Practice Problem Set 2 Solution

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1. For each of the following propositions:

$\forall x, \exists y, 2x - y = 0$

$\forall x, \exists y, 2y - x = 0$

determine which propositions are true when the domain of discourse is specified as:

- the nonnegative integers.
- the integers.
- the real numbers.

Solution

- When x and y are nonnegative integers :

  $\forall x, \exists y, 2x - y = 0$ is *true*. This is because for all nonnegative integers *there exists* $2x = y$.

  $\forall x, \exists y, 2y - x = 0$ is *false*. Counter example : 3 is a nonnegative integer. *There exists no* nonnegative integer such that $3 = 2y$.

- When x and y are integers :

  $\forall x, \exists y, 2x - y = 0$ is *true*. This is because for all integers *there exists* another integer y such that $2x = y$.

  $\forall x, \exists y, 2y - x = 0$ is *false*. Counter example : -3 is a integer. *There exists no* integer such that $-3 = 2y$.

- When x and y real numbers :

  $\forall x, \exists y, 2x - y = 0$ is *true*. This is because for all real numbers *there exists* $2x = y$.

  $\forall x, \exists y, 2y - x = 0$ is *true*. This is because for all real numbers *there exists* $y = x/2$.

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2. Let $Q(x, y)$ be the statement:

   x is a member of the current US Olympic National Team for sport y.

Let the domain of discourse for x be the set of students at ISU, and the domain of discourse for y be the set of Olympic sports.

Which of the following expressions are equivalent to the statement:

"No student at ISU is a member of the current US Olympic team for any sport."

Clearly state your reasoning.

- $\forall x, \forall y, \neg Q(x, y)$
- $\exists x, \exists y, \neg Q(x, y)$

- $\neg(\forall x, \forall y, Q(x, y))$
- $\neg(\exists x, \exists y, Q(x, y))$

Solution

The first statement $\forall x, \forall y, \neg Q(x, y)$ and the fourth statement $\neg(\exists x, \exists y, Q(x, y))$ is equivalent to the statement "No student at ISU is a member of the current US Olympic team for any sport".

Explanation:

$\forall x, \forall y, \neg Q(x, y)$

This expression translates to : *for all* students in ISU and *for all* Olympic sport the given statement $Q(x, y)$ is *false*. Hence this is equivalent to saying that "No student at ISU is a member of the current US Olympic team for any sport."

$\exists x, \exists y, \neg Q(x, y)$

This expression translates to *there exists* at least one student in ISU and *there exists* at least one Olympic sport for which the given statement $Q(x, y)$ is *false*. This expression still leaves room for some students to be a mamber of the US Olympic sport team.

$\neg(\exists x, \exists y, Q(x, y))$

If we consider the part inside the bracket this is what it says : *There exists a* student in ISU and *there exists a* sport in US Olympic team for which the given statement $Q(x, y)$ would be *true*. Now the negation of this statement would be: *There exists no* student in ISU and *there exists no* sport in US Olympic team for which the given statement $Q(x, y)$ would be *true*. This is equivalent to saying: *For all* students in ISU and *for all* sports in the US National Olympic team, the given statement $Q(x, y)$ is *false*. And hence this is equivalent to the statement : "No student at ISU is a member of the current US Olympic team for any sport."

$\neg(\forall x, \forall y, Q(x, y))$

If we consider the part inside the bracket this is what is says: *For all* students in ISU and *for all* sports in the US National Olympic team, the given statement $Q(x, y)$ is *true*. Now the negation of the statement would be : *There exists a* student in ISU and *there exists a* sport in US Olympic team for which the given statement $Q(x, y)$ would be *false*. Which again is not equivalent to the given statement.

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3. You write a software program but find out that it is buggy. There are 4 possible causes for the bug - undeclared variables, syntax errors within the first five lines, missing semicolons, or misspelled variable names. You do some basic debugging, and discover the following information:

- There is an undeclared variable or there is a syntax error in the first five lines.
- If there is a syntax error in the first five lines, then there is a missing semicolon or a variable name is misspelled.
- There is not a missing semicolon.
- There is not a misspelled variable name.

Using rules of inference, find out which of the sources of error caused the bug.

Solution

Define the following propositions:

- p: There is an undeclared variable in the first five lines.
- q: There is a syntax error in the first five lines.
- r: There is a missing semicolon.
- s: There is a variable name is misspelled.

Then, the premises are:

a. $p \vee q$

b. $q \implies (r \vee s)$

c. $\neg r$

d. $\neg s$

(Conjunction):-

$$\begin{array}{c} \neg r \\ \neg s \\ \hline \therefore \neg r \wedge \neg s \equiv \neg(r \vee s) \end{array}$$

(Modus tollens):-

$$\begin{array}{l} \quad\neg(r \vee s) \\ q \implies (r \vee s) \\ \hline \therefore \neg q \end{array}$$

(Disjunctive syllogism):-

$$\begin{array}{l} \neg q \\ \quad p \vee q \\ \hline \therefore p \end{array}$$

Hence, there is an undeclared variable in the first five lines.

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4. Use the rules of inference studied in class to deduce the conclusion from the hypotheses:

$$\begin{array}{l} p \vee q \\ q \implies r \\ p \wedge s \implies t \\ \neg r \\ \neg q \implies u \wedge s \\ \hline \therefore t \end{array}$$

Solution

(Modus tollens):

$$\begin{array}{l} \neg r \\ \quad q \implies r \\ \hline \therefore \neg q \end{array}$$

(Modus ponens):

$$\begin{array}{l} \neg q \\ \neg q \implies u \wedge s \\ \hline \therefore u \wedge s \end{array}$$

(Simplification):

$$\begin{array}{r} u \wedge s \\ \hline \therefore s \end{array}$$

(Disjunctive syllogism):

$$\begin{array}{l} \neg q \\ p \wedge q \\ \hline \therefore p \end{array}$$

(Conjunction):

$$\begin{array}{l} p \\ s \\ \hline \therefore p \wedge s \end{array}$$

(Modus ponens):

$$\begin{array}{l} p \wedge s \\ p \wedge s \implies t \\ \hline \therefore t \end{array}$$