Heuristics for Planning

Outline

I. Planning as boolean satisfiability

II. Heuristics for planning

III. Planning in nondeterministic domains

* Figures are from the textbook site.

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  - Define the initial state S_0: assert F^0 for every fluent F mentioned in S_0 and $\neg F^0$ not mentioned in S_0.

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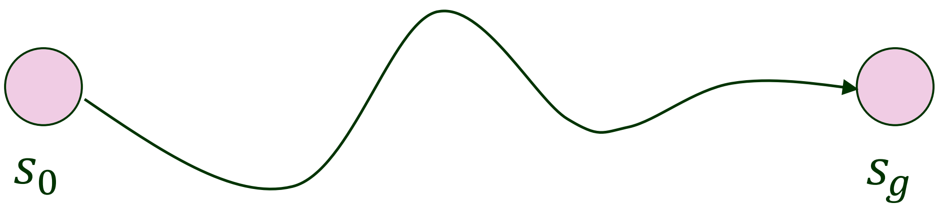
♣ Solve the resulting propositional logic satisfiability problem.

II. Heuristics for Planning

Search problem as conducted in a graph:

nodes $\Longleftrightarrow$ states

edges $\Longleftrightarrow$ actions



- Derive an admissible heuristic by relaxing the problem (which is easier) and using its solution.
  - Add more edges.
  - Group multiple nodes into one.

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8-puzzle:

Action(*Slide*(t, s_1, s_2))

PRECOND: *On*$(t, s_1) \wedge$ *Tile*$(t) \wedge$ *Blank*$(s_2) \wedge$ *Adjacent*(s_1, s_2)

EFFECT: *On*$(t, s_2) \wedge$ *Blank*$(s_1) \wedge \neg$*On*$(t, s_1) \wedge \neg$*Blank*(s_2)

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Action(*Slide*(t, s_1, s_2))

number of misplaced tiles

PRECOND: *On*$(t, s_1) \wedge$ *Tile*$(t) \wedge \cancel{Blank(s_2) \wedge Adjacent(s_1, s_2)}$

EFFECT: *On*$(t, s_2) \wedge$ *Blank*$(s_1) \wedge \neg$*On*$(t, s_1) \wedge \neg$*Blank*(s_2)

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$Action(Slide(t, s_1, s_2))$

Manhattan distance

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EFFECT: $On(t, s_2) \wedge Blank(s_1) \wedge \neg On(t, s_1) \wedge \neg Blank(s_2)$

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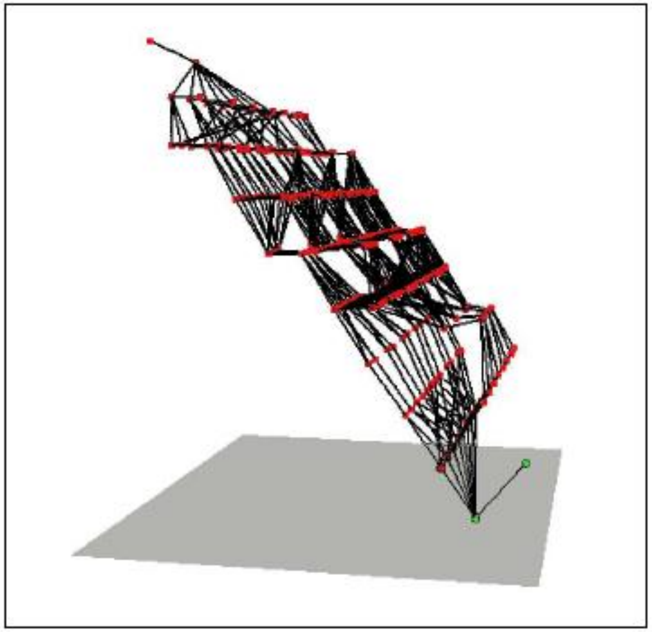

