Heuristic Functions

Outline

I. Variations of A* search

II. Generating heuristics

* Figures are from the textbook site (or drawn by the instructor) unless the source is specifically cited.

I. Sacrificing Search

- A* explores a lot of nodes due to equal weighting of g and h in $f = g + h$ which often distracts it from the optimal path.
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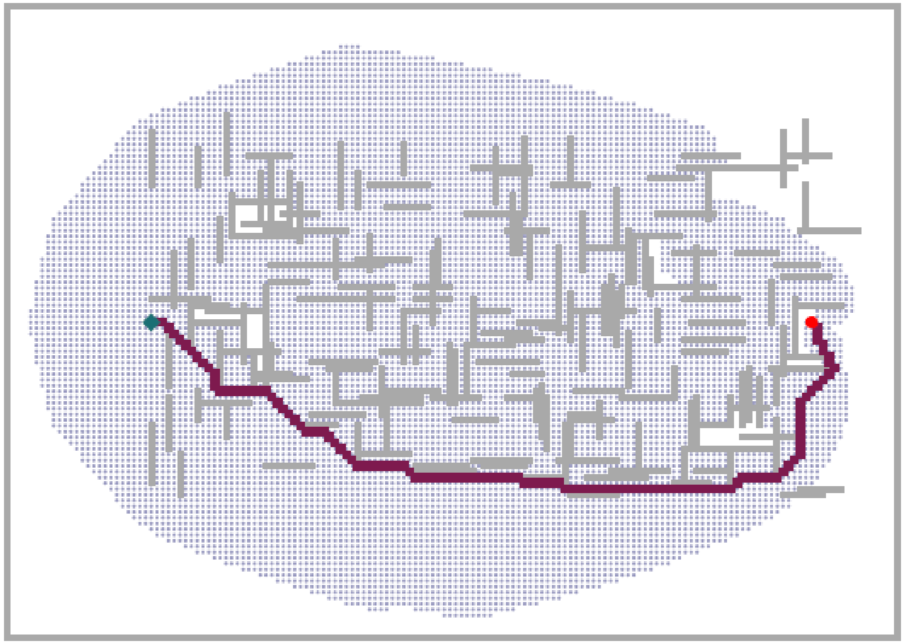
Idea of *detour index*: multiplier applied to the straight-line distance.

e.g., a detour index of 1.3 implies a good estimate of 13 miles between two cities that are 10 miles apart.

Weighted A*

Evaluation function: $f(n) = g(n) + W \times h(n)$ for some $W > 1$.



A* search

Gray bars: obstacles
Dots: reached states

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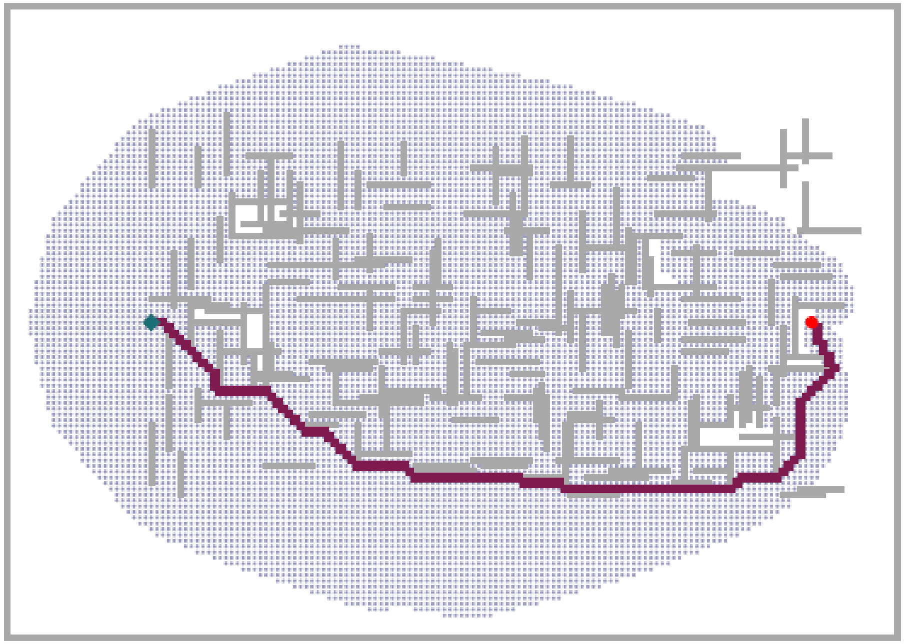

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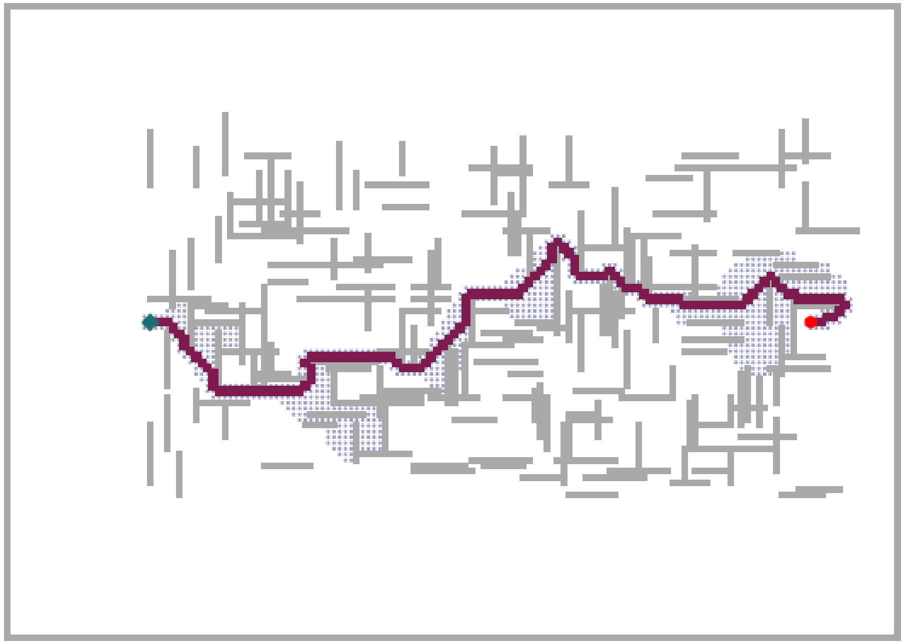

Weighted A* search
($W = 2$ on the same grid)

Weighted A* As a Generalization

- Weighted A* finds a solution with cost between C^* and $W \times C^*$.
- Cost is usually much closer to C^* in practice.

