Homework: Context-Free Grammar

Learning Objectives:

In this homework, we are going to exercise the following key knowledge points on the topic of context-free grammar (CFG)

1. understanding the relations of strings and grammars
2. performing derivations and constructing parse trees
3. determining and resolving ambiguity
4. designing a grammar to describe given string patterns

Instructions:

1. Total points: 40 pt
2. Early deadline: Sept 11 (Wed) 11:59 pm, Regular deadline Sept 13 (Fri) 11:59 pm (you can continue working on the homework till TA starts to grade the homework)
3. How to submit:
   - Submit your document to Canvas under Assignments, Homework 1
   - Please provide the complete solutions in one pdf file
   - You can write your solutions in latex or word and then convert it to pdf; or you can submit a scanned document with legible handwritten solutions

Questions:

1. (10 pt) Given a string $a0b10c$ and the context free grammar G:
   $S \rightarrow SA|A|SD$
   $A \rightarrow a|b|c$
   $D \rightarrow 0|1$
   (a) (2 pt) What are the terminals and non-terminals of the grammar?
   (b) (2 pt) Give a leftmost derivation for the string
   (c) (2 pt) Give a rightmost derivation for the string
   (d) (2 pt) Give a parse tree for the string
   (e) (2 pt) Write 3 strings using the terminals that do not belong to the language of the grammar $L(G)$
2. (10 pt) Consider the following grammar:
   - terminals: $x, y, z, >, <, 0, 1, (,)$, if, then, else

- non-terminals: S, F, B, T, E, N
- start symbol: S
- production rules:
  $S \rightarrow F|T\ N\ T$
  $F \rightarrow$if B then S|if B then S else S
  $B \rightarrow (T\ E\ T)$
  $T \rightarrow x|y|z|1|0$
  $E \rightarrow > \mid <$
  $N \rightarrow +|-|=$

(a) (4 pt) Draw two different parse trees for the string
if $(x > y)$ then if $(x < z)$ then $x = 1$ else $x = 0$

(b) (2 pt) Modify the grammar to remove ambiguity.

(c) (2 pt) Draw the parse tree for the string using new grammar

(d) (2 pt) Explain how your new grammar modifies the parse trees you drew in the first step to remove ambiguity

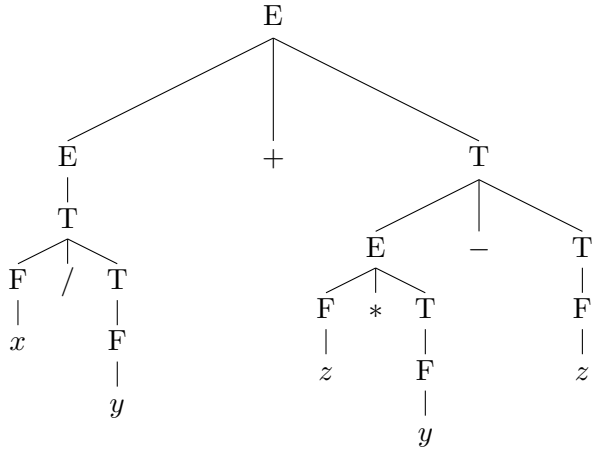
3. (10 pt) Consider the following grammar:

- terminals: x, y, z, +, -, *, /
- non-terminals: E, T, F, V
- start symbol: E
- production rules:
  $E \rightarrow E + T|E - T|T$
  $T \rightarrow F * T|F/T|F$
  $F \rightarrow x|y|z$

(a) (4 pt) What is the associativity of the operators $+$, $-$, $*$ and $/$; explain why.

(b) (3 pt) What is the precedence of $+$,$-$, $*$ and $/$; explain why.

(c) (3 pt) Given a parse tree



Explain how the value of the string is generated.

4. (10 pt) Design CFGs for the given languages:

   (a) (2 pt) Write a grammar that describes the strings $0^*1^+2^*$.

   (b) (3 pt) Write a grammar that describes the strings 0^n1^m, where $n > m$.

   (c) (5 pt) Given a graph below, where 1 is an entry and 7 is an exit, we can generate paths like 123467, 123567, 12343467, 12343567, 12353467 ... Write a grammar that describes these paths.