MATH 267 Section ___   **Final Exam**   12/13/2017
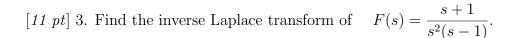
SHOW ALL YOUR WORK to avoid loss of points.

[*10 pt*] 1. Find the general solution of $x^2 y' + 2y = x e^{2/x}, \quad x > 0.$

[*10 pt*] 2. Find the solution of the initial value problem

$$y'' - 2y' + 2y = 0, \qquad y(0) = 2, \quad y'(0) = 1.$$

[*11 pt*] 3. Find the inverse Laplace transform of $\quad F(s) = \dfrac{s+1}{s^2(s-1)}$.

[*12 pt*] 4. Determine which of the following equations is/are exact and find the general solution of the exact one(s).

a) $(1 + 2y^2x)dx + (4x^2y)dy = 0$
b) $\dfrac{dy}{dx} = \dfrac{\cos y - 1/x}{x \sin y + 1/y}$
c) $\dfrac{dy}{dx} = \dfrac{x^2 + 2x - \ln|y|}{\arctan y}$

[*11 pt*] 5. Find the general solution of the equation $y'' - y = e^x - 2\sin x$, using the general theory of linear equations and the method of undetermined coefficients.

[*13 pt*] 6. Find the solution of the initial value problem

$$\vec{X}' = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \vec{X}, \qquad \vec{X}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

[*12 pt*] 7. Solve the IVP (using variation of parameters)

$$\vec{X}' = \begin{pmatrix} -1 & 3 \\ 4 & -2 \end{pmatrix} \vec{X} + \begin{pmatrix} 7e^{2t} \\ 14e^{-5t} \end{pmatrix}, \qquad \vec{X}(0) = \begin{pmatrix} 4/7 \\ -3/7 \end{pmatrix}$$

[Hint]: It is known that a fundamental set is: $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}, \begin{pmatrix} 3 \\ -4 \end{pmatrix} e^{-5t} \right\}$

[*13 pt*] 8. Use the Laplace transform method to solve the initial value problem

$$y'' + y = \delta(t - 2) + f(t), \qquad y(0) = 1, \quad y'(0) = 0,$$

where $f(t)$ is equal to zero everywhere except for $0 \leq t < 1$ where it is equal to 1.

[*13 pt*] 9. Find the first 6 terms of the power series solution (about zero) for the initial value problem:

$$y'' + x^2 y = 0, \qquad y(0) = 1, \quad y'(0) = 0.$$