

EE 330

Lecture 31

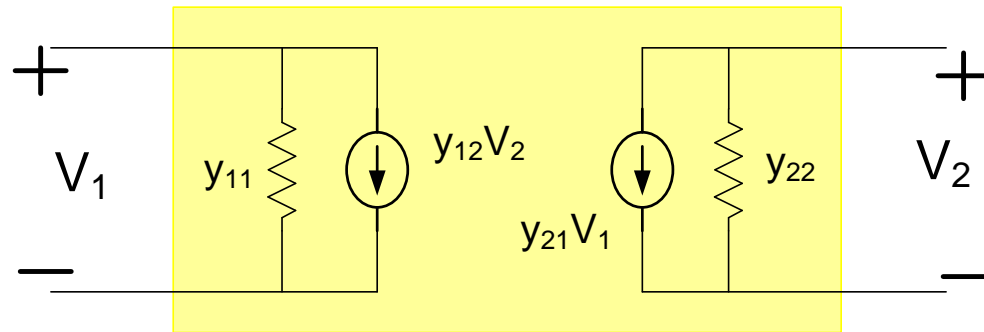
Two-Port Amplifier Models

Basic amplifier architectures

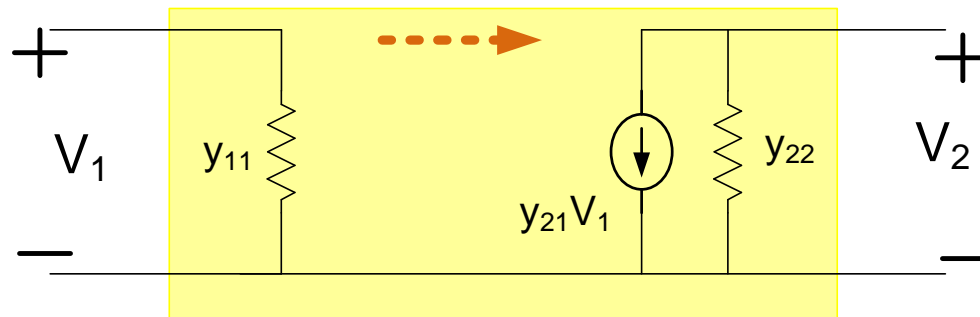
- Common Emitter/Source
- Common Collector/Drain
- Common Base/Gate

Two-port representation of amplifiers

Amplifiers can be modeled as a two-port

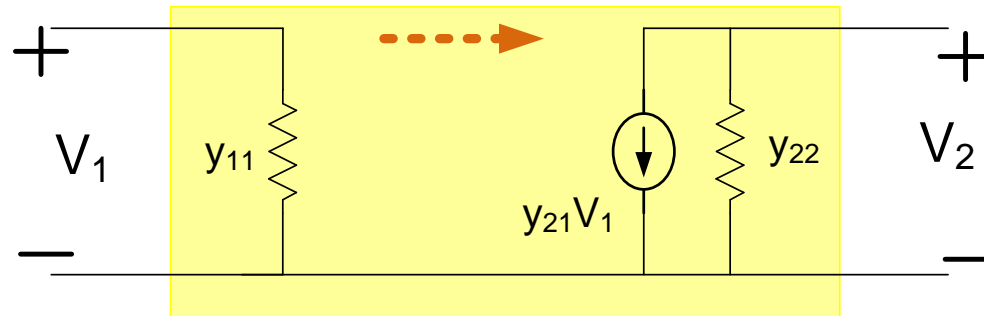


- Amplifier often **unilateral** (signal propagates in only one direction: wlog $y_{12}=0$)
- One terminal is often common

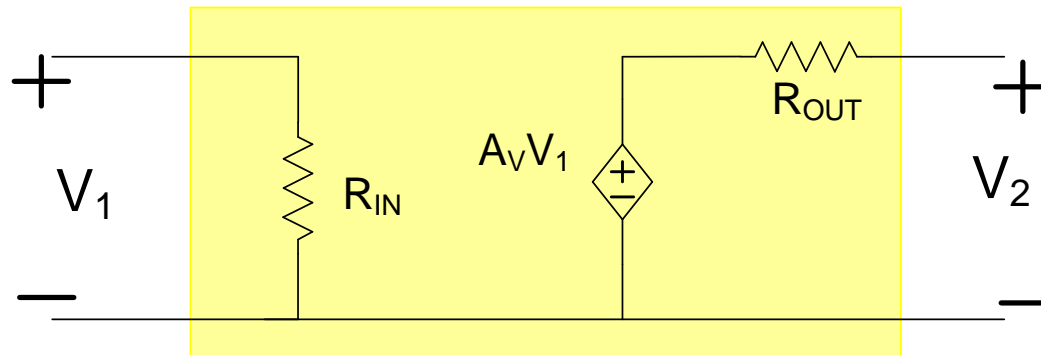


Two-port representation of amplifiers

Unilateral amplifiers:



- Thevenin equivalent output port often more standard
- R_{IN} , A_V , and R_{OUT} often used to characterize the two-port of amplifiers



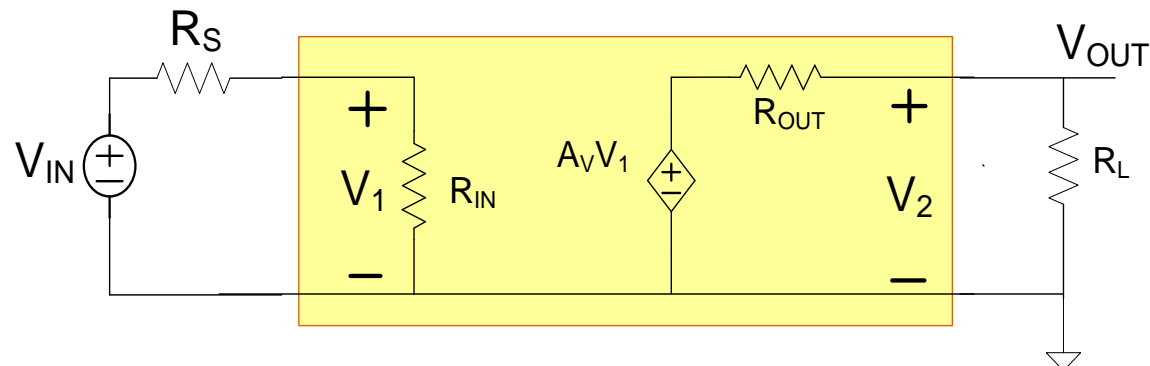
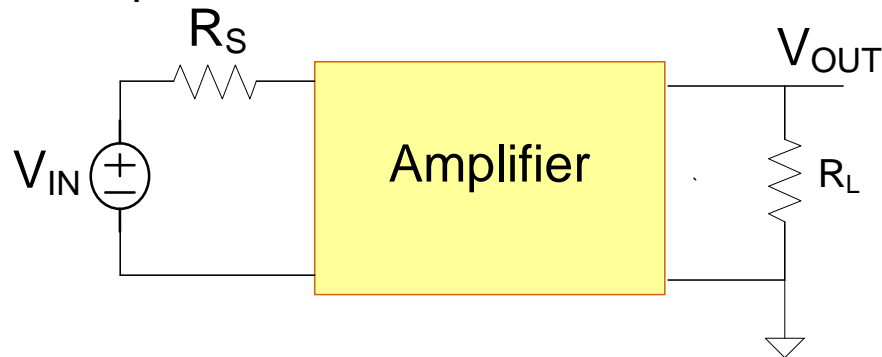
Unilateral amplifier in terms of “amplifier” parameters

$$R_{IN} = \frac{1}{y_{11}} \quad A_V = -\frac{y_{21}}{y_{22}} \quad R_{OUT} = \frac{1}{y_{22}}$$

Amplifier input impedance, output impedance and gain are usually of interest

Why?

Example 1: Assume amplifier is unilateral



$$V_{OUT} = \left(\frac{R_L}{R_L + R_{OUT}} \right) A_V \left(\frac{R_{IN}}{R_S + R_{IN}} \right) V_{IN}$$

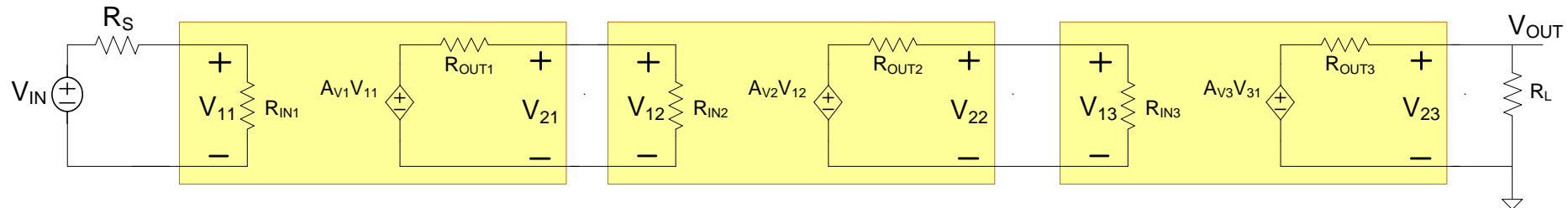
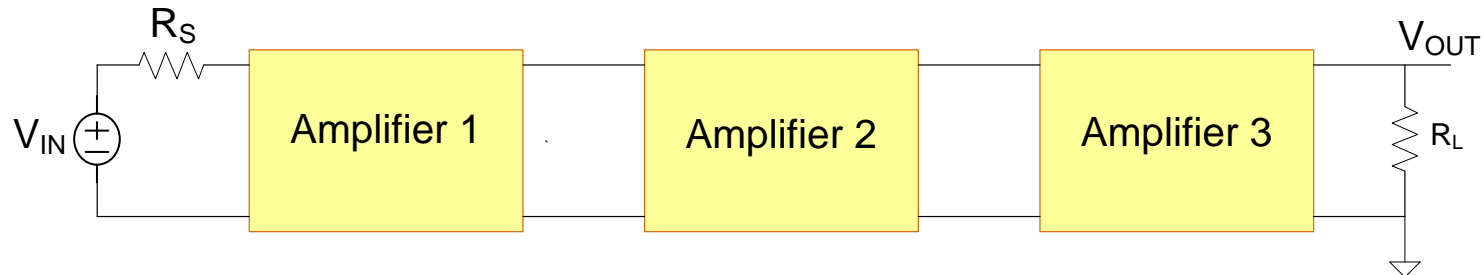
$$A_{VAMP} = \frac{V_{OUT}}{V_{IN}} = \left(\frac{R_L}{R_L + R_{OUT}} \right) \left(\frac{R_{IN}}{R_S + R_{IN}} \right) A_V$$

- Can get gain without reconsidering details about components internal to the Amplifier !!!
- Analysis more involved when not unilateral

Amplifier input impedance, output impedance and gain are usually of interest

Why?

Example 2: Assume amplifiers are unilateral



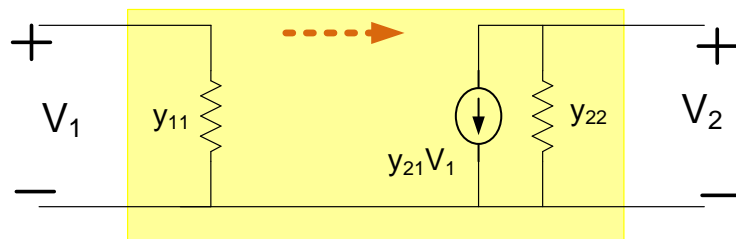
$$V_{OUT} = \left(\frac{R_L}{R_L + R_{OUT3}} \right) A_{V3} \left(\frac{R_{IN3}}{R_{OUT2} + R_{IN3}} \right) A_{V2} \left(\frac{R_{IN2}}{R_{OUT1} + R_{IN2}} \right) A_{V1} \left(\frac{R_{IN1}}{R_S + R_{IN1}} \right) V_{IN}$$

$$A_{VAMP} = \frac{V_{OUT}}{V_{IN}} = \left(\frac{R_L}{R_L + R_{OUT3}} \right) A_{V3} \left(\frac{R_{IN3}}{R_{OUT2} + R_{IN3}} \right) A_{V2} \left(\frac{R_{IN2}}{R_{OUT1} + R_{IN2}} \right) A_{V1} \left(\frac{R_{IN1}}{R_S + R_{IN1}} \right)$$

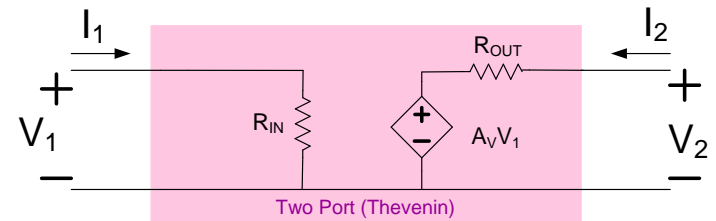
- Can get gain without reconsidering details about components internal to the Amplifier !!!
- Analysis more involved when not unilateral

Two-port representation of amplifiers

- Amplifier usually **unilateral** (signal propagates in only one direction: $y_{12}=0$)
- One terminal is often common
- “Amplifier” parameters often used

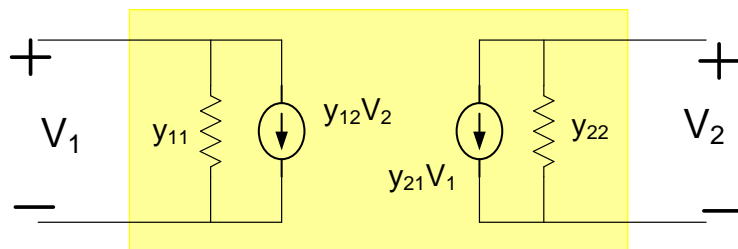


y parameters

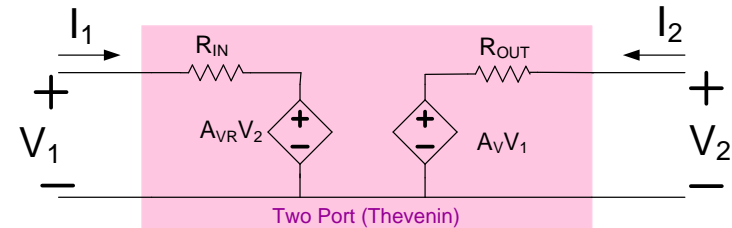


Amplifier parameters

- Amplifier parameters can also be used if not **unilateral**
- One terminal is often common



