Exercise 9.24

Let $\boldsymbol{L}$ be the first-order language with a single predicate $\boldsymbol{S(p,q)}$, meaning "$\boldsymbol{p}$ shaves $\boldsymbol{q}$." Assume a domain of people.

1. Consider the sentence "There exists a person $\boldsymbol{P}$ who shaves every one who does not shave themselves, and only people that do not shave themselves." Express this in $\boldsymbol{L}$.
2. Convert the sentence in (a) to clausal form.
3. Construct a resolution proof to show that the clauses in (b) are inherently inconsistent. (Note: you do not need any additional axioms.)

Exercise 9.29 [quantifier-order-exercise]

Here are two sentences in the language of first-order logic:

- **(A)** $\forall x \exists y (x \geq y)$
- **(B)** $\exists y \forall x (x \geq y)$

1. Assume that the variables range over all the natural numbers $\boldsymbol{0,1,2,\ldots,\infty}$ and that the "≥" predicate means "is greater than or equal to." Under this interpretation, translate (A) and (B) into English.
2. Is (A) true under this interpretation?
3. Is (B) true under this interpretation?
4. Does (A) logically entail (B)?
5. Does (B) logically entail (A)?
6. Using resolution, try to prove that (A) follows from (B). Do this even if you think that (B) does not logically entail (A); continue until the proof breaks down and you cannot proceed (if it does break down). Show the unifying substitution for each resolution step. If the proof fails, explain exactly where, how, and why it breaks down.
7. Now try to prove that (B) follows from (A).

Exercise 10.3

[strips-airport-exercise]Given the action schemas and initial state from Figure **airport-pddl-algorithm**, what are all the applicable concrete instances of $\text{Fly}(p,from,to)$Fly(p,from,to) in the state described by

$$At(P1,JFK) \wedge At(P2,SFO) \wedge Plane(P1) \wedge Plane(P2) \wedge Airport(JFK) \wedge Airport(SFO)?$$

Exercise 10.4

The monkey-and-bananas problem is faced by a monkey in a laboratory with some bananas hanging out of reach from the ceiling. A box is available that will enable the monkey to reach the bananas if he climbs on it. Initially, the monkey is at $\mathbf{A}$, the bananas at $\mathbf{B}$, and the box at $\mathbf{C}$. The monkey and box have height $\mathbf{Low}$, but if the monkey climbs onto the box he will have height $\mathbf{High}$, the same as the bananas. The actions available to the monkey include $\mathbf{Go}$ from one place to another, $\mathbf{Push}$ an object from one place to another, $\mathbf{ClimbUp}$ onto or $\mathbf{ClimbDown}$ from an object, and $\mathbf{Grasp}$ or $\mathbf{Ungrasp}$ an object. The result of a $\mathbf{Grasp}$ is that the monkey holds the object if the monkey and object are in the same place at the same height.

1. Write down the initial state description.
2. Write the six action schemas.
3. Suppose the monkey wants to fool the scientists, who are off to tea, by grabbing the bananas, but leaving the box in its original place. Write this as a general goal (i.e., not assuming that the box is necessarily at C) in the language of situation calculus. Can this goal be solved by a classical planning system?
4. Your schema for pushing is probably incorrect, because if the object is too heavy, its position will remain the same when the $\mathbf{Push}$ schema is applied. Fix your action schema to account for heavy objects.

Exercise 13.3

For each of the following statements, either prove it is true or give a counterexample.

a) If **P(a | b, c) = P(b | a, c)**, then **P(a | c) = P (b | c)**.

b) If **P(a | b, c) = P(a)**, then **P(b | c) = P(b)**.

c) If **P(a | b) = P(a)**, then **P(a | b, c) = P(a | c)**.

Exercise 13.6 [inclusion-exclusion-exercise]

Prove Equation (**kolmogorov-disjunction-equation**) from Equations (**basic-probability-axiom-equation**) and (**proposition-probability-equation**).

Exercise 13.8

Given the full joint distribution shown in Figure **dentist-joint-table**, calculate the following:

1. **P(toothache)**.
2. **P(Cavity)**.
3. **P(Toothachecavity)**.
4. **P(Cavitytoothache ∨ catch)**.