EE 330 HW 14 Solutions Spring 2018 TA: George Alphonse

Problem 1:

$$R_{pu} = \frac{L_p}{\mu_p C_{ox} W_p (V_{DD} + V_{tp})} = 4761.9\ \Omega$$

$$R_{pd} = \frac{L_n}{\mu_n C_{ox} W_n (V_{DD} - V_{tn})} = 952.4\ \Omega$$

$$t_{HL} = R_{pd} C_L = 3.81\ ns$$

$$t_{LH} = R_{pu} C_L = 19.0\ ns$$

Problem 2:

$$R_{pu} = \frac{L_p}{\mu_p C_{ox} W_p (V_{DD} + V_{tp})} = 7518.8\ \Omega$$

$$R_{pd} = \frac{L_n}{\mu_n C_{ox} W_n (V_{DD} - V_{tn})} = 2040.8\ \Omega$$

$$t_{HL} = R_{pd} C_L = 16.3\ ps$$

$$t_{LH} = R_{pu} C_L = 60.15\ ps$$

$$V_{trip} = \frac{V_{tn} + (V_{DD} + V_{tp})\sqrt{\frac{\mu_p}{\mu_n} * \frac{w_2}{w_1} * \frac{L_1}{L_2}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} * \frac{w_2}{w_1} * \frac{L_1}{L_2}}} = 1.002\ V$$

Problem 3:

$$\frac{V_{DD}}{2} = \frac{V_{tn} + (V_{DD} + V_{tp})\sqrt{\frac{\mu_p}{\mu_n} * \frac{w_2}{w_1} * \frac{L_1}{L_2}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} * \frac{w_2}{w_1} * \frac{L_1}{L_2}}} \rightarrow \frac{w_2}{w_1} = 0.988$$

$w_1 = 0.182\ \mu m, \quad w_2 = 0.18\ \mu m, \quad L_1 = L_2 = 0.18\ \mu m$

Problem 4:

$$R_{pu} = R_{pd} \rightarrow \frac{L_p}{\mu_p C_{ox} W_p (V_{DD} + V_{tp})} = \frac{L_n}{\mu_n C_{ox} W_n (V_{DD} - V_{tn})} \rightarrow \frac{w_p}{w_n} = 3.684$$

$w_p = 0.663\ \mu m, \quad w_n = 0.18\ \mu m, \quad L_1 = L_2 = 0.18\ \mu m$

Problem 5:

$$1 = \frac{0.5 + (2 - 0.5)\sqrt{\frac{1}{5} * \frac{w_2}{L_2}}}{1 + \sqrt{\frac{1}{5} * \frac{w_2}{L_2}}} \to \frac{w_2}{L_2} = 5$$

$L_2 = 0.18\ \mu m, \qquad w_2 = 0.9\ \mu m$

Problem 6:

$if\ V_{in} = 0V$

$$0 = \frac{u_p C_{ox}}{2} * \frac{W_2}{L_2} * \left(Vout - Vdd - V_{tp}\right)^2 \longrightarrow Vout = 1.5\ V$$

$if\ V_{in} = 2V$

$$\frac{u_n C_{ox} W_1}{L_1} * \left(V_{in} - V_{tn} - \frac{V_{out}}{2}\right) * Vout = \frac{u_p C_{ox}}{2} * \frac{W_2}{L_2} * \left(Vout - Vdd - V_{tp}\right)^2 \longrightarrow V_{out} = 0.439\ V$$

Problem 7:

a)

$$V_{trip} = \frac{V_{tn} + \left(V_{DD} + V_{tp}\right)\sqrt{\frac{\mu_p}{\mu_n} * \frac{w_p}{w_n} * \frac{L_n}{L_p}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} * \frac{w_p}{w_n} * \frac{L_n}{L_p}}} = \frac{0.5 + (3.5 - 0.5)\sqrt{\frac{1}{3} * \frac{3}{0.5} * \frac{1}{1}}}{1 + \sqrt{\frac{1}{3} * \frac{3}{0.5} * \frac{1}{1}}} = 1.96\ V$$

b)

$$C_L = C_{in} = 7 * C_{ox} * W_{min} * L_{min} = 7 * 2f * 0.5 * 0.5 = 3.5\ fF$$

$$R_{pu} = \frac{L_p}{\mu_p C_{ox} W_p \left(V_{DD} + V_{tp}\right)} = 10\ k\Omega$$

$$t_{LH} = R_{pu} C_L = 35\ ps$$

Problem 8:

