Resolution in FOL

♠ Forward and backward chaining work with definite clauses only.

♦ Resolution works for any knowledge base.

Outline

I. Conversion to CNF

II. Resolution inference

* Figures are from the textbook site.

I. Conjunctive Normal Form (CNF)

Before inference, we need to convert sentences to CNF.

$$(l_{11} \vee l_{12} \vee \cdots \vee l_{1n_1}) \wedge \cdots \wedge (l_{k1} \vee l_{k2} \vee \cdots \vee l_{kn_k})$$

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$$\Downarrow$$

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- Every FOL sentence can be converted into an inferentially equivalent CNF sentence.
- The conversion procedure is similar to the propositional logic case, except for the need to eliminate $\exists$.

Conversion to CNF

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Moving ¬ Inward

$\forall x \quad \neg(\forall y \quad \neg Animal(y) \lor Loves(x, y)) \lor (\exists y \; Loves(y, x))$

Moving $\neg$ Inward

$$\forall x \;\; \neg(\forall y \;\; \neg Animal(y) \lor Loves(x, y)) \lor (\exists y \; Loves(y, x))$$

$$\Downarrow$$

$$\forall x \;\; (\exists y \;\; \neg(\neg Animal(y) \lor Loves(x, y))) \lor (\exists y \; Loves(y, x))$$

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"*Either there is some animal a person doesn't love, or (otherwise) someone loves that person*."

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$\Updownarrow$

"*Everyone who loves all animals is loved by someone*."

Standardization

c) Standardize variables: If two quantified variables have the same name, as in $(\forall x\ P(x)) \Rightarrow (\exists x\ Q(x))$, rename of one of the variables.

$$\forall x\ \ (\exists y\ \ Animal(y) \land \neg Loves(x, y)) \lor (\exists y\ Loves(y, x))$$

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rename

$$\forall x\ \ (\exists y\ \ \mathit{Animal}(y) \wedge \neg \mathit{Loves}(x, y)) \vee (\exists z\ \mathit{Loves}(z, x))$$

Skolemization

d) Skolemize:

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$$\forall x\ \ (\exists y\ \ Animal(y) \wedge \neg Loves(x, y)) \vee (\exists z\ Loves(z, x))$$

1st try: introduce constants

A and *B* respectively for y and z.

Skolemization

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$$\forall x \ \ (\exists y \ \ \mathit{Animal}(y) \land \neg \mathit{Loves}(x, y)) \lor (\exists z \ \mathit{Loves}(z, x))$$

$\Downarrow$

1st try: introduce constants

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$$\forall x \ \ (\mathit{Animal}(A) \land \neg \mathit{Loves}(x, A)) \lor \mathit{Loves}(B, x)$$

Skolemization

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$$\forall x \ (\mathit{Animal}(A) \land \neg \mathit{Loves}(x, A)) \lor \mathit{Loves}(B, x)$$

"Everyone either fails to love an animal *A* or is loved by some particular entity *B*."

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Both y and z depends on x, and in different ways.

Skolemization (cont'd)

$\forall x \ \ (\exists y \ \ \mathit{Animal}(y) \wedge \neg \mathit{Loves}(x, y)) \vee (\exists z \ \mathit{Loves}(z, x))$

2nd try: introduce Skolem functions $F(x)$ and $G(x)$ respectively for y and z.

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$\forall x \ (Animal(F(x)) \land \neg Loves\big(x, F(x)\big)) \lor Loves(G(x), x)$

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General case:

$$\forall x_1, \dots., x_n \exists y\ \ P(y, x_1, \dots., x_n)$$

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$\forall x_1, \ldots, x_n \exists y \ \ P(y, x_1, \ldots, x_n)$ // y depends on $x_1, \ldots, x_n$

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$$\forall x_1, \ldots, x_n \exists y \;\; P(y, x_1, \ldots, x_n)$$

// y depends on $x_1, \ldots, x_n$

eliminate y by introducing function f

$$P(f(x_1, \ldots, x_n), x_1, \ldots, x_n)$$

Skolemization – One More Example

$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \;\; P(s, t, u, v, w, x, y, z)$

Skolemization – One More Example

$$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \;\; P(s, t, u, v, w, x, y, z)$$

Replace s with a constant C_1
(i.e., a function with no argument).

Skolemization – One More Example

$$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \ \ P(s,t,u,v,w,x,y,z)$$

$$\Downarrow$$

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Replace t with another constant C_2. (t depends s and is a function of C_1. It is thus a constant as well.)

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$\forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1, C_2, u, v, w, x, y, z)$

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$\Downarrow$ Replace t with another constant C_2. (t depends s and is a function of C_1. It is thus a constant as well.)

$$\forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1, C_2, u, v, w, x, y, z)$$

Eliminate the two universal quantifiers in front of u and v.

Skolemization – One More Example

$$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \ \ P(s,t,u,v,w,x,y,z)$$

⇓ Replace s with a constant C_1 (i.e., a function with no argument).

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⇓ Eliminate the two universal quantifiers in front of u and v.

$$\exists w \forall x \forall y \exists z \ \ P(C_1,C_2,u,v,w,x,y,z)$$

Skolemization – One More Example

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$\forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1,C_2,u,v,w,x,y,z)$

Eliminate the two universal quantifiers in front of u and v.

$\exists w \forall x \forall y \exists z \;\; P(C_1,C_2,u,v,w,x,y,z)$

w depends on $C_1, C_2, u, v,$ among which only u, v are variables. Introduce a Skolem function f_1.

Skolemization – One More Example

$$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \; P(s,t,u,v,w,x,y,z)$$

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(i.e., a function with no argument).

$$\exists t \forall u \forall v \exists w \forall x \forall y \exists z \; P(C_1,t,u,v,w,x,y,z)$$

Replace t with another constant C_2. (t depends s and is a function of C_1. It is thus a constant as well.)

