Recitation problems 1 - Solutions

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1. Which of the following sentences is a statement and why?

   (a) 1,024 is the smallest four-digit number that is a perfect square.

   (b) She is a mathematics major.

   (c) $128 = 26$.

   (d) $x = 26$.

   Solution

   (a) Yes. This is a true proposition

   (b) No. This is either true or false

   (c) Yes. This is a false proposition

   (d) No. This is either true or false

2. Let h = "John is healthy", w = "John is wealthy", and s = "John is wise". Express the following in symbolic form.

   (a) John is healthy and wealthy but not wise.

   (b) John is not wealthy but he is healthy and wise.

   (c) John is neither healthy, wealthy nor wise.

   Solution

   (a) $h \wedge w \wedge \neg s$

   (b) $\neg w \wedge h \wedge s$

   (c) $\neg h \wedge \neg w \wedge \neg s$

3. Write the truth table for

   $(p \vee (\neg p \vee q)) \wedge \neg(q \wedge \neg r)$

Solution

p	q	r	¬p ∨ q	p ∨ (¬p ∨ q)	q ∧ ¬r	(p ∨ (¬p ∨ q)) ∧ ¬(q ∧ ¬r)
T	T	T	T	T	F	T
T	T	F	T	T	T	F
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	F	T
F	T	F	T	T	T	F
F	F	T	T	T	F	T
F	F	F	T	T	F	T

4. This is a 2-part question.

   (a) Show that the following 3 statements are logically equivalent:

   $p \implies q \vee r$, $p \wedge \neg q \implies r$, $p \wedge \neg r \implies q$

   (b) Using the logical equivalences above, rewrite the following sentence in two different ways (assume that n here represents a fixed (and known) integer). "If n is prime, then n is odd or n is 2."

Solution

(a) These three statements are both equivalent to ¬p ∨ q ∨ r:

- p $\implies$ q ∨ r ≡ ¬p ∨ (q ∨ r) ≡ ¬p ∨ q ∨ r
- p ∧ ¬q $\implies$ r ≡ ¬(p ∧ ¬q) ∨ r ≡ ¬p ∨ q ∨ r
- p ∧ ¬r $\implies$ q ≡ ¬(p ∧ ¬r) ∨ q ≡ ¬p ∨ r ∨ q ≡ ¬p ∨ q ∨ r

(b) Let

p: n is prime.

q: n is odd.

r: n is 2.

Then the sentence can be rewritten as:

- If n is prime and not odd then n is 2.
- If n is prime and not 2 then n is odd.

5. Describe a simple algorithm which, given a positive integer n, produces a width n array of truth values whose rows would be all the possible truth values for n propositional variables. For example, for n=2, the array would be:

```
|-|-|
|T|T|
|T|F|
|F|T|
|F|F|
```

Your description can be in pseudocode, or in a familiar language like Java or Python.

Solution

Our truth table contains 2^n rows and n columns. This can be done in various recursive and non-recusive ways. For example the following pseudo code,

```
initialize row_index = 0, column_index = 0;

for row_index< 2^(n) , row_index++

    for column_index < n, column_index++

        if (row_index/(2^(n-1-column_index)))%2 == 0
            place 'T'
        else
```

```
            place 'F'
    end
end
```

A lot many different alogorithms can be written for the given question.