$w_n = 0.18\ \mu m, \qquad L_n = 0.18\ \mu m$

$w_p = 3 * 4 * W_{min} = 2.16\ \mu m, \qquad L_p = 0.18\ \mu m$

Problem 9:

a)

$V_L = 0.734\,V,\ \ V_H = 4V;\quad works\ as\ inverter$

b)

$V_L = 3.86\,V,\ \ V_H = 4V;\quad works\ as\ inverter$

c)

$V_L = 1.016\,V,\ \ V_H = 4V;\quad works\ as\ inverter$

Problem 10:

$$V_{trip} = \frac{V_{tn} + \left(V_{DD} + V_{tp}\right)\sqrt{\frac{\mu_p}{\mu_n} * \frac{w_p}{w_n} * \frac{L_n}{L_p}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} * \frac{w_p}{w_n} * \frac{L_n}{L_p}}} = \frac{1 + (5-1)\sqrt{1 * \frac{2}{w_n} * \frac{1}{1}}}{1 + \sqrt{1 * \frac{2}{w_n} * \frac{1}{1}}}$$

$if\ V_{in} = 5V$

$$\frac{u_n C_{ox} W_1}{L_1} * \left(V_{in} - V_{tn} - \frac{V_{out}}{2}\right) * Vout = \frac{u_p C_{ox}}{2} * \frac{W_2}{L_2} * \left(Vout - Vdd - V_{tp}\right)^2$$

$V_{trip} = 2.561\,V$

$Min\ val\ of\ W_1 = 1.697\ um$

Problem 11:

$$V_{trip} = \frac{0.5 + (2-0.5)\sqrt{\frac{1}{5} * 5 * 1}}{1 + \sqrt{\frac{1}{5} * 5 * 1}} = 1\,V$$

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$$V_{trip} = \frac{0.5 + (2-0.5)\sqrt{\frac{1}{5} * 1 * 1}}{1 + \sqrt{\frac{1}{5} * 1 * 1}} = 0.809\,V$$

$$V_{trip} = \frac{0.5 + (2 - 0.5)\sqrt{\frac{1}{5} * 3 * \frac{1}{10}}}{1 + \sqrt{\frac{1}{5} * 3 * \frac{1}{10}}} = 0.697\ V$$

Problem 12:

$$C_{in1} = \frac{3+8}{4} * C_{ref} = \frac{11}{4} C_{ref}$$

$$A_1 = 8 * \left(3W_{pmin} + 8W_{nmin}\right) * L_{min} = 88\ W_{min} L_{min}$$

$$C_{in2} = \frac{3+4}{4} * C_{ref} = \frac{7}{4} C_{ref}$$

$$A_2 = 2 * 4 * \left(3W_{pmin} + 4W_{nmin}\right) * L_{min} + 2 * 7 * W_{min} L_{min} + 4 * W_{min} L_{min} = 74\ W_{min} L_{min}$$

$$C_{in3} = \frac{3+2}{4} * C_{ref} = \frac{5}{4} C_{ref}$$

$$A_3 = 4 * 2 * \left(3W_{pmin} + 2W_{nmin}\right) * L_{min} + 4 * (13) * W_{min} L_{min} + 4 * W_{min} L_{min} = 96\ W_{min} L_{min}$$

$$C_{in4} = \frac{3+2}{4} * C_{ref} = \frac{5}{4} C_{ref}$$

$$A_1 = 4 * 2 * \left(3W_{pmin} + 2W_{nmin}\right) * L_{min} + 2 * 2 * 7W_{min} L_{min} + 2 * 5 * W_{min} L_{min} = 78\ W_{min} L_{min}$$

Problem 13:

For equal rise and fall time

$$t_{HL} = t_{LH} = R_{pd} C_L = \frac{0.18}{350e - 6 * 0.18 * (2V - 0.5)} * 40fF = 76.19\ ps$$

For minimum sized inverter

$$R_{pu} = \frac{L_p}{\mu_p C_{ox} W_p \left(V_{DD} + V_{tp}\right)} = 9523.8\ \Omega$$

$$R_{pd} = \frac{L_n}{\mu_n C_{ox} W_n (V_{DD} - V_{tn})} = 1904.8\ \Omega$$

$$t_{HL} = R_{pd} C_L = 76.19\ ps$$

$$t_{LH} = R_{pu} C_L = 380.95\ ps$$

Problem 14:

a)

