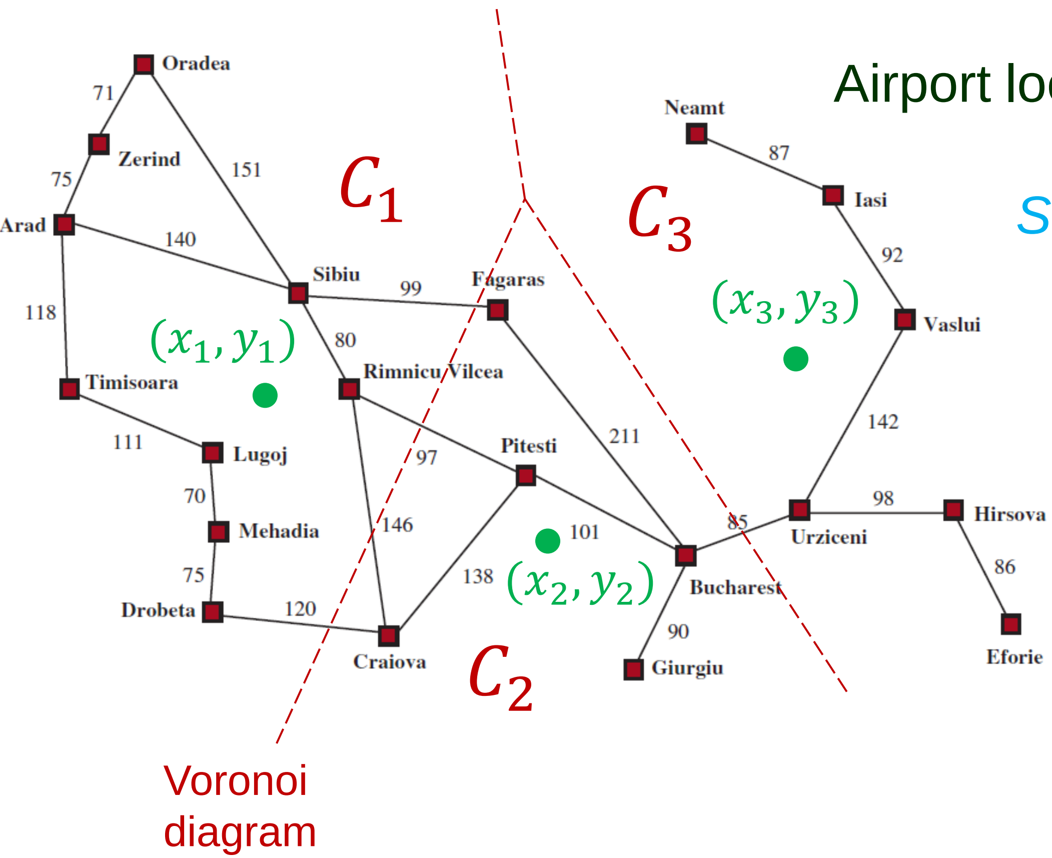

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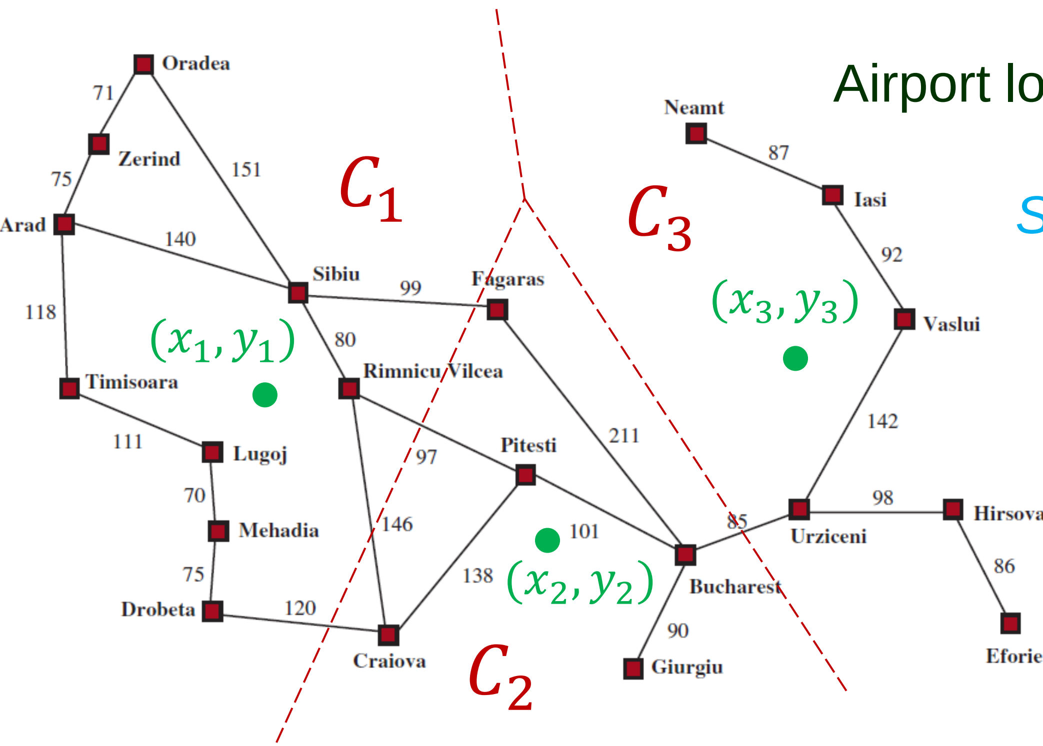

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$$\min_{x} f(\boldsymbol{x})$$

Gradient-Based Method

Use the *gradient*: $\nabla f(x_1, \ldots, x_n) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$

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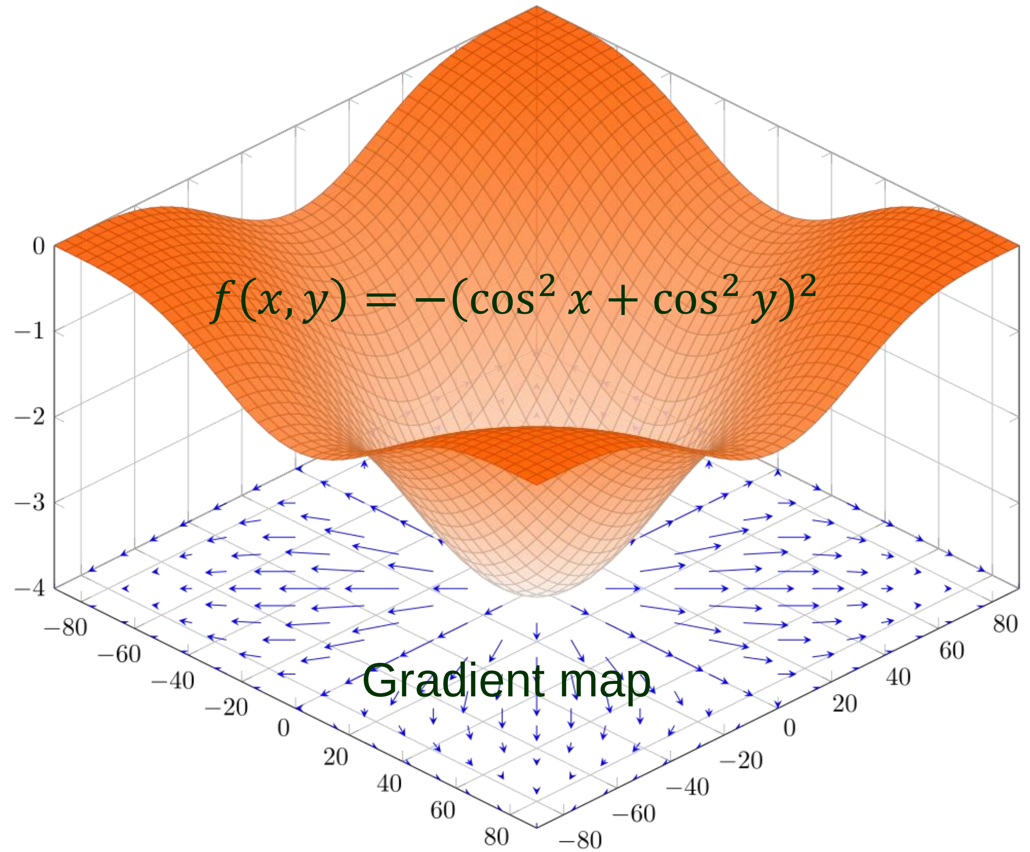

