Forward and Backward Chaining

Outline

I. Forward chaining in FOL

II. Backward chaining

III. Prolog

* Figures are from the textbook site.

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The *KB* consists of first-order definite clauses with no function symbols.
It is called a *Datalog*.

Simple Forward Chaining

1. Start from the known facts.
2. Trigger all the rules whose premises are satisfied.
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4. Repeat steps 2 and 3 until one of the following situations occurs:
    a. The query is answered.
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Likes(x,*IceCream*) is a renaming of *Likes*(y,*IceCream*).
Both have the meaning: "Everyone likes ice cream".

Execution of Forward Chaining

KB:

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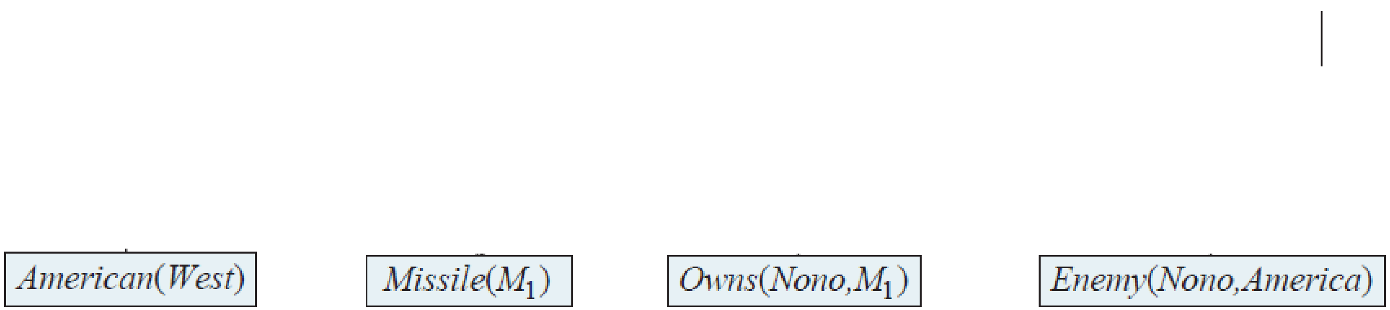
KB has now reached a *fixed point*, meaning that no new sentences are possible.

Proof Tree



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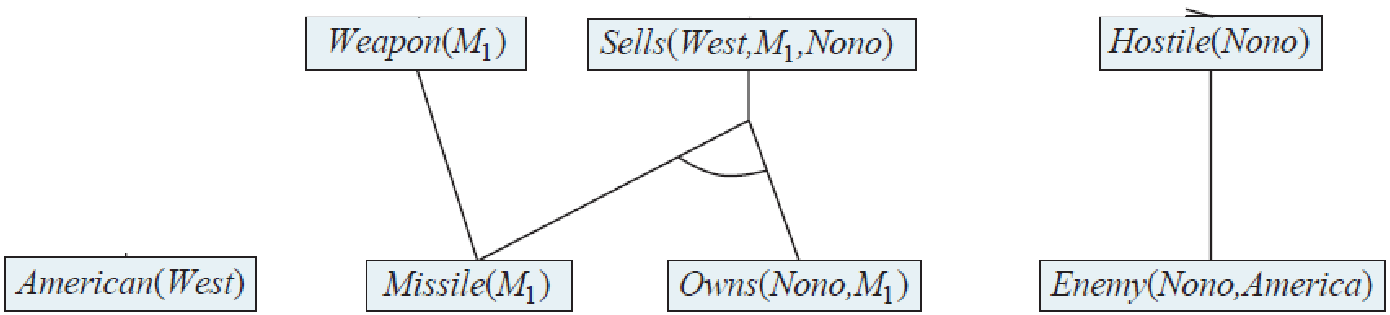

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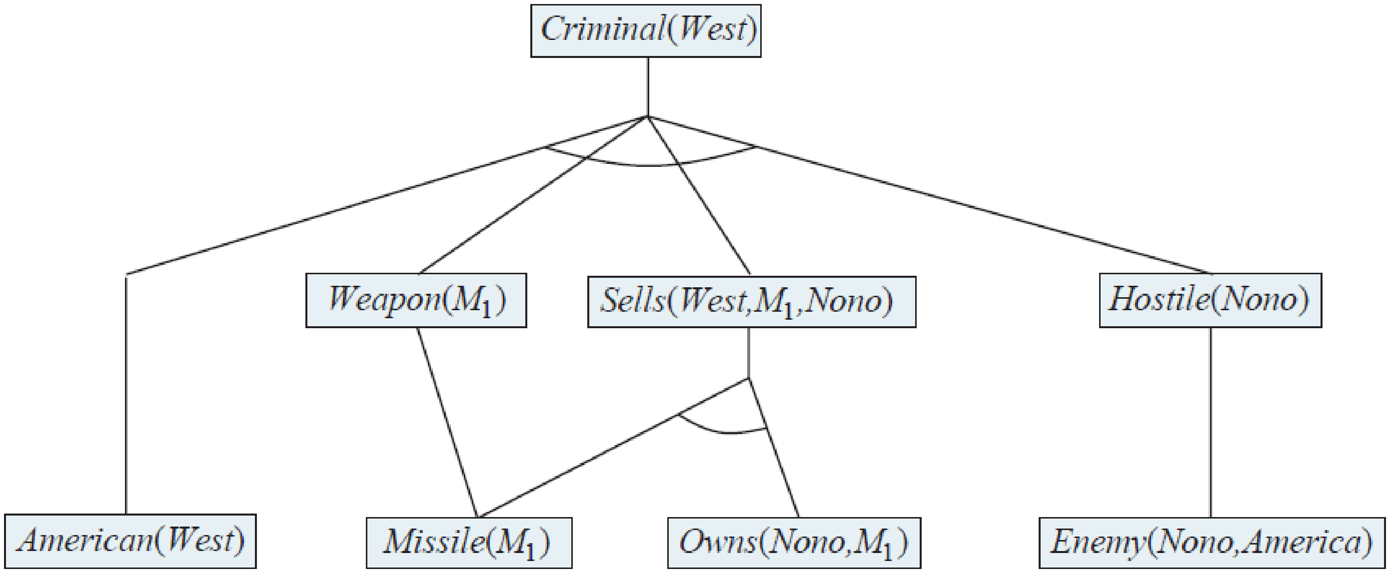
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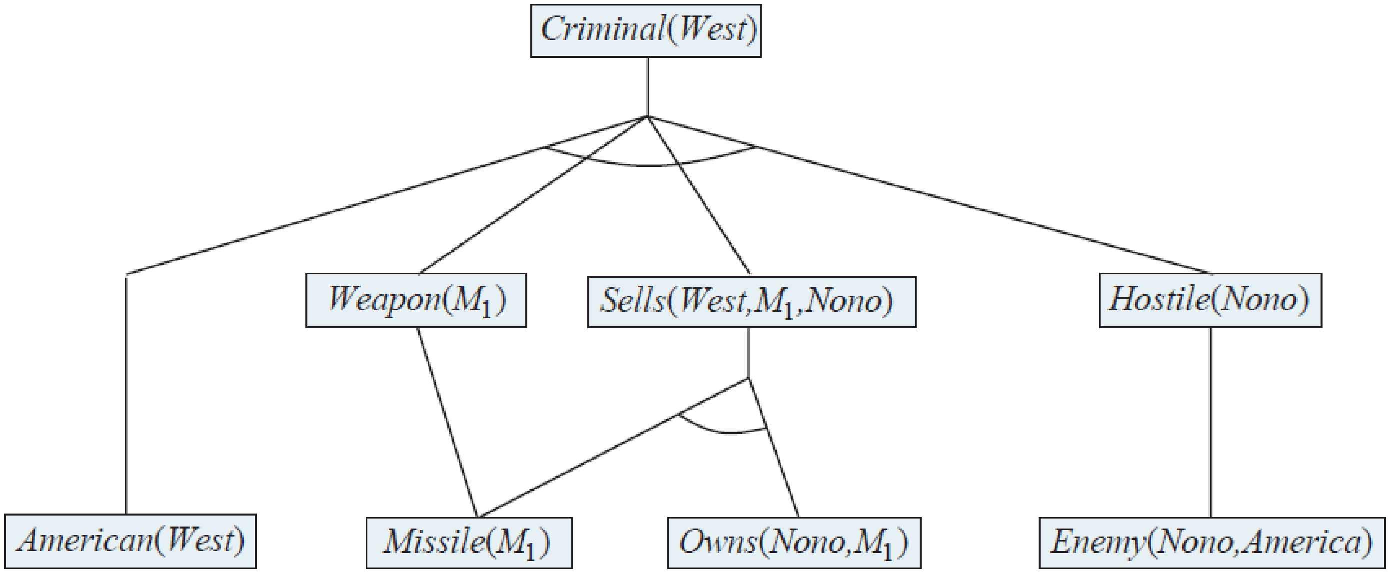

