EE330

HW 15

Solution

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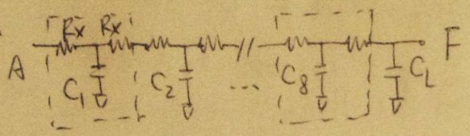
Problem #1.



$$R_x = \frac{20 \text{ ohms/square}}{2} \times \frac{50\mu \times 2\mu}{0.5\mu \times 0.5\mu} = \frac{20 \times 100}{0.25 \times 2}\Omega = \frac{8000\Omega}{2} = \underline{\underline{4K\Omega}}$$

$C_i = 0.5 \, fF/\mu^2 \times 50\mu \times 2\mu = 50fF = C_L$;

$t_1 = [4K\Omega \,//\, (8\times 8 - 4K\Omega)] \cdot C_1 = (4K\Omega \,//\, 60K\Omega) \cdot 50fF = 187.5ps$

$t_2 = (12K\Omega \,//\, 52K\Omega) \cdot 50fF = 487.5ps$

$t_3 = (20K\Omega \,//\, 44K\Omega) \cdot 50fF = 687.5ps$

$t_4 = (28K\Omega \,//\, 36K\Omega) \cdot 50fF = 787.5ps$

$t_5 = (36K\Omega \,//\, 28K\Omega) \cdot 50fF = t_4 = 787.5ps$

$t_6 = (44K\Omega \,//\, 20K\Omega) \cdot 50fF = t_3 = 687.5ps$

$t_7 = (52K\Omega \,//\, 12K\Omega) \cdot 50fF = t_2 = 487.5ps$

$t_8 = (60K\Omega \,//\, 4K\Omega) \cdot 50fF = t_1 = 187.5ps$

$t_9 = 64K\Omega \cdot 50fF = 3200ps$

$$\Rightarrow t_{PD} = \sum_{i=1}^{9} t_i = 7500ps = 7.5ns$$

Problem #2.

(a) $R_T = \frac{1}{2} \cdot 20\Omega \times \frac{50\mu \times 2\mu}{0.5\mu \times 0.5\mu} = 4K\Omega$

$C_T = 0.5fF/\mu^2 \times 2\mu \times 50\mu = 50fF$

(b) ~~$t_1 = (4K\Omega \,//\, 4K\Omega) \cdot 50fF = 100ps$~~

~~$t_2 = 8K\Omega \cdot 500fF = 4000ps$~~

~~$\Rightarrow t_{PD} = t_1 + t_2 = 4100ps = 4.1ns$;~~

$t_1 = (4K\Omega \,//\, 28K\Omega) \cdot 50fF = 175ps$

$t_2 = (12K\Omega \,//\, 20K\Omega) \cdot 50fF = 375ps$

Problem # 3 (continued)

$t_3 = (20k\Omega \parallel 12k\Omega) \cdot 50fF = t_2 = 375ps$

$t_4 = (28k\Omega \parallel 4k\Omega) \cdot 50fF = t_1 = 175ps$

$t_5 = 32k\Omega \cdot 500fF = 16000\,ps$

$\Rightarrow t_{PD} = \sum_{i=1}^{5} t_i = 17100ps = 17.1ns$

Problem # 4. (a) $P = f C \cdot V^2$; $V = V_{DD}$; $C = C_{ox} \cdot (W_n \cdot L_n + W_p \cdot L_p)$

$\Rightarrow P = 10kHz \times (2.5fF/\mu^2 \times 50\mu \times 0.5\mu) \times (3.5V)^2$

$P = 7.66\,nW$

(b) if $W_n = W_p = 0.5\mu$.

