EE330
Homework 8
Spring 2018

Problem 1

As long as we assume forward active the left one is simply,

$$I_C = \beta I_B = 100\left(\frac{12 - 0.6}{400k}\right) = 2.85mA$$
$$V_{out} = 12 - 3000 * 0.00285 = 3.45\ V$$

The right problem has an input of 0sin(1000t) into the circuit, which is equivalent to an open circuit, so the same as the left circuit,

$$V_{out} = 3.45\ V$$

Problem 2

With β=90,

$$V_{out} = 12 - \left(1500 * \left(90 * \left(\frac{12-0.6}{400k}\right)\right)\right) = 8.15V$$

With β=120,

$$V_{out} = 12 - \left(1500 * \left(120 * \left(\frac{12 - 0.6}{400k}\right)\right)\right) = 6.87V$$

Problem 3

In the previous circuit when β=100, $V_{out} = 7.725$.
Neglecting I_B in the fixed-biasing circuit,

$$V_B = 0.6 + 0.5k * I_C = 0.6 + 500 * 2.85 * 10^{-3} = 2.025V$$
$$V_B = 12\left(\frac{R_2}{R_1 + R_2}\right) = 12\left(\frac{R_2}{40k + R_2}\right) = 2.025V$$
$$R_2 = 8.12\ k\Omega$$

Problem 4

Continuing from problem 3, if we include I_B we can write the following equation to approximate I_B

$$\frac{12 - V_B}{40K} = I_B + \frac{V_B}{8.12k}\ and\ V_B - 0.6 = (I_B + \beta I_B)500\Omega$$
$$V_{OUT} = 12 - 1500I_C$$

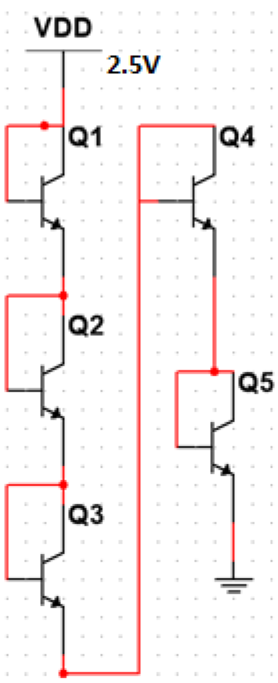
$$For\ \beta = 90$$
$$(I_B + 90I_B) * 500 = V_B - 0.6$$
$$V_B = 43000I_B + 0.6$$

$$\frac{12-43000I_B+0.6}{40000}=I_B+\frac{43000I_B+0.6}{8120}\rightarrow I_B=27.27\ \mu A$$
$I_C=\beta I_B=27.27*10^{-6}*90=2.454\ mA$
$V_{out}=12-1500*0.002454=8.32\ V$

With β=120,
$(I_B+90I_B)*500=V_B-0.6$
$V_B=60500I_B+0.6$
$$\frac{12-60500I_B+0.6}{40000}=I_B+\frac{60500I_B+0.6}{8120}\rightarrow I_B=21.19\ \mu A$$
$I_C=\beta I_B=2.54\ mA$
$V_{out}=12-1500*0.00254=8.191\ V$

Note the relatively small change in V_{out} with this compared to the much larger change from problem 6.

Problem 5



Using BJTs, like with MOSFETS we have only one degree of freedom, for BJTs this is the Area.

There is no limit to the number of BJTs and each BJT with the same Area works as a resistor with equal resistance. So 5 BJTs in series with equal area. Meaning that

$A_{E1}=A_{E2}=A_{E3}=A_{E4}=A_{E5}=100\ um^2$

Will give 1V output between third and the fourth BJT.

Problem 6

Left: Assume MOSFET is in saturation and BJT is in forward active
$V_{BE}=0.6V$
$$I=\frac{\mu_p C_{OX} W}{2L}\left(0.6-6-V_{T_P}\right)=\frac{33*10^{-6}*15}{2*10}\left(-5.4-(-1)\right)^2=0.479\ mA$$