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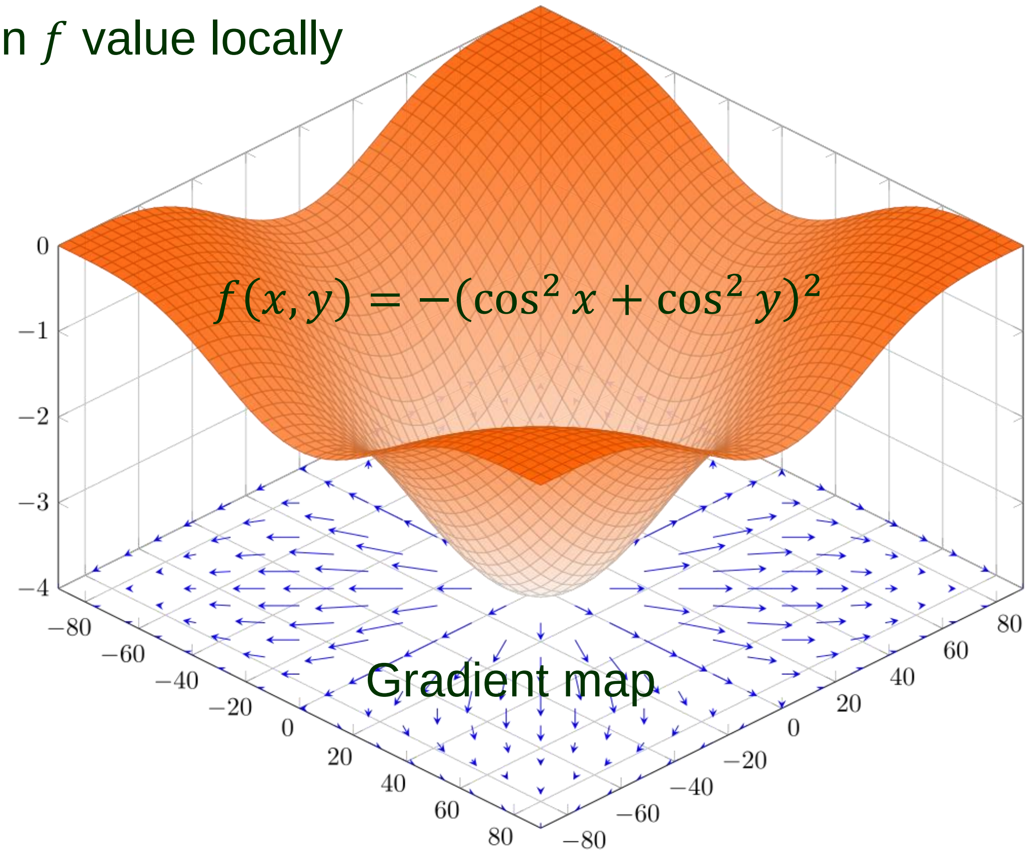
$-\nabla f$: direction of fastest decrease in f value locally



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Back to airport placements:

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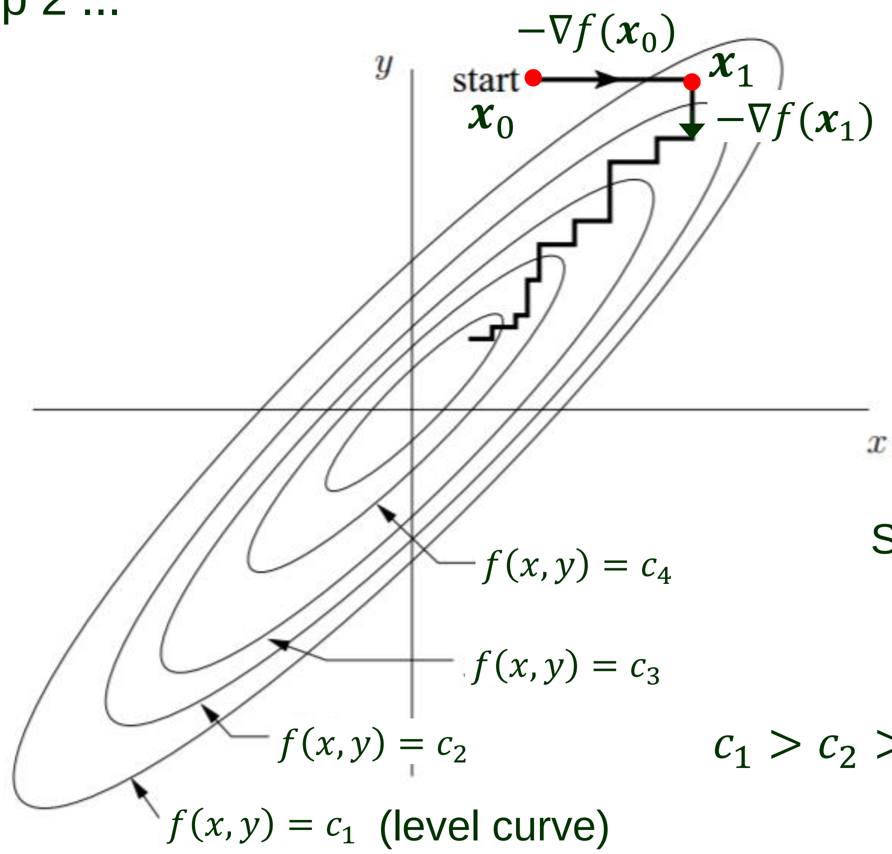
Line Search

1. Start at an initial state $\boldsymbol{x} = \boldsymbol{x}_0$.
2. Move along $-\nabla f(\boldsymbol{x})$ until f no longer decreases.
3. $\boldsymbol{x} \leftarrow$ new stopping point.
4. Go back to step 2 ...



Steepest descent (along $-\nabla f$)

$c_1 > c_2 > c_3 > c_4$

Newton-Raphson Method

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$$f(x) = 0 \qquad \text{// one variable}$$

with the iteration formula

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Hessian of f: matrix $\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)$

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To specify what to do depending on what percepts the agent receives while executing the plan.

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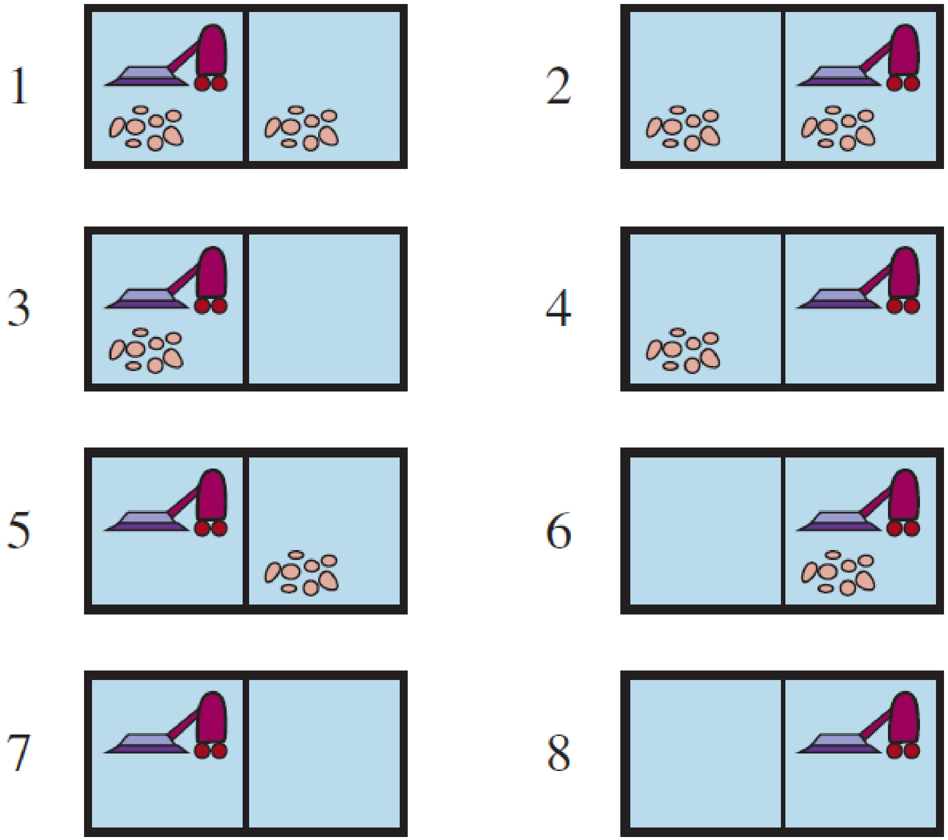
Fully observable, deterministic, and completely known



