ComS 472
Homework 4

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- 7.22 -

1) If the pair of clauses has no complimentary literals, there are no resolvents. ✓

   If the pair has one or more sets of complimentary literals, the resulting resolvents acquired from applying the same set of literals in any order will eventually reduce down to a single resolvent. ✓

2) A clause resolved with itself would contain complimentary literals left in the equation. As this clause does not contain any, it is impossible.

3) For a clause to resolve with a copy of itself, it must contain only complimentary literals. This would make the initial clause equivalent to True

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1)

F	P	D	$F \Rightarrow P$	$D \Rightarrow P$	$(F \Rightarrow P) \vee (D \Rightarrow P)$	$F \wedge D$	$(F \wedge D) \Rightarrow P$	$(F \Rightarrow P) \vee (D \Rightarrow P) \Rightarrow (F \wedge D) \Rightarrow P$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	F	F	F	T	F	T
T	F	F	F	T	T	F	T	T
F	T	T	T	T	T	F	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	F	T	F	T	T
F	F	F	T	T	T	F	T	T

The sentence is valid as it is true for all combinations of variables.

2) Original $(F \Rightarrow P) \vee (D \Rightarrow P) \Rightarrow (F \wedge D) \Rightarrow P$

Implication Elim: $(\neg F \vee P) \vee (D \Rightarrow P) \Rightarrow (F \wedge D) \Rightarrow P$
Implication Elim: $(\neg F \vee P) \vee (\neg D \vee P) \Rightarrow (F \wedge D) \Rightarrow P$
Implication Elim: $(\neg F \vee P) \vee (\neg D \vee P) \Rightarrow \neg(F \wedge D) \vee P$
De Morgan: $(\neg F \vee P) \vee (\neg D \vee P) \Rightarrow (\neg F \vee \neg D) \vee P$
Implication Elim: $\neg((\neg F \vee P) \vee (\neg D \vee P)) \vee (\neg F \vee \neg D) \vee P$
De Morgan: $\neg(\neg F \vee P) \wedge \neg(\neg D \vee P) \vee (\neg F \vee \neg D) \vee P$
De Morgan: $(F \wedge \neg P) \wedge (D \wedge \neg P) \vee (\neg F \vee \neg D) \vee P$
Associativity: $(F \wedge \neg P \wedge D \wedge \neg P) \vee (\neg F \vee \neg D \vee P)$
Duplicates: $(F \wedge \neg P \wedge D) \vee (\neg F \vee \neg D \vee P)$

Final Form (CNF): $\mathbf{(F \wedge \neg P \wedge D) \vee (\neg F \vee \neg D \vee P)}$

3)

F	P	D	$\neg F$	$\neg P$	$\neg D$	$F \wedge \neg P \wedge D$	$\neg F \vee P \vee \neg D$	$F \wedge \neg P \wedge D \vee \neg F \vee P \vee \neg D$
T	T	T	F	F	F	F	T	T
T	T	F	F	F	T	F	T	T
T	F	T	F	T	F	T	F	T
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	F	T	T
F	T	F	T	F	T	F	T	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	T	F	T	T

The resolved sentence is logically equivalent to the original.

- 7.26 -

S1) $A \Leftrightarrow (C \lor E)$ to...
$(A \Rightarrow (C \lor E)) \land ((C \lor E) \Rightarrow A)$
$(\neg A \lor (C \lor E)) \land (\neg(C \lor E) \lor A)$
$(\neg A \lor C \lor E) \land ((\neg C \land \neg E) \lor A)$
$(\neg A \lor C \lor E) \land (\neg C \lor A) \land (\neg E \lor A)$

S2) $E \Rightarrow D$ to...
$\neg E \lor D$

S3) $B \land F \Rightarrow \neg C$ to...
$\neg(B \land F) \lor \neg C$
$\neg B \lor \neg F \lor \neg C$

S4) $E \Rightarrow C$ to...
$\neg E \lor C$

S5) $C \Rightarrow F$ to...
$\neg C \lor F$

S6) $C \Rightarrow B$ to...
$\neg C \lor B$

- 8.11 -

1) $\text{Occupation}(\text{Emily}, \text{Surgeon}) \lor \text{Occupation}(\text{Emily}, \text{Lawyer})$
2) $\text{Occupation}(\text{Joe}, \text{Actor}) \land \exists\, j\ (\text{Occupation}(\text{Joe}, j) \land \neg(j{=}\text{Actor}))$
3) $\forall\, s\ (\text{Occupation}(s, \text{Surgeon}) \Rightarrow \text{Occupation}(s, \text{Doctor}))$
4) $\forall\, l\ (\text{Occupation}(l, \text{Lawyer}) \Rightarrow \neg\text{Customer}(\text{Joe}, l))$
5) $\exists\, b\ (\text{Boss}(b, \text{Emily}) \land \text{Occupation}(b, \text{Lawyer}))$
6) $\exists\, l\ \forall\, c\ (\text{Occupation}(l, \text{Lawyer}) \land (\text{Customer}(c, l) \Rightarrow \text{Occupation}(c, \text{Doctor})))$
7) $\forall\, s\ \exists\, l\ (\text{Occupation}(s, \text{Surgeon}) \Rightarrow (\text{Customer}(s, l) \land \text{Occupation}(l, \text{Lawyer})))$

