Homework 4

Please scan and upload your assignments to Canvas on or before March 6, 2018.

- You must do your work independently and on your own. That means no collaborations!
- However, you *can* ask questions about the homework on Piazza. You can also answer others' questions. It is possible that your question is already answered there, so check Piazza regularly.
- Scores on late submissions will be penalized by 50% for every day submitted late. Be on time!

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1. **(10 points)** Prove the *distributive law* for any three sets A, B, C:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

   You can use any method of proof. For example, for a formal logic proof you might want to consider an element $x \in A \cup (B \cap C)$ and construct a chain of logical deductions to show that x also belongs to $(A \cup B) \cap (A \cup C)$. Or you could use Venn diagrams.

2. **(10 points)** Let E be the set of even integers and O be the set of odd integers. Define a function:

$$f : E \times O \to \mathbb{Z}$$

   such that $f(x, y) = xy$. Is f one-to-one? Is f onto? For either question, if your answer is yes, then prove it; if not, then provide a counterexample.

3. **(10 points)** Let $f : A \to B$ and $g : B \to C$ be functions. Let $h : A \to C$ be their composition, i.e., $h(a) = g(f(a))$.
   (a) Prove that if f and g are surjections, then so is h.
   (b) Prove that if f and g are bijections then so is h.

4. **(10 points)** Consider an n-player round robin tournament where every pair of players play each other exactly once. Assume that there are no ties and every game has a winner. Then, the tournament can be represented via a directed graph with n nodes where the edge $x \to y$ means that x *has beaten* y in their game.
   (a) Explain why the tournament graph does not have cycles (loops) of size 1 or 2.
   (b) We can interpret this graph in terms of a relation where the domain of discourse is the set of n players. Explain whether the "beats" relation for any given tournament is always/sometimes/never:
      (i) asymmetric
      (ii) reflexive
      (iii) irreflexive
      (iv) transitive.

5. **(10 points)** Let W be the set of all words in the sentence, "The sky above the port was the color of television, tuned to a dead channel." Define a relation R on W as follows: for any words $w_1, w_2 \in W$, $w_1 R w_2$ if the first letter of w_1 is the same as the first letter of w_2 without regard to upper or lower cases.

(a) Prove that R is an equivalence relation.

(b) Enumerate all possible equivalence classes in R. (As per lecture, any equivalence class is the set of all elements in W that are related to each other via R.)