y parameters

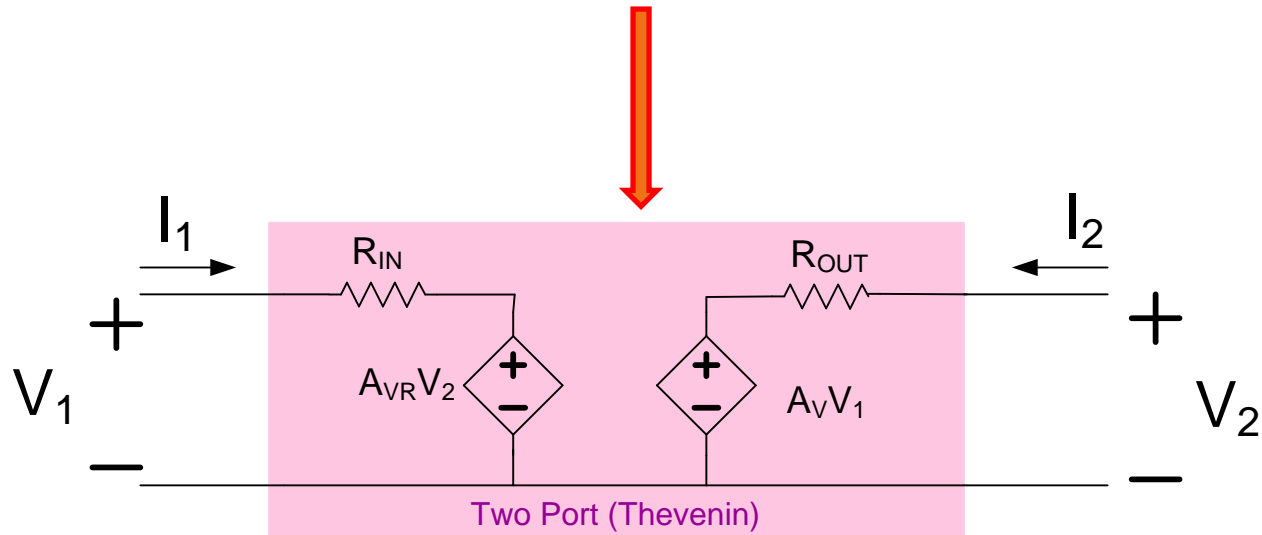
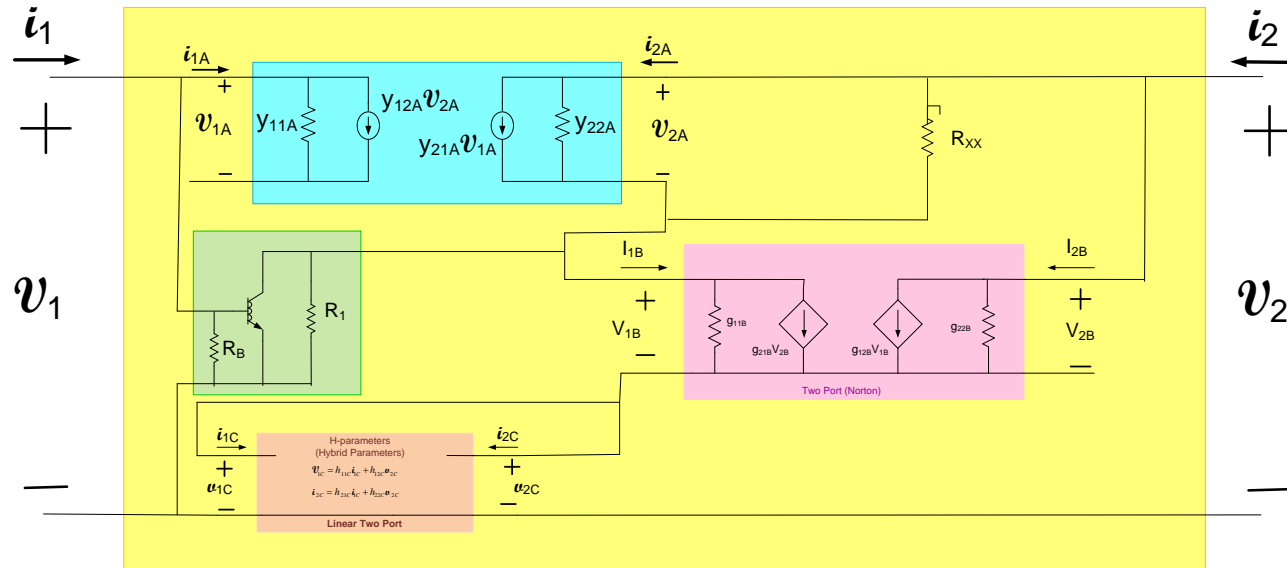


Amplifier parameters

Review from Previous Lecture

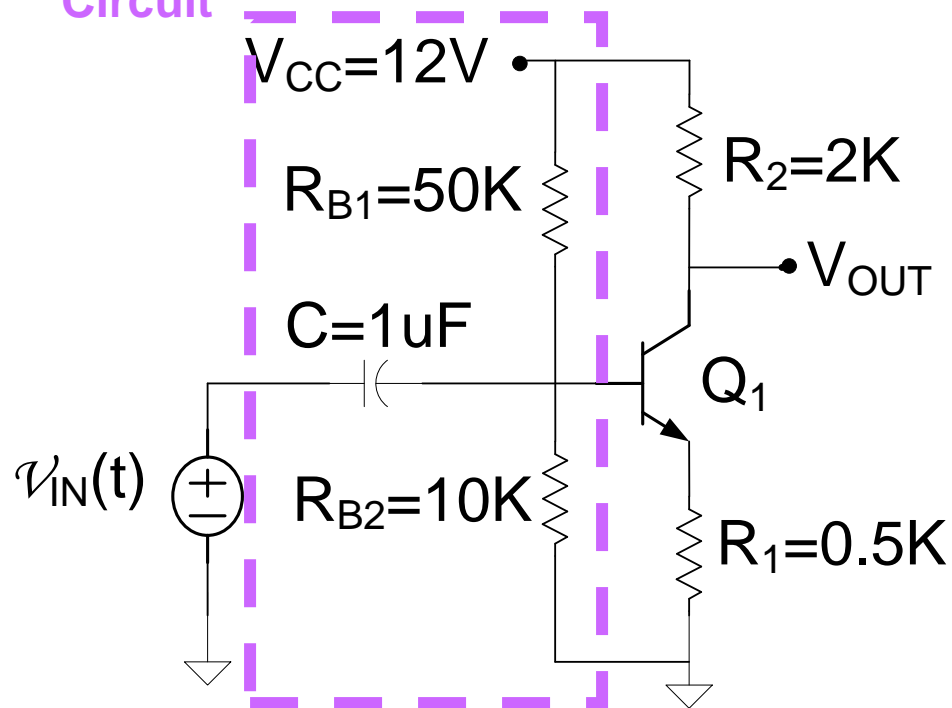
Two-Port Equivalents of Interconnected Two-ports

Example:



Examples

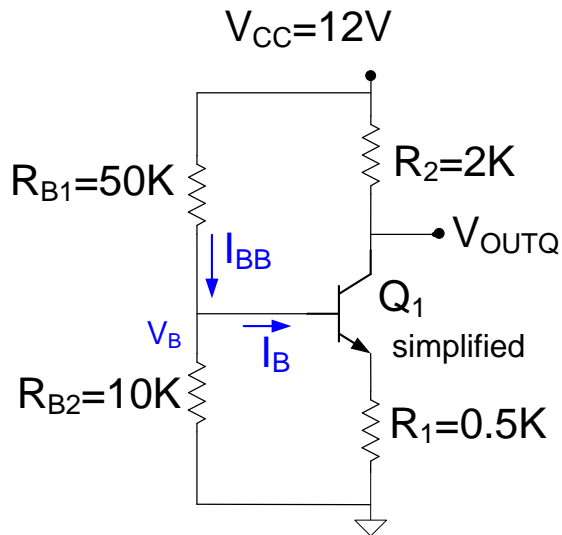
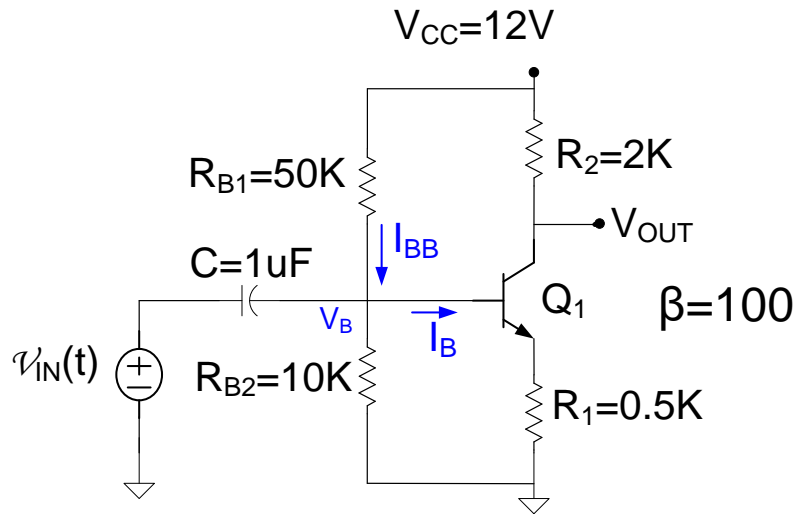
Biased
Circuit



Determine V_{OUTQ} and the SS voltage gain (A_V), assume $\beta=100$

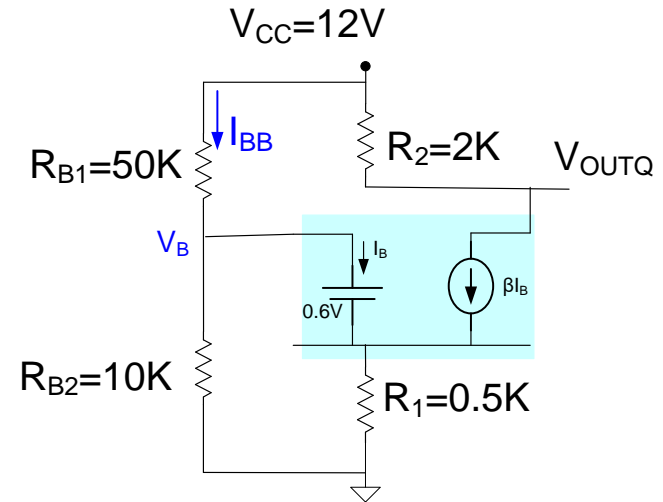
(A_V is one of the small-signal model parameters for this circuit)

Examples



dc equivalent circuit

Determine V_{OUTQ}



dc equivalent circuit

This circuit is most practical when $I_B \ll I_{BB}$

With this assumption,

$$V_B = \left(\frac{R_{B2}}{R_{B1} + R_{B2}} \right) 12V$$

$$I_{CQ} = I_{EQ} = \left(\frac{V_B - 0.6V}{R_1} \right) = \frac{1.4V}{.5K} = 2.8mA$$

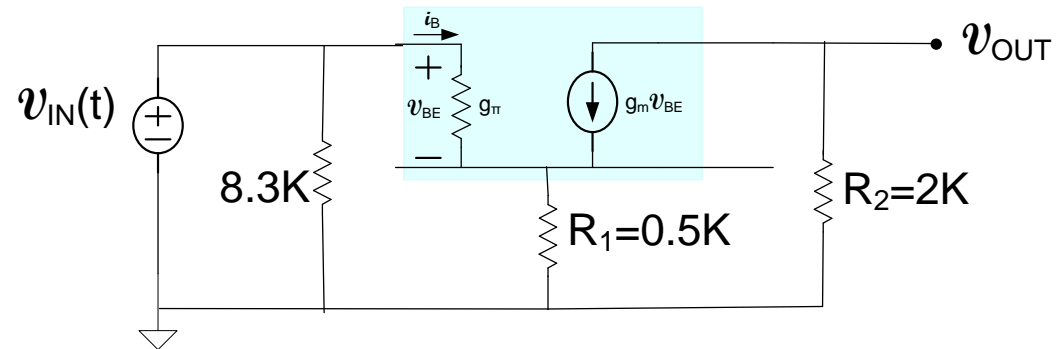
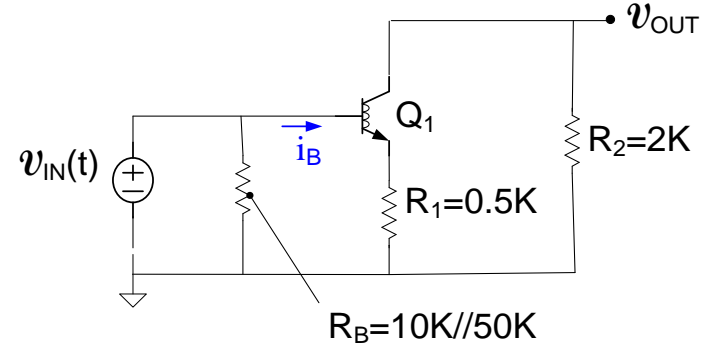
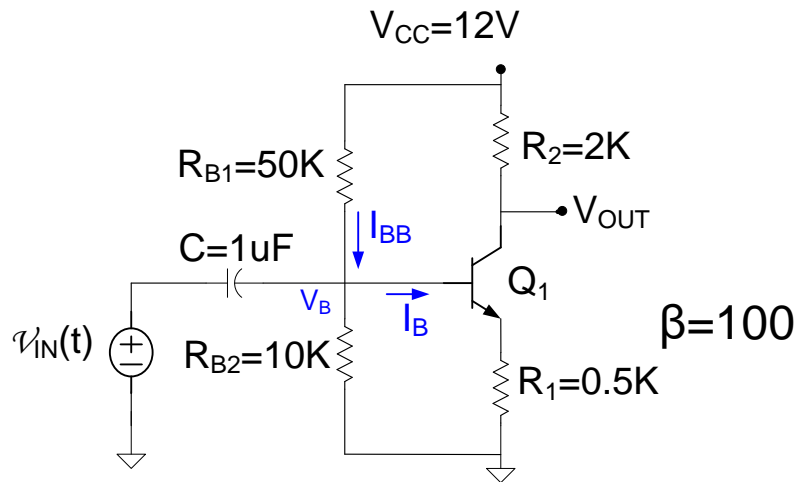
$$V_{OUTQ} = 12V - I_{CQ} R_1 = 6.4V$$

Note: This Q-point is nearly independent of the characteristics of the nonlinear BJT !

Review from Previous Lecture

Examples

Determine SS voltage gain



This voltage gain is nearly independent of the characteristics of the nonlinear BJT !

This is a fundamentally different amplifier structure

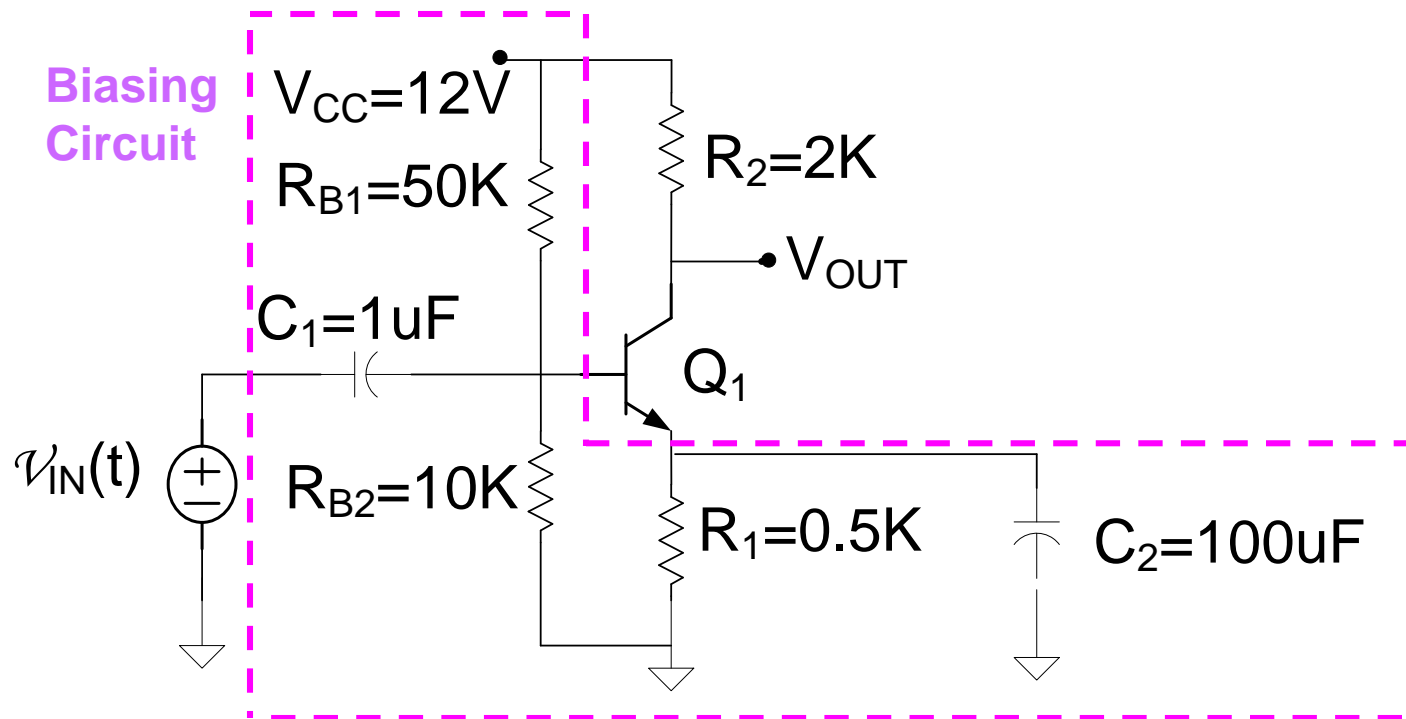
It can be shown that this is slightly non-unilateral

$$\left. \begin{aligned} v_{OUT} &= -g_m v_{BE} R_2 \\ v_{IN} &= v_{BE} + R_1 (v_{BE} [g_\pi + g_m]) \end{aligned} \right\}$$

$$A_V = \frac{-R_2 g_m v_{BE}}{v_{BE} + R_1 (v_{BE} [g_\pi + g_m])} = \frac{-R_2 g_m}{1 + R_1 ([g_\pi + g_m])}$$

