Other Search Strategies for Games

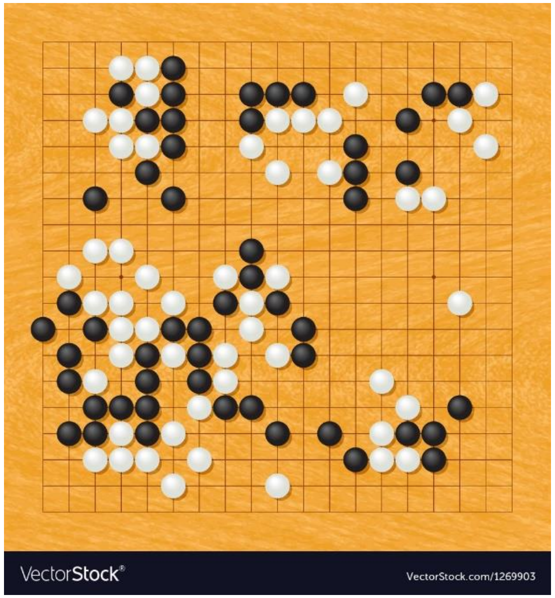
Outline

I. Monte Carlo tree search

II. Stochastic games

* Figures/images are from the textbook site (or by the instructor). Otherwise, the source is cited unless such citation would make little sense due to the triviality of generating the image.

I. Monte Carlo Tree Search



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♠ Its high branching factor ($b = 361 = 19^2$) limits the search to only 4 or 5 ply ($361^5 \approx 6.13 \times 10^{12}$).

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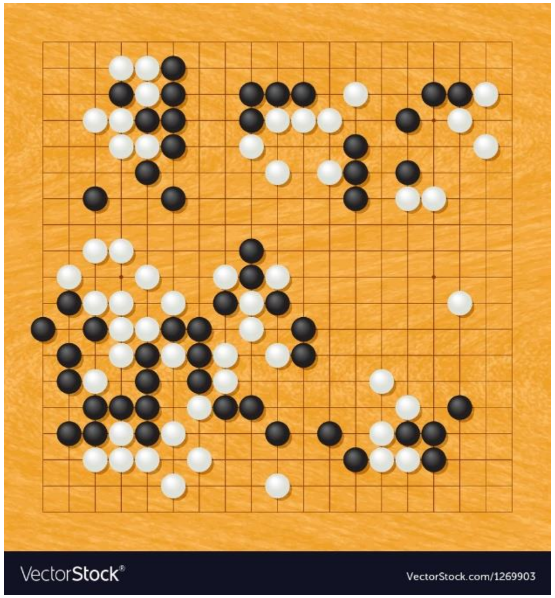

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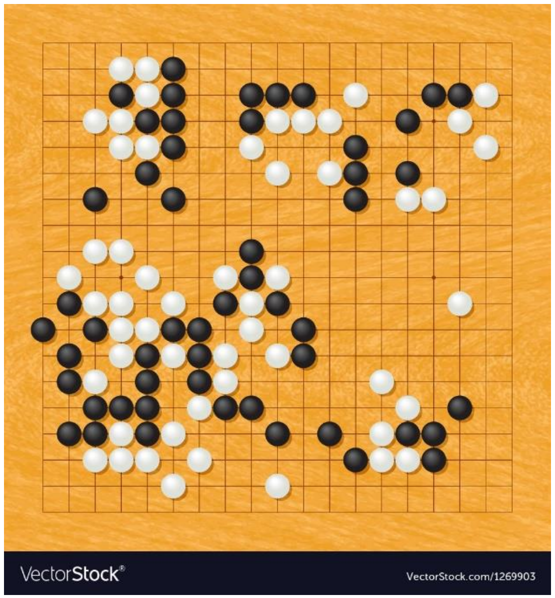

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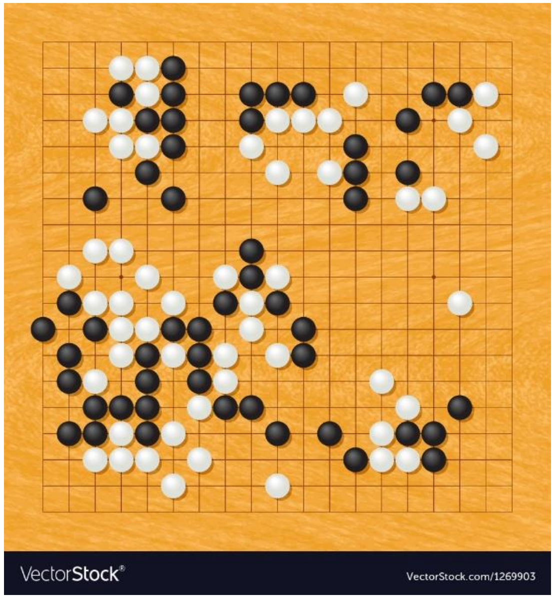

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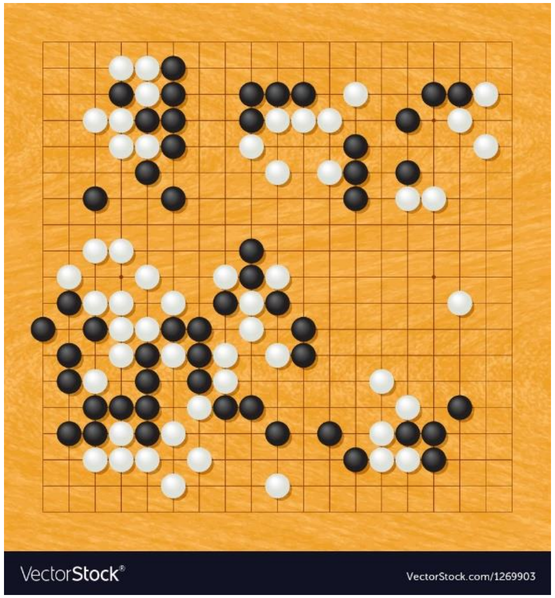

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Modern Go programs use Monte Carlo tree search (MCTS) instead of alpha-beta search.

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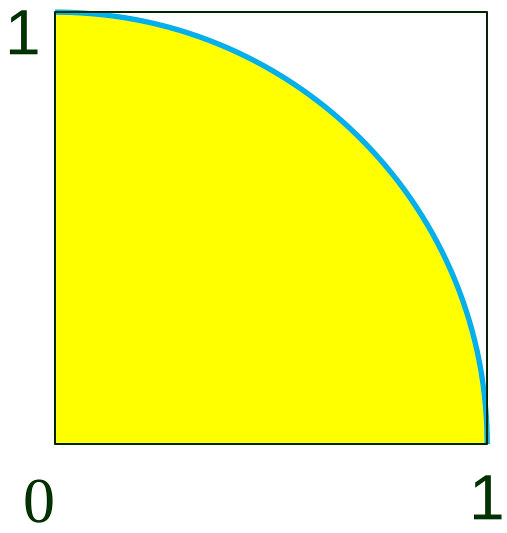

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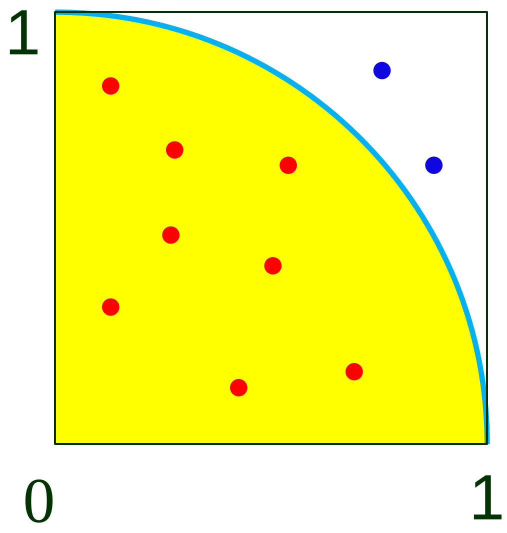

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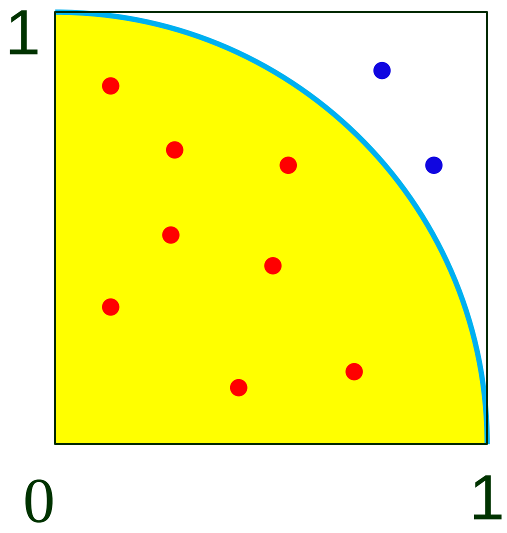

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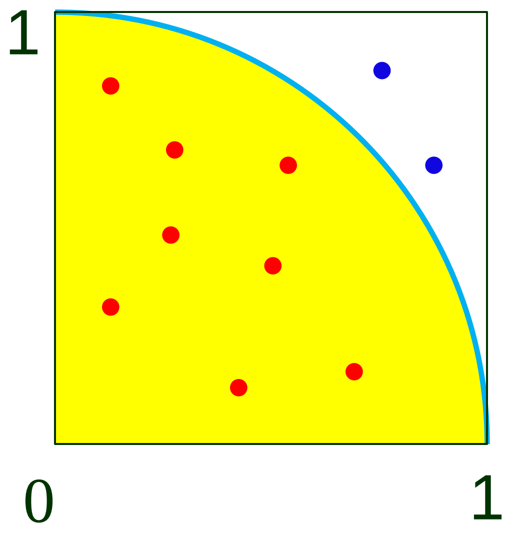

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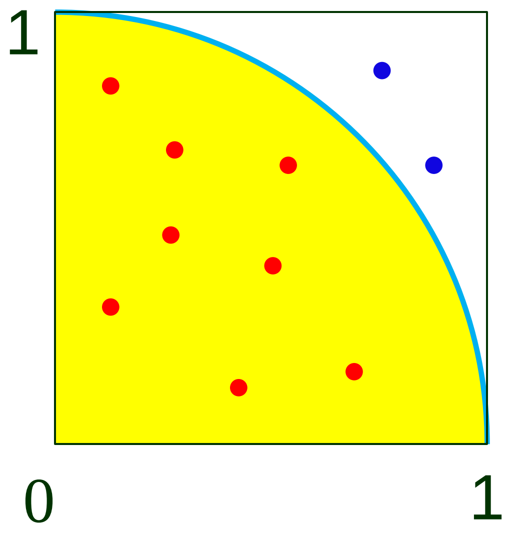