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  Every inference is an application of Generalized Modus Ponens.

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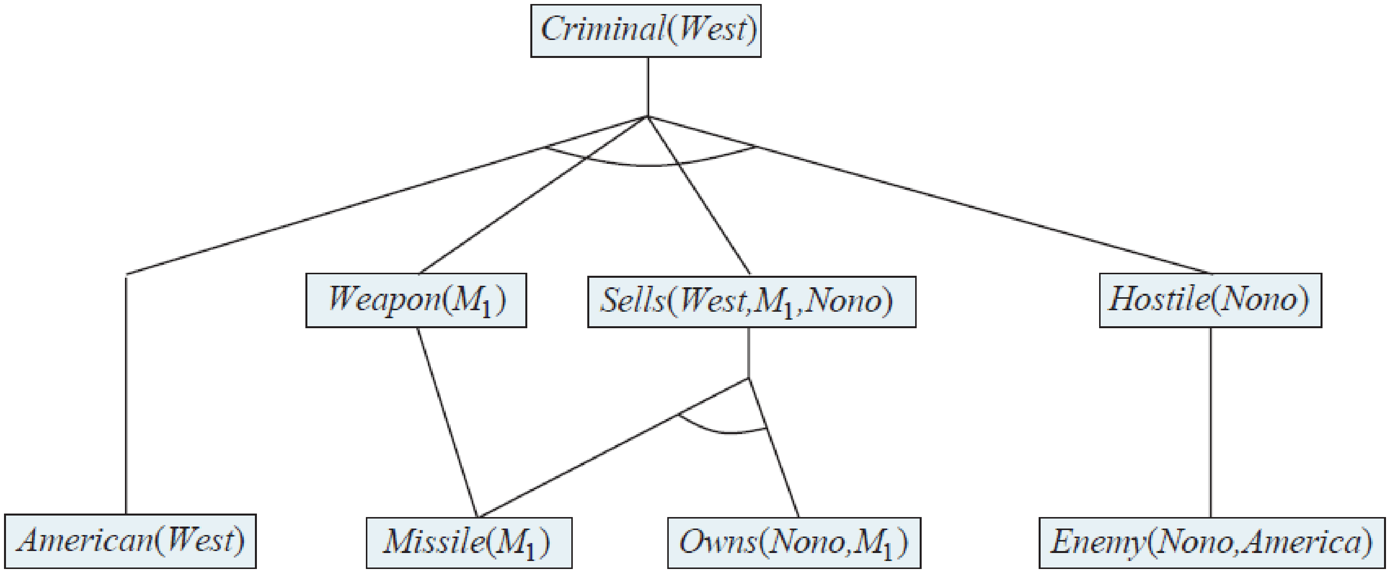

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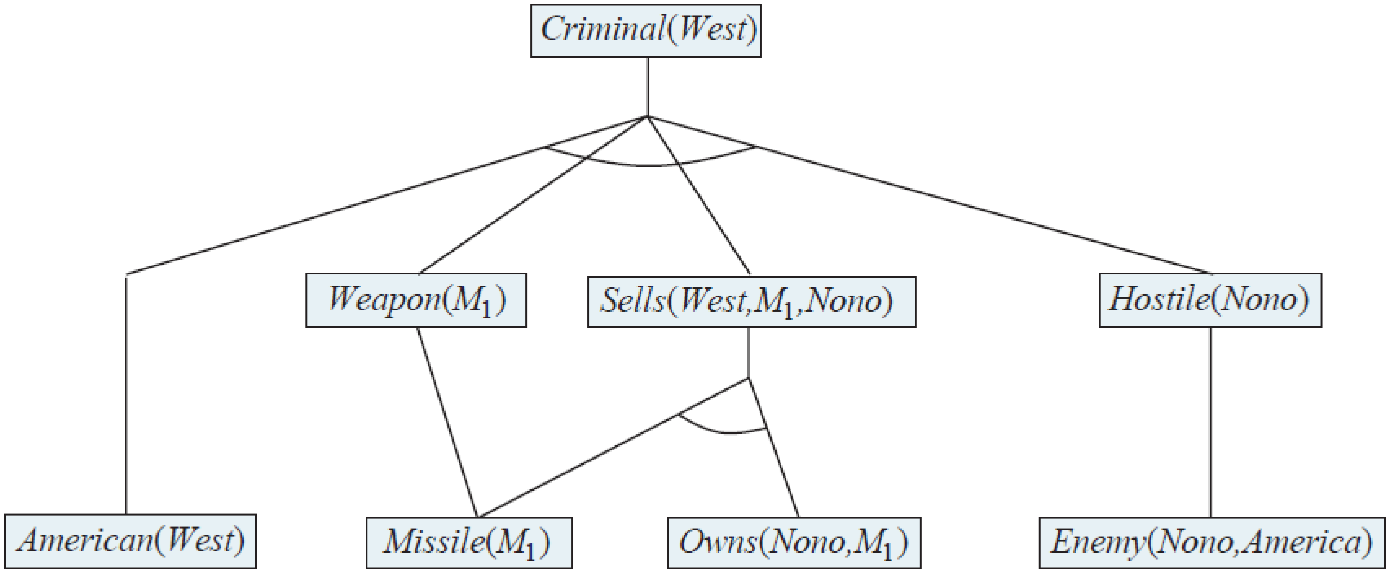

