

# Logical Agents

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## Outline

I. Knowledge-based agents

II. The Wumpus world

III. Logic

# I. Knowledge-Based Agents

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- ♣ Problem solving agents do not know general facts.

An 8-puzzle agent does not know that two tiles cannot occupy the same space.

- ♣ Their atomic representations are very limited.

e.g., a list of all possible concrete states.

# I. Knowledge-Based Agents

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An 8-puzzle agent does not know that two tiles cannot occupy the same space.

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e.g., a list of all possible concrete states.

- ♦ Intelligent agents need *knowledge about the world* in order to carry out reasoning for good decision making.

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An 8-puzzle agent does not know that two tiles cannot occupy the same space.

- ♣ Their atomic representations are very limited.

e.g., a list of all possible concrete states.

- ♦ Intelligent agents need *knowledge about the world* in order to carry out reasoning for good decision making.

- Represent states, actions, etc.
- Incorporate new percepts.
- Update internal representation of the world.
- Deduce hidden properties of the world.
- Deduce appropriate actions.

# Knowledge Base

---

A *knowledge base (KB)* is a set of sentences that represent some assertion about the world.

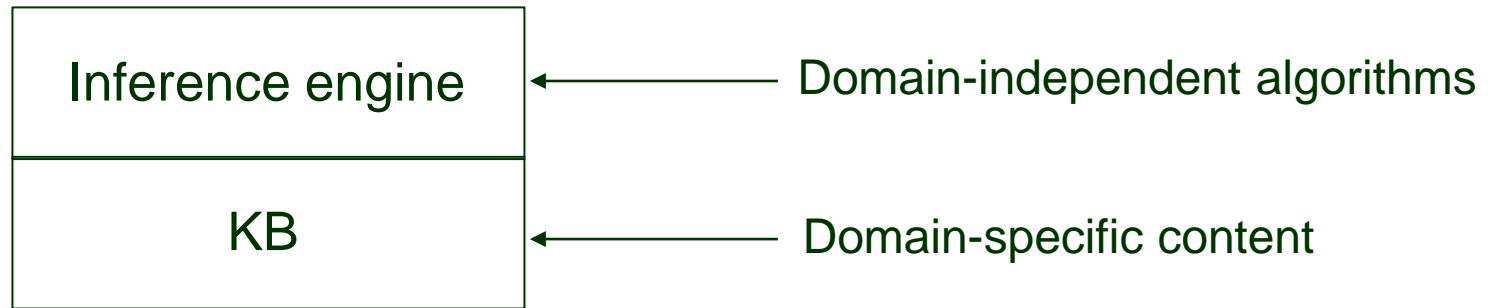
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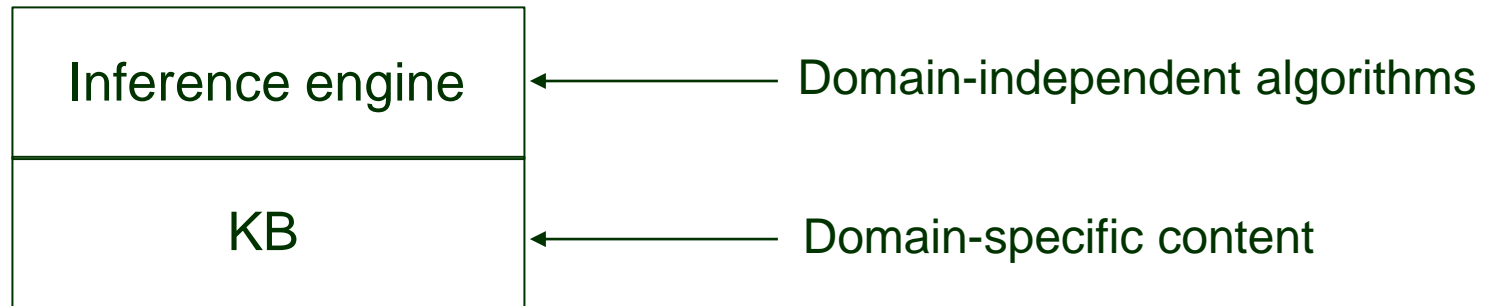


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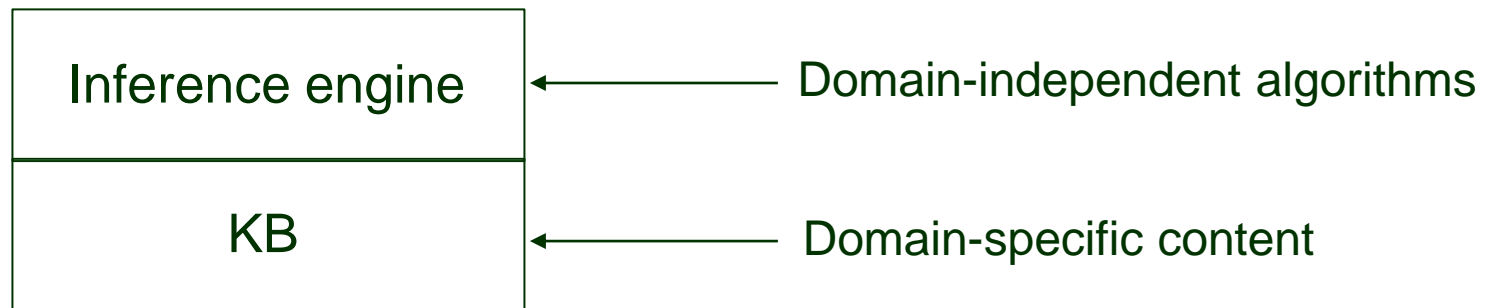
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**TELL:** Add new sentences to the KB.