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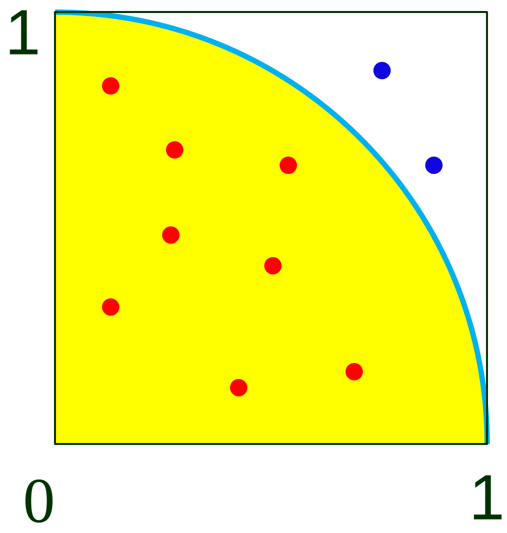

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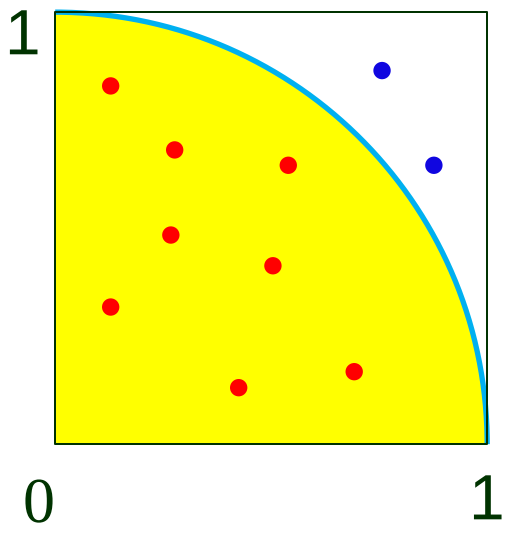

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For accuracy, many points should be generated.

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- No use of a heuristic evaluation function.
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A simulation (a playout or rollout) proceeds as below:

- Choose moves alternatively for the two players.
- Determine the outcome when a terminal position is reached.

Choice of a Move

What is the best move if both players play randomly?

What is the best move if both players play well?

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- These policies are often *learned from self-play* using neural networks.

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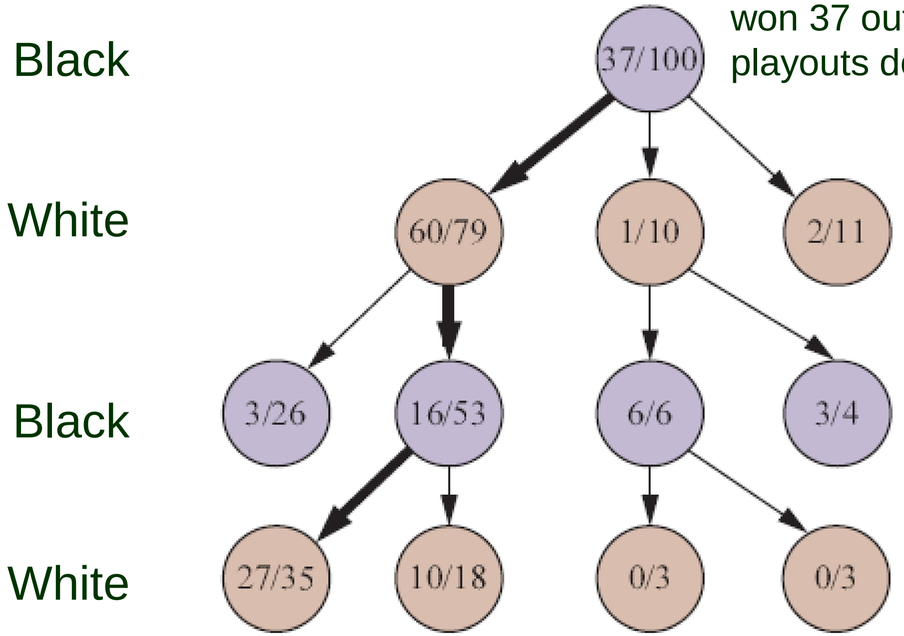
How to improve? Need a selection policy that balances

- ♦ exploration of states that have few playouts, and
- ♦ exploitation of states that have done well in the past.

Step 1 of MCTS (One Iteration): Selection

Which move should Black make (at the root)?

Root: state just after the move by white, who has won 37 out of the 100 playouts done so far.



(a) Selection

Step 1 of MCTS (One Iteration): Selection

Which move should Black make (at the root)?

Root: state just after the move by white, who has won 37 out of the 100 playouts done so far.

black has won 60 out of 79 playouts at the node

Black

White

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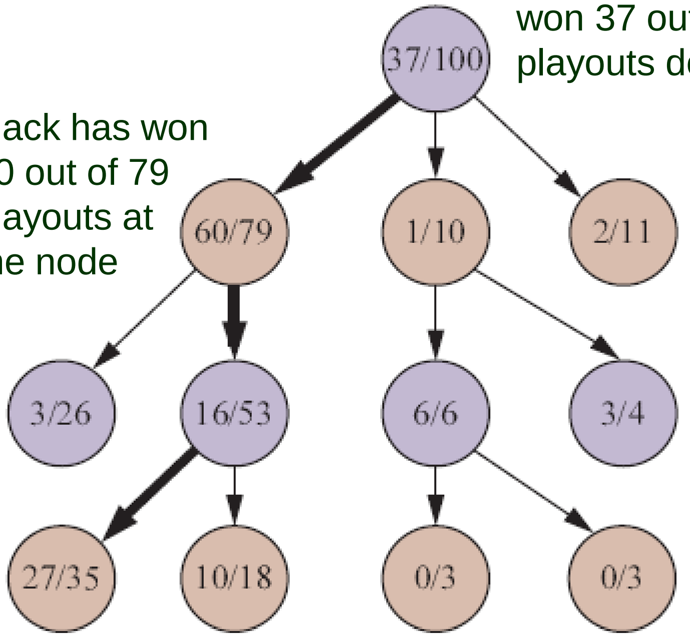

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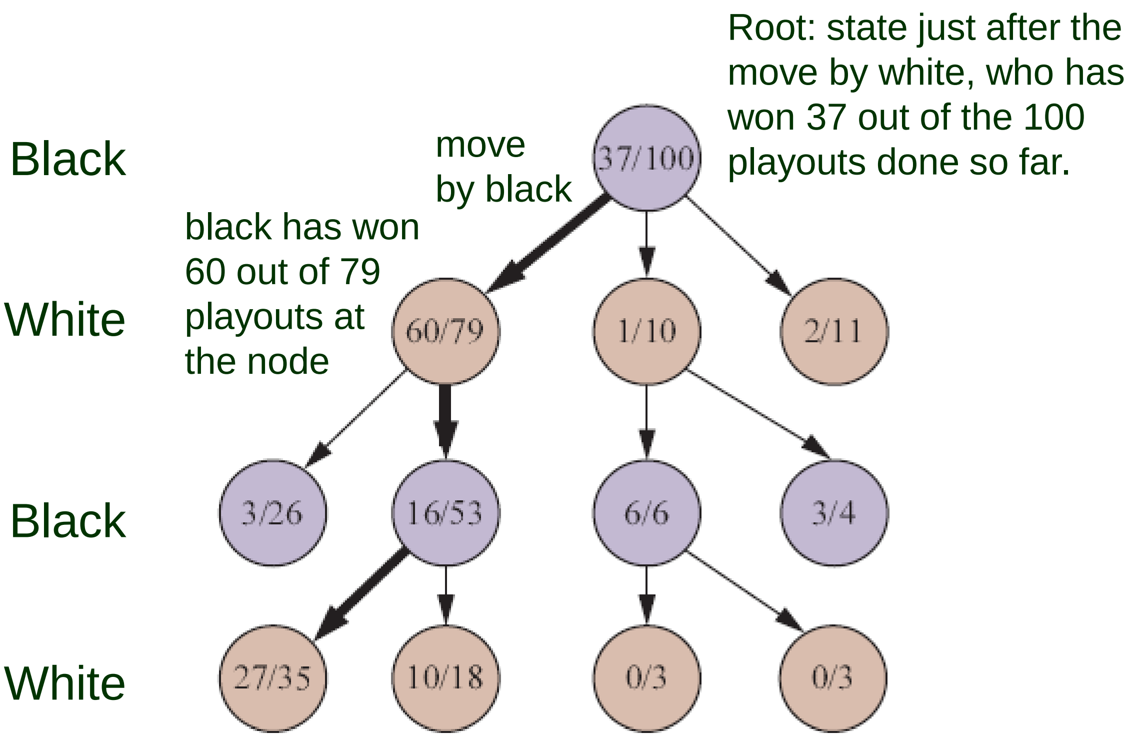