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  - Easy to establish if no function symbols appears in the KB.

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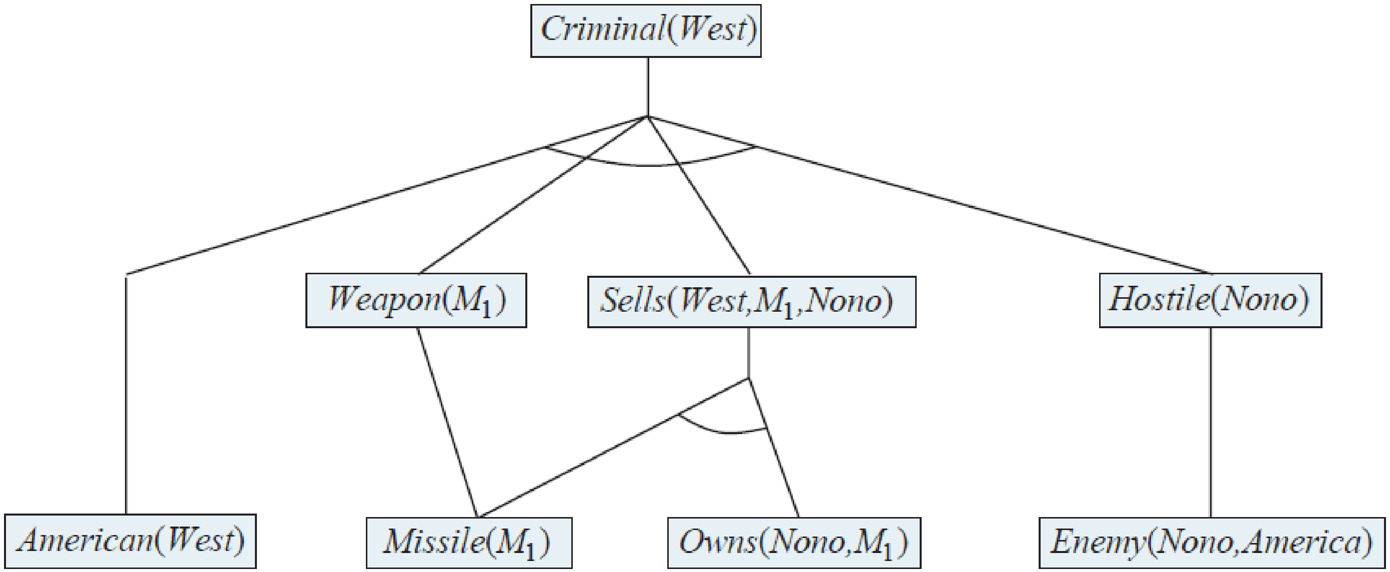

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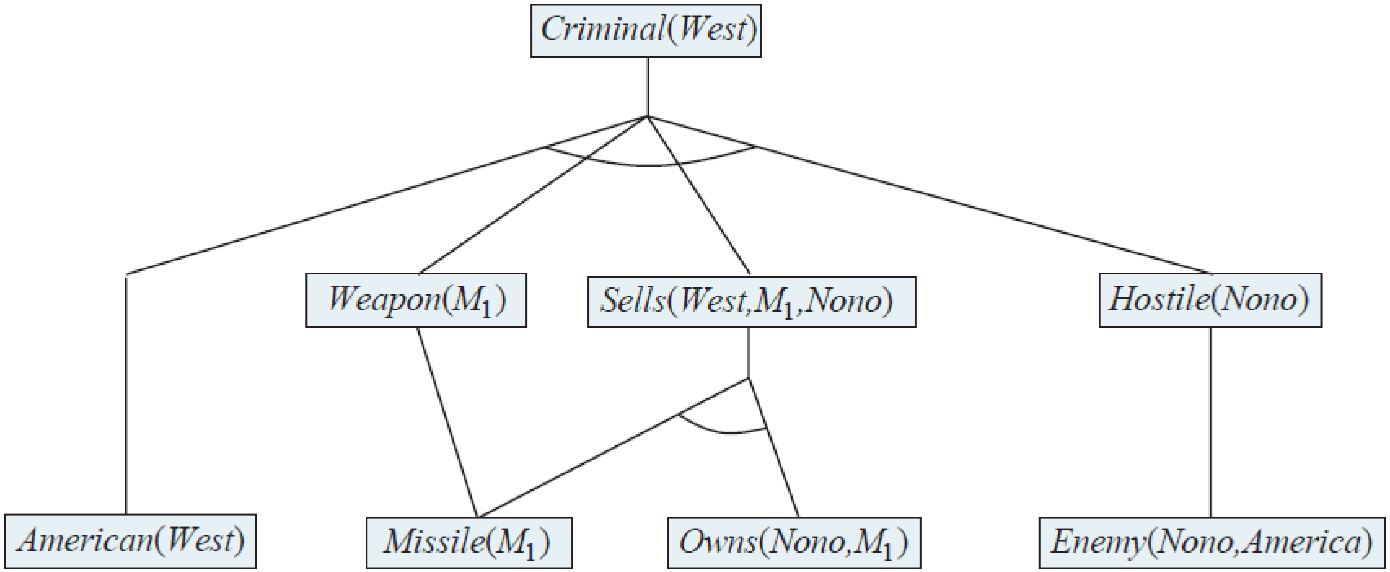

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  - Easy to establish if no function symbols appears in the KB.
  - Guaranteed except for a query with no answer, if function symbols appear in the KB.
  - Entailment with definite clauses is semi-decidable (Turing 1936, Church 1936).