Assuming both MOSFETs are in saturation

$$I_D = \frac{\mu_n C_{OX} W_1}{2L_1}(V_o - V_{TN})^2 = \frac{\mu_n C_{OX} W_2}{2L_2}(V_{dd} - V_o - V_{TN})^2$$

$$I_D = \frac{350 * 10^{-6} * 40}{2(5)}(V_o - 0.5)^2 = \frac{350 * 10^{-6} * 15}{2(5)}(8 - V_o - 0.5)^2 \rightarrow V_o = 3.159\,V$$

$$I_D = 9.90\,mA$$

Problem 7

For the MOSFET to be in saturation $V_{DS} \geq V_{GS} - V_T$

$$V_{out} + 2 \geq 2 - 0.5 \rightarrow V_{out} \geq -0.5$$

$$I_D = \frac{\mu_n C_{OX} W}{2L}(V_{GS} - V_T)^2 = \frac{4 - V_{out}}{R_1} \rightarrow V_{out} = 4 - 787.5 * 10^{-6} * R_1 \geq -0.5V$$

$$\rightarrow R_1 \leq 5.714k\Omega$$

Problem 8

$$\frac{R_1}{2} = 2.857k\Omega$$

$$A_V = \frac{2I_{DQ}R}{V_{SS} + V_T} = \frac{4.50\,V}{-1.5\,V} = -3$$

Problem 9

For quiescent values that capacitors act as open circuits, so the voltage is simply,

$$I_B = \frac{32 - V_B}{90\text{K}} - \frac{V_B}{10\text{K}} = \frac{32 - 10 * V_B}{90\text{K}}$$

$$I_E = (\beta + 1)I_B = (101) * \frac{32 - 10 * (V_E + 0.6)}{90\text{K}} = \frac{V_E}{1.5\text{K}} \rightarrow V_E = 2.454\text{ V} \rightarrow V_B = 3.054\text{ V}$$

$$I_C = 101 * 16.222\ \mu\text{A} \rightarrow V_C = 32 - 3000 * I_C = 27.085\text{ V}$$

$$V_{out} = 0V$$

Problem 10

$$R_{FET} = \frac{1}{\mu_n C_{OX}}\left(\frac{L}{W}\right)(2 - 1)$$

$$\frac{V_{out} - V_{in}}{R_F} = \frac{V_{in}}{R_{FET}} \rightarrow \frac{V_{out}}{V_{in}} = 1 + \frac{R_F}{R_{FET}}$$

$$\rightarrow \frac{V_{out}}{V_{in}} = 1 + \mu_n C_{OX} * \left(\frac{W}{L}\right) * R_F = 1 + \frac{7}{5000} * R_F$$

Problem 11-12

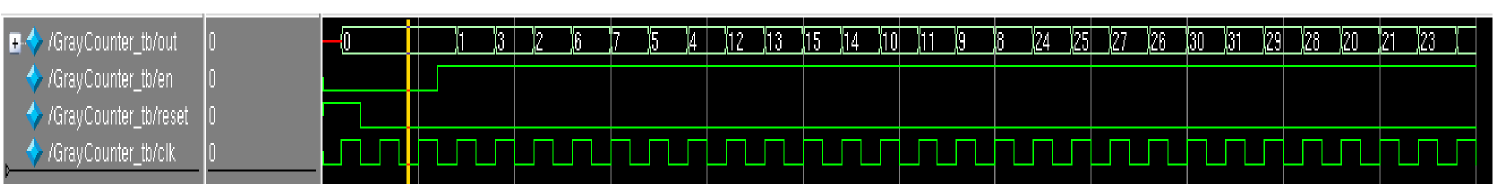
Code:

```
module GrayCounter(en, reset, clk, out);
  input en, reset, clk;
  output[7:0] out;

  wire[7:0] out;
  reg[7:0] count;

  always @(posedge clk) begin
    if(reset == 1)
      count <= 0;
    else if (en)
      count <= count +1;

  end

  assign out = {count[7], (count[7] ^ count[6]),(count[6] ^
                count[5]),(count[5] ^ count[4]), (count[4] ^
                count[3]),(count[3] ^ count[2]), (count[2] ^
                count[1]),(count[1] ^ count[0]) };

endmodule
```

Testbench:

```
`timescale 1ns/1ps

module GrayCounter_tb();
  wire[7:0] out;
  reg en, reset, clk;

  GrayCounter counter(.en(en), .reset(reset), .clk(clk), .out(out));

  initial begin
    en = 0;
    reset = 1;
    clk = 0;

    #20 reset = 0;
    #40 en = 1;
  end

  always #10 clk <= ~clk;

endmodule
```

Output:



Problem P1:

Schematic not shown.

