Propositional Model Checking

Outline

I. Horn Clauses

II. Effective Propositional Model Checking

III. Agents Based on Propositional Logic

* Figures are from the textbook site unless a source is specifically cited.

I. Horn Clause

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$$S$$

$$\neg P \vee \neg Q \vee R \qquad \neg R \vee \neg S$$

$$\neg P \vee \neg Q \vee \neg S$$

(cont'd)

- Inferences with Horn clauses are through forward- and backward-chaining algorithms.

Logic programming

(natural inference steps easy for humans to follow)

- Low computational complexity: deciding entailment with Horn clauses takes $O(n)$ time.

size of the KB

Forward Chaining

Question $KB \vDash q?$

single proposition symbol

- Begins from positive literals (facts).
- If all the premises of an implications are known, then add its conclusion to KB (as a new fact).
- Continues until q is added or no further inferences can be made.

Forward Chaining

Question $KB \models q$?

single proposition symbol

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- If all the premises of an implications are known, then add its conclusion to KB (as a new fact).
- Continues until q is added or no further inferences can be made.

function PL-FC-ENTAILS?(KB, q) **returns** *true* or *false*
  inputs: KB, the knowledge base, a set of propositional definite clauses
    q, the query, a proposition symbol
  $count \leftarrow$ a table, where $count[c]$ is initially the number of symbols in clause c's premise
  $inferred \leftarrow$ a table, where $inferred[s]$ is initially $false$ for all symbols
  $queue \leftarrow$ a queue of symbols, initially symbols known to be true in KB

  while $queue$ is not empty **do**
    $p \leftarrow$ POP($queue$)
    if $p = q$ **then return** *true*
    if $inferred[p] = false$ **then**
      $inferred[p] \leftarrow true$
      for each clause c in KB where p is in c.PREMISE **do**
        decrement $count[c]$
        if $count[c] = 0$ **then** add c.CONCLUSION to $queue$
  return *false*

Example of Forward Chaining

KB:

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

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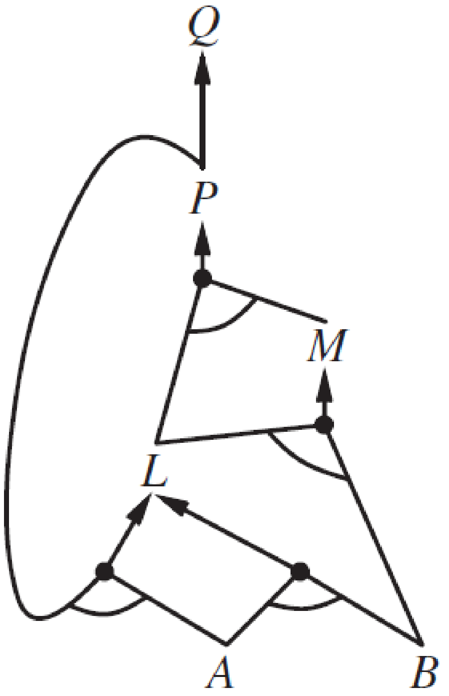
AND-OR graph representation



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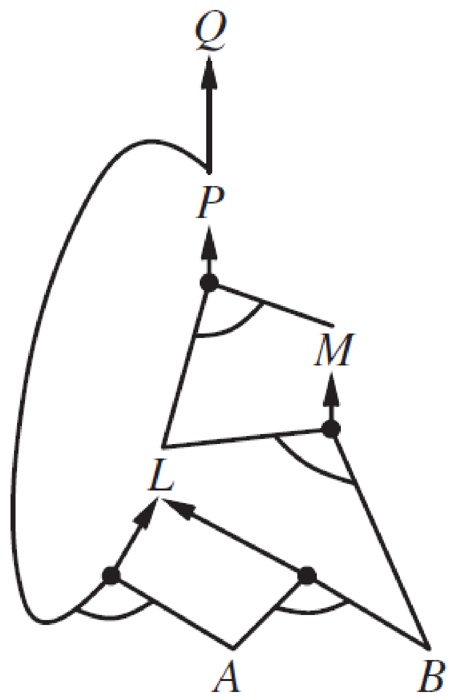

Q. *KB* $\models Q$?

Execution

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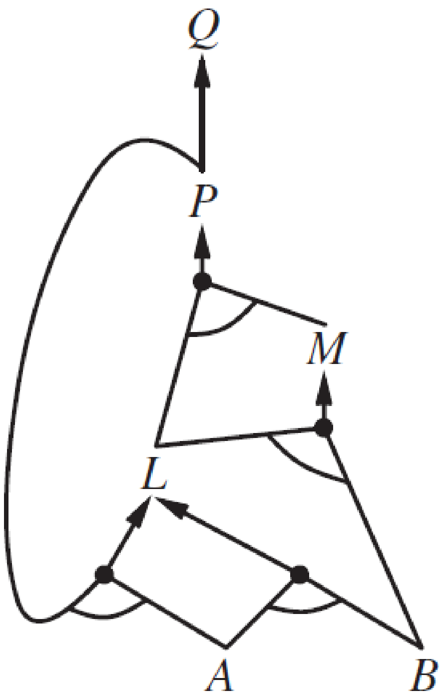