**ASK:** Query the KB.

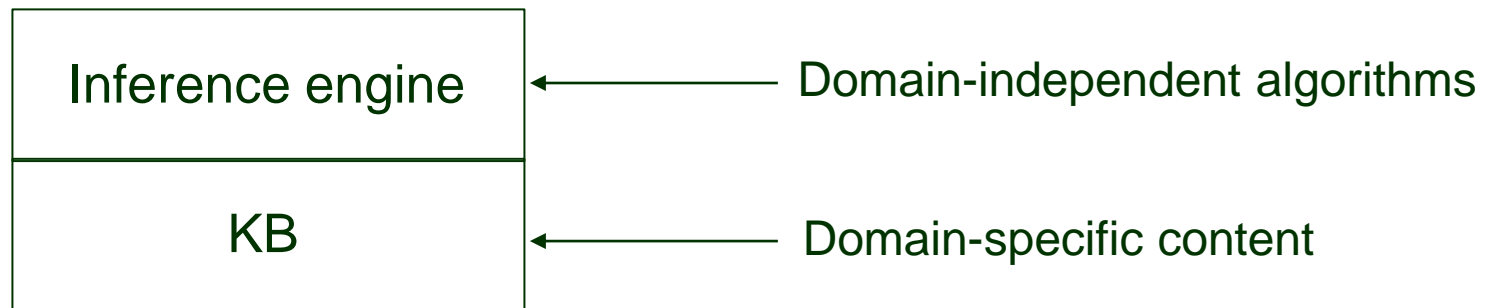


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An *axiom* is such a sentence that is taken to be true without being derived from other sentences.



**TELL:** Add new sentences to the KB.

**ASK:** Query the KB.

**Inference:** Derive new sentences from old.

# Generic Knowledge-Based Agent

---

**function** KB-AGENT(*percept*) **returns** an *action*

**persistent:** *KB*, a knowledge base

*t*, a counter, initially 0, indicating time

TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))

*action*  $\leftarrow$  ASK(*KB*, MAKE-ACTION-QUERY(*t*))

// asks what action  
// it should perform.

TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))

// tells what action

*t*  $\leftarrow$  *t* + 1

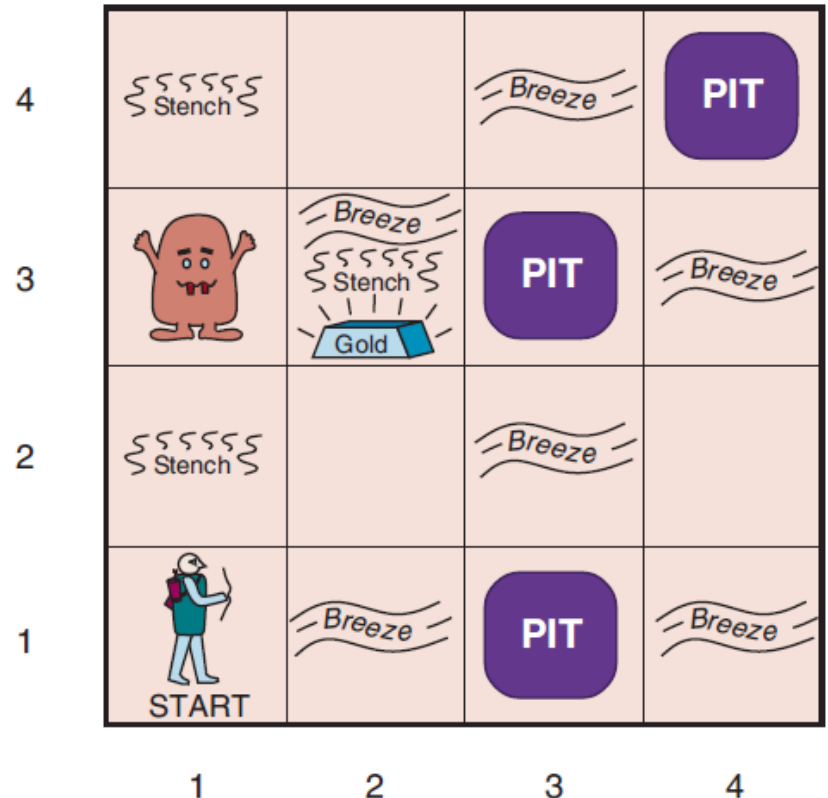
// was chosen

**return** *action*

## II. The Wumpus World

Cave consisting of connected rooms.

