Spring 2020 **Statistics 330** **Practice Exam II**
(– points)

Please write your name [First & Last] here:

Name _____

Instructions:

- You have 50 minutes to finish the exam.
- Partial credit will be given only if you show your work.
- Reason out your answers. In many cases, a line or two of justification is enough.
- If you get stuck on one, it may be a good idea to move on and come back to that question at the end.
- You may use your prepared notes (1 page, both sides) and a calculator only.

1. A video game store has one copy of a game that it rents out in whole hour increments with a maximum check out time of five hours. When a person comes by to rent the game we can define a random variable, X, as: X = time (in whole hours) that the game will be rented for. From past experience, the distribution of X is given by the following table.

x	1	2	3	4	5
$p_X(x)$	0.04	0.14	0.28	0.32	0.22

(a) Compute $\mathbb{E}(X)$ and $Var(X)$. (8 points)
Answer: $\mathbb{E}(X) = 1(.04) + 2(.14) + 3(.28) + 4(.32) + 5(.22) = 3.54$

$\mathbb{E}(X^2) = 1(.04) + 4(.14) + 9(.28) + 16(.32) + 25(.22) = 13.74$

So, $\text{Var}(X) = 13.74 - 3.54^2 = 1.21$

(b) The game store has the following pricing scheme for the game rental. \$5 flat fee plus \$2 per hour the game is rented for. Let Y = profit when a person come to rent the game. Write Y as a function of X and use the rules of expectation to find $\mathbb{E}(Y)$. (5 points)
Answer: $Y = 5 + 2X$. $\mathbb{E}(Y) = \mathbb{E}(5 + 2X) = 5 + 2\mathbb{E}(X) = 5 + 2(3.54) = 12.08$ (dollars)

(c) Suppose the game store adopts the following pricing scheme: no flat fee, \$7 per hour if rented for three or less hours, \$2 per hour if rented for four or five hours. Now, let Z = profit for this new pricing scheme. Fill in the rest of the table with the values Z can take on. (4 points)

z					
$p_Z(z)$	0.04	0.14	0.28	0.32	0.22

Answer:

z	7	14	21	8	10
$p_Z(z)$	0.04	0.14	0.28	0.32	0.22

(d) Calculate $\mathbb{E}(Z)$. Which pricing scheme is best for the game store in terms of highest expected profit from a game rental?
Answer:
$\mathbb{E}(Z) = 7(.04) + 14(.14) + 21(.28) + 8(.32) + 10(.22) = 12.88$ (dollars). The second pricing scheme is better.

2. A communications channel transmits the digits 0 and 1. However, due to static, the digit that is transmitted is incorrectly received with probability 0.05. Suppose we want to transmit an important message consisting of the digit 0. Thus we would have a 5% chance of the message being received incorrectly. To reduce the chance of error, we transmit the string 000 instead of just 0. **In this setup, each digit in the string can be "flipped" in transmission to a 1 with probability 0.05, independently of each other**. The receiver will then decode the message being sent after receiving the string of three digits. If there are more 0's in the string, the receiver will conclude that a 0 was originally sent. If there are more 1's in the string the receiver will conclude (incorrectly) that a 1 was originally sent. The process looks like this:

$$0 \to 000 \to 010 \to 0$$

Where the steps are replicate, transmit, decode.

(a) We send a 0 using the method described above. A sample space for this experiment is all three digit strings that the receiver can get in the transmission step. Write down the sample space here. (4 points)

Answer: $\Omega = \{000, 001, 010, 100, 111, 110, 101, 011\}$

(b) We send a 0 using the method described above. Instead of focusing on the sample space in part (a), let $X =$ the number of 1's that the receiver gets in the string. What is the distribution of X? Use the $X \sim$ notation with the numerical values of the parameter(s). (4 points)

Answer: $X \sim \text{Bin}(3, .05)$

(c) We send a 0 using the method described above, what is the probability that the receiver will (incorrectly) conclude that a 1 was originally sent? (4 points)

Answer: $\mathbb{P}(\text{conclude 1 was sent}) = \mathbb{P}(X \geq 2) = 1 - \mathbb{P}(X \leq 1) = 1 - .9928 = .0072$ using the binomial cdf table. Notice this is an 86% reduction in the probability of an error.