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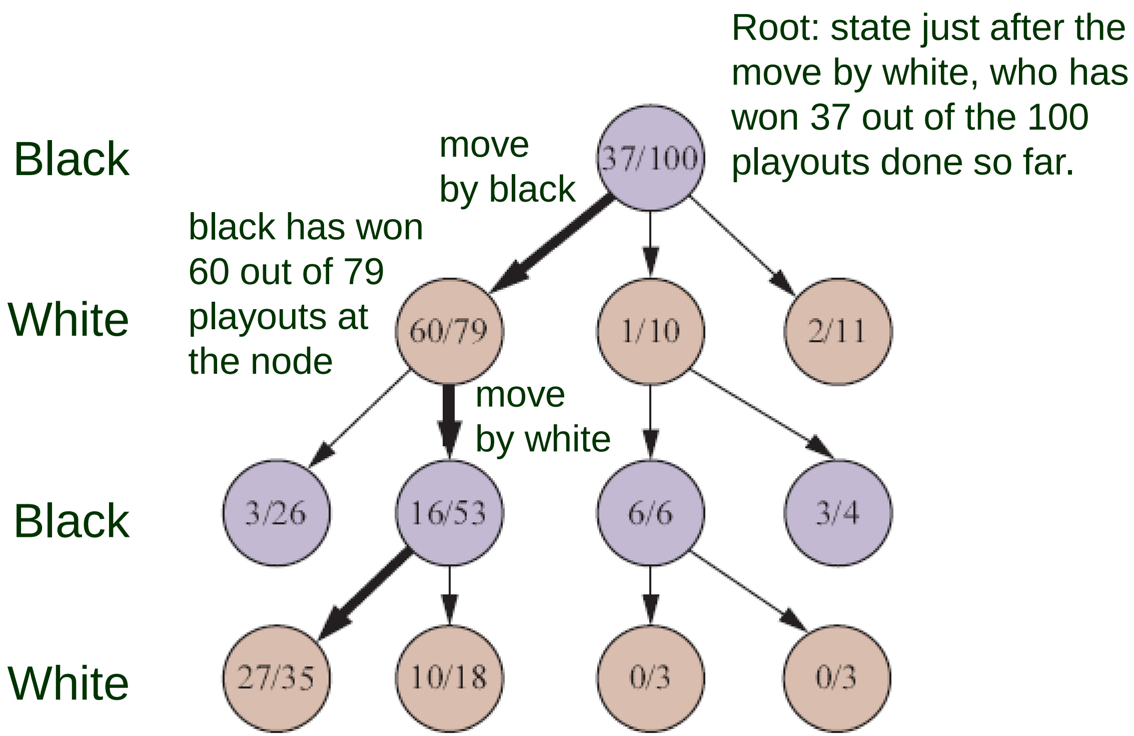

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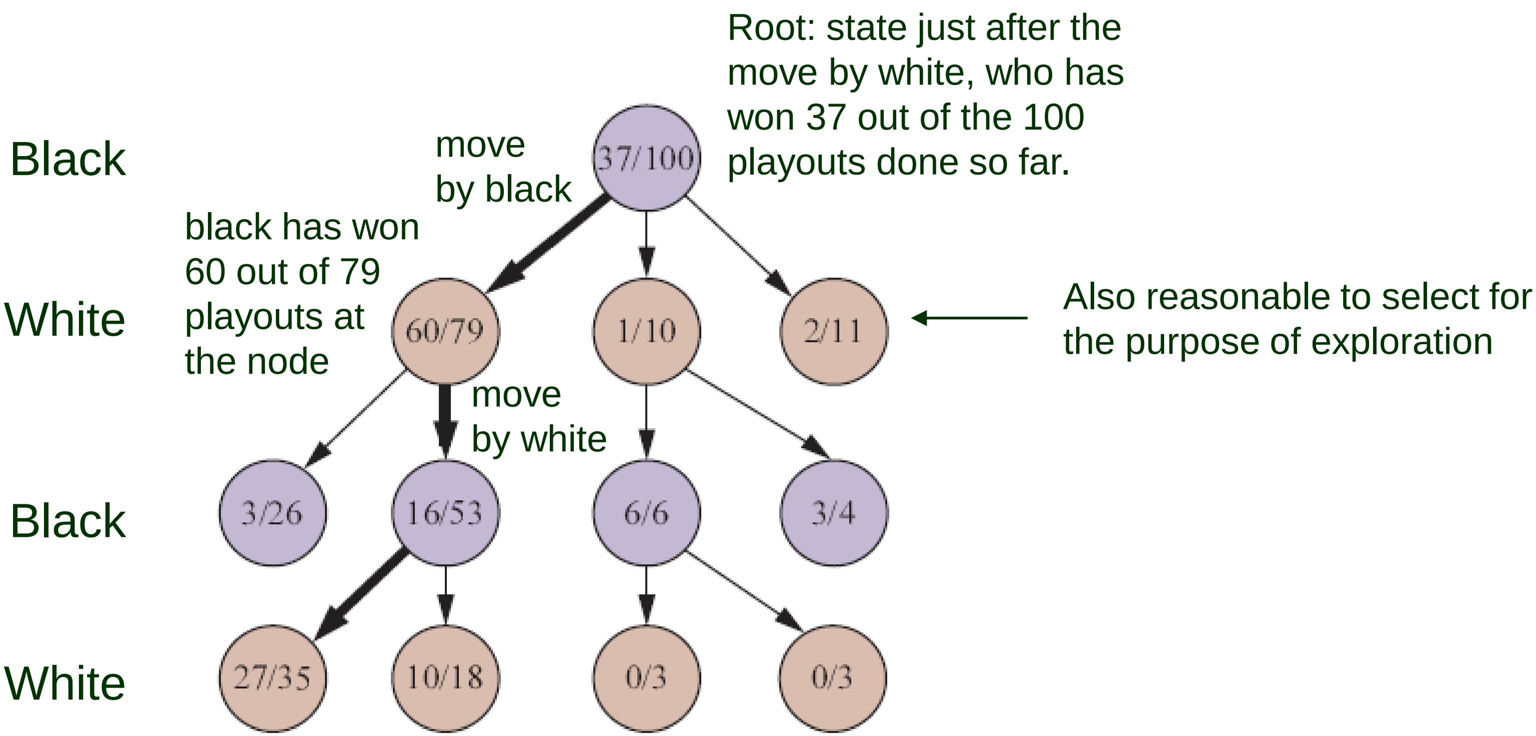

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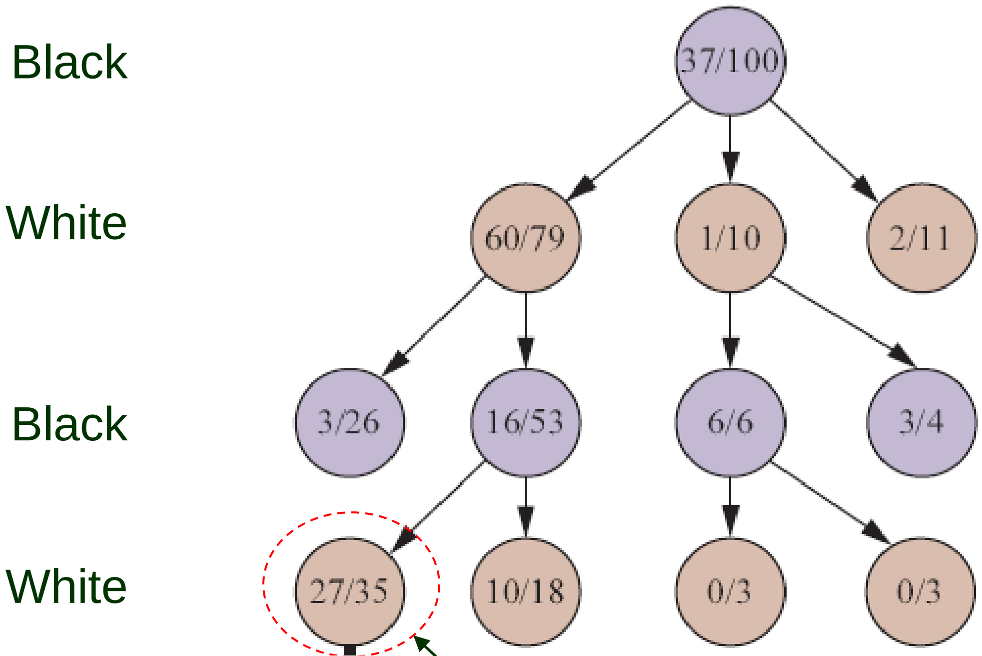

selected (27 wins for black out of 35 playouts)

(b) Expansion and simulation

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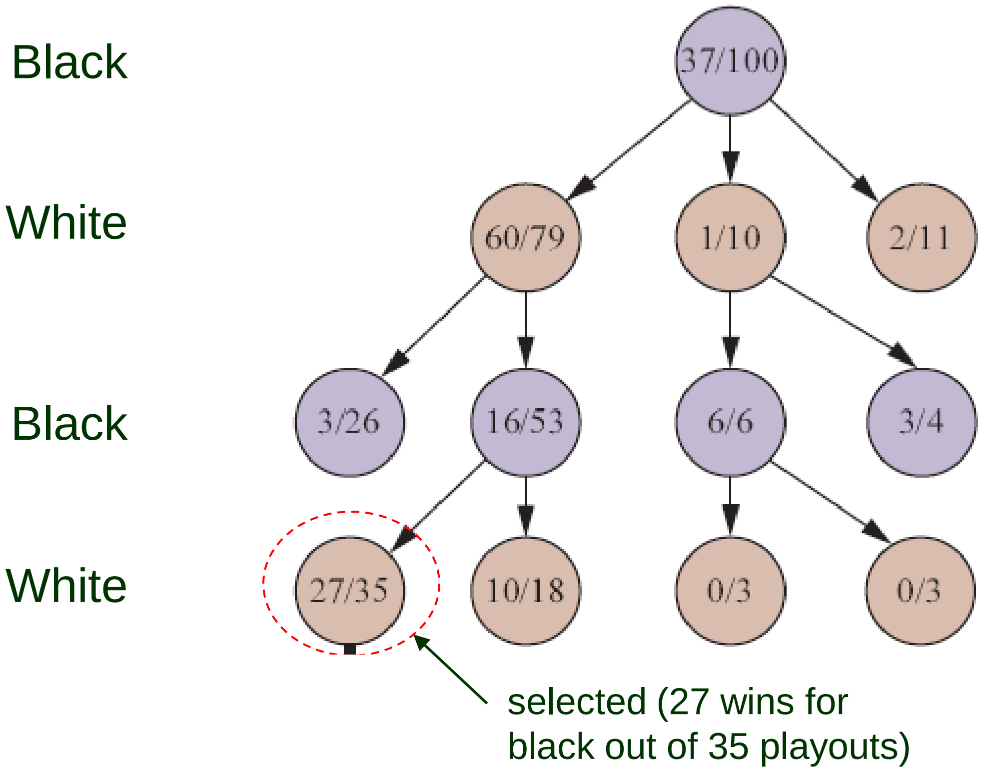