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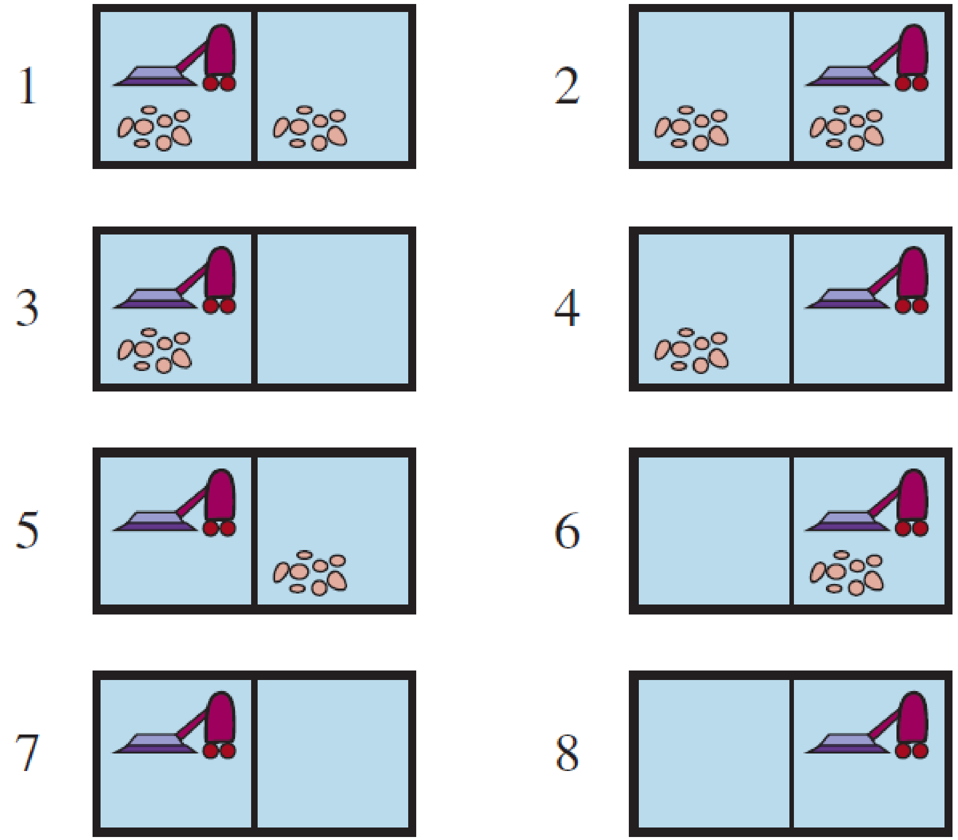
Solution: *Suck, Right, Suck* ⇐



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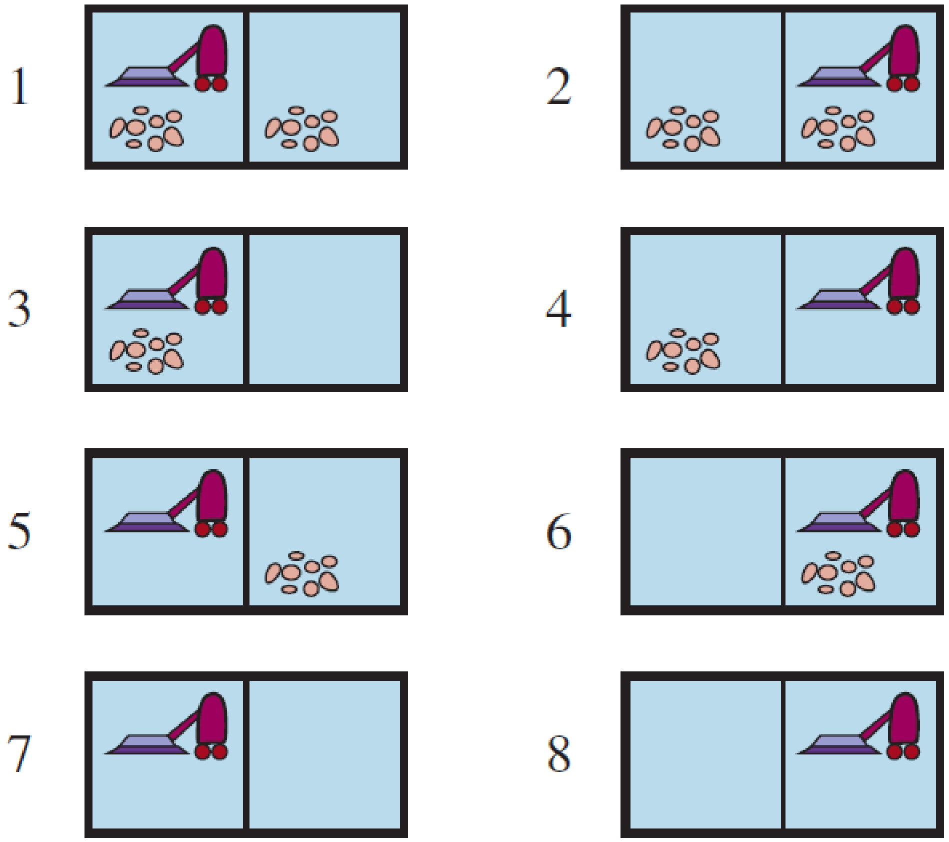

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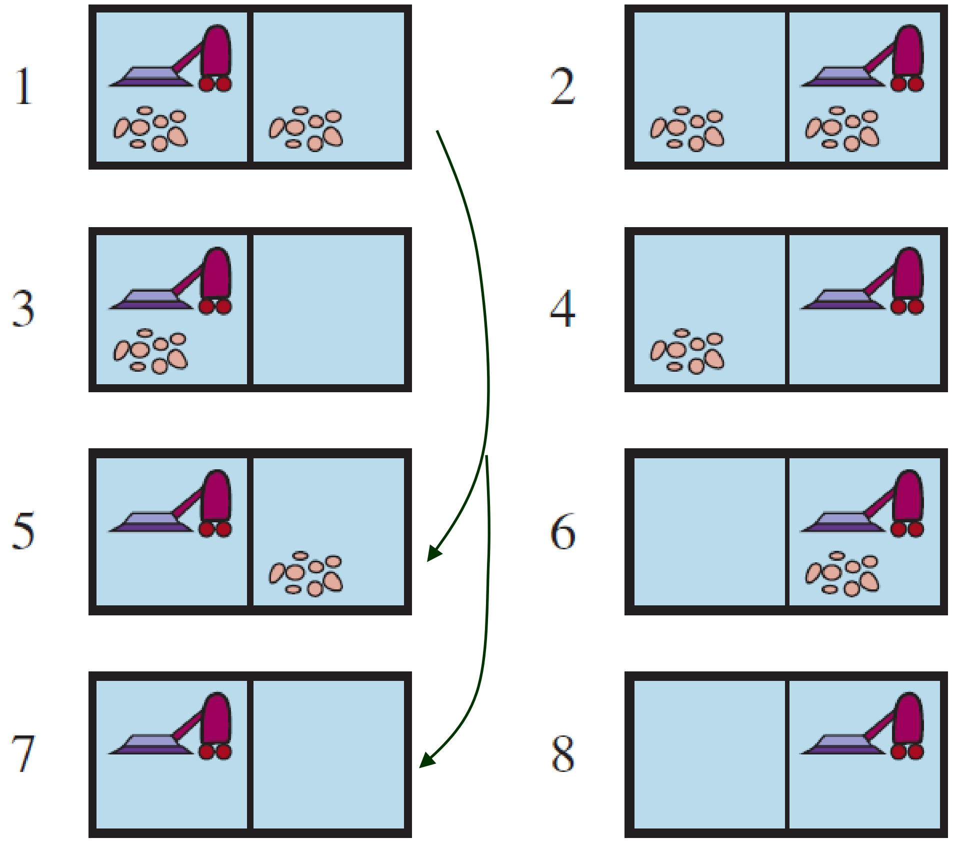

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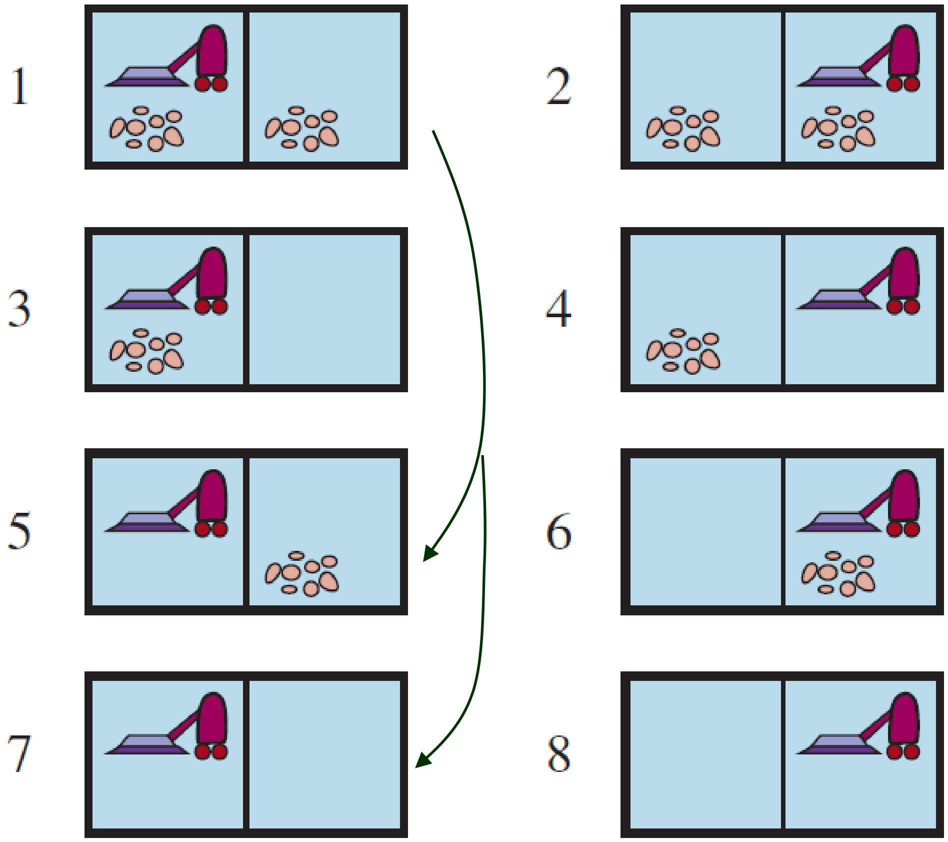