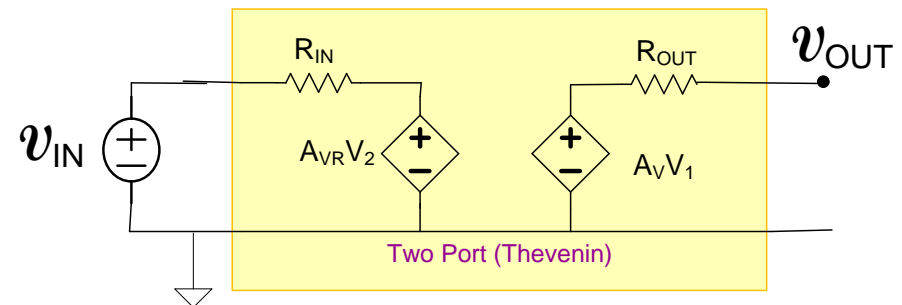
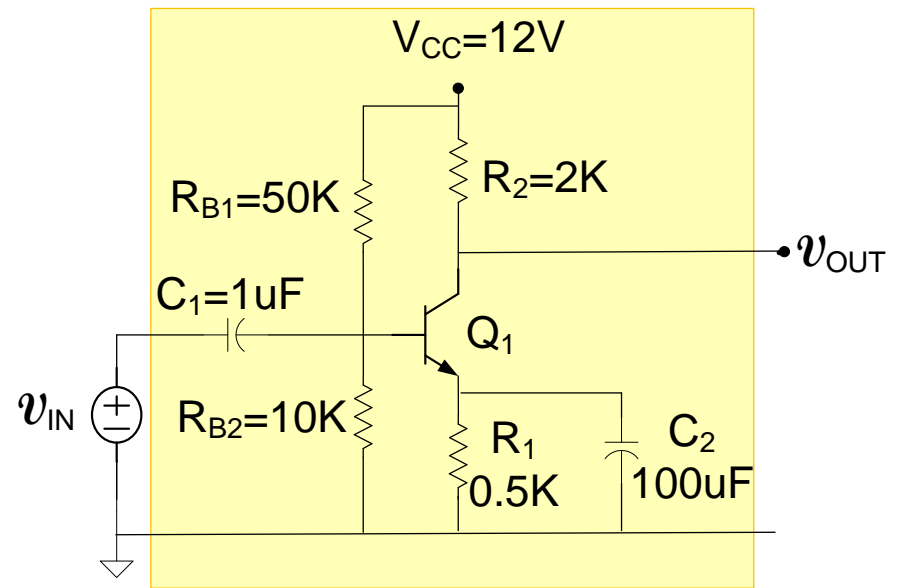
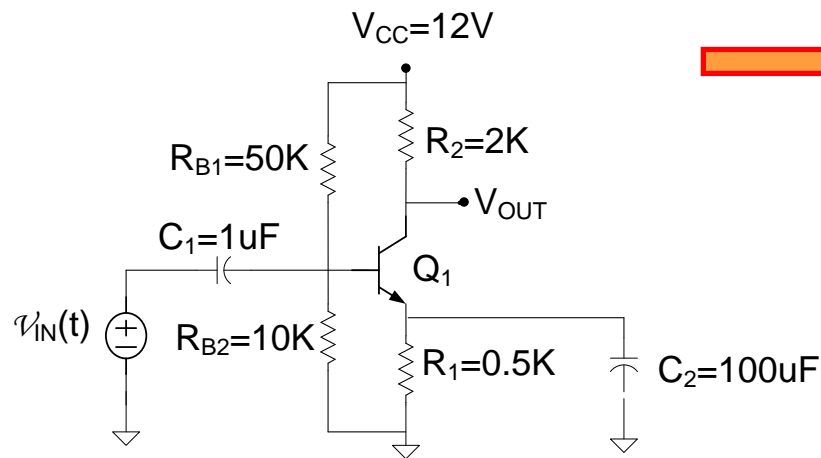
$$A_V \cong \frac{-R_2 g_m}{R_1 g_m} = \frac{-R_2}{R_1} = -4$$

Examples



Determine V_{OUTQ} , R_{IN} , R_{OUT} , and the SS voltage gain, and A_{VR} assume $\beta = 100$

Examples

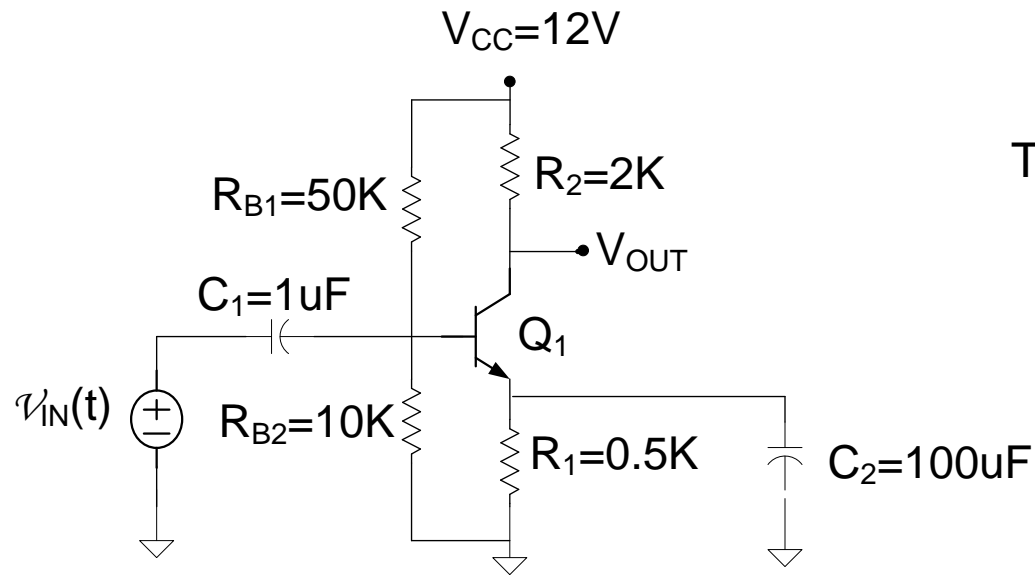


Determine V_{OUTQ} , R_{IN} , R_{OUT} , A_V , and A_{VR} ; assume $\beta=100$

(A_V , R_{IN} , R_{OUT} , and A_{VR} are the small-signal model parameters for this circuit)

Examples

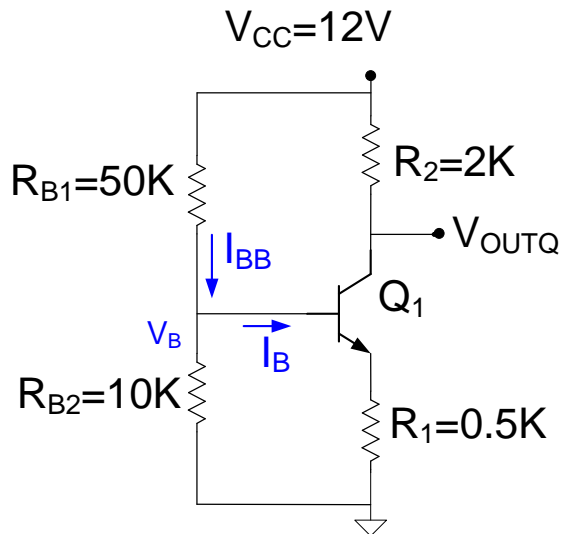
Determine V_{OUTQ} ✓



This is the same as the previous circuit !

$$V_{OUTQ} = 6.4V$$

$$I_{CQ} = \frac{5.6V}{2K} = 2.8mA$$

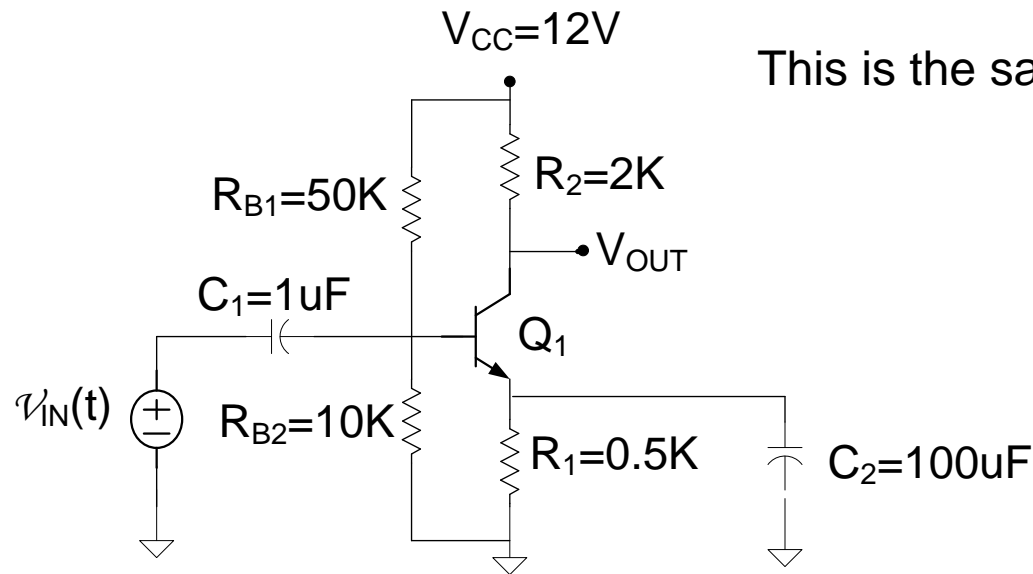


Note: This Q-point is nearly independent of the characteristics of the nonlinear BJT !

The dc equivalent circuit

Examples

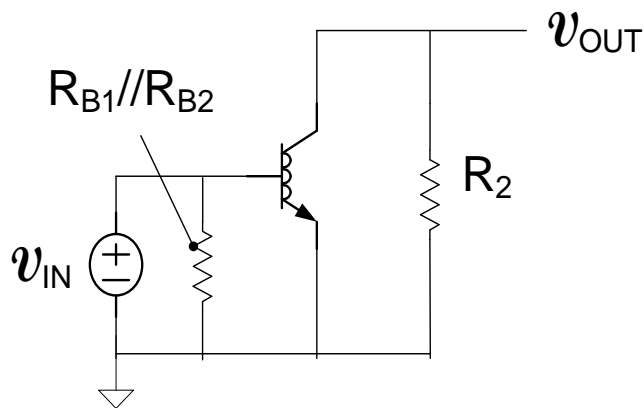
Determine the SS voltage gain A_V



This is the same as another previous-previous circuit !

$$A_V \cong -g_m R_2$$

$$A_V \cong -\frac{I_{CQ} R_2}{V_t}$$



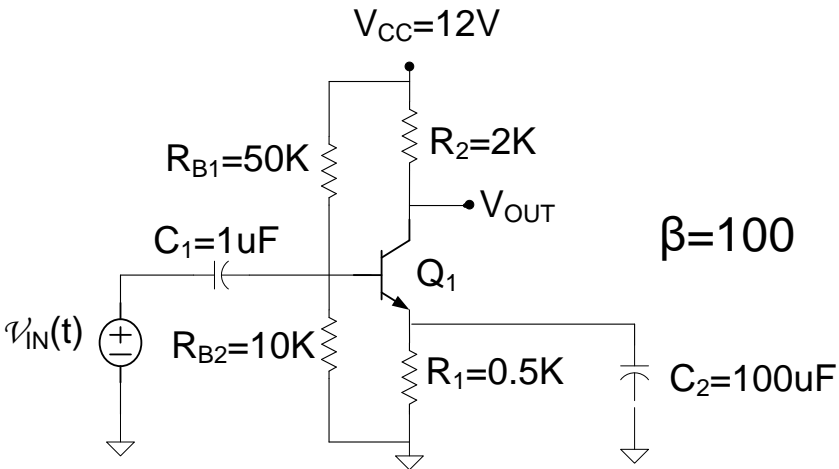
$$A_V \cong -\frac{5.6V}{26mV} = -215$$

The SS equivalent circuit

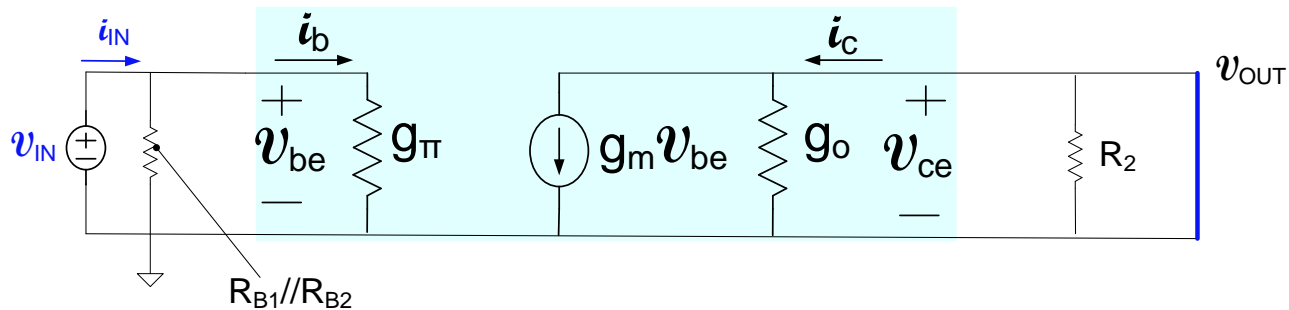
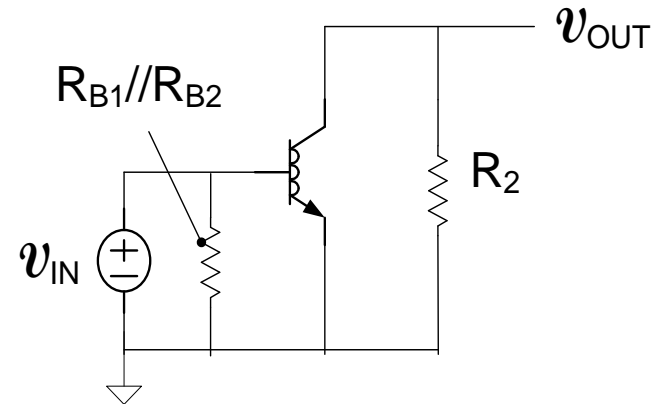
Note: This Gain is nearly independent of the characteristics of the nonlinear BJT !