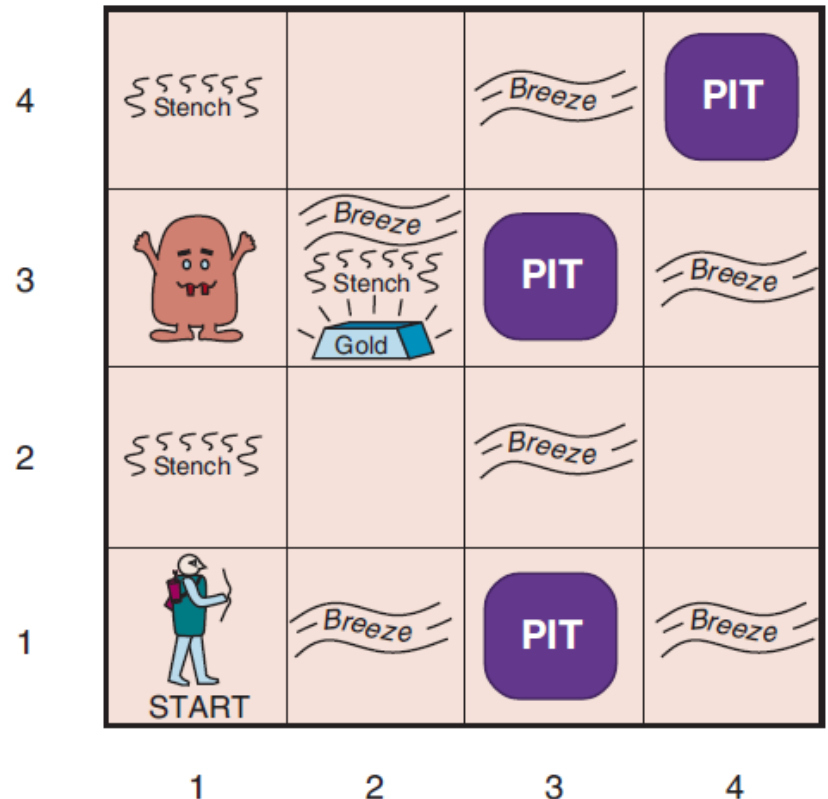
- Some rooms contain pits that will trap whoever enters them.
- The wumpus lurks in one room ready to eat whoever enters the room.
- The wumpus can be shot by the agent, who has **only one** arrow.
- A heap of gold is in a different room than where the wumpus lurks.



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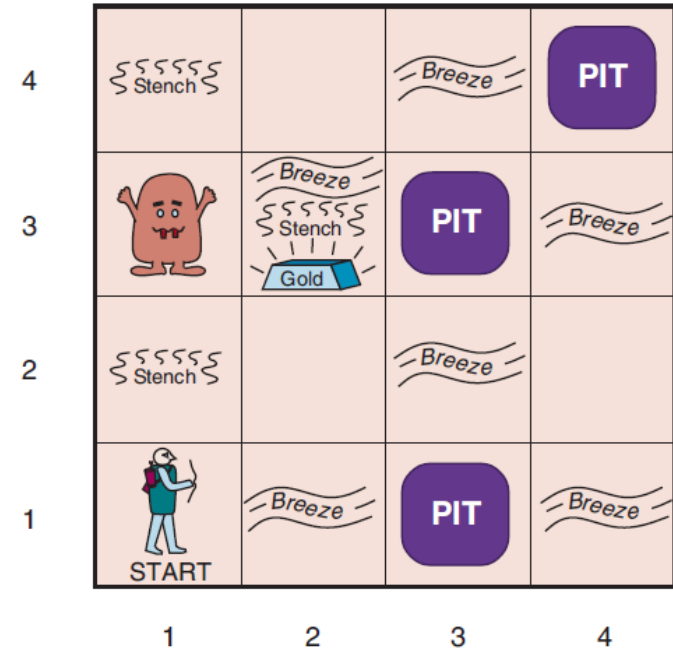


**Goal:** Find the gold and bring it back to the start without getting killed.

# Task Environment

## Performance measure

- +1000 (climbing out of the cave with the gold)
- -1000 (falling into a pit or being eaten by the wumpus)
- -1 (each action taken)
- -10 (using up the arrow)



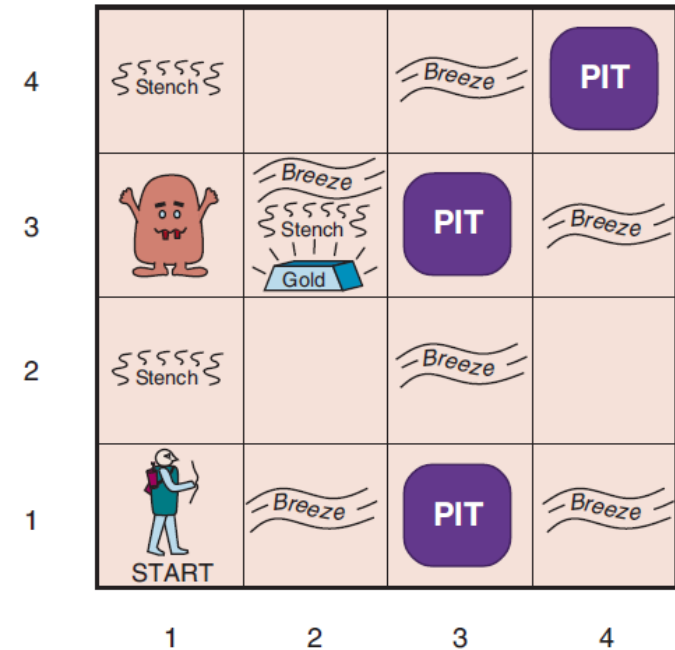
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## Environment

- $4 \times 4$  grid surrounded by walls
- [1,1]: the starting square for the agent, who faces east
- locations of the gold and the wumpus:
  - different from [1, 1]
  - otherwise randomly generated under uniform distribution
- 0.2 probability for a square other than [1, 1] and without gold or wumpus to be pit



# Actuators

## Actuators:

### 1) *Forward*, *TurnLeft* by 90°, *TurnRight* by 90°

- Death of the agent if it enters a square containing a pit or a live wumpus.
- No movement if bumping into a wall.

