Recitation 3

- Here is a set of additional problems. They range from being very easy to very tough. The best way to learn the material in 310 is to solve problems on your own.
- Feel free to ask (and answer) questions about this problem set on Piazza. The TAs will discuss these problems during office hours.
- This is an **optional** problem set; do not turn this in for grading.
- While you don't have to turn this in, be warned that this material **can** appear in a quiz or exam.

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1. Prove the statements that are true, and give a counterexample to disprove those that are false:

   (a) The product of any two odd integers is odd.
   (b) The sum of any even and any odd integer is odd.
   (c) The difference of any two odd integers is odd.
   (d) The product of any even integer and any integer is even.

2. Recall: a *rational number* is defined as a real number r that can be written as the ratio of two integers $\frac{p}{q}$ with q not equal to 0. An irrational number is defined as a real number that is not rational. Prove that the sum of a rational number with an irrational number is always irrational. Clearly state your method of proof in the beginning.

3. Identify precisely the conceptual bug(s) in the following "proof" -

- **Claim**: 1/8 > 1/4.
- **Proof**: The proof proceeds as follows.
  - We know that 3 > 2;
  - therefore, $3\log_{10}(1/2) > 2\log_{10}(1/2)$;
  - therefore, using the properties of logarithms, $\log_{10}(1/2)^3 > \log_{10}(1/2)^2$;
  - therefore, $(1/2)^3 > (1/2)^2$;
  - therefore, $1/8 > 1/4$.

4. Find a counterexample for each of the following statements:
   (a) If n is prime, then $2^n - 1$ is prime.
   (b) Every triangle has an obtuse angle.
   (c) For all real numbers x, $x^2 \geq x$.
   (d) For every nonprime positive integer n, if some prime p divides n then some other prime q (where $q \neq p$) also divides n.