

Homework 3

Com S 311

Due: Monday, September 30, 11:59 pm

5% bonus for submission by Sunday, September 29, 11:59 pm

No Late submissions accepted

There are 3 problems and each problem is worth 40 points.

For problems 2 and 3, you must use the *divide and conquer* paradigm. Note that a typical design and conquer algorithm consists of 3 steps.

- Dividing the problem into sub problems
- Solving sub problems recursively.
- Combining solutions of sub problems.

Your algorithm must clearly and precisely describe each of these three steps. In addition, you must derive a recurrence relation for the runtime of the algorithm and solve the recurrence relation. (If you do not use divide and conquer, you will receive zero credit.)

1. Derive a solution to each of the following recurrence relations, using the technique of "unrolling" the recurrence as shown in sections 5.1 and 5.2 in the text¹ (in particular, see the subsections "Unrolling the Mergesort recurrence" on p. 212 and "The case of $q > 2$ subproblems" on p. 215). You must show your work, e.g. as on p. 216 of the text, but you are not required to provide a proof by induction of correctness (though you may wish to do so, in order to check your result).
 - (a) $T(n) \leq 3T(n/2) + cn^2$, $T(2) \leq c$.
 - (b) $T(n) \leq 2T(n/2) + cn \log n$. $T(2) \leq c$.
2. An array $A = \{a_1, a_2, \dots, a_n\}$ of integers is defined to be *k-sorted* if $A[i] \leq A[i+k]$ for every i in the range $[1, n-k]$. Give a divide and conquer algorithm that takes a *k-sorted* array of size n as input, and sorts it. Express the run-time as a function of n and k .

¹The required textbook for the course is *Algorithm Design* by Kleinberg and Tardos. There is a copy available on reserve at Parks library; ask at the circulation desk.

3. Let S be a set of two-dimensional points. Assume that all x -coordinates are distinct and all y -coordinates are distinct. A point $\langle x, y \rangle \in S$ is *purple* if there exists a point $\langle p, q \rangle$ in S such that $x < p$ and $y < q$. Give a divide and conquer algorithm that gets a set of points as input and outputs all purple points.

GUIDE LINES:

- Please write your recitation number, time and TA name.
- You must work on the homework problems on your own. You should write the final solutions alone, without consulting any one. Your writing should demonstrate that you understand the proofs completely.
- When proofs are required, you should make them clear and rigorous.
- Any concerns about grading should be made within one week of returning the homework.
- **Please submit your HW via Canvas. If you type your solutions, then please submit pdf version. If you hand-write your solutions, then please scan your solutions and submit a pdf version. Please make sure that the quality of the scan is good, and your hand writing is legible.**
- If your hand writing is not legible or the quality of the scan is poor, your homework will not be graded.