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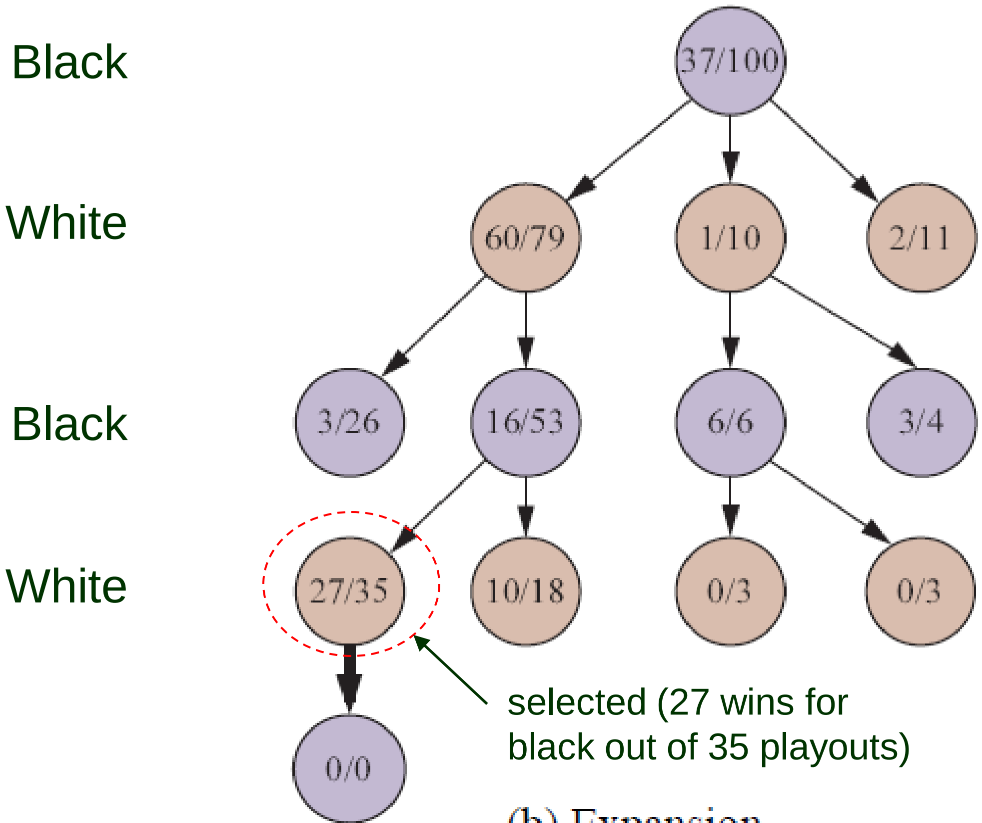

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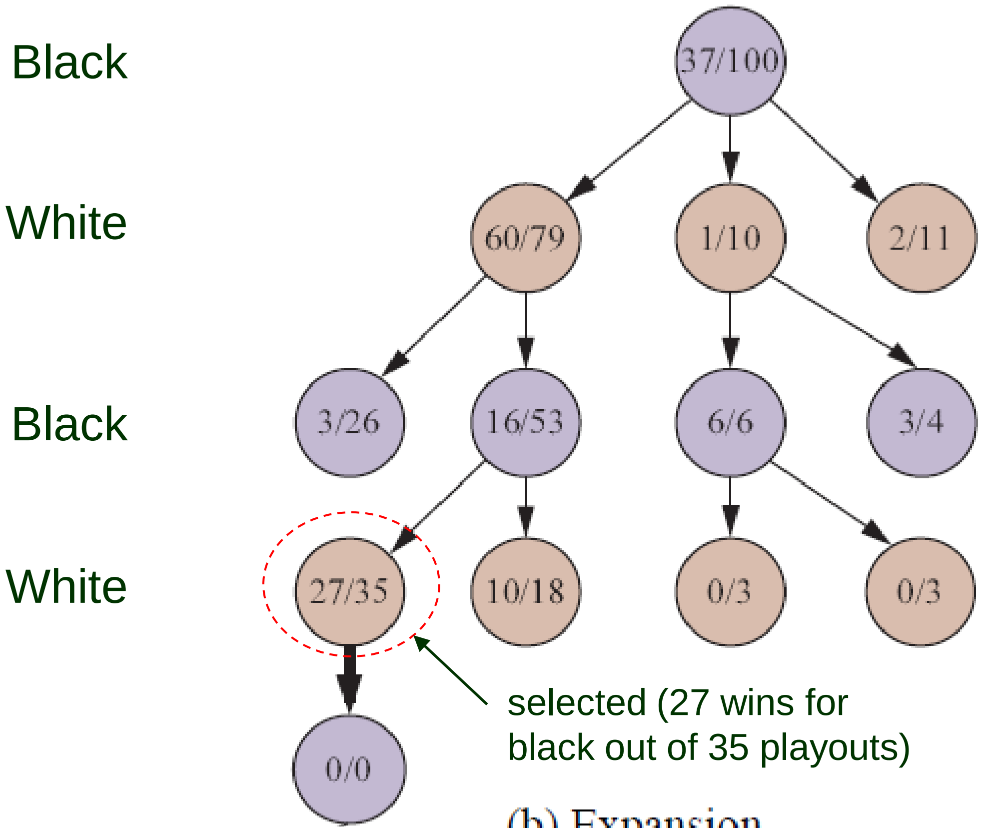

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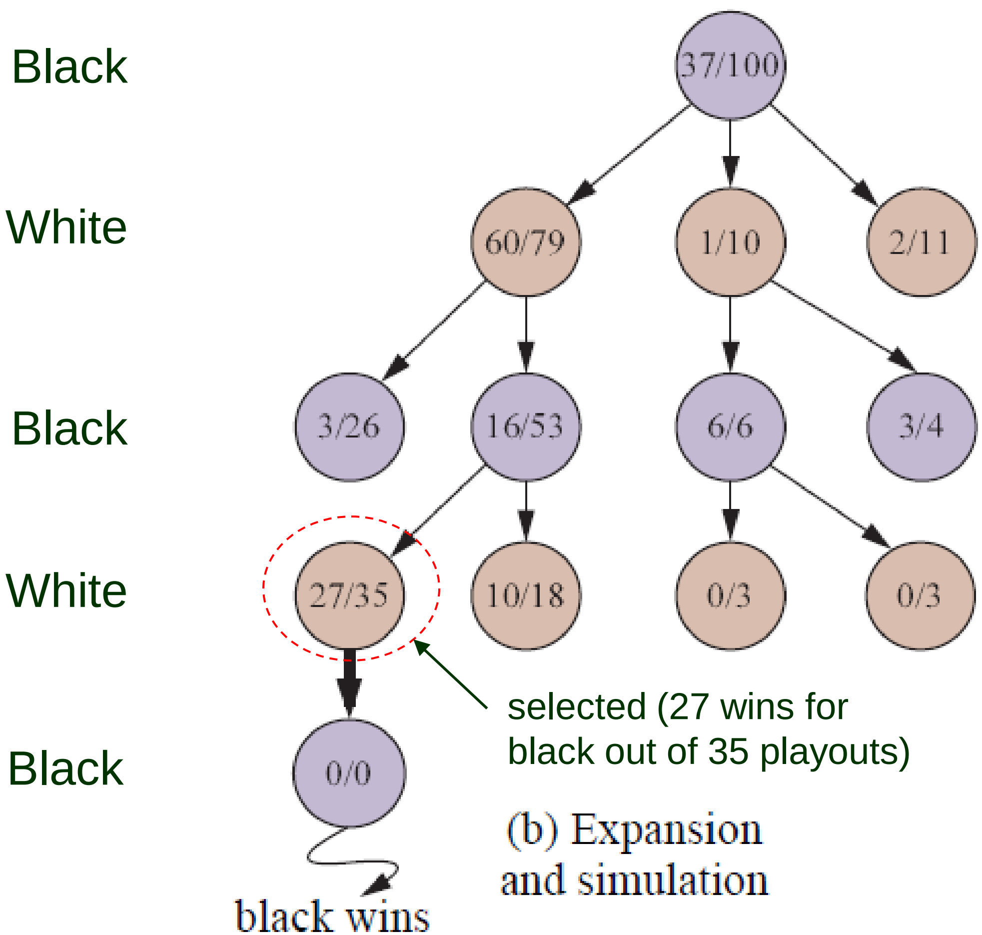

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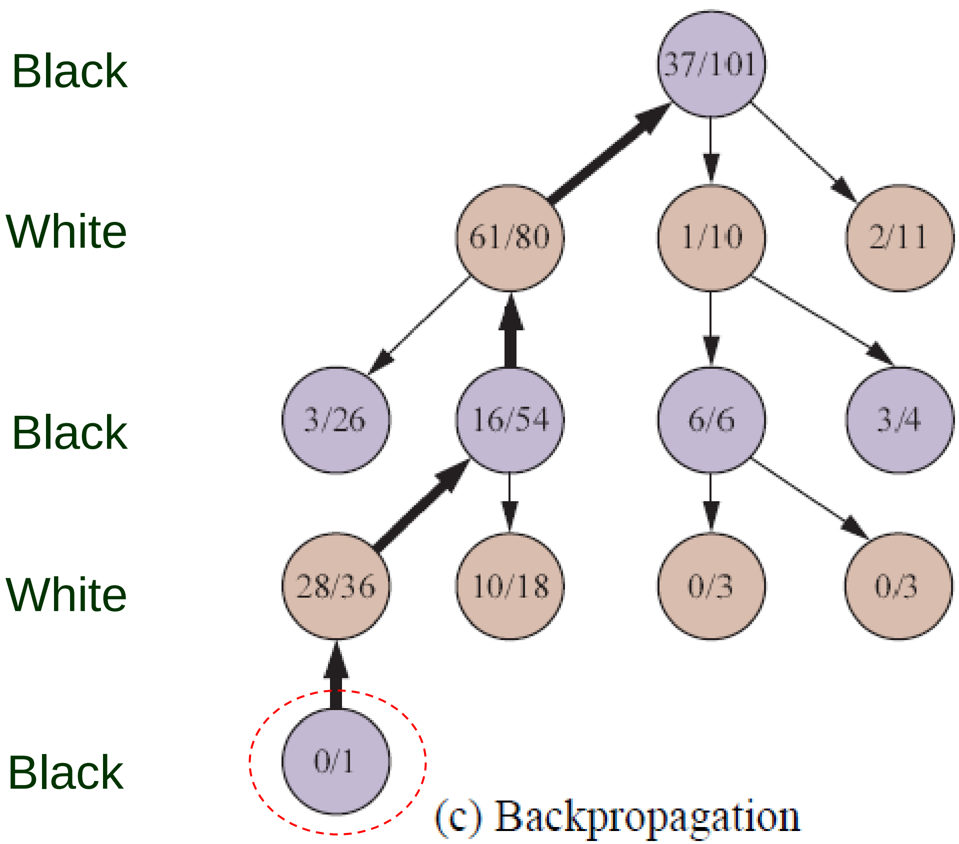

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- Update all the nodes upward along the path until the root.

