Homework 1 - Solutions

0. **(5 points)** Log in to Piazza and post something. You can post a question (or an answer) about a lecture or homework problem, or even simply introduce yourself to the rest of the class.

1. **(5 points)** Determine whether or not the following expression is a tautology by writing down its truth table:

   Solution

   $p \implies (q \wedge \neg(p \implies q))$

p	q	$\neg p$	$\neg q$	$p \implies q$	$q \wedge \neg(p \implies q)$	$p \implies (q \wedge \neg(p \implies q))$
T	T	F	F	T	F	F
T	F	F	T	F	F	F
F	T	T	F	T	F	T
F	F	T	T	T	F	T

   Here from the truth table, we can easily see that the given expression assumes 'False' (F) values for some values of p and q. Thus, **the given expression is not a tautology.**

2. **(10 points)** The NAND logical connective $\uparrow$ is defined by the truth table written below. Using this definition, show the following (using any technique you like):

a. $p \uparrow q$ is logically equivalent to $\neg(p \wedge q)$. (This explains the name "NAND", i.e., "NOT AND").

b. $(p \uparrow q) \uparrow (p \uparrow q)$ is logically equivalent to $p \wedge q$.

c. $(p \uparrow p) \uparrow (q \uparrow q)$ is logically equivalent to $p \vee q$.

d. $(p \uparrow p)$ is logically equivalent to $\neg p$.

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

(The last 3 properties shows that it is possible to build the other fundamental logical connectives entirely out of NAND gates.)

Solution

Using Truth table,

p	q	$p \wedge q$	$p \vee q$	$p \uparrow q$	$\neg(p \wedge q)$	$(p \uparrow q) \uparrow (p \uparrow q)$	$(p \uparrow p) \uparrow (q \uparrow q)$	$(p \uparrow p)$
T	T	T	T	F	F	T	T	F
T	F	F	T	T	T	F	T	F
F	T	F	T	T	T	F	T	T
F	F	F	F	T	T	F	F	T

Here, comparing the columns of the truth table corrosponding to the different expressions, we can deduce that each expression (a,b, c and d) is valid.

3. **(10 points)** Suppose Ava and Bob are students in a class. Curiously, the following events always happen:

a. If Ava is late, then Bob is late as well.
b. If both Ava and Bob are late, then class ends early.

Suppose that class does not end early. What can you conclude about Ava?

Provide your reasoning using propositional logic by (i) clearly defining the various propositions, and (ii) writing down a truth table, and (iii) coming to a logical conclusion.

Solution

(i) defining $p :$ Ava is late $q :$ Bob is late $r :$ Class ends early

So, implications a and b are, $a : p \implies q$ $b : (p \wedge q) \implies r$

(ii) Truth table :

Below is the complete truth table, but it is specified in the question that events a and b always happen. It means we need to consider only those cases in the truth table for which $p \implies q$ and $(p \wedge q) \implies r$ both are true. Which happens in the rows *1,5,6,7 and 8* .

no.	p	q	r	$p \implies q$	$p \wedge q$	$(p \wedge q) \implies r$
1	**T**	**T**	**T**	**T**	**T**	**T**
2	T	T	F	T	T	F
3	T	F	T	F	F	T
4	T	F	F	F	F	T
5	**F**	**T**	**T**	**T**	**F**	**T**
6	***F***	**T**	***F***	**T**	**F**	**T**
7	**F**	**F**	**T**	**T**	**F**	**T**
8	***F***	**F**	***F***	**T**	**F**	**T**

(iii) Now, in the given case, Class does not end early. It means truth value of r is **F**. Which occurs in row numbers *6 and 8*. Examining truth value of p in *6 and 8*, it is **F** in both the columns.

Therefore, we can surely say that Ava is not late when class does not end early.

4. **(10 points)** Suppose you are taking CprE 310. Let *p, q, r* denote propositional variables as below. Write down the symbolic expressions of sentences (a) to (d) using the propositional variables *p, q, r* .

p: "You get an A on the final exam"
q: "You do every exercise in the textbook"
r: "You get an A overall"

Solution

a. You get an A on the final, you do every exercise in the book, and you get an A overall.
   - $(p \wedge q \wedge r)$
b. You do not get an A on the final exam, but you get an A overall.
   - $\neg p \wedge r$
c. To get an A in the class, it is necessary for you to get an A on the final.
   - $\neg p \implies \neg r$
d. To get an A in the class, it is sufficient for you to get an A on the final exam.
   - $(p \implies r)$
e. You getting an A on the final exam is a necessary and sufficient condition for you getting an A overall.
   - $p \Leftrightarrow r$

5. **(10 points)** Use the contrapositive to rewrite the following statements in if-then form in two ways.

   Solution

   a. Being divisible by 3 is a necessary condition for a number to be divisible by 9.
      - If a number is divisible by 9, then it is divisible by 3.
      - If a number is not divisible by 3, then it is not divisible by 9.
   b. Doing her homework regularly is a necessary condition for Jane to pass the course.
      - If Jane wants to pass the course, then she needs to do homework regularly.
      - If Jane does not do homework regularly, she will not pass the course.
   c. A sufficient condition for Jack's team to win the championship is that it wins the rest of its games.
      - If Jack's team wins the rest of its games, then they will win the championship.
      - If Jack's team does not win the championship, then they did not win the rest of its games.