EE 330 Lecture 27

Small-Signal Analysis

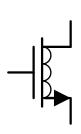
- Examples
- Graphical Analysis

Bipolar Processes

- Device Sizes
- Parasitic Devices
 - JFET
 - Thyristors

Review from last lecture

Small-Signal Model of MOSFET



$$g_{m} = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_{T})$$

Alternate equivalent expressions for g_m :

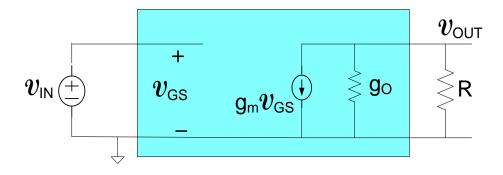
$$I_{\text{\tiny DQ}} = \mu C_{\text{\tiny OX}} \frac{W}{2L} \left(V_{\text{\tiny GSQ}} - V_{\text{\tiny T}} \right)^2 \left(1 + \lambda V_{\text{\tiny DSQ}} \right) \cong \mu C_{\text{\tiny OX}} \frac{W}{2L} \left(V_{\text{\tiny GSQ}} - V_{\text{\tiny T}} \right)^2$$

$$g_{m} = \mu C_{OX} \frac{W}{L} (V_{GSQ} - V_{T})$$

$$g_{m} = \sqrt{2\mu C_{OX} \frac{W}{L}} \bullet \sqrt{I_{DQ}}$$

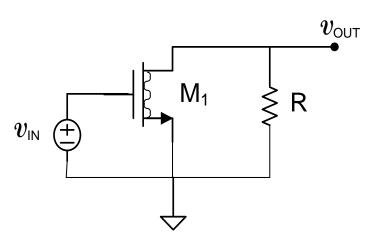
$$g_{m} = \frac{2I_{DQ}}{V_{GSQ} - V_{T}}$$

Small-signal analysis example



$$A_{V} = \frac{V_{OUT}}{V_{IN}} = -\frac{g_{m}}{g_{O} + 1/R}$$

For
$$\lambda=0$$
, $g_O = \lambda I_{DQ} = 0$

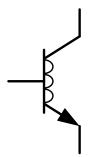


$$\longrightarrow$$

$$A_{v} = \frac{2I_{DQ}R}{\left[V_{SS} + V_{T}\right]}$$

- Same expression as derived before!
- More accurate gain can be obtained if
 λ effects are included and does not significantly
 increase complexity of small-signal analysis

Small Signal Model of BJT

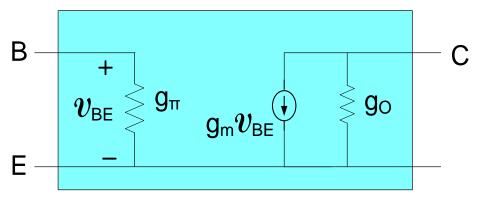


$$g_{\pi} = \frac{I_{CQ}}{\beta V_{\perp}}$$
 $g_{m} = \frac{I_{CQ}}{V_{\perp}}$ $g_{o} = \frac{I_{CQ}}{V_{AF}}$

$$g_{\scriptscriptstyle m} = \frac{I_{\scriptscriptstyle CQ}}{V}$$

$$g_o = \frac{I_{CQ}}{V_{AF}}$$

$$\mathbf{i}_{B} = g_{\pi} \mathbf{V}_{BE}$$
 $\mathbf{i}_{C} = g_{m} \mathbf{V}_{BE} + g_{O} \mathbf{V}_{CE}$

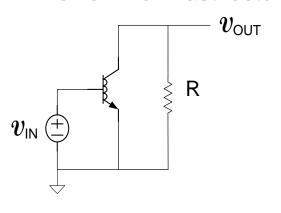


An equivalent circuit

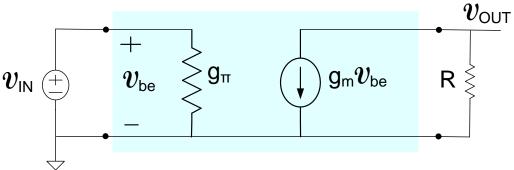
y-parameter model using "g" parameter notation

Review from last lecture

Neglect V_{AF} effects (i.e. $V_{AF} = \infty$) to be consistent with earlier analysis



$$g_o = \frac{1_{CQ}}{V_{AF}} = 0$$



$$v_{ ext{OUT}} = -g_{ ext{m}}Rv_{ ext{BE}}$$
 $A_{ ext{V}} = \frac{v_{ ext{OUT}}}{v_{ ext{IN}}} = -g_{ ext{m}}R$

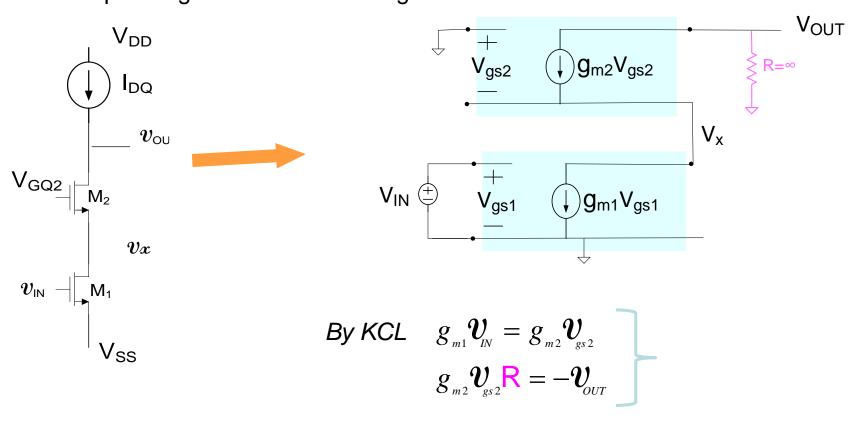
$$A_{v} = \frac{v_{OUT}}{v_{IN}} = -g_{m}R$$