Execution

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

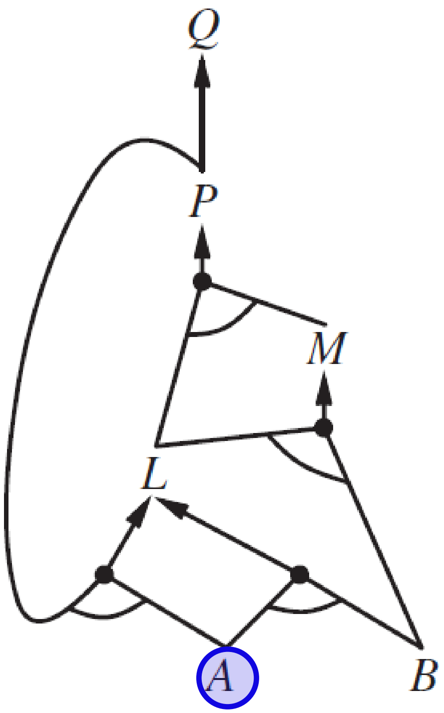
$B \wedge L \Rightarrow M$

$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

$\boxed{A}$

B



Execution

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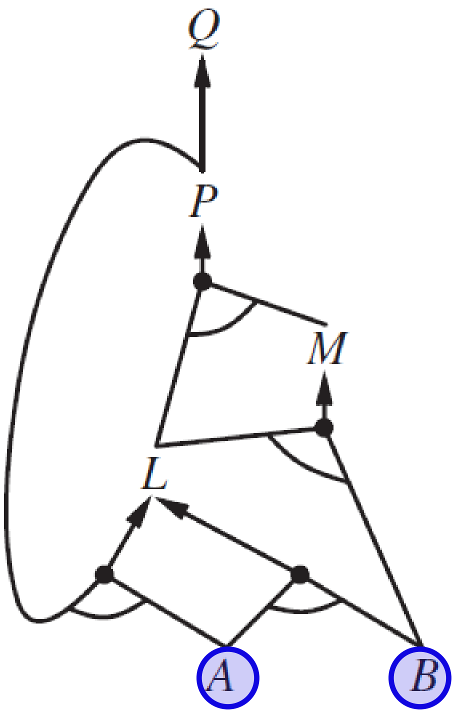

Execution

$P \Rightarrow Q$

$L \wedge M \Rightarrow P$

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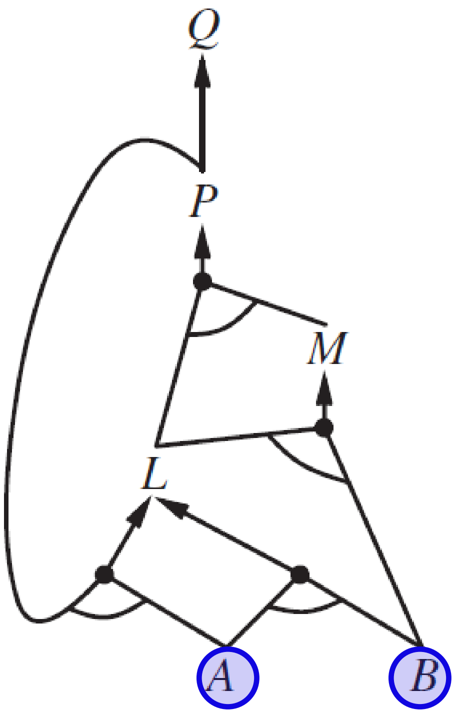
$A \wedge P \Rightarrow L$

$\boxed{A \wedge B \Rightarrow L}$

A

B



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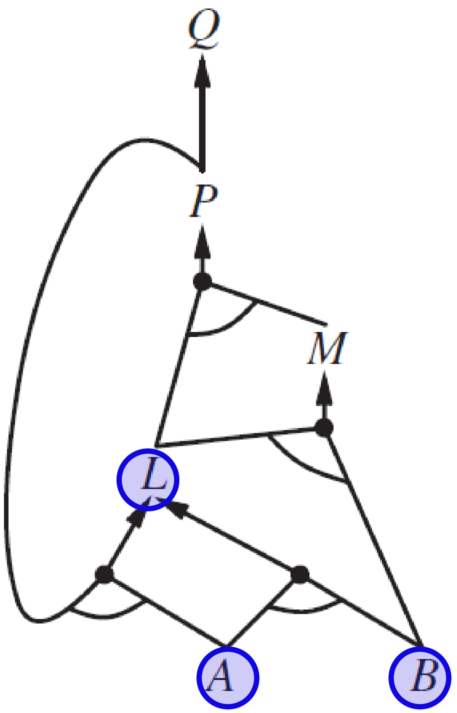
A

B



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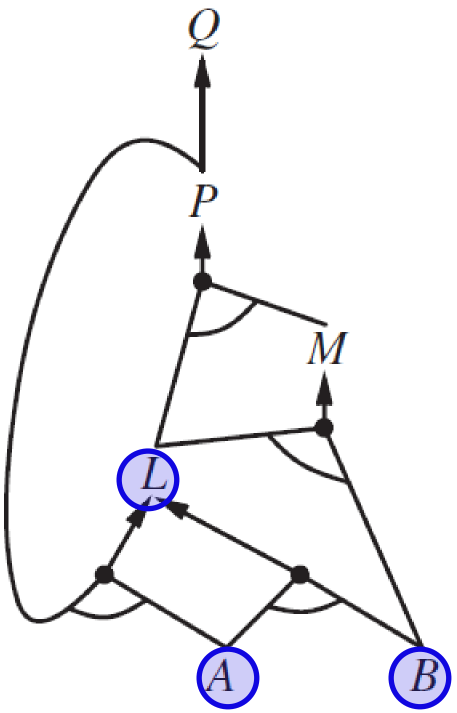

Execution

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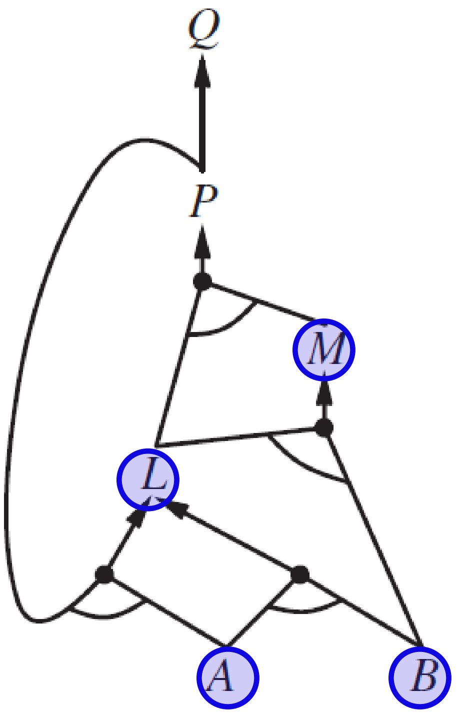
$A \wedge P \Rightarrow L$