Weighted A* search $g(n) + W \times h(n) \quad (1 \leq W < \infty)$

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A* search	$g(n) + h(n)$	$(W = 1)$
Uniform-cost search	$g(n)$	$(W = 0)$
Best-cost search	$h(n)$	$(W = \infty)$
Weighted A* search	$g(n) + W \times h(n)$	$(1 \leq W < \infty)$

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  - ♦ Less memory and faster execution.
  - ♣ Incomplete and suboptimal.

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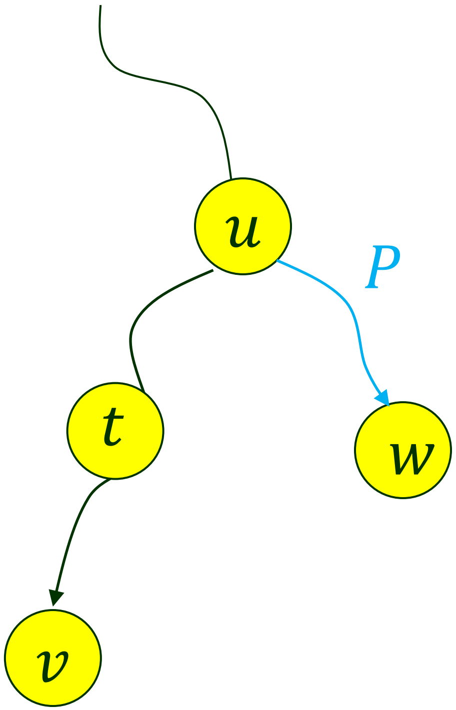
  ♣ In the worst case, #iterations = #states

Recursive Best-First Search (RBFS)



$f_{limit}(v)$: f-value of the *best alternative path* from any *ancestor*

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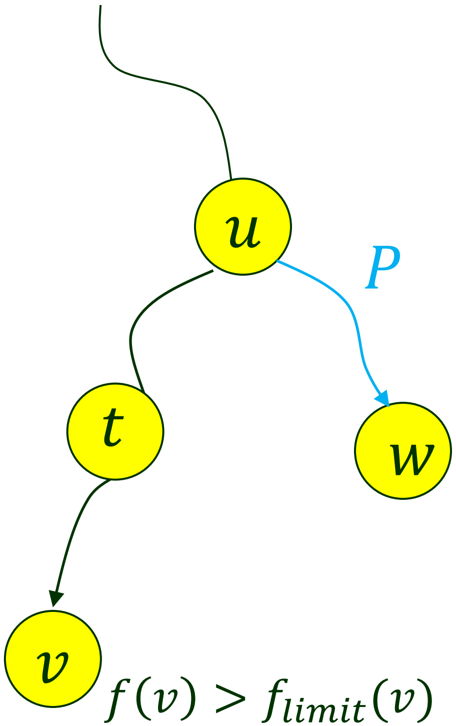

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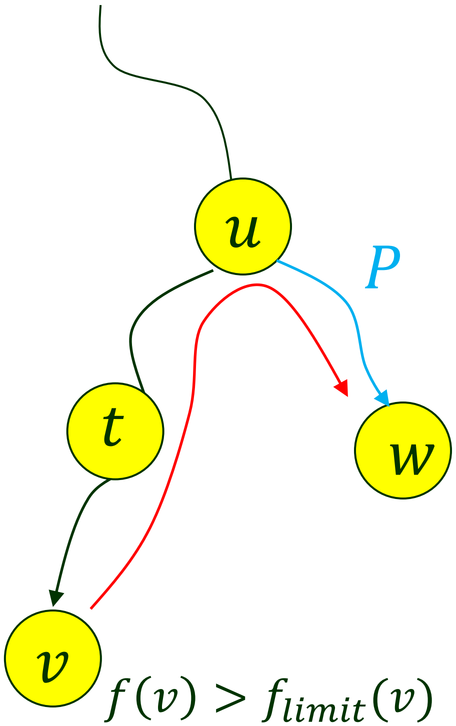

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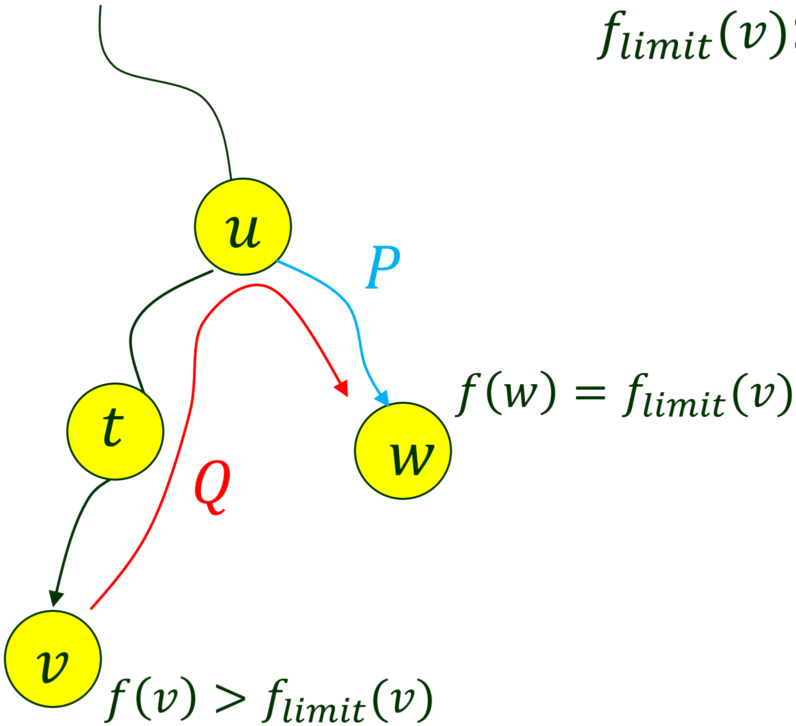