$$g_{m} = \frac{I_{CQ}}{V_{t}}$$

$$A_{V} = -\frac{I_{CQ}R}{V_{t}}$$

Note this is identical to what was obtained with the direct nonlinear analysis

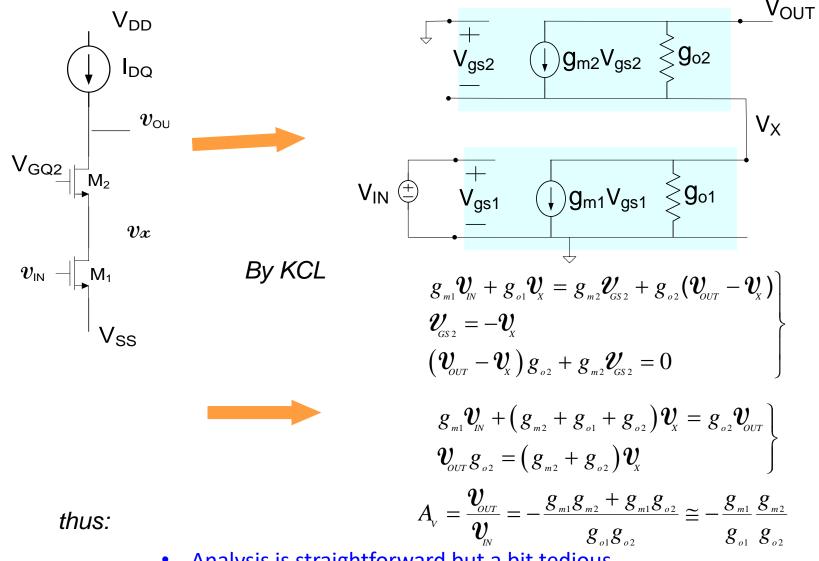
Example: Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda=0$



Solving obtain:
$$A_{V} = \frac{\mathbf{v}_{OUT}}{\mathbf{v}_{N}} = -g_{M1} R \xrightarrow{R=\infty} \infty$$

Unexpectedly large, need better device models!

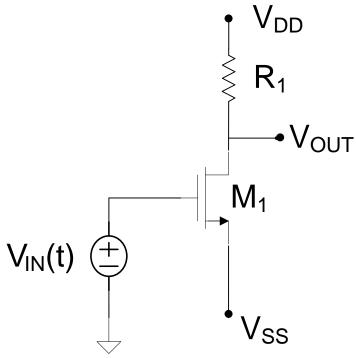
Example: Determine the small signal voltage gain $A_V = v_{OUT}/v_{IN}$. Assume M_1 and M_2 are operating in the saturation region and that $\lambda \neq 0$



- Analysis is straightforward but a bit tedious
- A_V is very large and would go to ∞ if g_{01} and g_{02} were both 0
- Will look at how big this gain really is later

Graphical Analysis and Interpretation

Consider Again



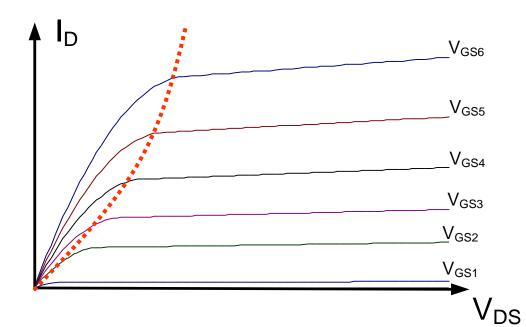
$$V_{OUT} = V_{DD} - I_{D}R$$

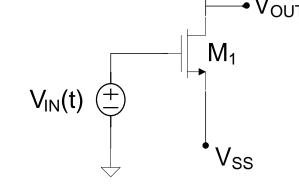
$$I_{D} = \frac{\mu C_{OX}W}{2L} (V_{IN} - V_{SS} - V_{T})^{2}$$

$$\boldsymbol{I}_{\text{\tiny DQ}} = \frac{\mu C_{\text{\tiny OX}} W}{2L} \big(\boldsymbol{V}_{\text{\tiny SS}} \boldsymbol{+} \boldsymbol{V}_{\!\scriptscriptstyle T} \big)^{\!\scriptscriptstyle 2}$$

Graphical Analysis and Interpretation

Device Model (family of curves)
$$I_{D} = \frac{\mu C_{Ox}W}{2L} (V_{GS} - V_{T})^{2} (1 + \lambda V_{DS})$$





Load Line

Device Model at Operating Point

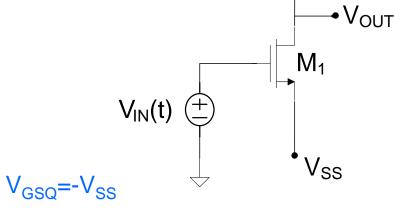
$$V_{OUT} = V_{DD} - I_{D}R$$

$$I_{D} = \frac{\mu C_{OX}W}{2L} (V_{IN} - V_{SS} - V_{T})^{2}$$

$$I_{DQ} = \frac{\mu C_{OX} W}{2I} (V_{SS} + V_{T})$$

Graphical Analysis and Interpretation

Device Model (family of curves)
$$I_{\scriptscriptstyle D} = \frac{\mu \; C_{\scriptscriptstyle Ox} W}{2L} \big(V_{\scriptscriptstyle GS} - V_{\scriptscriptstyle T} \big)^{\scriptscriptstyle 2} \big(1 + \lambda V_{\scriptscriptstyle DS} \big)$$



 V_{DD}

$$I_{\text{DQ}} \cong \frac{\mu \, C_{\text{ox}} W}{2L} \big(V_{\text{ss}} + V_{\text{T}} \big)^2$$
 Load Line