### 2) *Grab*

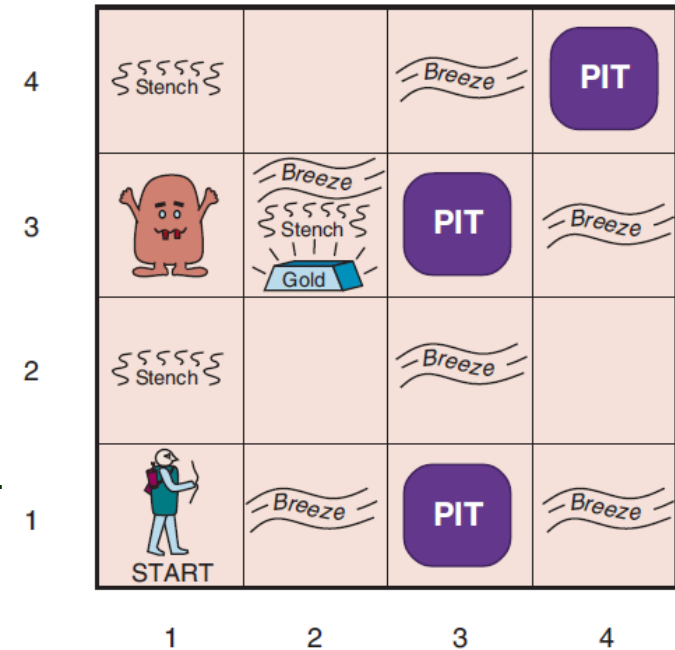
- Picks up the gold if it is in the same square as the agent.

### 3) *Shoot*

- Fire an arrow in the direction the agent is facing.
- The arrow continues until hitting the wumpus (who gets killed consequently) or a wall.

### 4) *Climb*

- Climb out of the cave if at [1, 1].



# Sensors

---

5 Sensors, each providing one bit of information:

1) *Stench*

- in the squares directly (not diagonally) adjacent to the wumpus

2) *Breeze*

- in the squares directly (not diagonally) adjacent to a pit

3) *Glitter*

- in the square where the gold is

4) *Bump*

- when the agent walks into a wall

5) *Scream*

- when the wumpus is killed



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- in the square where the gold is

4) *Bump*

- when the agent walks into a wall

5) *Scream*

- when the wumpus is killed

Percepts in the form of a 5-vector:

e.g., [*Stench*, *Breeze*, *None*, *None*, *None*]

# Characteristics of WW

---

- ◆ Deterministic, discrete, static, and single-agent

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Outcome specified.

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The wumpus does not move.

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Locations of the pits and the wumpus are unknown.

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# Solution by a Knowledge-Based Agent

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1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2	3,2	4,2
1,1 A OK	2,1 OK	3,1	4,1

**A** = Agent  
**B** = Breeze  
**G** = Glitter, Gold  
**OK** = Safe square  
**P** = Pit  
**S** = Stench  
**V** = Visited  
**W** = Wumpus



# Solution by a Knowledge-Based Agent

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1,4	2,4	3,4	4,4
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Percept: [None, None, None, None, None]

# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
1,1 <b>A</b> OK	2,1 OK	3,1	4,1

**A** = Agent  
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Percept: [None, None, None, None, None]



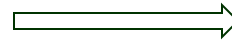
[1,2] and [2, 1] are free of dangers.

# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
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Forward



1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

Percept: [None, None, None, None, None]



[1,2] and [2, 1] are free of dangers.

# Solution by a Knowledge-Based Agent

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
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Percept: [None, None, None, None, None]



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[None, Breeze, None, None, None]

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Percept: [None, None, None, None, None]



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Percept: [None, None, None, None, None]



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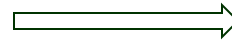
A pit in [1,1], [2,2], or [3, 1].

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1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

Percept: [None, None, None, None, None]



[1,2] and [2, 1] are free of dangers.

[None, Breeze, None, None, None]



A pit in [1,1], [2,2], or [3, 1].



[1,1] has just been visited.

A pit in [2,2] or [3, 1].

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1,3	2,3	3,3	4,3
1,2  OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 <div>A</div> B OK	3,1 P?	4,1

Only one unexplored square [1,2] is OK.

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1,4	2,4	3,4	4,4
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1,3	2,3	3,3	4,3
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1,1 OK V OK	2,1 A B OK	3,1 P?	4,1



1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 A S OK	2,2 OK	3,2	4,2
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1,3	2,3	3,3	4,3
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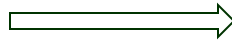
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*[Stench, None, None, None, None]*

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1,3	2,3	3,3	4,3
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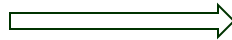
*[Stench, None, None, None, None]*



The wumpus is in [1,1], [2,2], or [1, 3].

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1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2 P?	3,2	4,2
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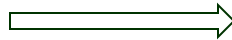
The wumpus is in [1,1] , [2,2] , or [1, 3].

[1,1] is OK

[2,2] is impossible because no stench was detected at [2, 1].

# Next Step

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1



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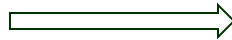
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Only one unexplored square [1,2] is OK.

Be prudent: Turn around, go back to [1,1] and move onto [1,2].

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The wumpus is in [1,1] , [2,2] , or [1, 3].

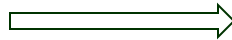


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The wumpus is in [1, 3].