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Eliminate the two universal quantifiers in front of u and v.

$$\exists w \forall x \forall y \exists z \; P(C_1,C_2,u,v,w,x,y,z)$$

w depends on $C_1, C_2, u, v,$ among which only u, v are variables. Introduce a Skolem function f_1.

$$\forall x \forall y \exists z \; P(C_1,C_2,u,v,f_1(u,v),x,y,z)$$

Skolemization – One More Example

$$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \; P(s, t, u, v, w, x, y, z)$$

Replace s with a constant C_1
(i.e., a function with no argument).

$$\exists t \forall u \forall v \exists w \forall x \forall y \exists z \; P(C_1, t, u, v, w, x, y, z)$$

Replace t with another constant C_2. (t depends s and is a function of C_1. It is thus a constant as well.)

$$\forall u \forall v \exists w \forall x \forall y \exists z \; P(C_1, C_2, u, v, w, x, y, z)$$

Eliminate the two universal quantifiers in front of u and v.

$$\exists w \forall x \forall y \exists z \; P(C_1, C_2, u, v, w, x, y, z)$$

w depends on $C_1, C_2, u, v,$ among which only u, v are variables. Introduce a Skolem function f_1.

$$\forall x \forall y \exists z \; P(C_1, C_2, u, v, f_1(u, v), x, y, z)$$

Eliminate two more universal quantifiers.

Skolemization – One More Example

$$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \;\; P(s,t,u,v,w,x,y,z)$$

Replace s with a constant C_1 (i.e., a function with no argument).

$$\exists t \forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1,t,u,v,w,x,y,z)$$

Replace t with another constant C_2. (t depends s and is a function of C_1. It is thus a constant as well.)

$$\forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1,C_2,u,v,w,x,y,z)$$

Eliminate the two universal quantifiers in front of u and v.

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w depends on $C_1, C_2, u, v,$ among which only u, v are variables. Introduce a Skolem function f_1.

$$\forall x \forall y \exists z \;\; P(C_1,C_2,u,v,f_1(u,v),x,y,z)$$

Eliminate two more universal quantifiers.

$$\exists z \;\; P(C_1,C_2,u,v,f_1(u,v),x,y,z)$$

Skolemization – One More Example

$$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \;\; P(s,t,u,v,w,x,y,z)$$

$\Downarrow$ Replace s with a constant C_1 (i.e., a function with no argument).

$$\exists t \forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1,t,u,v,w,x,y,z)$$

$\Downarrow$ Replace t with another constant C_2. (t depends s and is a function of C_1. It is thus a constant as well.)

$$\forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1,C_2,u,v,w,x,y,z)$$

$\Downarrow$ Eliminate the two universal quantifiers in front of u and v.

$$\exists w \forall x \forall y \exists z \;\; P(C_1,C_2,u,v,w,x,y,z)$$

$\Downarrow$ w depends on $C_1, C_2, u, v,$ among which only u, v are variables. Introduce a Skolem function f_1.

$$\forall x \forall y \exists z \;\; P(C_1,C_2,u,v,f_1(u,v),x,y,z)$$

$\Downarrow$ Eliminate two more universal quantifiers.

$$\exists z \;\; P(C_1,C_2,u,v,f_1(u,v),x,y,z)$$

z depends on C_1, C_2, u, v, x, y, among which only u, v, x, y are variables. Introduce a second Skolem function f_2.

Skolemization – One More Example

$$\exists s \exists t \forall u \forall v \exists w \forall x \forall y \exists z \;\; P(s,t,u,v,w,x,y,z)$$

Replace s with a constant C_1
(i.e., a function with no argument).

$$\exists t \forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1,t,u,v,w,x,y,z)$$

Replace t with another constant C_2. (t depends s and is a function of C_1. It is thus a constant as well.)

$$\forall u \forall v \exists w \forall x \forall y \exists z \;\; P(C_1,C_2,u,v,w,x,y,z)$$

Eliminate the two universal quantifiers in front of u and v.

$$\exists w \forall x \forall y \exists z \;\; P(C_1,C_2,u,v,w,x,y,z)$$

w depends on $C_1, C_2, u, v,$ among which only u, v are variables. Introduce a Skolem function f_1.

$$\forall x \forall y \exists z \;\; P(C_1,C_2,u,v,f_1(u,v),x,y,z)$$

Eliminate two more universal quantifiers.

$$\exists z \;\; P(C_1,C_2,u,v,f_1(u,v),x,y,z)$$

z depends on C_1, C_2, u, v, x, y, among which only u, v, x, y are variables. Introduce a second Skolem function f_2.

$$P(C_1,C_2,u,v,f_1(u,v),x,y,f_2(u,v,x,y))$$

Handling ∀, ∨, and ∧

e) Drop universal quantifiers:

$$\forall x \ \ (Animal(F(x)) \land \neg Loves\big(x, F(x)\big)) \lor Loves(G(x), x)$$

Handling ∀, ∨, and ∧

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$$\forall x \ \ (Animal(F(x)) \wedge \neg Loves\big(x, F(x)\big)) \vee Loves(G(x), x)$$

$$\Downarrow$$

$$(Animal(F(x)) \wedge \neg Loves\big(x, F(x)\big)) \vee Loves(G(x), x)$$

Handling ∀, ∨, and ∧

e) Drop universal quantifiers:

$$\forall x \;\; (\mathit{Animal}(F(x)) \land \neg \mathit{Loves}\big(x, F(x)\big)) \lor \mathit{Loves}(G(x), x)$$

$$\Downarrow$$

$$(\mathit{Animal}(F(x)) \land \neg \mathit{Loves}\big(x, F(x)\big)) \lor \mathit{Loves}(G(x), x)$$

f) Distribute ∨ over ∧ :