 V_{GS6}

 V_{GS5}

 V_{GS4}

 V_{GS3}

$$V_{\text{OUT}} = V_{\text{DD}} - I_{\text{D}}R$$

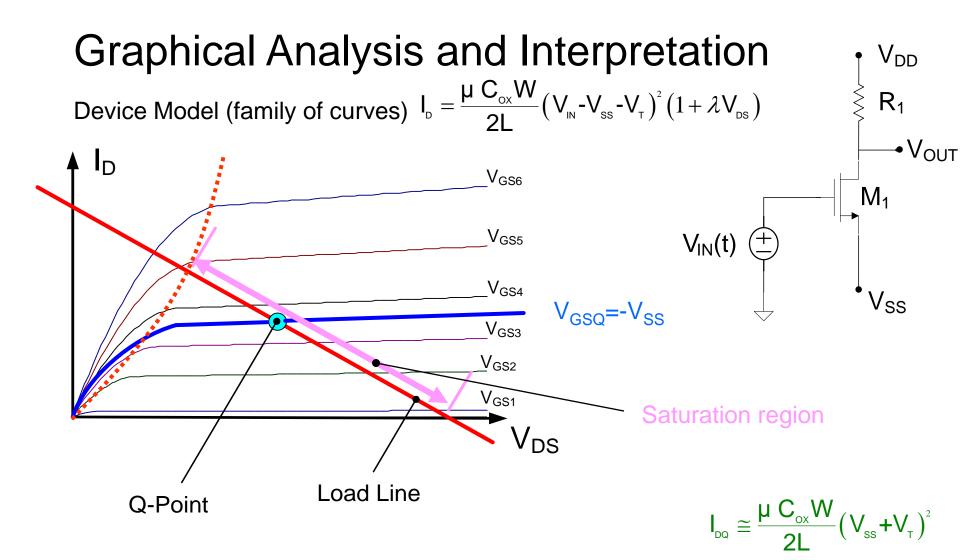
Q-Point

$$I_{D} = \frac{\mu C_{OX} W}{2L} (V_{IN} - V_{SS} - V_{T})^{2}$$

Must satisfy both equations all of the time!

Graphical Analysis and Interpretation V_{DD} Device Model (family of curves) $I_D = \frac{\mu C_{ox}W}{2I} (V_{IN} - V_{ss} - V_{T})^2 (1 + \lambda V_{DS})$ V_{GS6} M₁ V_{GS5} $V_{IN}(t)$ V_{GS4} $V_{GSQ} = -V_{SS}$ V_{GS2} V_{GS1} **Load Line** Q-Point

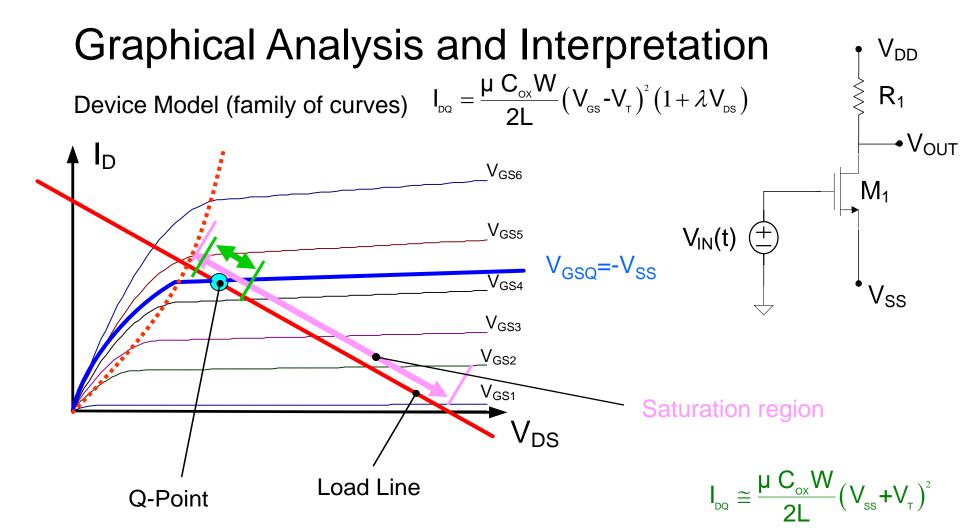
- As V_{IN} changes around Q-point, V_{IN} induces changes in V_{GS} . The operating point must remain on the load line!
- Small sinusoidal changes of V_{IN} will be nearly symmetric around the V_{GSO} line
- This will cause nearly symmetric changes in both I_D and V_{DS}!
- Since V_{SS} is constant, change in V_{DS} is equal to change in V_{OUT}



As V_{IN} changes around Q-point, due to changes V_{IN} induces in V_{GS} , the operating point must remain on the load line!

Graphical Analysis and Interpretation V_{DD} Device Model (family of curves) $I_{DQ} = \frac{\mu C_{Ox}W}{2I} (V_{GS} - V_{T})^{2} (1 + \lambda V_{DS})$ V_{GS6} M_1 V_{GS5} $V_{IN}(t)$ V_{GS4} $V_{GSO} = -V_{SS}$ V_{GS2} V_{GS1} Saturation region $I_{DQ} \cong \frac{\mu C_{OX}W}{2I} (V_{SS} + V_{T})^{2}$ Load Line Q-Point

- Linear signal swing region smaller than saturation region
- Modest nonlinear distortion provided saturation region operation maintained
- Symmetric swing about Q-point
- Signal swing can be maximized by judicious location of Q-point



Very limited signal swing with non-optimal Q-point location

Graphical Analysis and Interpretation V_{DD} Device Model (family of curves) $I_{\text{\tiny DQ}} = \frac{\mu \ C_{\text{\tiny OX}} W}{2I} \big(V_{\text{\tiny GS}} - V_{\text{\tiny T}} \big)^2 \big(1 + \lambda V_{\text{\tiny DS}} \big)$ V_{GS6} M_1 V_{GS5} $V_{IN}(t)$ V_{GS4} V_{GS3} $V_{GSQ} = -V_{SS}$ V_{GS2} V_{GS1} Saturation region Load Line Q-Point $I_{DQ} \cong \frac{\mu C_{OX}W}{2I} (V_{SS} + V_{T})^{2}$

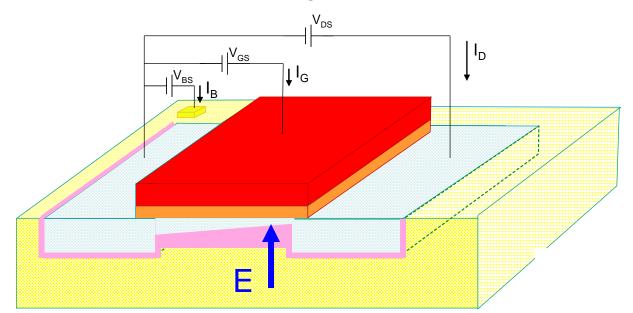
- Signal swing can be maximized by judicious location of Q-point
- Often selected to be at middle of load line in saturation region

Small-Signal MOSFET Model Extension

Existing 3-terminal small-signal model does not depend upon the bulk voltage!