$A \wedge B \Rightarrow L$

A

B



Execution

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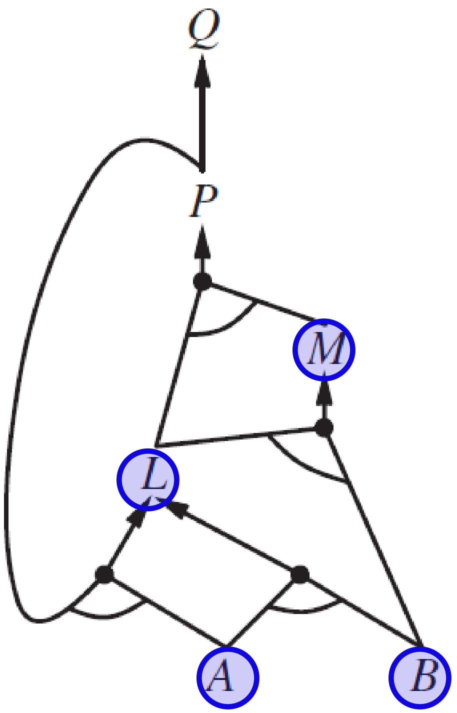

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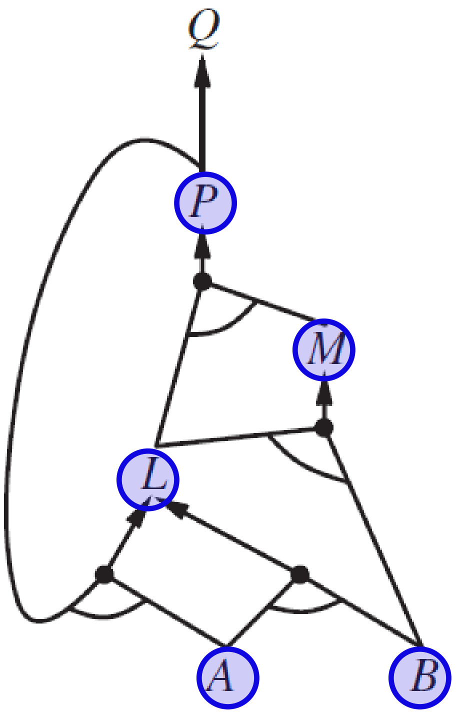

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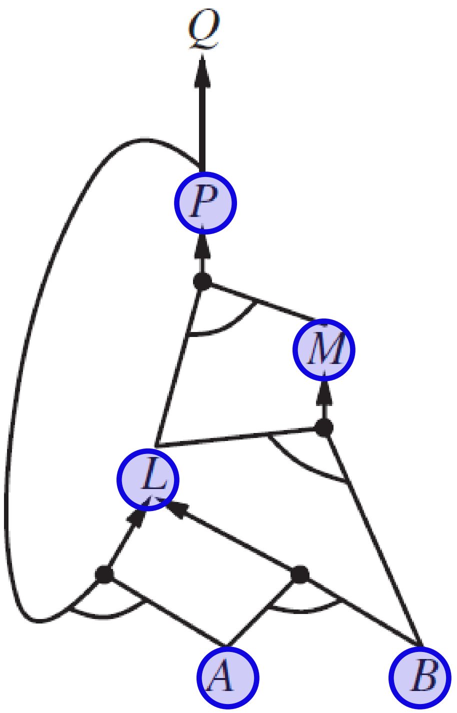

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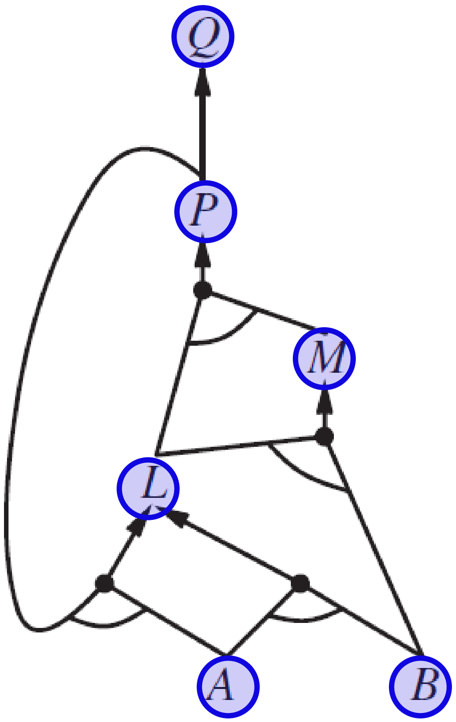

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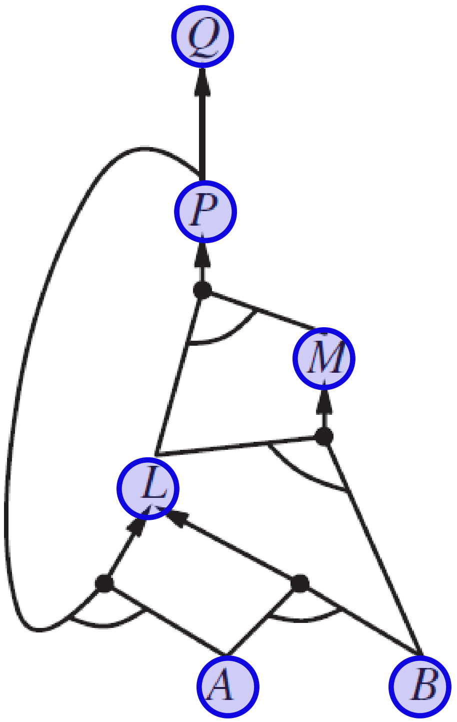

Soundness of forward chaining: every inference is an application of Modus Ponens.

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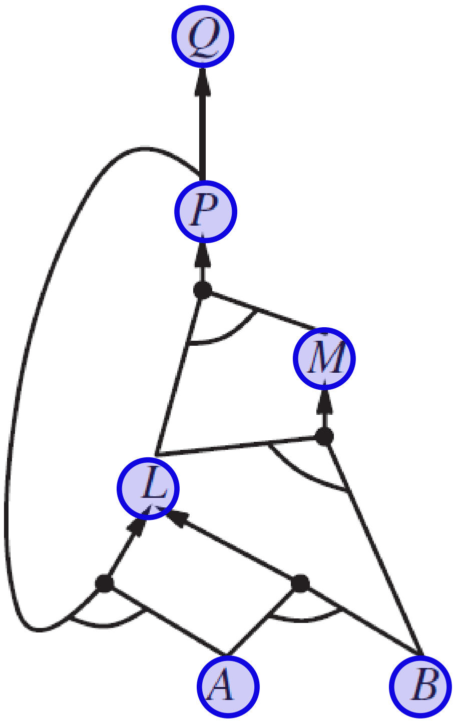

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Completeness: every entailed atomic sentences will be derived.