Examples

Determination of R_{IN}



The SS equivalent circuit



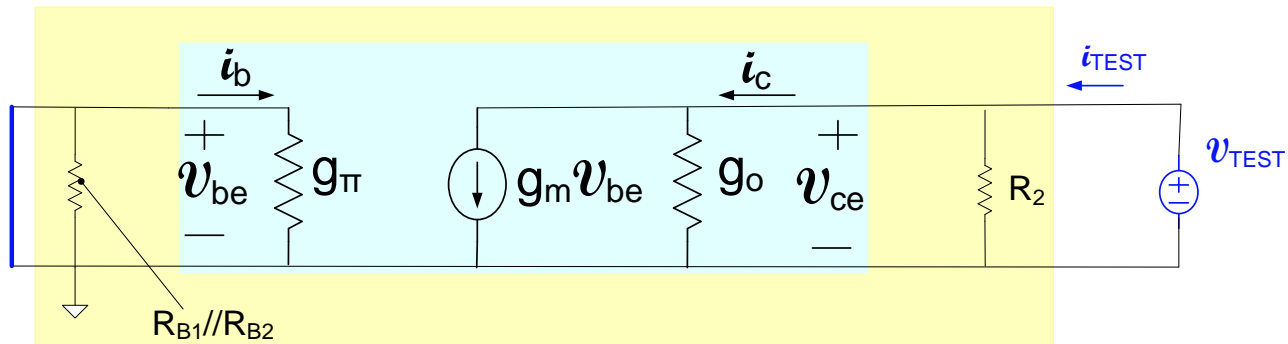
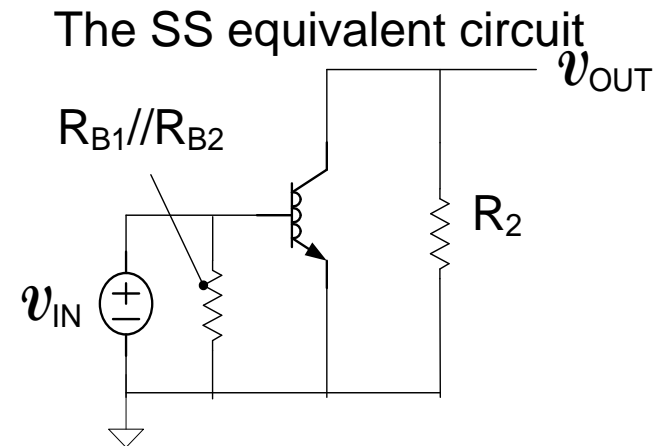
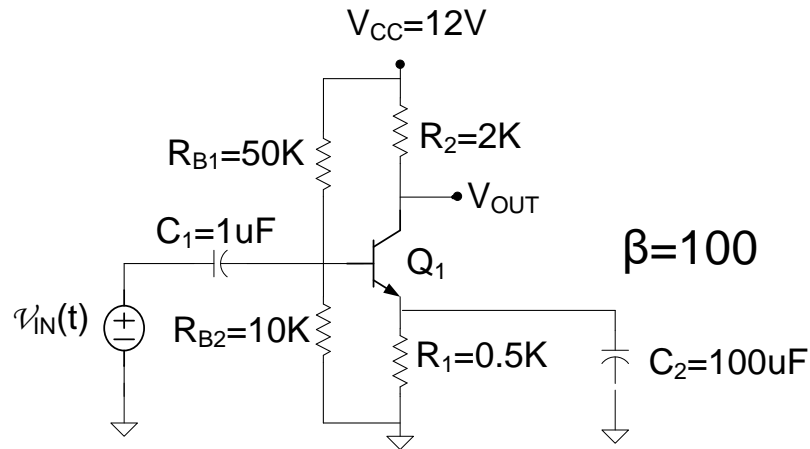
$$R_{IN} = R_{B1} // R_{B2} // r_{\pi} \cong r_{\pi}$$

$$r_{\pi} = \left(\frac{I_{CQ}}{\beta V_t} \right)^{-1} = \left(\frac{2.8 \text{ mA}}{100 \cdot 26 \text{ mV}} \right)^{-1} = 928 \Omega$$

$$R_{IN} = R_{B1} // R_{B2} // r_{\pi} \cong r_{\pi} = 930 \Omega$$

Examples

Determination of R_{OUT}

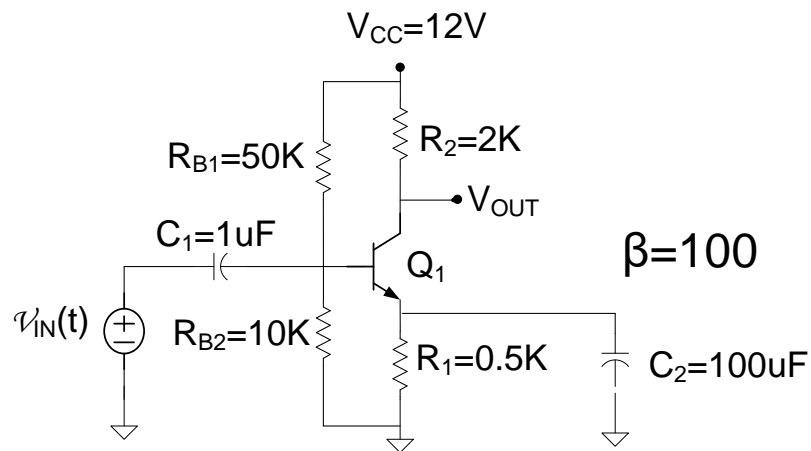


$$R_{OUT} = \frac{v_{TEST}}{i_{TEST}} = R_2 // r_o$$

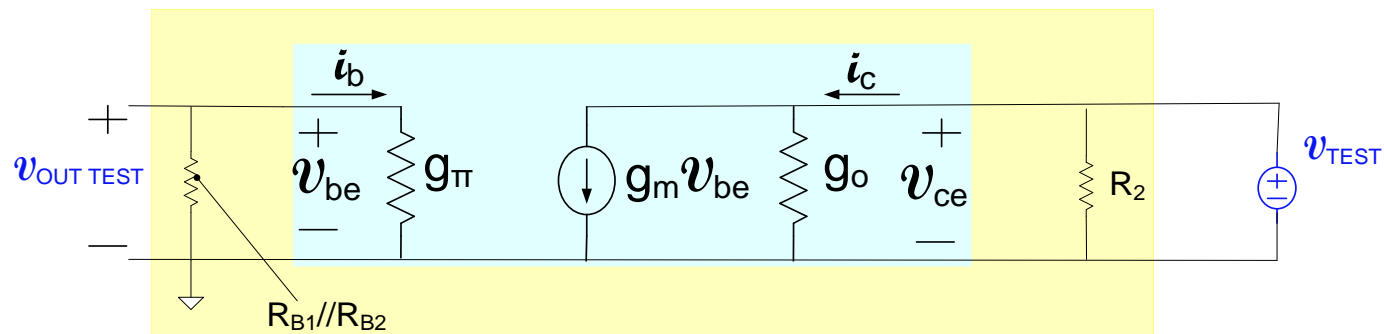
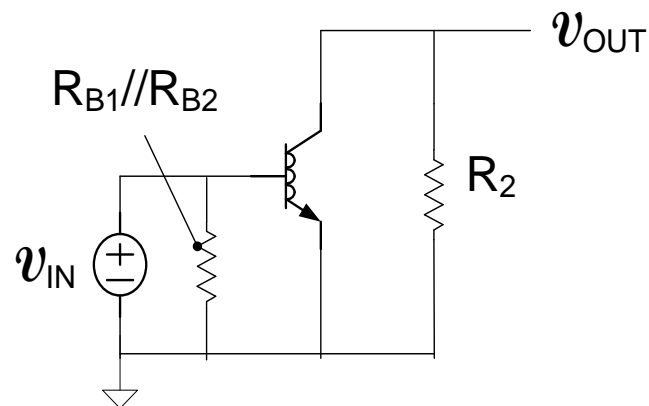
$$r_o = \left(\frac{I_{CQ}}{V_{AF}} \right)^{-1} = \left(\frac{2.8\text{mA}}{200\text{V}} \right)^{-1} = (1.4\text{E-}5)^{-1} = 71\text{K}\Omega$$

$$R_{OUT} = R_2 // r_o \cong R_2 = 2\text{K}$$

Examples Determine A_{VR}



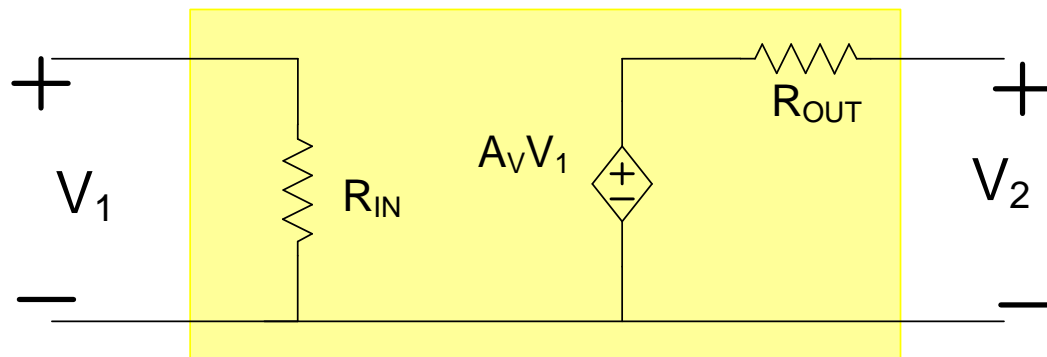
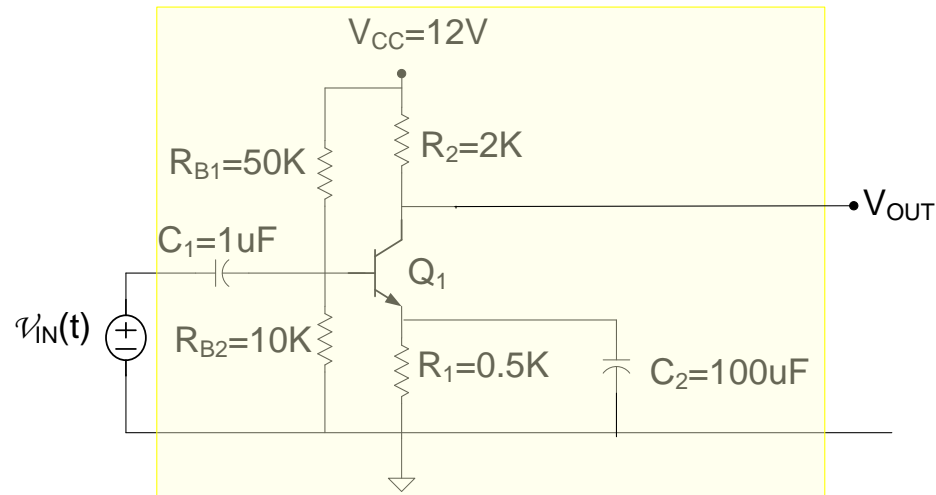
The SS equivalent circuit



$$v_{OUT\ TEST} = 0$$

$$A_{VR} = 0$$

Determination of small-signal two-port representation



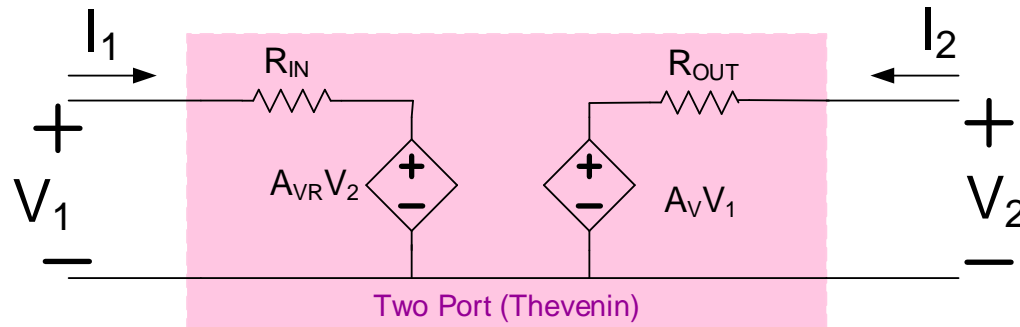
$$A_V \cong -215$$

$$R_{IN} \cong r_{\pi} = 930\Omega$$

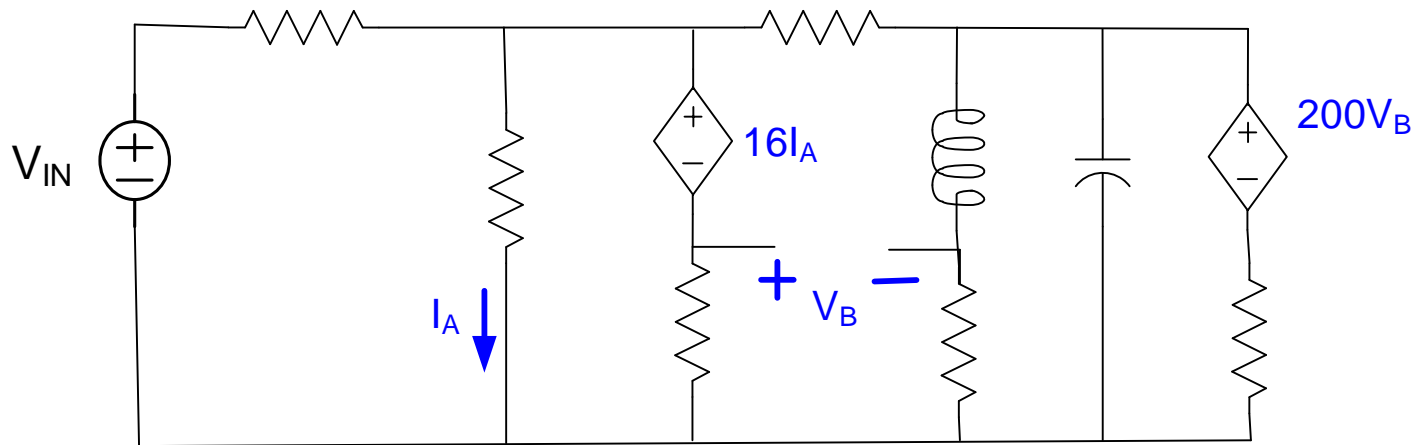
$$R_{OUT} \cong R_2 = 2K$$

This is the same basic amplifier that was considered many times

Relationship with Dependent Sources ?

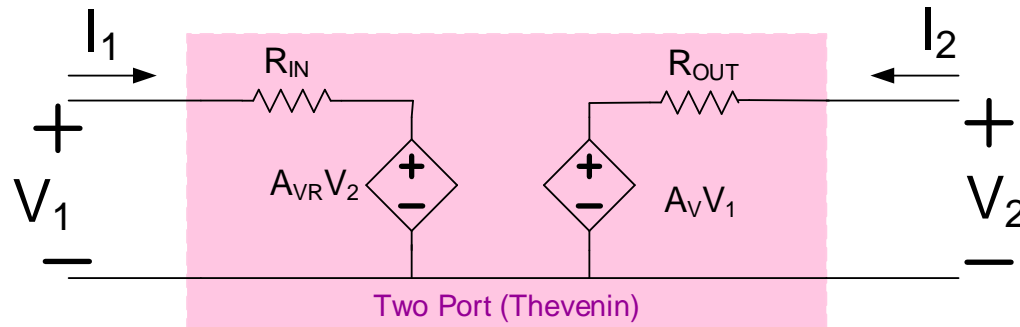


Dependent sources from EE 201



Example showing two dependent sources

Relationship with Dependent Sources ?