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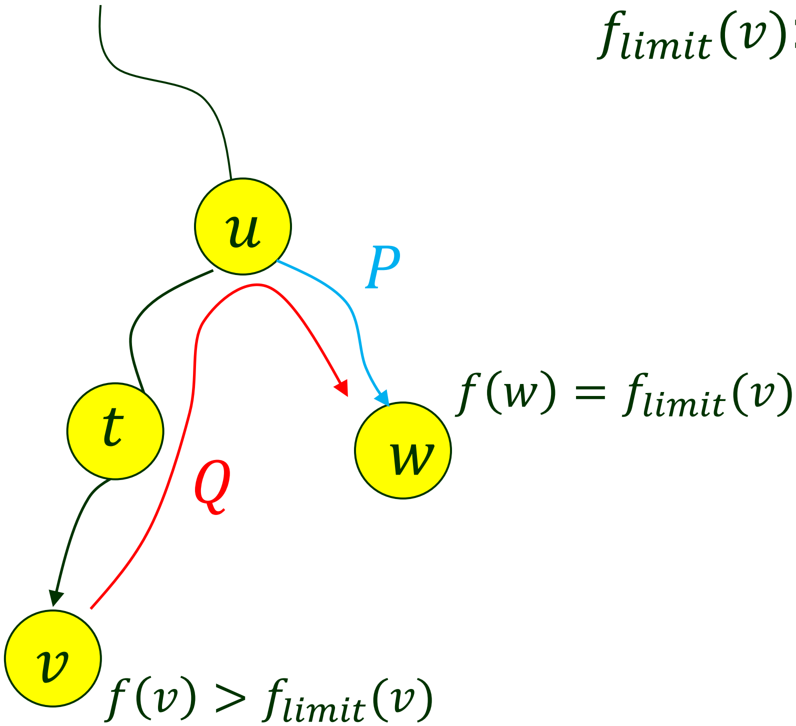

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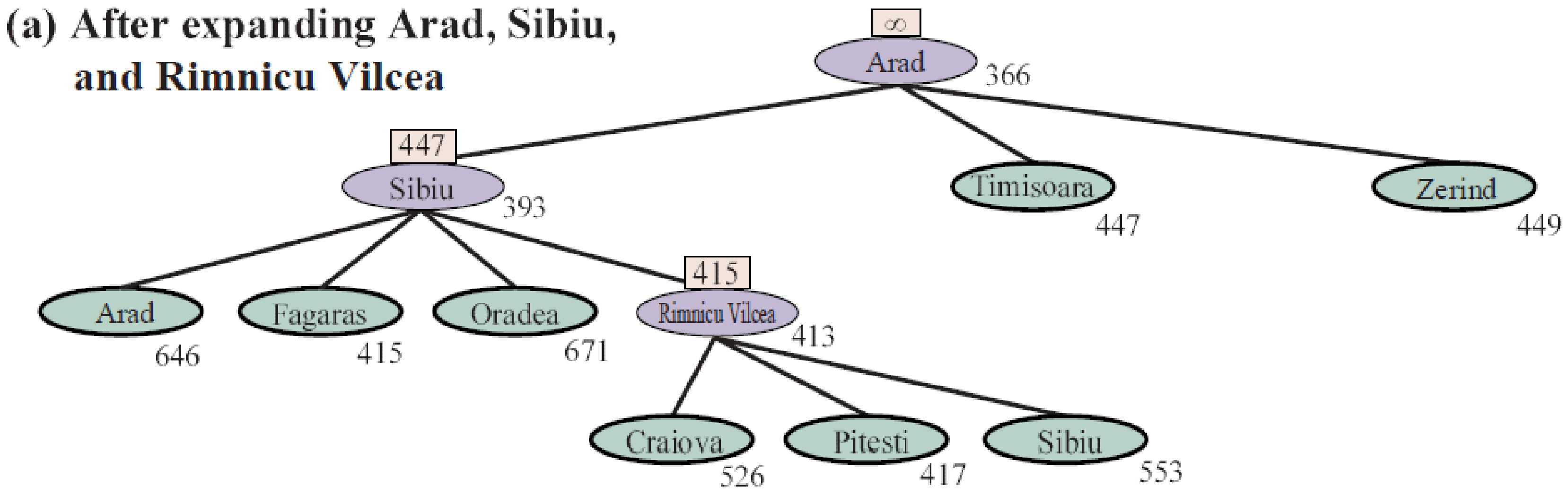
- Best-first search if at the currently visited node v, $f(v) \leq f_{limit}(v)$.
- Otherwise,
  - unwinds back to the alternative path P;
  - update the f-value of every node along the path Q (until u).

RBFS Example



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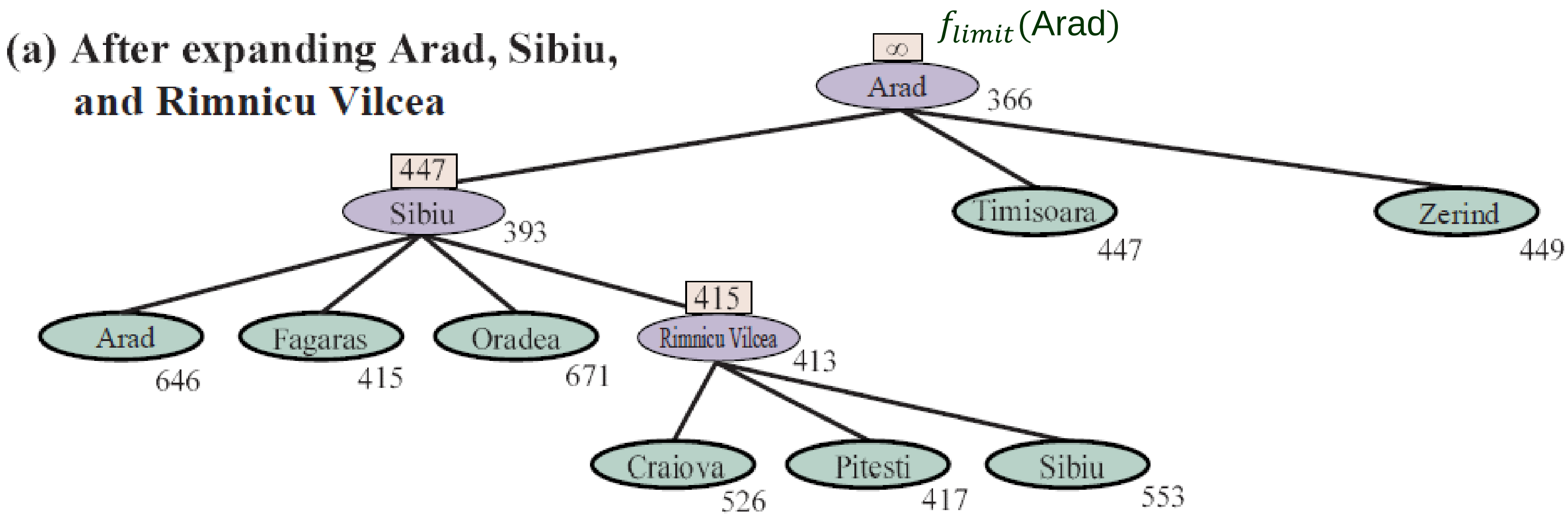