For 4-input NAND:

$$W_n = 4 * W_{min}, \qquad L_n = L_{min}$$

$$W_p = 3 * W_{min}, \qquad L_p = L_{min}$$

$$R_{pu} = \frac{L_p}{\mu_p C_{ox} W_p (V_{DD} + V_{tp})} = 3.33\ k\Omega$$

$$C_L = \frac{10(3+4)}{4} * 400f + 5 * 400f = 9000\ fF$$

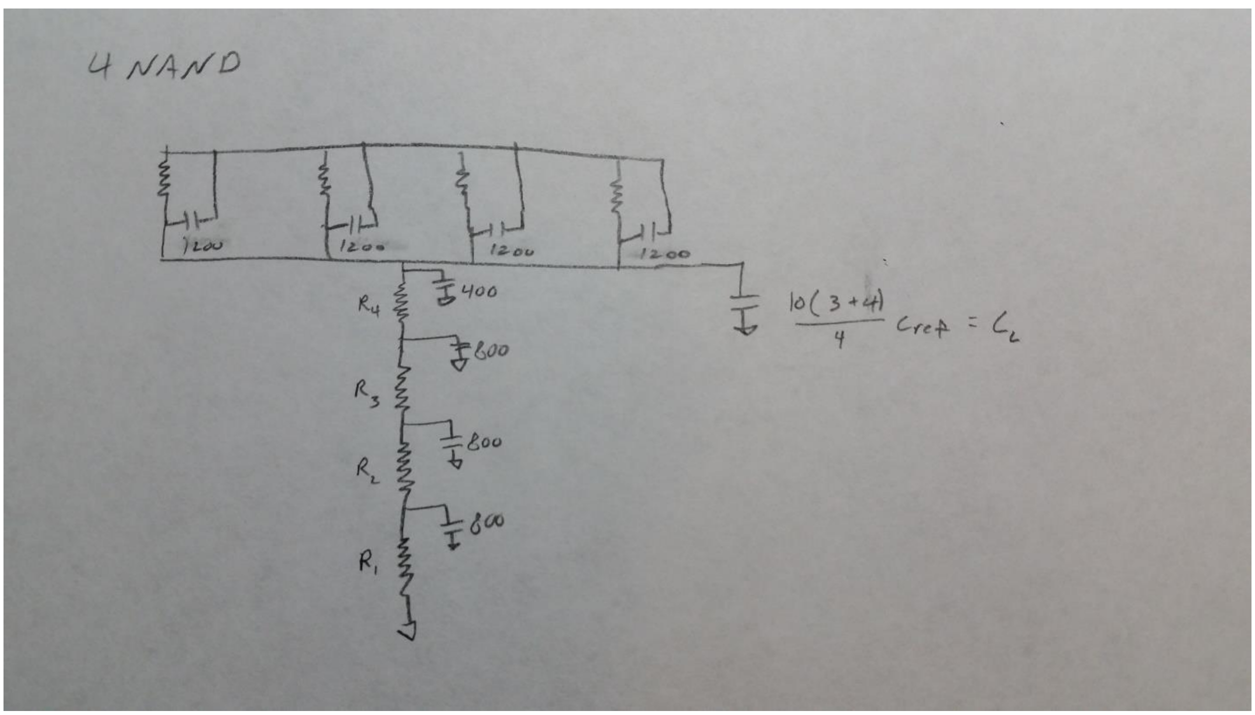
$$t_{HL} = t_{LH} = R_{pu} C_L = 30\ ns$$

b)

$$\begin{aligned} Elmore's\ Delay &= R_1(3*800 + 400 + 4*1200 + C_L) + R_2(2*800 + 400 + 4*1200 + C_L) \\ &+ R_3(800 + 400 + 4*1200 + C_L) + R_4(400 + 4*1200 + C_L) \end{aligned}$$

$$R_{pu} = R_1 = R_2 = R_3 = R_4$$

$$t_{HL} = t_{LH} = 178.7\ ns$$



Problem 15:

$$t_{prop12} = \frac{3+3}{4}\, t_{ref}$$

$$t_{prop23} = \frac{1+6}{4}\, t_{ref}$$

$$t_{prop34} = \frac{20}{4}\, t_{ref}$$

$$t_{propBF} = (1.5 + 1.75 + 5)t_{ref} = 8.25\, t_{ref}$$

Problem 16:

$$t_{prop1} = \frac{800}{4}\, t_{ref} = 200\, t_{ref}$$

$$t_{prop1} = \frac{8}{1}\, t_{ref} + \frac{64}{8}\, t_{ref} + \frac{800}{64}\, t_{ref} = 28.5\, t_{ref}$$

$$t_{prop1} = \frac{1}{1}\, t_{ref} + \frac{64}{1}\, t_{ref} + \frac{800}{64}\, t_{ref} = 77.5\, t_{ref}$$