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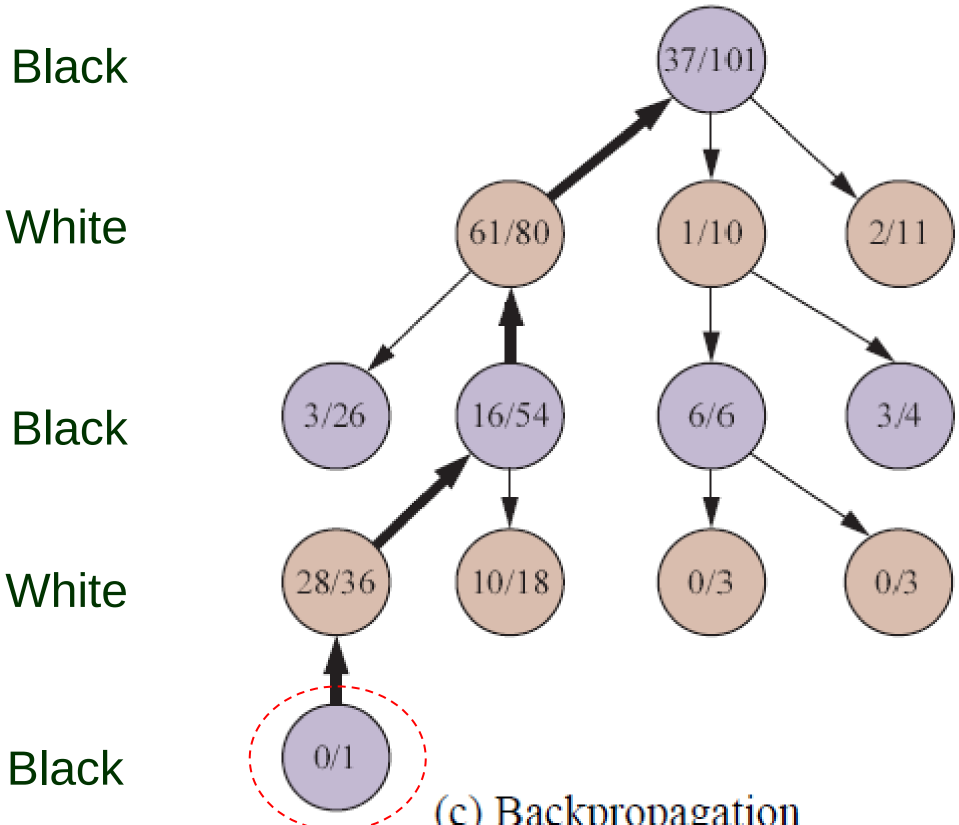

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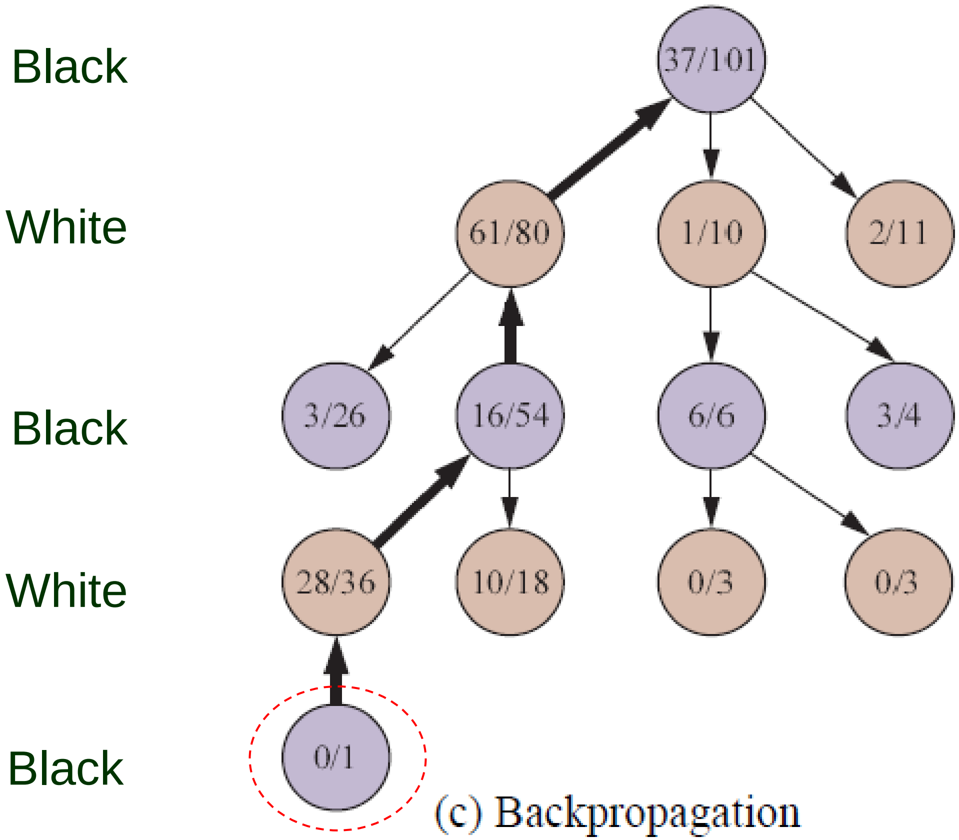

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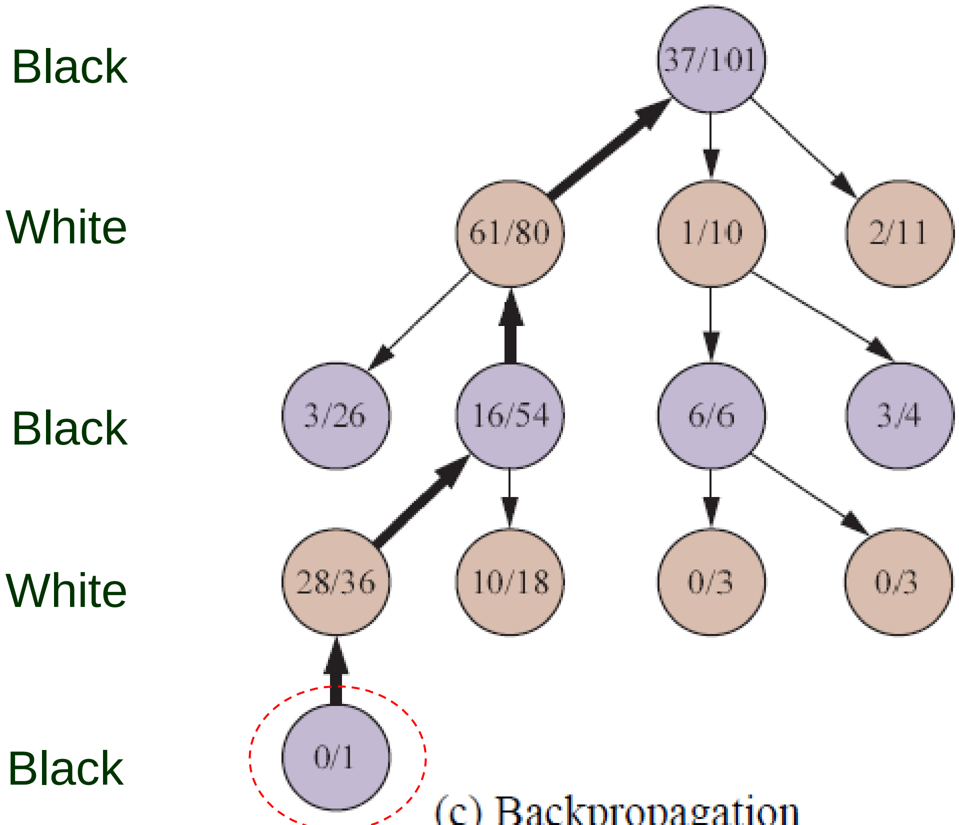

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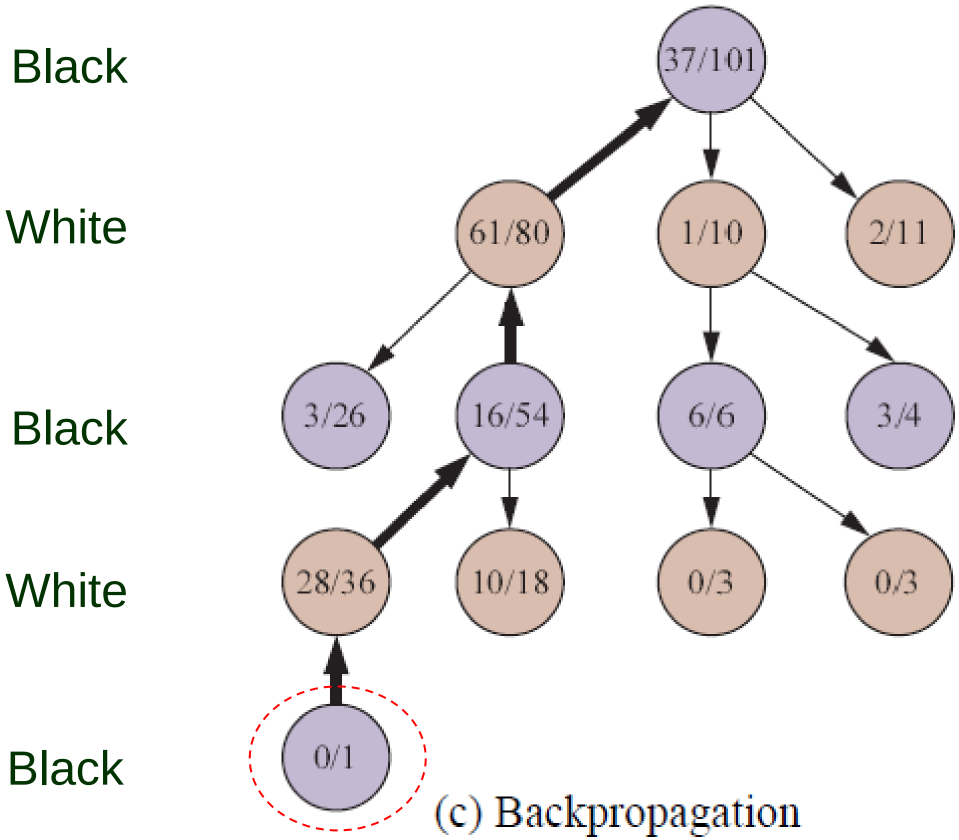

