Sgordon4
Section D

1. *Prove, using mathematical induction, that for any n ≥ 1:*

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

We prove this statement by induction.

P(n) = $\frac{n(n+1)(2n+1)}{6}$

Base case: P(1) = 1 ✓

Suppose P(k) = $\frac{k(k+1)(2k+1)}{6}$ = $\frac{2k^3+3k^2+k}{6}$

P(k+1) = $1^2 + 2^2 + \ldots + k^2 + (k+1)^2$ = P(k) + $(k+1)^2$

$(k+1)^2$ = $k^2 + 2k + 1$

P(k+1) = $\frac{(k+1)(k+2)(2k+3)}{6}$

P(k) + $(k+1)^2$ = $\frac{k(k+1)(2k+1)}{6}$ + $(k+1)^2$ = $\frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$ =

$\frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$ = $\frac{2k^3+3k^2+k + 6k^2+12k+6}{6}$ = $\frac{2k^3+9k^2+13k+6}{6}$ = $\frac{(k+1)(k+2)(2k+3)}{6}$ ✓

By induction, P(n) is true for all n ≥ *1*

2. *Prove the Prime Factorization Theorem (PFT) using strong induction.*

We prove this statement by strong induction.

P(n) = Every integer n ≥ 2 can be written as the product of prime numbers

Base Case: P(2) is true. 2 is prime, so it is the product of itself and 1.

Suppose for 2 ≤ n ≤ k, n can be written as a product of prime numbers.

To prove k+1 is a product of primes: k+1 = a*b

By strong induction, as 2 ≤ a ≤ k and 2 ≤ b ≤ k, both a and b are themselves products of primes. Therefore, because k+1 = a*b, k+1 is the product of products of primes, and is therefore a product of primes.

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3. *Prove that the number of sides for the nth Koch snowflake is given by 3 * 4n.*

We prove this statement by induction.
P(n) = $3 * 4^n$

Base case: P(0) = 3 ✓

Suppose P(k) is also true: P(k) = $3 * 4^k$ ✓
P(k+1) = $3 * 4^{k+1}$ = $3 * 4^k * 4$ = P(k) * 4

By induction, P(n) is true for all n.

4.

Attempt 1:

- Initial pile size: 7, 0 points
- Move 1: Split the pile; pile sizes: (4, 3); points tally: 4 x 3 = 12 points
- Move 2: Split second pile; pile sizes: (4, 2, 1); points tally: 12 + (2 x 1) = 14 points
- Move 3: Split second pile; pile sizes: (4, 1, 1, 1); points tally: 14 + (1 x 1) = 15 points
- Move 4: Split first pile; pile sizes: (2, 2, 1, 1, 1); points tally: 15 + (2 x 2) = 19 points
- Move 5: Split second pile; pile sizes: (2, 1, 1, 1, 1, 1); points tally: 19 + (1 x 1) = 20 points
- Move 6: Split first pile; pile sizes: (1, 1, 1, 1, 1, 1, 1); points tally: 20 + (1 x 1) = 21 points

P(7) = 7(7-1)/2 = 21 ✓

Attempt 2:

- Initial pile size: 7, 0 points
- Move 1: Split the pile; pile sizes: (5, 2); points tally: 5 x 2 = 10 points
- Move 2: Split second pile; pile sizes: (5, 1, 1); points tally: 10 + (1 x 1) = 11 points
- Move 3: Split first pile; pile sizes: (3, 2, 1, 1); points tally: 11 + (3 x 2) = 17 points
- Move 4: Split second pile; pile sizes: (3, 1, 1, 1, 1); points tally: 17 + (1 x 1) = 18 points
- Move 5: Split first pile; pile sizes: (2, 1, 1, 1, 1, 1); points tally: 18 + (2 x 1) = 20 points
- Move 6: Split first pile; pile sizes: (1, 1, 1, 1, 1, 1, 1); points tally: 20 + (1 x 1) = 21 points

P(7) = 7(7-1)/2 = 21 ✓

We prove this statement by strong induction.

P(n) = 0+1+3+6+...+(n-1) = n(n-1)/2

Base cases: P(1) = 0 ✓, P(2) = 1 ✓

Suppose P(3) → P(k) is also true: P(k) = k(k-1)/2 ✓

P(k+1) = $\frac{k(k+1)}{2}$ = $\frac{k^2+k}{2}$ = $\frac{k^2-k+2k}{2}$ = $\frac{k(k-1)+2k}{2}$ = $\frac{k(k-1)}{2}$ + k = $\frac{k(k-1)}{2}$ + ((k+1)-1) = P(k) + ((k+1)-1)

By strong induction, P(n) is true for all n.