Write the Null Hypothesis and Alternative Hypothesis for each of the problems below.

1. A start-up company purchases commercials during the Super Bowl to increase their name recognition among the public. The company's goal is to have over 40% of the public recognize its brand name and associate it with computer equipment. The day after the game, a pollster contacts 200 randomly selected adults and finds that 84 of them know this computer makes printers. Is this evidence that the company met their goal?

   Answer:

   $$H_0 : p = 0.4$$
   $$H_A : p > 0.4$$

2. An administrator at a large school district believes the average IQ of the district's students is 110. In a random sample of 100 students from the school district, the mean IQ was found to be 112 with a standard deviation of 10. Is there enough evidence to dispute the administrator's claim?

   Answer:

   $$H_0 : \mu = 110$$
   $$H_A : \mu \neq 110$$

3. A survey of 66 randomly chosen adults was taken as part of the National Health and Nutrition Examination Survey (NHANES) from 2009-2010. In the survey, 33 adults said they smoke and 33 said they do not smoke. The mean age of the adults that smoke was 48.18 with a standard deviation of 18.07. The mean age of adults that do not smoke was 57.39 with a standard deviation of 15.44. Is there evidence that the mean age of smokers is smaller than non-smokers?

   Answer: Let group 1 be smoking group, and group 2 be non-smoking group.

   $$H_0 : \mu_1 = \mu_2$$
   $$H_A : \mu_1 < \mu_2$$

4. Large-scale surveys were done with randomly selected American teenagers from across the United States: 2928 teens in 1988-1994 and 1771 teens in 2005-2006. The researchers found that 14.9% of the teens in the 1988-1994 sample group had some hearing loss, compared to 19.5% of teens in the 2005-2006 sample group. Is the population proportion of teens that had hearing loss greater in 2005-2006 compared to 1988-1994?

   Answer: Let group 1 be $2005-2006$ group, and group 2 be $1988-1994$ group.

   $$H_0 : p_1 = p_2$$
   $$H_A : p_1 > p_2$$

1 Start-up Company

Hypotheses

$H_0: p = 0.4$

$H_A: p > 0.4$

Test-statistic

$$Z = \frac{\hat{p} - 0.4}{\sqrt{\frac{0.4(1-0.4)}{n}}} \sim N(0,1)$$

plugging in data

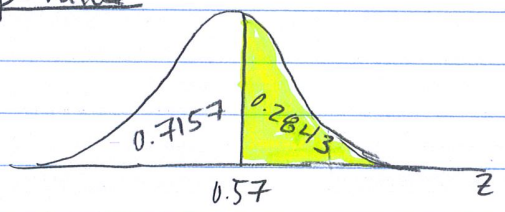
$$Z = \frac{0.42 - 0.4}{\sqrt{\frac{0.4(1-0.4)}{200}}} = 0.57$$

data

$n = 200$

$\hat{p} = 84/200 = 0.42$

p-value



since ">" in H_A, we want right-hand area.

p-value $= P(Z > 0.57) = 1 - P(Z < 0.57) = 1 - 0.7157 = 0.2843$

→ since p-value is not very small, do not reject H_0.
→ There is not significant evidence that more than 40% of public recognize company's name
→ There is not evidence that company met their goal.

2. IQ

Hypotheses

$H_0: \mu = 110$

$H_A: \mu \neq 110$

Test-statistic

$$z = \frac{\bar{x} - 110}{s/\sqrt{n}} \sim N(0,1)$$

plugging in data

$$z = \frac{112 - 110}{10/\sqrt{100}} = 2.00$$

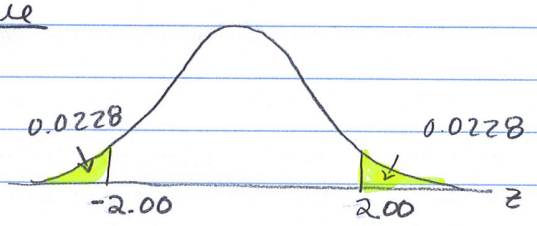
data

$\bar{x} = 112$

$s = 10$

$n = 100$

p-value



since "$\neq$" in H_A, we want right-hand area & left-hand area

p-value

$= P(|z| > 2) = P(z > 2) + P(z < -2)$

$= 2P(z < -2)$

$= 2(0.0228)$

$= 0.0456$ = p-value

→ Since the p-value is pretty small, we reject the null hypothesis in favor of alternative hypothesis.

→ There is evidence that true mean IQ of students is not 110.

→ There is evidence to dispute the administrator's claim.

3 Smoking vs. Non-smoking

group 1 = smoking group 2 = non-smoking

Hypotheses

$H_0: \mu_1 = \mu_2$

$H_A: \mu_1 < \mu_2$

Test-Statistic

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim N(0,1)$$

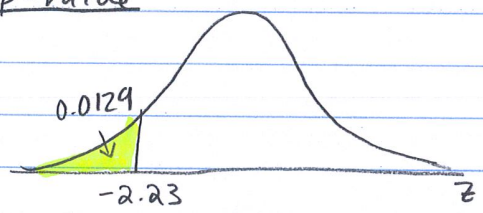
plugging in data

$$Z = \frac{48.18 - 57.39}{\sqrt{\frac{18.07^2}{33} + \frac{15.44^2}{33}}} = -2.23$$

data

$n_1 = 33$	$n_2 = 33$
$\bar{x}_1 = 48.18$	$\bar{x}_2 = 57.39$
$s_1 = 18.07$	$s_2 = 15.44$

p-value



since "<" in H_A, we want left-hand area.

p-value = $P(Z < -2.23) = 0.0129$

→ Since p-value is small, reject H_0 in favor of H_A

→ There's significant evidence that true mean age of smokers is smaller than non-smokers in 2009-2010.

4. Hearing Loss

group 1 = 2005 - 2006 group
group 2 = 1988 - 1994 group

Hypotheses

$H_0 : p_1 = p_2$

$H_A : p_1 > p_2$

Test-statistic

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}_{pool}(1-\hat{p}_{pool})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$$

plugging in data

$$Z = \frac{0.195 - 0.149}{\sqrt{0.166(1-0.166)}\sqrt{\frac{1}{1771} + \frac{1}{2928}}}$$

$= 4.11$

data

$n_1 = 1771 \quad n_2 = 2928$

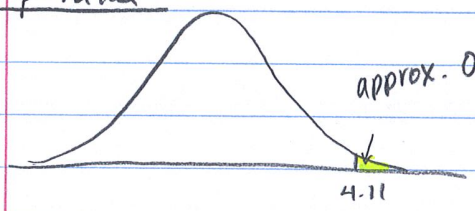
$\hat{p}_1 = 0.195 \quad \hat{p}_2 = 0.149$

$$\hat{p}_{pooled} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

$$= \frac{1771(0.195) + 2928(0.149)}{1771 + 2928}$$

$= 0.166$

p-value



since " > " in H_A, we want right-hand area.

p-value $= P(Z > 4.11) = 1 - P(Z < 4.11) \approx 1 - 1 = 0$

→ Since p-value is very small, reject H_0 in favor of H_A

→ There's strong evidence that pop. proportion of teens with hearing loss is larger in 2005-2006 compared to 1988-1994.