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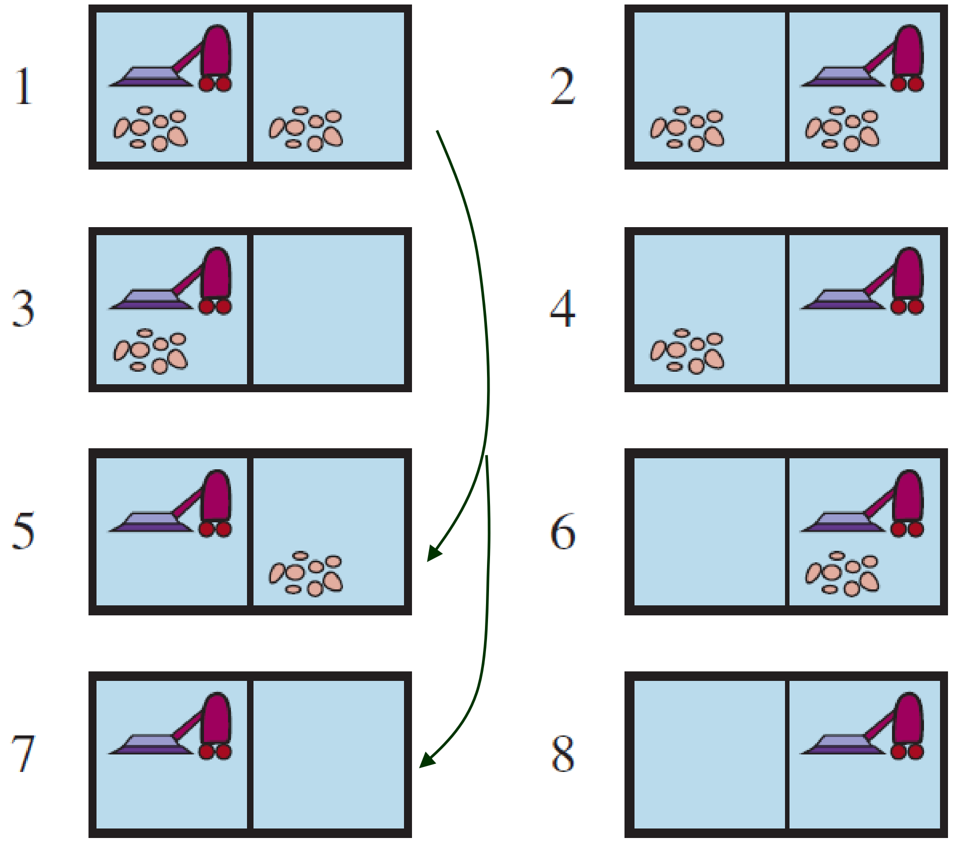

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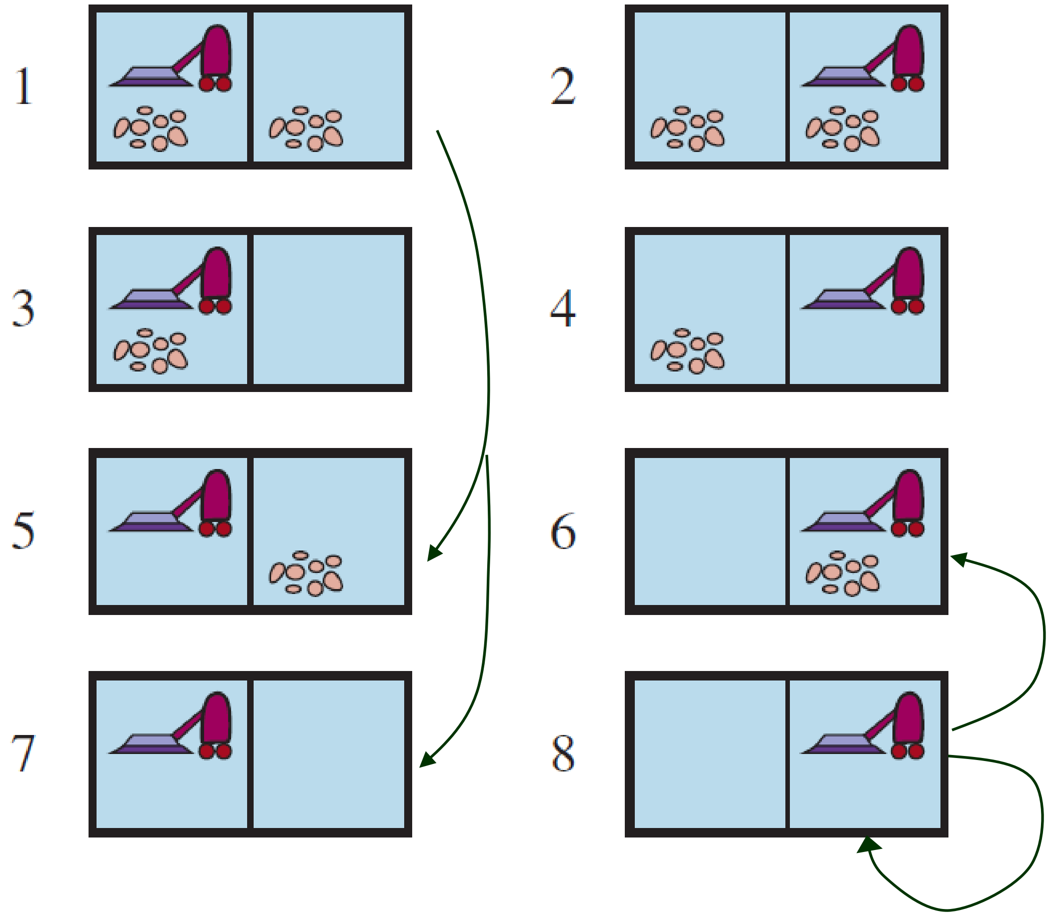

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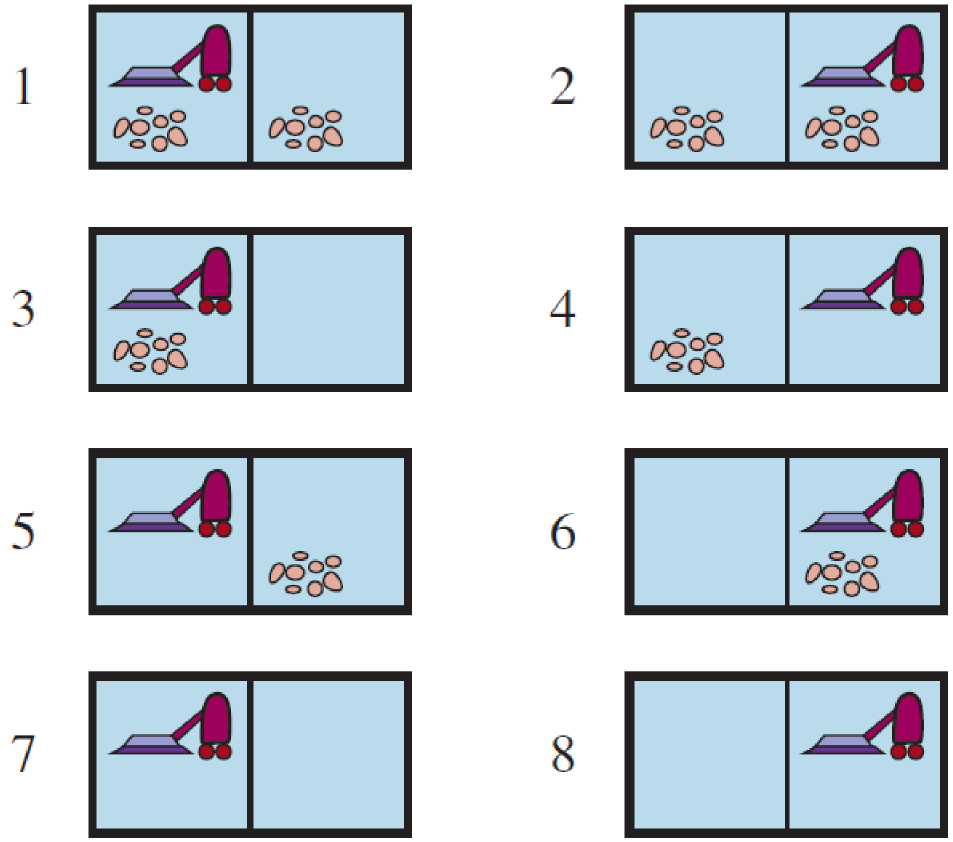