$P = 7.66nW \times \frac{1\mu}{50\mu} = 0.15\,nW$

(c) $R_{PU} = \frac{L_p}{\mu_p \cdot C_{ox} \cdot W_p \cdot (V_{DD} + V_{TP})} = \frac{\cancel{0.5}\ \cancel{40\mu}\ 0.5\mu}{\frac{100\mu A}{3V^2} \times 40\mu \times 3V} = \frac{3 \times 0.5}{100\mu \times 40 \times 3}\,\Omega$

$R_{PU} = 0.125k\Omega = \frac{1}{8}k\Omega$

$R_{PD} = \frac{L_n}{\mu_n \cdot C_{ox} \cdot W_n \cdot (V_{DD} - V_{Tn})} = \frac{0.5\mu}{100\mu \times 10\mu \times 3V} = \frac{1}{6}k\Omega$

$C_L = 600fF + (2.5fF \times 50 \times 0.5) = 662.5fF$

$t_{HL} = R_{PD} \cdot C_L = 0.11ns$

$t_{LH} = R_{PU} \cdot C_L = 0.0828ns$

$\Rightarrow clk_0 = \frac{1}{t_{HL}} = \frac{1}{0.11ns} = 9.09\,GHz$

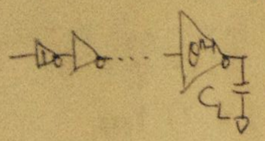
Problem # 5. $P = f \cdot C \cdot V^2$; $V = V_{DD} = 3.5V$; $C = \frac{1}{2} fF \times 0.5 \times 10^6 = 0.5nF$

$\Rightarrow P = 1.5 \times 10^9 Hz \times 0.5nF \times (3.5V)^2$

$P = 1.5 \times 10^9 \times 0.5 \times 10^{-9} \times 3.5 \times 3.5\,W = \underline{9.2}\,W$

Problem #6. $C_{REF} = 2fF \times 0.5 \times 0.5 \times 2 = 1fF$

$n_{OPT} = \ln\left(\frac{25pF}{1fF}\right) = \ln(25 \times 10^3) = 10.1 \approx 10$



$\theta^n = \frac{25pF}{1fF} = 25000 = \theta^{10} \Rightarrow \theta = \underline{\underline{2.753}}$

$C_{final} = \frac{C}{\theta}$; $P = f \cdot C_{final} \cdot V_{DD}^2$

$P = 300MHz \times \left(\frac{25pF}{2.753}\right) \times (3.5V)^2 = 33.37mW$

Problem #7. $P = f \cdot C \cdot V^2$, $V = 1.5V$; $f = 400MHz$

$C = 32 \times 4pf = 128pf$

$\Rightarrow P = 128pf \times 400MHz \times (1.5V)^2 = 0.1152W$

Problem #8. The more stages we use, the more power the circuit consumes. We can assume that $clk \approx 100MHz$

namely, $t_{PROP} = \frac{1}{clk} = \frac{1}{100MHz} = \underline{\underline{10ns}}$

$t_{PROP} = t_{REF} \cdot \frac{\theta}{\ln(\theta)} \left[\ln\frac{C_L}{C_{REF}}\right]$

$C_L = 600fF$; $C_{REF} = 2 \times 2fF \times 0.5 \times 0.5 \times 2 = 2fF$

$t_{REF} = R_{REF}\left(\frac{C_{REF}}{2}\right) = \frac{0.5\mu}{100\mu A \times 0.5\mu \times 3V} \times 2fF = 6.67 \times 10^{-12}s$

$\Rightarrow \frac{\theta}{\ln\theta} = \frac{10ns}{40.68ps} = 234$ $\Rightarrow \theta \approx 300$

$n = \frac{1}{\ln\theta} \cdot \ln\left(\frac{600fF}{1fF}\right) = 2$

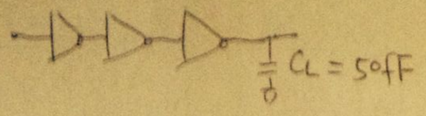
Problem #9. $clk = \frac{1}{t_{PROP}}$; $t_{PROP} = t_{REF} \cdot \frac{\theta}{\ln\theta}\left(\ln\frac{C_L}{C_{REF}}\right)$

