Problem Solving via Search

Outline

I. Problem formulation

II. Example problems

III. Search algorithms

I. Problem-Solving Agent

♣ A reflex agent cannot operate when the mapping from states to to actions becomes too large to store.

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if current percepts then action
```

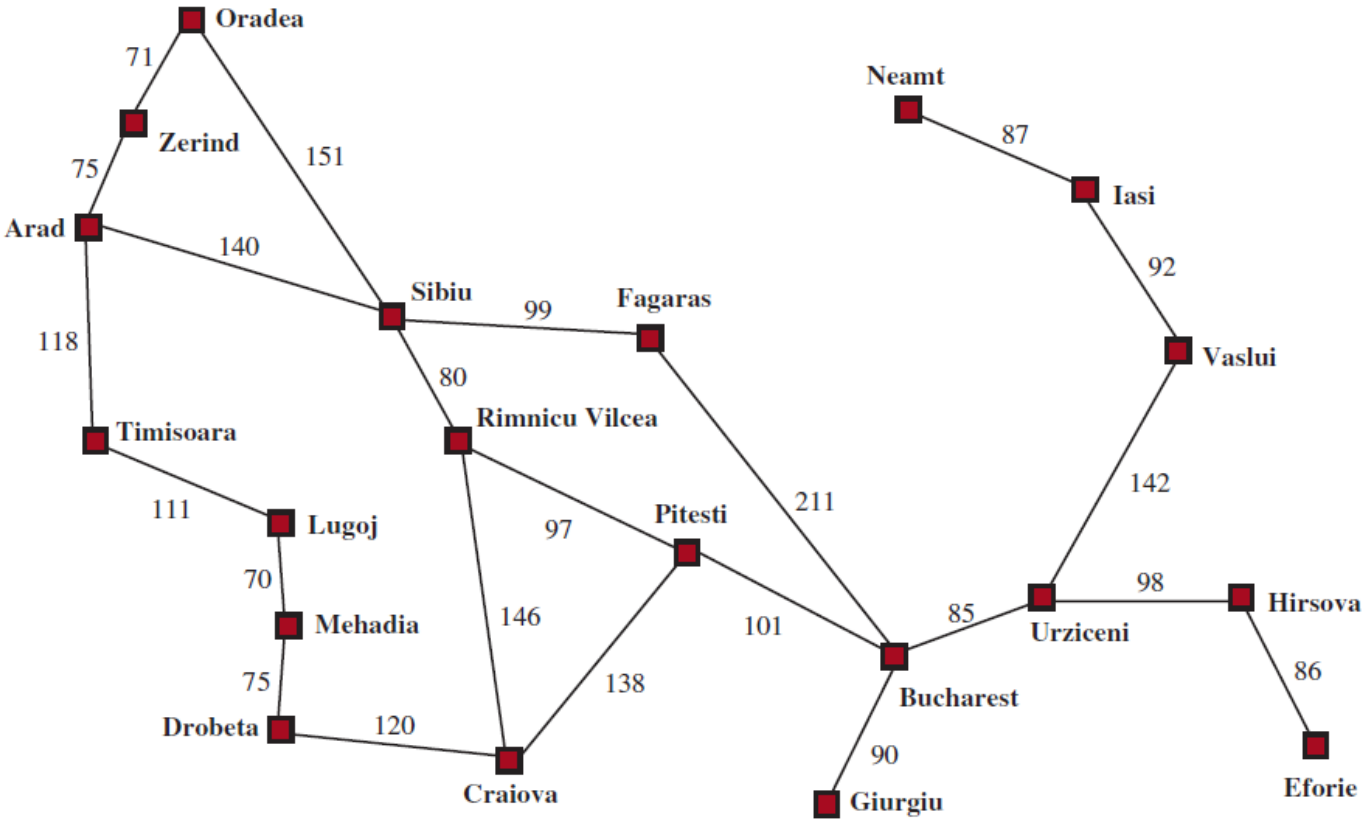
- A problem-solving agent is one kind of goal-based agent:
  - ♦ It considers states with no internal structure, i.e., in *atomic* representations.
- Problems are solved by general-purpose *search* algorithms.

1975 ACM Turing Award Lecture:

Computer science as empirical inquiry: symbols and search

Allen Newell and Herbert Simon

Sightseeing Trip in Romania



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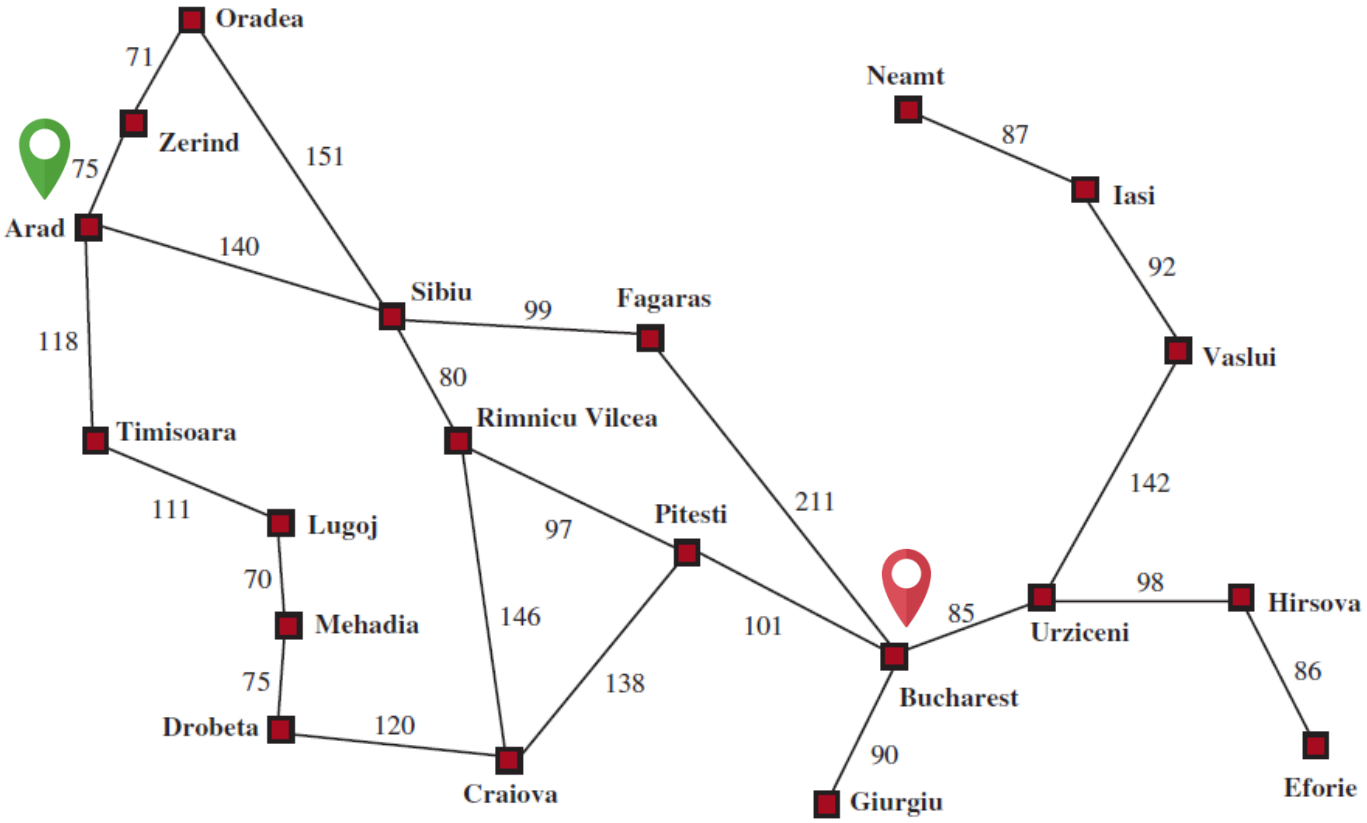

How to get to Bucharest from Arad?

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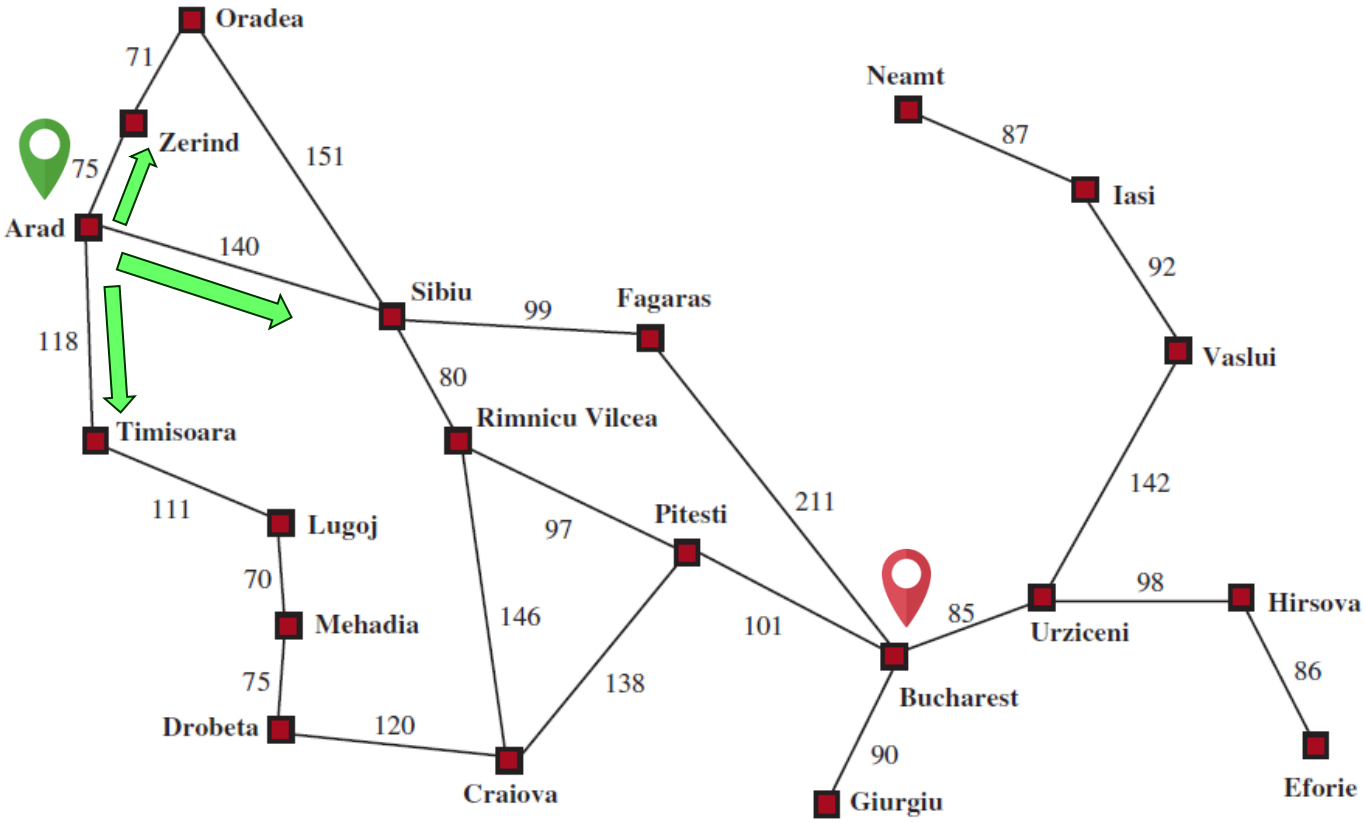

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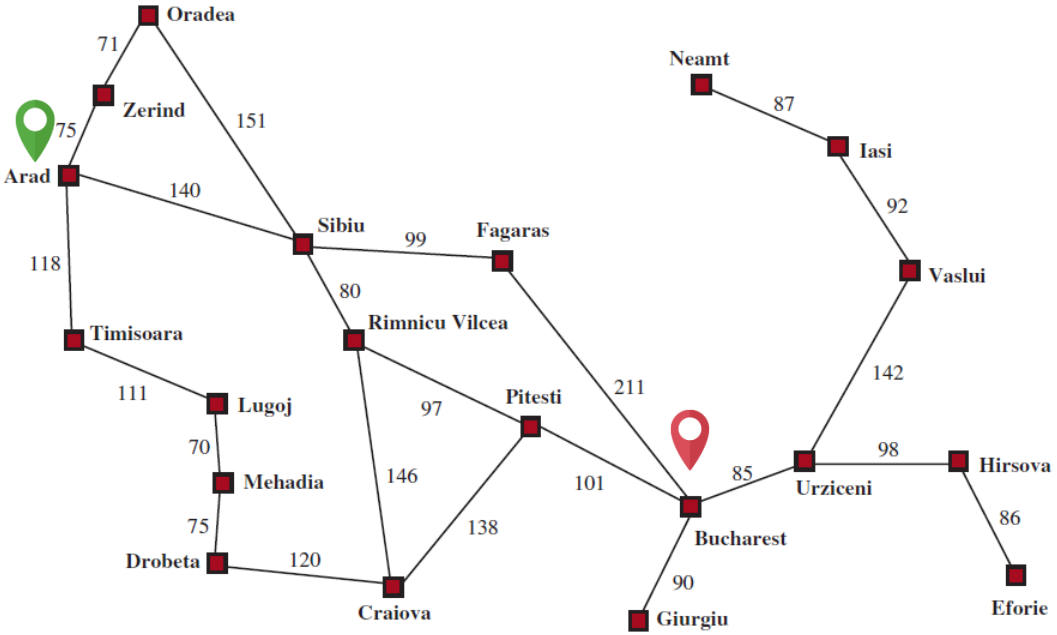
Four-Phase Problem Solving



- Goal formulation
- Problem formulation

  states: cities

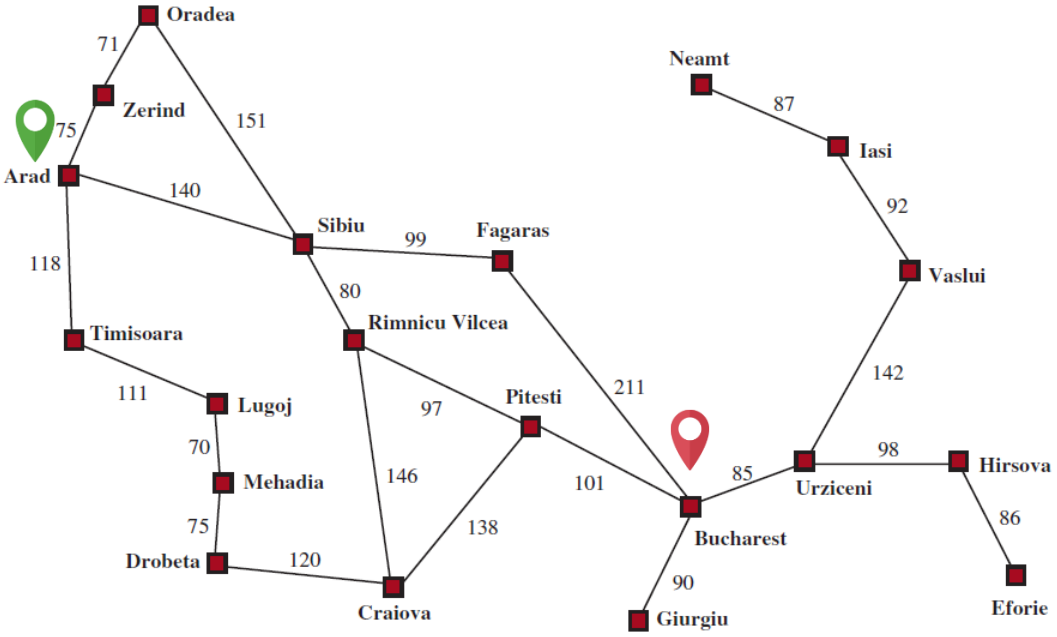
  action: travel from one city to an adjacent city
- Search

  Find a solution.

  ↑

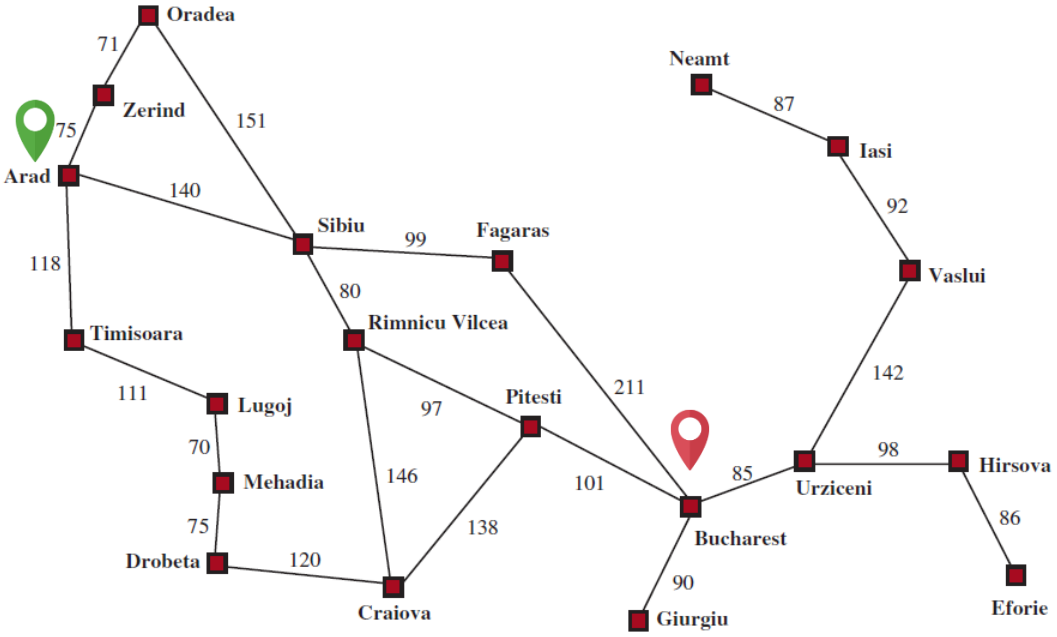
  A sequence of actions to reach the goal
- Execution

Search Problem



- State space (as a graph)

Search Problem



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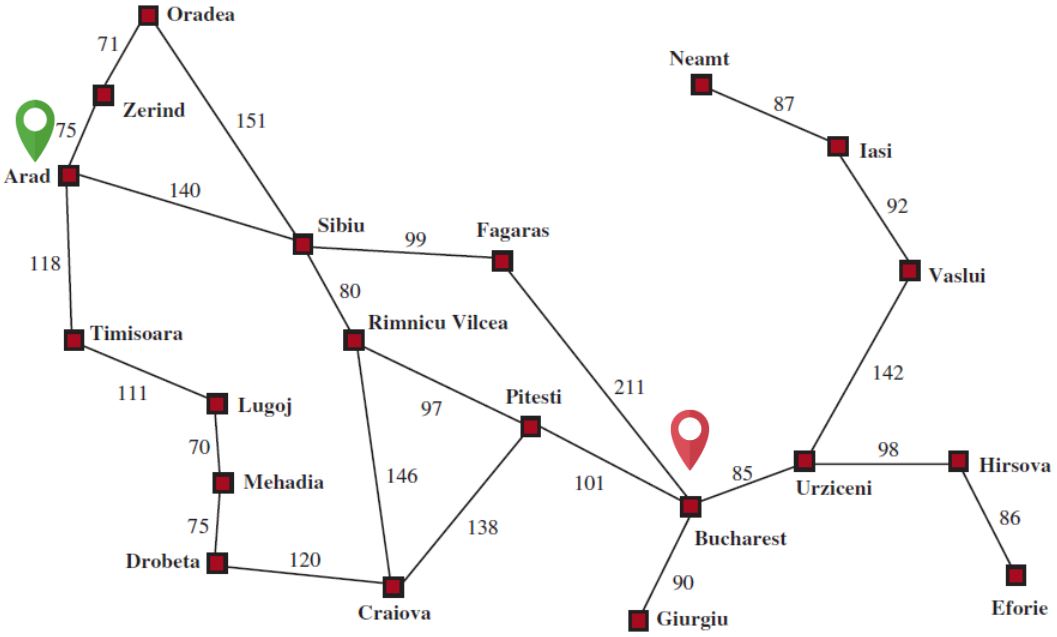

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- Goal state(s) (e.g., Bucharest)

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IS-GOAL(Fagaras)
```