(d) We send a 0, but suppose instead of replicating it as 000, we replicate it as 0000000 and send that string. Now, let $Y =$ the number of 1's that the receiver gets in the string. What is the probability that the receiver will (correctly) conclude that a 0 was originally sent? (hint: what is the distribution of Y here) (4 points)

Answer: $\mathbb{P}(\text{conclude 0 was sent}) = \mathbb{P}(X \leq 3)$ where now $X \sim \text{Bin}(7, .05)$. This is .9998 using the binomial cdf table.

3. The joint probability mass function of 2 random variables, X and Y, is given below:

	X		
Y	0	1	2
0	0.12	0.08	0.2
1	0.18	0.12	0.3

(a) Calculate $\mathbb{P}(X > Y)$ [2 points]
Answer:

$$\begin{aligned}\mathbb{P}(X > Y) &= \mathbb{P}(X = 1, Y = 0) + \mathbb{P}(X = 2, Y = 0) + \mathbb{P}(X = 2, Y = 1) \\ &= 0.08 + 0.2 + 0.3 \\ &= 0.58\end{aligned}$$

(b) Find the marginal probability mass functions for X and Y. [4 points]
Answer:

x	0	1	2
$p_X(x)$	0.3	0.2	0.5

y	0	1
$p_Y(y)$	0.4	0.6

(c) Calculate E(X) and E(Y). [4 points] **Answer:**

$$E(X) = \sum_x x p_X(x) = (0)(0.3) + (1)(0.2) + (2)(0.5) = 1.2$$
$$E(Y) = \sum_y y p_Y(y) = (0)(0.4) + (1)(0.6) = 0.6$$

(d) Calculate the covariance between X and Y. [4 points]
Answer:

$$\begin{aligned}E(XY) &= \sum_{xy} xy p_{X,Y}(x, y) \\ &= (0)(0)(0.12) + (0)(1)(0.08) + (0)(2)(0.2) \\ &\quad + (1)(0)(0.18) + (1)(1)(0.12) + (1)(2)(0.3) = 0.72 \\ Cov(X, Y) &= E(XY) - E(X)E(Y) = 0.72 - (1.2)(0.6) = 0\end{aligned}$$

(e) Are X and Y independent? Justify your answer. [4 points]
Answer: X and Y are independent because $p_{X,Y}(x, y) = p_X(x)p_Y(y)$ for all (x, y) pairs.

4. Suppose the number of goals scored in a game by your soccer team follows a Poisson distribution. Your team has played 10 games in total, and scored a total of 8 goals.

   (a) Based on your team's history, what is the average number of goals you expect your team to score in a single game? [2 points]
   Answer: $\frac{8}{10} = 0.8$

   (b) Define X as the number of goals scored in a single game by your team. Give the distribution of X and value(s) of any parameter(s). [2 points]
   Answer:
   $$X \sim Pois(0.8)$$

   (c) What is the probability that your team scores no goals in a game? [4 points]
   Answer:
   $$\mathbb{P}(X = 0) = \frac{e^{-0.8}0.8^0}{0!} = 0.4493$$

   (d) What is the probability that your team scores at least 1 goal in a game? [4 points]
   Answer:
   $$\mathbb{P}(X \geq 1) = 1 - \mathbb{P}(X < 1) = 1 - \mathbb{P}(X = 0) = 1 - 0.4493 = 0.5507$$

   (e) What is the probability that your team scores between 1 and 3 goals (inclusive) in a game? [4 points]
   Answer:
   $$\mathbb{P}(1 \leq X \leq 3) = \mathbb{P}(X \leq 3) - \mathbb{P}(X = 0) = 0.9909 - 0.4493 = 0.5416$$

   (f) During the season, your team plays several games, where each game is independent and identically distributed. Define a new random variable, Y, as the number of games until your team starts scoring goals (i.e. get at least one goal).