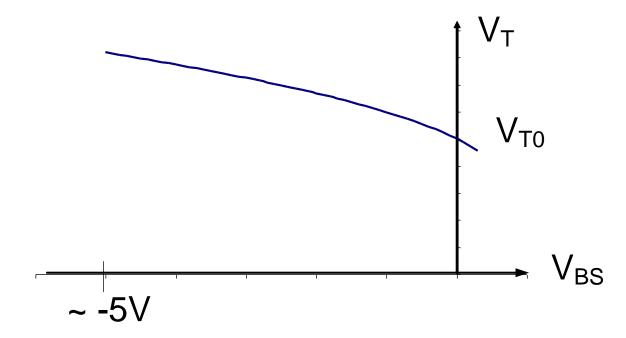
Recall that changing the bulk voltage changes the electric field in the channel region and thus the threshold voltage!



Recall: Typical Effects of Bulk on Threshold Voltage for n-channel Device

$$V_T = V_{T0} + \gamma \left[\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right]$$

$$\gamma \cong 0.4 V^{\frac{1}{2}} \qquad \phi \cong 0.6 V$$



Bulk-Diffusion Generally Reverse Biased (V_{BS}< 0 or at least less than 0.3V) for n-channel

Shift in threshold voltage with bulk voltage can be substantial Often V_{RS} =0

Recall: Typical Effects of Bulk on Threshold Voltage for p-channel Device

$$V_{T} = V_{T0} - \gamma \left[\sqrt{\phi + V_{BS}} - \sqrt{\phi} \right]$$

$$\gamma \cong 0.4V^{\frac{1}{2}} \qquad \phi \cong 0.6V$$

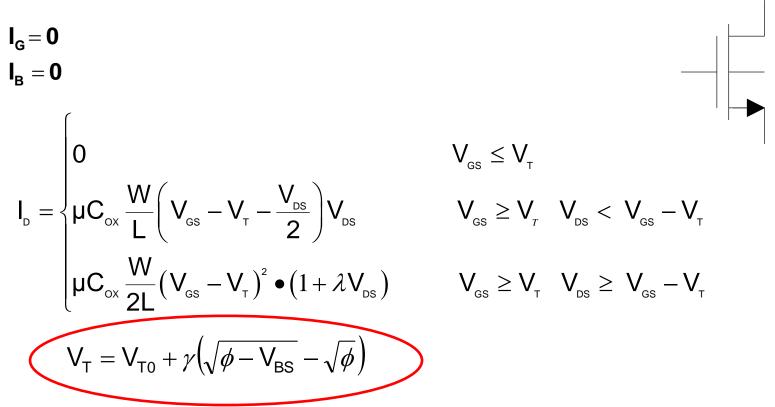
$$V_{BS}$$

Bulk-Diffusion Generally Reverse Biased ($V_{BS} > 0$ or at least greater than -0.3V) for n-channel

Same functional form as for n-channel devices but V_{T0} is now negative and the magnitude of V_T still increases with the magnitude of the reverse bias

Recall:

4-terminal model extension



Model Parameters : $\{\mu, C_{OX}, V_{T0}, \phi, \gamma, \lambda\}$

Design Parameters : {W,L} but only one degree of freedom W/L biasing or quiescent point

Small-Signal 4-terminal Model Extension

$$\begin{split} & \mathbf{I}_{\mathrm{g}} = \mathbf{0} \\ & \mathbf{I}_{\mathrm{B}} = \mathbf{0} \\ & \mathbf{I}_{\mathrm{D}} = \begin{cases} \mathbf{0} & \mathbf{V}_{\mathrm{GS}} \leq \mathbf{V}_{\mathrm{T}} \\ \mathbf{\mu} \mathbf{C}_{\mathrm{ox}} \frac{\mathbf{W}}{\mathbf{L}} \bigg(\mathbf{V}_{\mathrm{GS}} - \mathbf{V}_{\mathrm{T}} - \frac{\mathbf{V}_{\mathrm{DS}}}{2} \bigg) \mathbf{V}_{\mathrm{DS}} & \mathbf{V}_{\mathrm{GS}} \geq \mathbf{V}_{\mathrm{T}} & \mathbf{V}_{\mathrm{DS}} < \mathbf{V}_{\mathrm{GS}} - \mathbf{V}_{\mathrm{T}} \\ \mathbf{\mu} \mathbf{C}_{\mathrm{ox}} \frac{\mathbf{W}}{2\mathbf{L}} \Big(\mathbf{V}_{\mathrm{GS}} - \mathbf{V}_{\mathrm{T}} \Big)^{2} \bullet \Big(\mathbf{1} + \lambda \mathbf{V}_{\mathrm{DS}} \Big) & \mathbf{V}_{\mathrm{GS}} \geq \mathbf{V}_{\mathrm{T}} & \mathbf{V}_{\mathrm{DS}} \leq \mathbf{V}_{\mathrm{GS}} - \mathbf{V}_{\mathrm{T}} \\ \mathbf{V}_{\mathrm{T}} = \mathbf{V}_{\mathrm{T0}} + \gamma \Big(\sqrt{\phi - \mathbf{V}_{\mathrm{BS}}} - \sqrt{\phi} \Big) & \mathbf{V}_{\mathrm{GS}} \geq \mathbf{V}_{\mathrm{T}} & \mathbf{V}_{\mathrm{DS}} \geq \mathbf{V}_{\mathrm{GS}} - \mathbf{V}_{\mathrm{T}} \\ \mathbf{V}_{\mathrm{T}} = \mathbf{V}_{\mathrm{T0}} + \gamma \Big(\sqrt{\phi - \mathbf{V}_{\mathrm{BS}}} - \sqrt{\phi} \Big) & \mathbf{V}_{\mathrm{13}} = \frac{\partial \mathbf{I}_{\mathbf{G}}}{\partial \mathbf{V}_{\mathrm{GS}}} \Big|_{\mathbf{V} = \mathbf{V}_{\mathbf{Q}}} = \mathbf{0} \\ \mathbf{V}_{\mathrm{11}} = \frac{\partial \mathbf{I}_{\mathbf{G}}}{\partial \mathbf{V}_{\mathrm{GS}}} \Big|_{\mathbf{V} = \mathbf{V}_{\mathbf{Q}}} = \mathbf{0} & \mathbf{V}_{\mathrm{12}} = \frac{\partial \mathbf{I}_{\mathbf{G}}}{\partial \mathbf{V}_{\mathrm{DS}}} \Big|_{\mathbf{V} = \mathbf{V}_{\mathbf{Q}}} = \mathbf{0} \\ \mathbf{V}_{\mathrm{21}} = \frac{\partial \mathbf{I}_{\mathbf{G}}}{\partial \mathbf{V}_{\mathrm{GS}}} \Big|_{\mathbf{V} = \mathbf{V}_{\mathbf{Q}}} = \mathbf{0} & \mathbf{V}_{\mathrm{32}} = \frac{\partial \mathbf{I}_{\mathbf{G}}}{\partial \mathbf{V}_{\mathrm{DS}}} \Big|_{\mathbf{V} = \mathbf{V}_{\mathbf{Q}}} = \mathbf{0} \\ \mathbf{V}_{\mathrm{33}} = \frac{\partial \mathbf{I}_{\mathbf{B}}}{\partial \mathbf{V}_{\mathrm{GS}}} \Big|_{\mathbf{V} = \mathbf{V}_{\mathbf{Q}}} = \mathbf{0} \\ \mathbf{V}_{\mathrm{33}} = \frac{\partial \mathbf{I}_{\mathbf{B}}}{\partial \mathbf{V}_{\mathrm{GS}}} \Big|_{\mathbf{V} = \mathbf{V}_{\mathbf{Q}}} = \mathbf{0} \end{aligned}$$