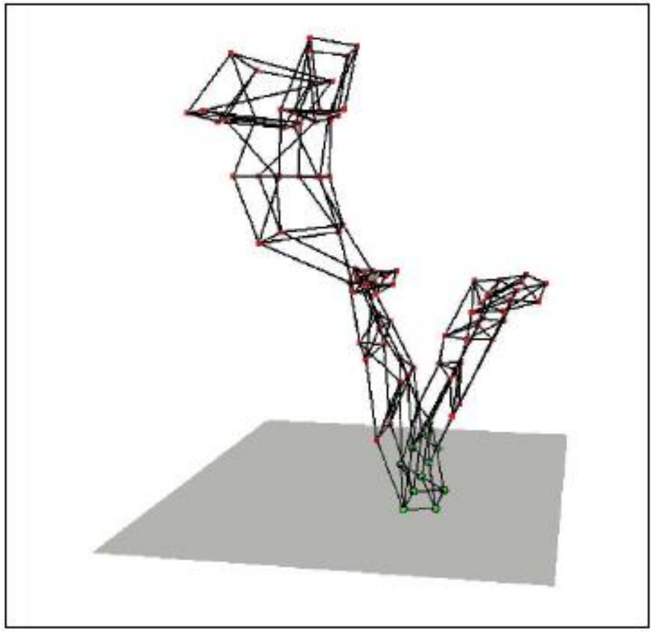

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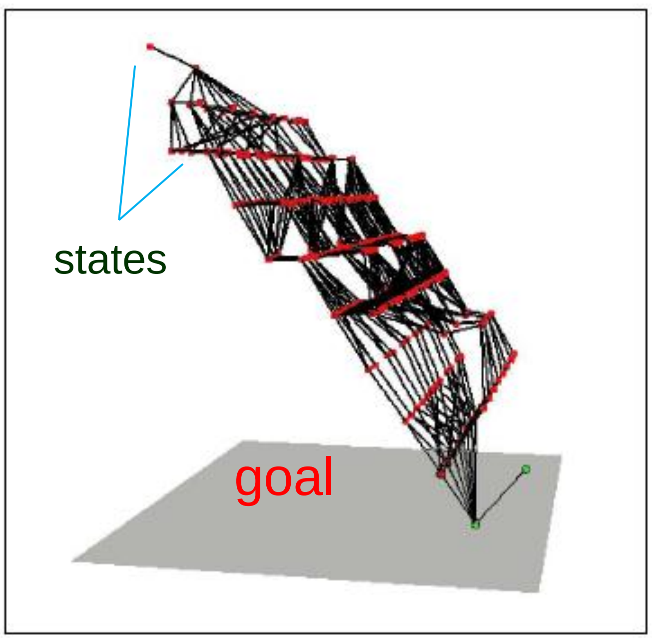

