

Stat 330

Homework 6

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1)

	X \ Y	Y			$p_x(x)$
		0	1	2	
(a)	0	0.3	0.1	0.1	0.5
	1	0.2	0.1	0	0.3
	2	0.1	0.1	0	0.2
	$p_y(y)$	0.6	0.3	0.1	1

(b) $E(X) = (0)(0.5) + (1)(0.3) + (2)(0.2) = 0.7$
 $E(Y) = (0)(0.6) + (1)(0.3) + (2)(0.1) = 0.5$

$E(X^2) = (0)^2(0.5) + (1)^2(0.3) + (2)^2(0.2) = 1.1$
 $E(Y^2) = (0)^2(0.5) + (1)^2(0.3) + (2)^2(0.2) = 0.7$

$\text{Var}(X) = E(X^2) - |E(X)|^2 = 1.1 - 0.7^2 = 0.61$
 $\text{Var}(Y) = E(Y^2) - |E(Y)|^2 = 0.7 - 0.5^2 = 0.45$

(c) $E(XY) = (0)(0)(0.3) + (1)(0)(0.1) + (2)(0)(0.1) +$
 $(0)(1)(0.2) + (1)(1)(0.1) + (2)(1)(0) +$
 $(0)(2)(0.1) + (1)(2)(0.1) + (2)(2)(0) = 0.3$

$\text{Cov}(X, Y) = E(XY) - E(X)E(Y) = 0.3 - (0.7)(0.5) = -0.05$

$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) * \text{Var}(Y)}} = \frac{-0.05}{\sqrt{0.61 * 0.45}} = -0.095$

(d) Covariance $\neq 0$, and $p_{x,y}(1, 0) \neq p_x(1) * p_y(0)$.
Therefore the two days are not independent.

2)

$$(a) P(X=Y) = p_{x,y}(0,0) + p_{x,y}(1,1) + p_{x,y}(2,2) = 0.3 + 0.1 + 0 = 0.4$$

$$(b) P(X<Y) = p_{x,y}(0,1) + p_{x,y}(0,2) + p_{x,y}(1,2) = 0.1 + 0.1 + 0 = 0.2$$

$$(c) P(X>Y) = p_{x,y}(1,0) + p_{x,y}(2,0) + p_{x,y}(2,1) = 0.2 + 0.1 + 0.1 = 0.4$$

$$(d) p_{x,y}(0,0) = 0.3$$

$$(e) p_{x,y}(1,2) = 0$$

3)

$$(a) X \sim \text{Bin}(15, .85). \text{ Then, } P(X \geq 13) = P(X=13) + P(X=14) + P(X=15)$$

$$\binom{15}{13} (0.85)^{13} (1-0.15)^{15-13} + \binom{15}{14} (0.85)^{14} (1-0.15)^{15-14} + \binom{15}{15} (0.85)^{15} (1-0.15)^{15-15} = .2856 + .2312 + .0874 = .6042$$

$$(b) Y \sim \text{Geo}(.85) \Rightarrow E(Y) = 1/.85 = 1.176$$

4)

$$(a) X \sim \text{Pois}(10)$$

$$(b) \frac{e^{-10}(10)^8}{8!} = .113$$

$$(c) X \sim \text{Pois}(10/(60/12)) = \text{Pois}(2)$$

$$(d) \frac{e^{-2}(2)^3}{3!} = .18$$

$$(e) E(X) = \lambda = 2$$

5)

$$(a) 3 \text{ goals in the next 5 games} \Rightarrow \lambda = 1.1*5 = 5.5$$

$$P(X>3) = 1 - P(X \leq 3). \quad \text{Using CDF table, } P(X \leq 3) = 0.2017$$

$$P(X>3) = 1 - 0.2017 = .7983$$

$$(b) \text{ As the team averages 1.1 goals per game, the probability of } P(Y=0) = .3329$$

$$\text{Thus, } Y \sim \text{Bin}(5, .3329) \Rightarrow P(Y<2) = P(Y<0) + P(Y<1) =$$

$$\binom{5}{0} (0.3329)^0 (1-0.3329)^{5-0} + \binom{5}{1} (0.3329)^1 (1-0.3329)^{5-1} = .1321 + .3296 = .4618$$

6)

$$(a) X \sim \text{Pois}(1). \Rightarrow P(\text{High risk} \mid 0 \text{ accidents}) = P(X=0) = e^{-1} = .3679$$