Improvement 1: Matching Rules Against Known Facts

Inefficiency of simple forward chaining:

- ♠ Exhaustively matches every rule against every fact.
- ♠ Rechecks every rule on each iteration (even with very few additions to *KB*).
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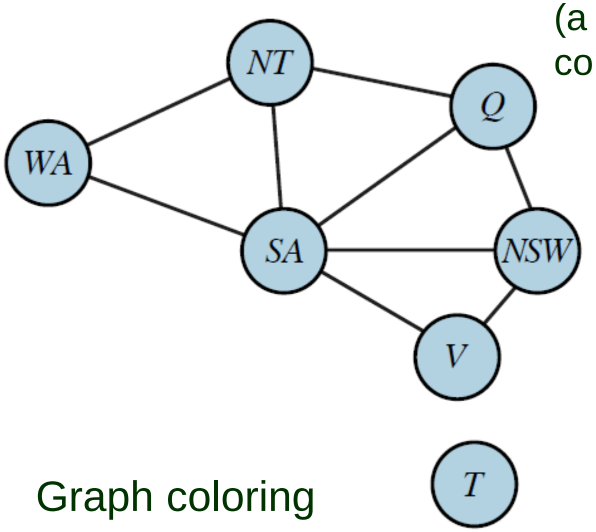
Use a heuristic, e.g., the minimum-remaining-values (MRV) heuristic for CSPs.

CSP as a Definite Clause

View every conjunct in the premise as a constraint on the variables it contains.



Graph coloring

color of WA (a Skolem constant)

$Diff(wa, nt) \wedge Diff(wa, sa) \wedge$
$Diff(nt, q) \wedge Diff(nt, sa) \wedge$
$Diff(q, nsw) \wedge Diff(q, sa) \wedge$
$Diff(nsw, v) \wedge Diff(nsw, sa) \wedge$
$Diff(v, sa) \Rightarrow true$

$Diff(Red, Blue)$	$Diff(Red, Green)$
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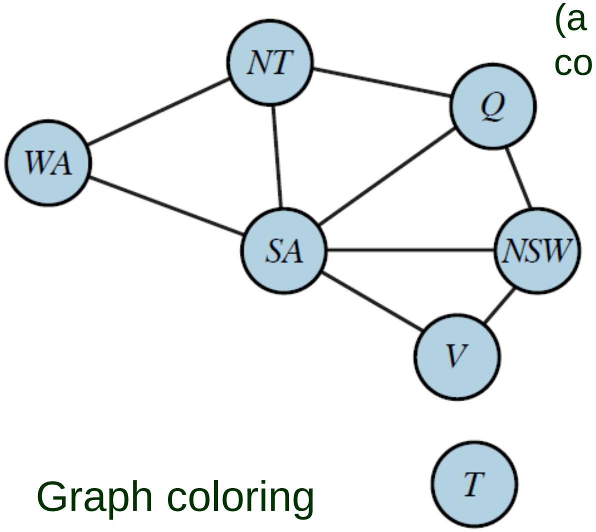

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Constraint satisfaction is NP-hard.

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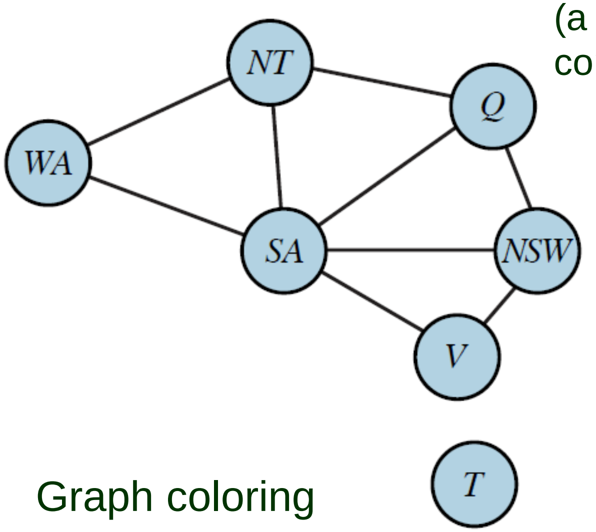

Graph coloring

color of WA (a Skolem constant)

$Diff(wa, nt) \land Diff(wa, sa) \land$
$Diff(nt, q) \land Diff(nt, sa) \land$
$Diff(q, nsw) \land Diff(q, sa) \land$
$Diff(nsw, v) \land Diff(nsw, sa) \land$
$Diff(v, sa) \Rightarrow true$

$Diff(Red, Blue)$	$Diff(Red, Green)$
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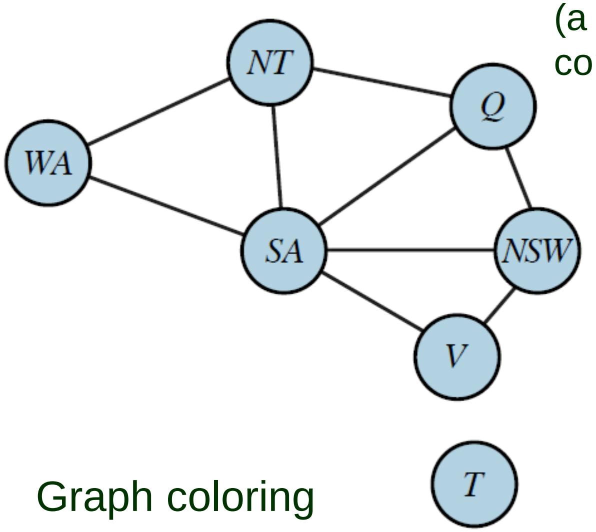

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Diff(*v*, *sa*) ⇒ *true*

Diff(*Red*, *Blue*)	*Diff*(*Red*, *Green*)
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Diff(*Blue*, *Red*)	*Diff*(*Blue*, *Green*)

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Good news View every Datalog clause as a CSP and apply heuristics for CSPs (e.g., tree structure, cutset conditioning, etc.).

Improvement 2: Incremental FC

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II. Backward Chaining

Works like AND/OR search:

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  - ♣ The goal query can be proved by any rule in the *KB*.
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How does it work?

- ♦ Fetch all clauses that unify with the goal.
- ♦ Rename the variables in every such clause to be brand-new.
- ♦ Prove every conjunct in the clause by keeping track of the expanded substitution as it goes.