Problem P2:

Assuming both are in Forward Active Region, starting with the left circuit

$$I_B = \frac{(8-0.6)}{600k} = 12.3\ \mu A, Using\ \beta = 100$$

$$I_C = \beta I_B = 100 * 12.3\ \mu A = 1.23\ mA$$
$$V_{out} = 8V - 4k * 0.00123 = 3.08\ V$$

The right circuit is very similar, but

$$I_C = \beta I_B = 100 * \frac{(8-0.6)}{300k} = 2.47\ mA$$
$$V_{out} = 8V - 12k * 0.00247 = -26.2\ V$$

As we can see, the circuit isn't in forward active so guess saturation

$$I_B = \frac{(8-0.6)}{300k} = 24.7\ \mu A$$

$$I_C = \frac{8-.2}{12k} = 650\ \mu A$$
$$V_{out} = 8V - 12k * 650 * 10^{-6} = 0.2\ V$$

Verify saturation region $\beta I_B = 2.47\ mA > 650\ \mu A$

Problem P3:

Assume BJT works in forward active region

$$I_B = \left(\frac{10-0.6}{500k}\right) = 18.8\mu A$$
$$I_C = \beta I_B = 100 * 18.8\mu A = 1.88mA$$
$$V_C = 10 - 4000 * 0.00188 = 2.48V$$
$$V_{out} = 0V \text{ (there is a capacitor creating an open circuit in DC.)}$$

Problem P4:

Assuming that M_1 and M_2 are in saturation

$$I_{D_1} = I_{D_2} \rightarrow \frac{\mu_n C_{OX} W_n}{2L_n}(V_{GS} - V_T)^2 = \frac{\mu_p C_{OX} W_p}{2L_p}(V_{GS} - V_T)^2$$

$$\rightarrow \frac{350 * 10^{-6} * 10}{2 * 2}(0 - (-2) - 0.5)^2 = \frac{70 * 10^{-6} * 3}{2 * 1}\left(V_{out} - 5 - (-0.5)\right)^2$$
$$\rightarrow V_{out} = 0.170V$$

Problem P5:

$$V_{out} = 12 - (12000 * i_{DQ})$$
$$I_{DQ} = 350 * 10^{-6} * \left(\frac{6}{2*4}\right) * (0 - (-2) - 1)^2$$
$$I_{DQ} = 262.5\mu A$$
$$V_{out} = 8.85V$$

Problem P6:

a)

$$I_{DQ} = 350 * 10^{-6} * \left(\frac{6}{2*3}\right)(2 - 1)^2$$
$$I_{DQ} = 350\ \mu A$$
$$V_{outq} = 4 + 100\ \mu * 20k = 11V$$

b)
When $V_{in} = 0V, V_{out1} = V_{outQ} = 11V$
When $V_{in} = 25mV, V_{out2} = V_{outQ} + \Delta V$
$$g_m = 100 * 10^{-6}\left(\frac{6}{3}\right)(1) = 200\frac{\mu A}{V}$$
$$\Delta V = (g_m * \Delta V_{in}) * 20k = 0.1V$$
$$V_{out2} = 11.1V$$

Problem P7:

a)$\frac{I_{B1}}{I_{B2}} = \frac{A_{E1}}{A_{E2}} = \frac{1}{4}$
$$I_B = I_{B1} + I_{B2} = 5\ I_{B1}$$
$$I_{IN} = I_{C1} + \beta I_B = \beta I_{B1} + 5I_{B1}$$
$$I_{B1} = I_{in}\left(\frac{1}{\beta + 5}\right) -> I_{out} = \beta I_{B2} = \beta * 4I_{B1} = I_{in}\left(\frac{4}{1 + \frac{5}{\beta}}\right)$$
Assuming that β is large $\rightarrow I_{out} = 4 * I_{in} = 4\ mA$

b)

$$\frac{I_{D1}}{I_{D2}} = \frac{\frac{W_1}{L_1}}{\frac{W_2}{L_2}} = \frac{10}{20} = \frac{1}{2}$$

$$I_{out} = 2I_{in} = 2\ mA$$

Problem P8:

$BJT: I_{out} = \frac{A_{E2}}{A_{E1}} I_{in}$

$MOSFET: I_{out} = \frac{\frac{W_2}{L_2}}{\frac{W_1}{L_1}} I_{in}$

Problem P9:

At the basics, $I_d = \mu C_{ox}\left(\frac{w}{2L}\right)\left(V_{gs} - V_T\right)^2$, and all three have the same total length and width. Because the length/width is the one degree of freedom we have to modify the MOSFET, they should behave the same.

Problem P10:

Yes, this does behave as a rectifier, but it does not work particularly well. It is “Diode Connected” and behaves as a diode, but it’s I-V curve is not as good as the standard diodes used in class but may be better than some LEDs.