$\theta = e$, $n = \ln\left(\frac{C_L}{C_{REF}}\right)$; $t_{PROP} = t_{REF} \cdot e \cdot \ln\left(\frac{C_L}{C_{REF}}\right)$

$n = 8 \Rightarrow t_{PROP} = 1.67ps \times e \times 8 = 35ps$

$\Rightarrow clk = 29GHz$

Problem # 10. (a)



Without OD, $F_{I2}=1$, $F_{I3}=1$, $F_{I4}=\frac{50}{2}=25$

$t_{PROP} = t_{REF}\cdot(1\cdot1+1\cdot1+25\cdot\frac{1}{1}) = 27\cdot t_{REF}$

(b) $F_{I2}=\frac{2}{3}$ $\quad F_{I3}=\frac{2}{3}$ $\quad F_{I4}=25$

$t_{PROP}=t_{REF}\cdot(\frac{2}{3}\cdot1+\frac{2}{3}\cdot1+25\cdot\frac{1}{1})=\frac{79}{3}\cdot t_{REF}$

(c) $P=C\cdot f\cdot V^2$, $V=V_{DD}=3.5V$

$C=4W_{min}\cdot L_{min}\cdot C_{ox}\cdot3+C_L=6fF+50fF=56fF$

$P=56fF\times10MHz\times(3.5V)^2=6.86\mu W$

(d) $C=2W_{min}\cdot L_{min}\cdot C_{ox}\cdot3+C_L=3fF+50fF=53fF$

$P=53fF\times10MHz\times(3.5V)^2=6.49\mu W$

Problem # 11. For 6-input Nor gate: $W_n=W_{min}$, $W_p=18W_{min}$, $L_n=L_p=L_{min}$

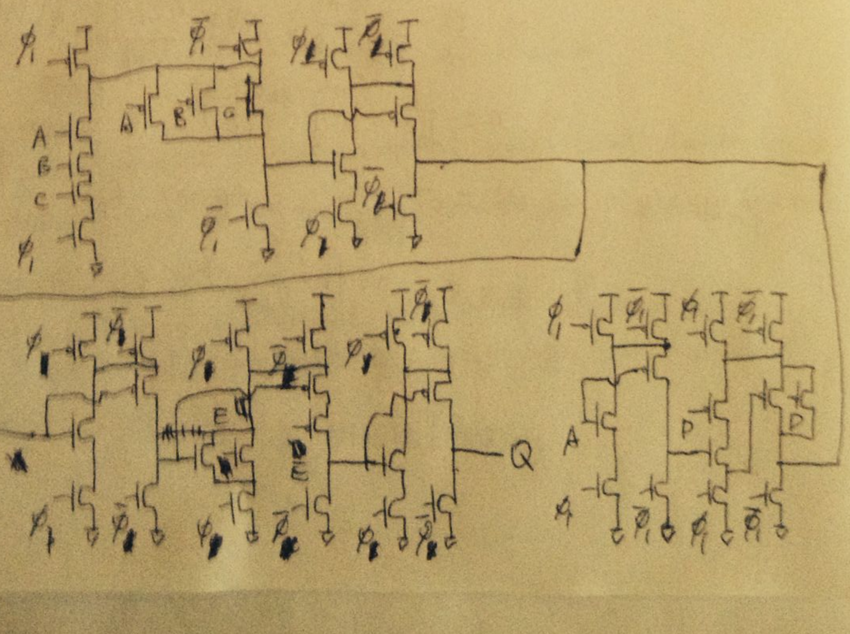
$P_1=f\cdot C_1V^2$, $V=V_{DD}$, $C=6\times19W_{min}\cdot L_{min}C_{ox}=\underline{114W_{min}L_{min}C_{ox}}$

For domino logic Nor gate:

$P_2=f\cdot C_2\cdot V^2$, $V=V_{DD}/2$, $\Rightarrow P_2=\underline{\frac{1}{4}}\cdot P_1$

Problem #12.