Depth-First Proof Tree

$American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$

$Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$

$Missile(x) \Rightarrow Weapon(x)$

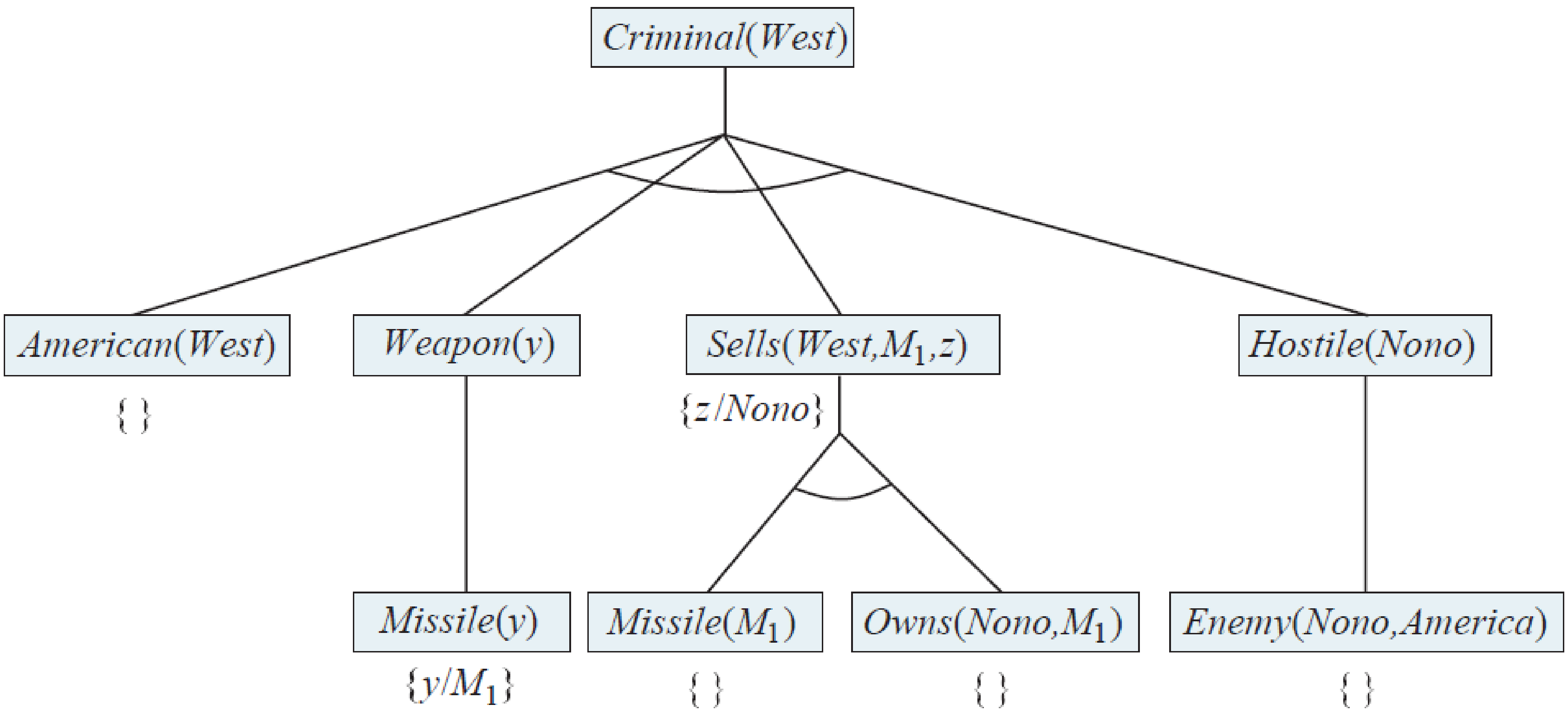
$Enemy(x,America) \Rightarrow Hostile(x)$

$Owns(Nono,M_1)$

$Missile(M_1)$

$American(West)$

$Enemy(Nono,America)$



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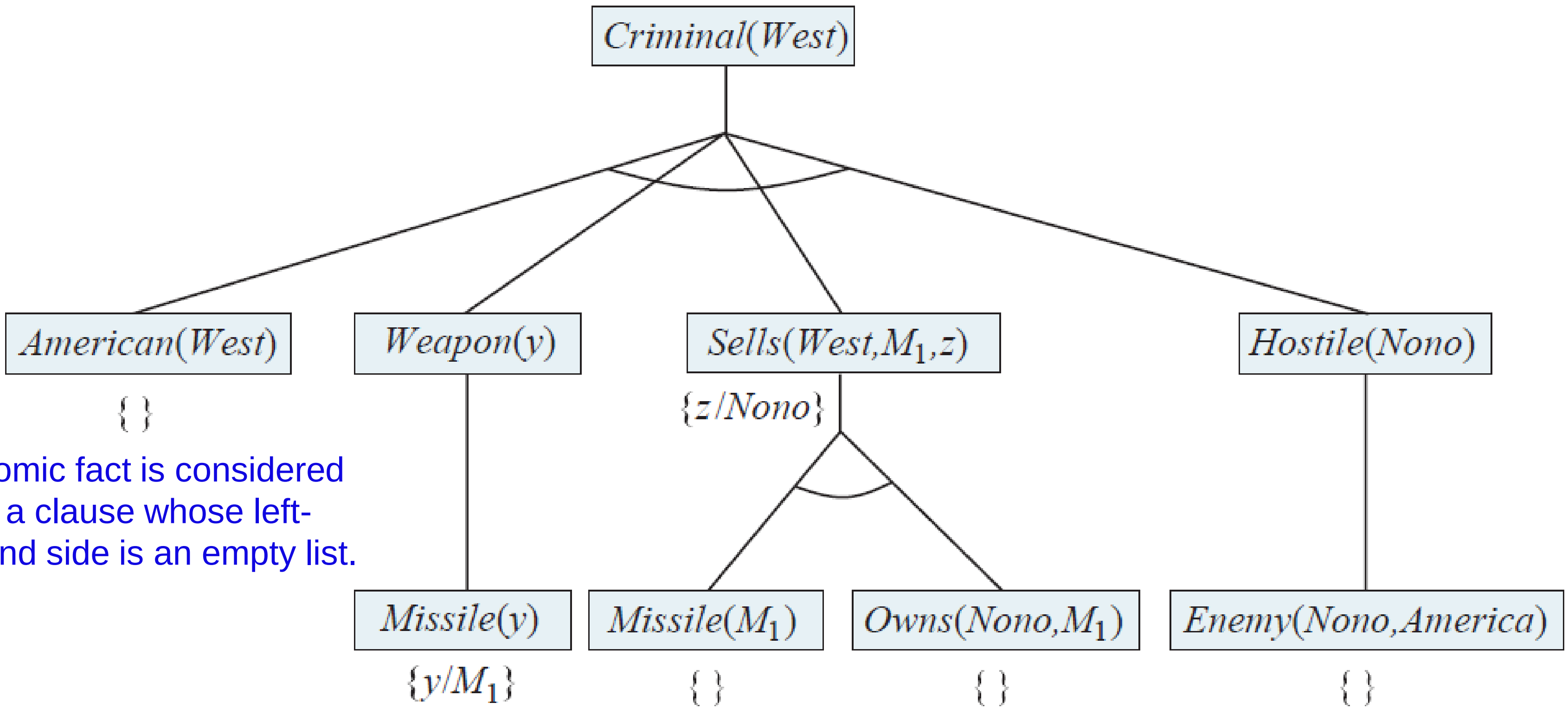

Atomic fact is considered as a clause whose left-hand side is an empty list.