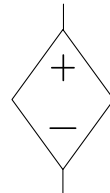


Dependent sources from EE 201

Voltage
Amplifier

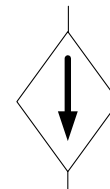
$$v_s = \mu v_x$$

Voltage Dependent
Voltage Source



$$I_s = \alpha v_x$$

Voltage Dependent
Current Source

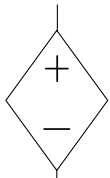


Transconductance
Amplifier

Transresistance
Amplifier

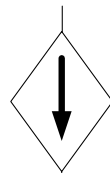
$$v_s = \rho I_x$$

Current Dependent
Voltage Source



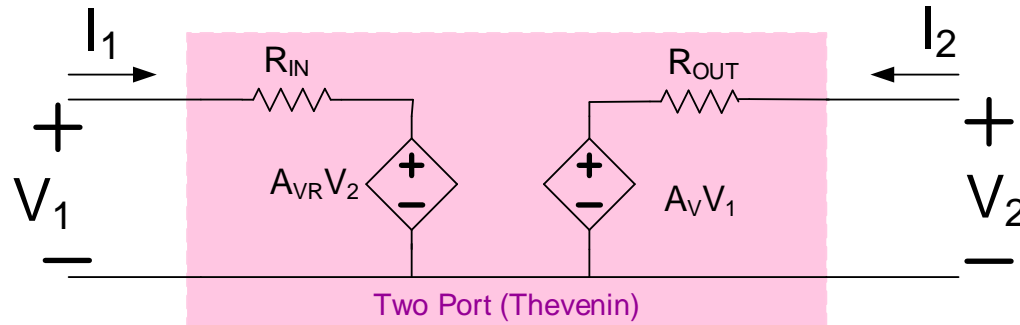
$$I_s = \beta I_x$$

Current Dependent
Current Source

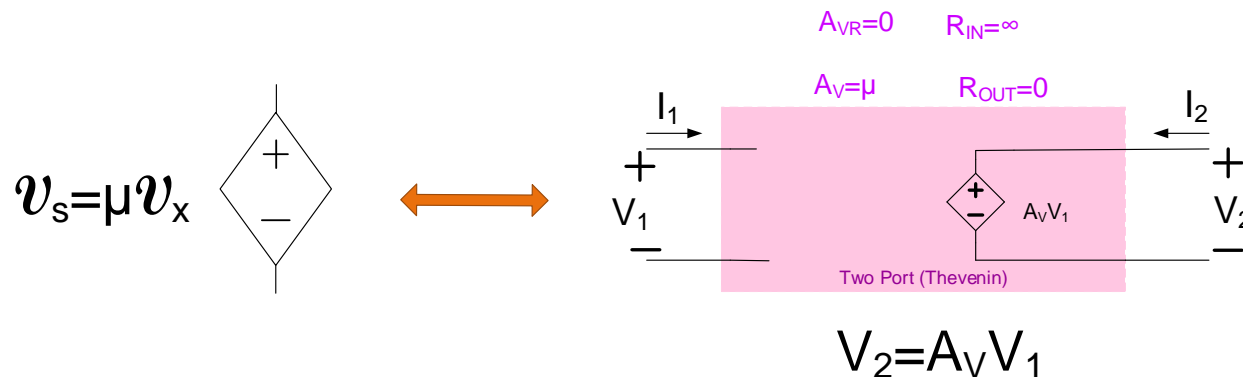


Current
Amplifier

Relationship with Dependent Sources ?

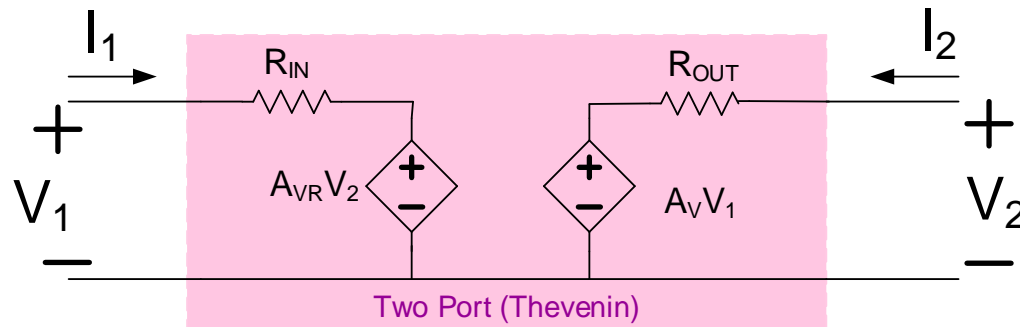


It follows that

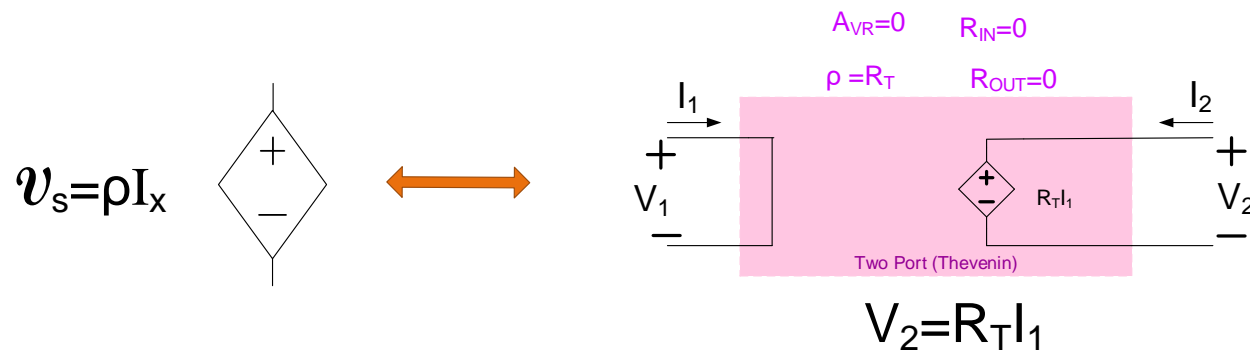


Voltage dependent voltage source is a unilateral floating two-port voltage amplifier with $R_{IN} = \infty$ and $R_{OUT} = 0$

Relationship with Dependent Sources ?

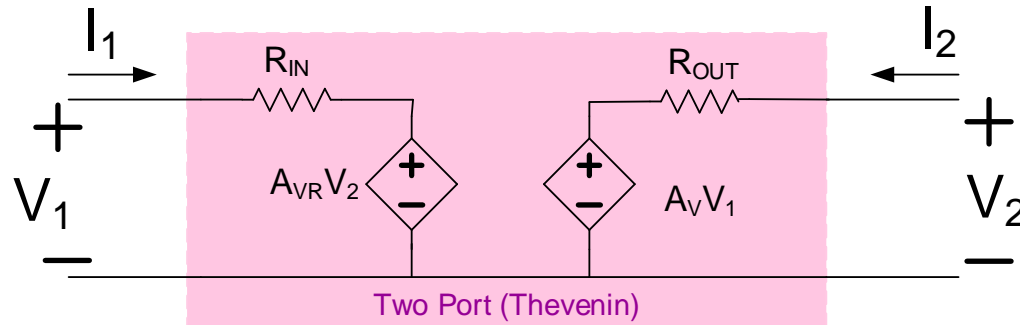


It follows that

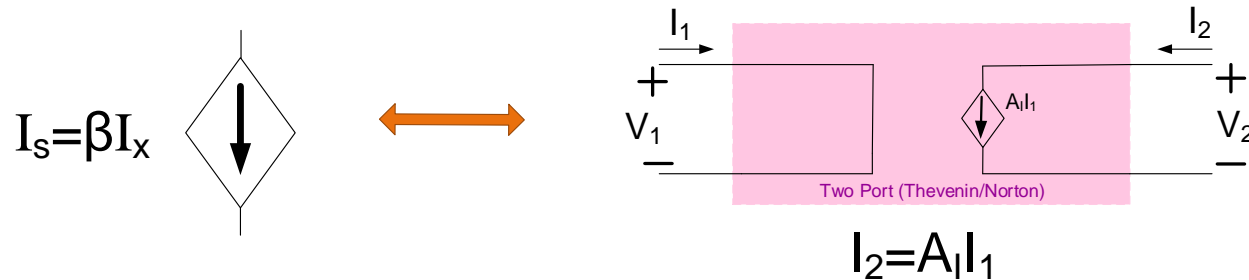


Current dependent voltage source is a unilateral floating two-port transresistance amplifier with $R_{IN}=0$ and $R_{OUT}=0$

Relationship with Dependent Sources ?

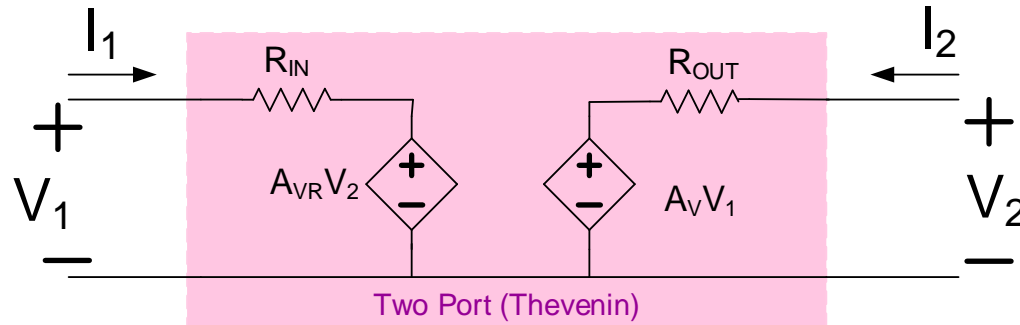


It follows that

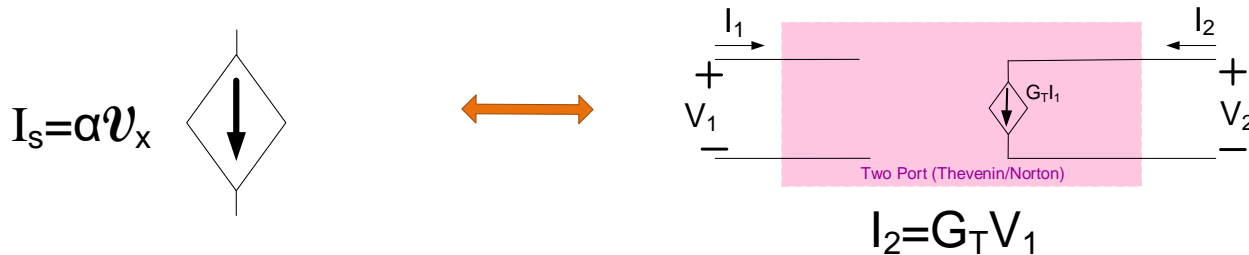


Current dependent current source is a floating unilateral two-port current amplifier with $R_{IN}=0$ and $R_{OUT}=\infty$

Relationship with Dependent Sources ?

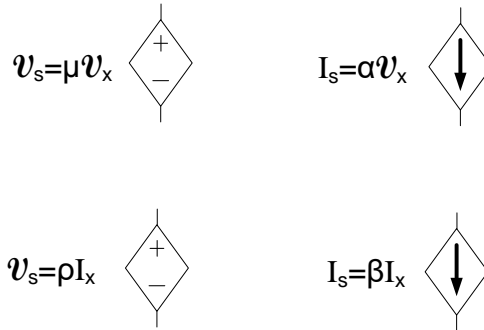


It follows that



Voltage dependent current source is a floating unilateral two-port transconductance amplifier with $R_{IN} = \infty$ and $R_{OUT} = \infty$

Dependent Sources



Dependent sources are unilateral two-port amplifiers with ideal input and output impedances

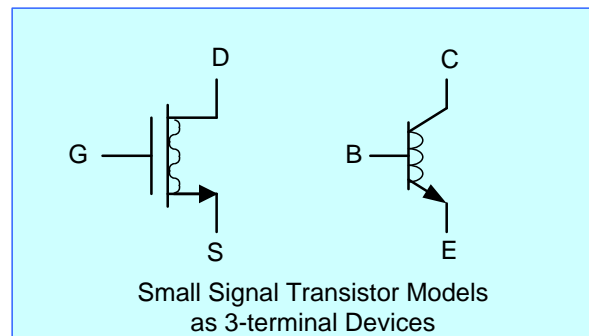
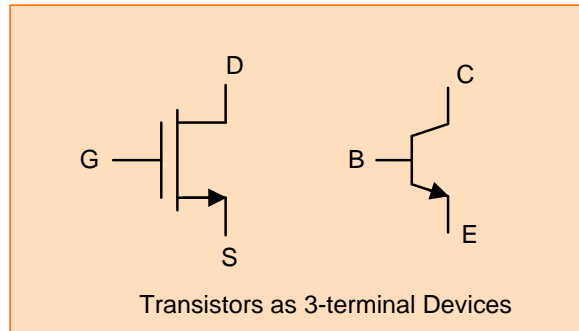
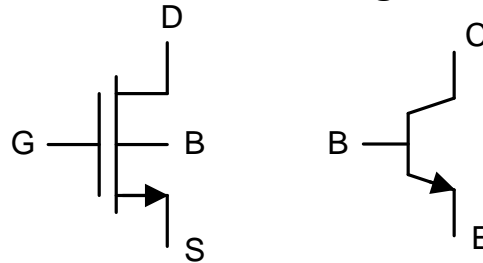
Dependent sources do not exist as basic circuit elements but amplifiers can be designed to perform approximately like a dependent source

- Practical dependent sources typically are not floating on input or output
- One terminal is usually grounded
- Input and output impedances of realistic structures are usually not ideal

Why were “dependent sources” introduced as basic circuit elements instead of two-port amplifiers???

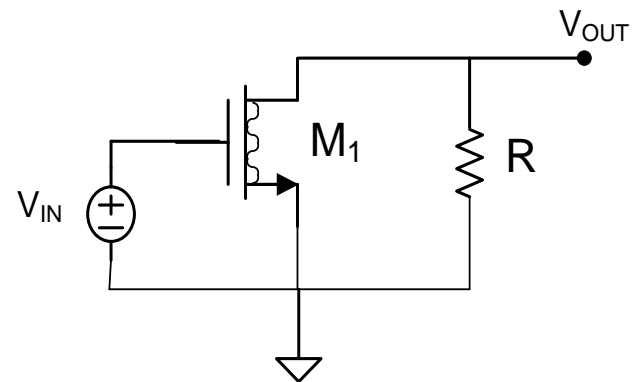
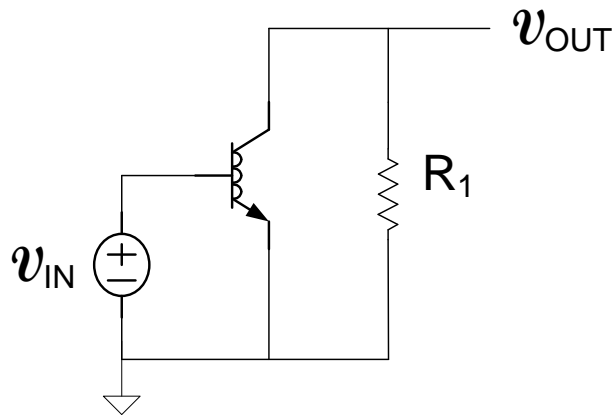
Basic Amplifier Structures

- MOS and Bipolar Transistors Both have 3 primary terminals
- MOS transistor has a fourth terminal that is generally considered a parasitic terminal



Basic Amplifier Structures

Observation:



These circuits considered previously have a terminal (emitter or source) common to the input and output in the small-signal equivalent circuit

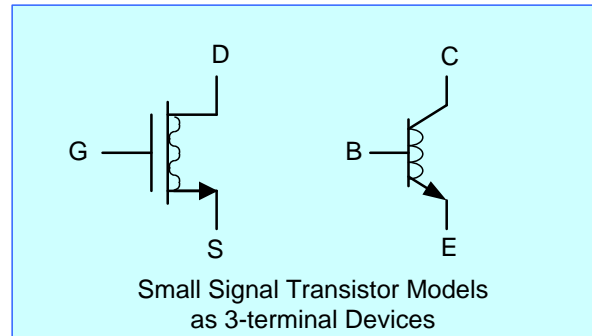
For BJT, E is common, input on B, output on C

Termed “Common Emitter”

For MOSFET, S is common, input on G, output on D

Termed “Common Source”

Basic Amplifier Structures



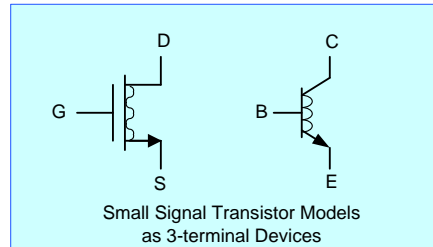
Amplifiers using these devices generally have one terminal common and use remaining terminals as input and output

Since devices are unilateral, designation of input and output terminals is uniquely determined

Three different ways to designate the common terminal

Source or Emitter	termed Common Source or Common Emitter
Gate or Base	termed Common Gate or Common Base
Drain or Collector	termed Common Drain or Common Collector

Basic Amplifier Structures

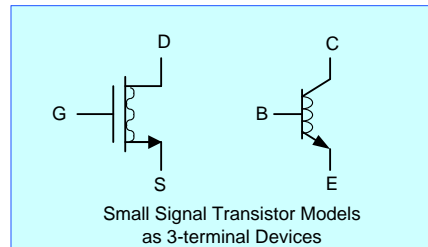


	MOS			BJT		
	Common	Input	Output	Common	Input	Output
Common Source or Common Emitter	S	G	D	E	B	C
Common Gate or Common Base	G	S	D	B	E	C
Common Drain or Common Collector	D	G	S	C	B	E

Identification of Input and Output Terminals is not arbitrary

It will be shown that all 3 of the basic amplifiers are useful !