Handling ∀, ∨, and ∧

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$$\forall x \ \ (\mathit{Animal}(F(x)) \wedge \neg \mathit{Loves}\big(x, F(x)\big)) \vee \mathit{Loves}(G(x), x)$$

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$$(\mathit{Animal}(F(x)) \wedge \neg \mathit{Loves}\big(x, F(x)\big)) \vee \mathit{Loves}(G(x), x)$$

$$\Downarrow$$

f) Distribute ∨ over ∧ :

$$(\mathit{Animal}\big(F(x)\big) \vee \mathit{Loves}(G(x), x)) \wedge (\neg \mathit{Loves}\big(x, F(x)\big) \vee \mathit{Loves}(G(x), x))$$

Handling ∀, ∨, and ∧

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$$\forall x \ \ (Animal(F(x)) \land \neg Loves(x, F(x))) \lor Loves(G(x), x)$$

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$$(Animal(F(x)) \land \neg Loves(x, F(x))) \lor Loves(G(x), x)$$

f) Distribute ∨ over ∧ :

$$\Downarrow$$

$$\underbrace{(Animal(F(x)) \lor Loves(G(x), x))}_{\text{clause 1}} \land \underbrace{(\neg Loves(x, F(x)) \lor Loves(G(x), x))}_{\text{clause 2}}$$

Handling ∀, ∨, and ∧

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$$\underbrace{(Animal(F(x)) \vee Loves(G(x), x))}_{\text{clause 1}} \wedge \underbrace{(\neg Loves(x, F(x)) \vee Loves(G(x), x))}_{\text{clause 2}}$$

Resolution Inference Rule

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \qquad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_k}{\text{SUBST}(\theta, l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)}$$

where $\theta = \text{UNIFY}(l_i, m_j)$.

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where $\theta = \textsc{Unify}\left(l_i, m_j\right)$.

$\mathit{Animal}\left(F(x)\right) \vee \mathit{Loves}(G(x), x) \qquad (\neg \mathit{Loves}(u, v) \vee \neg \mathit{Kills}(u, v))$

Resolution Inference Rule

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \qquad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_k}{\text{SUBST}(\theta, l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)}$$

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$$\underbrace{\mathit{Animal}\big(F(x)\big) \vee \mathit{Loves}(G(x), x) \qquad (\neg \mathit{Loves}(u, v)}_{\text{unifier: } \theta = \{u/G(x), v/x\}} \vee \neg \mathit{Kills}(u, v))$$

Resolution Inference Rule

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \qquad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_k}{\textsc{Subst}(\theta, l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)}$$

where $\theta = \textsc{Unify}(l_i, m_j)$.

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resolvent: $\boxed{\mathit{Animal}(F(x)) \vee \neg \mathit{Kills}(G(x), x)}$

Resolution Inference Rule

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \qquad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_k}{\text{SUBST}(\theta, l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)}$$

where $\theta = \text{UNIFY}(l_i, m_j)$.

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resolvent: $\mathit{Animal}(F(x)) \vee \neg \mathit{Kills}(G(x), x)$

♣ Binary resolution as given above does not yield a complete inference procedure.

Resolution Inference Rule

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \qquad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_k}{\mathrm{S{\scriptsize UBST}}(\theta, l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \vee \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n)}$$

where $\theta = \mathrm{U{\scriptsize NIFY}}(l_i, m_j)$.

$Animal(F(x)) \vee Loves(G(x), x) \qquad (\neg Loves(u, v) \vee \neg Kills(u, v))$

unifier: $\theta = \{u/G(x), v/x\}$

---

resolvent: $\boxed{Animal(F(x)) \vee \neg Kills(G(x), x)}$

- Binary resolution as given above does not yield a complete inference procedure.
- *Full resolution* does. It resolves subsets of literals in each clause that are unifiable.

Example Proof 1

The crime example:

$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x, y, z) \lor \neg Hostile(z) \lor Criminal(x)$
$\neg\, Missile(x) \lor \neg Owns(Nono, x) \lor Sells(West, x, Nono)$
$\neg Missile(x) \lor Weapon(x)$
$\neg Enemy(x, America) \lor Hostile(x)$

