Stat 330
Exam 4 (final)

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1) $Q_1 - 1.5(IQR) = 20 - 1.5(14) = -1$
$Q_3 + 1.5(IQR) = 34 + 1.5(14) = 55$
As **58** is the only number outside the range, it is the only outlier.

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2)

(a) Mean $= \frac{24+31+32+33+35+37+49}{7} = 34.43$
Median $= 33$
$Q_1 = 31, Q_3 = 37, \qquad IQR = Q_3 - Q_1 = 6$

(b) $Q_1 - 1.5(IQR) = 31 - 1.5(6) = 22$
$Q_3 + 1.5(IQR) = 37 + 1.5(6) = 46$
49 is the only point outside the range.

(c) Mean $= \frac{24+31+32+33+35+37}{6} = 32$
Median $= \frac{33+32}{2} = 31.5$
$Q_1 = 31, Q_3 = 35, \qquad IQR = Q_3 - Q_1 = 4$

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3)

(a) $\hat{\lambda}_1 = (1/3)E(X_1) + (2/3)E(X_2) = (3/3)\theta = \theta$
$\hat{\lambda}_2 = E(\bar{X}) = \theta$
$\hat{\lambda}_3 = 5$
Estimator $\hat{\lambda}_3$ is biased, while $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are unbiased

(b) $\mathrm{Var}(\hat{\lambda}_1) = (1/9)\mathrm{Var}(X_1) + (4/9)\mathrm{Var}(X_2) = (5/9)\sigma^2$
$\mathrm{Var}(\hat{\lambda}_2) = \mathrm{Var}(\bar{X}) = \sigma^2/n$
$\mathrm{Var}(\hat{\lambda}_3) = 0$

(c) $\mathrm{MSE}(\hat{\lambda}_1) = 0^2 + (5/9)\sigma^2 = (5/9)\sigma^2$
$\mathrm{MSE}(\hat{\lambda}_2) = 0^2 + \sigma^2/n = \sigma^2/\mathrm{n}$
$\mathrm{MSE}(\hat{\lambda}_3) = 5^2 - \theta + 0 = 5^2 - \theta$

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4)

(a) $\hat{\theta}_{MOM} \Rightarrow \sqrt{\dfrac{\theta\pi}{2}} = \bar{X} \Rightarrow \theta = \dfrac{2\bar{X}}{\pi}$

$\bar{X} = 2.32 \Rightarrow \theta = \dfrac{2 * 2.23}{\pi} = 1.48$

(b)

(i) $log(L(\theta)) = log(x_i\theta^{-n} * e^{-\frac{\Sigma x_i^2}{2\theta}}) = log(x_i) + log(\theta^{-n}) + log(e^{-\frac{\Sigma x_i^2}{2\theta}}) =$

$log(x_i) - nlog(\theta) - \dfrac{\Sigma x_i^2}{2\theta} log(e)$

(ii) $-\dfrac{n}{\theta} + \dfrac{\Sigma x_i^2}{2\theta^2}\mathrm{log(e)} = 0 \Rightarrow \dfrac{\Sigma x_i^2}{2\theta^2}\mathrm{log(e)} = \dfrac{n}{\theta} \Rightarrow \Sigma x_i^2 log(e)\theta = 2n\theta^2 \Rightarrow \Sigma x_i^2 log(e) = 2n\theta \Rightarrow$

$\theta = \dfrac{\Sigma x_i^2 log(e)}{2n} \Rightarrow \hat{\theta}_{mle} = \dfrac{\Sigma x_i^2 log(e)}{2n}$

(iii) $\Sigma x_i^2 = \dfrac{2.15^2 + 2.68^2 + 2.17^2 + 2.28^2}{4} = 5.42$

$\hat{\theta}_{mle} = \dfrac{5.42 * 0.434}{2(4)} = 0.294$

5a)

(i) $H_0 : \mu = 50$
$H_A : \mu \neq 50$

(ii) $s_1 = \sqrt{64} = 8$, $Z = \dfrac{52-50}{8/\sqrt{80}} = 2.24$

(iii) Using Z table, 2*P(Z<-2.24) = 2*(0.0125) = 0.025
This p value is small, so we reject H_0 in favor of H_A

5b) (i) $H_0 : \mu_1 = \mu_2$
$H_A : \mu_1 > \mu_2$

(ii) $Z = \dfrac{49-52-0}{\sqrt{(64/80)+(130/10)}} = -.22$

(iii) Using Z table, P(Z<-.22) = 0.4129
This p value is not small, so we do not reject H_0 in favor of H_A

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6)

(a) $.42 \pm 2.326\dfrac{\sqrt{.42(1-.42)}}{7276} = .42 \pm .0015 = (.4185,\ .4215)$

(b) $(.49\text{-}.35) \pm 1.96\sqrt{\dfrac{.49(1-.49)}{3638} + \dfrac{.35(1-.35)}{3638}} = .14 \pm .0225 = (.1175,\ .1625)$

(c) As we are fairly certain the lowest the difference in proportion goes is .1175, it is safe to say the proportions are not equal.