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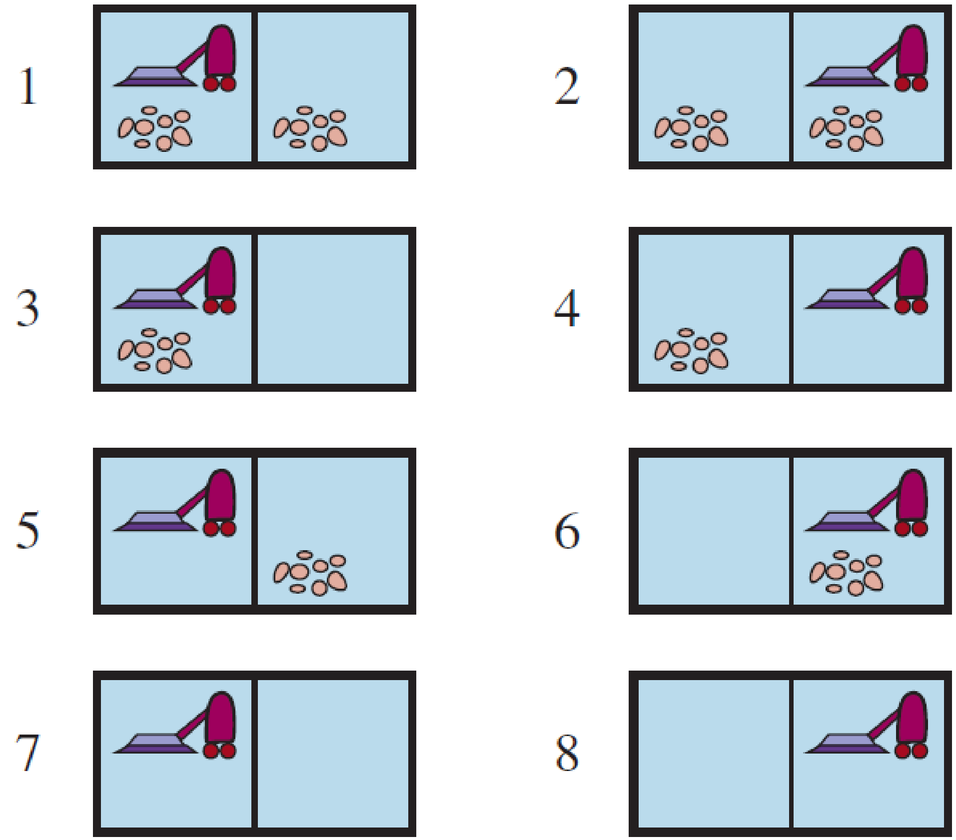

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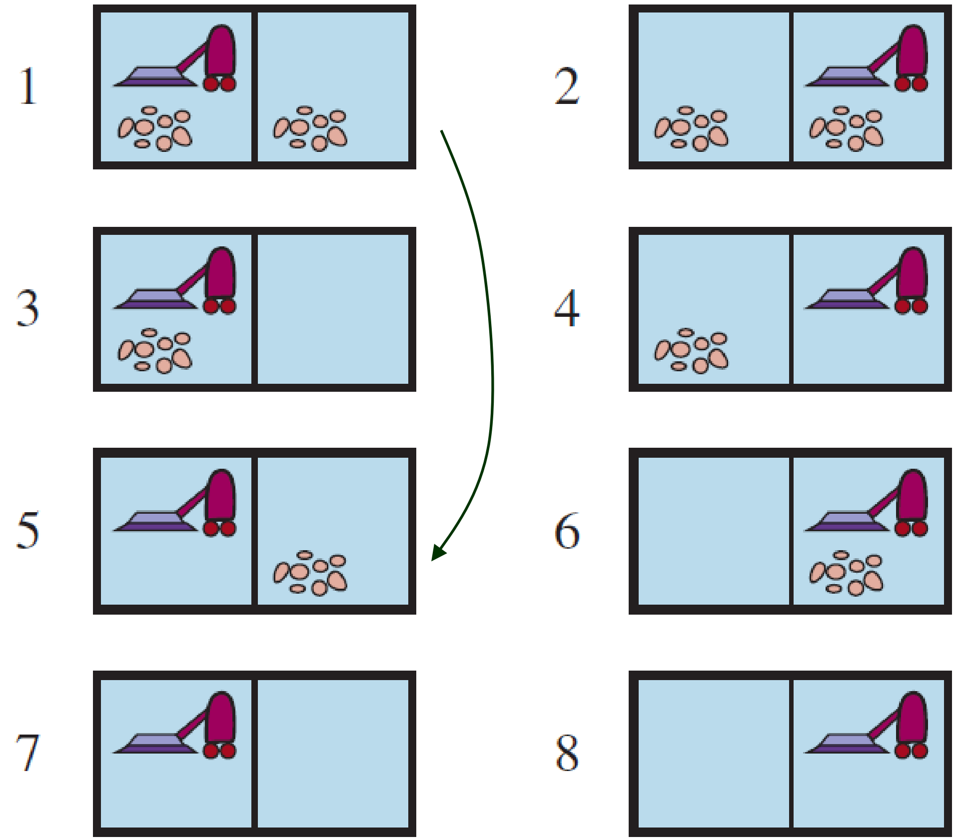

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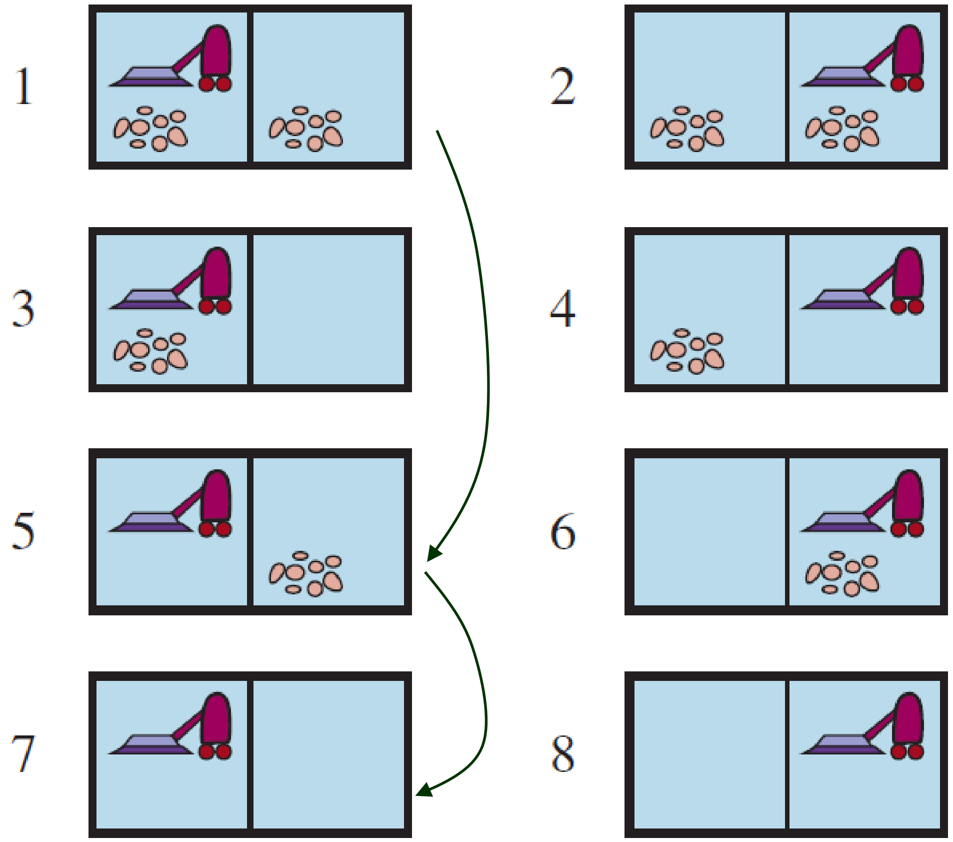

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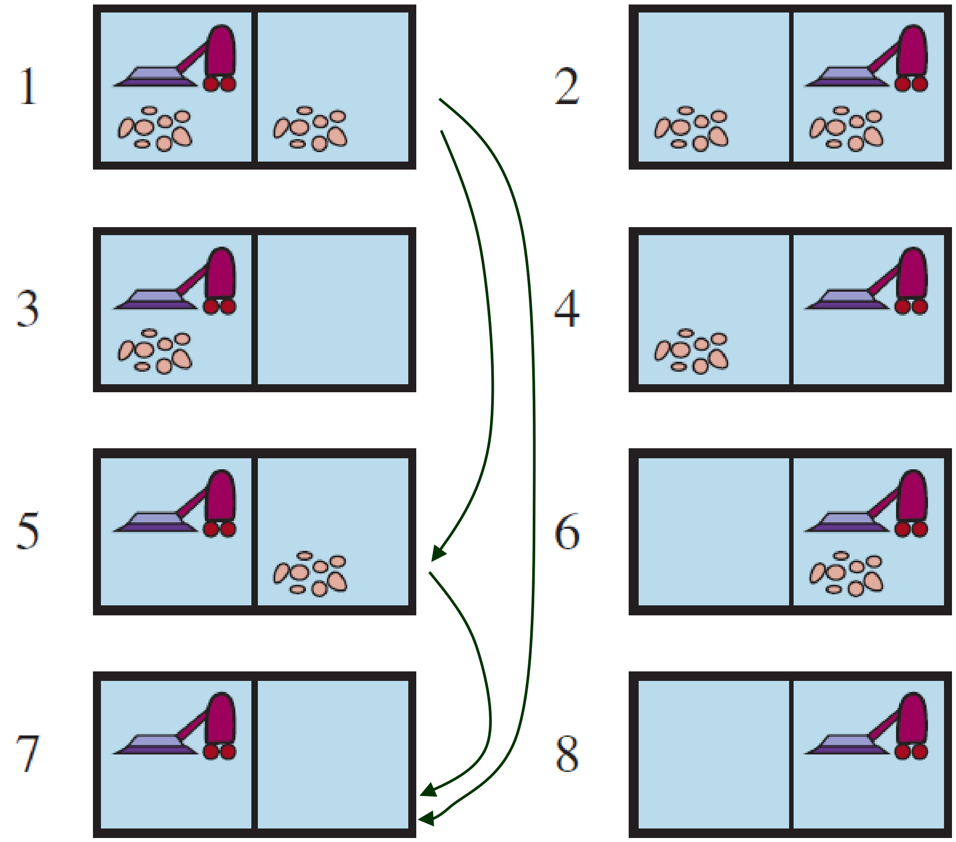