- 8.23 -

1) $\exists$ d Parent(Joan, d) $\land$ Female(d)

2) $\exists!$ d Parent(Joan, d) $\land$ Female(d)

3) ($\exists!$ d Parent(Joan, d)) $\land$ ($\forall$ d Parent(Joan, d) $\Rightarrow$ Female(d))

4) $\exists!$ c Parent(Joan, c) $\land$ Parent(Kevin, c)

5) ($\exists$ c Parent(Joan, c) $\land$ Parent(Kevin, c)) $\land$ $\neg$($\exists$ c Parent(Joan, c) $\land$ $\neg$Parent(Kevin, c))

- 8.29 -

1) This is a good translation

2) This is not a good translation, as it doesn't specify only 1 apartment
   $\exists!$ a Apt(a) $\land$ In(a, Paris) $\land$ (Rent(a) $<$ Dollars(1000))

3) This is a good translation

- 9.4 -

1) P(A, B, B), P(A, B, z) : {x/A, y/B, z/B}

2) Does not exist.

3) Older(Father(y), y), Older(Father(x), John) : {x/John, y/John}

4) Does not exist.

- 9.7 -

1) $\forall$ m (Horse(m) $\lor$ Cow(m) $\lor$ Pig(m)) $\Rightarrow$ Mammal(m)

2) $\forall$ h Horse(h) $\land$ ($\forall$ o Offspring(o, h) $\Rightarrow$ Horse(o))

3) Horse(Bluebeard)

4) Parent(Bluebeard, Charlie)

5) $\forall$ p $\forall$ o Offspring(o, p) $\Leftrightarrow$ Parent(p, o)

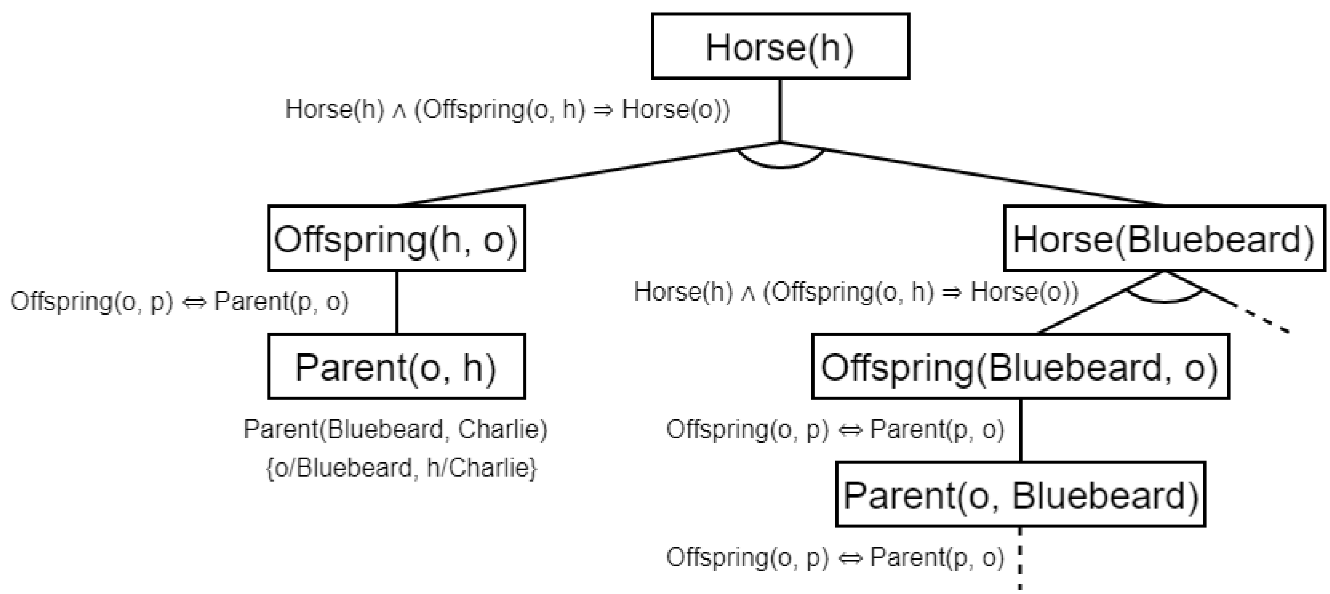
6) $\forall$ m $\exists$ p Parent(p, m)

- 9.9 -

1) False
2) True
3) True
4) False
5) False

- 9.16 -

1)



2) The domain extends forever, and because of that there are two infinite loops.
3) 2 solutions, Charlie and Bluebeard.

- 9.18 -

1)

P(A, [1, 2, 3])	Goal
P(1, [1—2, 3])	Solution with A=1
P(1, [1—2, 3])	
P(2, [2, 3])	Solution with A=2
P(2, [2, 3])	
P(3, [3])	Solution with A=3
P(3, [3])	

2)

P(2, [1, A, 3])	Goal
P(2, [1—2, 3])	
P(2, [1—2, 3])	
P(2, [2, 3])	Solution with A=2
P(2, [2, 3])	
P(3, [3])	
P(3, [3])	