

CIS 554: Conversion to Clause Form

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These rules are adapted from *Artificial Intelligence*, by Elaine Rich and Kevin Knight.

1. Eliminate logical implications, \Rightarrow , using the fact that $A \Rightarrow B$ is equivalent to $\neg A \vee B$.
2. Reduce the scope of each negation to a single term, using the following facts:

$$\neg(\neg P) = P$$

$$\neg(A \vee B) = \neg A \wedge \neg B$$

$$\neg(A \wedge B) = \neg A \vee \neg B$$

$$\neg \forall x: P(x) = \exists x: \neg P(x)$$

$$\neg \exists x: P(x) = \forall x: \neg P(x)$$

3. Standardize variables so that each quantifier binds a unique variable.
4. Move all quantifiers to the left, maintaining their order.
5. Eliminate existential quantifiers, using Skolem functions (functions of the preceding universally quantified variables). Examples:

$$\exists x: P(x) \text{ becomes } x'$$

$$\forall x: \exists y: P(y) \text{ becomes } \forall x: y'(x)$$

6. Drop the prefix; assume universal quantification.
Note: The term *prefix* refers to all the quantifiers; the *matrix* is everything else.
7. Convert the matrix into a conjunction of disjuncts.
8. Create a separate clause corresponding to each conjunct.
9. Standardize apart the variables in the clauses.