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- If q is true, no work is needed.
- Otherwise, finds implications in the KB whose conclusion is q.
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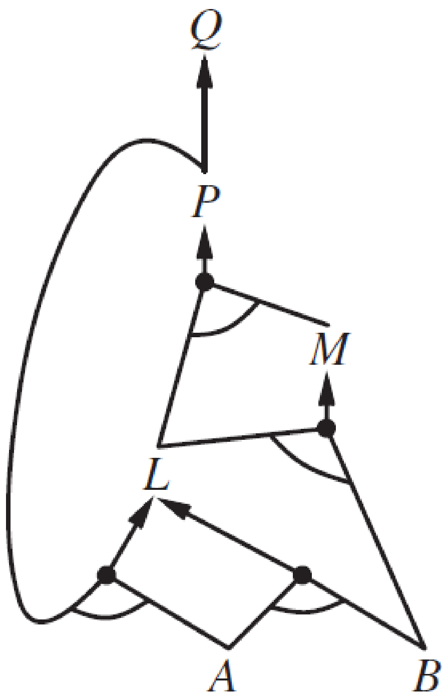

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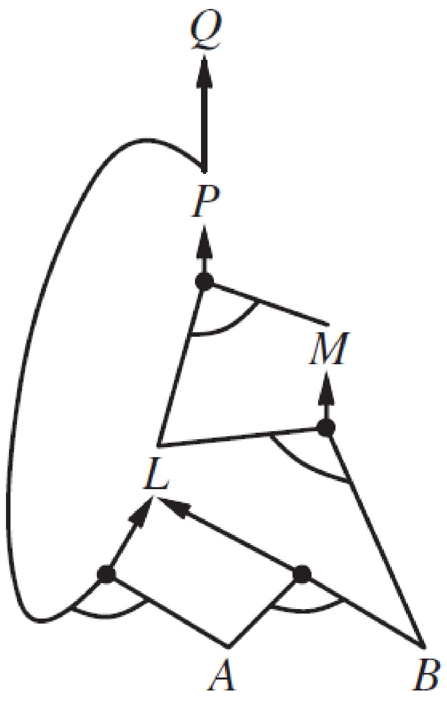

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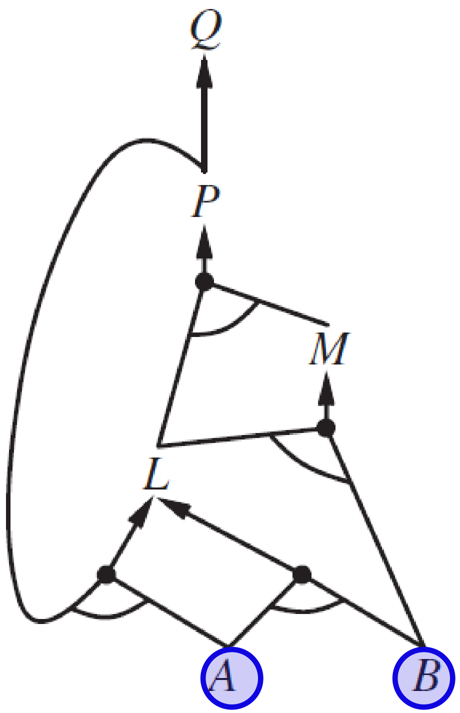

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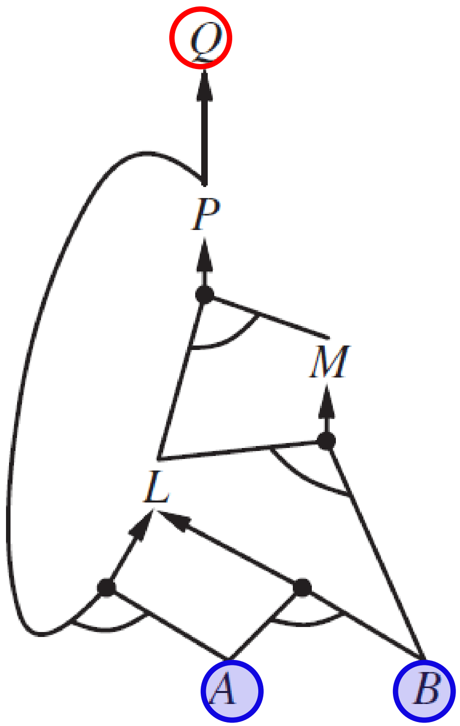

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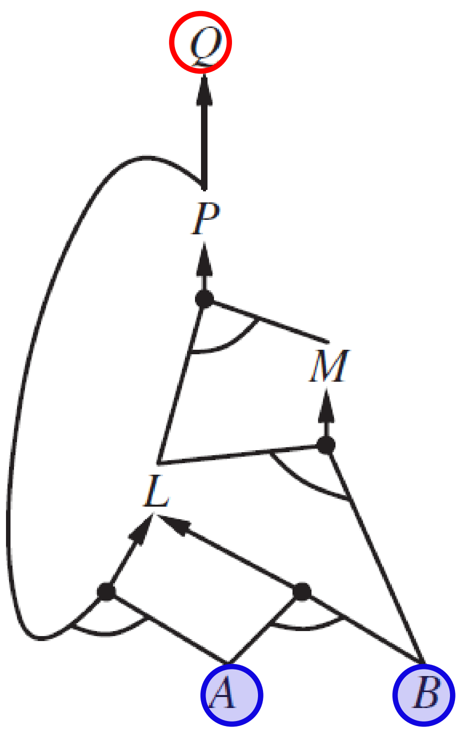

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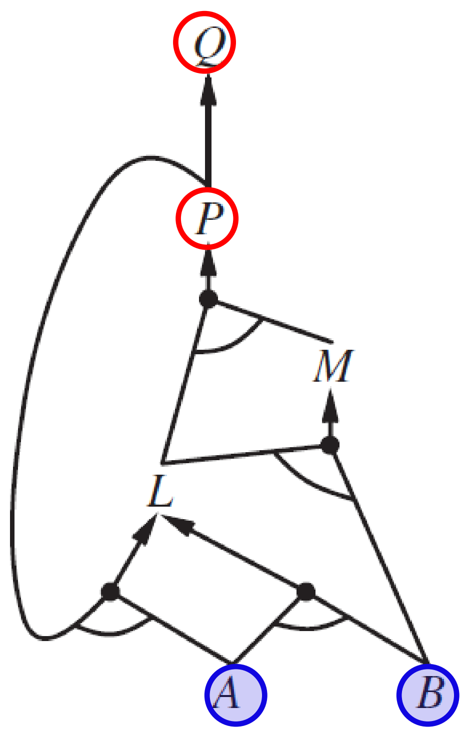

