

## Homework 2

1. (10 points) Consider the following relation  $r$  with attributes  $A$ ,  $B$ ,  $C$ , and  $D$ .

$r$			
$A$	$B$	$C$	$D$
6	2	3	4
1	3	8	5
6	7	8	5

(a) Somehow we know that  $r$  has a key consisting of a single attribute. With this clue state which single attribute is a key and which cannot be. In each case informally provide good reasons.

(b) Prove that the functional dependency  $C \rightarrow D$  is satisfied by  $r$ . Give the most concise answer you can.

(c) Prove that  $r$  does not satisfy  $C \rightarrow B$ . Give the most concise answer you can.

(d) Prove that  $B \rightarrow ACD$  is satisfied by  $r$ . Give the most concise answer you can.

2. (15 points) Given a set of functional dependencies  $F = \{AG \rightarrow B, B \rightarrow CD, BD \rightarrow E, CE \rightarrow F\}$  over  $R = ABCDEFG$ .

(a) Prove that  $F \models AG \rightarrow BDF$ . (This also means that  $F$  logically implies  $AG \rightarrow BDF$ , or  $AG \rightarrow BDF$  can be deduced from  $F$ ).

(b) Compute  $(B)^+$ . ( $X^+$  is the set of all attributes  $A$  for which  $X \rightarrow A$  can be deduced from  $F$ .)

(c) Find a key of  $R$ .

3. (20 points) Give minimal covers of the following sets of functional dependencies

- (a)  $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
- (b)  $\{ABCD \rightarrow CDEF\}$
- (c)  $\{A \rightarrow BC, C \rightarrow D\}$
- (d)  $\{AB \rightarrow CD, A \rightarrow B, B \rightarrow C\}$
- (e)  $\{A \rightarrow B, ABCD \rightarrow E, EF \rightarrow GH, ACDF \rightarrow EG\}$

4. (15 points) Prove or disprove the following rules of inference:

- (a)  $XY \rightarrow Z$  infer  $X \rightarrow Z$ .
- (b)  $X \rightarrow YZ$  infer  $X \rightarrow Y$
- (c)  $\{X \rightarrow YZ, Y \rightarrow W\}$  infer  $Y \rightarrow Z$ .

5. (15 points) Given a relational schema R with attributes A, B, C, and D, where functional dependencies  $B \rightarrow ACD$  and  $C \rightarrow D$  are supposed to hold.

- (a) What are all the keys in ABCD?
- (b) Give example of a superkey in ABCD that is not a key.
- (c) Give example of a trivial functional dependency over ABCD.

6. (25 points) Given a relational schema R with attributes A, B, C and D where the functional dependencies  $AB \rightarrow C$ ,  $C \rightarrow D$ , and  $D \rightarrow A$  are supposed to hold.

- (a) Is R in BCNF? If yes, explain why. If not, list all violations.
- (b) If R is NOT in BCNF, give it a lossless BCNF decomposition.
- (c) Does your decomposition in (b) preserve the given functional dependencies? Explain.
- (d) Give a 3NF decomposition for R.
- (e) Does your decomposition in (d) preserve the given functional dependencies? Explain.

### Submission Instruction

*Please use Microsoft Words or other tools to type your answer. Don't handwrite. Submit your file in pdf format through Canvas.*