   Give the distribution of Y and value(s) of any parameter(s). [2 points]
   Answer:
   Y = number of games until 1^{st} "success" (where "success" is scoring at least one goal)
   $\mathbb{P}$(Success) = $\mathbb{P}$(score at least 1 goal in a game) = 0.5507
   $$Y \sim Geo(p) = Geo(0.5507)$$

   (g) What is the probability that you only start scoring goals after your 3^{rd} game? [4 points]
   Answer:
   $$\begin{aligned}\mathbb{P}(Y > 3) &= 1 - \mathbb{P}(Y \leq 3) \\ &= 1 - F_Y(3) \\ &= 1 - (1 - (1-p)^3) \\ &= 1 - (1 - (1 - 0.5507)^3) \\ &= 0.0907\end{aligned}$$

5. A continuous random variable X has the probability density function (pdf)

$$f_X(x) = \begin{cases} 6x(1-x)\ , & \text{if } \ 0 < x < 1, \\ 0\ , & \text{otherwise.} \end{cases}$$

(a) Find $P(X < 0.4)$. (4 points)

Answer: $P(X < 0.4) = 6\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^{0.4} = 0.352$

(b) Find $P(X < 0.4 \mid X < 0.8)$. (4 points)

Answer: $P(X < 0.8) = 6\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^{0.8} = 6 \times 0.1493 = 0.896$

$$P(X < 0.4 \mid X < 0.8) = \frac{P(X < 0.4)}{P(X < 0.8)} = \frac{0.352}{0.896} = 0.393.$$

(c) Compute $E(X)$. (4 points)

Answer:

$$E(X) = \int_0^1 x \times 6x(1-x)\ dx = \int_0^1 (6x^2 - 6x^3)\ dx = \left(2x^3 - \frac{3}{2}x^4\right)\Big|_0^1 = 0.5$$

Appendix A

Distribution Tables

Binomial Distribution

$$B_{n,p}(x) = \sum_{i=0}^{\lfloor x \rfloor} \binom{n}{i} p^i (1-p)^{n-i}$$

n=1	p=0.01	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.4	0.5
x=0	0.99	0.95	0.9	0.85	0.8333333	0.8	0.75	0.7	0.6666667	0.6	0.5

n=2	p=0.01	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.4	0.5
x=0	0.9801	0.9025	0.81	0.7225	0.6944444	0.64	0.5625	0.49	0.4444444	0.36	0.25
1	0.9999	0.9975	0.99	0.9775	0.9722222	0.96	0.9375	0.91	0.8888889	0.84	0.75

n=3	p=0.01	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.4	0.5
x=0	0.970299	0.857375	0.729	0.614125	0.5787037	0.512	0.421875	0.343	0.2962963	0.216	0.125
1	0.999702	0.992750	0.972	0.939250	0.9259259	0.896	0.843750	0.784	0.7407407	0.648	0.500
2	0.999999	0.999875	0.999	0.996625	0.9953704	0.992	0.984375	0.973	0.9629630	0.936	0.875

n=4	p=0.01	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.4	0.5
x=0	0.960596	0.8145062	0.6561	0.5220063	0.4822531	0.4096	0.3164063	0.2401	0.1975309	0.1296	0.0625
1	0.999408	0.9859812	0.9477	0.8904813	0.8680556	0.8192	0.7382812	0.6517	0.5925926	0.4752	0.3125
2	0.999996	0.9995188	0.9963	0.9880187	0.9837963	0.9728	0.9492188	0.9163	0.8888889	0.8208	0.6875
3	1.000000	0.9999938	0.9999	0.9994937	0.9992284	0.9984	0.9960938	0.9919	0.9876543	0.9744	0.9375

n=5	p=0.01	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.4	0.5
x=0	0.9509900	0.7737809	0.59049	0.4437053	0.4018776	0.32768	0.2373047	0.16807	0.1316872	0.07776	0.03125
1	0.9990199	0.9774075	0.91854	0.8352100	0.8037551	0.73728	0.6328125	0.52822	0.4609053	0.33696	0.18750
2	0.9999901	0.9988419	0.99144	0.9733881	0.9645062	0.94208	0.8964844	0.83692	0.7901235	0.68256	0.50000
3	1.0000000	0.9999700	0.99954	0.9977725	0.9966564	0.99328	0.9843750	0.96922	0.9547325	0.91296	0.81250
4	1.0000000	0.9999997	0.99999	0.9999241	0.9998714	0.99968	0.9990234	0.99757	0.9958848	0.98976	0.96875