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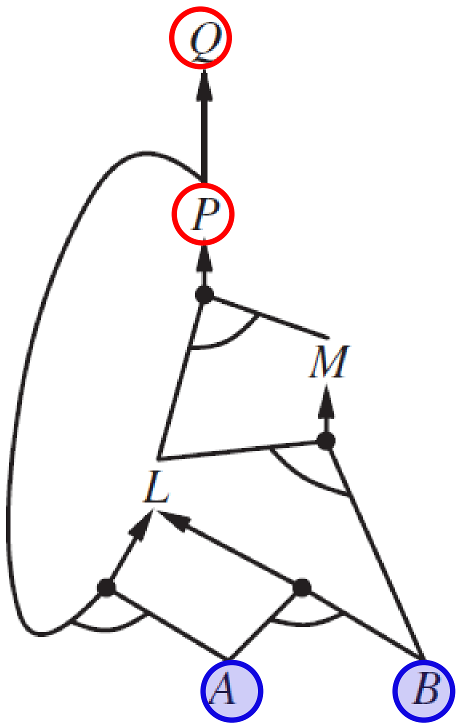

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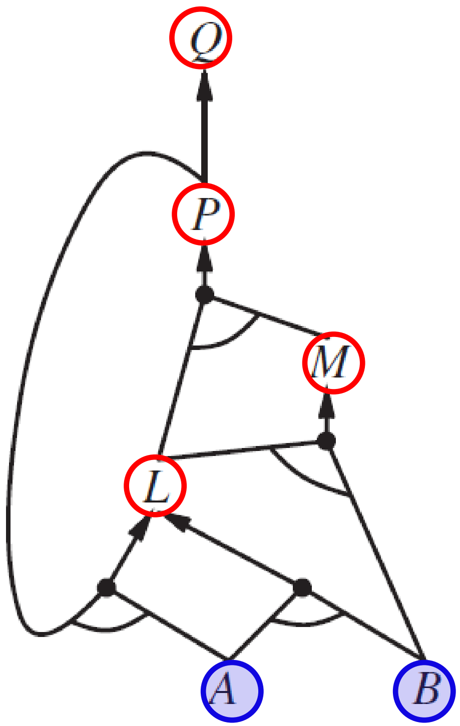
B



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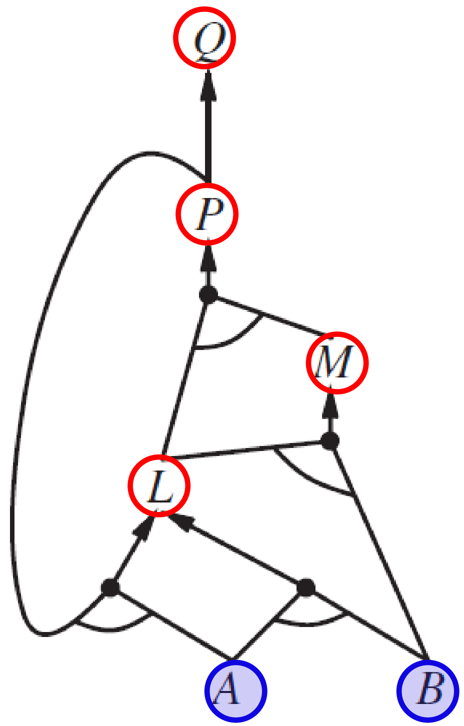

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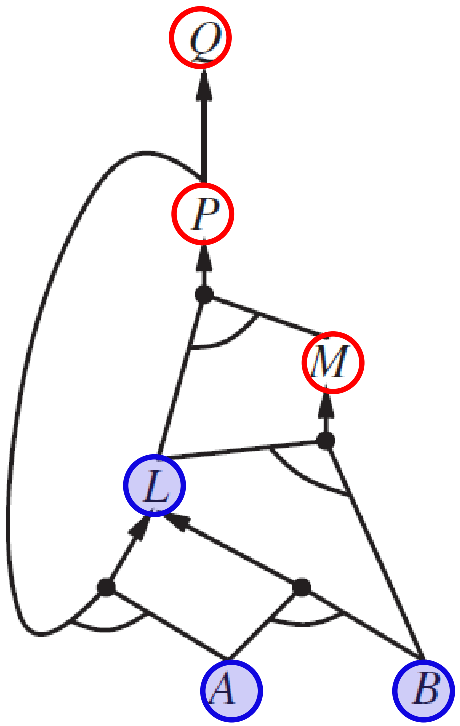

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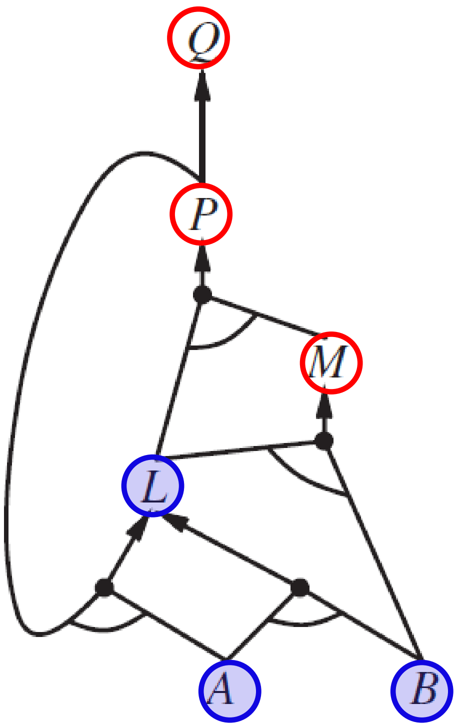