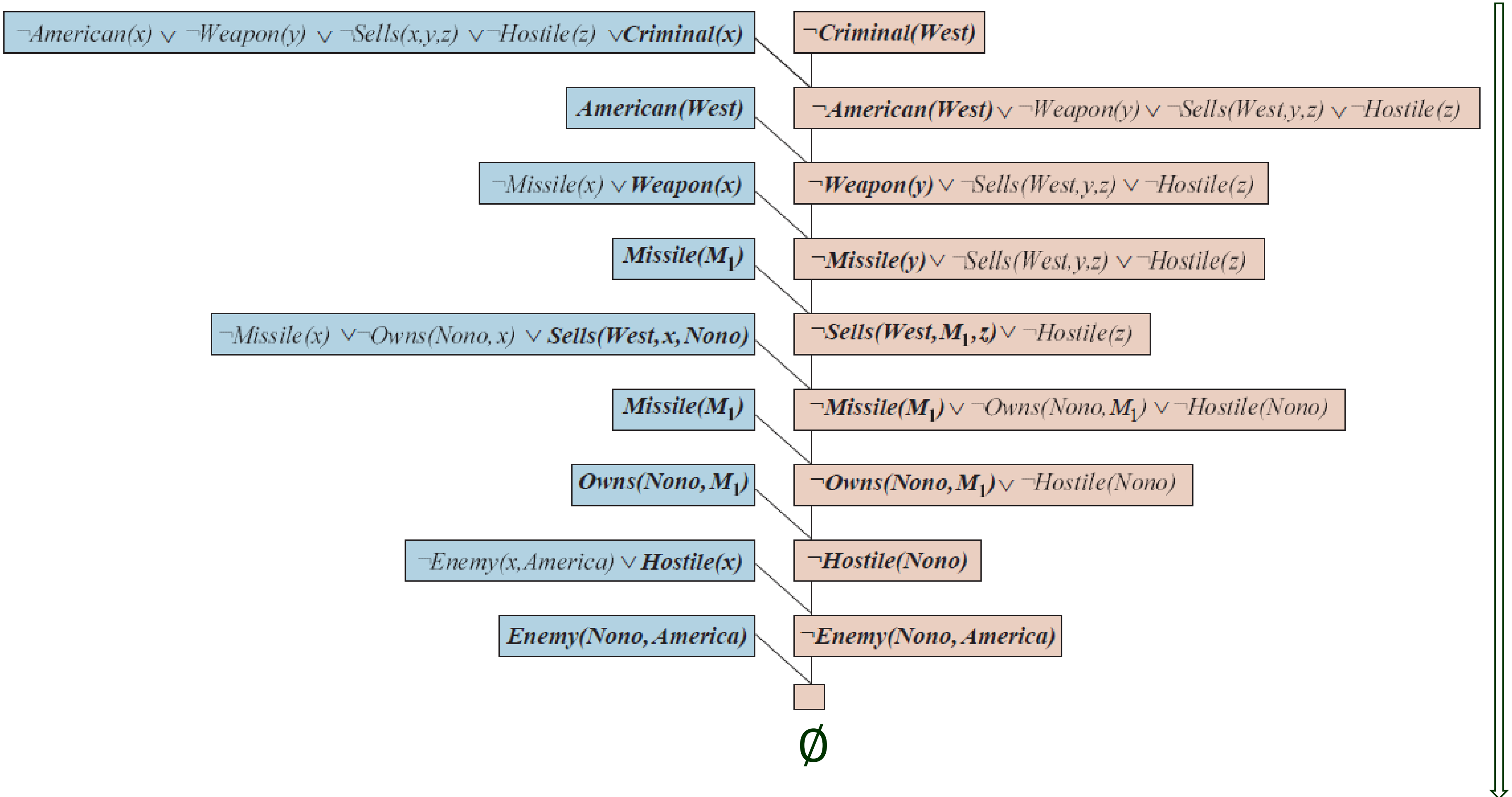
$Owns(Nono, M_1)$
$Missile(M_1)$
$American(West)$
$Enemy(Nono, America)$

We prove *Criminal*(*West*) by adding

$$\neg\, Criminal(West)$$

and deriving the empty clause ∅.

Resolution Proof 1



Like backward chaining.

Example Proof 2 (with Skolemization)

Everyone who loves all animals is loved by someone.
Anyone who kills an animal is loved by no one.
Jack loves all animals.
Either Jack or Curiosity killed the cat, who is named Tuna.
Did Curiosity kill the cat?

Example Proof 2 (with Skolemization)

Everyone who loves all animals is loved by someone.
Anyone who kills an animal is loved by no one.
Jack loves all animals.
Either Jack or Curiosity killed the cat, who is named Tuna.
Did Curiosity kill the cat?

A. $\forall x \;\; (\forall y \;\; \mathit{Animal}(y) \Rightarrow \mathit{Loves}(x, y)) \Rightarrow (\exists y \;\; \mathit{Loves}(y, x))$

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A. $\forall x \;\; (\forall y \;\; \mathit{Animal}(y) \Rightarrow \mathit{Loves}(x, y)) \Rightarrow (\exists y \;\; \mathit{Loves}(y, x))$

B. $\forall x \;\; (\exists z \;\; \mathit{Animal}(z) \wedge \mathit{Kills}(x, z)) \Rightarrow (\forall y \;\; \neg \mathit{Loves}(y, x))$

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A. $\forall x \;\; (\forall y \;\; Animal(y) \Rightarrow Loves(x, y)) \Rightarrow (\exists y \;\; Loves(y, x))$

B. $\forall x \;\; (\exists z \;\; Animal(z) \wedge Kills(x, z)) \Rightarrow (\forall y \;\; \neg Loves(y, x))$

C. $\forall x \;\; Animal(x) \Rightarrow Loves(Jack, x)$

Example Proof 2 (with Skolemization)

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Either Jack or Curiosity killed the cat, who is named Tuna.
Did Curiosity kill the cat?

A. $\forall x \ \ (\forall y \ \ \mathit{Animal}(y) \Rightarrow \mathit{Loves}(x, y)) \Rightarrow (\exists y \ \ \mathit{Loves}(y, x))$

B. $\forall x \ \ (\exists z \ \ \mathit{Animal}(z) \land \mathit{Kills}(x, z)) \Rightarrow (\forall y \ \ \neg \mathit{Loves}(y, x))$

C. $\forall x \ \ \mathit{Animal}(x) \Rightarrow \mathit{Loves}(\mathit{Jack}, x)$

D. $\mathit{Kills}(\mathit{Jack}, \mathit{Tuna}) \lor \mathit{Kills}(\mathit{Curiosity}, \mathit{Tuna})$

Example Proof 2 (with Skolemization)

Everyone who loves all animals is loved by someone.
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Did Curiosity kill the cat?

A. $\forall x \;\; (\forall y \;\; Animal(y) \Rightarrow Loves(x, y)) \Rightarrow (\exists y \;\; Loves(y, x))$

B. $\forall x \;\; (\exists z \;\; Animal(z) \wedge Kills(x, z)) \Rightarrow (\forall y \;\; \neg Loves(y, x))$

C. $\forall x \;\; Animal(x) \Rightarrow Loves(Jack, x)$

D. $Kills(Jack, Tuna) \vee Kills(Curiosity, Tuna)$

Background knowledge

E. $Cat(Tuna)$

F. $\forall x \;\; Cat(x) \Rightarrow Animal(x)$

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Everyone who loves all animals is loved by someone.
Anyone who kills an animal is loved by no one.
Jack loves all animals.
Either Jack or Curiosity killed the cat, who is named Tuna.
Did Curiosity kill the cat?