Basic Amplifier Structures



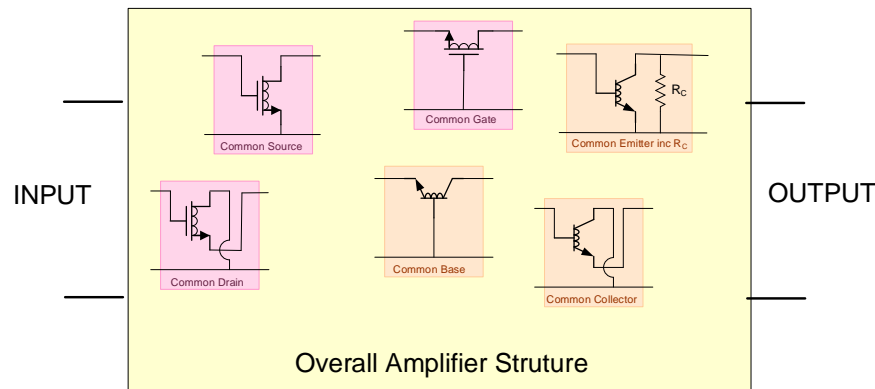
Common Source or Common Emitter

Common Gate or Common Base

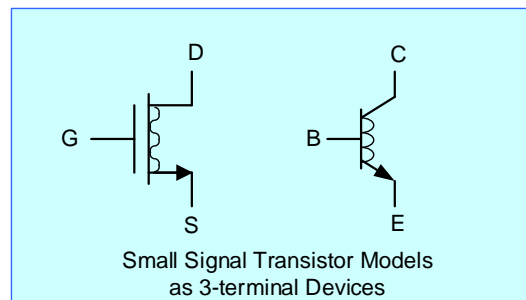
Common Drain or Common Collector

Objectives in Study of Basic Amplifier Structures

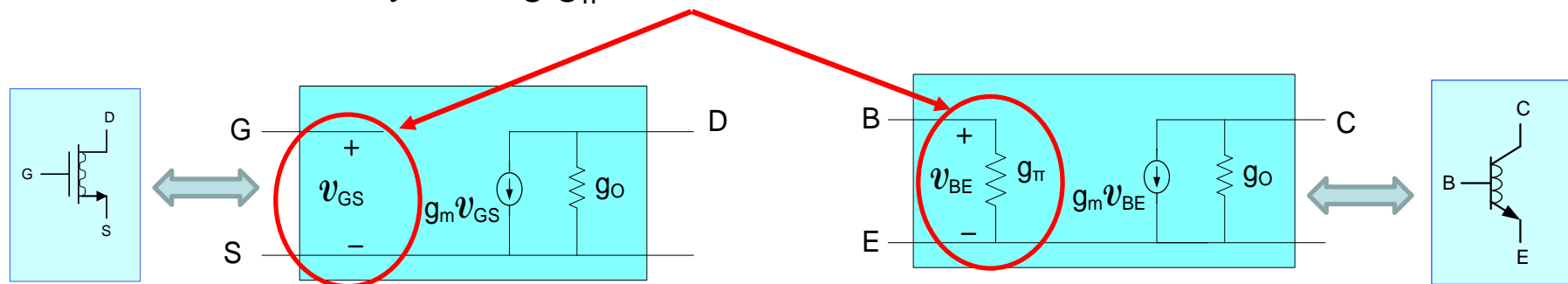
1. Obtain key properties of each basic amplifier
2. Develop method of designing amplifiers with specific characteristics using basic amplifier structures



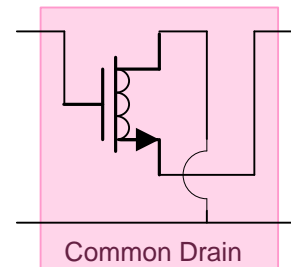
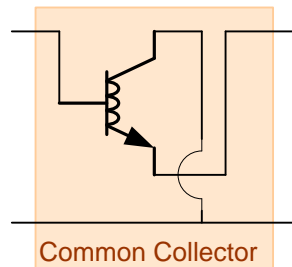
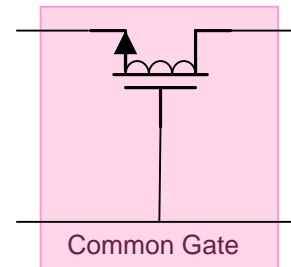
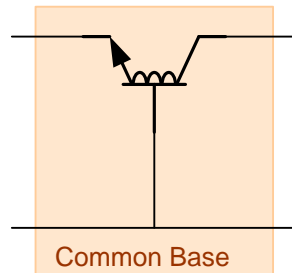
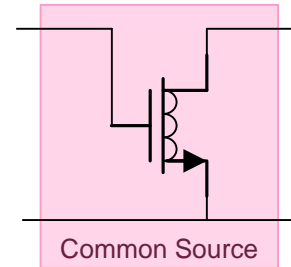
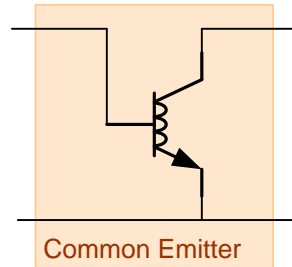
Characterization of Basic Amplifier Structures



- Observe that the small-signal equivalent of any 3-terminal network is a two-port
- Thus to characterize any of the 3 basic amplifier structures, it suffices to determine the two-port equivalent network
- Since small signal model when expressed in terms of small-signal parameters of BJT and MOSFET differ only in the presence/absence of g_{π} term, can analyze the BJT structures and then obtain characteristics of corresponding MOS structure by setting $g_{\pi}=0$

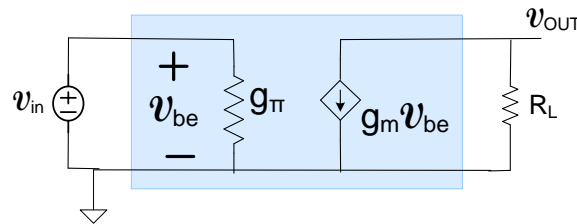
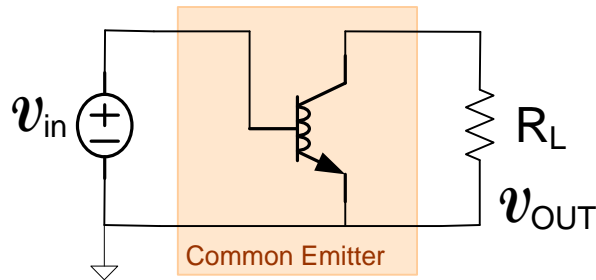


The three basic amplifier types for both MOS and bipolar processes



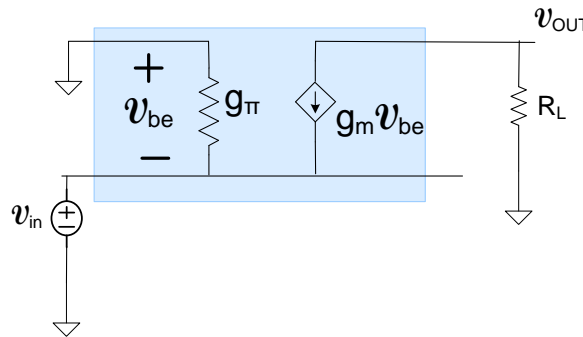
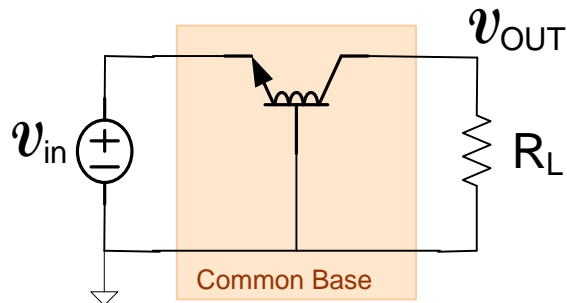
Will focus on the performance of the bipolar structures and then obtain performance of the MOS structures by observation

The three basic amplifier types for both MOS and bipolar processes



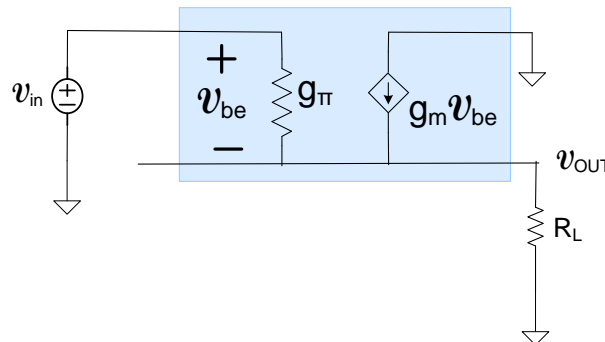
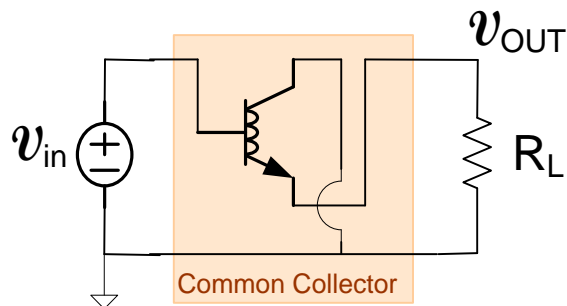
$$\left. \begin{aligned} v_{OUT} &= -g_m R_L v_{be} \\ v_{IN} &= v_{be} \end{aligned} \right\}$$

$$A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R_L$$



$$\left. \begin{aligned} v_{OUT} &= -g_m R_L v_{be} \\ v_{IN} &= -v_{be} \end{aligned} \right\}$$

$$A_V = \frac{v_{OUT}}{v_{IN}} = g_m R_L$$

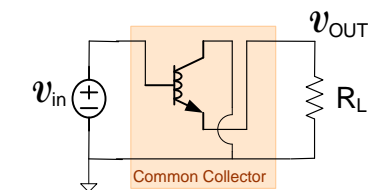
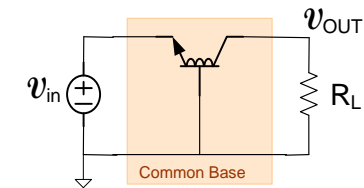
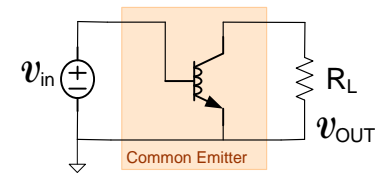
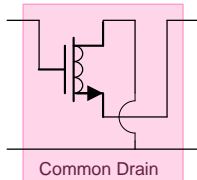
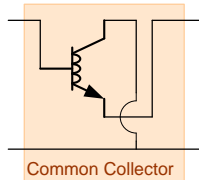
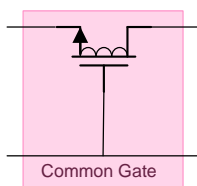
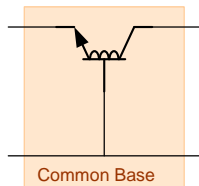
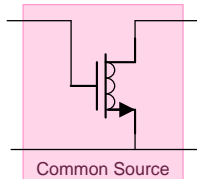
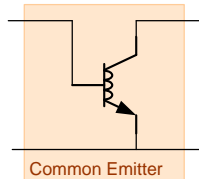


$$\left. \begin{aligned} v_{OUT} &= (g_m + g_\pi) v_{be} R_L \\ v_{IN} &= v_{be} + (g_m + g_\pi) v_{be} R_L \end{aligned} \right\}$$

$$A_V = \frac{v_{OUT}}{v_{IN}} = \frac{(g_m + g_\pi) R_L}{1 + (g_m + g_\pi) R_L} \cong 1$$

- Significantly different gain characteristics for the three basic amplifiers
- There are other significant differences too (R_{IN} , R_{OUT} , ...) as well

The three basic amplifier types for both MOS and bipolar processes



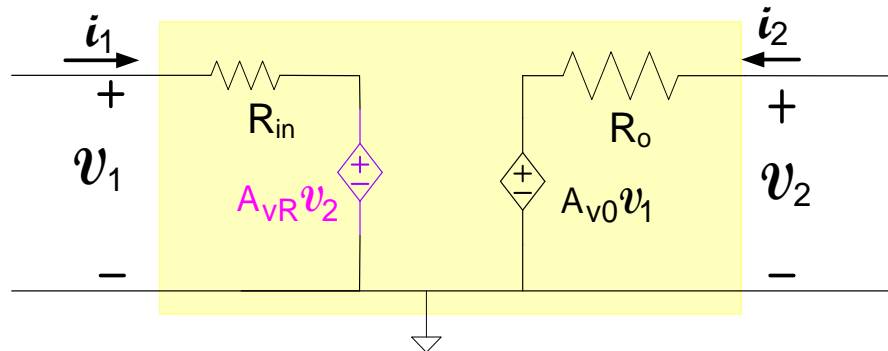
More general models are needed to accommodate biasing, understand performance capabilities, and include effects of loading of the basic structures

Two-port models are useful for characterizing the basic amplifier structures

How can the two-port parameters be obtained for these or any other linear two-port networks?

Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network



1. $v_{TEST} : i_{TEST}$ Method (considered in last lecture)

2. Write $v_1 : v_2$ equations in standard form

$$v_1 = i_1 R_{IN} + A_{VR} v_2$$

$$v_2 = i_2 R_O + A_{V0} v_1$$

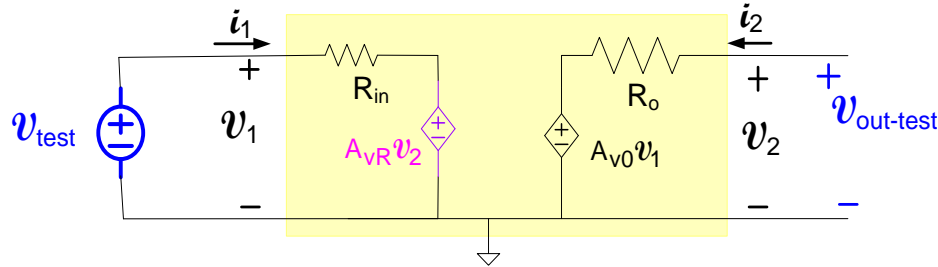
3. Thevenin-Norton Transformations

4. Ad Hoc Approaches

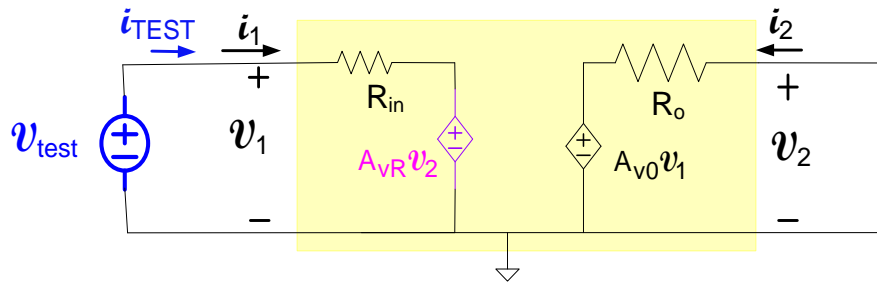
Any of these methods can be used to obtain the two-port model

$v_{\text{test}} : i_{\text{test}}$ Method for Obtaining Two-Port Amplifier Parameters