Small-Signal 4-terminal Model Extension

$$I_{D} = \mu C_{OX} \frac{W}{2L} (V_{GS} - V_{T})^{2} \bullet (1 + \lambda V_{DS})$$

$$V_{EB} = V_{GS} - V_{T}$$

$$V_{T} = V_{T0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$$
Definition:
$$V_{EBQ} = V_{GSQ} - V_{TQ}$$

$$g_{_{m}} = \frac{\partial I_{_{D}}}{\partial V_{_{GS}}}\bigg|_{_{\vec{V} = \vec{V}_{_{Q}}}} = \mu C_{_{OX}} \frac{W}{2L} 2 \left(V_{_{GS}} - V_{_{T}}\right)^{_{1}} \bullet \left(1 + \lambda V_{_{DS}}\right)\bigg|_{_{\vec{V} = \vec{V}_{_{Q}}}} \cong \mu C_{_{OX}} \frac{W}{L} V_{_{EBQ}}$$
Same as 3-term

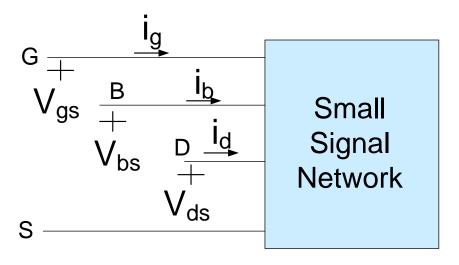
$$g_{o} = \frac{\partial I_{D}}{\partial V_{DS}}\bigg|_{\vec{V} = \vec{V}_{Q}} = \mu C_{ox} \frac{W}{2L} 2(V_{GS} - V_{T})^{2} \bullet \lambda \bigg|_{\vec{V} = \vec{V}_{Q}} \cong \lambda I_{DQ}$$
Same as 3-term

$$g_{mb} = \frac{\partial I_{D}}{\partial V_{BS}}\Big|_{\vec{V} = \vec{V}_{Q}} = \mu C_{OX} \frac{W}{2L} 2(V_{GS} - V_{T})^{1} \cdot \left(-\frac{\partial V_{T}}{\partial V_{BS}}\right) \cdot (1 + \lambda V_{DS})\Big|_{\vec{V} = \vec{V}_{Q}}$$

$$g_{mb} = \frac{\partial I_{D}}{\partial V_{BS}}\Big|_{\vec{V} = \vec{V}_{L}} \cong \mu C_{OX} \frac{W}{L} V_{EBQ} \cdot \frac{\partial V_{T}}{\partial V_{BS}}\Big|_{\vec{V} = \vec{V}_{L}} = \left(\mu C_{OX} \frac{W}{L} V_{EBQ}\right) (-1) \gamma \frac{1}{2} (\phi - V_{BS})^{-\frac{1}{2}}\Big|_{\vec{V} = \vec{V}_{Q}} (-1)$$

$$g_{\scriptscriptstyle mb}\cong \mathsf{g}_{\scriptscriptstyle m}\,rac{\gamma}{2\sqrt{\phi extsf{-}\mathsf{V}_{\scriptscriptstyle \mathsf{BSQ}}}}$$

Small Signal Model Summary



$$i_g = 0$$

$$i_b = 0$$

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

$$g_{m} = \frac{\mu C_{ox} W}{L} V_{EBQ}$$

$$g_{o} = \lambda I_{DQ}$$

$$g_{mb} = g_{m} \left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right)$$

Relative Magnitude of Small Signal MOS **Parameters**

Consider:

$$i_{d} = g_{m} v_{gs} + g_{mb} v_{bs} + g_{o} v_{ds}$$

3 alternate equivalent expressions for g_m

$$\begin{split} g_{_{m}} &= \frac{\mu C_{_{OX}}W}{L} V_{_{EBQ}} \quad g_{_{m}} = \sqrt{\frac{2\mu C_{_{OX}}W}{L}} \sqrt{I_{_{DQ}}} \qquad g_{_{m}} = \frac{2I_{_{DQ}}}{V_{_{EBQ}}} \end{split}$$
 If $\mu C_{_{OX}} = 100\mu A/V^2$, $\lambda = .01V^{-1}$, $\gamma = 0.4V^{0.5}$, $V_{EBQ} = 1V$, $W/L = 1$, $V_{BSQ} = 0V$

$$I_{_{DQ}} &\cong \frac{\mu C_{_{OX}}W}{2L} V_{_{EBQ}}^2 = \frac{10^4 \mathcal{W}}{2\mathcal{V}} (1V)^2 = 5E-5 \qquad \qquad \begin{array}{c} \text{In this example} \\ g_{_{0}} << g_{_{m}}, g_{_{mb}} \\ g_{_{o}} &= \lambda I_{_{DQ}} = 5E-7 \\ g_{_{o}} &= \lambda I_{_{DQ}} = 5E-7 \\ g_{_{mb}} &= g_{_{m}} \left(\frac{\gamma}{2\sqrt{\phi-V}}\right) = .26g_{_{m}} & \text{In many circuits,} \\ v_{_{BS}} = 0 \text{ as well} \end{split}$$

