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1. (8 points) Find the solution of

$$y' = (y^2 + 1) \cos x \quad y\left(\frac{\pi}{2}\right) = 1$$

2. (8 points) Find the general solution of

$$y' + \frac{2y}{x} = \frac{\sin 2x}{x}$$

3. (8 points) Census Bureau estimates of the United States population were 281.4 million for April 1, 2000 and 308.7 million for April 1, 2010. Assuming that the rate of population growth is proportional to population present, estimate what the United States population will be on April 1, 2016 to the nearest million.

4. (8 points) Find the general solution of

$$y' = \frac{y^2 + xy}{x^2}$$

(Suggestion: use the substitution $v = \frac{y}{x}$.)

5. (10 points) Solve the initial value problem

$$y''' - 2y'' - 24y' = 0 \quad y(0) = 1 \quad y'(0) = 2 \quad y''(0) = 0$$

6. (10 points) Find the general solution of

$$y'' + 2y' + 6y = 4x + e^{-2x}$$

7. (18 points) Let

$$A = \begin{bmatrix} -4 & 6 \\ -3 & 5 \end{bmatrix} \quad \mathbf{x}_0 = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad \mathbf{f}(t) = \begin{bmatrix} e^t \\ 1 \end{bmatrix}$$

(a) Find the general solution of $\mathbf{x}' = A\mathbf{x}$.

(b) Find the solution of the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \mathbf{x}_0$.

(c) Find the general solution of $\mathbf{x}' = A\mathbf{x} + \mathbf{f}(t)$. *(Note: \mathbf{x} is a vector, $\mathbf{f}(t)$ is a vector)*

8. (8 points) Find the general solution of

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & -5 \\ 6 & 2 & -3 \end{bmatrix} \mathbf{x}$$

9. (12 points) Use the Laplace transform technique to obtain the solution of

$$y'' + y' - 2y = 3e^t \quad y(0) = 1 \quad y'(0) = 0$$

10. (10 points) Find the recurrence relation and the first four terms in the power series solution

$$y(x) = \sum_{n=0}^{\infty} a_n x^n \text{ of}$$

$$y'' - (x+1)y = 0 \quad y(0) = 1 \quad y'(0) = 3$$