$Q=(ABC+\bar{A}D)\cdot E$



Problem # 13. ring oscillator: $f = \frac{1}{2 \cdot t \cdot n}$

t: time delay of single inverter;

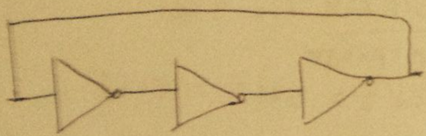
n: number of inverter stages;

$$2 \cdot t \cdot n = \frac{1}{80MHz} \Rightarrow t \cdot n = \frac{1}{2 \times 80 \times 10^6 Hz} = 6.25\ ns$$

$t_{ref} = \frac{1}{300} ns$ and $t \propto L^2$

So, we can use $L_n = 2L_{min}$ $W_n = W_{min}$

$\Rightarrow n = 3$ $L_p = L_n$ $W_p = 3 \cdot W_n$



Problem # 15 $R = \frac{L_{min}}{\mu_n \cdot C_{ox} \cdot W_5 \cdot (V_{DD} - V_{Tn})}$

$C = 2 \cdot W_{min} \cdot L_{min} \cdot C_{ox}$

$\frac{1}{2} \cdot \frac{1}{Clk} = R \cdot C$

$$\Rightarrow \frac{L_{min}}{\mu_n \cdot C_{ox} \cdot (V_{DD} - V_{Tn})} \cdot \frac{1}{W_5} \cdot 2 \cdot W_{min} \cdot L_{min} \cdot C_{ox} = \frac{1}{2 \cdot Clk}$$

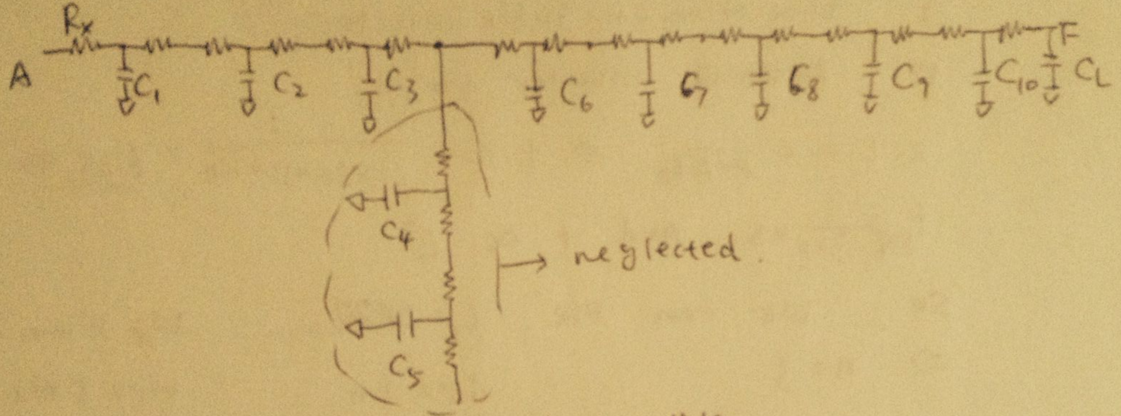
$$\Rightarrow W_5 = \frac{L_{min}^2 \cdot 4 \cdot Clk}{\mu_n \cdot (V_{DD} - V_{Tn})} \cdot W_{min}$$

Problem # 14.

~~for~~ the difference between unfooted and footed domino logics is that there is no the foot transistor in the unfooted domino logic gate.

the foot transistor ensure nMOS array disabled during precharge. Thus the unfooted domino logic gate consumes less energy.

Problem # 16.



$$R_x = \frac{200hms}{2} \times \frac{50\mu \times 2\mu}{0.5\mu \times 0.5\mu} = 4\,K\Omega$$

$$C_1 = 0.5fF \times 50 \times 2 = 50fF = C_L = 50fF$$

the problem is the same with the problem # 1.

So the delay $t_{PD} = 7.5ns$