# Next Step

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1



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# More Inference

1,4	2,4 P?	3,4	4,4
1,3 W!	2,3 A S G B	3,3 P?	4,3
1,2 S V OK	2,2 V OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

[Stench, None, None, None, None]



The wumpus is in [1, 3].

# More Inference

1,4	2,4 P?	3,4	4,4
1,3 W!	2,3 A S G B	3,3 P?	4,3
1,2 S V OK	2,2 V OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

[Stench, None, None, None, None]



The wumpus is in [1, 3].



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A conclusion drawn is guaranteed if the available information is correct.

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Every sentence must be either true or false in each possible world.



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Model  $m$ : assigns values to variables.

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**Example**  $x = 0$  entails  $xy = 0$ .

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1,3	2,3	3,3	4,3
1,2 OK	2,2 P? 	3,2	4,2
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Knowledge base (KB) includes

- All the rules.
- Percepts:
  - [None, None, None, None, None]* in [1,1]
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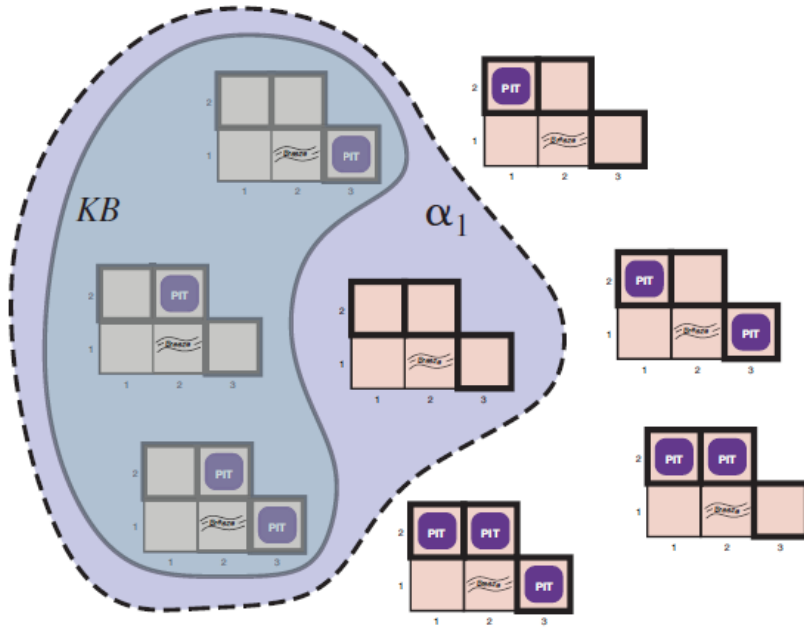
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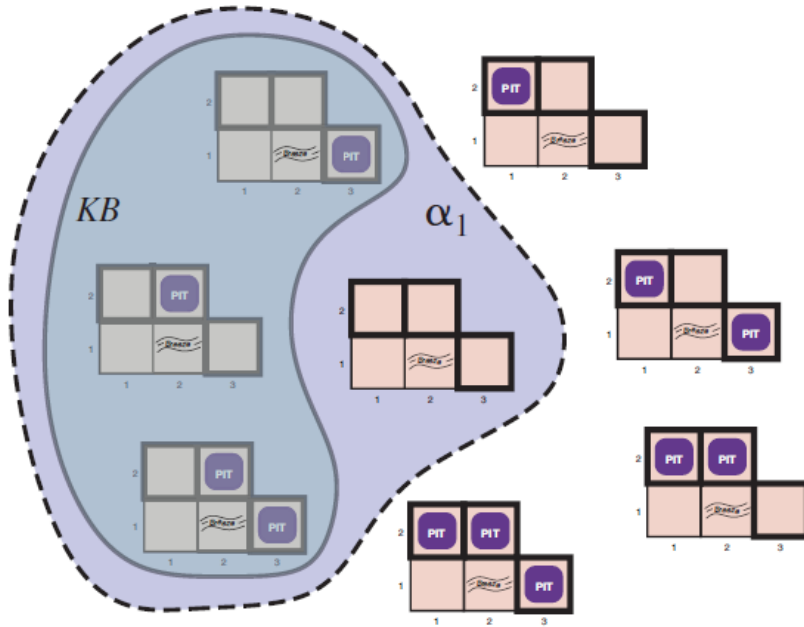
8 possibilities if ignoring the KB.

# Which Neighbors Contain a pit?



8 possible models for the presence of pits in squares [1,2], [2,2], and [3,1].