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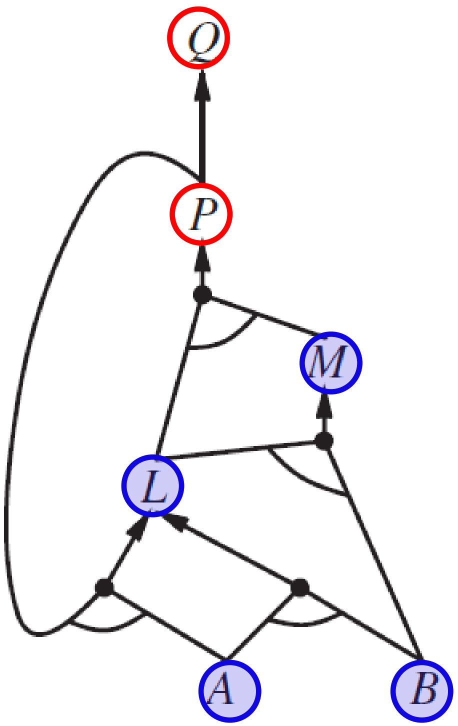

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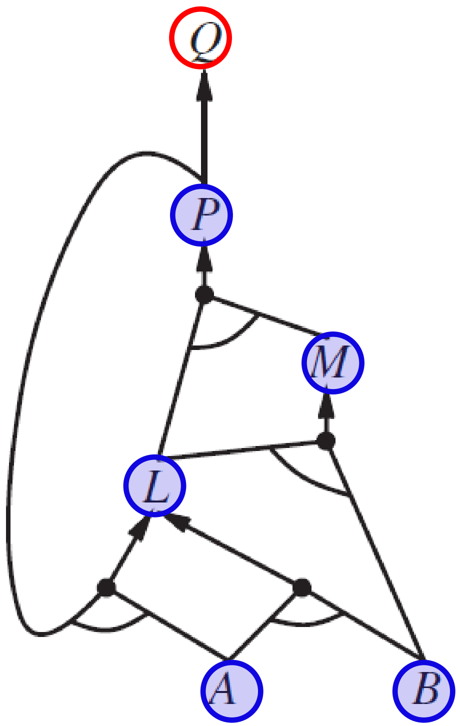

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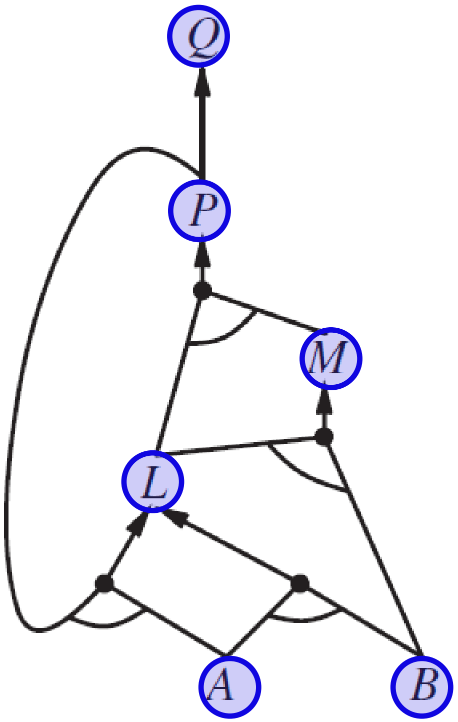

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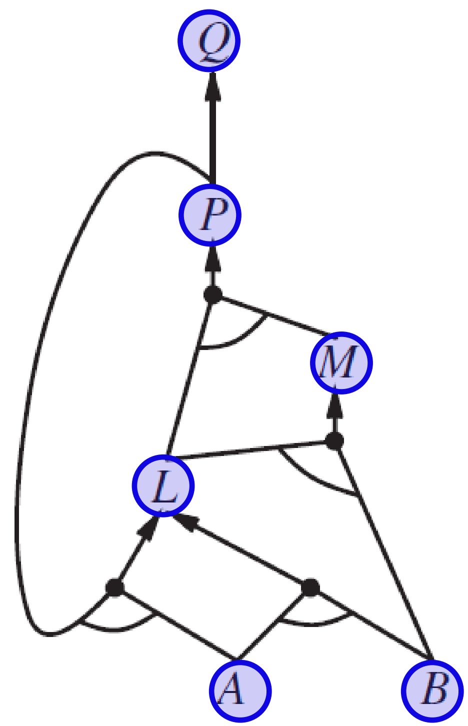

AND-OR graph search!

Forward vs. Backward Chaing

- Forward chaining is *data-driven*, automatic, unconscious processing.
- It may perform a lot of work that is irrelevant to the goal.
- Backward chaining is *goal-driven*, and appropriate for problem solving.
- It may run in time sublinear in the size of KB, since it touches only relevant facts.

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II. Effective Propositional Model Checking

$KB \models \beta$ if and only if $KB \wedge \neg\beta$ is unsatisfiable.

- Cast the problem as one of constraint satisfaction.

Many combinatorial problems in computer science can be reduced to checking the satisfiability of a propositional sentence.

♦ Complete backtracking search (DPLL algorithm)

♦ Incomplete local search (WALKSAT algorithm)

DPLL Algorithm

Davis, Putnam, Logemann, and Loveland (1960, 1962)

With enhancements, modern solvers can handle a problem with a multiple of 10^7 variables.

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function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of s
  $symbols \leftarrow$ a list of the proposition symbols in s
  return DPLL($clauses, symbols, \{\ \}$)

function DPLL($clauses, symbols, model$) **returns** *true* or *false*

  if every clause in $clauses$ is true in $model$ **then return** *true*
  if some clause in $clauses$ is false in $model$ **then return** *false*
  $P, value \leftarrow$ FIND-PURE-SYMBOL($symbols, clauses, model$)
  if P is non-null **then return** DPLL($clauses, symbols - P, model \cup \{P{=}value\}$)
  $P, value \leftarrow$ FIND-UNIT-CLAUSE($clauses, model$)
  if P is non-null **then return** DPLL($clauses, symbols - P, model \cup \{P{=}value\}$)
  $P \leftarrow$ FIRST($symbols$); $rest \leftarrow$ REST($symbols$)
  return DPLL($clauses, rest, model \cup \{P{=}true\}$) **or**
      DPLL($clauses, rest, model \cup \{P{=}false\}$))

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Unit clause propagation on a clause in which all literals but one are assigned *false*. E.g., $\neg B \vee \neg C$ simplifies to the unit clause $\neg C$ if B = *true*.

Local Search Algorithms

- Take steps in the space of complete assignments, flipping the truth value of one symbol at a time.
- Use an evaluation that counts the number of unsatisfied clauses.
- Escape local minima using various forms of randomness.
- Find a good balance between greediness and randomness.

