Cpr E 489 Spring 2020
Homework #2 Solution

1. (40 points)

Consider the 2-out-of-5 error detection code. In this code, each codeword is 5-bit long; 2 out of 5 bits are "1"s and the others are "0"s. For example, 11000 is a valid codeword, but 01110 is not.

a. (10 points) List all the valid codewords.

Answer:

Total number of codewords = $\binom{5}{2} = 10$. The codewords are:
00011 00101 01001 10001 00110 01010 10010 01100 10100 11000

b. (15 points) What fraction of errors is undetectable by this code, i.e., what is the FUE of this code? Justify your answer.

Answer:

Total number of valid errors = $2^5 - 1 = 31$. Total number of codewords = $\binom{5}{2} = 10$.
Therefore, FUE = (10 – 1) / 31 = 9/31.

c. (15 points) What fraction of *4-bit errors* is undetectable by this code, i.e., what is the FUE(M=4) of this code? Justify your answer.

Answer:

Total number of 4-bit errors $= \binom{5}{4} = 5$.
For a 4-bit error to be undetectable, it needs to flip both "1"s to "0"s, and 2 out of 3 "0"s to "1"s.
This means that the total number of undetectable 4-bit errors $= \binom{2}{2}\binom{3}{2} = 3$.
Therefore, FUE(M=4) = 3/5.

2. (60 points)

Consider a CRC code with a generator polynomial of $g(x) = x^4 + x^3 + 1$.

a. (20 points) Show step by step (using the longhand division) how to find the codeword that corresponds to information bits of 1101.

Answer:

We know: $g(x) = x^4 + x^3 + 1$

Information bits of 1101 $\Rightarrow$ $i(x) = x^3 + x^2 + 1$

$\Rightarrow$ dividend polynomial = $x^4 * i(x) = x^7 + x^6 + x^4$

Next, perform the long-hand division:

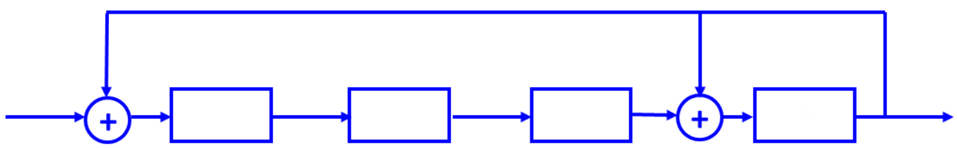
$$
\begin{array}{r|ccccc}
 & x^3 & & +\,1 & & \\ \hline
x^4 + x^3 + 1 & x^7 & +\,x^6 & +\,x^4 & & \\
 & x^7 & +\,x^6 & & +\,x^3 & \\ \cline{2-5}
 & & & x^4 & +\,x^3 & \\ \cline{4-5}
 & & & x^4 & +\,x^3 & +\,1 \\ \cline{4-6}
 & & & & & 1
\end{array}
$$

So, we have: $r(x) = 1$ $\Rightarrow$ $b(x) = x^4 * i(x) + r(x) = x^7 + x^6 + x^4 + 1$

$\Rightarrow$ codeword is (1101 0001)

b. (20 points) Show the shift-register circuit that implements this CRC code.

Answer:



c. Suppose the codeword length is 8. Answer the following questions, with proper justifications.

i. (10 points) Give an example of undetectable *error burst of length 6*;

Answer:

$$
\left.\begin{array}{r}
e(x) \text{ is an error burst of length 6} \Rightarrow e(x) = x^i(x^5 + \cdots + 1) \\
e(x) \text{ is undetectable} \Rightarrow e(x) = x^i g(x) c(x) \\
g(x) = x^4 + x^3 + 1
\end{array}\right\} \Rightarrow c(x) = x + 1
$$

Let's pick i = 1. Then, we have:

- $e(x) = x^1(x^4 + x^3 + 1)(x + 1) \Rightarrow \underline{e} = [01010110]$

ii. (10 points) Give an example of undetectable *6-bit error*.

Answer:

One such an example is $\underline{e} = [01111101]$ because:
The corresponding $e(x) = x^6 + x^5 + x^4 + x^3 + x^2 + 1 = g(x) * (x^2 + 1)$.