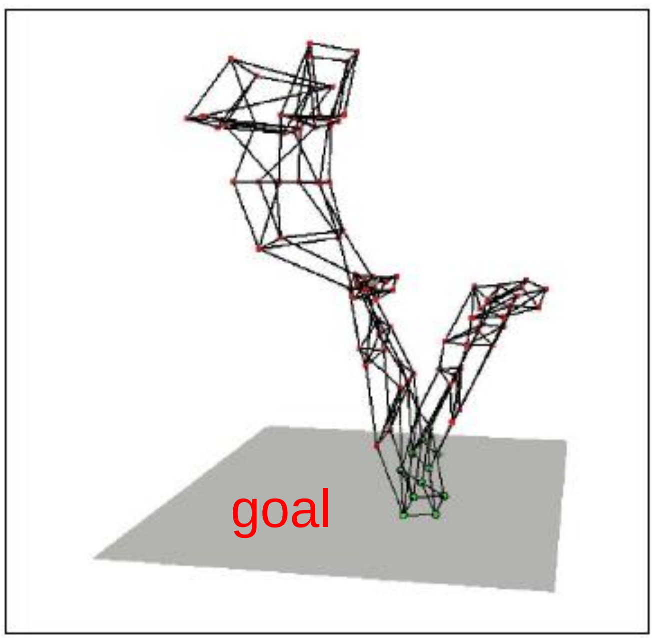

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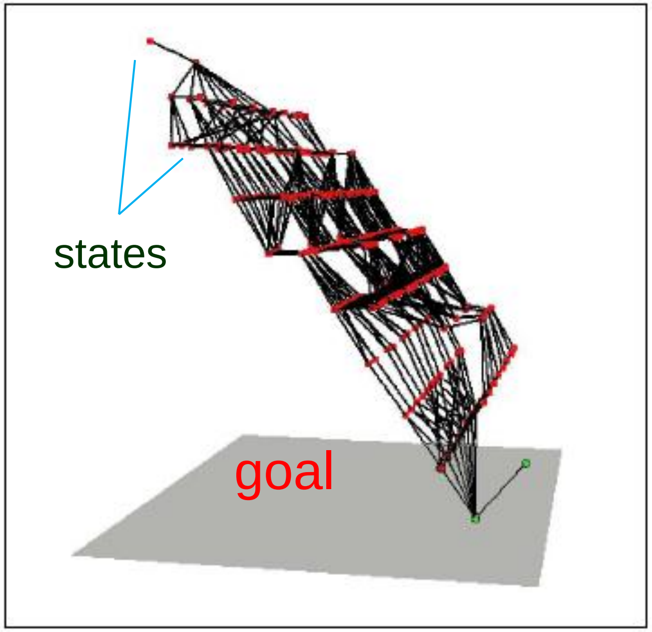

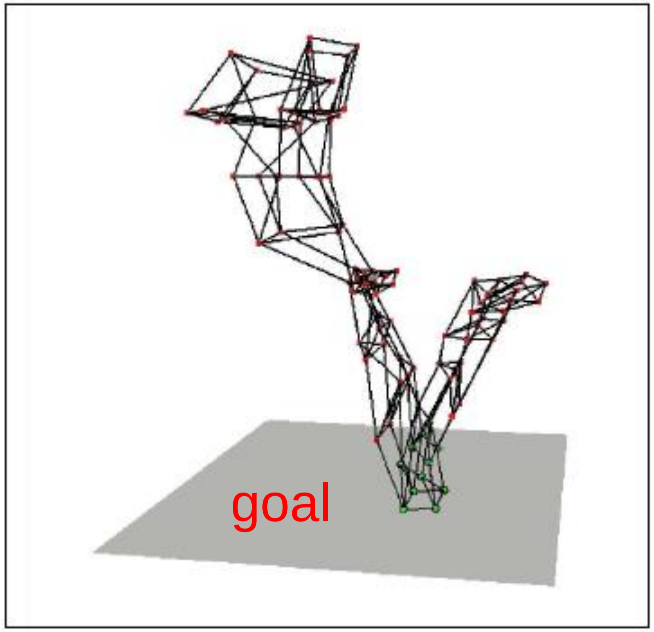

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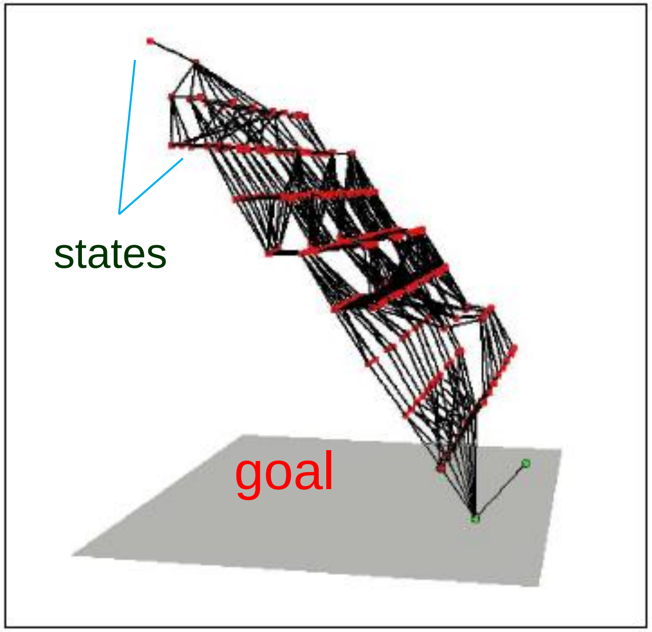