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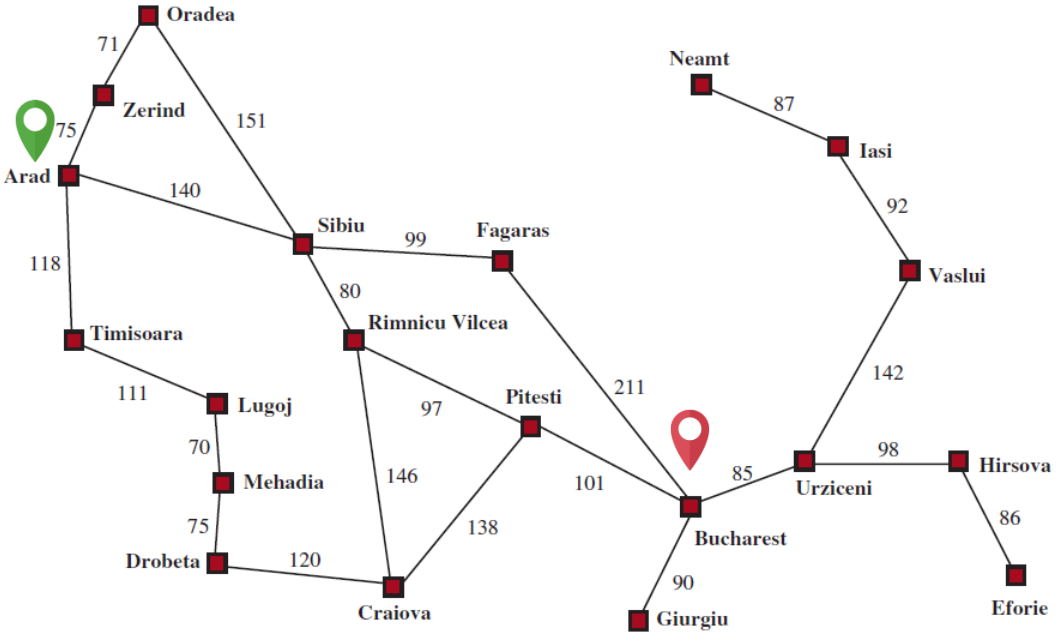

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- Actions

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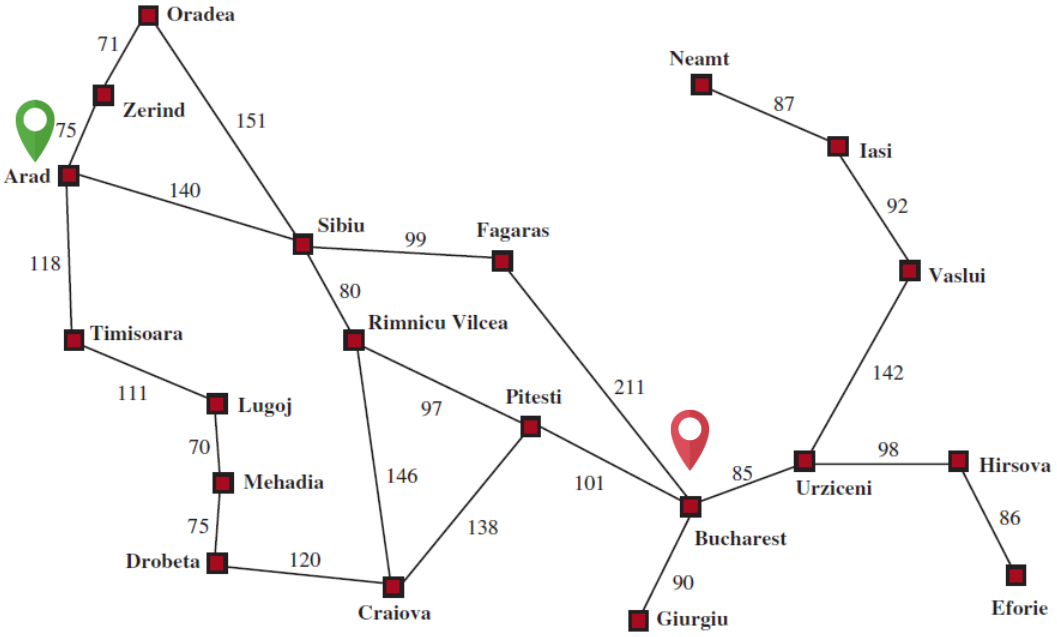

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ACTIONS(s): a finite set of actions executable at state s.

ACTIONS($Arad$) = $\{ToSibiu, ToTimisoara, ToZerind\}$

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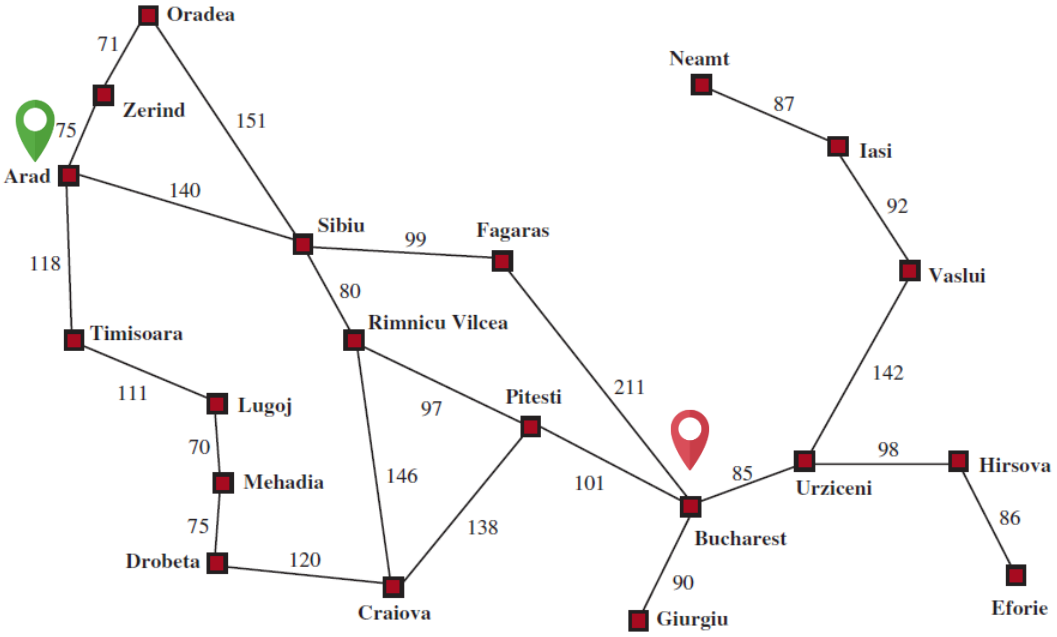

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RESULT($Arad, ToZerind$) = $Zerind$

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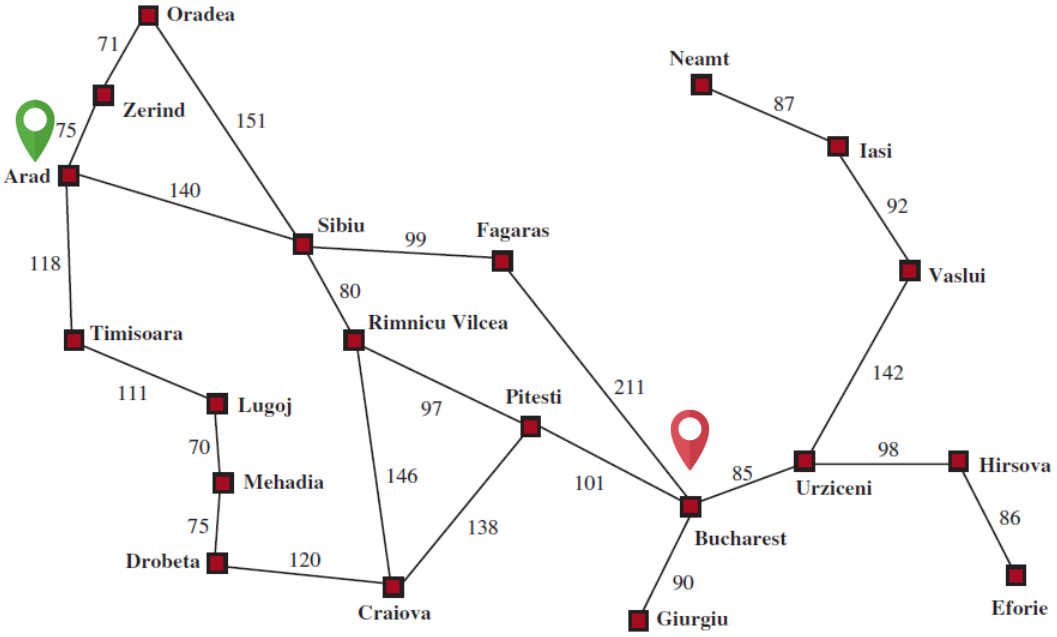

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$c(s, a, s')$: cost of applying action a in state s to reach state s'.