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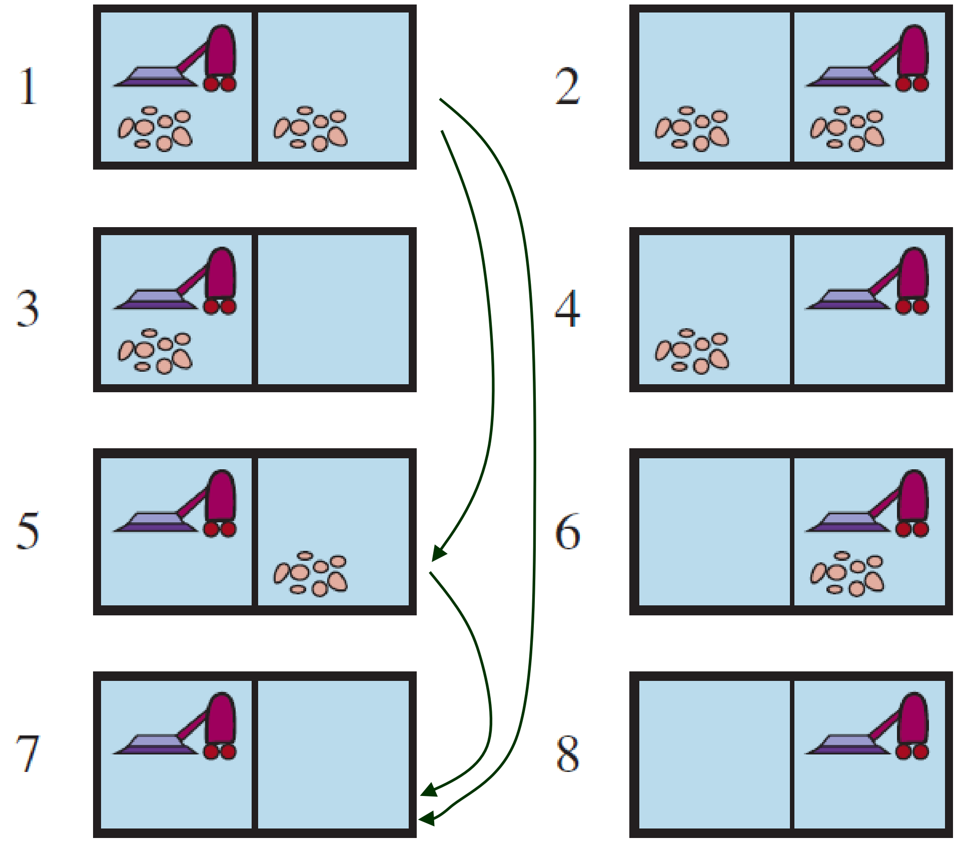

Solution is a tree – of a different character!

AND-OR Search Tree

OR-node (*deterministic*): the agent chooses an action.

e.g., *Left*, *Right*, or *Suck*

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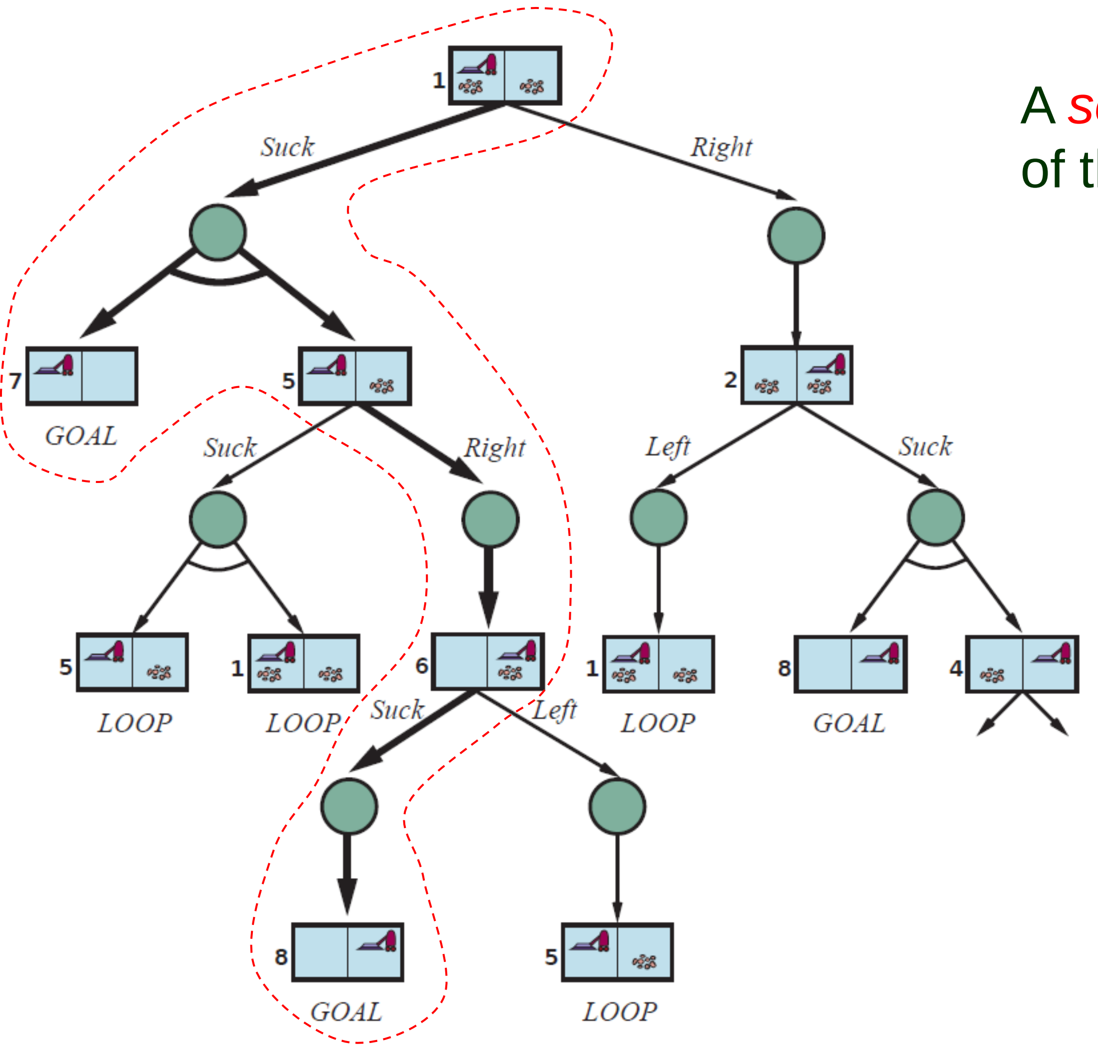
OR- and AND-nodes alternate in the tree.