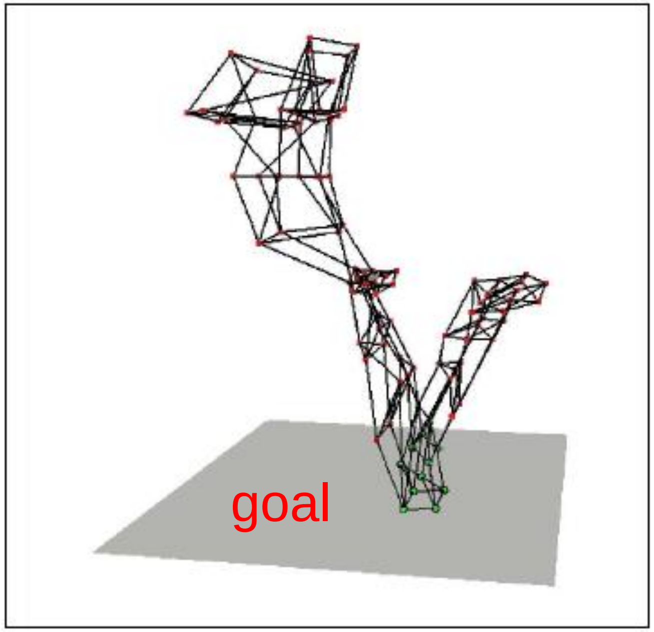

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Hill-climbing search works!

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Admissible but too low.

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Not admissible.

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♣ $\max_i \text{COST}(P_i)$? Admissible but too low.

♣ $\sum_i \text{COST}(P_i)$? Not admissible.

♣ $\text{COST}(P_i) + \text{COST}(P_j)$? when P_i and P_j are independent.

III. Nondeterministic Domains

Task Make a chair and a table have the same color.

$Init(Object(Table) \land Object(Chair) \land Can(C_1) \land Can(C_2) \land InView(Table))$

$Goal(Color(Chair, c) \land Color(Table, c))$

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Task Make a chair and a table have the same color.

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$\mathit{Action}(\mathit{Paint}(x, \mathit{can})$, // paint x using the paint from an open can
  PRECOND: $\mathit{Object}(x) \wedge \mathit{Can}(\mathit{can}) \wedge \mathit{Color}(\mathit{can}, c) \wedge \mathit{Open}(\mathit{can})$
  EFFECT: $\mathit{Color}(x, c)$

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Action($Paint(x, can)$), // paint x using the paint from an open can
  PRECOND: $Object(x) \wedge Can(can) \wedge Color(can, c) \wedge Open(can)$
  EFFECT: $Color(x, c)$

No color argument c in $Paint(x, can)$.
// it would be $Paint(x, can, c)$ in the fully observable case.

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- If the colors are not the same, open the two paint cans.

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 EFFECT: *InView*(x) $\wedge$ $\neg$*InView*(y))

- Look at the table and chair to get their colors.
- If the colors are not the same, open the two paint cans.
  - If the paint in a can has the same color as one furniture piece, apply it to the other piece.

Percept Schema

The agent needs to be able to reason about its percepts as it executes the plan.

Percept($Color(x, c)$),
  PRECOND: *Object*(x) ∧ *InView*(x)

// the agent will perceive the color of
// an object in view.

Percept(*Color*(*can*, c)),
  PRECOND: *Can*(*can*) ∧ *Inview*(*can*) ∧ *Open*(*can*)

// the agent will perceive the color
// of an open can in view.

Action(*LookAt*(x),
  PRECOND: *InView*(y) ∧ $x \neq y$
  EFFECT: *InView*(x) ∧ ¬*InView*(y))

- Look at the table and chair to get their colors.
- If the colors are not the same, open the two paint cans.
  - If the paint in a can has the same color as one furniture piece, apply it to the other piece.
  - Otherwise, paint both pieces with the same color.

Sensorless Planning

The belief state can now be represented as a logical formula instead of a set of enumerated states.

// these facts hold in every belief state.

$Object(Table) \land Object(Chair) \land Can(C_1) \land Can(C_2)$

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$\mathit{Object}(\mathit{Table}) \wedge \mathit{Object}(\mathit{Chair}) \wedge \mathit{Can}(C_1) \wedge \mathit{Can}(C_2)$

// objects and cans have colors.

