Stat 330 (Spring 2020): Homework 3 Due: February 5, 2020

Show all of your work, and *please* staple your assignment if you use more than one sheet. Write your name, the course number and the section on every sheet. Problems marked with * will be graded and one additional randomly chosen problem will be graded.

1. Proof Questions:

   (a) With conditional probability, $\mathbb{P}(A|B)$, the axioms of probability hold for the event on the left side of the bar. A useful consequence is applying the complement rule to conditional probability. We have that $\mathbb{P}(A|B) = 1 - \mathbb{P}(\overline{A}|B)$.

   Prove this by showing that $\mathbb{P}(A|B) + \mathbb{P}(\overline{A}|B) = 1$ (Hint: just use the definition of conditional probability, a proof should be very short).

   (b) If two events A and B are independent, then we know $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$. A fact is that if A and B are independent, then so are all combinations of $A, \overline{B}, \ldots$ etc.

   Show that if events A and B are independent, then $\mathbb{P}(\overline{A} \cap \overline{B}) = \mathbb{P}(\overline{A})\mathbb{P}(\overline{B})$, and thus $\overline{A}$ and $\overline{B}$ are independent. (Hint: $\mathbb{P}(\overline{A} \cap \overline{B}) = 1 - \mathbb{P}(A \cup B)$. Then use addition rule and simplify.)

2. A computer has a dual-core processor. At any time, the probability that each of the processors are active is

		Processor 2 In Use	Not In Use	
Processor 1	In Use	0.28	0.12	0.40
	Not In Use	0.42	0.18	0.60
		0.70	0.30	

   Let A be the event that processor 1 is in use and B be the event that processor 2 is in use.

   (a) From the table, give $\mathbb{P}(A), \mathbb{P}(B)$, and $\mathbb{P}(A \cap B)$

   (b) Calculate $\mathbb{P}(A|B)$.

   (c) Calculate $\mathbb{P}(B|A)$

   (d) Are the events A and B independent? Why or why not?

3. Suppose you have two urns with poker chips in them. Urn I contains two red chips and four white chips. Urn II contains three red chips and one white chip. You randomly select one chip from urn I and put it into urn II. Then you randomly select a chip from urn II.

   (a) What is the probability that the chip you select from urn II is white?

   (b) Is selecting a white chip from urn I and selecting a white chip from urn II independent? Justify your answer numerically.

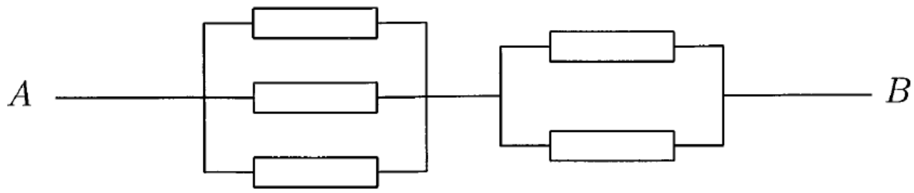
4. * A diagnostic test has a 98% probability of giving a positive result when given to a person who has a certain disease. It has a 10% probability of giving a (false) positive result when given to a person who does not have the disease. It is estimated that 15% of the population suffers from this disease.

   (a) What is the probability that a test result is positive?

   (b) A person receives a positive test result. What is the probability that this person actually has the disease?

   (c) A person receives a positive test result. What is the probability that this person does not actually have the disease?

5. In the following system, each component *fails* with probability 0.3 independently of other components. Compute the systems reliability.



6. * Calculate the reliability of each system show below, if components A, B, C, and D function properly (independently of each other) with probabilities 0.95, 0.9, 0.8, and 0.7 respectively.