Example of a (Partial) Tree



A *solution* is a connected portion of the AND-OR tree such that

- its root is the tree's root;
- every OR node has exactly one child (i.e., one of the actions);
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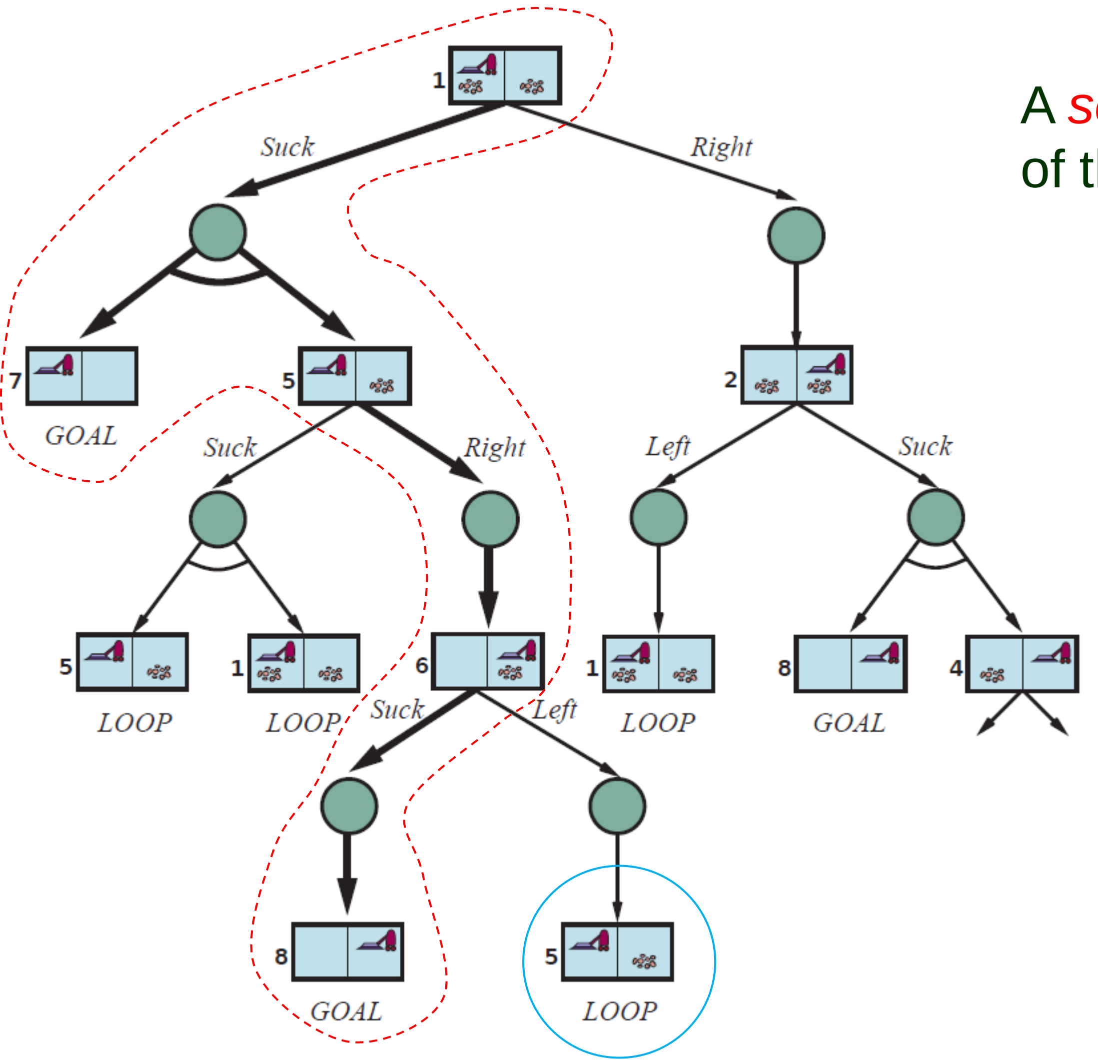

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DFS Implementation of AND-OR Tree Search

```
function AND-OR-SEARCH(problem) returns a conditional plan, or failure
   return OR-SEARCH(problem, problem.INITIAL, [])

function OR-SEARCH(problem, state, path) returns a conditional plan, or failure
   if problem.IS-GOAL(state) then return the empty plan
   if IS-CYCLE(path) then return failure  // ignore a solution with a cycle. such a solution would
   for each action in problem.ACTIONS(state) do  // imply the existence of a non-cyclic solution
       plan ← AND-SEARCH(problem, RESULTS(state, action), [state] + path])  // which can
       if plan ≠ failure then return [action] + plan]  // be found.
   return failure

function AND-SEARCH(problem, states, path) returns a conditional plan, or failure
   for each s_i in states do
       plan_i ← OR-SEARCH(problem, s_i, path)
       if plan_i = failure then return failure
   return [if s_1 then plan_1 else if s_2 then plan_2 else ... if s_{n-1} then plan_{n-1} else plan_n]
```

Solution plan

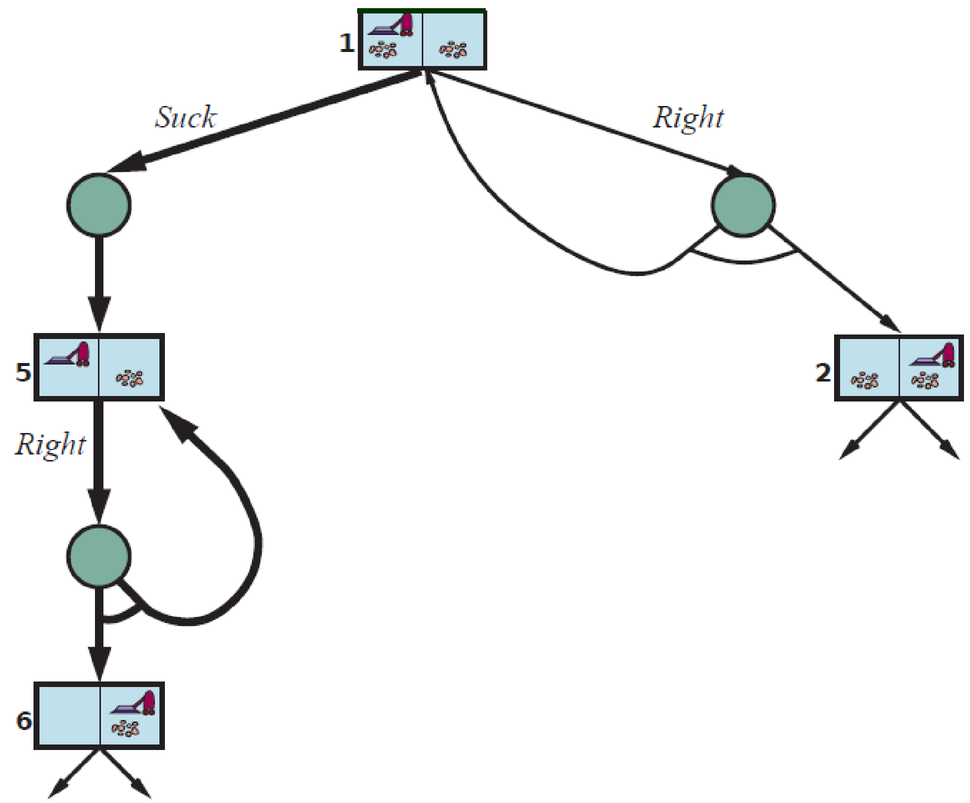
Cyclic Plan for a Solution

What if an action fails and the state is not changed?

Slippery vacuum world.

$$\text{State 1} \xrightarrow{\textit{Right}} \{1, 2\}$$

$$\text{State 5} \xrightarrow{\textit{Right}} \{5, 6\}$$



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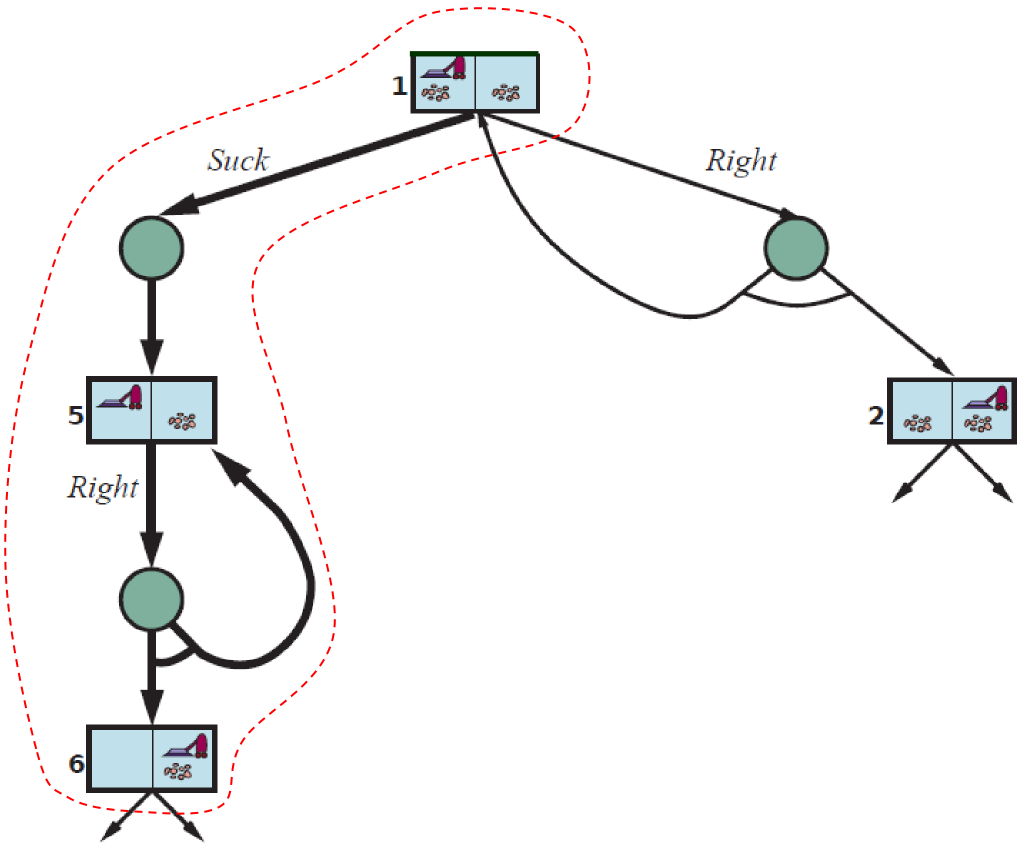
Cyclic solution



