

Stat 330

Homework 4

Sean Gordon

February 26, 2020

1)

(a) X = # of drivers until one doesn't make a full stop.

$$X \sim \text{Geo}(1 - .85) = \text{Geo}(.15)$$

$$P(X < 10) = ?$$

(b) X = # of correct answers out of total answers.

$$X \sim \text{Bin}(20, .6)$$

$$P(X \geq 12) = ?$$

(c) X = # of customers that arrive between 1:00 pm and 2:00 pm.

$$X \sim \text{Pois}(16)$$

$$P(X = 14) = ?$$

2)

(a) There are 6 possible doubles rolls out of 36 possible rolls. Thus, $p = 6/36 = .16\bar{6}$

$$(b) E(X) = 0(1-p) + 1(p) = 0(.83\bar{3}) + 1(.16\bar{6}) = .16\bar{6}$$

$$E(X^2) = 0^2(1-p) + 1^2(p) = 0^2(.83\bar{3}) + 1^2(.16\bar{6}) = .16\bar{6}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 = .16\bar{6} - (.16\bar{6})^2 = .16\bar{6} - 0.027\bar{7} = 0.138\bar{8}$$

x		-2	-1	0	1	2
(a, b)	$P_X(x)$	0.1	0.3	0.3	0.1	0.2
	$F_X(x)$	0.1	0.4	0.7	0.8	1.0

(c)

- i. $P(X \leq 1) \Rightarrow P(X = 1) + P(X = 2) = 0.1 + 0.2 = 0.3$
- ii. $P(-1 < X \leq 1) \Rightarrow P(X = 0) + P(X = 1) = 0.3 + 0.1 = 0.4$
- iii. $P(X < 0) \Rightarrow P(X = -2) + P(X = -1) = 0.1 + 0.3 = 0.4$

(d)

- i. $F(1) = 0.8$
- ii. $F(0.5) = F(0) = 0.7$
- iii. $P(X \geq 0) = 1 - F(-1) = 0.6$

(e) $E(X) = -2(0.1) + -1(0.3) + 0(0.3) + 1(0.1) + 2(0.2) = 0.2$

$$E(X^2) = -2^2(0.1) + -1^2(0.3) + 0^2(0.3) + 1^2(0.1) + 2^2(0.2) = 0.6$$

$$\text{Variance} = E(X^2) - E(X)^2 = 0.6 - (0.2)^2 = 0.56$$

3)

(a) $\text{Im}(Y) = \{8, 6, 4, 2, 0\}$

(b) $E(X) = 8(0.1) + 6(0.3) + 4(0.3) + 2(0.1) + 0(0.2) = 4$

$$E(X^2) = 8^2(0.1) + 6^2(0.3) + 4^2(0.3) + 2^2(0.1) + 0^2(0.2) = 22.4$$

$$\text{Variance} = E(X^2) - E(X)^2 = 22.4 - (4)^2 = 38.4$$

4) $\text{Var}(aX) = E([aX]^2) - [E(aX)]^2 = a^2E(X^2) - a^2E(X)^2 = a^2(E(X^2) - E(X)^2) = a^2\text{Var}(X)$

5)

(a) $P(X=2) = \binom{6}{2}(0.05)^2(0.95)^{6-2}$

$$15 \cdot (0.05)^2(0.95)^4 = .0305$$

(b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) \Rightarrow$

$$\binom{6}{0}(0.05)^0(0.95)^{6-0} + \binom{6}{1}(0.05)^1(0.95)^{6-1} + \binom{6}{2}(0.05)^2(0.95)^{6-2} =$$

$$0.735 + 0.232 + 0.031 = 0.998$$

6)

$$(a) P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10) \Rightarrow$$

$$\binom{10}{5} (0.2)^5 (0.8)^{10-5} + \binom{10}{6} (0.2)^6 (0.8)^{10-6} + \binom{10}{7} (0.2)^7 (0.8)^{10-7} +$$

$$\binom{10}{8} (0.2)^8 (0.8)^{10-8} + \binom{10}{9} (0.2)^9 (0.8)^{10-9} + \binom{10}{10} (0.2)^{10} (0.8)^{10-10} =$$

$$0.02642 + 0.00551 + .00079 + .00007 + \sim 0 + \sim 0 = .033$$

$$(b) P(X \geq 5) = 1 - P(X \leq 4) = (.2)(.8)^{4-1} + (.2)(.8)^{3-1} + (.2)(.8)^{2-1} + (.2)(.8)^{1-1} =$$

$$.1024 + .128 + .16 + .2 = .5904$$