$\forall x\ \exists c\ \ \mathit{Color}(x, c)$

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$\Downarrow$

$b_0 = Color(x, C(x))$ Initial belief state

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$[RemoveLid(Can_1), Paint(Chair, Can_1), Paint(Table, Can_1)]$ Solution plan

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$[\mathit{RemoveLid}(\mathit{Can}_1), \mathit{Paint}(\mathit{Chair}, \mathit{Can}_1), \mathit{Paint}(\mathit{Table}, \mathit{Can}_1)]$ Solution plan

Open-world assumption:

If a fluent does not appear, its value is unknown (rather than false).

Updating the Belief State

In a belief state b, we consider any action a whose preconditions are satisfied.

$$b' = \text{RESULT}(b, a) = \{s' \mid s' = \text{RESULT}_P\,(s, a) \text{ and } s \in b\}$$

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physical transition model

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Consider $b = l_1 \wedge \cdots \wedge l_k$ (1-CNF), where $l_1, \ldots, l_k$ are literals.

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Consider $b = l_1 \wedge \cdots \wedge l_k$ (1-CNF), where $l_1, \ldots, l_k$ are literals.

- l_i has known truth value v in b (i.e., in all physical states $s \in b$).

  Compute the truth value in b' from v and the action's add and delete lists.

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  - If a adds l_i , then l_i will be true in b'.

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  - If a adds l_i , then l_i will be true in b'.
  - If a deletes l_i , then l_i will be false in b'.
  - If a does not affect l_i , then l_i will retain its unknown value and not appear in b'.

Updating the Belief State (cont'd)

Calculation of b' is almost the same as in the observable case.

$$b' = \text{RESULT}(b, a) = (b - \text{DEL}(a)) \cup \text{ADD}(a)$$

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$$b' = \text{RESULT}(b, a) = (b - \text{DEL}(a)) \cup \text{ADD}(a)$$

$$b_0 = \mathit{Color}(x, C(x))$$

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$$\mathit{RemoveLid}(\mathit{Can}_1)$$

Action(*RemoveLid*(*can*),
PRECOND: *Can*(*can*)
EFFECT: *Open*(*can*))

Updating the Belief State (cont'd)

Calculation of b' is almost the same as in the observable case.

$$b' = \text{RESULT}(b, a) = (b - \text{DEL}(a)) \cup \text{ADD}(a)$$

$$b_0 = \mathit{Color}(x, C(x))$$

$$\Downarrow \mathit{RemoveLid}(\mathit{Can}_1)$$

$$b_1 = \mathit{Color}(x, C(x)) \land \mathit{Open}(\mathit{Can}_1)$$

Action(*RemoveLid*(*can*),
PRECOND: *Can*(*can*)
EFFECT: *Open*(*can*))

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$$b_1 = \mathit{Color}(x, C(x)) \land \mathit{Open}(\mathit{Can}_1)$$

$\mathit{Paint}(\mathit{Chair}, \mathit{Can}_1)$

$\{x/\ \mathit{Chair}, \text{can}/\ \mathit{Can}_1\}$

Action(*RemoveLid*(*can*),
    PRECOND: *Can*(*can*)
    EFFECT: *Open*(*can*))

Action(*Paint*(x, *can*),
    PRECOND: *Object*(x) ∧ *Can*(*can*) ∧
        Color(*can*, c) ∧ *Open*(*can*)
    EFFECT: *Color*(x, c)

Updating the Belief State (cont'd)

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$$b' = \text{RESULT}(b, a) = (b - \text{DEL}(a)) \cup \text{ADD}(a)$$

$$b_0 = \mathit{Color}(x, C(x))$$

$$\Downarrow \; \mathit{RemoveLid}(\mathit{Can}_1)$$

$$b_1 = \mathit{Color}(x, C(x)) \wedge \mathit{Open}(\mathit{Can}_1)$$

$$\Downarrow \; \mathit{Paint}(\mathit{Chair}, \mathit{Can}_1) \; \{x/\,\mathit{Chair}, \text{can}/\,\mathit{Can}_1\}$$

$$b_2 = \mathit{Color}(x, C(x)) \wedge \mathit{Open}(\mathit{Can}_1) \wedge \mathit{Color}(\mathit{Chair}, C(\mathit{Can}_1))$$