Cyclic Plan for a Solution

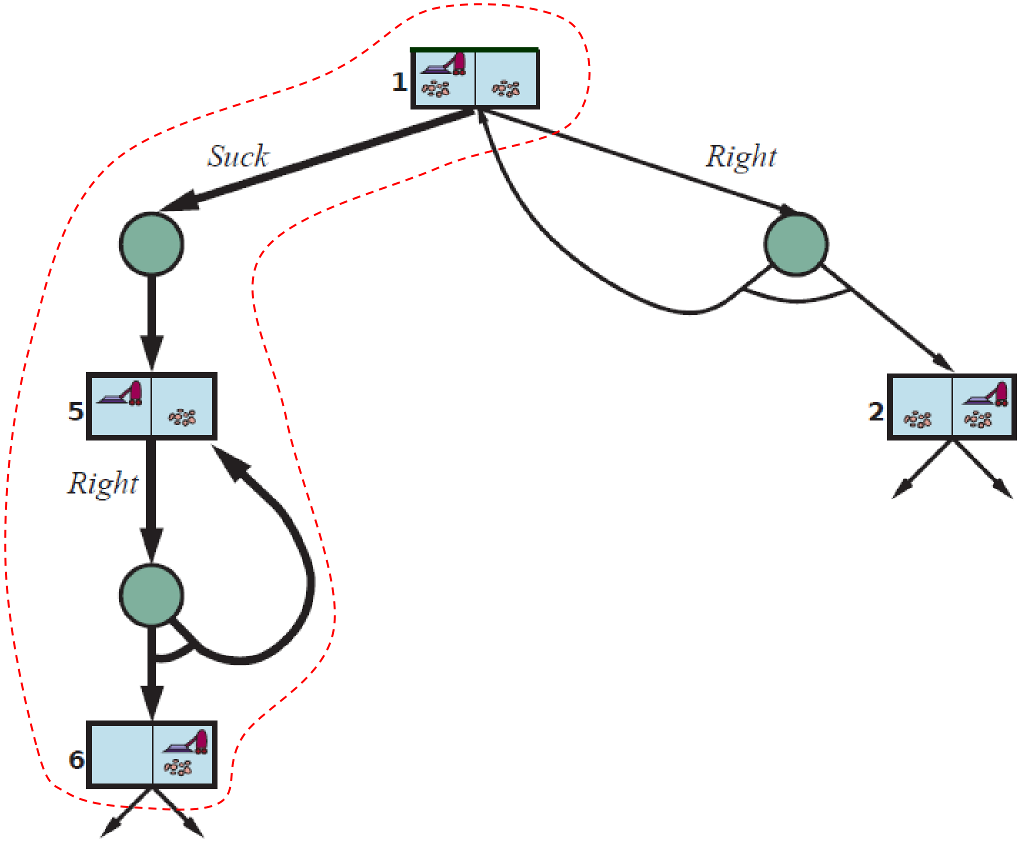
What if an action fails and the state is not changed?

Slippery vacuum world.

$$\text{State 1} \xrightarrow{\textit{Right}} \{1, 2\}$$

$$\text{State 5} \xrightarrow{\textit{Right}} \{5, 6\}$$

Cyclic solution



do
Suck;
if State = 5 then *Right*

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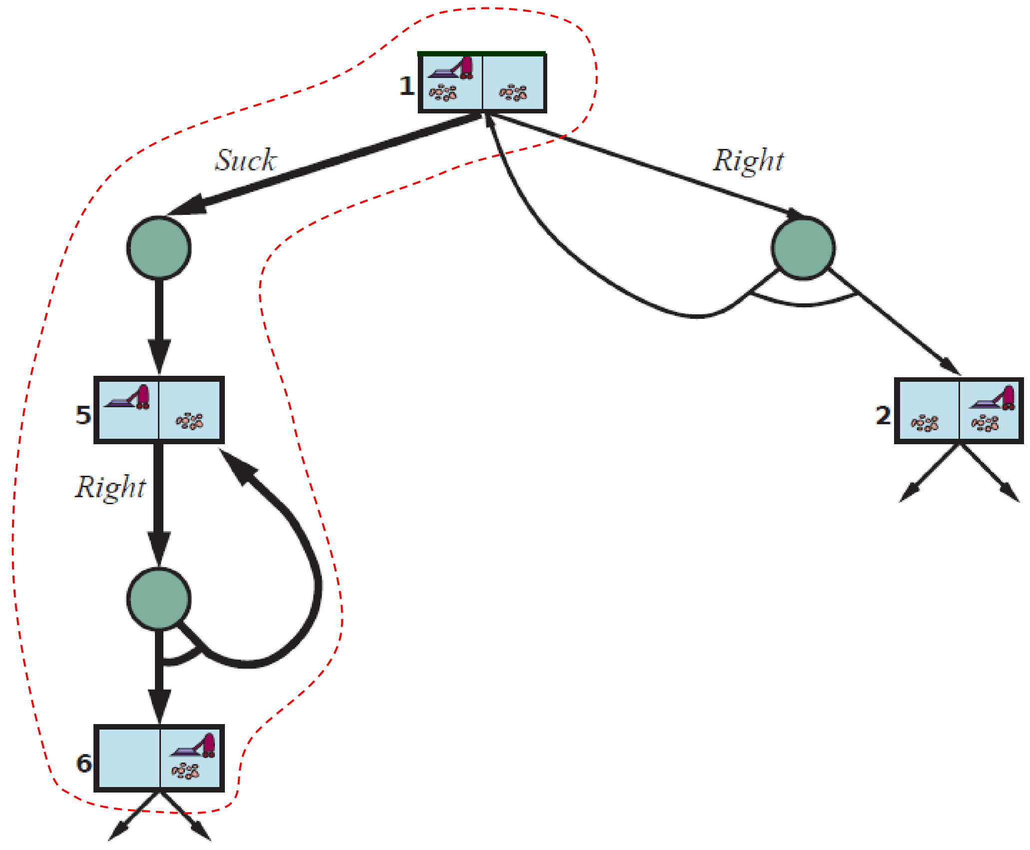

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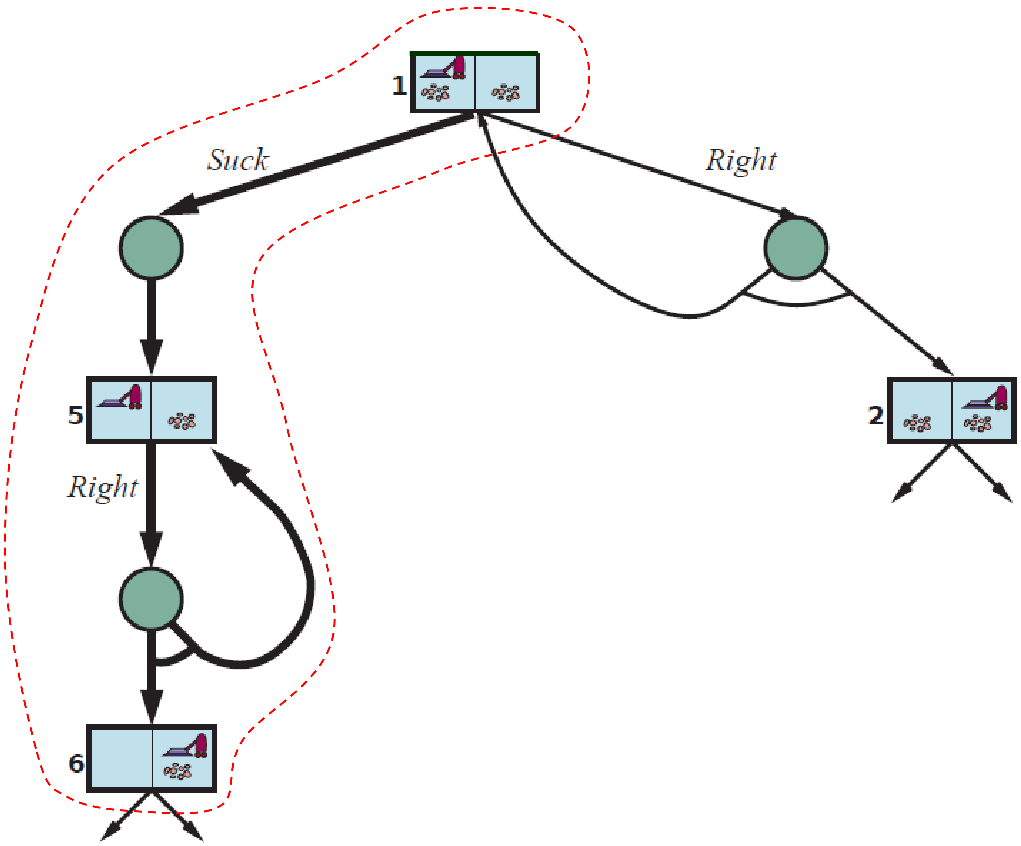

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