n=6	p=0.01	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.4	0.5
x=0	0.9414801	0.7350919	0.531441	0.3771495	0.3348980	0.262144	0.1779785	0.117649	0.0877915	0.046656	0.015625
1	0.9985396	0.9672262	0.885735	0.7764843	0.7367755	0.655360	0.5339355	0.420175	0.3511660	0.233280	0.109375
2	0.9999804	0.9977702	0.984150	0.9526614	0.9377143	0.901120	0.8305664	0.744310	0.6803841	0.544320	0.343750
3	0.9999999	0.9999136	0.998730	0.9941148	0.9912980	0.983040	0.9624023	0.929530	0.8998628	0.820800	0.656250
4	1.0000000	0.9999982	0.999945	0.9996013	0.9993356	0.998400	0.9953613	0.989065	0.9821674	0.959040	0.890625
5	1.0000000	1.0000000	0.999999	0.9999886	0.9999786	0.999936	0.9997559	0.999271	0.9986283	0.995904	0.984375

n=7	p=0.01	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.4	0.5
x=0	0.9320653	0.6983373	0.4782969	0.3205771	0.2790816	0.2097152	0.1334839	0.0823543	0.05852766	0.0279936	0.0078125
1	0.9979690	0.9556195	0.8503056	0.7165841	0.6697960	0.5767168	0.4449463	0.3294172	0.26337449	0.1586304	0.0625000
2	0.9999660	0.9962430	0.9743085	0.9262348	0.9042245	0.8519680	0.7564087	0.6470695	0.57064472	0.4199040	0.2265625
3	0.9999997	0.9998064	0.9972720	0.9878968	0.9823674	0.9666560	0.9294434	0.8739640	0.82670325	0.7102080	0.5000000
4	1.0000000	0.9999940	0.9998235	0.9987784	0.9979960	0.9953280	0.9871216	0.9712045	0.95473251	0.9037440	0.7734375
5	1.0000000	0.9999999	0.9999936	0.9999305	0.9998714	0.9996288	0.9986572	0.9962092	0.99314129	0.9811584	0.9375000
6	1.0000000	1.0000000	0.9999999	0.9999983	0.9999964	0.9999872	0.9999390	0.9997813	0.99954275	0.9983616	0.9921875

A.1 Poisson Distribution

$$Po_\lambda(x) = \sum_{k=0}^{\lfloor x \rfloor} e^{-\lambda} \frac{\lambda^k}{k!}$$

x	λ =0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048374	0.8187308	0.7408182	0.6703200	0.6065307	0.5488116	0.4965853	0.4493290	0.4065697	0.3678794
1	0.9953212	0.9824769	0.9630637	0.9384481	0.9097960	0.8780986	0.8441950	0.8087921	0.7724824	0.7357589
2	0.9998453	0.9988515	0.9964005	0.9920737	0.9856123	0.9768847	0.9658584	0.9525774	0.9371431	0.9196986
3	0.9999962	0.9999432	0.9997342	0.9992237	0.9982484	0.9966419	0.9942465	0.9909201	0.9865413	0.9810118
4	0.9999999	0.9999977	0.9999842	0.9999388	0.9998279	0.9996055	0.9992145	0.9985887	0.9976559	0.9963402
5	1.0000000	0.9999999	0.9999992	0.9999960	0.9999858	0.9999611	0.9999100	0.9998157	0.9996565	0.9994058
6		1.0000000	1.0000000	0.9999998	0.9999990	0.9999967	0.9999911	0.9999793	0.9999566	0.9999168
7				1.0000000	0.9999999	0.9999998	0.9999992	0.9999979	0.9999952	0.9999898
8					1.0000000	1.0000000	0.9999999	0.9999998	0.9999995	0.9999989
9							1.0000000	1.0000000	1.0000000	0.9999999
10										1.0000000