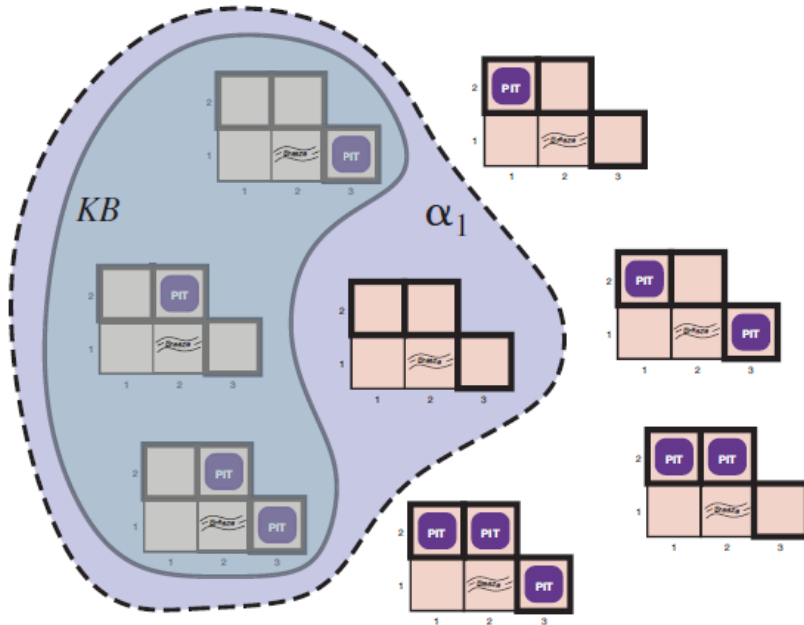
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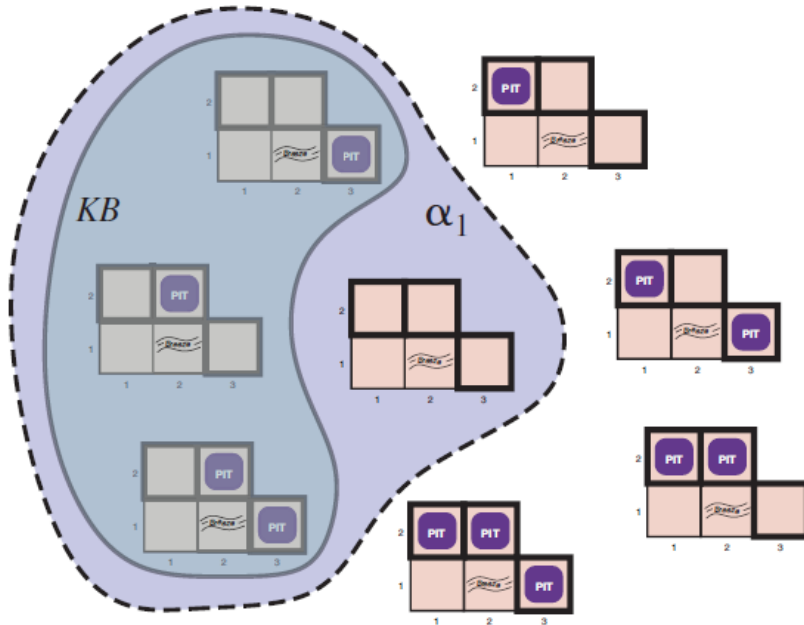