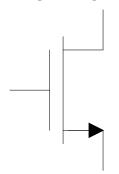
Often the go term can be neglected in the small signal model because it is so small

 $v_{\scriptscriptstyle{\mathrm{RS}}}$ =0 as well

Be careful about neglecting go prior to obtaining a final expression

Large and Small Signal Model Summary

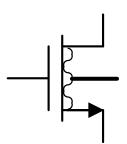
Large Signal Model



 $V_{T} = V_{T0} + \gamma \left(\sqrt{\phi - V_{BS}} - \sqrt{\phi} \right)$

$$\begin{split} I_{_{D}} = & \begin{cases} 0 & V_{_{GS}} \leq V_{_{T}} \\ \mu C_{_{OX}} \frac{W}{L} \bigg(V_{_{GS}} - V_{_{T}} - \frac{V_{_{DS}}}{2} \bigg) V_{_{DS}} & V_{_{GS}} \geq V_{_{T}} & V_{_{DS}} < V_{_{GS}} - V_{_{T}} \\ \mu C_{_{OX}} \frac{W}{2L} \Big(V_{_{GS}} - V_{_{T}} \Big)^2 \bullet \Big(1 + \lambda V_{_{DS}} \Big) & V_{_{GS}} \geq V_{_{T}} & V_{_{DS}} \geq V_{_{GS}} - V_{_{T}} \\ & \text{saturation} \end{cases} \end{split}$$

Small Signal Model



saturation

$$i_g = 0$$

$$i_b = 0$$

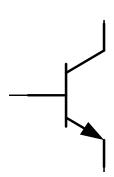
$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

where

$$\begin{split} g_{_{m}} &= \frac{\mu C_{_{OX}} W}{L} \, V_{_{EBQ}} \\ g_{_{mb}} &= g_{_{m}} \! \left(\frac{\gamma}{2 \sqrt{\phi - V_{_{BSQ}}}} \right) \\ g_{_{o}} &= \lambda I_{_{DQ}} \end{split}$$

Large and Small Signal Model Summary

Large Signal Model



$$\begin{split} I_{C} &= \beta I_{B} \Biggl(1 + \frac{V_{CE}}{V_{AF}} \Biggr) \\ I_{B} &= \frac{J_{S} A_{E}}{\beta} e^{\frac{V_{BE}}{V_{t}}} \end{split} \qquad \begin{aligned} V_{BC} &> 0.4V \\ V_{BC} &< 0 \end{aligned}$$

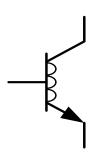
$$V_{BE}=0.7V$$

 $V_{CE}=0.2V$

$$I_C < \beta I_B$$

$$I_C=I_B=0$$
 $V_{BC}<0$
 $V_{BC}<0$

Small Signal Model



Forward Active

$$i_b = g_{\pi} v_{be}$$

$$i_c = g_m v_{be} + g_0 v_{ce}$$

where

$$\mathbf{g}_{\mathsf{m}} = rac{\mathbf{I}_{\mathsf{CQ}}}{\mathsf{V}_{\mathsf{t}}}$$
 $\mathbf{g}_{\pi} = rac{\mathbf{I}_{\mathsf{CQ}}}{\mathsf{\beta}\mathsf{V}_{\mathsf{t}}}$ $\mathbf{g}_{o} \cong rac{\mathbf{I}_{\mathsf{CQ}}}{\mathsf{V}_{\mathsf{AF}}}$

Relative Magnitude of Small Signal BJT Parameters

$$\begin{split} g_m &= \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}} \\ &\frac{g_m}{g_\pi} = \frac{\begin{bmatrix} I_Q \\ V_t \end{bmatrix}}{\begin{bmatrix} I_Q \\ \beta V_t \end{bmatrix}} \\ &\frac{g_m}{g_o} = \frac{\begin{bmatrix} I_Q \\ \beta V_t \end{bmatrix}}{\begin{bmatrix} I_Q \\ \gamma V_{AF} \end{bmatrix}} \end{split}$$



$$g_m >> g_\pi >> g_o$$

Often the go term can be neglected in the small signal model because it is so small

Relative Magnitude of Small Signal Parameters

$$g_{m} = \frac{I_{CQ}}{V_{t}} \qquad g_{\pi} = \frac{I_{CQ}}{\beta V_{t}} \qquad g_{o} \cong \frac{I_{CQ}}{V_{AF}}$$

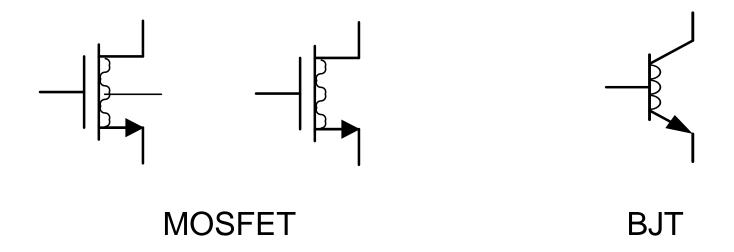
$$\frac{g_{m}}{g_{\pi}} = \frac{\begin{bmatrix} I_{Q} \\ V_{t} \end{bmatrix}}{\begin{bmatrix} I_{Q} \\ \beta V_{t} \end{bmatrix}} = \beta$$

$$\frac{g_{\pi}}{g_{o}} = \frac{\begin{bmatrix} I_{Q} \\ \beta V_{t} \end{bmatrix}}{\begin{bmatrix} I_{Q} \\ V_{AF} \end{bmatrix}} = \frac{V_{AF}}{\beta V_{t}} \approx \frac{200V}{100 \cdot 26mV} = 77$$

$$g_{m} >> g_{\pi} >> g_{o}$$

- Often the g_o term can be neglected in the small signal model because it is so small
- Be careful about neglecting g_o prior to obtaining a final expression

Small Signal Model Simplifications for the MOSFET and BJT

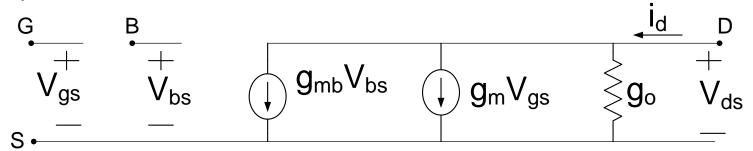


Often simplifications of the small signal model are adequate for a given application

