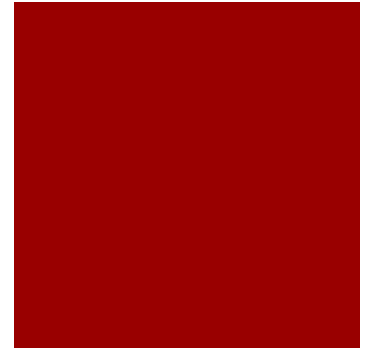# COM S 342

Recitation 10/28/2019 – 10/30/2019

Topic

- Lambda Calculus
  - Beta Reduction
  - Church Encoding

- Q&A

Lambda Calculus

- Formal mathematical system

- Simplest programming language

- Invented in 1936 by Alonzo Church(1903-1995)

Lambda Calculus

$$e ::= x \mid \lambda x.e \mid e_0 \ e_1$$

<expression>	:=	<name> | <function> | <application>
<function>	:=	λ <name>.<expression>
<application>	:=	<expression><expression>

Syntax implemented in Scheme:

$$
\begin{aligned}
e \quad \rightarrow \quad & x & \text{Variable} \\
\mid \quad & (_x \ \lambda \ (x) \ e \ )_x & \text{a lambda expression} \\
\mid \quad & (e \ e) & \text{Application}
\end{aligned}
$$

Beta-reduction Exercises

$((\lambda(x)\ e1\ )\ e2)$ evaluates the expression e1 by replacing every ("free") occurrences of x in e1 using e2, denoted as $e1[x{\rightarrow}e2]$

(1) $(((\lambda(x)\ (\lambda(y)\ (y\ y)))\ (\lambda(a)\ a))\ b)$

(2) $(((\lambda(x)\ (\lambda(y)\ (x\ y)))\ w)\ z)$

Beta-reduction Exercises

$((\lambda(x)\ e1\ )\ e2)$ evaluates the expression e1 by replacing every ("free") occurrences of x in e1 using e2, denoted as $e1[x \rightarrow e2]$

(1) $(((\lambda(x)\ (\lambda(y)\ (y\ y)))\ (\lambda(a)\ a))\ b) = (b\ b)$

(2) $(((\lambda(x)\ (\lambda(y)\ (x\ y)))\ w)\ z) = (w\ z)$

Church Numbers

○ Encoding of numbers: 0, 1, 2, ... , as functions such that their semantics follows the natural number semantics

○ Intuition: The number n means how many times one can do certain operation.

○ A natural number encoding function takes two arguments (function and entity on which the function is to be applied)

Church Numbers

zero $(_f\ \lambda\ (f)\ (_x\ \lambda\ (x)\ x\ )_x\ )_f$

one $(_f\ \lambda\ (f)\ (_x\ \lambda\ (x)\ (f\ x)\ )_x\ )_f$

two $(_f\ \lambda\ (f)\ (_x\ \lambda\ (x)\ (f\ (f\ x))\ )_x\ )_f$

n $(_f\ \lambda\ (f)\ (_x\ \lambda\ (x)\ (f\ \ldots(f\ x)\ldots))\ )_x\ )_f$

((two g) x): two applications of g on x

Church Encoding

Representing data and operations using functions

- ▶ non-negative integers, 0, 1, 2 ...
- ▶ booleans, true, false, and, or, not, ite
- ▶ pairs
- ▶ Rational numbers may be encoded as a pair of signed numbers.
- ▶ Real numbers may be encoded by a limiting process that guarantees that the difference from the real value differs by a number which may be made as small as we need

Q&A