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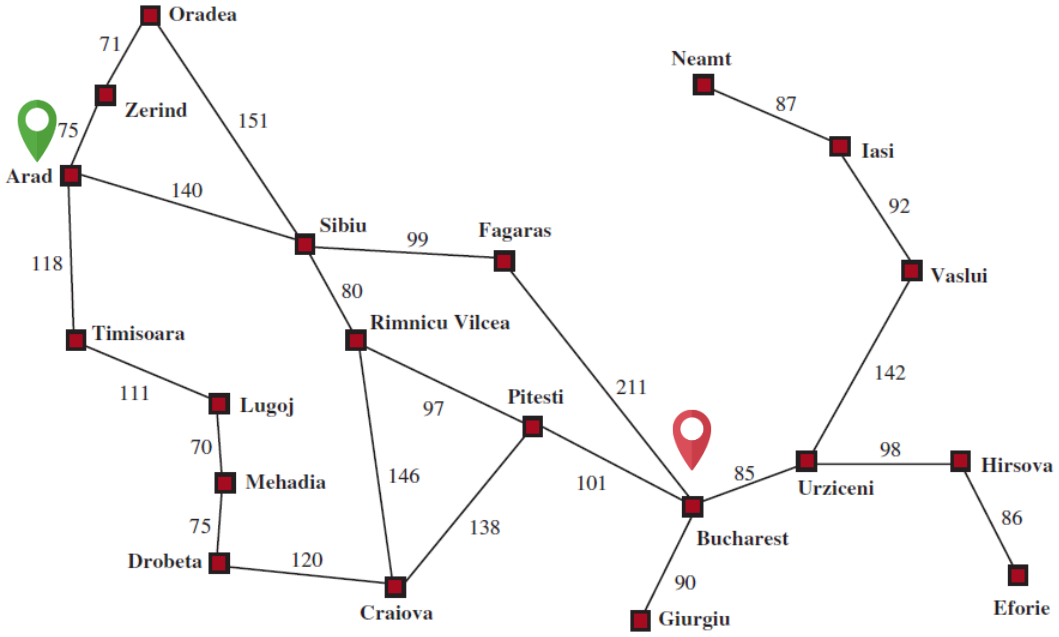

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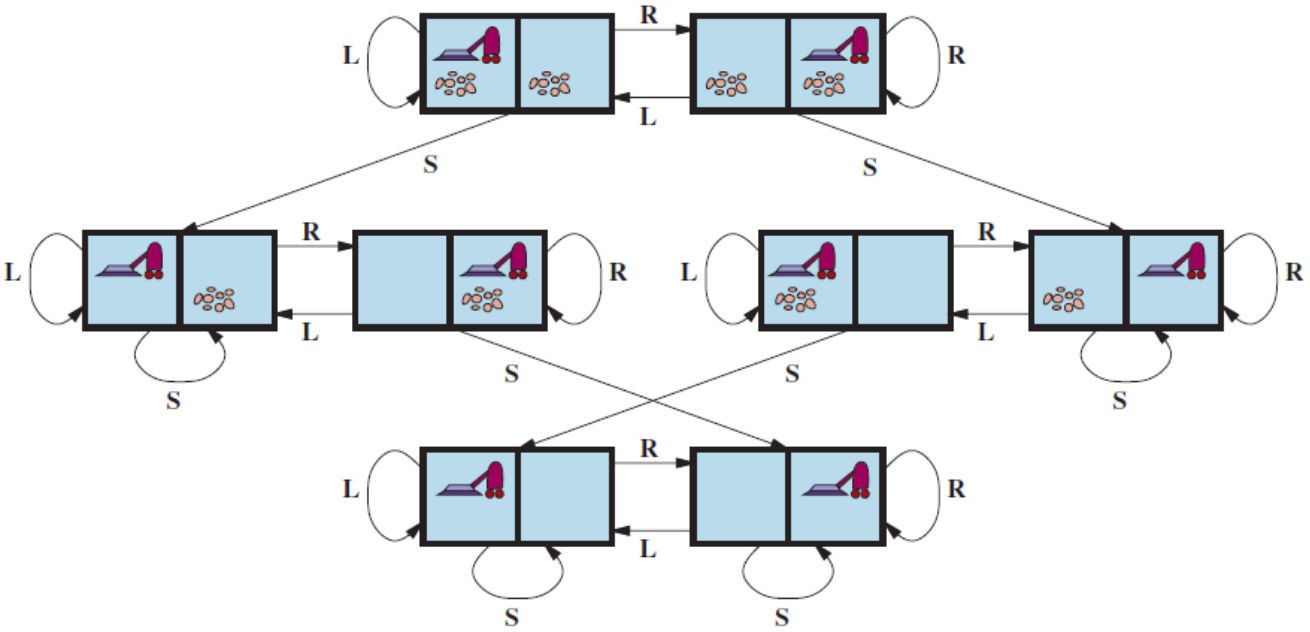
- Solution: initial state ⇝ goal state (e.g., Arad – Sibiu – Fagaras – Bucharest)

II. Example 1: Vacuum World

State-space graph:



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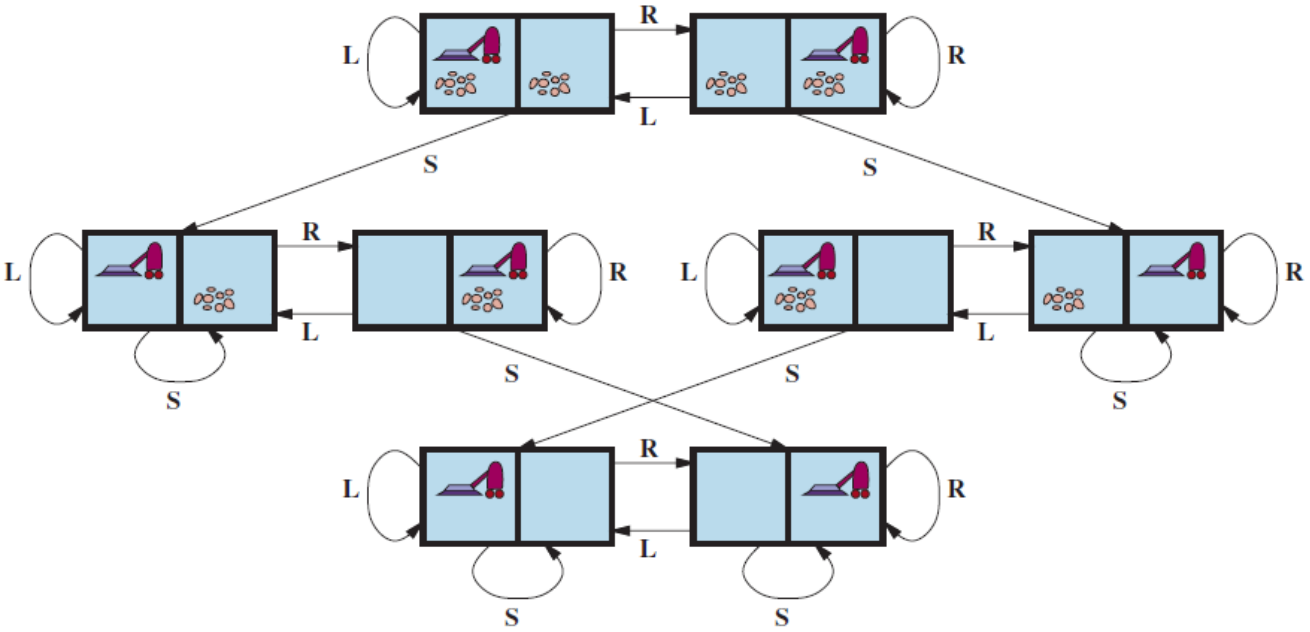

Agent in either cell

↓

2

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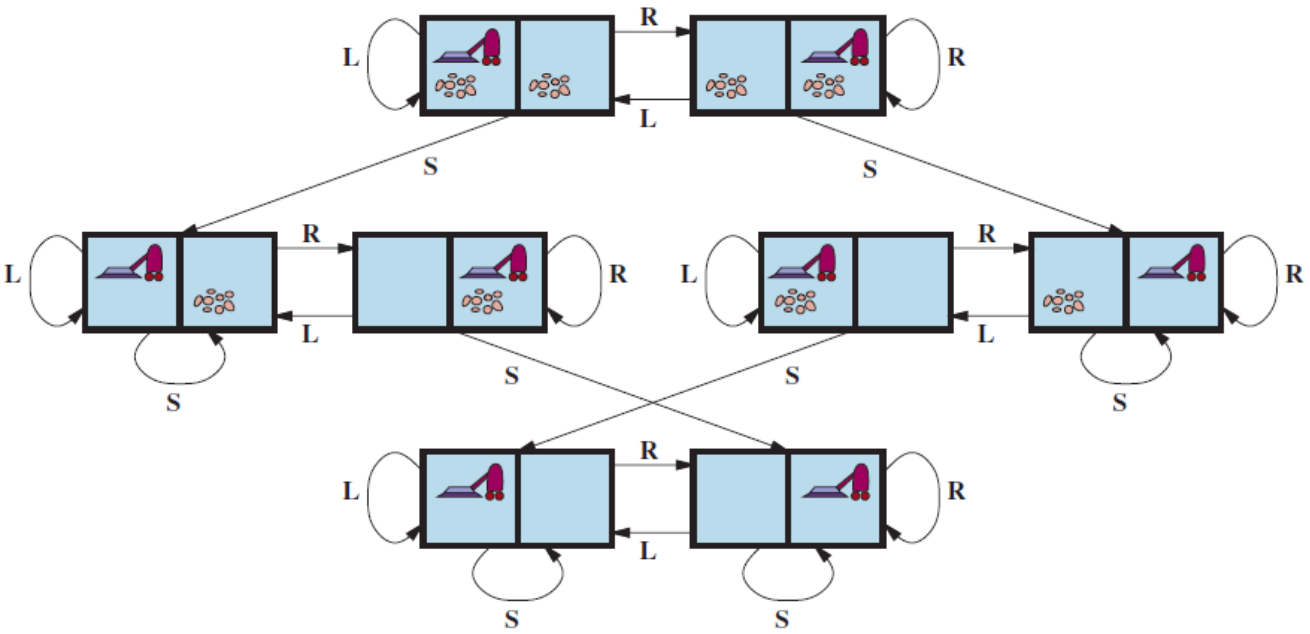

Agent in either cell

$2 \cdot 2$

Left cell has dirt or not

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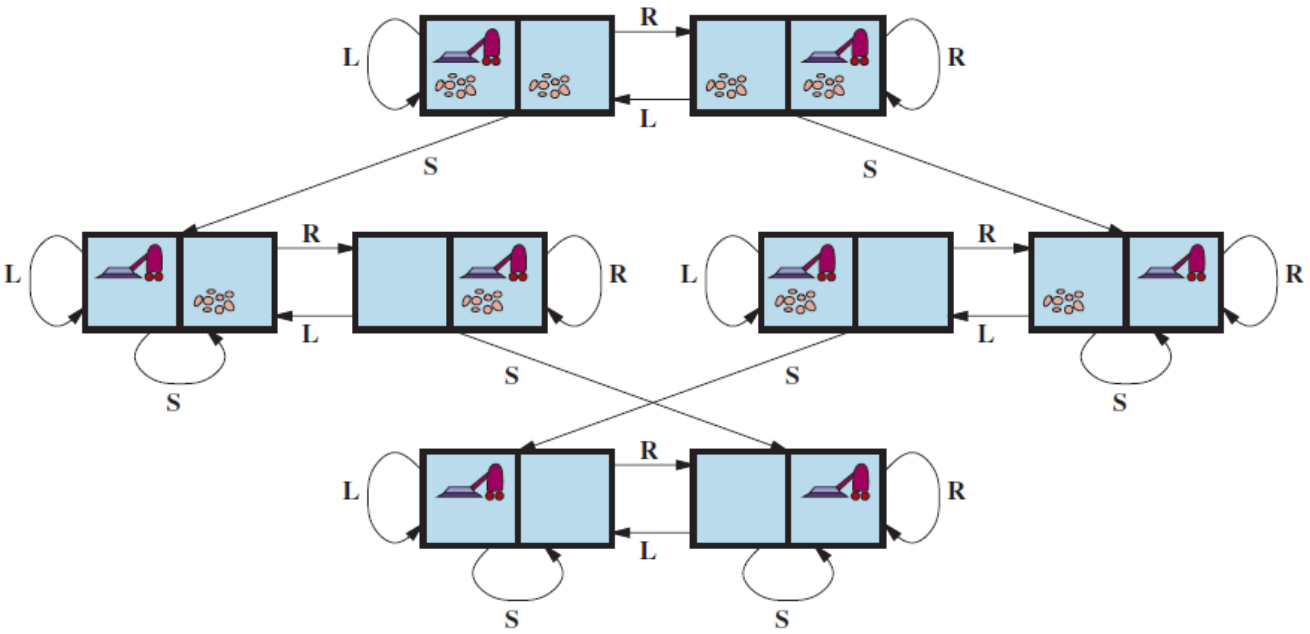