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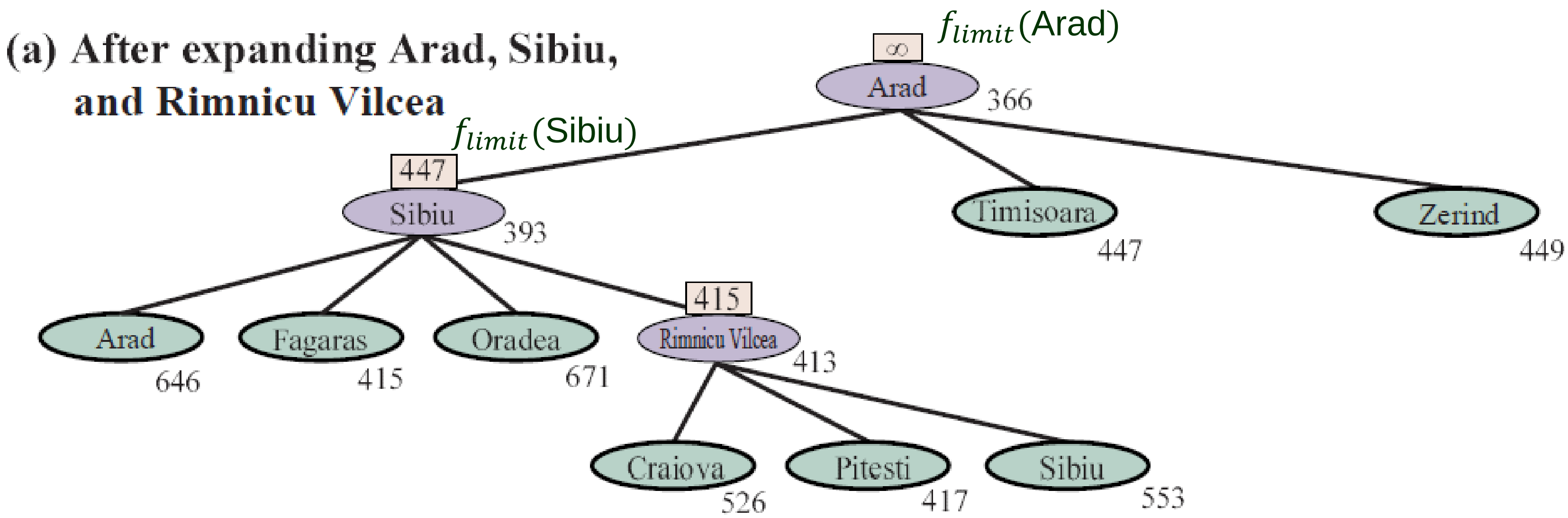

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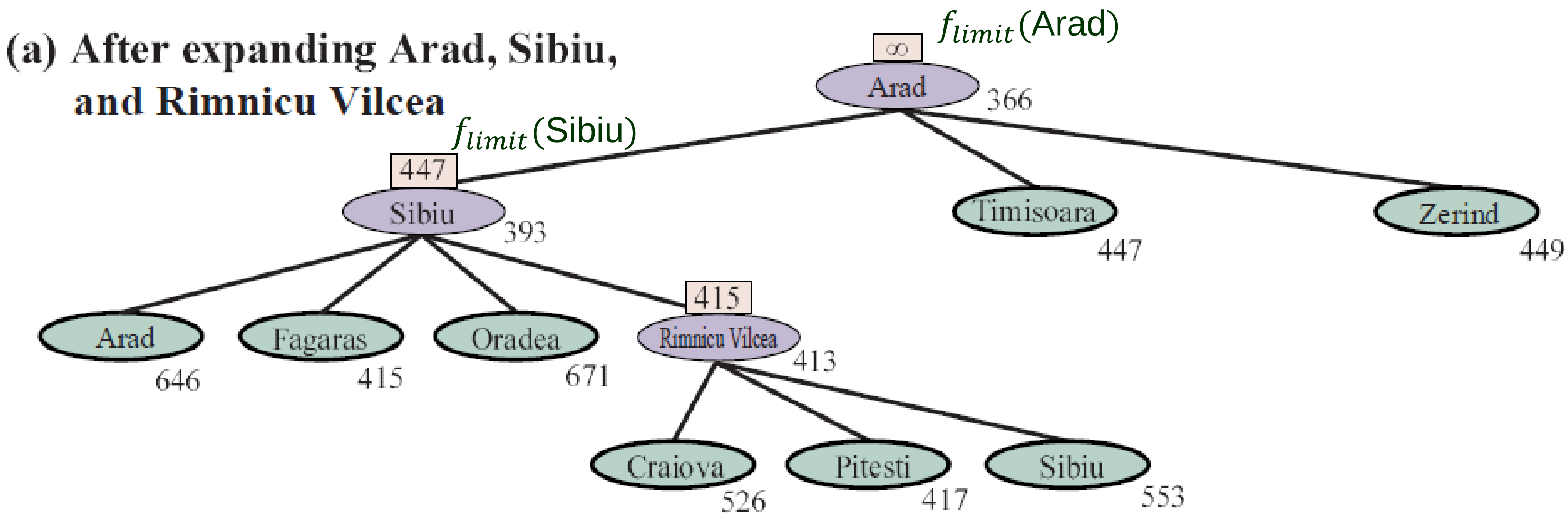