III. Logic Programming

Algorithm = Logic + Control (Robert Kowalski)

Prolog (1972) is the most widely used logic programming language.

- Rapid prototyping
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Backward Chaining in Prolog

- Prolog recursively defines a function.

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Example List appending.

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append( [A|X], Y, [A|Z]) :- append(X,Y,Z)
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Describes the relations among the three arguments of append.

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Query: `append(X, Y, [1, 2, 3])`

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Infinite Loop

Finds if a path exists between two nodes in a directed graph.

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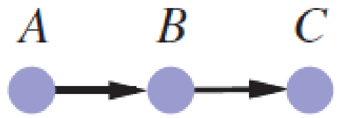

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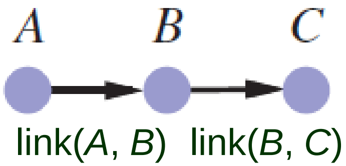

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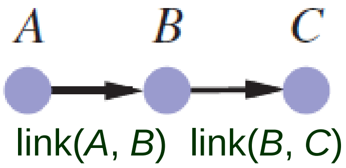

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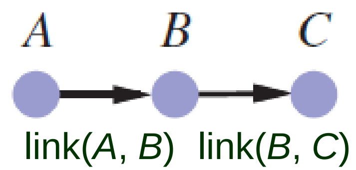

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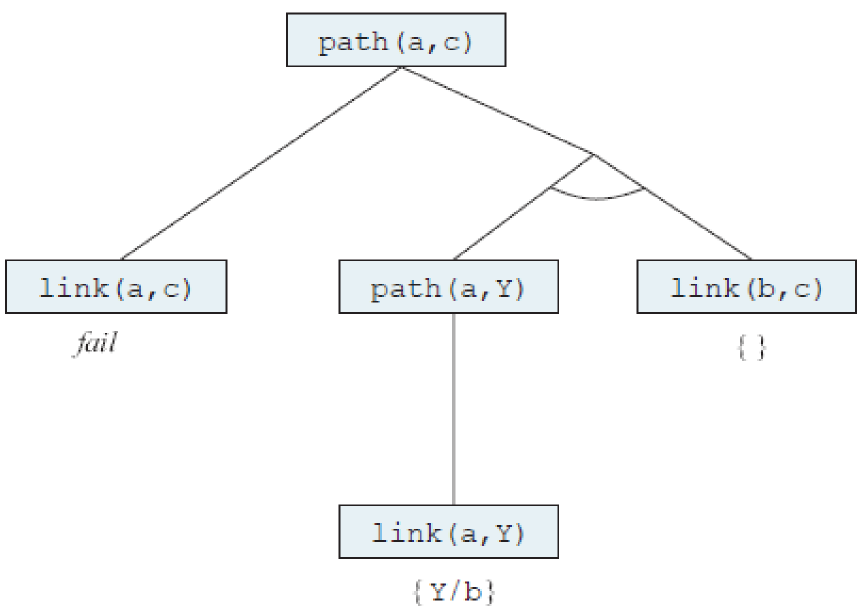

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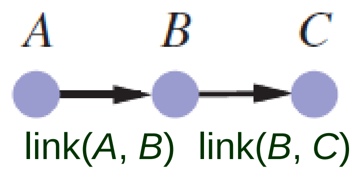

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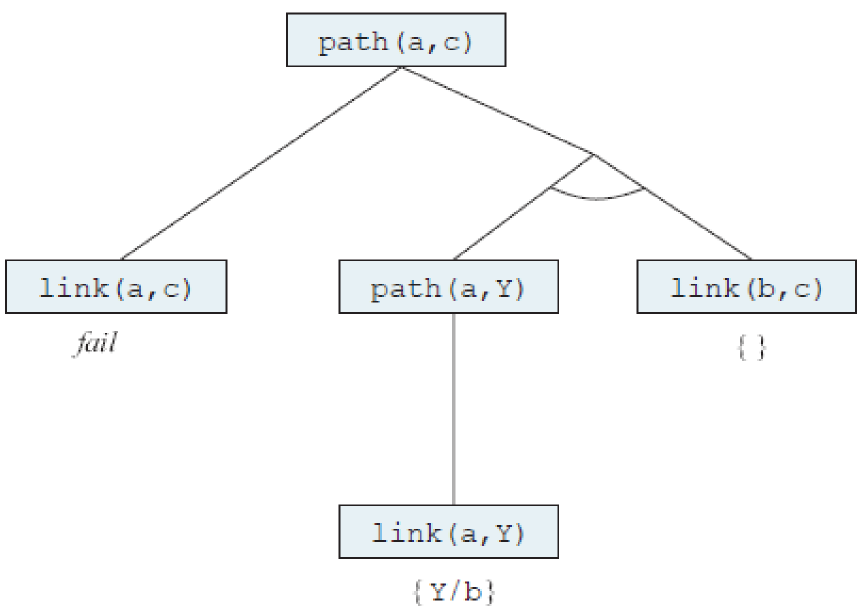

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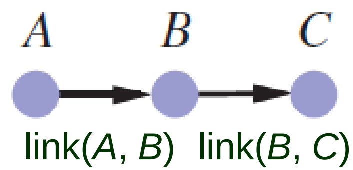