Agent in either cell

$$2 \cdot 2 \cdot 2 = 8 \text{ states}$$

Left cell has dirt or not

Right cell has dirt or not

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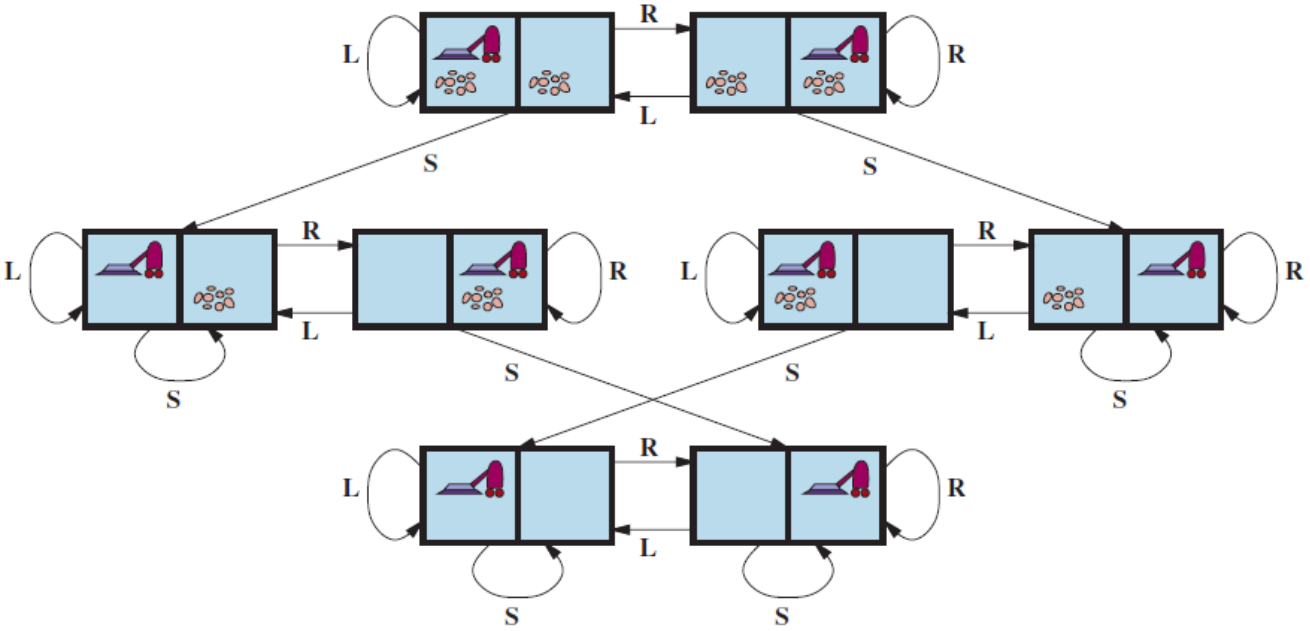

- Actions: *Suck*, *Left*, *Right*

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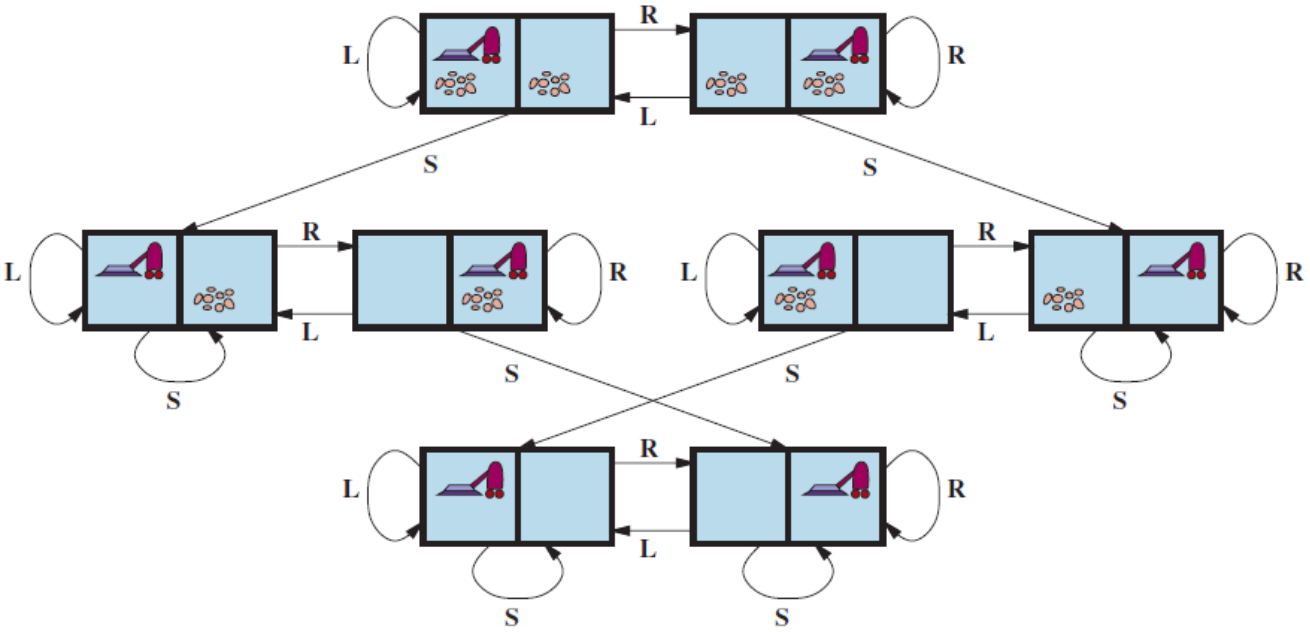

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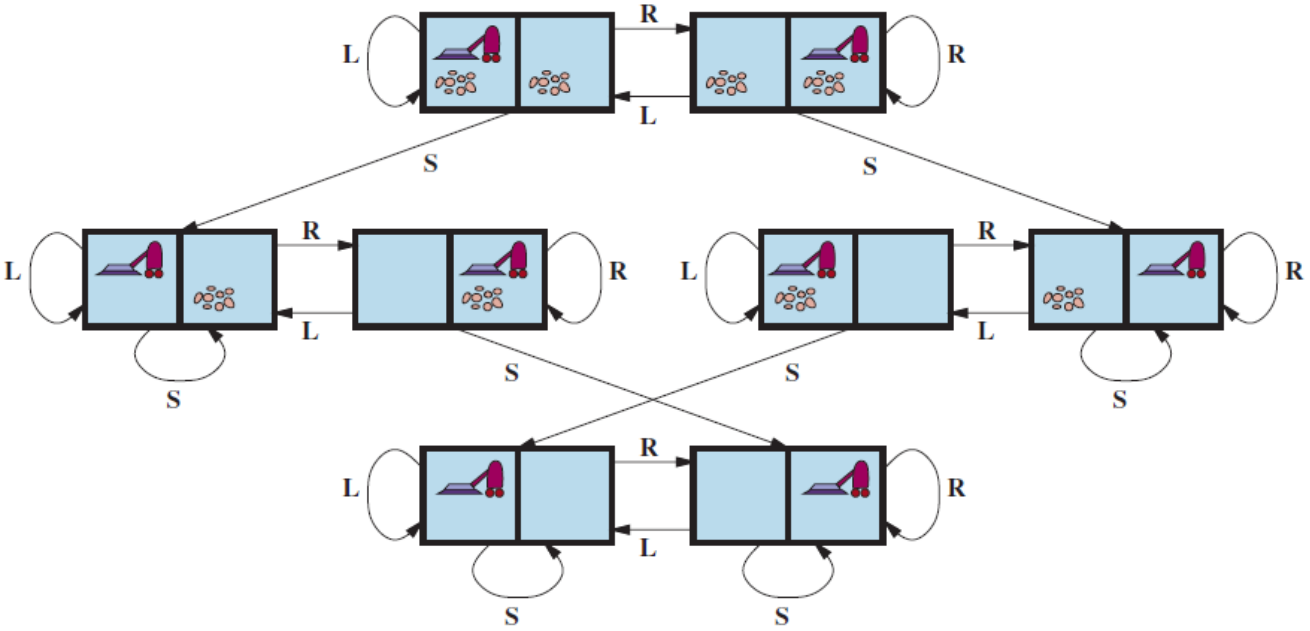

- Actions: *Suck*, *Left*, *Right*
- Goal: every cell is clean.
- Cost: 1 for each action

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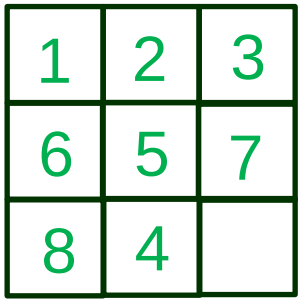
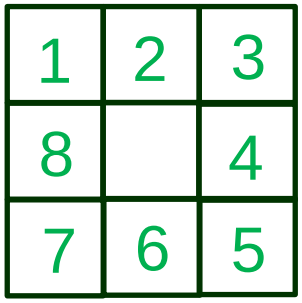
Example 2: 8-Puzzle



Initial state



Goal state

- Actions: ways of sliding a tile (adjacent to the blank space).

Left, Right, Up, Down

Applied to the
right neighbor
of the blank space

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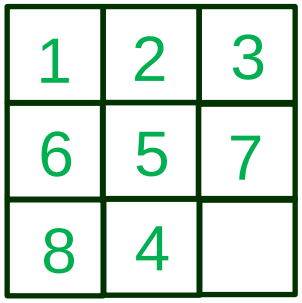

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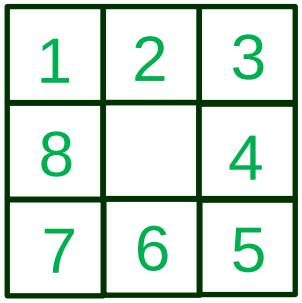

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Only two possible actions at the initial state: *Right* and *Down.*

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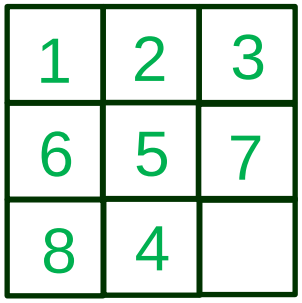

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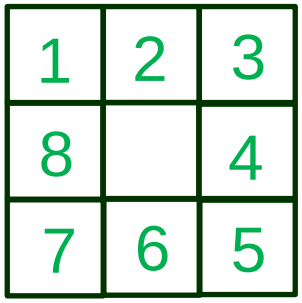

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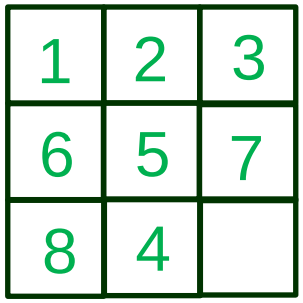
Cost 1 for each action.