x	λ =1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3328711	0.3011942	0.2725318	0.2465970	0.2231302	0.2018965	0.1826835	0.1652989	0.1495686	0.1353353
1	0.6990293	0.6626273	0.6268231	0.5918327	0.5578254	0.5249309	0.4932455	0.4628369	0.4337490	0.4060058
2	0.9004163	0.8794871	0.8571125	0.8334977	0.8088468	0.7833585	0.7572232	0.7306211	0.7037204	0.6766764
3	0.9742582	0.9662310	0.9569045	0.9462747	0.9343575	0.9211865	0.9068106	0.8912916	0.8747022	0.8571235
4	0.9945647	0.9922542	0.9893370	0.9857467	0.9814241	0.9763177	0.9703852	0.9635933	0.9559186	0.9473470
5	0.9990321	0.9984998	0.9977694	0.9967989	0.9955440	0.9939597	0.9920006	0.9896220	0.9867808	0.9834364
6	0.9998512	0.9997489	0.9995964	0.9993777	0.9990740	0.9986642	0.9981249	0.9974306	0.9965539	0.9954662
7	0.9999799	0.9999630	0.9999357	0.9998935	0.9998304	0.9997396	0.9996123	0.9994385	0.9992065	0.9989033
8	0.9999976	0.9999951	0.9999909	0.9999837	0.9999723	0.9999546	0.9999283	0.9998903	0.9998366	0.9997626
9	0.9999997	0.9999994	0.9999988	0.9999978	0.9999959	0.9999929	0.9999880	0.9999806	0.9999696	0.9999535
10	1.0000000	0.9999999	0.9999999	0.9999997	0.9999994	0.9999990	0.9999982	0.9999969	0.9999948	0.9999917
11		1.0000000	1.0000000	1.0000000	0.9999999	0.9999999	0.9999997	0.9999995	0.9999992	0.9999986
12					1.0000000	1.0000000	1.0000000	0.9999999	0.9999999	0.9999998
13								1.0000000	1.0000000	1.0000000

x	λ =2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1224564	0.1108032	0.1002588	0.09071795	0.0820850	0.07427358	0.06720551	0.06081006	0.05502322	0.04978707
1	0.3796149	0.3545701	0.3308542	0.30844104	0.2872975	0.26738488	0.24866040	0.23107824	0.21459056	0.19914827
2	0.6496314	0.6227137	0.5960388	0.56970875	0.5438131	0.51842958	0.49362449	0.46945368	0.44596320	0.42319008
3	0.8386428	0.8193524	0.7993471	0.77872291	0.7575761	0.73600164	0.71409218	0.69193743	0.66962342	0.64723189
4	0.9378739	0.9275037	0.9162493	0.90413141	0.8911780	0.87742349	0.86290786	0.84767606	0.83177708	0.81526324
5	0.9795509	0.9750902	0.9700243	0.96432749	0.9579790	0.95096285	0.94326833	0.93488969	0.92582620	0.91608206
6	0.9941379	0.9925387	0.9906381	0.98840592	0.9858127	0.98282990	0.97943055	0.97558938	0.97128327	0.96649146
7	0.9985140	0.9980224	0.9974112	0.99666138	0.9957533	0.99466624	0.99337883	0.99186926	0.99011549	0.98809550
8	0.9996627	0.9995305	0.9993584	0.99913802	0.9988597	0.99851305	0.99808637	0.99756722	0.99694217	0.99619701
9	0.9999307	0.9998991	0.9998561	0.99979846	0.9997226	0.99962435	0.99949864	0.99933991	0.99914188	0.99889751
10	0.9999870	0.9999802	0.9999705	0.99995696	0.9999384	0.99991329	0.99987995	0.99983627	0.99977979	0.99970766
11	0.9999978	0.9999964	0.9999944	0.99999155	0.9999874	0.99998158	0.99997354	0.99996261	0.99994797	0.99992861
12	0.9999996	0.9999994	0.9999990	0.99999846	0.9999976	0.99999638	0.99999460	0.99999209	0.99998861	0.99998385
13	0.9999999	0.9999999	0.9999998	0.99999974	0.9999996	0.99999934	0.99999897	0.99999844	0.99999768	0.99999660
14	1.00000000	1.0000000	1.0000000	0.99999996	0.9999999	0.99999989	0.99999982	0.99999971	0.99999956	0.99999933
15				1.00000000	1.0000000	0.99999998	0.99999997	0.99999995	0.99999992	0.99999988
16						1.00000000	1.00000000	0.99999999	0.99999999	0.99999998
17								1.00000000	1.00000000	1.00000000