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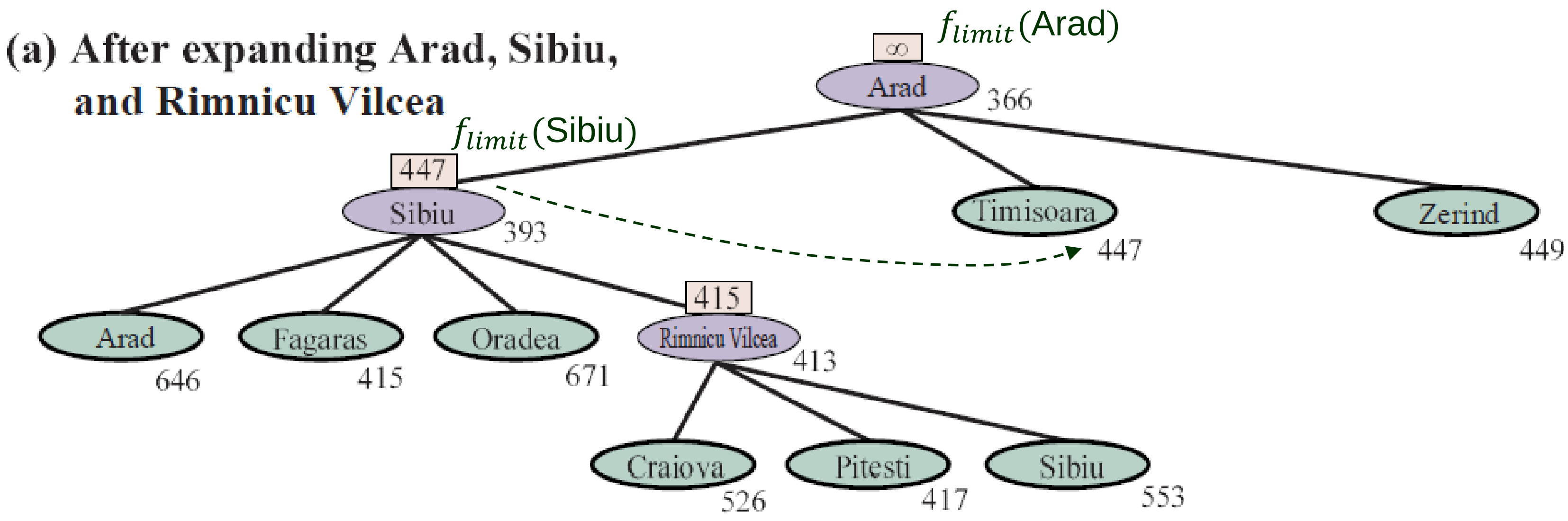
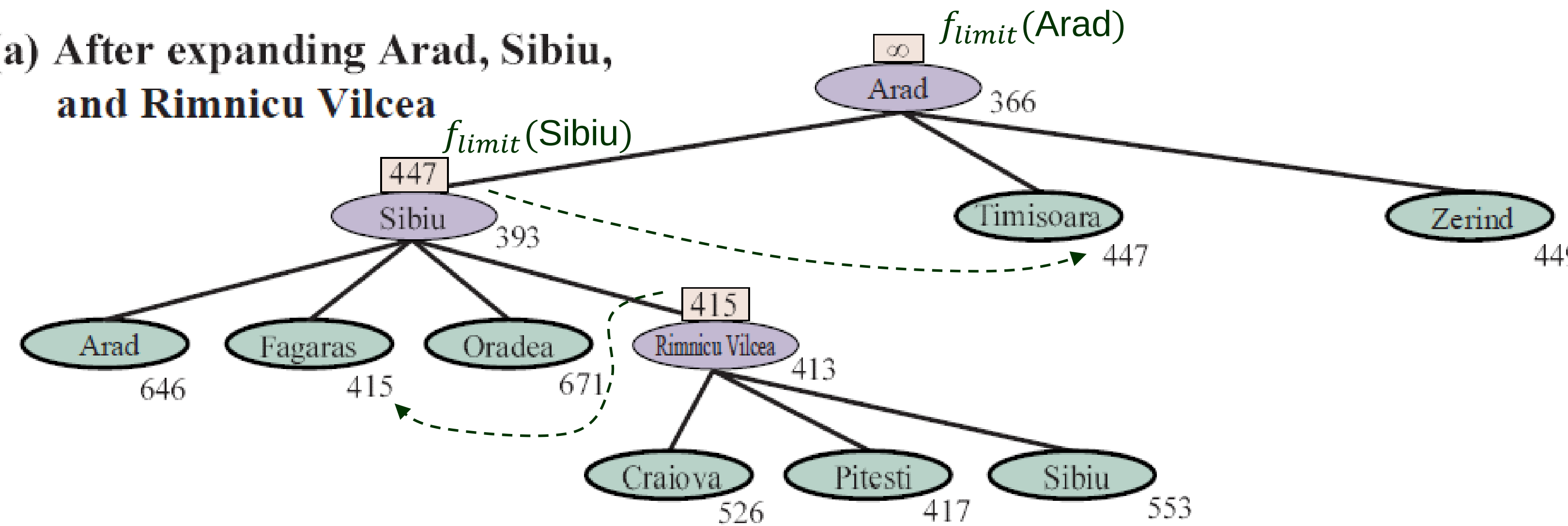