Solution to an 8-Puzzle



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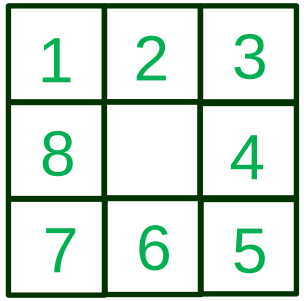

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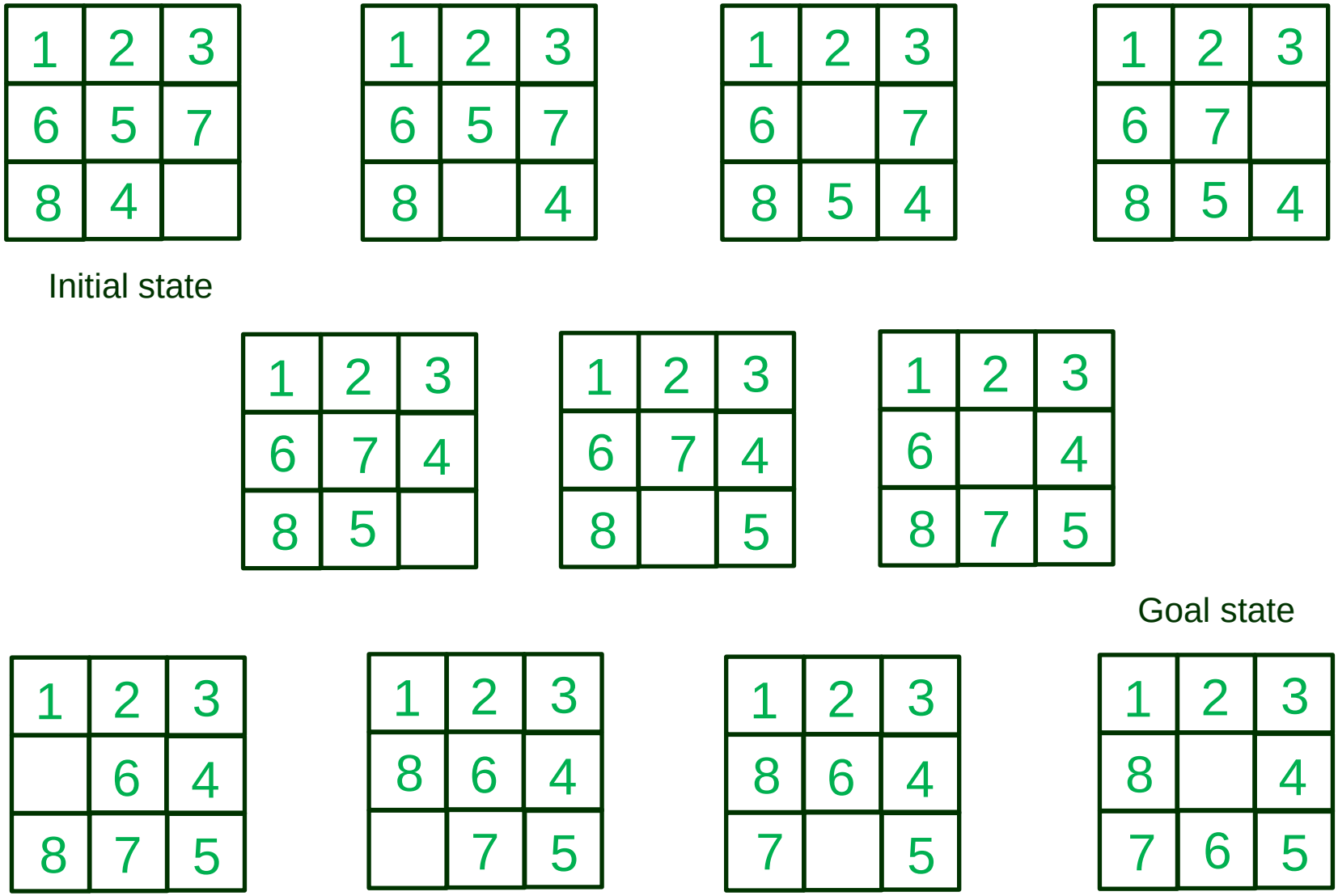
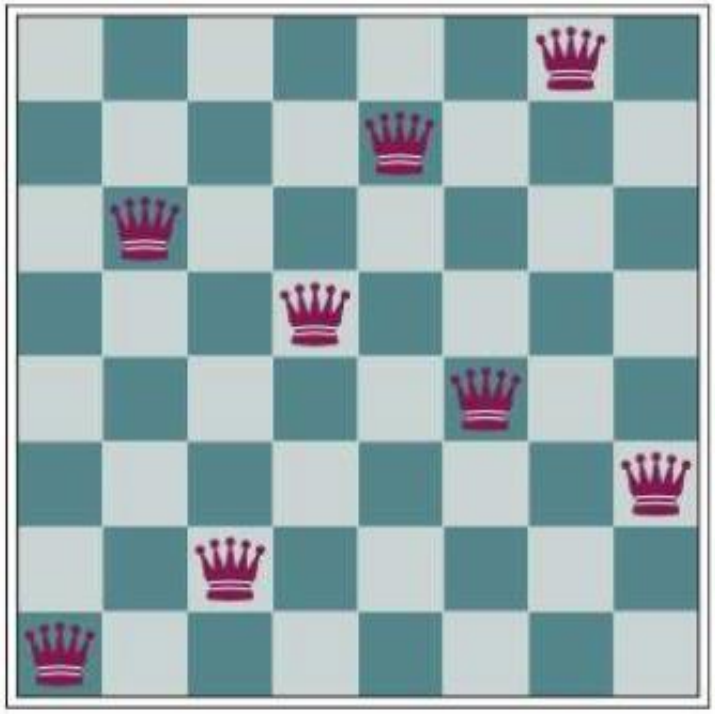


8-Queens Problem



- Goal: A placement of 8 queens on the chess board in which no queen attacks another.

  ↓

  same row, column, or diagonal

- Initial state: no queen on the board.

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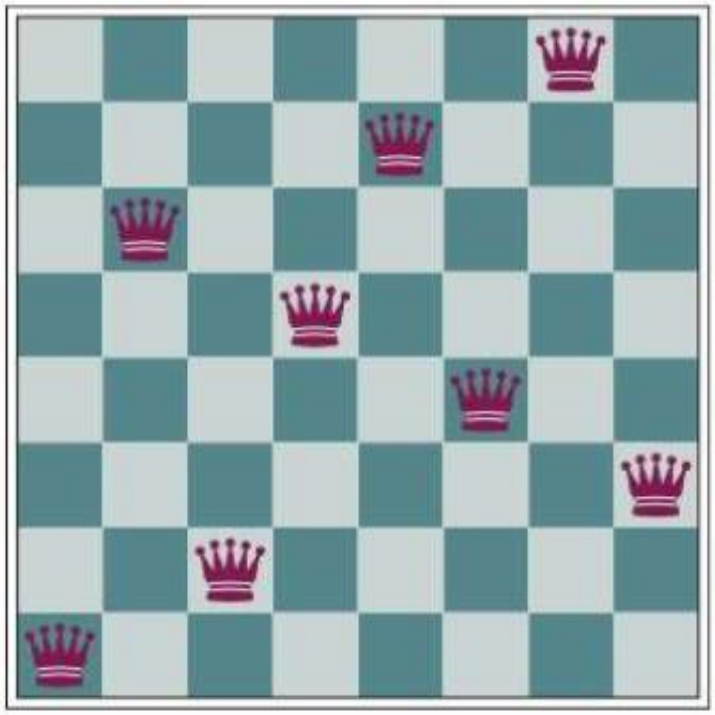

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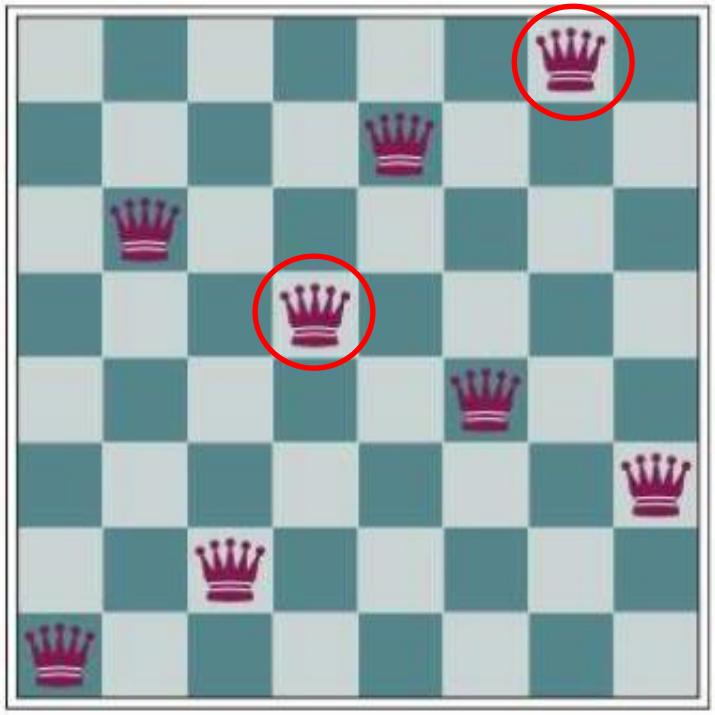

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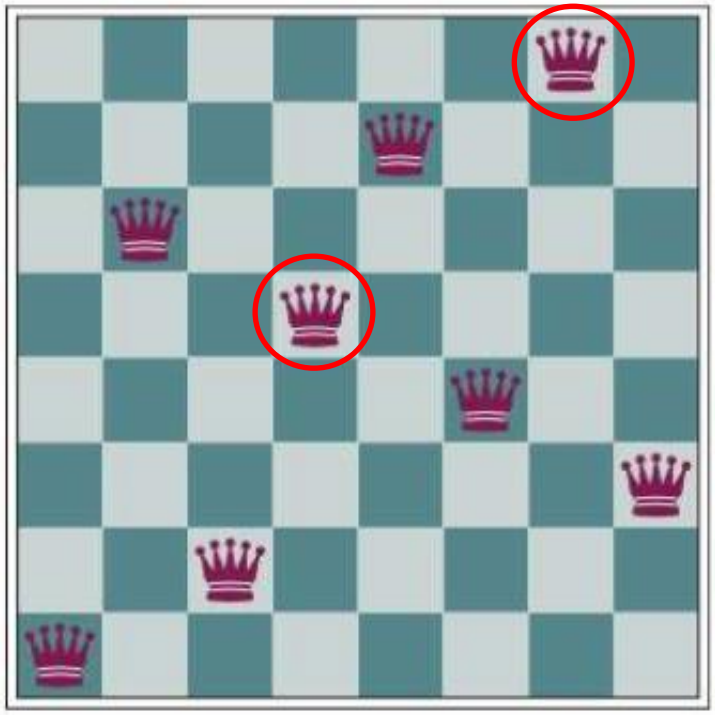

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Constraint satisfaction!

Knuth's Conjecture (1964)

Donald Knuth (Stanford)

"father of the analysis of algorithms"
ACM Turing Award (1974)
National Medal of Science (1979)

Any integer > 4 can be reached from 4 via a sequence of square root, floor, and factorial operations.

$$\left\lfloor \sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}} \right\rfloor = 5$$

- States: positive real numbers.
- Actions: square root, floor, or factorial operation.
- Action cost: 1.

* Photo from https://amturing.acm.org/byyear.cfm.

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Real-World Problems

- Route finding (e.g., from Arad to Bucharest)
- Traveling salesman problem

Round trip visiting 13,509 US cities
with population size > 500 exactly once*

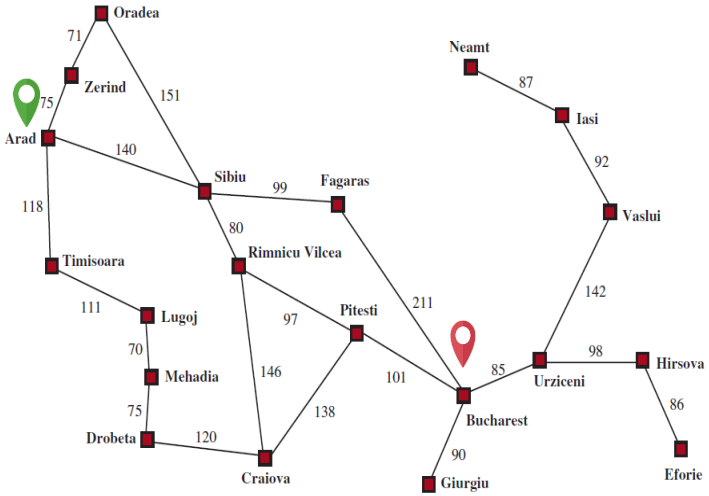
- VLSI layout

Positioning of millions of components and connections on a chip.

- Robot navigation
- Autonomous assembly sequencing (e.g., protein design)

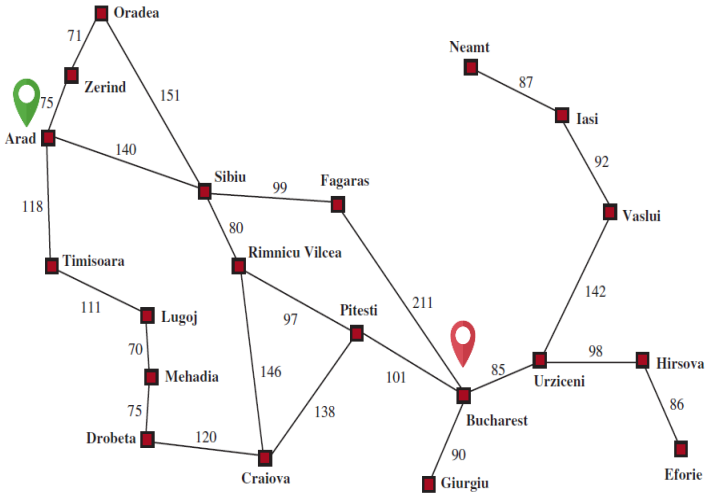
* Figure from http://www.crpc.rice.edu/CRPC/newsletters/sum98/news_tsp.html.

III. Tree Search



Superimpose a search tree over the state space graph and find a path from the initial state to the goal state.