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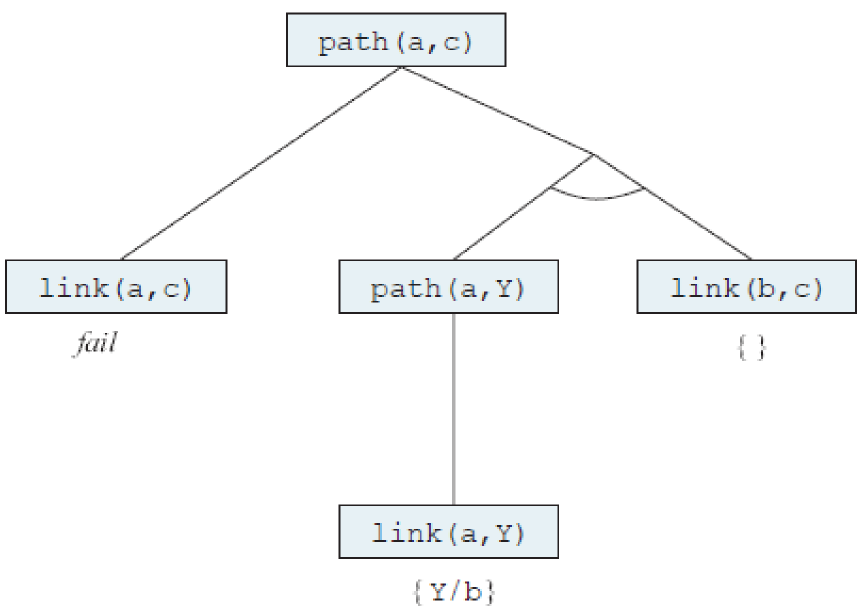

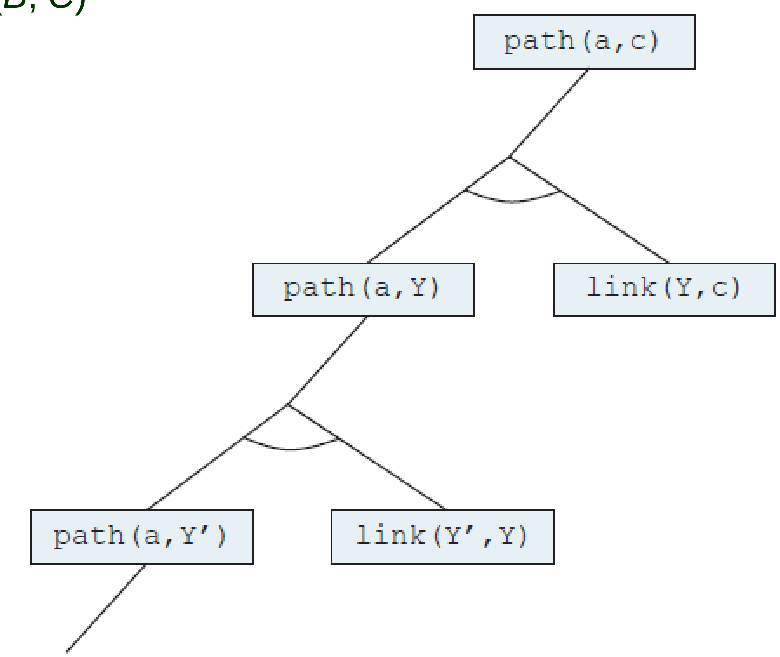

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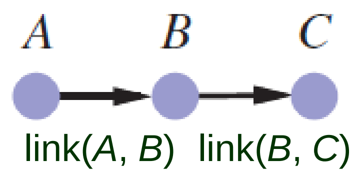

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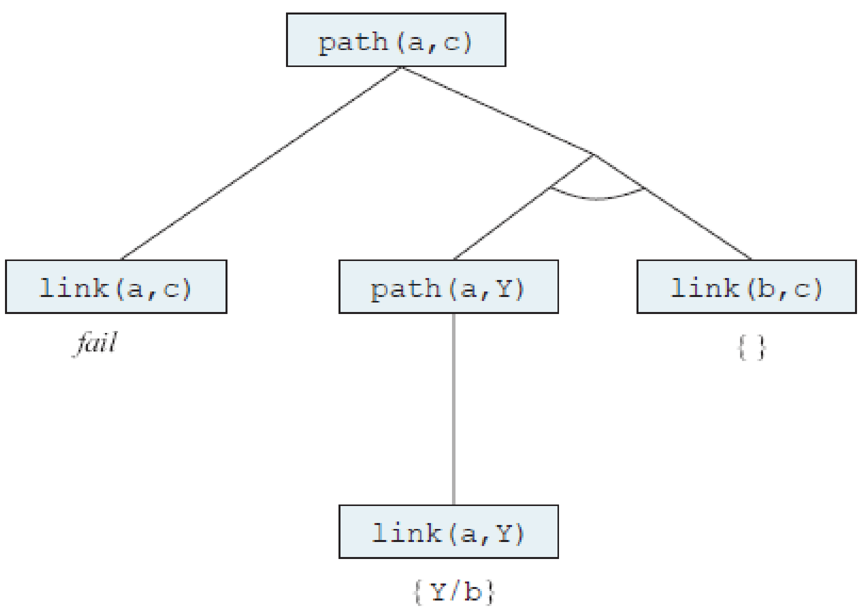
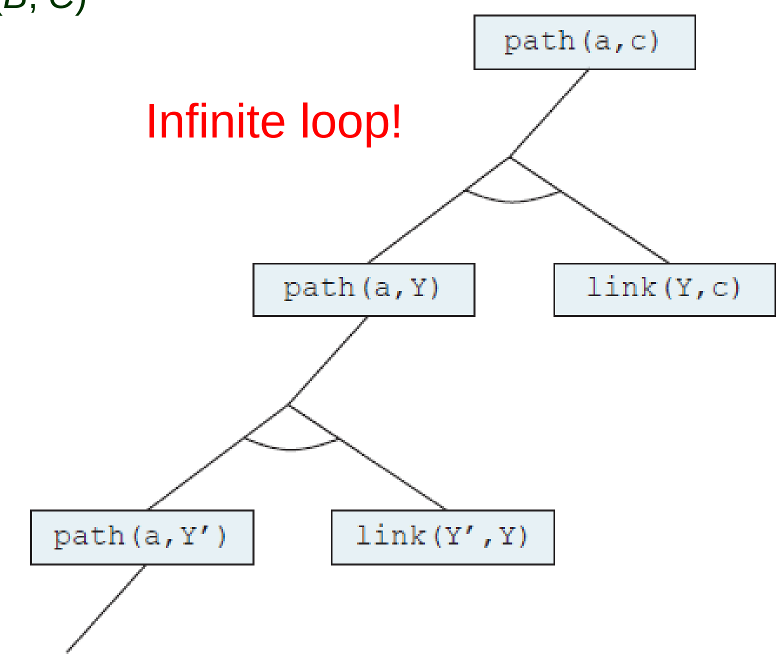


Infinite loop!