Problem 17:

a)

$$t_{prop} = \left(\left(\frac{2*(3*3+1)}{4} * \frac{1}{4}\right) + \left(\left(\frac{3+3}{4} + 1 + 14\right) * \frac{1}{2}\right) + \frac{500}{1.5}\right) t_{ref} = 342.8\, t_{ref} = 6.86\, ns$$

b)

$$t_{prop} = (\frac{1}{2} + \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right) + \frac{500}{0.8})\, t_{ref} = 627\, t_{ref} = 12.5\, ns$$

Problem 18:

a)

For 3-input NAND with overdrive 6:

$W_n = 3 * 6 * W_{min}, \qquad L_n = L_{min}$

$W_p = 5 * 6 * W_{min}, \qquad L_p = L_{min}$

For 2-input NOR with overdrive 3:

$W_n = 1 * 3 * W_{min}, \qquad L_n = L_{min}$

$W_p = 10 * 3 * W_{min}, \qquad L_p = L_{min}$

b)

$$t_{prop} = \left(\left(3 * \frac{3 * 2 + 1}{4}\right) + 4 + 2\right) * \frac{1}{6} t_{ref} + 4 * \frac{1}{3} t_{ref} = 3.208\ t_{ref}$$

Problem 19:

$$C_{ref} = 4 * C_{ox} W_{min} L_{min} = 4 * \left(\frac{8fF}{um^2}\right) * (0.18um) * (0.18um) = 1.037\ fF$$

Total effort of inverter chain is F = 280p/1.037f = 270010

$$Delay = N * \sqrt{F}^{\frac{1}{N}} + P, where\ N = number\ of\ stages, and\ P = parasitic\ delay$$

$$n_{OPT} = \ln\left(\frac{C_L}{C_{REF}}\right) = \ln(270010) = 12.5 \rightarrow 13$$

Overdrive of each stage is $\sqrt[13]{270010} = 2.617$

Problem 20:

a)

$$V_{trip} = \frac{V_{tn} + (V_{DD} + V_{tp})\sqrt{\frac{\mu_p}{\mu_n} * \frac{w_p}{w_n} * \frac{L_n}{L_p}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} * \frac{w_p}{w_n} * \frac{L_n}{L_p}}} = \frac{1.5 + (5 - 0.5)\sqrt{\frac{1}{1} * \frac{w_p}{w_n} * \frac{L_n}{L_p}}}{1 + \sqrt{\frac{1}{1} * \frac{w_p}{w_n} * \frac{L_n}{L_p}}} = 2.5\ V$$

$\rightarrow \frac{w_p}{w_n} = \frac{1}{4}, \quad L = 1\mu$

b) $t_{HL} = t_{LH} \rightarrow \frac{w_p}{w_n} = \frac{1}{4}, \quad L = 1\mu$

Problem 21:

$$C_{ref} = 4 * W_{min} L_{min} C_{ox} = 4 * .5 * .5 * 2f = 2fF$$

$$F_{I1} = 5 * \frac{3+3}{4}$$

$$F_{I2} = \frac{1}{5}\left(\frac{(1+1)}{4} + \frac{30f}{2f} + 3 + \frac{2}{4}\right)$$

$$F_{I3} = \frac{1}{2}\left(\frac{6(3+2)}{4} + \frac{3(1+1)}{4}\right)$$

$$F_{I4} = \frac{1}{6}\left(\frac{(9+1)}{4} + \frac{1+1}{4}\right)$$

$$F_{I5} = \frac{\frac{5}{2}(3+2)}{4}$$

$$F_{I6} = \frac{2}{5}\left(\frac{100f}{2f}\right)$$

$$t_{prop} = t_{ref} * \sum_{i=1}^{6} F_{Ii} = 39.425 * t_{ref}$$