RBFS Example

(a) After expanding Arad, Sibiu, and Rimnicu Vilcea



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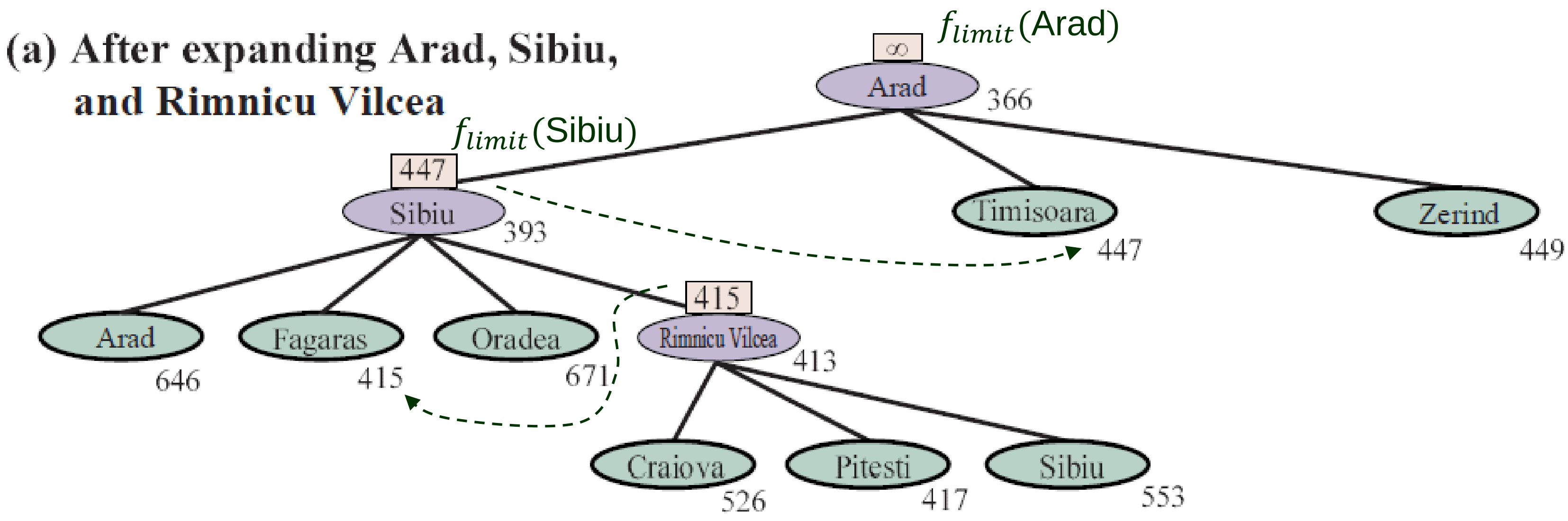

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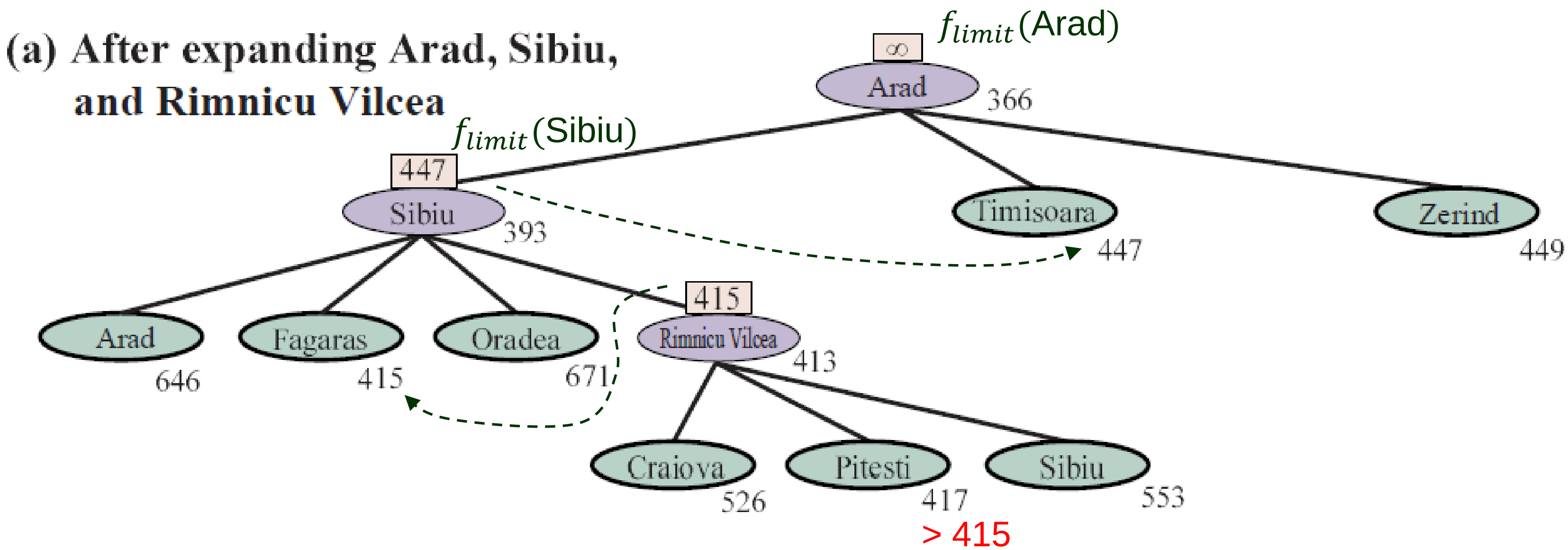