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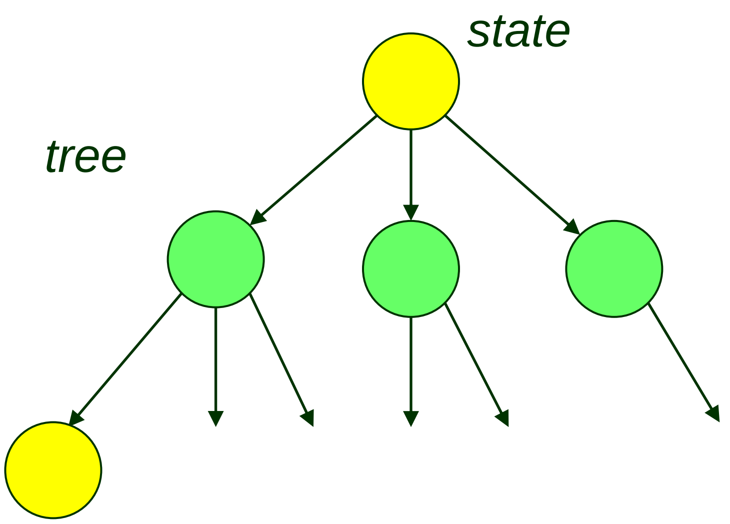
Monte Carlo Tree Search Algorithm

For deciding a move at the current state during the game:

function MONTE-CARLO-TREE-SEARCH(*state*) **returns** *an action*
  tree ← NODE(*state*)
  while IS-TIME-REMAINING() **do**
    leaf ← SELECT(*tree*)
    child ← EXPAND(*leaf*)
    result ← SIMULATE(*child*)
    BACK-PROPAGATE(*result*, *child*)
  return the move in ACTIONS(*state*) whose node has highest number of playouts



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Upper confidence bound formula:

$$\text{UCB}(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(PARENT(n))}{N(n)}}$$

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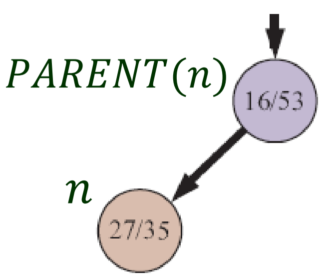

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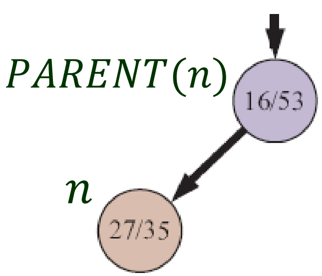

$$U(n) = 27$$
$$N(n) = 35$$
$$N\big(PARENT(n)\big) = 53$$
$$C = \sqrt{2}$$ (choice by a theoretical argument)

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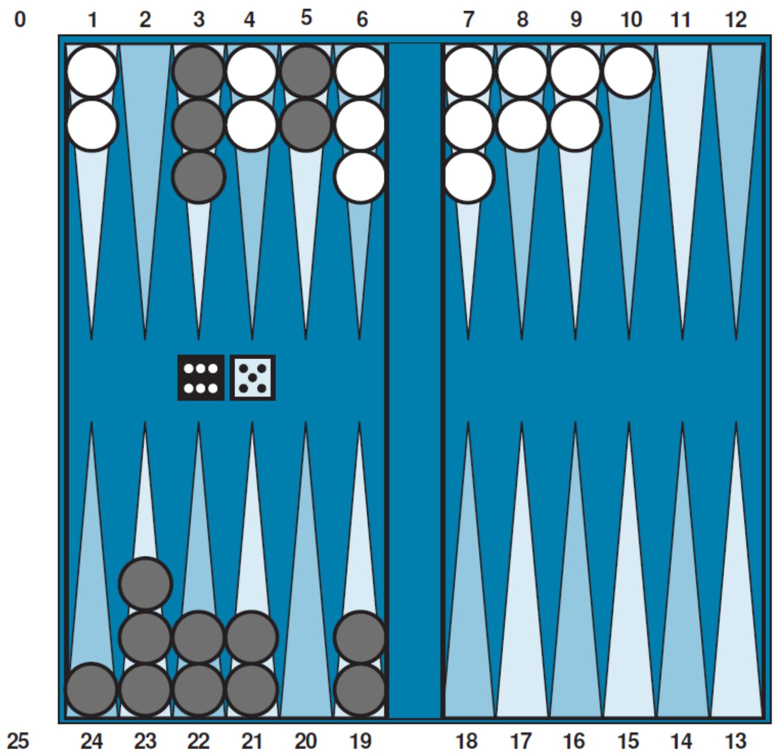
- MCTS is less desired than alpha-beta on a game like chess with low b and good evaluation function.

Stochastic Games

Some games (e.g., backgammon) have randomness due to throwing of dice. They combine both luck and skill.



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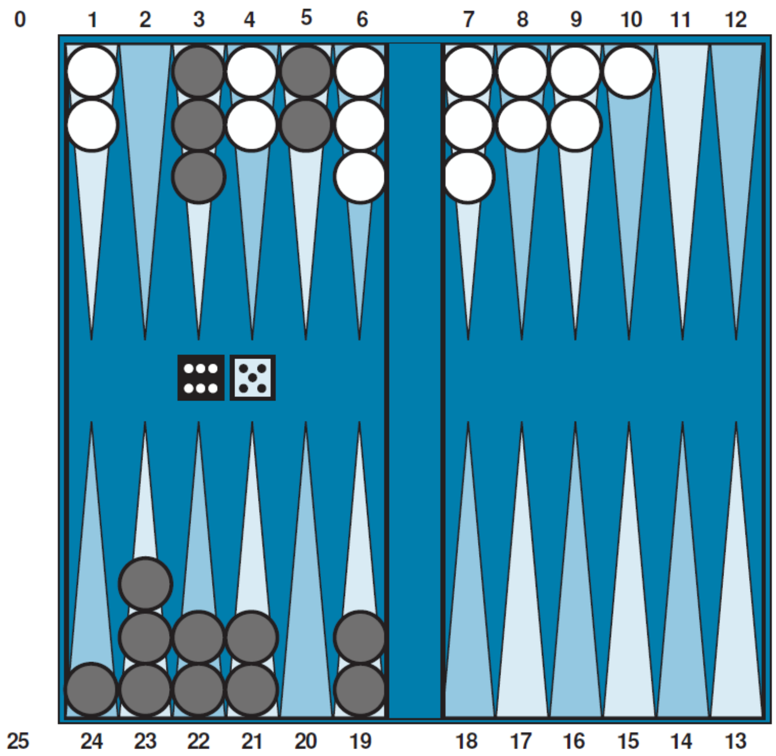

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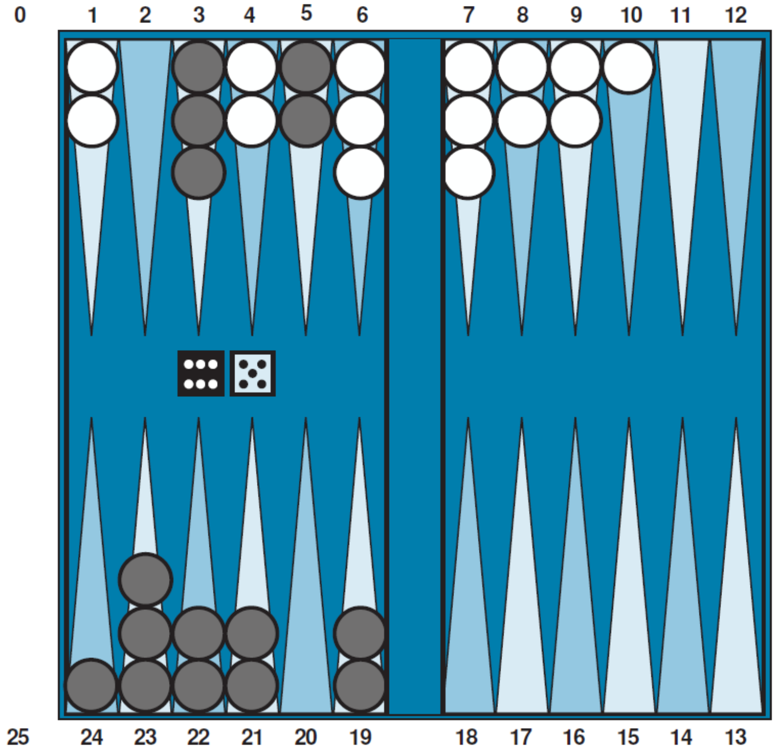

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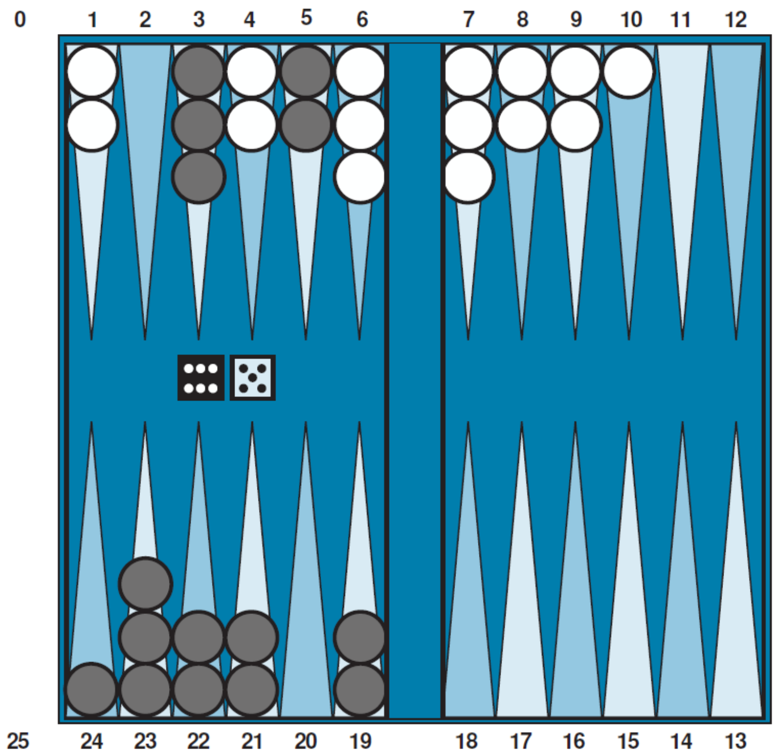