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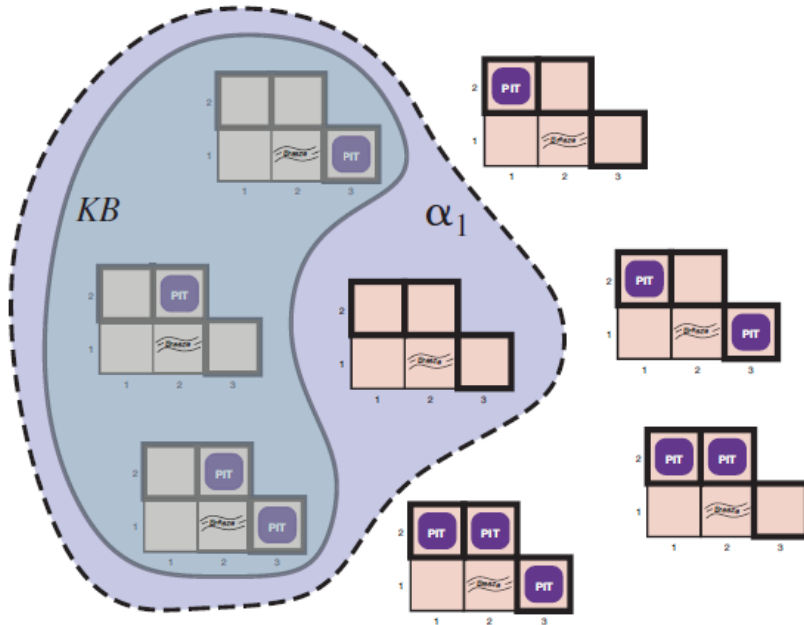
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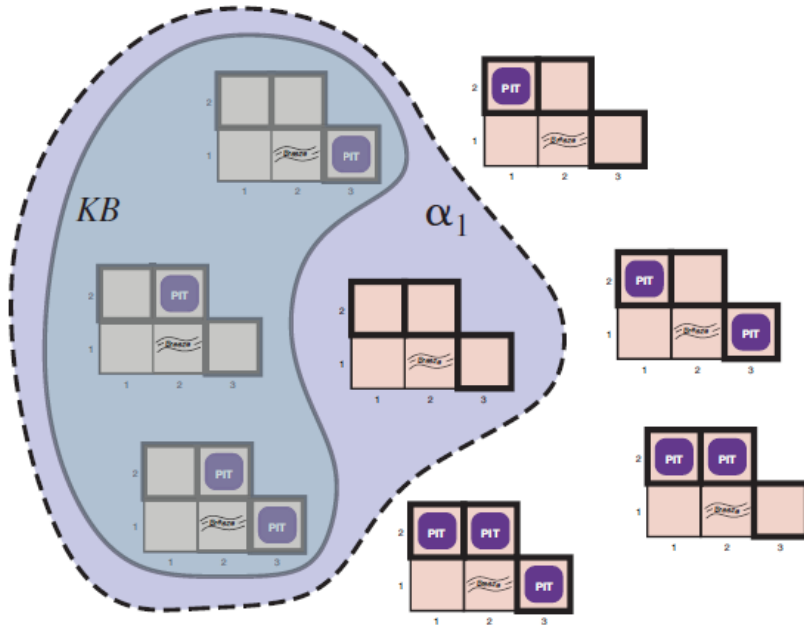
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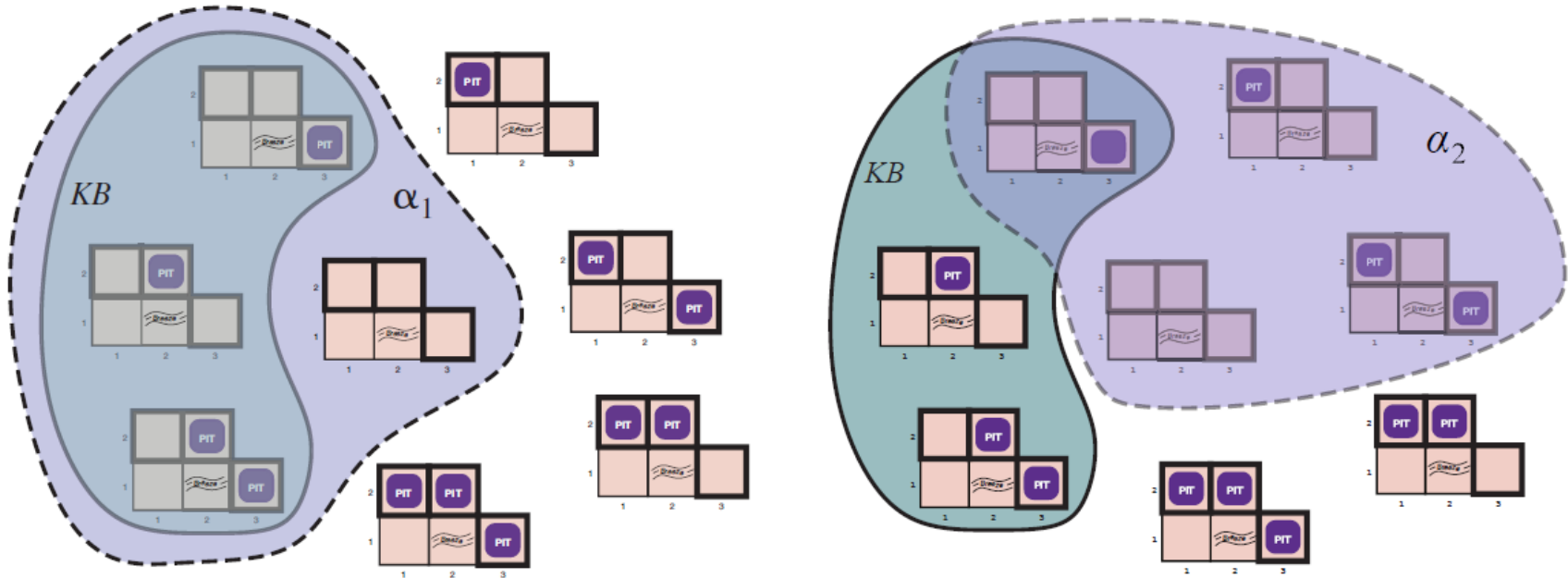
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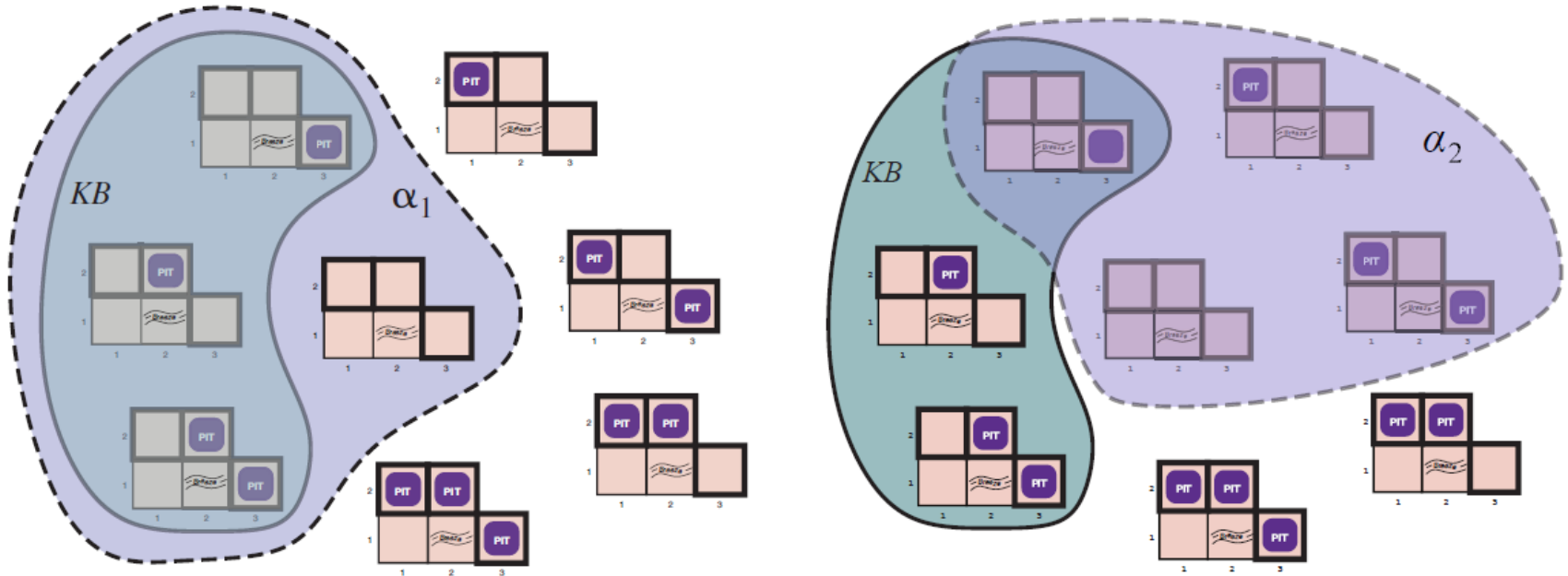
$\alpha_2$  = "There is no pit in [2, 2]."