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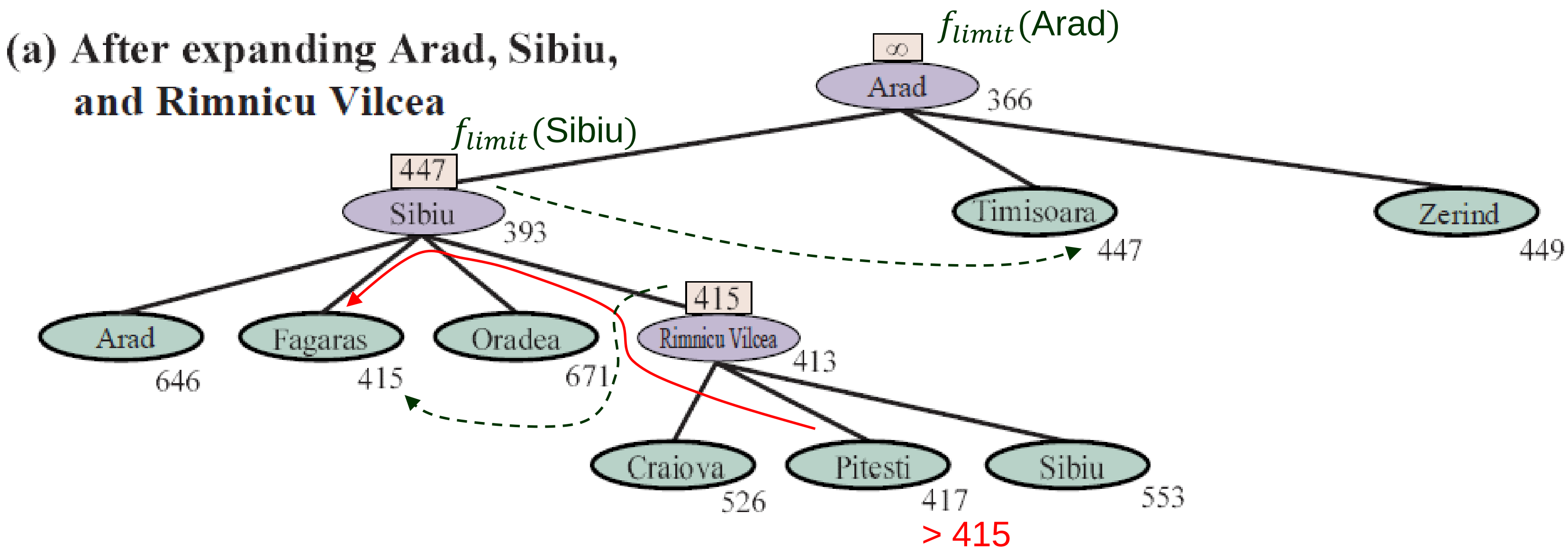

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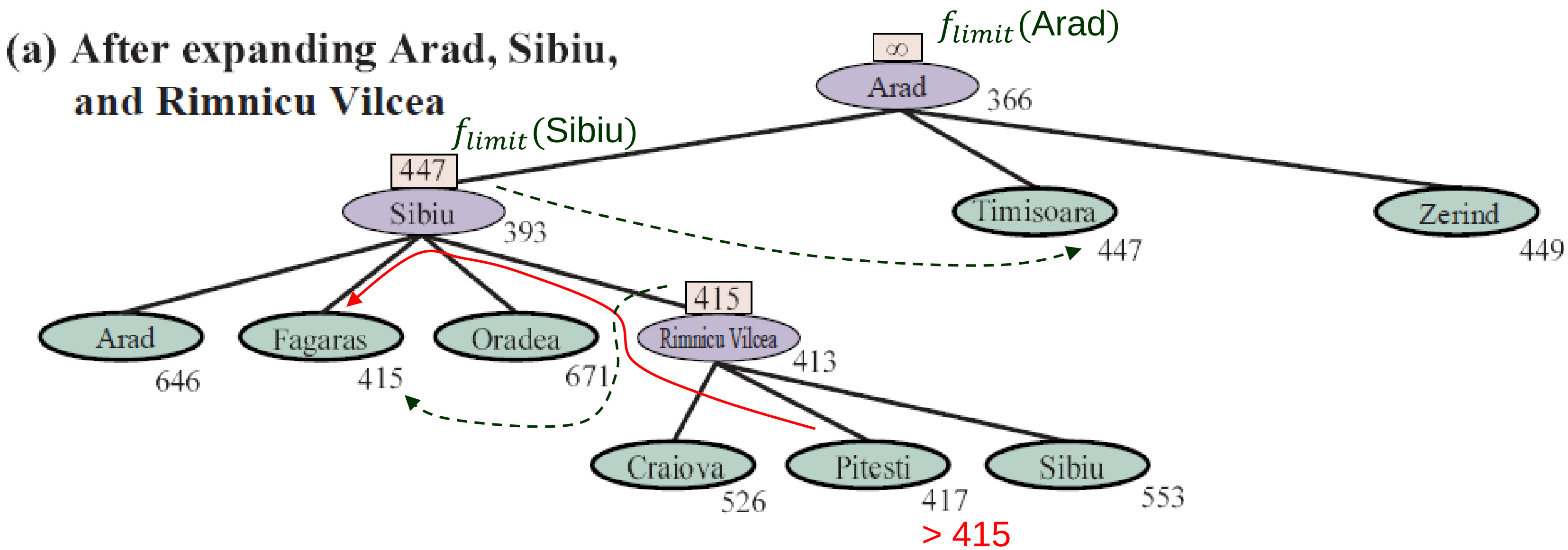