A. $\forall x \;\; (\forall y \;\; Animal(y) \Rightarrow Loves(x, y)) \Rightarrow (\exists y \;\; Loves(y, x))$

B. $\forall x \;\; (\exists z \;\; Animal(z) \wedge Kills(x, z)) \Rightarrow (\forall y \;\; \neg Loves(y, x))$

C. $\forall x \;\; Animal(x) \Rightarrow Loves(Jack, x)$

D. $Kills(Jack, Tuna) \vee Kills(Curiosity, Tuna)$

Background knowledge

E. $Cat(Tuna)$

F. $\forall x \;\; Cat(x) \Rightarrow Animal(x)$

Negated goal

¬ G. $\neg Kills(Curiosity, Tuna)$

Converting to CNF

A. $\forall x \quad (\forall y \;\; Animal(y) \Rightarrow Loves(x, y)) \Rightarrow (\exists y \;\; Loves(x, y))$

$F(x)$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $G(x)$

Converting to CNF

A. $\forall x \ \ (\forall y \ \ Animal(y) \Rightarrow Loves(x, y)) \Rightarrow (\exists y \ \ Loves(x, y))$

$F(x)$ $G(x)$

A1. $Animal(F(x)) \lor Loves(G(x), x))$

A2. $\neg Loves(x, F(x)) \lor Loves(G(x), x))$

Converting to CNF

A. $\forall x \ \ (\forall y \ \ Animal(y) \Rightarrow Loves(x,y)) \Rightarrow (\exists y \ \ Loves(x,y))$

$F(x)$ $\qquad$ $G(x)$

A1. $Animal(F(x)) \lor Loves(G(x),x))$

A2. $\neg Loves(x,F(x)) \lor Loves(G(x),x))$

B. $\neg Animal(z) \lor \neg Kills(x,z) \lor \neg Loves(y,x))$

C. $\neg Animal(x) \lor Loves(Jack,x))$

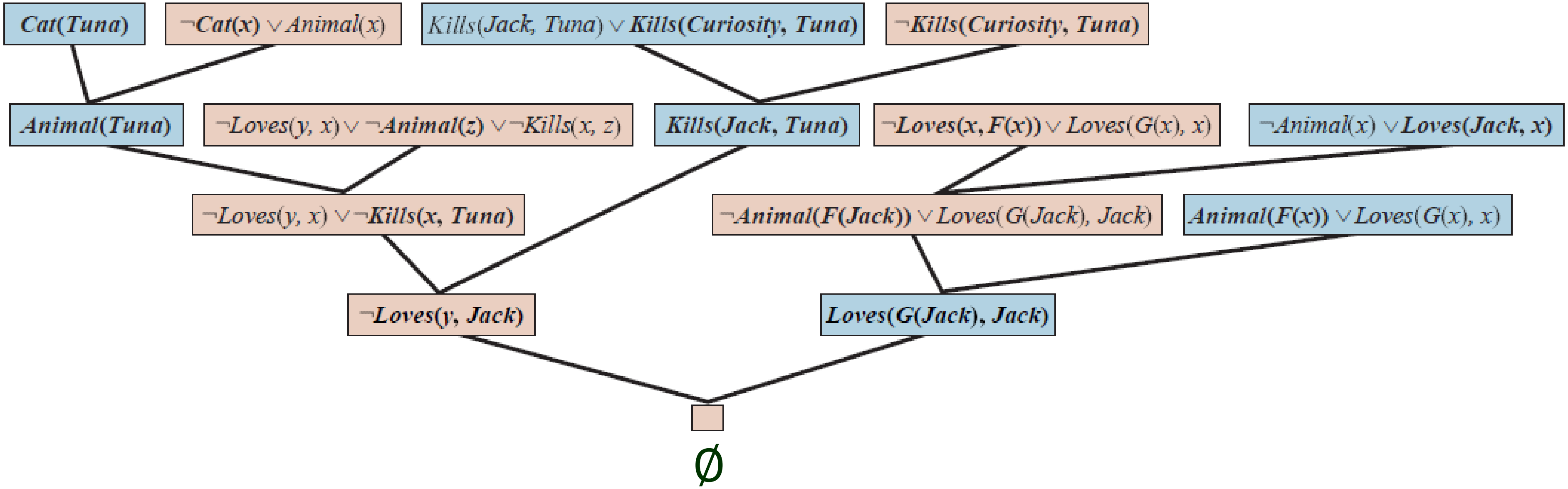
D. $Kills(Jack,Tuna) \lor Kills(Curiosity,Tuna)$