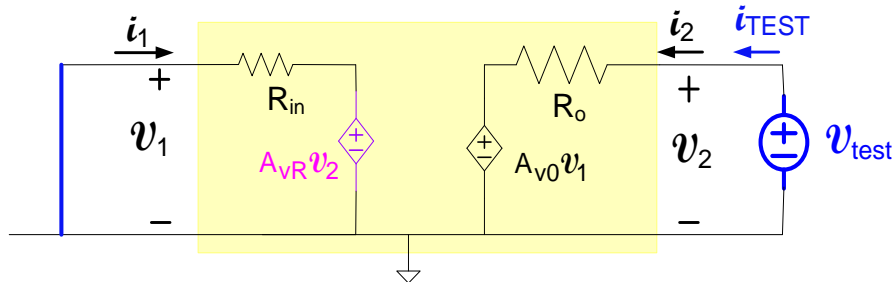
SUMMARY from PREVIOUS LECTURE



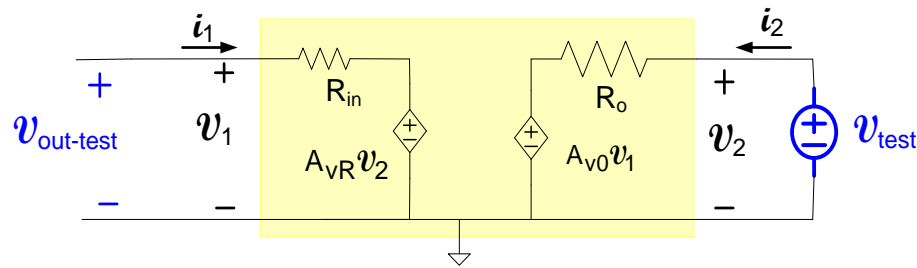
$$A_{v0} = \frac{v_{\text{out-test}}}{v_{\text{test}}}$$



$$R_{\text{in}} = \frac{v_{\text{test}}}{i_{\text{test}}}$$



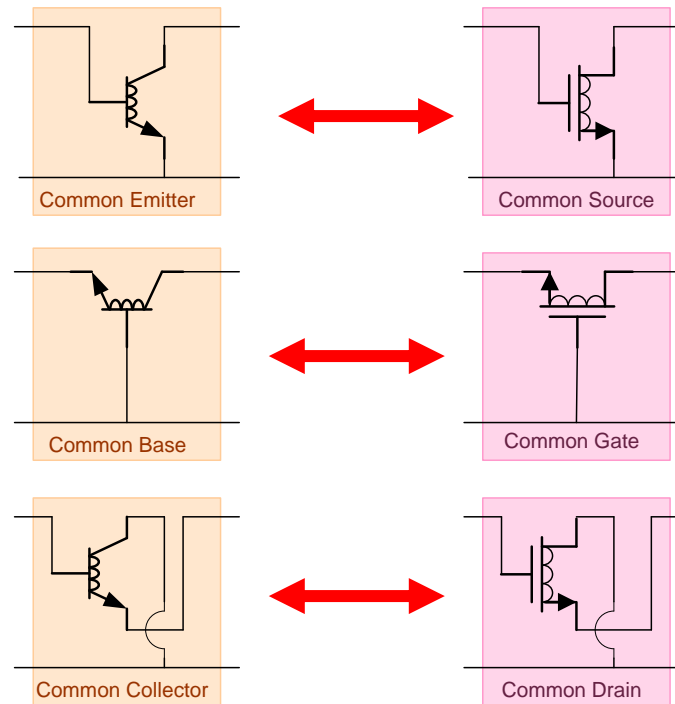
$$R_0 = \frac{v_{\text{test}}}{i_{\text{test}}}$$



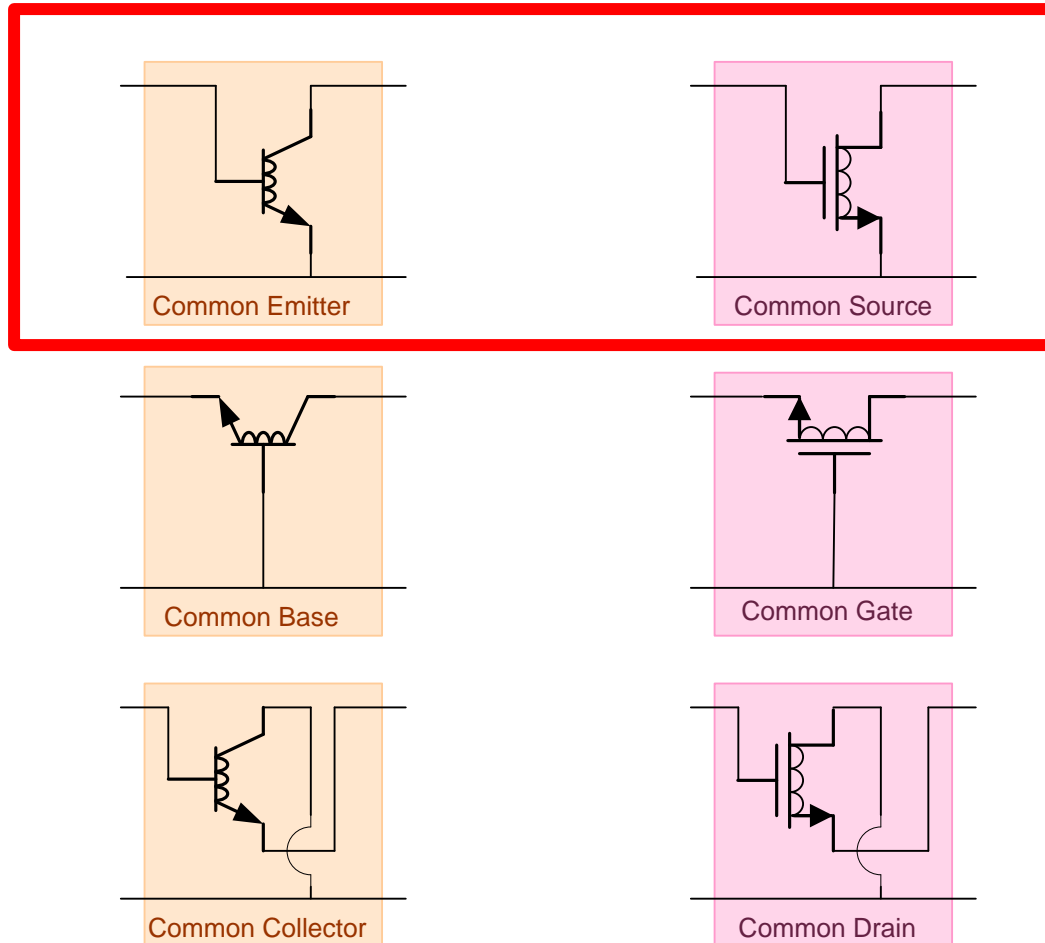
$$A_{vR} = \frac{v_{\text{out-test}}}{v_{\text{test}}}$$

If Unilateral $A_{vR} = 0$

Will now develop two-port model for each of the three basic amplifiers and look at one widely used application of each

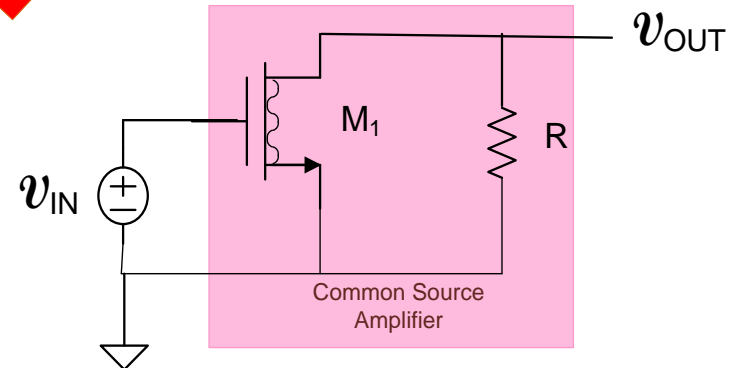
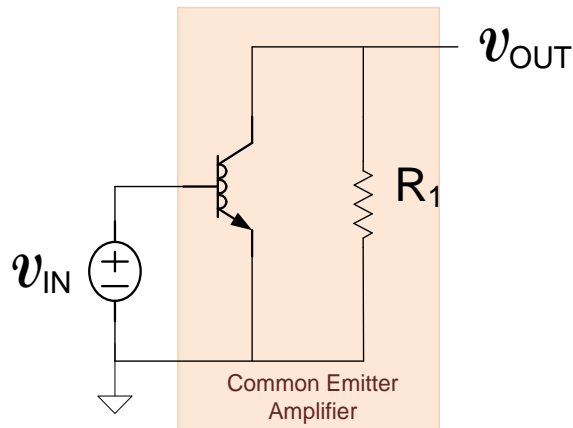
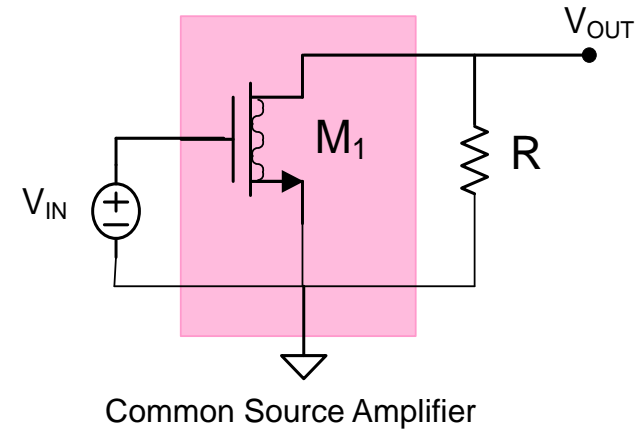
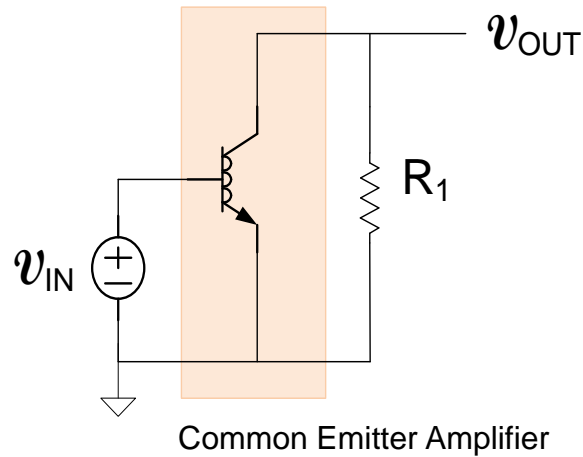


Consider Common Emitter/Common Source Two-port Models



Will focus on Bipolar Circuit since MOS counterpart is a special case obtained by setting $g_{\pi}=0$

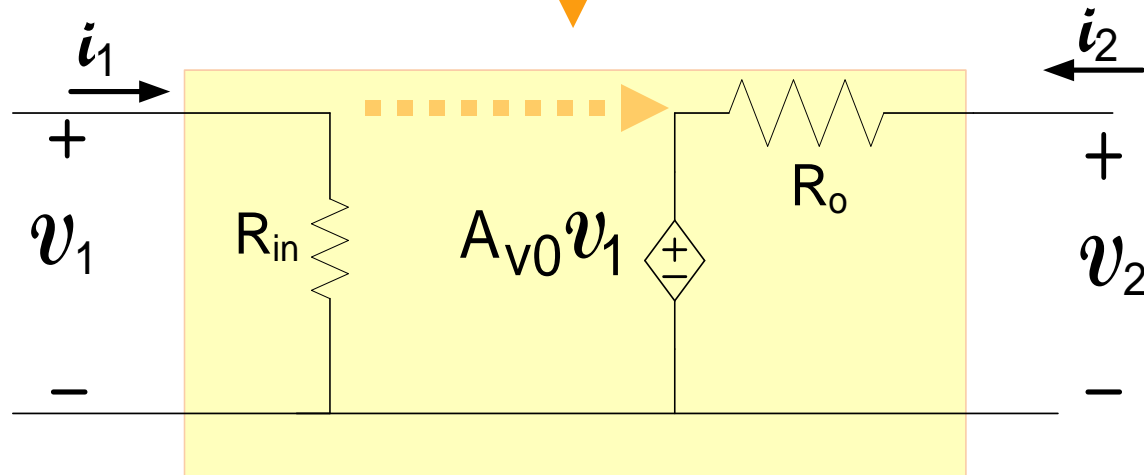
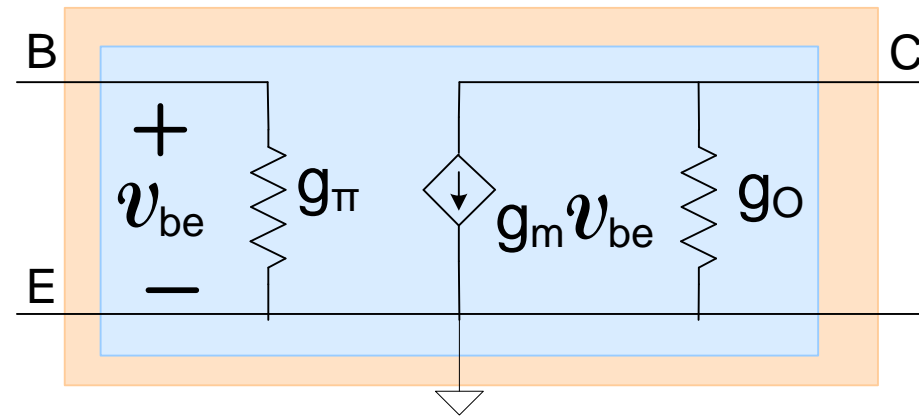
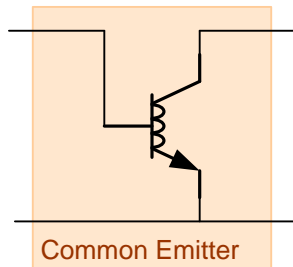
Basic CE/CS Amplifier Structures



Can include or exclude R and R_1 in two-port models (of course they are different circuits)

The CE and CS amplifiers are themselves two-ports !