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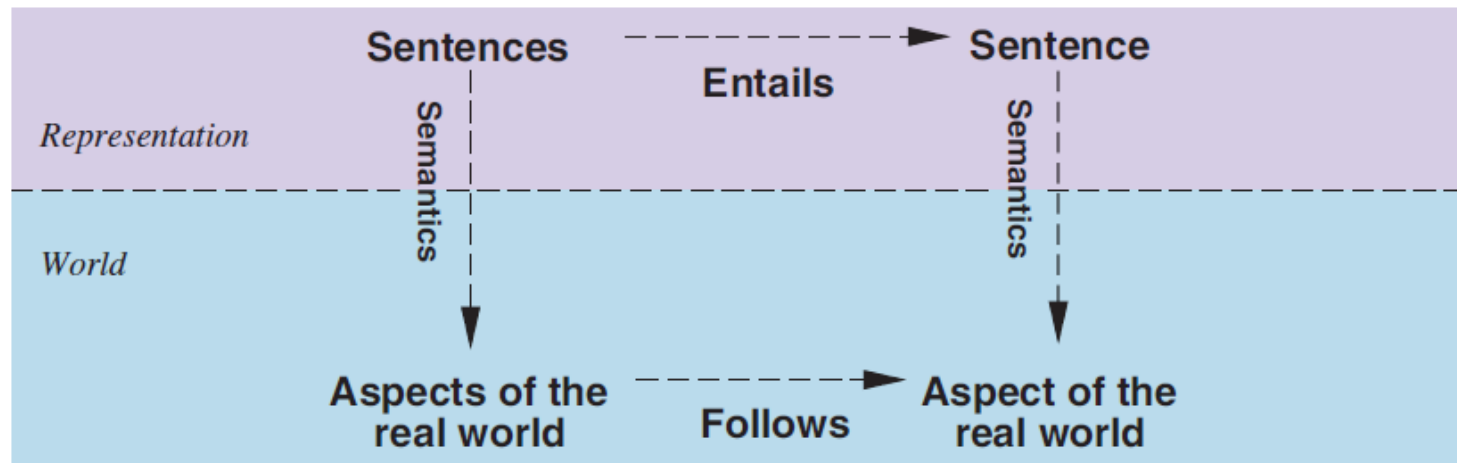
- ♣ It is *complete* if it can derive any sentence that is entailed, that is,

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# Logical Reasoning

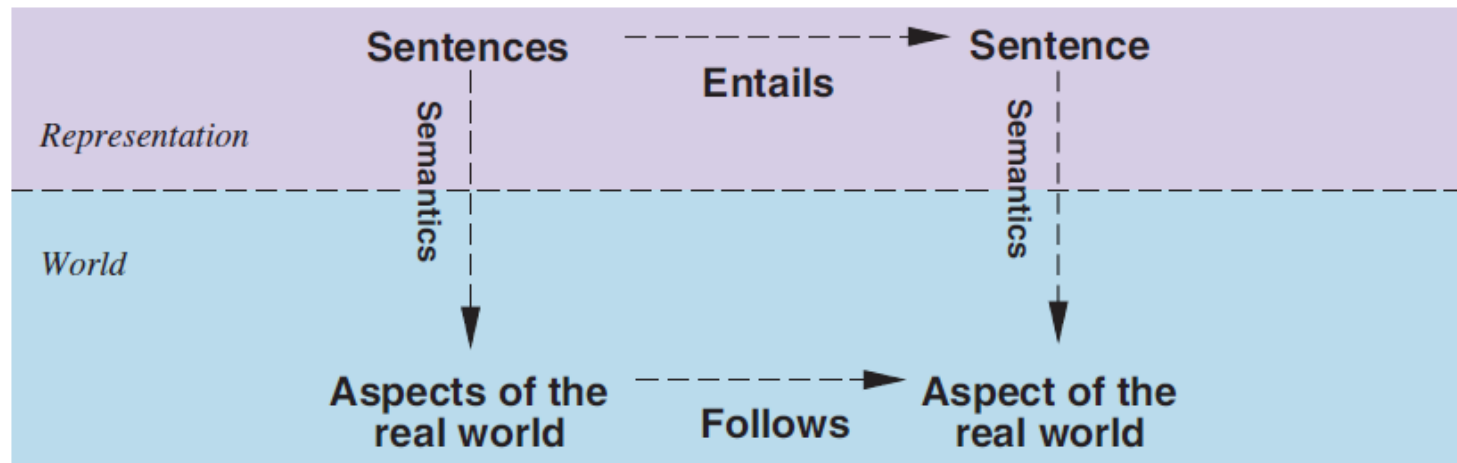
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