E. $Cat(Tuna)$

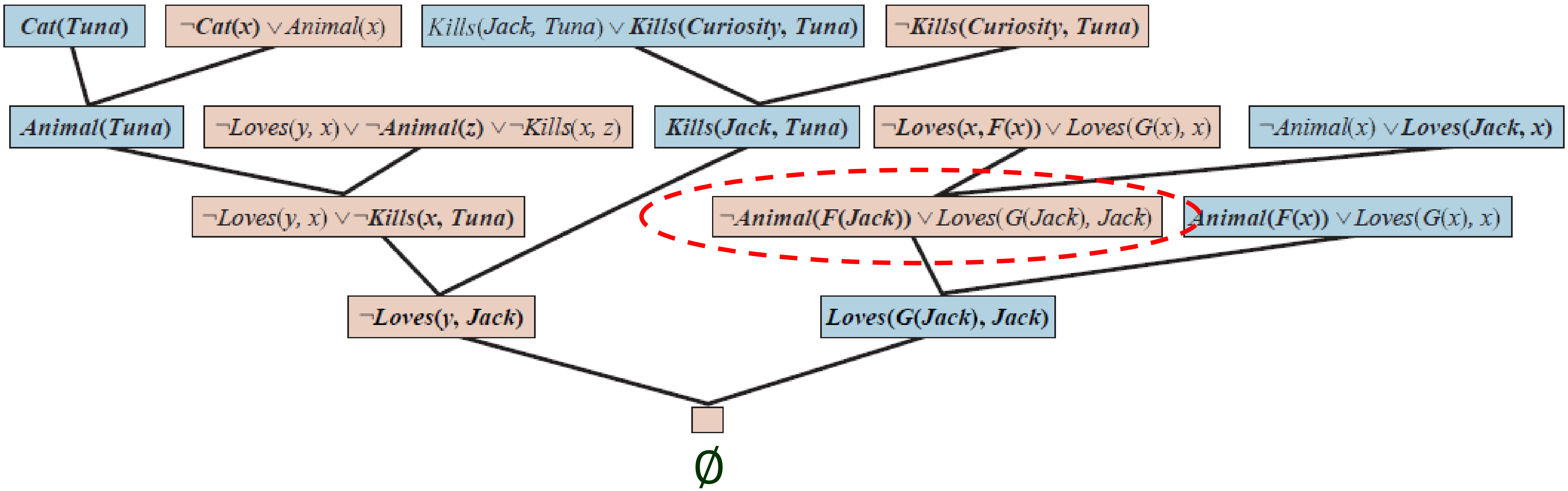
F. $\neg Cat(x) \lor Animal(x)$

$\neg$ G. $\neg Kills(Curiosity,Tuna)$

Resolution Proof 2



Resolution Proof 2



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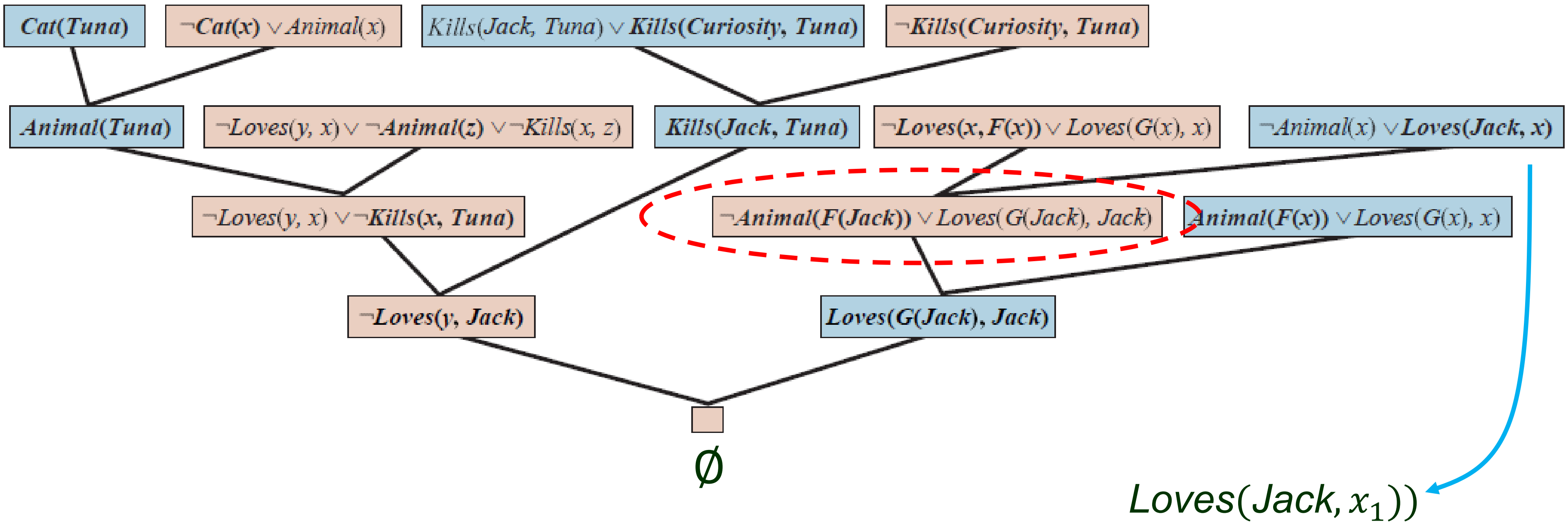
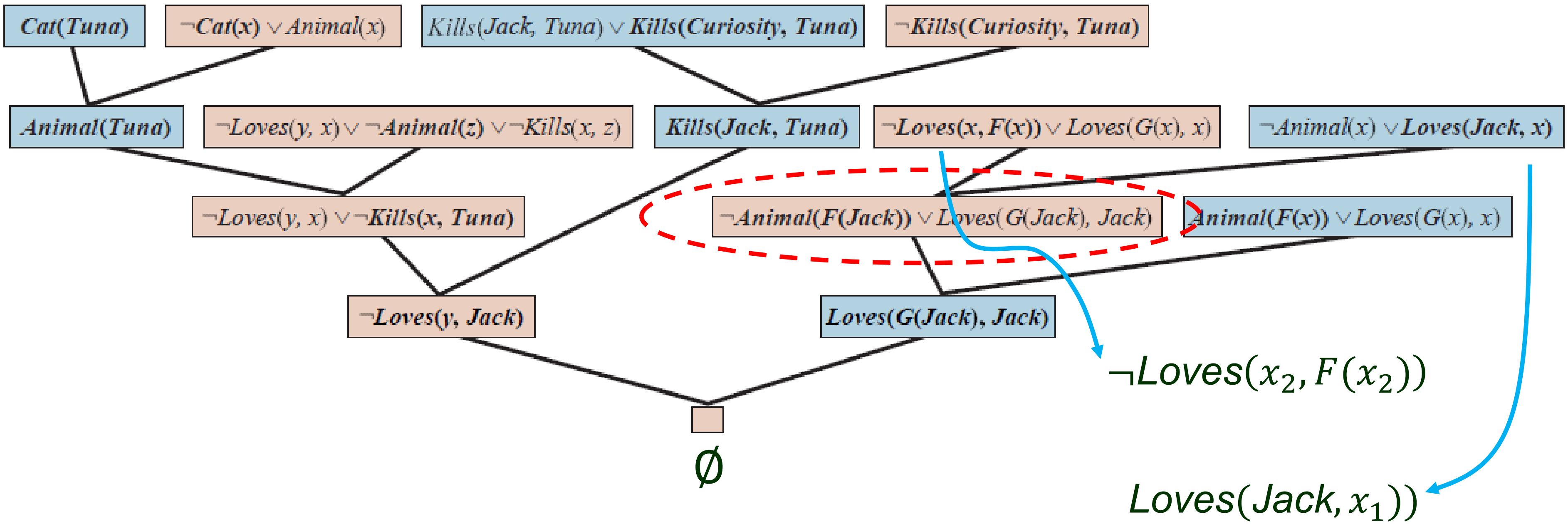