The WALKSAT Algorithm

function WALKSAT(*clauses*, *p*, *max_flips*) **returns** a satisfying model or *failure*
  inputs: *clauses*, a set of clauses in propositional logic
    p, the probability of choosing to do a "random walk" move, typically around 0.5
    max_flips, number of value flips allowed before giving up

  model ← a random assignment of *true*/*false* to the symbols in *clauses*
  for each $i = 1$ **to** *max_flips* **do**
    if *model* satisfies *clauses* **then return** *model*
    clause ← a randomly selected clause from *clauses* that is false in *model*
    if RANDOM(0, 1) $\leq p$ **then**
      flip the value in *model* of a randomly selected symbol from *clause*
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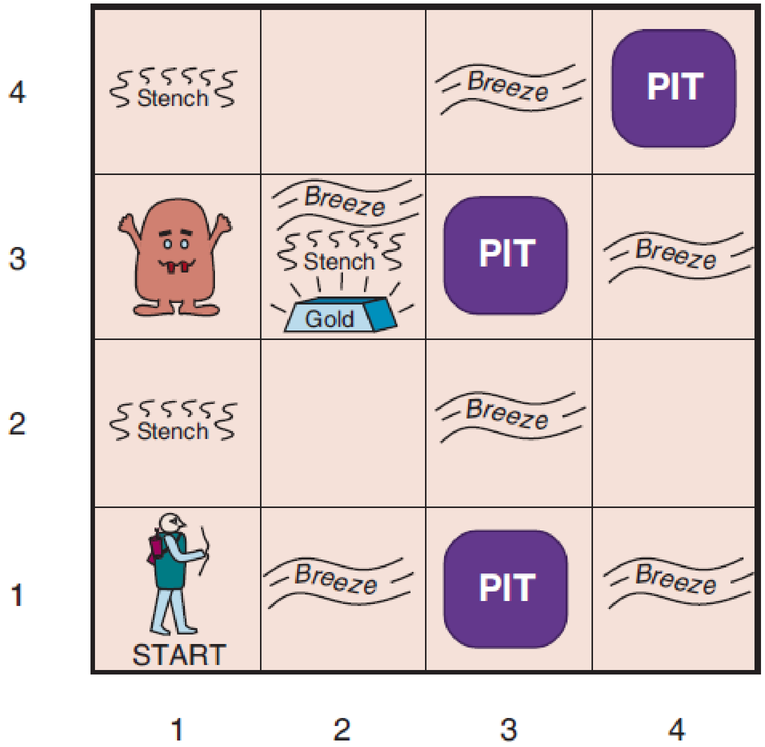
III. Agent Based on Propositional Logic

- Write down a complete logical model of the effects of action.
- How logical inference can be used by an agent?.
- How to keep track of the world without resorting to inference history?
- How to use logical inference to construct plans based on the KB?

Knowledge base (KB):

- ♣ general knowledge about how the world works
- ♣ percept sentences obtained in a particular world

Current State in the Wumpus World



Axioms:

$\neg P_{1,1}$ $\neg W_{1,1}$

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ // 16 rules of this type

$S_{1,1} \Leftrightarrow (W_{1,2} \vee W_{2,1})$ // 16

…

- $P_{x,y}$ = *true* if there is a pit in $[x, y]$.
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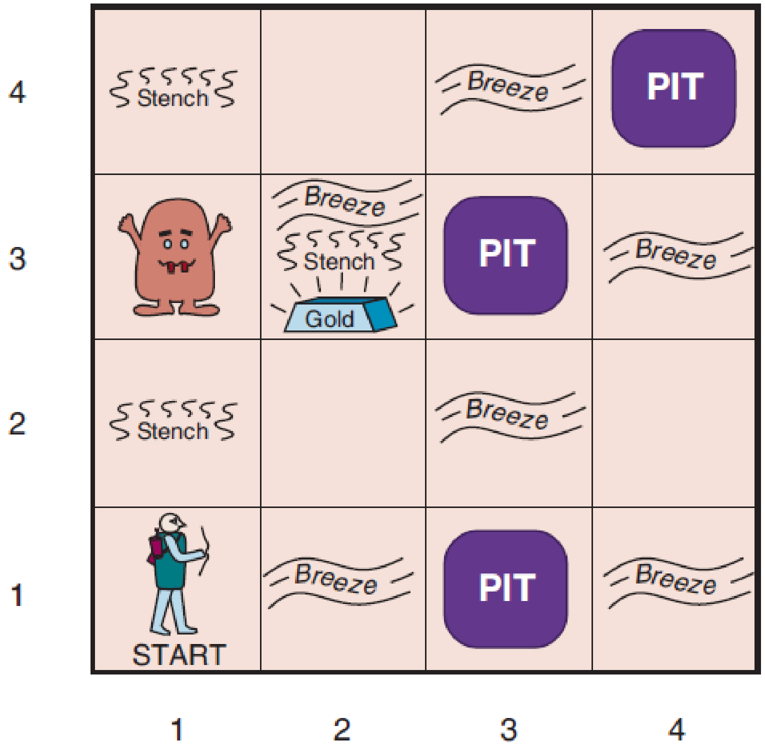

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$\ldots$

- Exactly one wumpus

$W_{1,1} \vee W_{1,2} \vee \cdots \vee W_{4,3} \vee W_{4,4}$ $\qquad$ // ≥ 1 wumpus

$\neg W_{i,j} \vee \neg W_{k,l}$ $\qquad$ $1 \leq i, j, k, l \leq 4$ and $(i, j) \neq (k, l)$

// ≤ 1 Wumpus;

// $\frac{16 \times 15}{2} = 120$ rules

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Representing Percepts

A percept asserts something only about the current time.

$Stench^4$: the agent senses stench at time step 4 (in square A).

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$O(mn)$ frame axioms for m actions and n fluents

Axioms for Successor States

Successor-state axiom, one for every fluent F, states that

- either the action at t causes F to be true at $t+1$,
- or F was already true at t and the action does not cause it to be false.