Action(*RemoveLid*(*can*),
PRECOND: *Can*(*can*)
EFFECT: *Open*(*can*))

Action(*Paint*(x, *can*),
PRECOND: *Object*(x) ∧ *Can*(*can*) ∧ *Color*(*can*, c) ∧ *Open*(*can*)
EFFECT: *Color*(x, c)

Updating the Belief State (cont'd)

Calculation of b' is almost the same as in the observable case.

$$b' = \textsc{Result}(b, a) = (b - \textsc{Del}(a)) \cup \textsc{Add}(a)$$

$$b_0 = \mathit{Color}(x, C(x))$$

$$\Downarrow \; \mathit{RemoveLid}(\mathit{Can}_1)$$

$$b_1 = \mathit{Color}(x, C(x)) \wedge \mathit{Open}(\mathit{Can}_1)$$

$$\Downarrow \; \mathit{Paint}(\mathit{Chair}, \mathit{Can}_1) \quad \{x/\,\mathit{Chair},\ \text{can}/\,\mathit{Can}_1\}$$

$$b_2 = \mathit{Color}(x, C(x)) \wedge \mathit{Open}(\mathit{Can}_1) \wedge \mathit{Color}(\mathit{Chair}, C(\mathit{Can}_1))$$

$$\Downarrow \; \mathit{Paint}(\mathit{Table}, \mathit{Can}_1)$$

$$b_3 = \mathit{Color}(x, C(x)) \wedge \mathit{Open}(\mathit{Can}_1) \wedge \mathit{Color}(\mathit{Chair}, C(\mathit{Can}_1)) \wedge \mathit{Color}(\mathit{Table}, C(\mathit{Can}_1))$$

Action(*RemoveLid*(*can*),
PRECOND: *Can*(*can*)
EFFECT: *Open*(*can*))

Action(*Paint*(x, *can*),
PRECOND: *Object*(x) $\wedge$ *Can*(*can*) $\wedge$ *Color*(*can*, c) $\wedge$ *Open*(*can*)
EFFECT: *Color*(x, c))

Updating the Belief State (cont'd)

Calculation of b' is almost the same as in the observable case.

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$$b_0 = \mathit{Color}(x, C(x))$$

$$\Downarrow \ \mathit{RemoveLid}(\mathit{Can}_1)$$

Action(*RemoveLid*(*can*),
PRECOND: *Can*(*can*)
EFFECT: *Open*(*can*))

$$b_1 = \mathit{Color}(x, C(x)) \land \mathit{Open}(\mathit{Can}_1)$$

$$\Downarrow \ \mathit{Paint}(\mathit{Chair}, \mathit{Can}_1) \ \{x/\,\mathit{Chair},\ \text{can}/\,\mathit{Can}_1\}$$

Action(*Paint*(x, *can*),
PRECOND: *Object*(x) ∧ *Can*(*can*) ∧
Color(*can*, c) ∧ *Open*(*can*)
EFFECT: *Color*(x, c)

$$b_2 = \mathit{Color}(x, C(x)) \land \mathit{Open}(\mathit{Can}_1) \land \mathit{Color}(\mathit{Chair}, C(\mathit{Can}_1))$$

$$\Downarrow \ \mathit{Paint}(\mathit{Table}, \mathit{Can}_1)$$

$$b_3 = \mathit{Color}(x, C(x)) \land \mathit{Open}(\mathit{Can}_1) \land \mathit{Color}(\mathit{Chair}, C(\mathit{Can}_1)) \land \mathit{Color}(\mathit{Table}, C(\mathit{Can}_1))$$

$$\mathit{Goal}(\mathit{Color}(\mathit{Chair}, c) \land \mathit{Color}(\mathit{Table}, c))$$

Updating the Belief State (cont'd)

Calculation of b' is almost the same as in the observable case.

$$b' = \textsc{Result}(b, a) = (b - \textsc{Del}(a)) \cup \textsc{Add}(a)$$

$b_0 = \mathit{Color}(x, C(x))$

$\Downarrow$ $\mathit{RemoveLid}(\mathit{Can}_1)$

$b_1 = \mathit{Color}(x, C(x)) \wedge \mathit{Open}(\mathit{Can}_1)$

$\Downarrow$ $\mathit{Paint}(\mathit{Chair}, \mathit{Can}_1)$ $\{x/\ \mathit{Chair}, \text{can}/\ \mathit{Can}_1\}$