Resolution Proof 2



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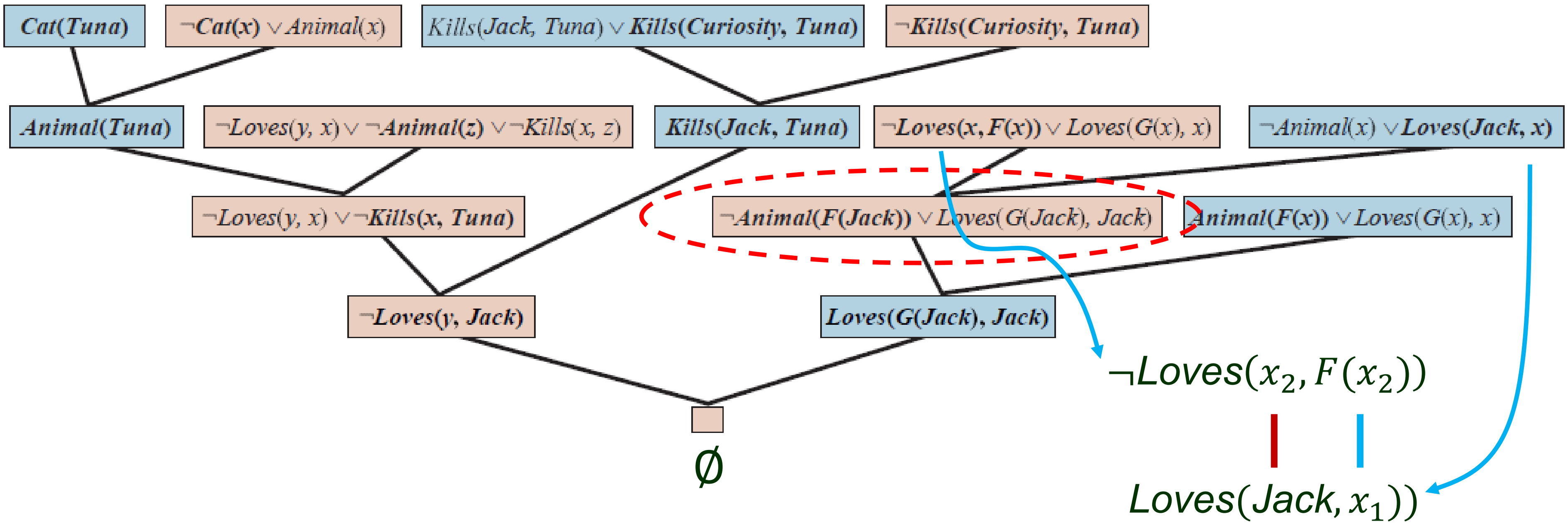
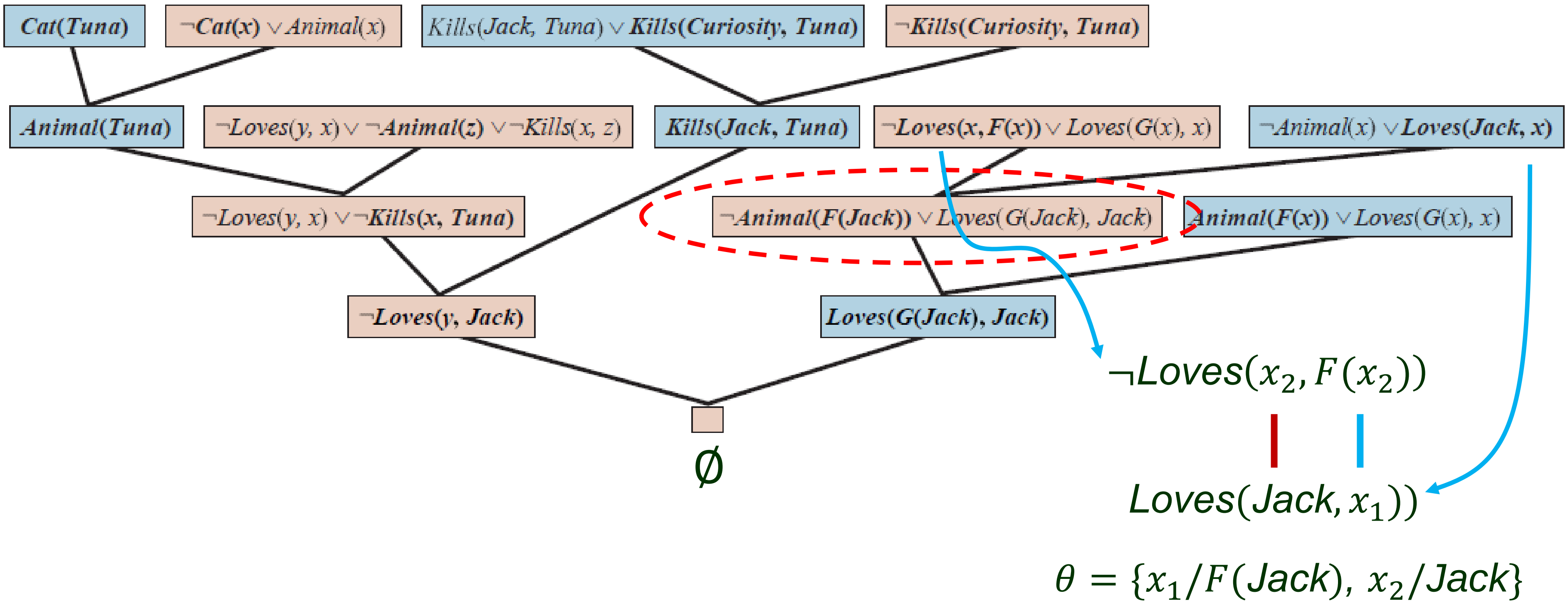