x	λ =3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
0	0.03019738	0.01831564	0.01110900	0.00673795	0.00408677	0.00247875	0.00150344	0.00091188	0.00055308	0.00033546
1	0.13588823	0.09157819	0.06109948	0.04042768	0.02656401	0.01735127	0.01127579	0.00729506	0.00470122	0.00301916
2	0.32084720	0.23810331	0.17357807	0.12465202	0.08837643	0.06196880	0.04303595	0.02963616	0.02025672	0.01375397
3	0.53663267	0.43347012	0.34229596	0.26502592	0.20169920	0.15120388	0.11184961	0.08176542	0.05914546	0.04238011
4	0.72544495	0.62883694	0.53210358	0.44049329	0.35751800	0.28505650	0.22367182	0.17299161	0.13206186	0.09963240
5	0.85761355	0.78513039	0.70293043	0.61596066	0.52891869	0.44567964	0.36904068	0.30070828	0.24143645	0.19123606
6	0.93471190	0.88932602	0.83105058	0.76218346	0.68603598	0.60630278	0.52652362	0.44971106	0.37815469	0.31337428
7	0.97326108	0.94886638	0.91341353	0.86662833	0.80948528	0.74397976	0.67275778	0.59871384	0.52463853	0.45296081
8	0.99012634	0.97863657	0.95974269	0.93190637	0.89435668	0.84723749	0.79157303	0.72909127	0.66196712	0.59254734
9	0.99668506	0.99186776	0.98290727	0.96817194	0.94622253	0.91607598	0.87738405	0.83049594	0.77640761	0.71662426
10	0.99898061	0.99716023	0.99333133	0.98630473	0.97474875	0.95737908	0.93316121	0.90147921	0.86223798	0.81588579
11	0.99971101	0.99908477	0.99759572	0.99454691	0.98901186	0.97990804	0.96612044	0.94665038	0.92075869	0.88807600
12	0.99992404	0.99972628	0.99919486	0.99798115	0.99554912	0.99117252	0.98397336	0.97300023	0.95733413	0.93620280
13	0.99998140	0.99992367	0.99974841	0.99930201	0.99831488	0.99637151	0.99289982	0.98718861	0.97843535	0.96581930
14	0.99999574	0.99998007	0.99992634	0.99977375	0.99940143	0.99859965	0.99704424	0.99428280	0.98973957	0.98274301
15	0.99999908	0.99999511	0.99997972	0.99993099	0.99979983	0.99949090	0.99884016	0.99759342	0.99539168	0.99176900
16	0.99999981	0.99999887	0.99999473	0.99998013	0.99993678	0.99982512	0.99956975	0.99904182	0.99804111	0.99628200
17	0.99999996	0.99999975	0.99999870	0.99999458	0.99998109	0.99994308	0.99984872	0.99963822	0.99921000	0.99840574
18	0.99999999	0.99999995	0.99999970	0.99999860	0.99999463	0.99998240	0.99994945	0.99987015	0.99969700	0.99934963
19	1.00000000	0.99999999	0.99999993	0.99999966	0.99999855	0.99999482	0.99998391	0.99995560	0.99988925	0.99974706
20		1.00000000	0.99999999	0.99999992	0.99999963	0.99999855	0.99999511	0.99998551	0.99996134	0.99990603
21			1.00000000	0.99999998	0.99999991	0.99999961	0.99999858	0.99999547	0.99998709	0.99996659
22				1.00000000	0.99999998	0.99999990	0.99999961	0.99999865	0.99999587	0.99998861
23					1.00000000	0.99999998	0.99999990	0.99999961	0.99999873	0.99999627
24						0.99999999	0.99999997	0.99999989	0.99999963	0.99999883
25						1.00000000	0.99999999	0.99999997	0.99999989	0.99999964
26							1.00000000	0.99999999	0.99999997	0.99999990
27								1.00000000	0.99999999	0.99999997
28									1.00000000	0.99999999
29										1.00000000