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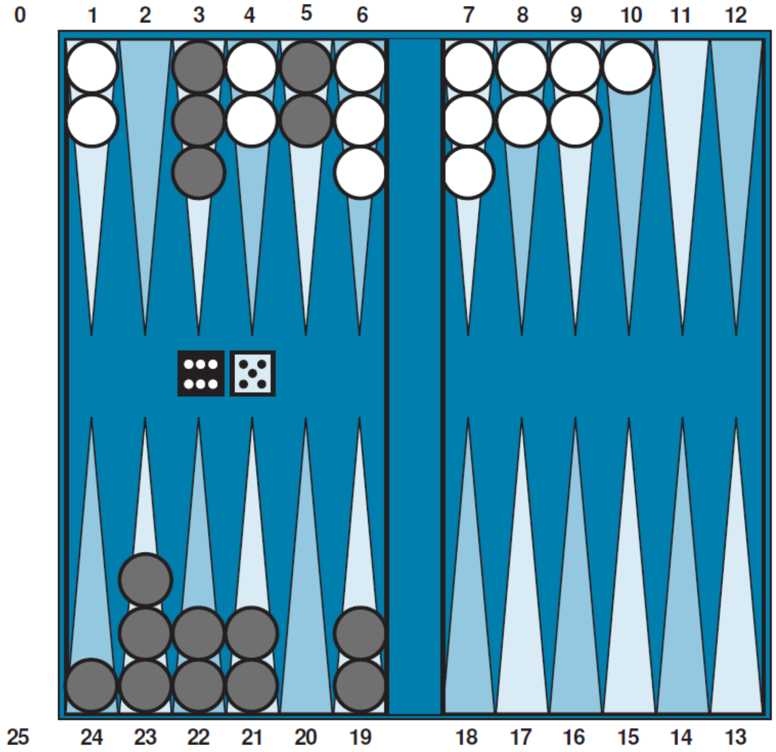

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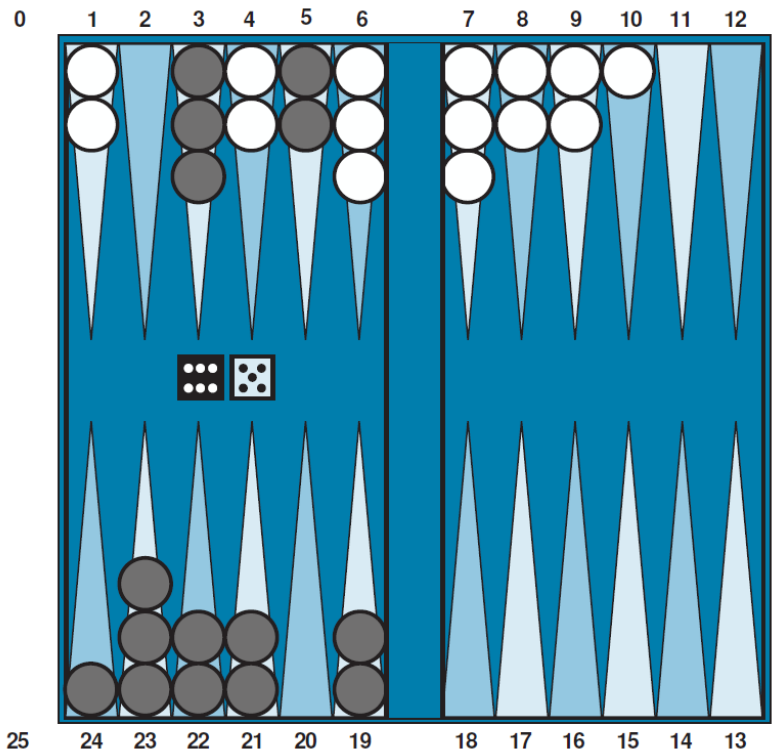

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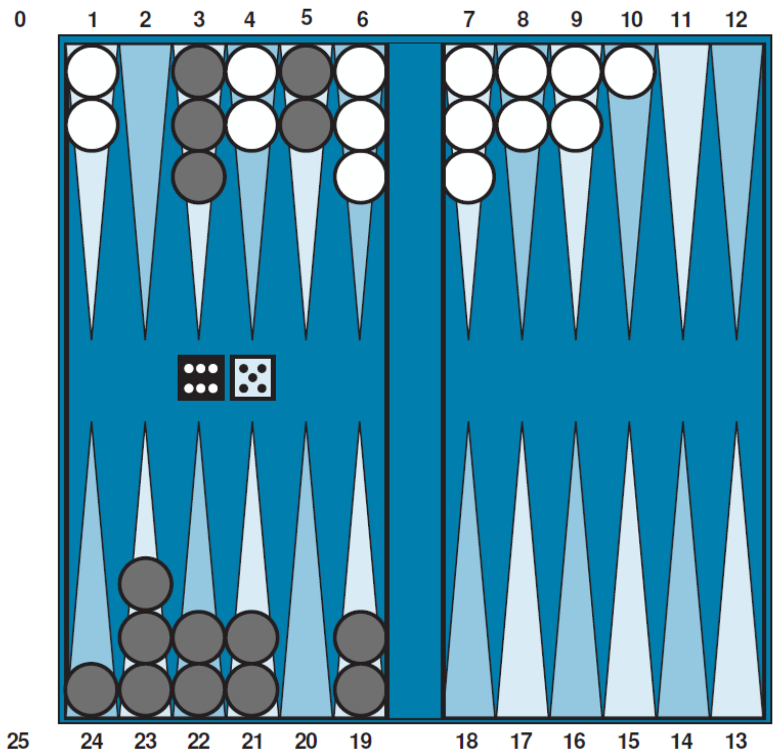

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- Calculate expected value (called *expectiminimax* value) of a position.

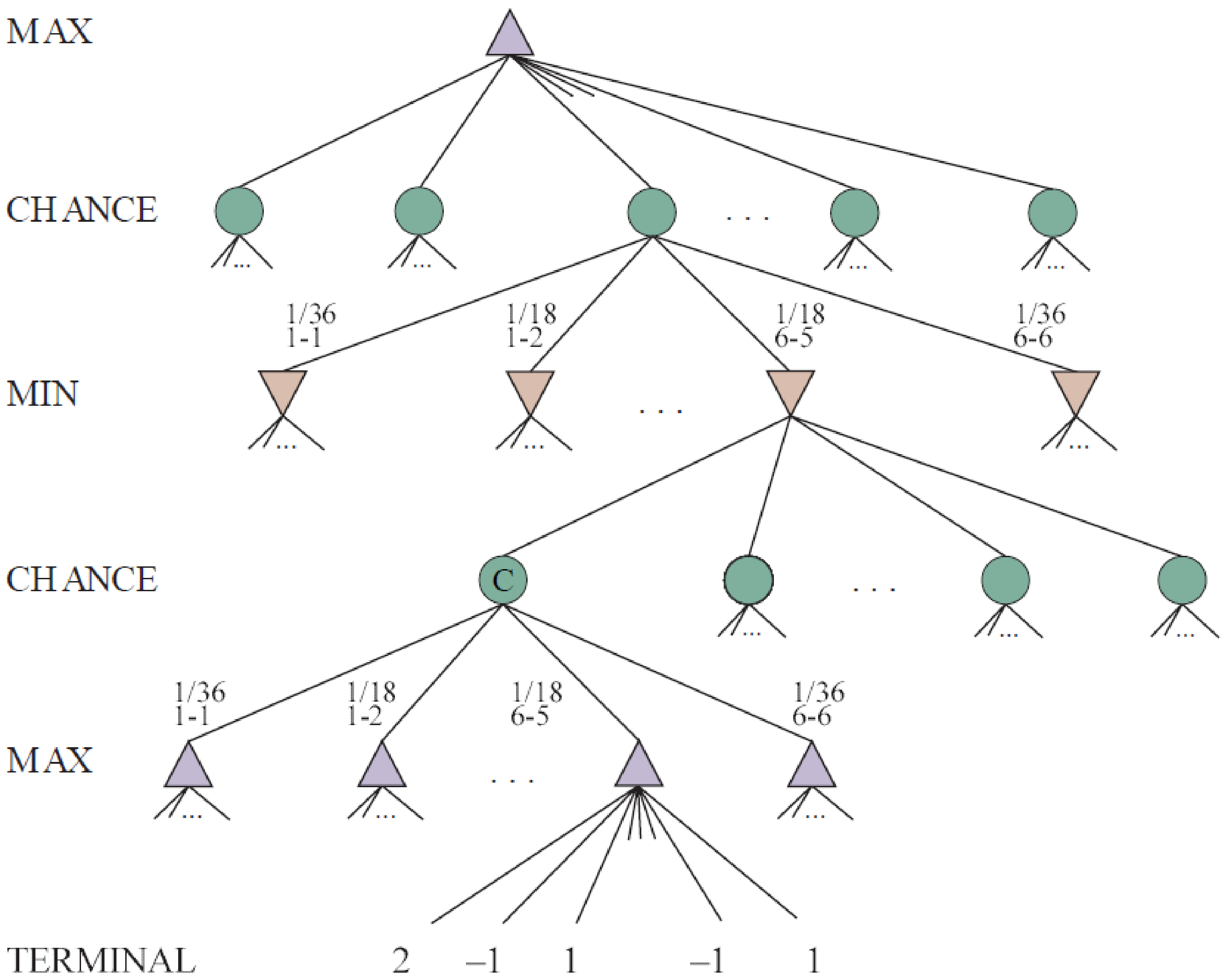
Expectiminimax Value

$$\textsc{Expectiminimax}(s) = \begin{cases} \textsc{Utility}(s, \textsc{Max}) & \text{if } \textsc{Is-Terminal}(s) \\ \max_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{To-Move}(s) = \textsc{Max} \\ \min_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{To-Move}(s) = \textsc{Min} \\ \boxed{\sum_r P(r)\, \textsc{Expectiminimax}(\textsc{Result}(s,r))} & \text{if } \textsc{To-Move}(s) = \textsc{Chance} \end{cases}$$