These simplifications will be discussed next

Small Signal MOSFET Model Summary

An equivalent Circuit:



$$g_{m} = \frac{\mu C_{OX} W}{L} (V_{GSQ} - V_{T})$$

$$g_o = \lambda I_{DQ}$$

$$g_{\text{mb}} = g_{\text{m}} \left(\frac{\gamma}{2\sqrt{\phi - V_{\text{BSQ}}}} \right)$$

Alternate equivalent representations for $g_{\rm m}$

$$I_D \cong \mu C_{OX} \frac{W}{2I} (V_{GS} - V_T)^2$$

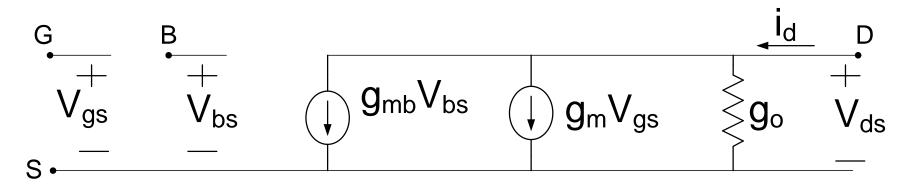
$$g_{m} = \sqrt{\frac{2\mu C_{OX}W}{I}}\sqrt{I_{DQ}}$$

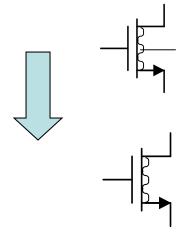
$$g_{m} = \frac{2I_{DQ}}{V_{GSQ} - V_{T}} = \frac{2I_{DQ}}{V_{FBQ}}$$

$$g_{mb} < g_{mb}$$

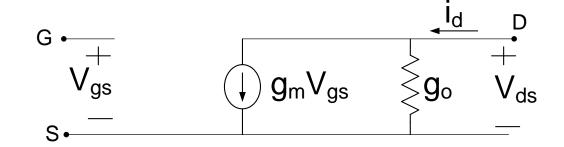
$$\boldsymbol{g}_{\scriptscriptstyle 0} \!<\! < \! \boldsymbol{g}_{\scriptscriptstyle m}, \boldsymbol{g}_{\scriptscriptstyle mb}$$

Small Signal Model Simplifications

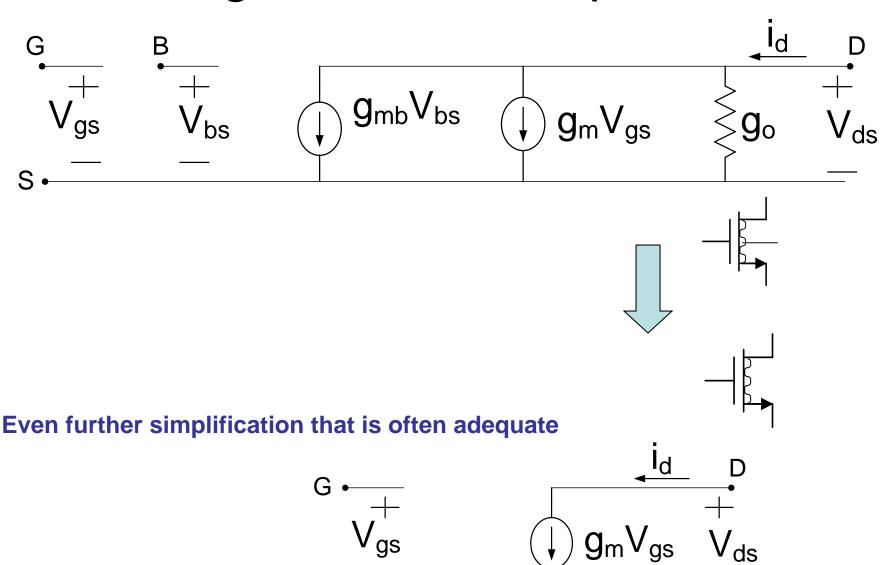




Simplification that is often adequate

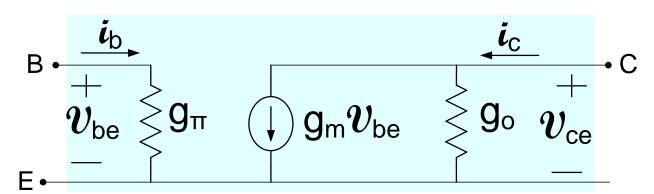


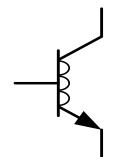
Small Signal Model Simplifications



Small Signal BJT Model Summary

An equivalent circuit





$$g_m = \frac{I_{CQ}}{V_t}$$

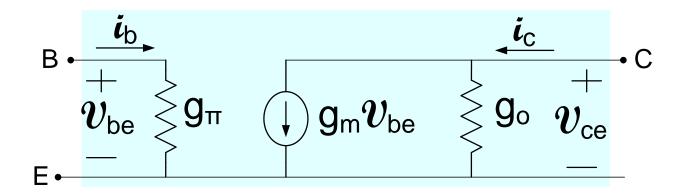
$$g_{\pi} = \frac{I_{CQ}}{\beta V_{\star}}$$

$$g_o \cong \frac{I_{CQ}}{V_{AF}}$$

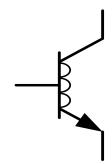
$$g_m >> g_\pi >> g_o$$

This contains absolutely no more information than the set of small-signal model equations

