Problem Set 3

Due: Monday, October 26th

Exercise 7.22

Prove each of the following assertions:

1. Every pair of propositional clauses either has no resolvents, or all their resolvents are logically equivalent.
2. There is no clause that, when resolved with itself, yields (after factoring) the clause $(\neg P \vee \neg Q)$.
3. If a propositional clause C can be resolved with a copy of itself, it must be logically equivalent to $True$.

Exercise 7.23

Consider the following sentence:
$[(Food \;\Rightarrow\; Party) \vee (Drinks \;\Rightarrow\; Party)] \;\Rightarrow\; [(Food \wedge Drinks) \;\Rightarrow\; Party]$.

1. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.
2. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).
3. Prove your answer to (a) using resolution.

Exercise 7.26 [convert-clausal-exercise]

Convert the following set of sentences to clausal form.

> *S1:* $A \;\Leftrightarrow\; (C \vee E)$.

> *S2:* $E\colon \;\Rightarrow : D$.

> *S3:* $B \wedge F\colon \;\Rightarrow : \neg C$.

> *S4:* $E\colon \;\Rightarrow : C$.

> *S5:* $C\colon \;\Rightarrow : F$.

> *S6:* $C\colon \;\Rightarrow : B$

Give a trace of the execution of DPLL on the conjunction of these clauses.

Exercise 8.11

Consider a vocabulary with the following symbols:

> $Occupation(p, o)$: *Predicate. Person p has occupation o.*

> $Customer(p1, p2)$: *Predicate. Person $p1$ is a customer of person $p2$.*

> $Boss(p1, p2)$: *Predicate. Person $p1$ is a boss of person $p2$.*

> $Doctor, Surgeon, Lawyer, Actor$: *Constants denoting occupations.*

> $Emily, Joe$: *Constants denoting people.*

Use these symbols to write the following assertions in first-order logic:

1. Emily is either a surgeon or a lawyer.
2. Joe is an actor, but he also holds another job.
3. All surgeons are doctors.
4. Joe does not have a lawyer (i.e., is not a customer of any lawyer).
5. Emily has a boss who is a lawyer.
6. There exists a lawyer all of whose customers are doctors.
7. Every surgeon has a lawyer.

Exercise 8.23

Assuming predicates $Parent(p, q)$ and $Female(p)$ and constants $Joan$ and $Kevin$, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviation $\exists^1$ to mean "there exists exactly one.")

1. Joan has a daughter (possibly more than one, and possibly sons as well).
2. Joan has exactly one daughter (but may have sons as well).
3. Joan has exactly one child, a daughter.
4. Joan and Kevin have exactly one child together.
5. Joan has at least one child with Kevin, and no children with anyone else.

Exercise 8.29

For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it.

1. Any apartment in London has lower rent than some apartments in Paris.

$$\forall x [Apt(x) \land In(x, London)] \implies \exists y ([Apt(y) \land In(y, Paris)] \implies (Rent(x) < Rent(y)))$$

1. There is exactly one apartment in Paris with rent below \$1000.

$$\exists x \, Apt(x) \land In(x, Paris) \land \forall y [Apt(y) \land In(y, Paris) \land (Rent(y) < Dollars(1000))] \implies (y = x)$$

1. If an apartment is more expensive than all apartments in London, it must be in Moscow.

$$\forall x \, Apt(x) \land [\forall y Apt(y) \land In(y, London) \land (Rent(x) > Rent(y))] \implies In(x, Moscow).$$

Exercise 9.4

For each pair of atomic sentences, give the most general unifier if it exists:

1. $P(A, B, B)$, $P(x, y, z)$.
2. $Q(y, G(A, B))$, $Q(G(x, x), y)$.
3. $Older(Father(y), y)$, $Older(Father(x), John)$.
4. $Knows(Father(y), y)$, $Knows(x, x)$.

Exercise 9.7 [fol-horses-exercise]

Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens:

1. Horses, cows, and pigs are mammals.
2. An offspring of a horse is a horse.
3. Bluebeard is a horse.
4. Bluebeard is Charlie's parent.
5. Offspring and parent are inverse relations.
6. Every mammal has a parent.

Exercise 9.9

This question considers Horn KBs, such as the following:
$$\begin{array}{l} P(F(x)) \;\Rightarrow\; P(x) \\ Q(x) \;\Rightarrow\; P(F(x)) \\ P(A) \\ Q(B) \end{array}$$
Let FC be a breadth-first forward-chaining algorithm that repeatedly adds all consequences of currently satisfied rules; let BC be a depth-first left-to-right backward-chaining algorithm that tries clauses in the order given in the KB. Which of the following are true?

1. FC will infer the literal $Q(A)$.
2. FC will infer the literal $P(B)$.
3. If FC has failed to infer a given literal, then it is not entailed by the KB.
4. BC will return $true$ given the query $P(B)$.
5. If BC does not return $true$ given a query literal, then it is not entailed by the KB.

Exercise 9.16

In this exercise, use the sentences you wrote in Exercise **fol-horses-exercise** to answer a question by using a backward-chaining algorithm.

1. Draw the proof tree generated by an exhaustive backward-chaining algorithm for the query $\exists\, h \;\; Horse(h)$, where clauses are matched in the order given.
2. What do you notice about this domain?
3. How many solutions for h actually follow from your sentences?
4. Can you think of a way to find all of them? (*Hint*: See @Smith+al:1986.)

Exercise 9.18

The following Prolog code defines a predicate P. (Remember that uppercase terms are variables, not constants, in Prolog.)

```
P(X,[X|Y]).
P(X,[Y|Z]) :- P(X,Z).
```

1. Show proof trees and solutions for the queries P(A,[2,1,3]) and P(2,[1,A,3]).
2. What standard list operation does P represent?