Redundant Inference

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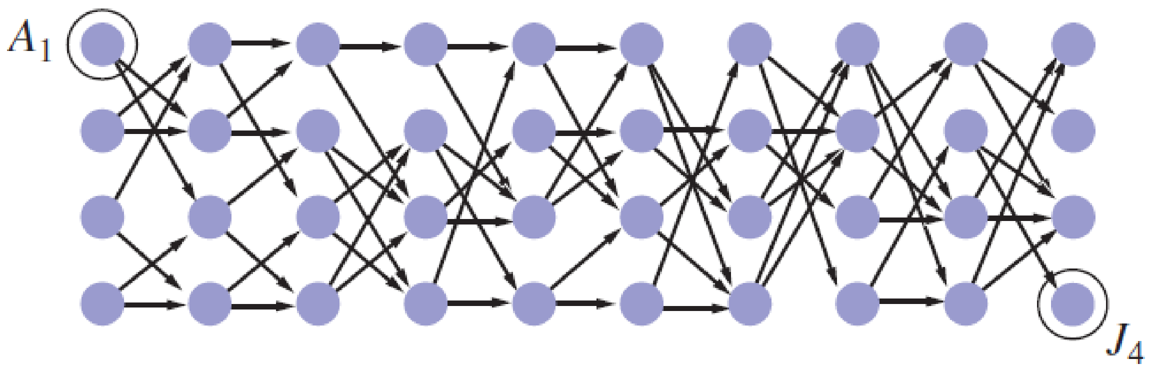

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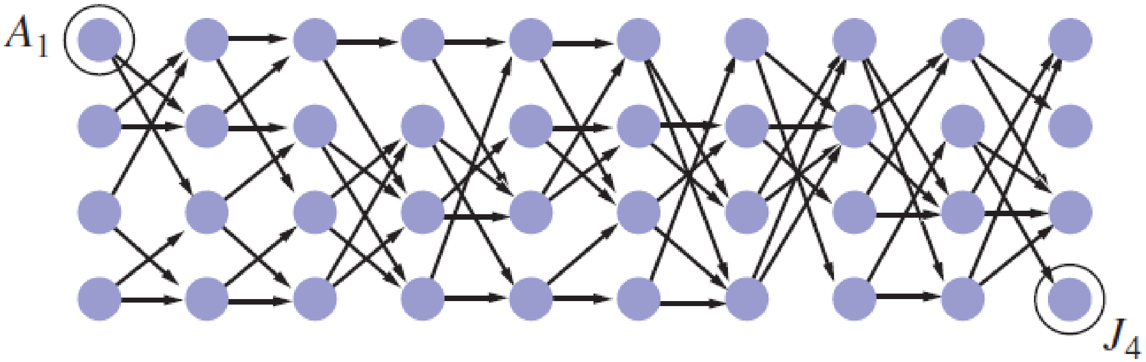

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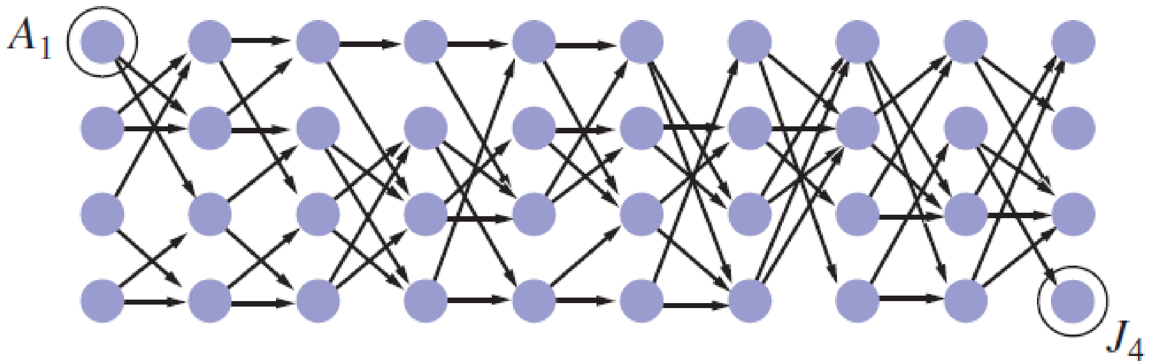

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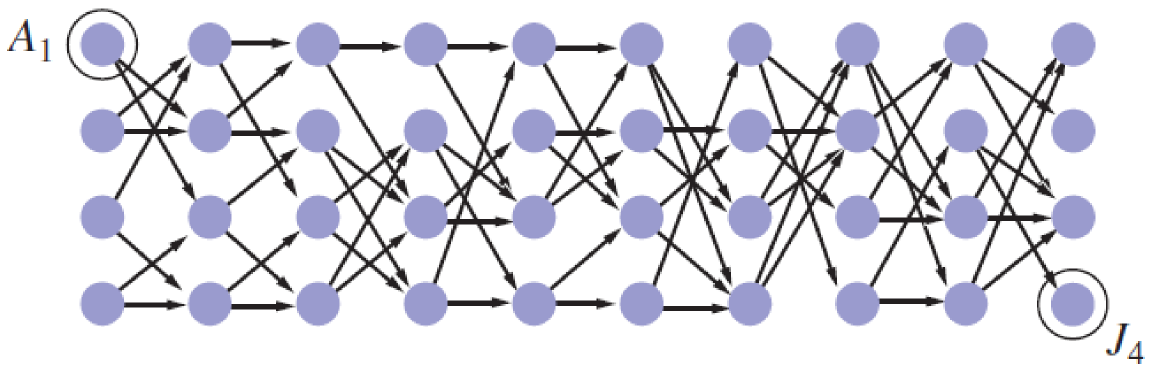

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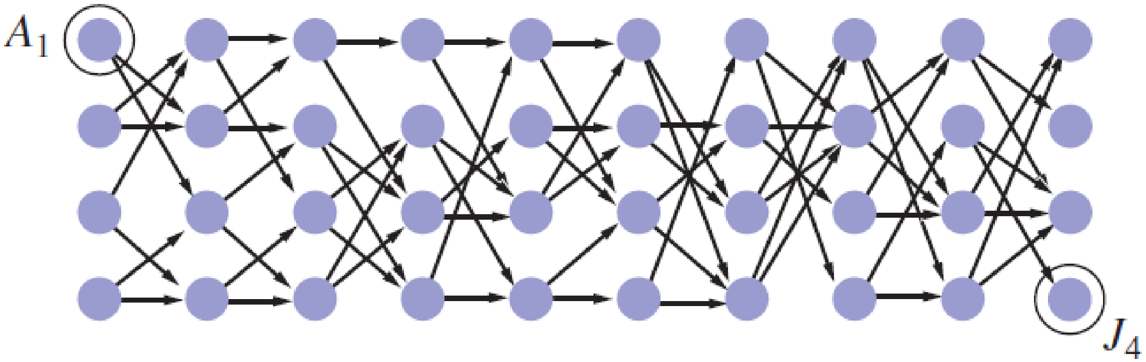

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