Resolution Proof 2



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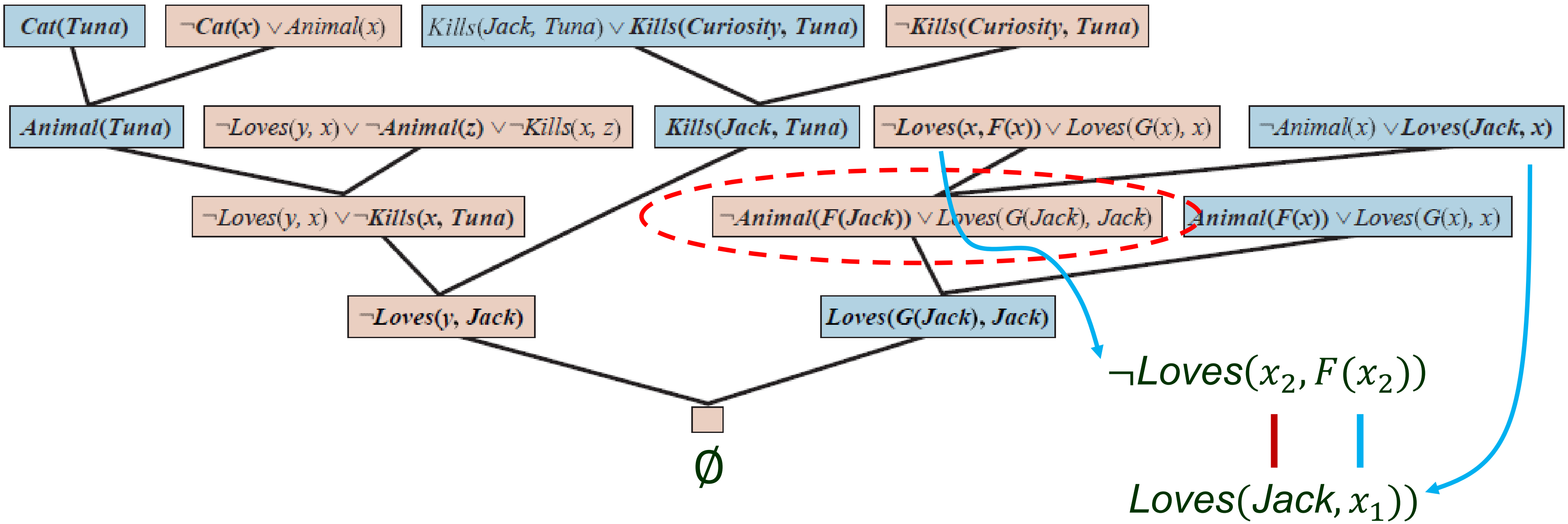

Paraphrased in English:

$\theta = \{x_1/F(Jack),\ x_2/Jack\}$

Suppose Curiosity did not kill Tuna. We know that either Jack or Curiosity did; thus Jack must have. Now, Tuna is a cat and cats are animals, so Tuna is an animal. Because anyone who kills an animal is loved by no one, we know that no one loves Jack. On the other hand, Jack loves all animals, so someone loves him; so we have a contradiction. Therefore, Curiosity killed the cat.

Completeness of Resolution

Theorem If a set S of sentences is unsatisfiable, then resolution will always be able to derive a contradiction.

♠ Not all logical consequences of S can be generated using resolution.

♣ A sentence entailed by S can always be established using resolution.

We can use resolution to find all answers to a question $Q(x)$ by proving that $KB \wedge \neg Q(x)$ is unsatisfiable.

Equality

Axiomatize equality: write down sentences about equality in the *KB.*

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- One for each basic axioms.

$\forall x \;\; x = x$ // reflexive

$\forall x, y \;\; x = y \Leftrightarrow y = x$ // symmetric

$\forall x, y, z \;\; x = y \wedge y = z \Rightarrow x = z$ // transitive

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- One for each predicate.

$$\forall x, y \;\; x = y \Rightarrow (P_1(x) \Leftrightarrow P_1(y))$$

$$\vdots$$

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- One for each predicate.

$$\forall x, y \;\; x = y \Rightarrow (P_1(x) \Leftrightarrow P_1(y))$$

$$\vdots$$

- One for each function.

$$\forall x, y \;\; x = y \Rightarrow (F_1(x) = F_1(y))$$

$$\vdots$$