x	λ=9	10	11	12	13	14	15	20	25	30
0	0.00012341	0.00004540	0.00001670	0.00000614	0.00000226	0.00000083	0.00000031	0.00000000	0.00000000	0.00000000
1	0.00123410	0.00049940	0.00020042	0.00007987	0.00003164	0.00001247	0.00000489	0.00000004	0.00000000	0.00000000
2	0.00623220	0.00276940	0.00121087	0.00052226	0.00022264	0.00009396	0.00003931	0.00000046	0.00000000	0.00000000
3	0.02122649	0.01033605	0.00491587	0.00229179	0.00105030	0.00047425	0.00021138	0.00000320	0.00000004	0.00000000
4	0.05496364	0.02925269	0.01510460	0.00760039	0.00374019	0.00180525	0.00085664	0.00001694	0.00000027	0.00000000
5	0.11569052	0.06708596	0.03751981	0.02034103	0.01073389	0.00553205	0.00279243	0.00007191	0.00000140	0.00000002
6	0.20678084	0.13014140	0.07861437	0.04582231	0.02588692	0.01422792	0.00763190	0.00025512	0.00000611	0.00000012
7	0.32389696	0.22022060	0.14319153	0.08950450	0.05402825	0.03161966	0.01800219	0.00077859	0.00002292	0.00000052
8	0.45565260	0.33281970	0.23198513	0.15502780	0.09975791	0.06205520	0.03744649	0.00208726	0.00007548	0.00000205
9	0.58740824	0.45792970	0.34051064	0.24239220	0.16581190	0.10939940	0.06985366	0.00499541	0.00022148	0.00000712
10	0.70598832	0.58303980	0.45988870	0.34722940	0.25168200	0.17568120	0.11846440	0.01081172	0.00058646	0.00002235
11	0.80300838	0.69677610	0.57926676	0.46159730	0.35316490	0.26003990	0.18475180	0.02138682	0.00141597	0.00006388
12	0.87577343	0.79155650	0.68869665	0.57596520	0.46310470	0.35845840	0.26761100	0.03901199	0.00314412	0.00016770
13	0.92614923	0.86446440	0.78129117	0.68153560	0.57304460	0.46444760	0.36321780	0.06612764	0.00646748	0.00040728
14	0.95853367	0.91654150	0.85404401	0.77202450	0.67513150	0.57043670	0.46565370	0.10486430	0.01240206	0.00092068
15	0.97796434	0.95125960	0.90739609	0.84441570	0.76360690	0.66935990	0.56808960	0.15651310	0.02229302	0.00194748
16	0.98889409	0.97295840	0.94407565	0.89870900	0.83549310	0.75591770	0.66412320	0.22107420	0.03774765	0.00387273
17	0.99468043	0.98572240	0.96780948	0.93703370	0.89046500	0.82720060	0.74885880	0.29702840	0.06047504	0.00727022
18	0.99757360	0.99281350	0.98231349	0.96258350	0.93016690	0.88264290	0.81947170	0.38142190	0.09204086	0.01293270
19	0.99894405	0.99654570	0.99071054	0.97872020	0.95733130	0.92349510	0.87521880	0.47025730	0.13357480	0.02187347
20	0.99956075	0.99841170	0.99532892	0.98840230	0.97498820	0.95209160	0.91702910	0.55909260	0.18549230	0.03528462
21	0.99982505	0.99930030	0.99774808	0.99393490	0.98591860	0.97115590	0.94689360	0.64369760	0.24729880	0.05444340
22	0.99993317	0.99970430	0.99895765	0.99695260	0.99237750	0.98328780	0.96725580	0.72061130	0.31753350	0.08056902
23	0.99997548	0.99987990	0.99953614	0.99852710	0.99602820	0.99067240	0.98053540	0.78749280	0.39387550	0.11464590
24	0.99999135	0.99995310	0.99980129	0.99931440	0.99800570	0.99498010	0.98883520	0.84322740	0.47339850	0.15724200
25	0.99999706	0.99998230	0.99991795	0.99969220	0.99903400	0.99739240	0.99381510	0.88781500	0.55292140	0.20835740
26	0.99999904	0.99999360	0.99996731	0.99986670	0.99954810	0.99869130	0.99668810	0.92211320	0.62938580	0.26733660
27	0.99999969	0.99999770	0.99998742	0.99994420	0.99979570	0.99936490	0.99828420	0.94751930	0.70018610	0.33286910
28	0.99999991	0.99999920	0.99999532	0.99997740	0.99991060	0.99970160	0.99913930	0.96566650	0.76340070	0.40308250
29	0.99999997	0.99999970	0.99999831	0.99999110	0.99996210	0.99986420	0.99958160	0.97818180	0.81789610	0.47571700
30	0.99999999	0.99999990	0.99999941	0.99999660	0.99998440	0.99994010	0.99980270	0.98652530	0.86330890	0.54835150
31	1.00000000	1.00000000	0.99999980	0.99999880	0.99999380	0.99997430	0.99990970	0.99190820	0.89993210	0.61864300