$b_2 = \mathit{Color}(x, C(x)) \wedge \mathit{Open}(\mathit{Can}_1) \wedge \mathit{Color}(\mathit{Chair}, C(\mathit{Can}_1))$

$\Downarrow$ $\mathit{Paint}(\mathit{Table}, \mathit{Can}_1)$

$b_3 = \mathit{Color}(x, C(x)) \wedge \mathit{Open}(\mathit{Can}_1) \wedge \mathit{Color}(\mathit{Chair}, C(\mathit{Can}_1)) \wedge \mathit{Color}(\mathit{Table}, C(\mathit{Can}_1))$

$\mathit{Action}(\mathit{RemoveLid}(\mathit{can}),$
  $\textsc{Precond}$: $\mathit{Can}(\mathit{can})$
  $\textsc{Effect}$: $\mathit{Open}(\mathit{can}))$

$\mathit{Action}(\mathit{Paint}(x, \mathit{can}),$
  $\textsc{Precond}$: $\mathit{Object}(x) \wedge \mathit{Can}(\mathit{can}) \wedge$
    $\mathit{Color}(\mathit{can}, c) \wedge \mathit{Open}(\mathit{can})$
  $\textsc{Effect}$: $\mathit{Color}(x, c)$

$\mathit{Goal}(\mathit{Color}(\mathit{Chair}, c) \wedge \mathit{Color}(\mathit{Table}, c))$

satisfied under substitution $\{\ c\ /\ C(\mathit{Can}_1)\}$!

Contingent Planning

Generate a plan with conditional branching based on percepts.

A conditional solution:

[*LookAt*(*Table*), *LookAt*(*Chair*),
 if *Color*(*Table*, c) $\wedge$ *Color*(*Chair*, c) then *NoOp*
  else [*RemoveLid* (Can_1), *LookAt* (Can_1), *RemoveLid*(Can_2), *LookAt* (Can_2),
   if *Color*(*Table*, c) $\wedge$ *Color*(*can*, c) then *Paint*(*Chair*, *can*)
   else if *Color*(*Chair*, c) $\wedge$ *Color*(*can*, c) then *Paint*(*Table*, *can*)
   else [*Paint*(*Chair*, Can_1), *Paint*(*Table*, Can_1)]]]

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   if $Color(Table, c) \wedge Color(can, c)$ then $Paint(Chair, can)$
   else if $Color(Chair, c) \wedge Color(can, c)$ then $Paint(Table, can)$
   else [$Paint(Chair, Can_1)$, $Paint(Table, Can_1)$]]]

- ♣ Variables in the solution are existentially quantified.

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 else if *Color*(*Chair*, c) $\wedge$ *Color*(*can*, c) then *Paint*(*Table*, *can*)
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Generate a plan with conditional branching based on percepts.

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else [*RemoveLid* (*Can*$_1$), *LookAt* (*Can*$_1$), *RemoveLid*(*Can*$_2$), *LookAt* (*Can*$_2$),
if *Color*(*Table*, c) $\wedge$ *Color*(*can*, c) then *Paint*(*Chair*, *can*)
else if *Color*(*Chair*, c) $\wedge$ *Color*(*can*, c) then *Paint*(*Table*, *can*)
else [*Paint*(*Chair*, *Can*$_1$), *Paint*(*Table*, *Can*$_1$)]]]

- ♣ Variables in the solution are existentially quantified.
- ♣ A conditional formula can be satisfied in multiple ways depending on variable substitutions.
- ♣ The planning algorithm has to avoid a belief state in which the condition's truth value is unknown.

How to Update the Belief State?

Contingent planning carries out the following two steps for each action:

- Calculate the belief state ($b = l_1 \wedge \cdots \wedge l_k$) after the action.

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  - If some p_i, $1 \le i \le k$, has one percept schema, then add all the literals in the preconditions of the schema.

Percept(*Color*(*can*, c),
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