Homework 5

Please scan and upload your assignments to BBLearn on or before March 27, 2018.

- You must do your work independently and on your own. That means no collaborations!
- However, you *can* ask questions about the homework on Piazza. You can also answer others' questions. It is possible that your question is already answered there, so check Piazza regularly.
- Scores on late submissions will be penalized by 50% for every day submitted late. Be on time!

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1. **(10 points)** Suppose you want to write a program that collects information on a customer's tastes, and customize web content. By monitoring online shopping habits, you are able to collect pairwise preferences of a customer on some given set of products X. If x and y are two products in X, we say that $x \preceq y$ if the customer prefers y over x. (For simplicity, also assume by convention that $x \preceq x$ for all $x \in X$, and also that $\preceq$ is a transitive relation.) Suppose you know that your customer prefers:
   - lettuce over broccoli
   - cabbage over broccoli
   - tomatoes over cabbage
   - carrots over cabbage
   - carrots over lettuce
   - asparagus over lettuce
   - mushrooms over tomatoes
   - corn over tomatoes
   - corn over carrots
   - eggplant over carrots
   - eggplant over asparagus
   - onions over mushrooms
   - onions over corn

Answer the following questions: a. Draw the Hasse diagram for $(X, \preceq)$. b. What is/are the customer's favorite vegetable(s)? (i.e., what is/are the maximal elements?) What is/are the least favorite vegetables? c. Use topological sorting to produce an ordering the vegetables according to the customer's preferences. d. Is this ordering unique?

2. **(10 points)** The National Quidditch League contains 32 teams.

   a. Suppose each team plays every other team exactly once in a given season. Model the season as a relation R in terms of its graph representation. Clearly say what the nodes and edges represent in this graph.
   b. Calculate how many games in total need to be played.
   c. How would your answer to part (b) change if each team now plays every other team *twice* every season (one at home and one away)?
   d. Generalize your answers to (b) and (c) if the League contains n teams, where n is any integer greater than or equal to 2.

3. **(10 points)** Consider the set $A = \{1, 2, 3\}$.

a. Enumerate all elements of its power set, pow(A).
b. Clearly express a bijective function between pow(A) and the set of all possible 3-bit strings. (E.g. 000, 001, 010, ... are 3-bit strings.)
c. Suppose we construct an (undirected) graph where nodes correspond to 3-bit strings, and an edge between nodes indicates that the two corresponding strings differ in exactly 1 location. Draw this graph, and clearly label each vertex.
d. What is the degree of each node in this graph?

4. **(10 points)** Recall that a *complete binary tree* is a binary tree where every non-leaf node has two children, while every leaf node has no children.

    a. Draw complete binary trees with 1, 2, 3, and 4 levels.
    b. If a complete binary tree has k internal (non-leaf) nodes, then verify that it has $k + 1$ leaf nodes (no need for a proof; you can just check your answers.)
    c. *By applying the First Degree theorem*, prove that a complete binary tree with k internal nodes has $2k + 1$ total nodes.

5. **(10 points)** Recall that an *Euler* path is one that traverses all edges of a (connected) graph without repeating any edge. Using properties of Euler paths, figure out whether which of the following figures can be drawn without lifting your pen from the paper and without repeating any line.
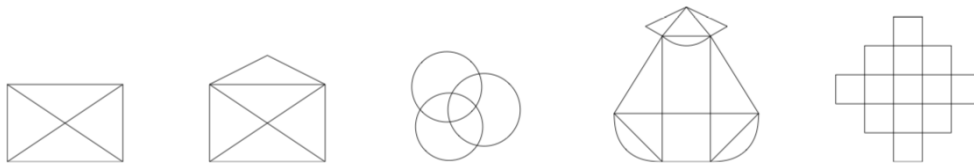

Figure 1: Can you draw these shapes without lifting your pen and repeating lines?