expected value

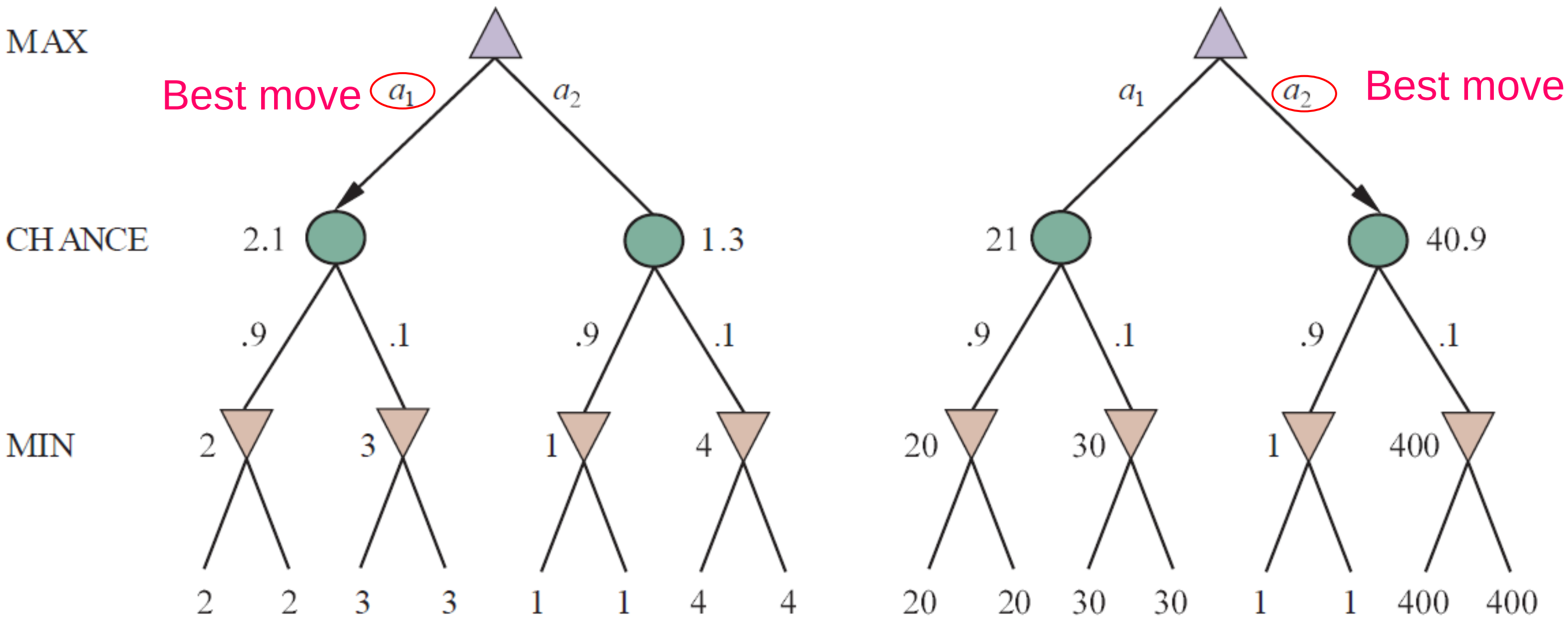
one possible dice roll

Game Tree for a Backgammon Position



Evaluation Functions

Evaluation functions with the same order of leaf values can yield different move choices at a state.



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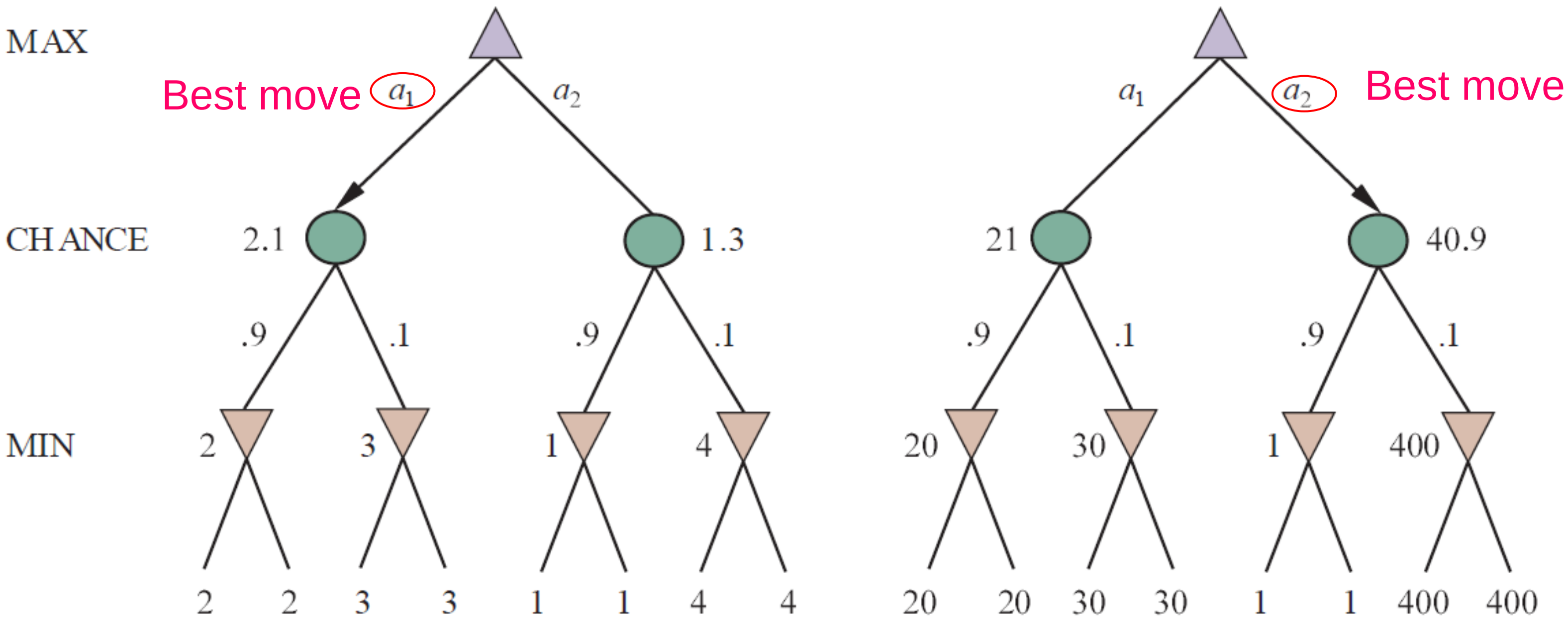

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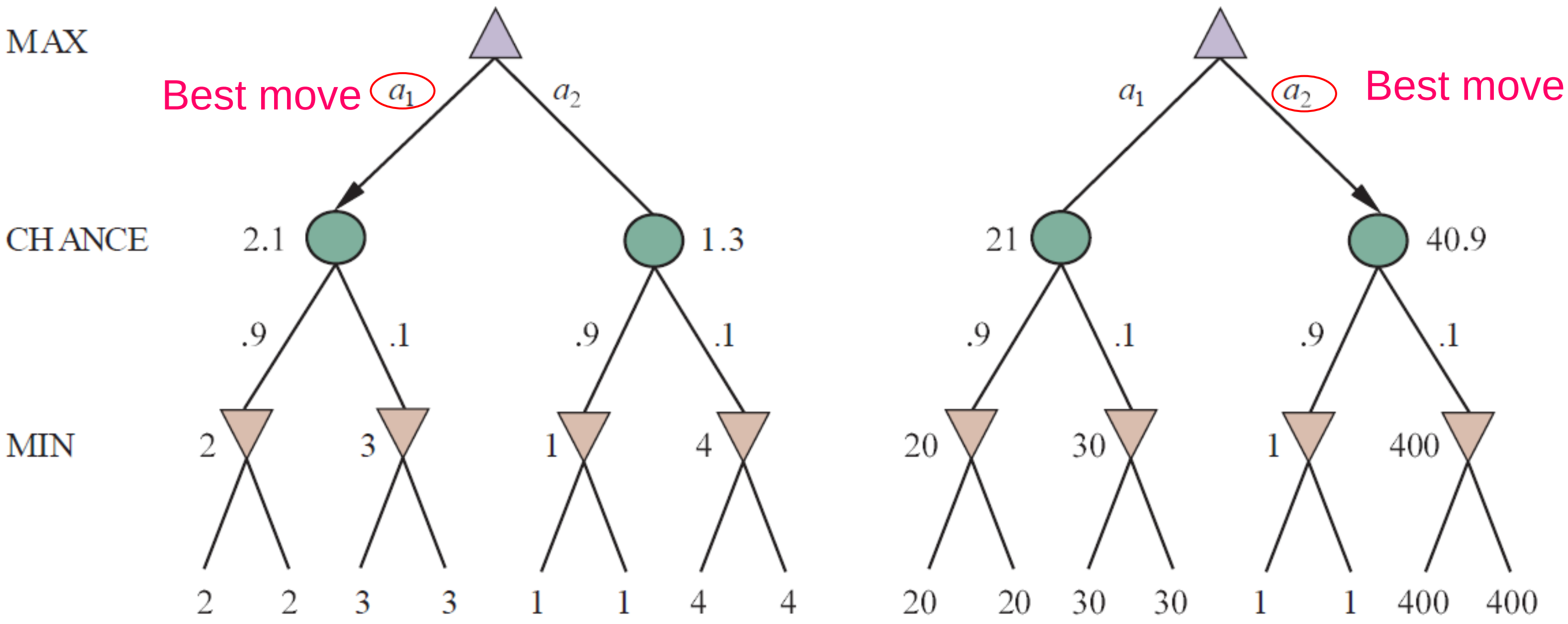

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