III. Tree Search



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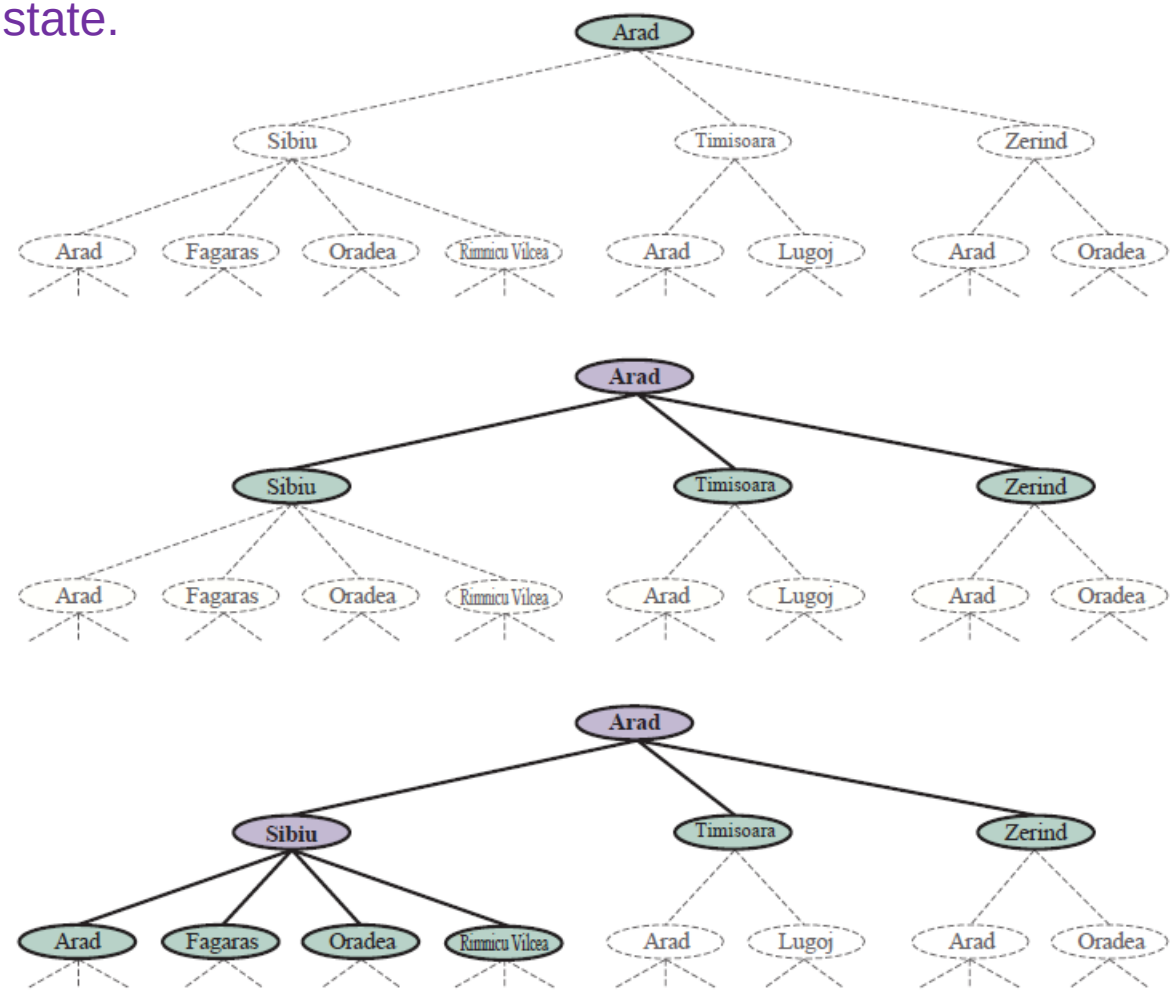

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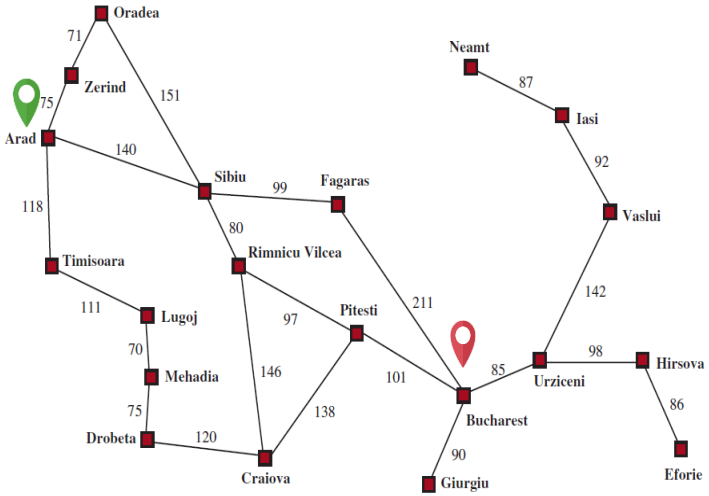

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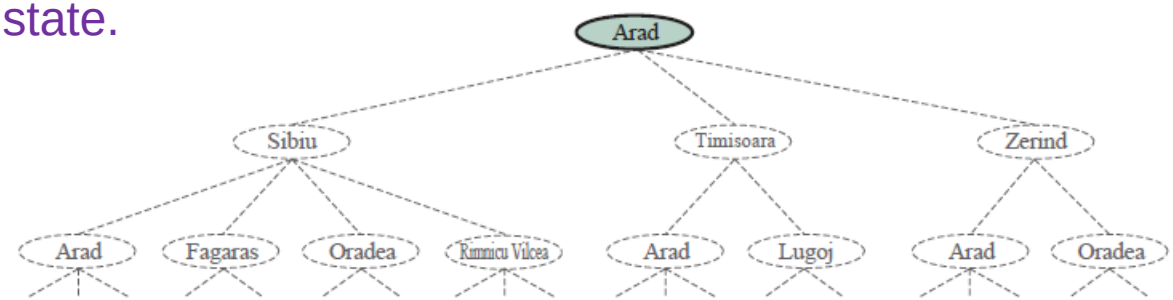
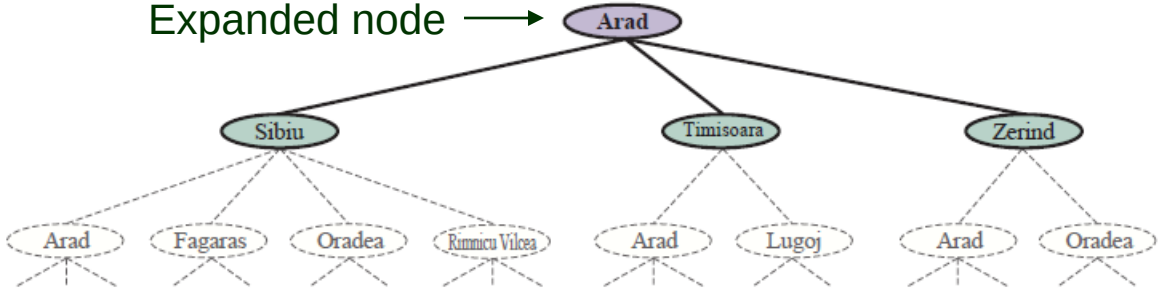
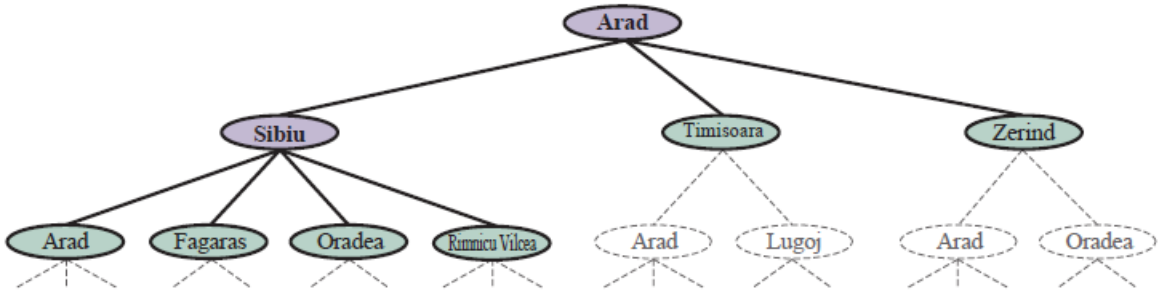


Expanded node →





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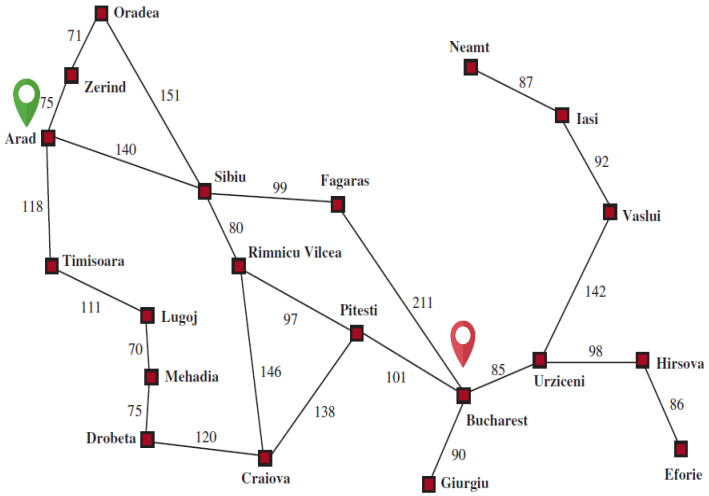

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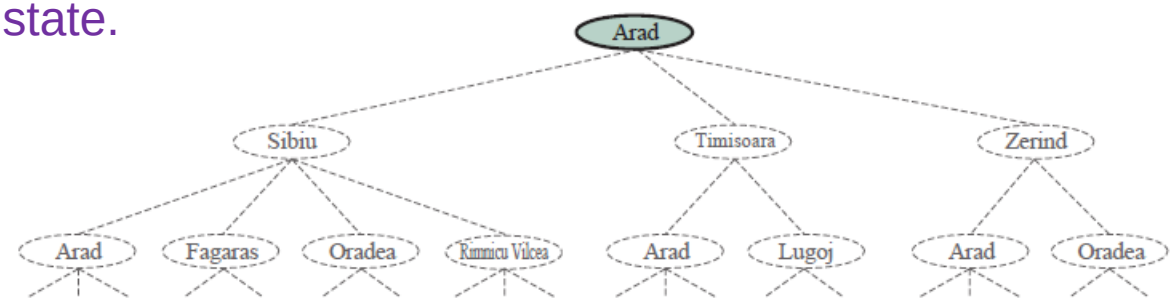
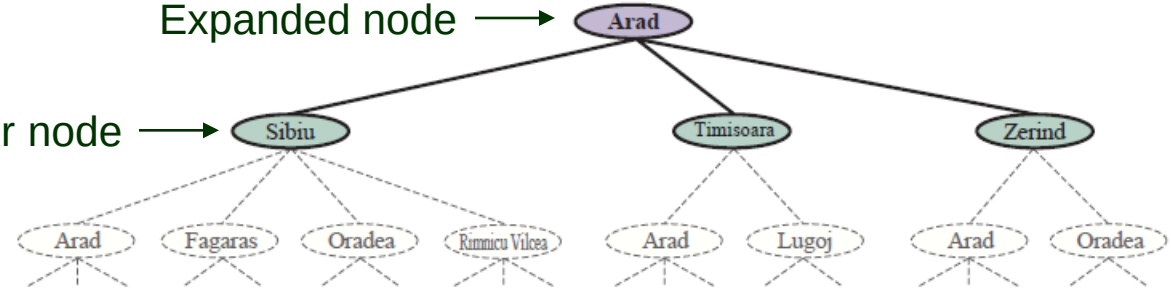
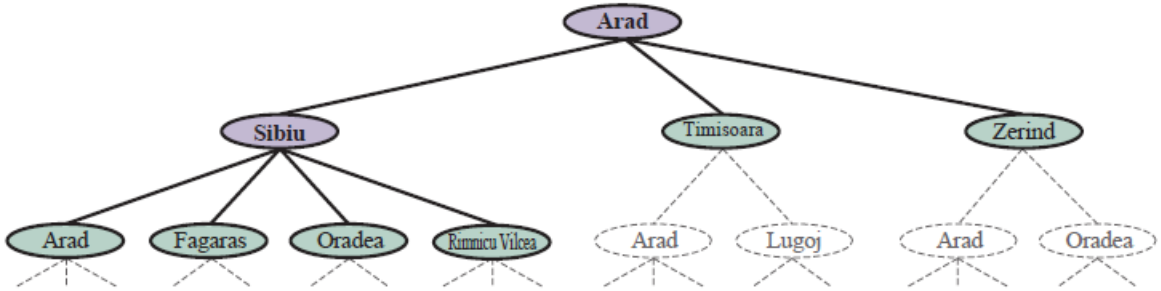


Expanded node →

Frontier node →





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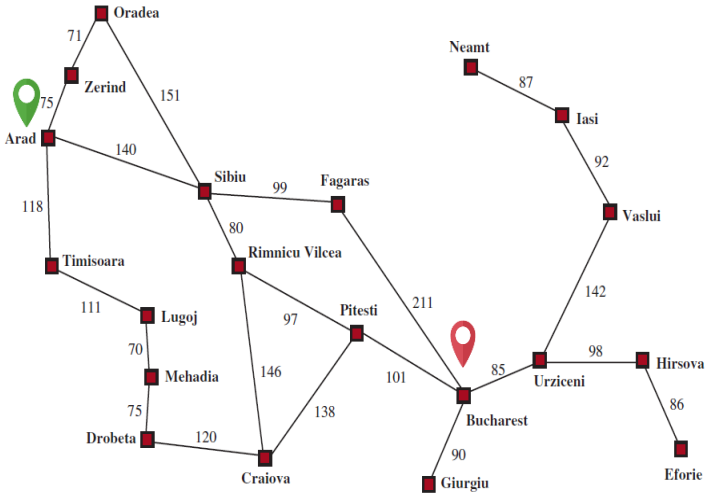
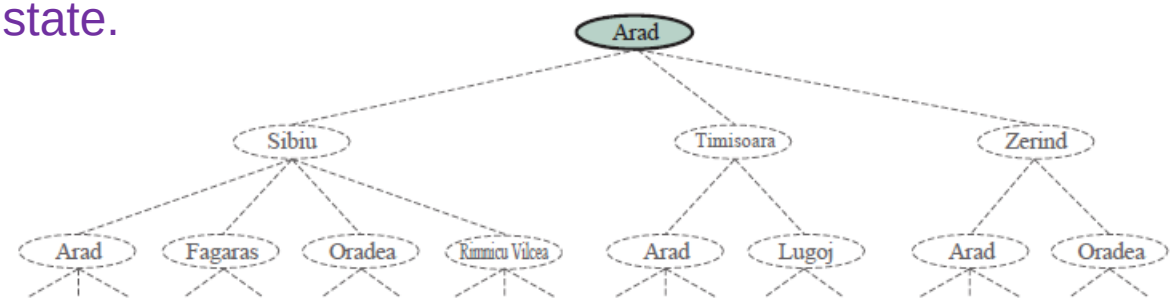
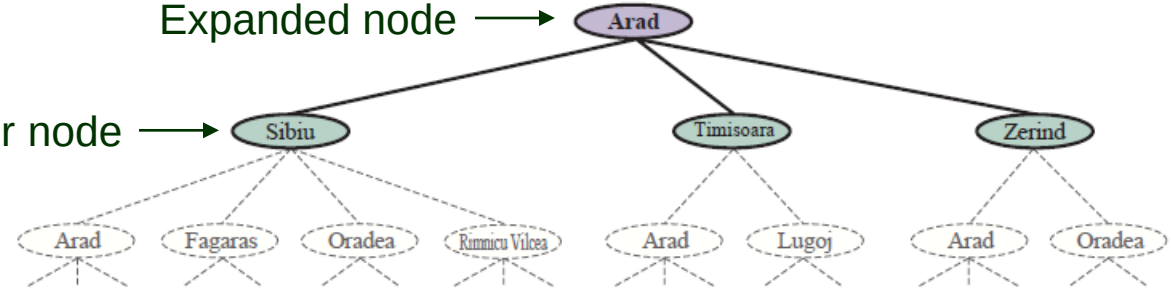


Superimpose a search tree over the state space graph and find a path from the initial state to the goal state.