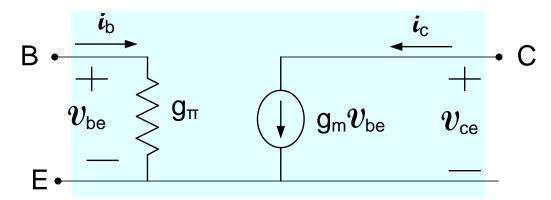
Small Signal BJT Model Simplifications



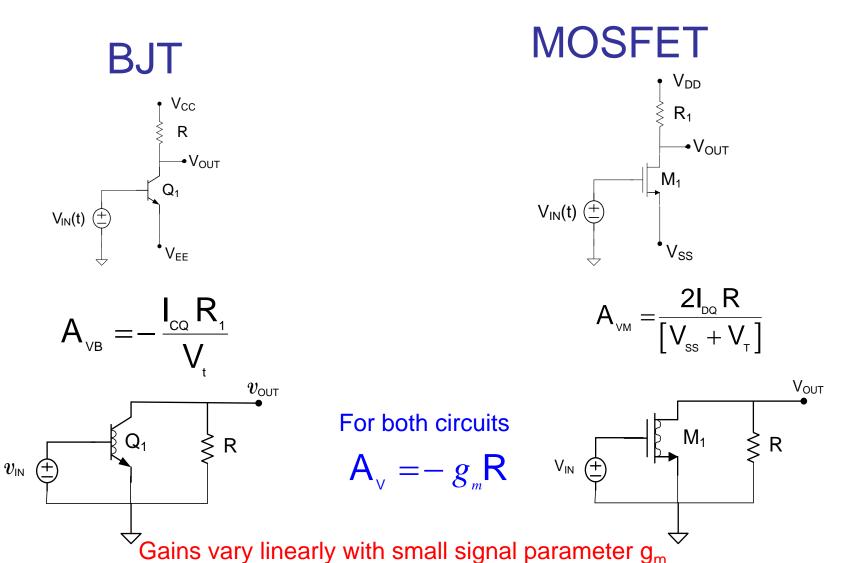




Simplification that is often adequate



Gains for MOSFET and BJT Circuits



Power is often a key resource in the design of an integrated circuit In both circuits, power is proportional to I_{CQ} , I_{DQ}

How does g_m vary with I_{DO} ?

$$g_{m} = \sqrt{\frac{2\mu C_{OX}W}{L}} \sqrt{I_{DQ}}$$

Varies with the square root of I_{DO}

$$g_{m} = \frac{2I_{DQ}}{V_{GSQ} - V_{T}} = \frac{2I_{DQ}}{V_{EBQ}}$$

Varies linearly with I_{DO}

$$g_{m} = \frac{\mu C_{OX} W}{L} \left(V_{GSQ} - V_{T} \right)$$

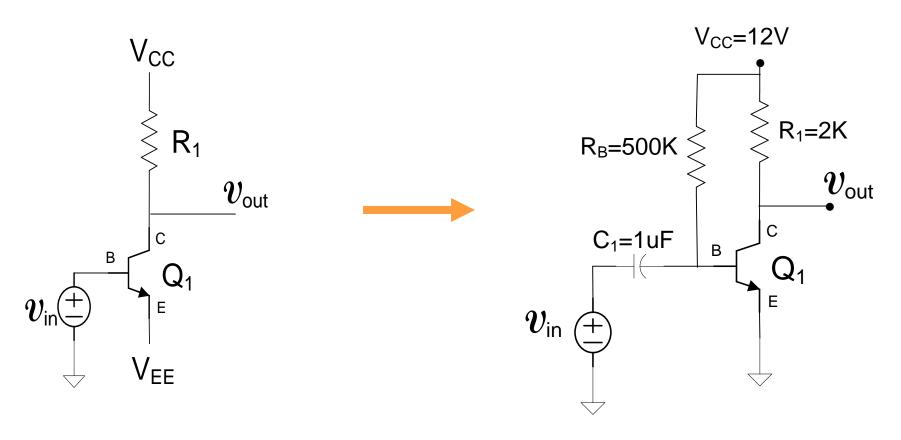
Doesn't vary with I_{DO}

How does g_m vary with I_{DQ} ?

All of the above are true – but with qualification

 g_m is a function of more than one variable (I_{DQ}) and how it varies depends upon how the remaining variables are constrained

Amplifier Biasing (precursor)



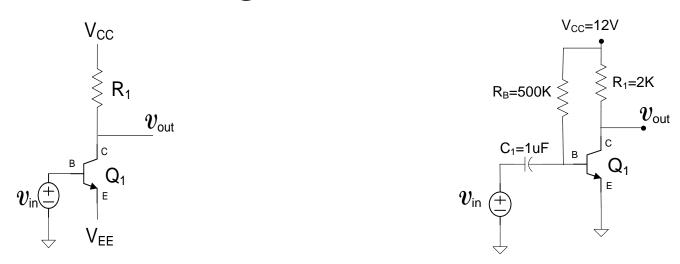
Not convenient to have multiple dc power supplies V_{OUTQ} very sensitive to V_{EE}

Single power supply Additional resistor and capacitor

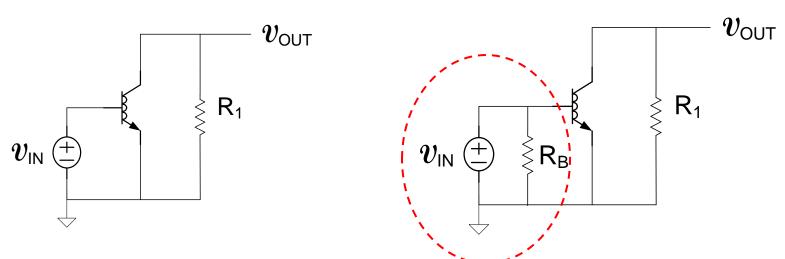
Compare the small-signal equivalent circuits of these two structures

Compare the small-signal voltage gain of these two structures

Amplifier Biasing (precursor)



Compare the small-signal equivalent circuits of these two structures



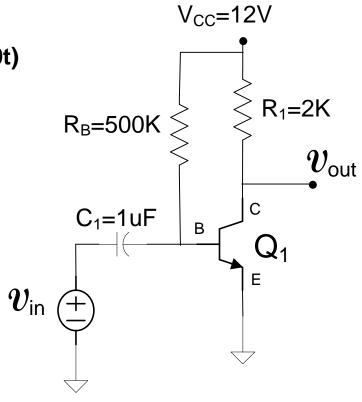
Since Thevenin equivalent circuit in red circle is V_{IN} , both circuits have same voltage gain

But the load placed on V_{IN} is different

Method of characterizing the amplifiers is needed to assess impact of difference

Amplifier Characterization (an example)

Determine $V_{\rm OUTQ}$, $A_{\rm V}$, $R_{\rm IN}$ Determine $v_{\rm OUT}$ and $v_{\rm OUT}$ (t) if $v_{\rm IN}$ =.002sin(400t)



In the following slides we will analyze this circuit

End of Lecture 27