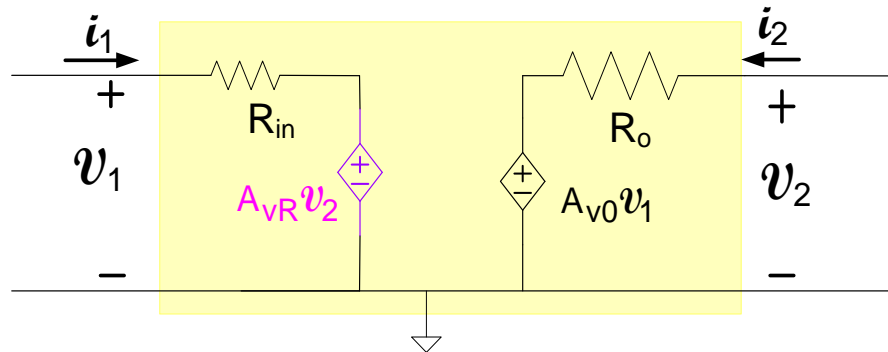
Two-port model for Common Emitter Configuration



$\{R_i, A_{v0} \text{ and } R_o\}$

Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network

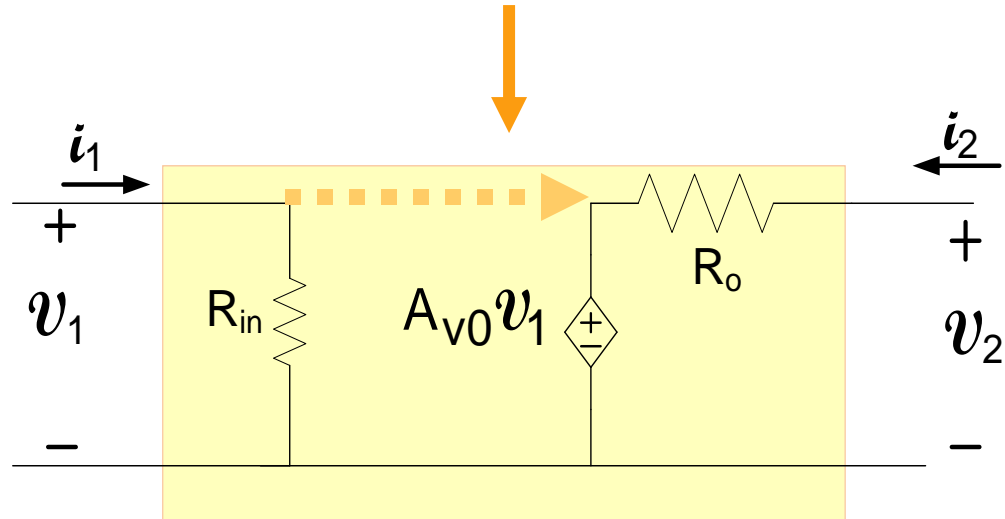
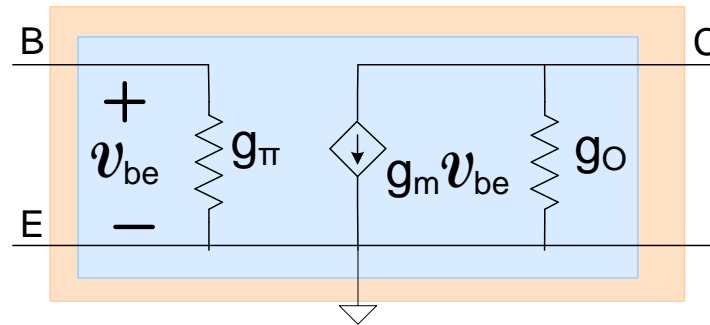
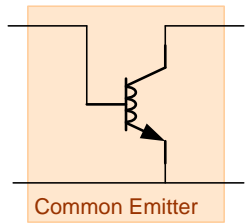


1. $v_{TEST} : i_{TEST}$ Method
2. Write $v_1 : v_2$ equations in standard form
$$v_1 = i_1 R_{IN} + A_{VR} v_2$$
$$v_2 = i_2 R_O + A_{V0} v_1$$



3. Thevenin-Norton Transformations
4. Ad Hoc Approaches

Two-port model for Common Emitter Configuration



By Thevenin : Norton Transformations

$$R_{in} = \frac{1}{g_{\pi}}$$

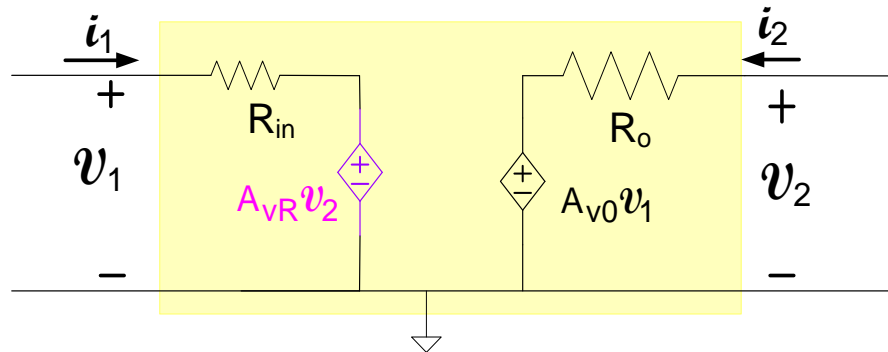
$$A_{V0} = -\frac{g_m}{g_o}$$

$$R_o = \frac{1}{g_o}$$

$$A_{VR} = 0$$

Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network

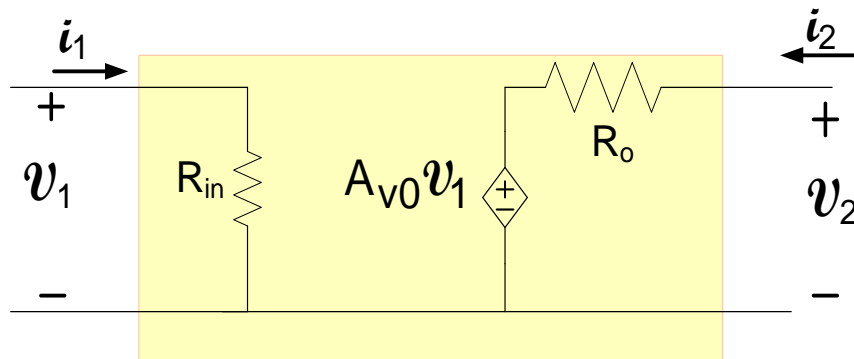
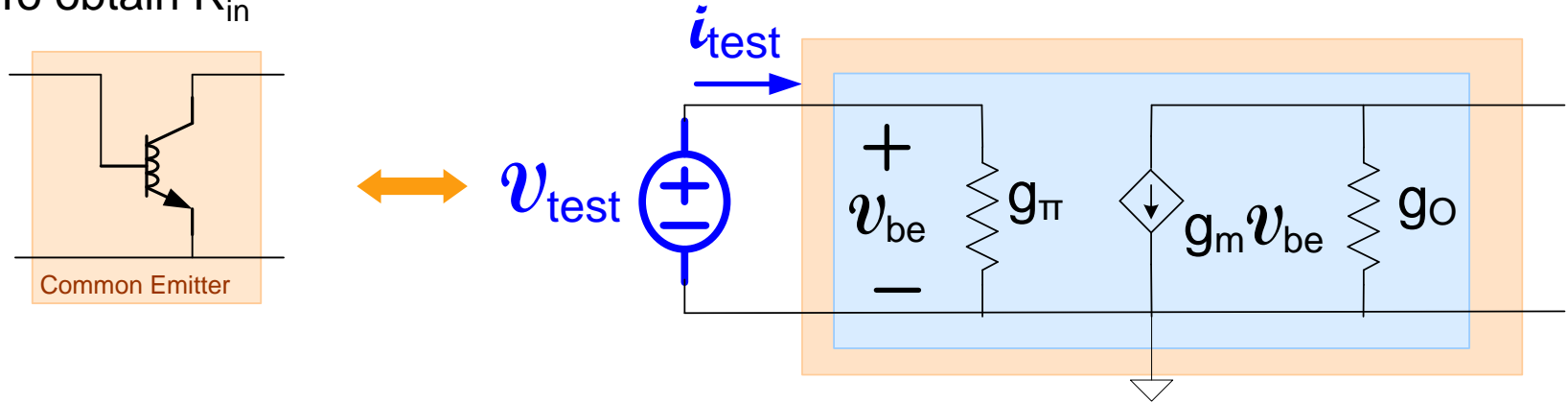


-
1. $v_{TEST} : i_{TEST}$ method
 2. Write $v_1 : v_2$ equations in standard form
$$v_1 = i_1 R_{IN} + A_{VR} v_2$$
$$v_2 = i_2 R_O + A_{VO} v_1$$
 3. Thevenin-Norton Transformations
 4. Ad Hoc Approaches
- ↓

Two-port model for Common Emitter Configuration

Alternately, by $v_{\text{TEST}} : i_{\text{TEST}}$ Method

To obtain R_{in}



$$R_{\text{in}} = \frac{v_{\text{test}}}{i_{\text{test}}}$$

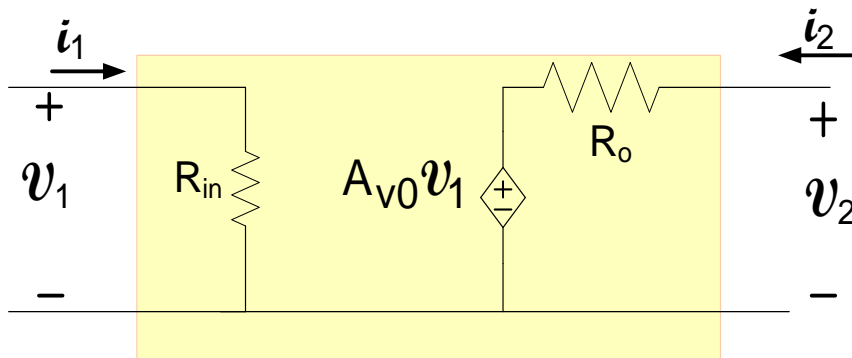
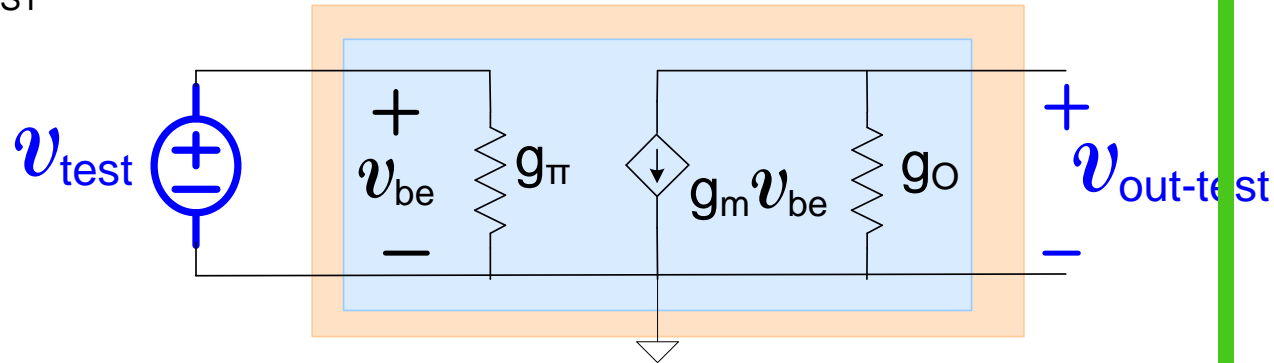
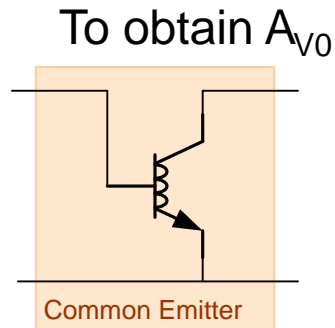
$$R_{\text{in}} = \frac{1}{g_{\pi}}$$

$\{R_{\text{in}}, A_{v0} \text{ and } R_o\}$



Two-port model for Common Emitter Configuration

Alternately, by $v_{\text{TEST}} : i_{\text{TEST}}$ Method



$$A_{V0} = \frac{v_{\text{out-test}}}{v_{\text{test}}}$$

$$v_{\text{out-test}} = v_{\text{test}} \left(-\frac{g_m}{g_o} \right)$$

$$A_{V0} = -\frac{g_m}{g_o}$$

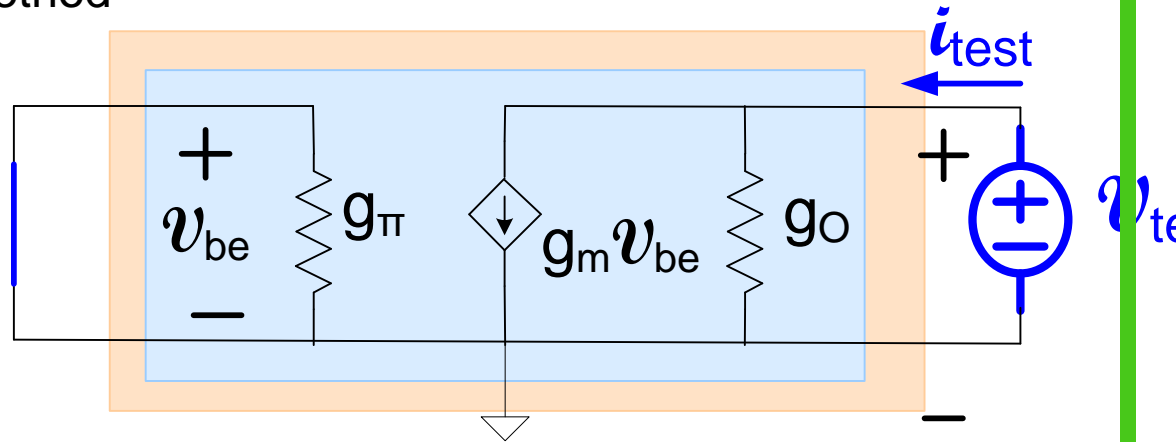
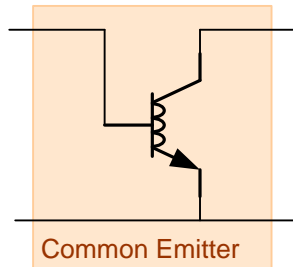
$\{R_{\text{in}}, A_{V0} \text{ and } R_o\}$



Two-port model for Common Emitter Configuration

Alternately, by $v_{\text{TEST}} : i_{\text{TEST}}$ Method

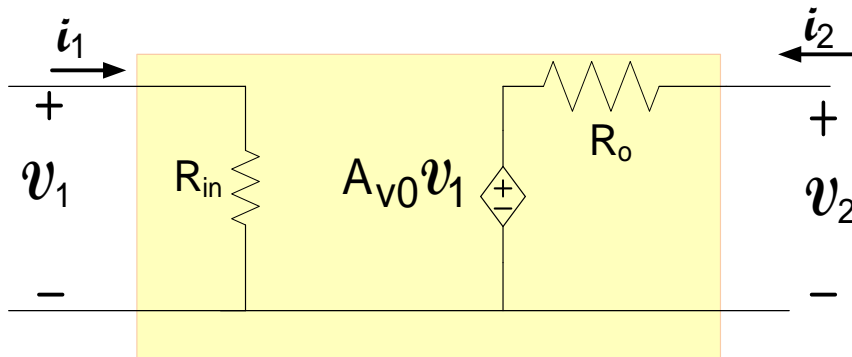
To obtain g_0



$$R_0 = \frac{v_{test}}{i_{test}}$$

$$v_{test} = i_{test} (g_0)$$

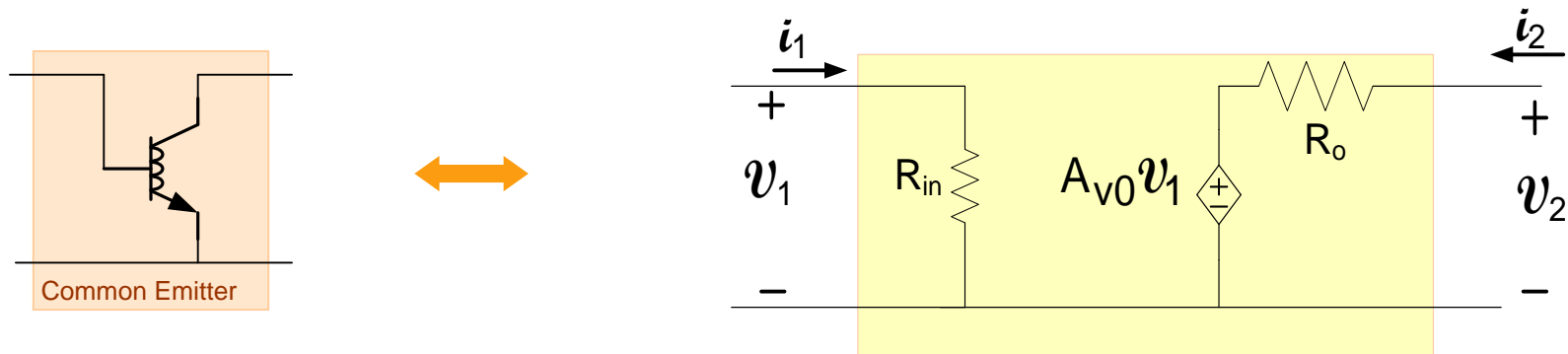
$$R_0 = \frac{1}{g_0}$$



$\{R_{in}, A_{v0} \text{ and } R_o\}$



Two-port model for Common Emitter Configuration



In terms of small signal model parameters:

$$R_{in} = \frac{1}{g_{\pi}} \quad A_{V0} = -\frac{g_m}{g_o} \quad R_o = \frac{1}{g_o} \quad A_{VR} = 0$$

In terms of operating point and model parameters:

$$R_i = \frac{\beta V_t}{I_{CQ}} \quad A_{V0} = -\frac{V_{AF}}{V_t} \quad R_o = \frac{V_{AF}}{I_{CQ}} \quad A_{VR} = 0$$

Characteristics:

- Input impedance is mid-range
- Voltage Gain is Large and Inverting
- Output impedance is large
- Unilateral
- Widely used to build voltage amplifiers

End of Lecture 31