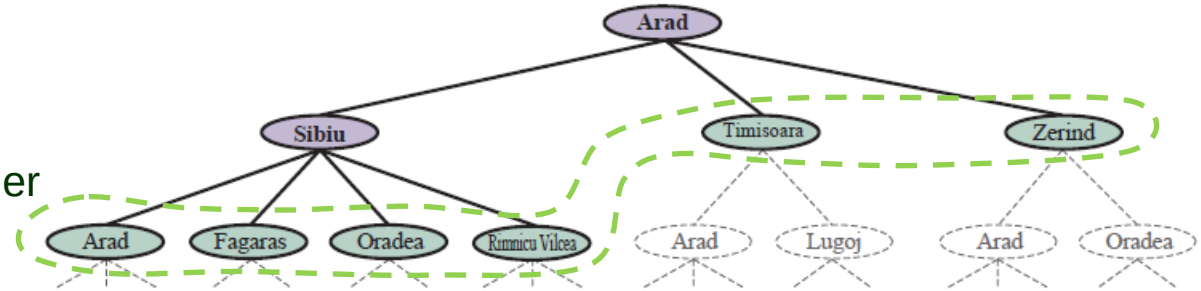


Expanded node →

Frontier node →



Frontier



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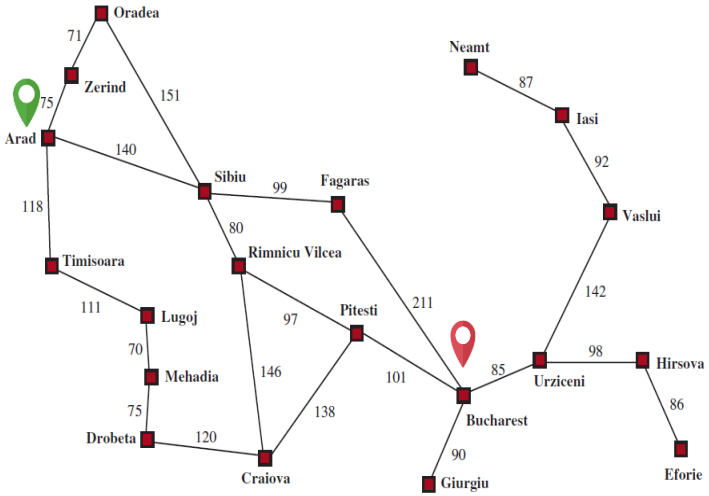

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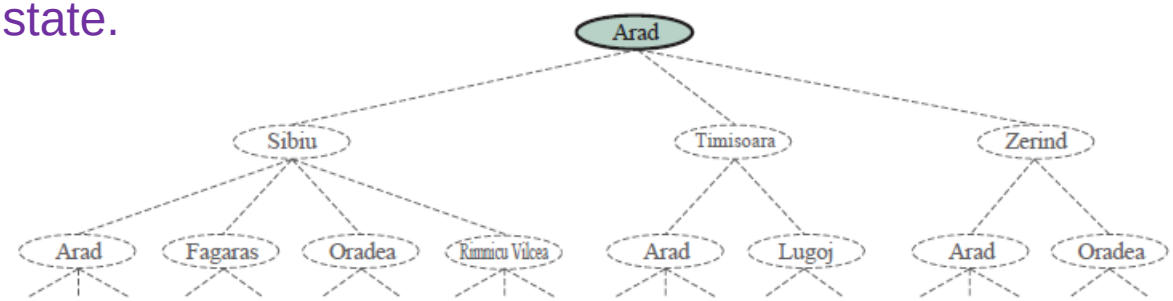
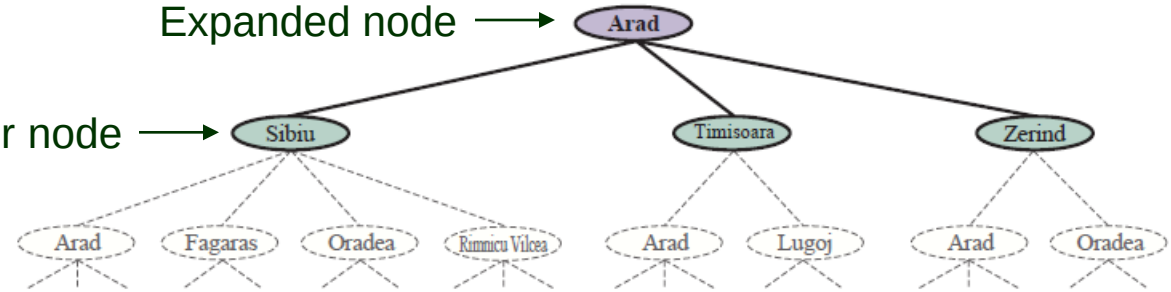


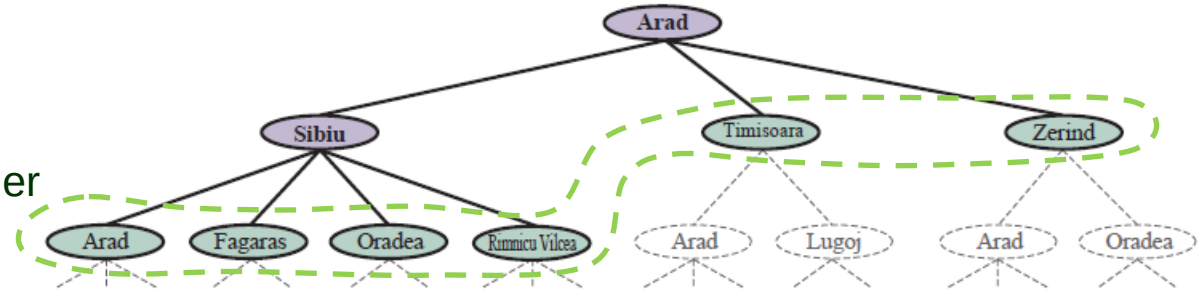
Expanded node →

Frontier node →

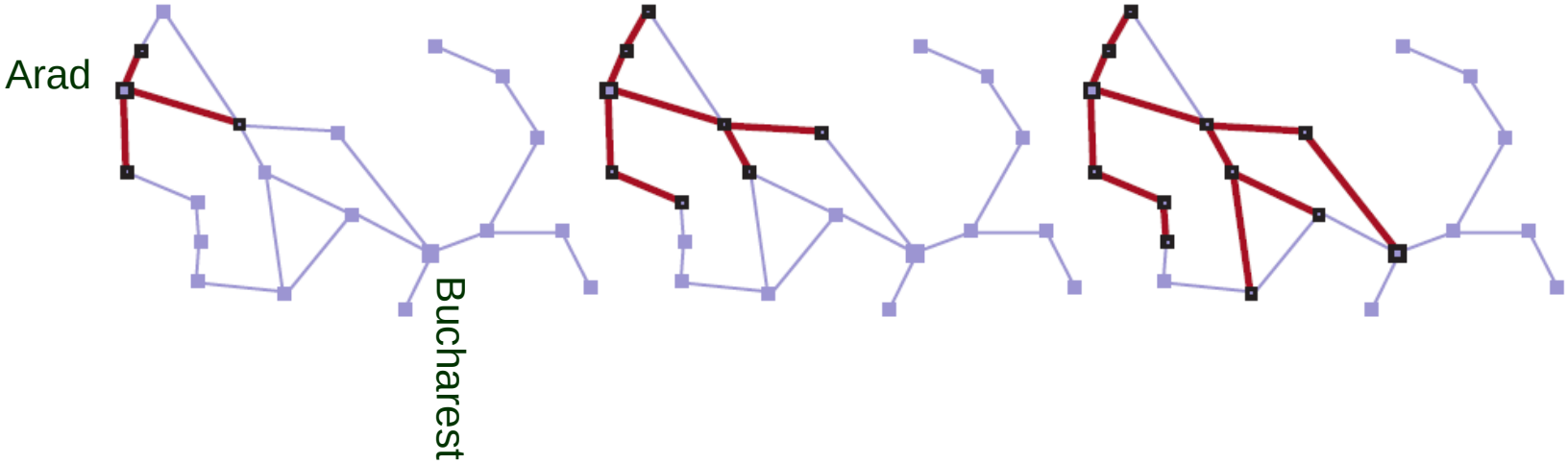
Node to be generated next →



Frontier



Generated Search Trees



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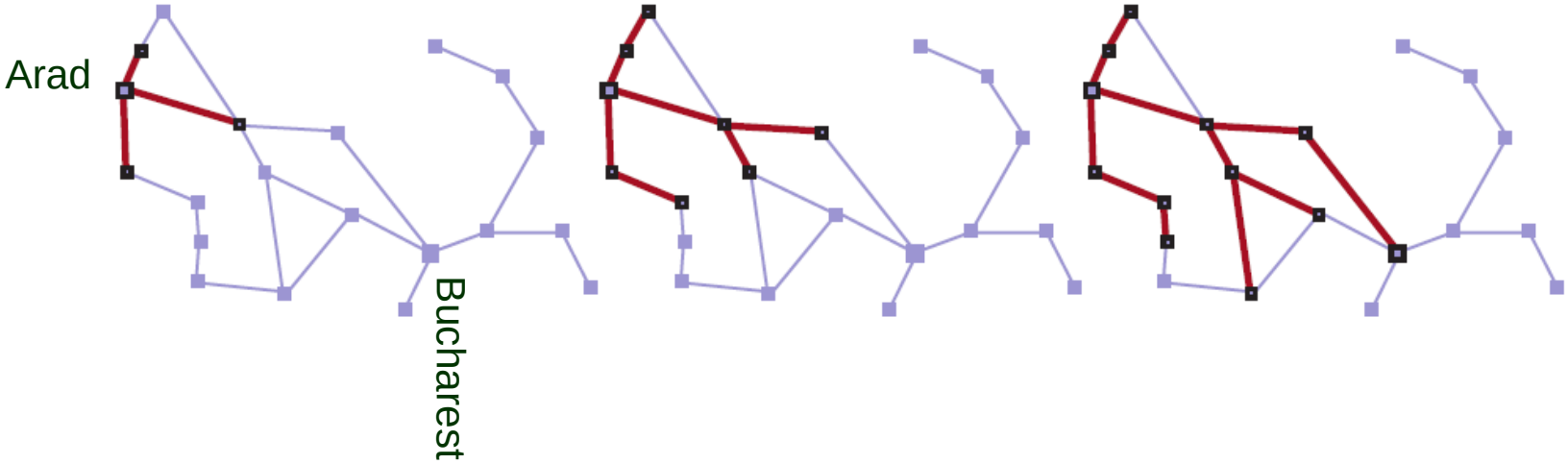
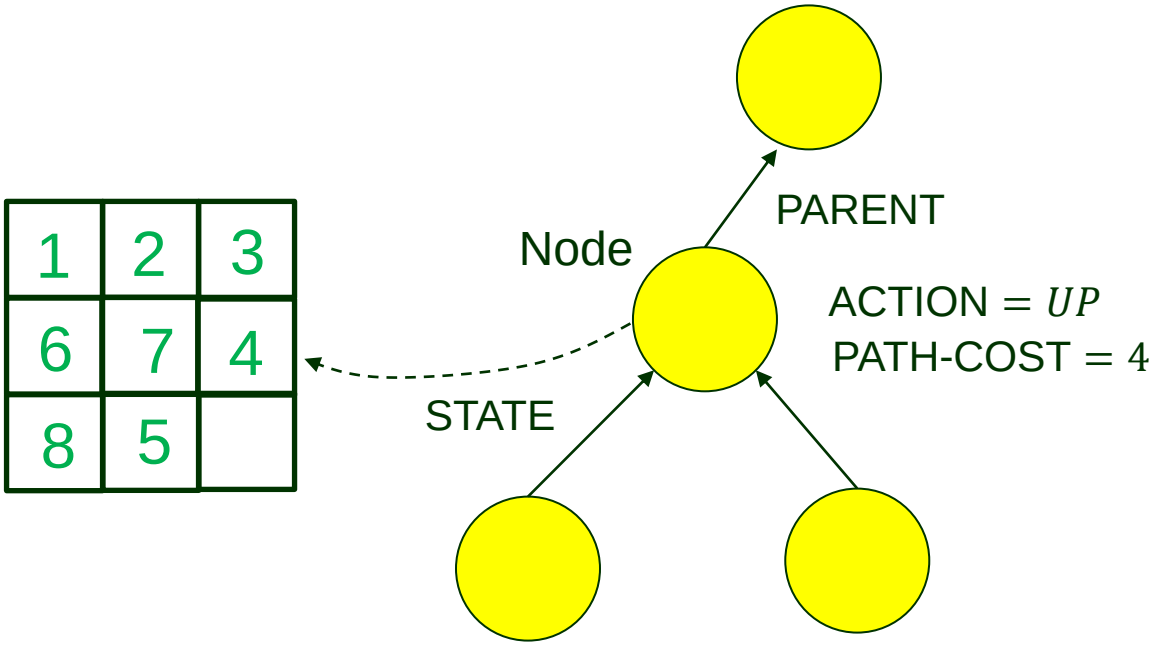


Which node on the frontier to expand next?