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$$L_{1,1}^{t+1} \Leftrightarrow (L_{1,1}^{t} \wedge (\neg \mathit{Forward}^{t} \vee \mathit{Bump}^{t+1})) \vee (L_{1,2}^{t} \wedge (\mathit{FacingSouth}^{t} \vee \mathit{Forward}^{t})) \vee (L_{2,1}^{t} \wedge (\mathit{FacingWest}^{t} \vee \mathit{Forward}^{t}))$$

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$$L_{1,1}^{t+1} \Leftrightarrow (L_{1,1}^t \wedge (\neg \mathit{Forward}^t \vee \mathit{Bump}^{t+1})) \vee (L_{1,2}^t \wedge (\mathit{FacingSouth}^t \vee \mathit{Forward}^t)) \vee (L_{2,1}^t \wedge (\mathit{FacingWest}^t \vee \mathit{Forward}^t))$$

Square-OK axiom asserts that a square is free of a pit or live Wumpus.

$$\mathit{OK}_{x,y}^t \Leftrightarrow \neg P_{x,y} \wedge \neg (W_{x,y} \wedge \mathit{WumpusAlive}^t)$$

Initial Percepts and Actions

$\neg Stench^0 \wedge \neg Breeze^0 \wedge \neg Glitter^0 \wedge \neg Bump^0 \wedge \neg Scream^0 ;\ Forward^0$

$\neg Stench^1 \wedge Breeze^1 \wedge \neg Glitter^1 \wedge \neg Bump^1 \wedge \neg Scream^1 ;\ TurnRight^1$

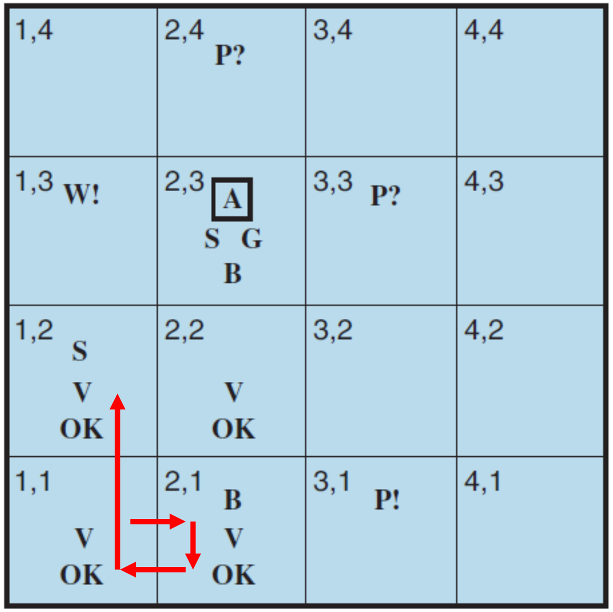
$\neg Stench^2 \wedge Breeze^2 \wedge \neg Glitter^2 \wedge \neg Bump^2 \wedge \neg Scream^2 ;\ TurnRight^2$

$\neg Stench^3 \wedge Breeze^3 \wedge \neg Glitter^3 \wedge \neg Bump^3 \wedge \neg Scream^3 ;\ Forward^3$

$\neg Stench^4 \wedge \neg Breeze^4 \wedge \neg Glitter^4 \wedge \neg Bump^4 \wedge \neg Scream^4 ;\ TurnRight^4$

$\neg Stench^5 \wedge \neg Breeze^5 \wedge \neg Glitter^5 \wedge \neg Bump^5 \wedge \neg Scream^5 ;\ Forward^5$

$Stench^6 \wedge \neg Breeze^6 \wedge \neg Glitter^6 \wedge \neg Bump^6 \wedge \neg Scream^6$



Query the knowledge base:

$\text{ASK}(KB, L^6_{1,2}) = true$

$\text{ASK}(KB, W_{1,3}) = true$

$\text{ASK}(KB, P_{3,1}) = true$

$\text{ASK}(KB, OK^6_{2,2}) = true$

// the square [2,2] is OK to move into.

Initial Percepts and Actions

$\neg Stench^0 \wedge \neg Breeze^0 \wedge \neg Glitter^0 \wedge \neg Bump^0 \wedge \neg Scream^0$; $Forward^0$

$\neg Stench^1 \wedge Breeze^1 \wedge \neg Glitter^1 \wedge \neg Bump^1 \wedge \neg Scream^1$; $TurnRight^1$

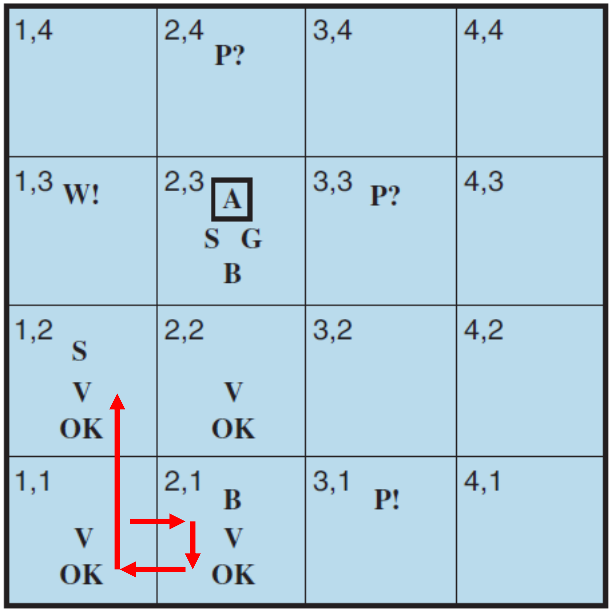
$\neg Stench^2 \wedge Breeze^2 \wedge \neg Glitter^2 \wedge \neg Bump^2 \wedge \neg Scream^2$; $TurnRight^2$

$\neg Stench^3 \wedge Breeze^3 \wedge \neg Glitter^3 \wedge \neg Bump^3 \wedge \neg Scream^3$; $Forward^3$

$\neg Stench^4 \wedge \neg Breeze^4 \wedge \neg Glitter^4 \wedge \neg Bump^4 \wedge \neg Scream^4$; $TurnRight^4$

$\neg Stench^5 \wedge \neg Breeze^5 \wedge \neg Glitter^5 \wedge \neg Bump^5 \wedge \neg Scream^5$; $Forward^5$

$Stench^6 \wedge \neg Breeze^6 \wedge \neg Glitter^6 \wedge \neg Bump^6 \wedge \neg Scream^6$



Query the knowledge base:

$$\text{ASK}(KB, L^6_{1,2}) = true$$
$$\text{ASK}(KB, W_{1,3}) = true$$
$$\text{ASK}(KB, P_{3,1}) = true$$

$$\text{ASK}(KB, OK^6_{2,2}) = true$$

// the square [2,2] is OK to move into.