State vs. Node



Best-First Search

Choose a node n which minimum value $f(n)$.

evaluation function

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```
function BEST-FIRST-SEARCH(problem, f) returns a solution node or failure
  node ← NODE(STATE=problem.INITIAL)
  frontier ← a priority queue ordered by f, with node as an element // states on the frontier
  reached ← a lookup table, with one entry with key problem.INITIAL and value node // states
  while not IS-EMPTY(frontier) do                                   // that have been reached
    node ← POP(frontier)
    if problem.IS-GOAL(node.STATE) then return node
    for each child in EXPAND(problem, node) do
      s ← child.STATE
      if s is not in reached or child.PATH-COST < reached[s].PATH-COST then
        reached[s] ← child
        add child to frontier
  return failure

function EXPAND(problem, node) yields nodes
  s ← node.STATE
  for each action in problem.ACTIONS(s) do
    s' ← problem.RESULT(s, action)
    cost ← node.PATH-COST + problem.ACTION-COST(s, action, s')
    yield NODE(STATE=s', PARENT=node, ACTION=action, PATH-COST=cost)
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Can implement BFS and DFS.

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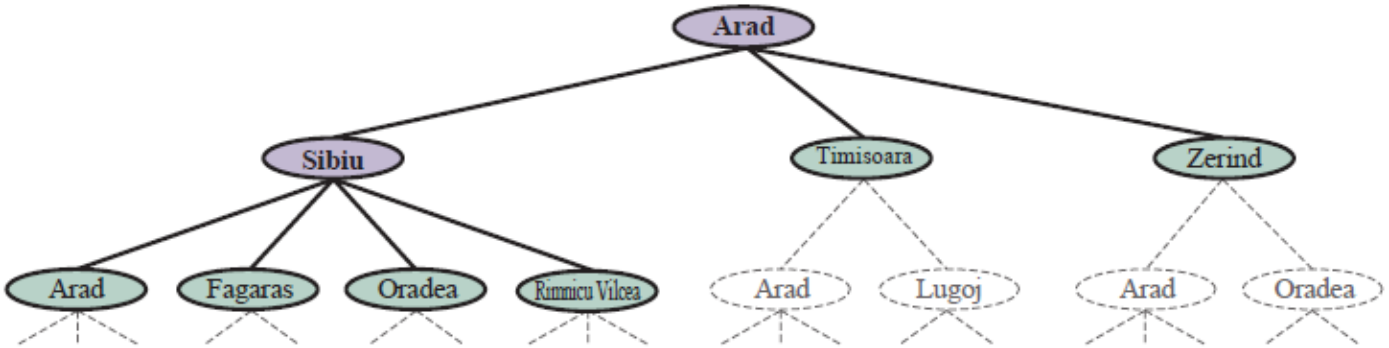
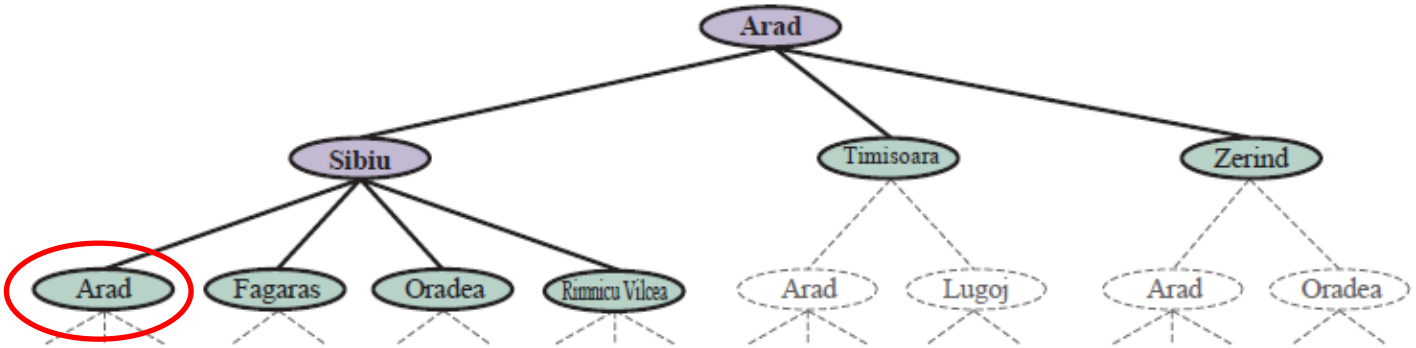
Repeated State



Repeated State



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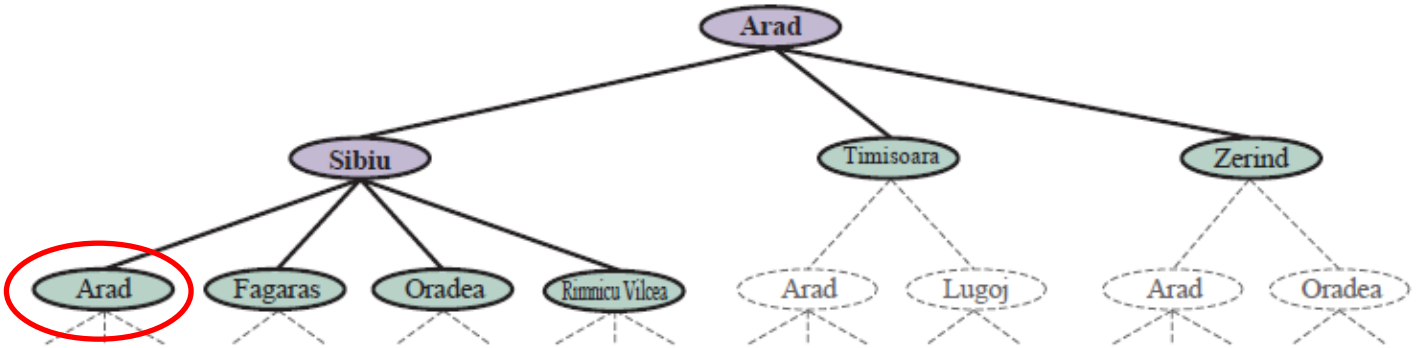

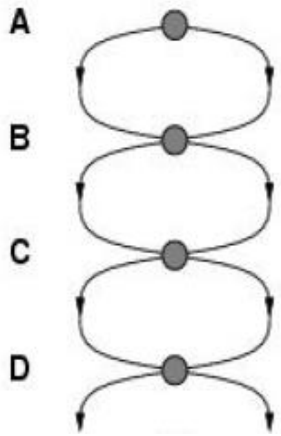

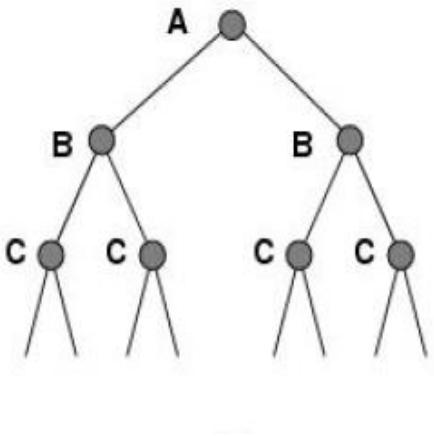

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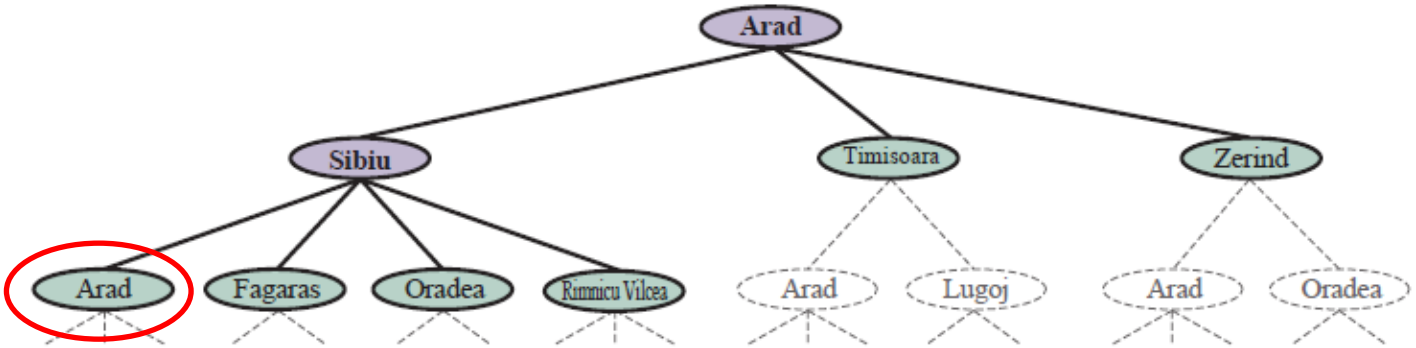

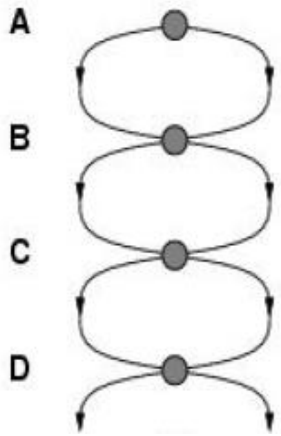

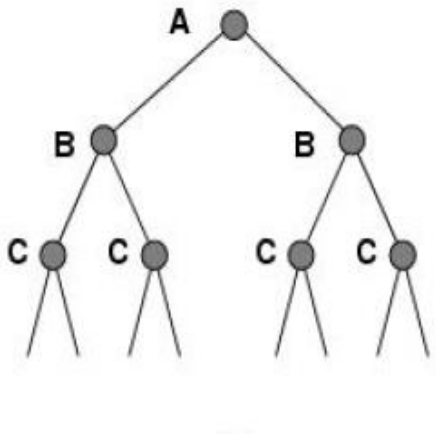

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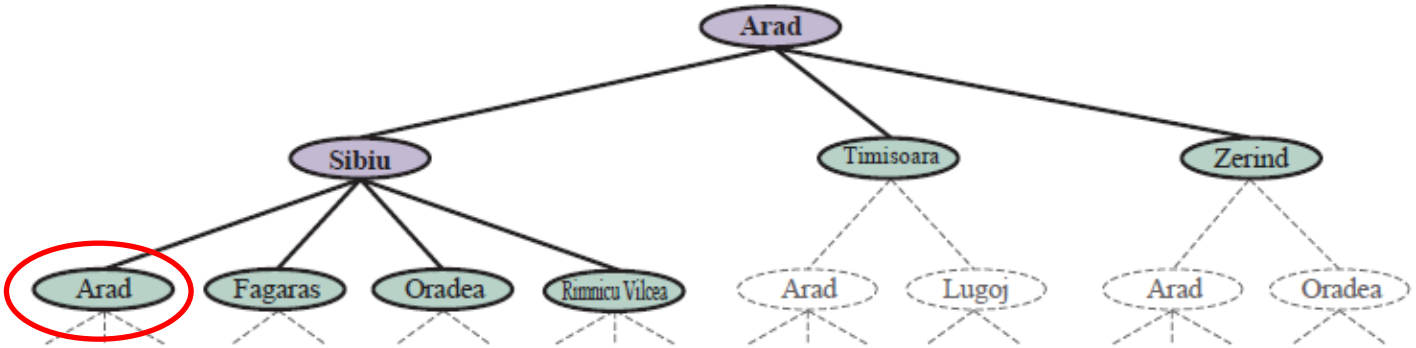

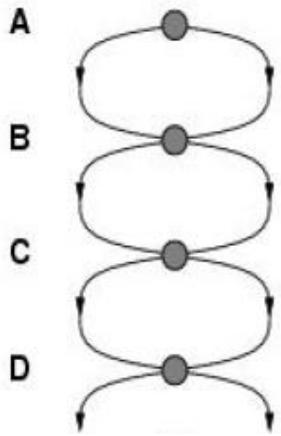

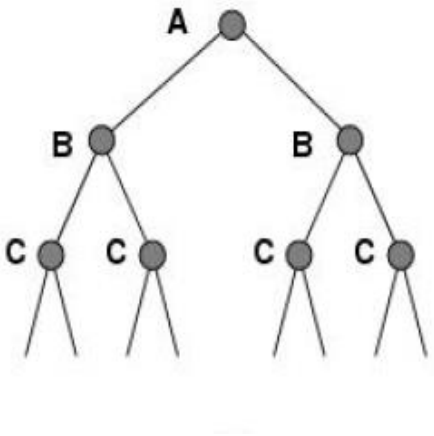

- ♠ Failure to detect repeated states can turn a solvable problem into an unsolvable one.
- ♦ Keep only the best path to each state.

Performance Measures

- Completeness: Is the algorithm guaranteed to find a solution whenever one exists, and to report failure otherwise?

The state space may be infinite!

- Cost optimality: Does it find a solution with the lowest path cost of all solutions?

- Time complexity: Physical time or the number of states and actions.

- Space complexity: Memory needed for the search.

$$|V| + |E|?$$

Depth and Branching Factor

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- Accordingly, complexity is measured in terms of
  - d: *depth* (number of actions in the optimal solution)
  - m: *maximum number of actions* in any path
  - b: *branching factor* (number of successors of a node).