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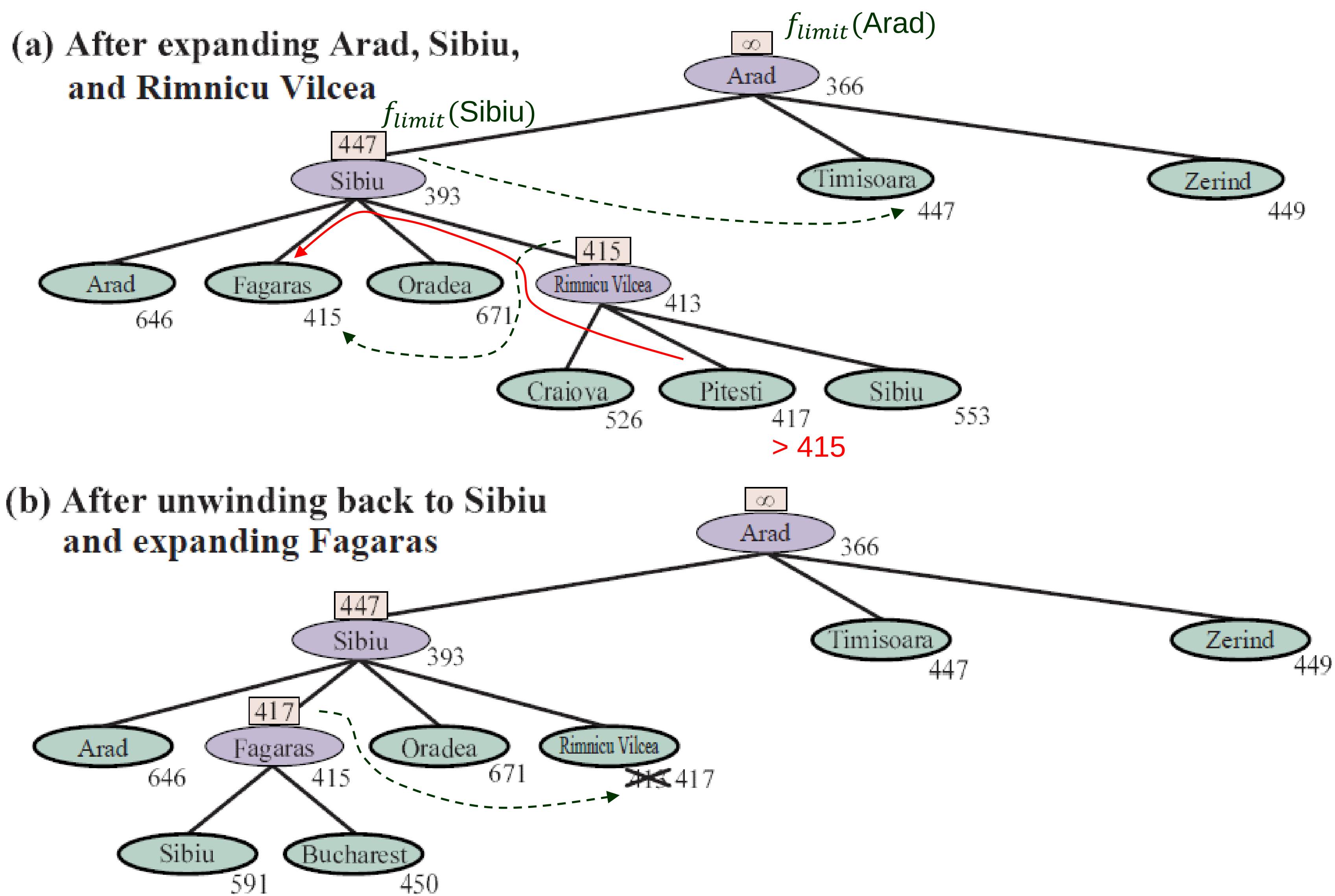

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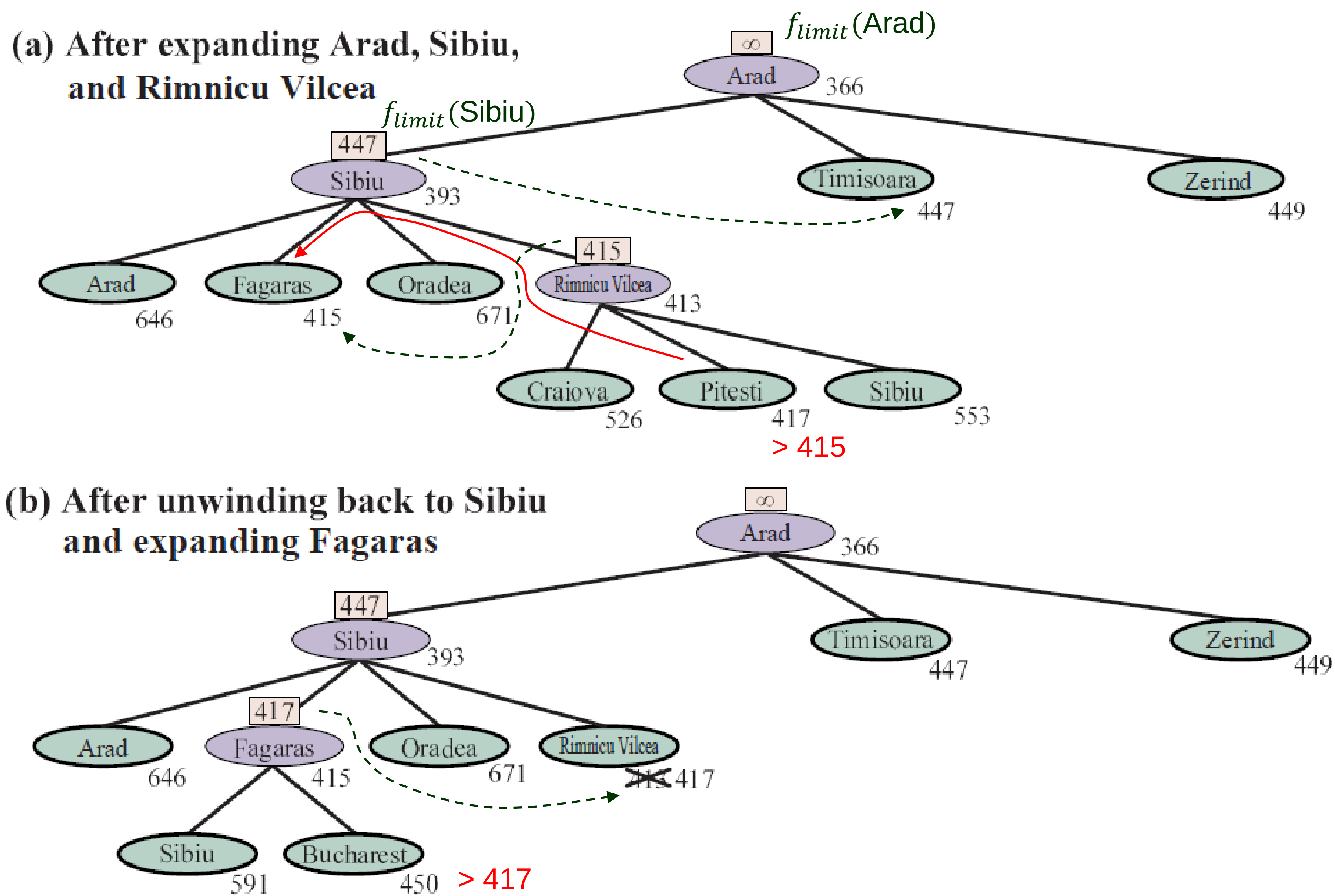

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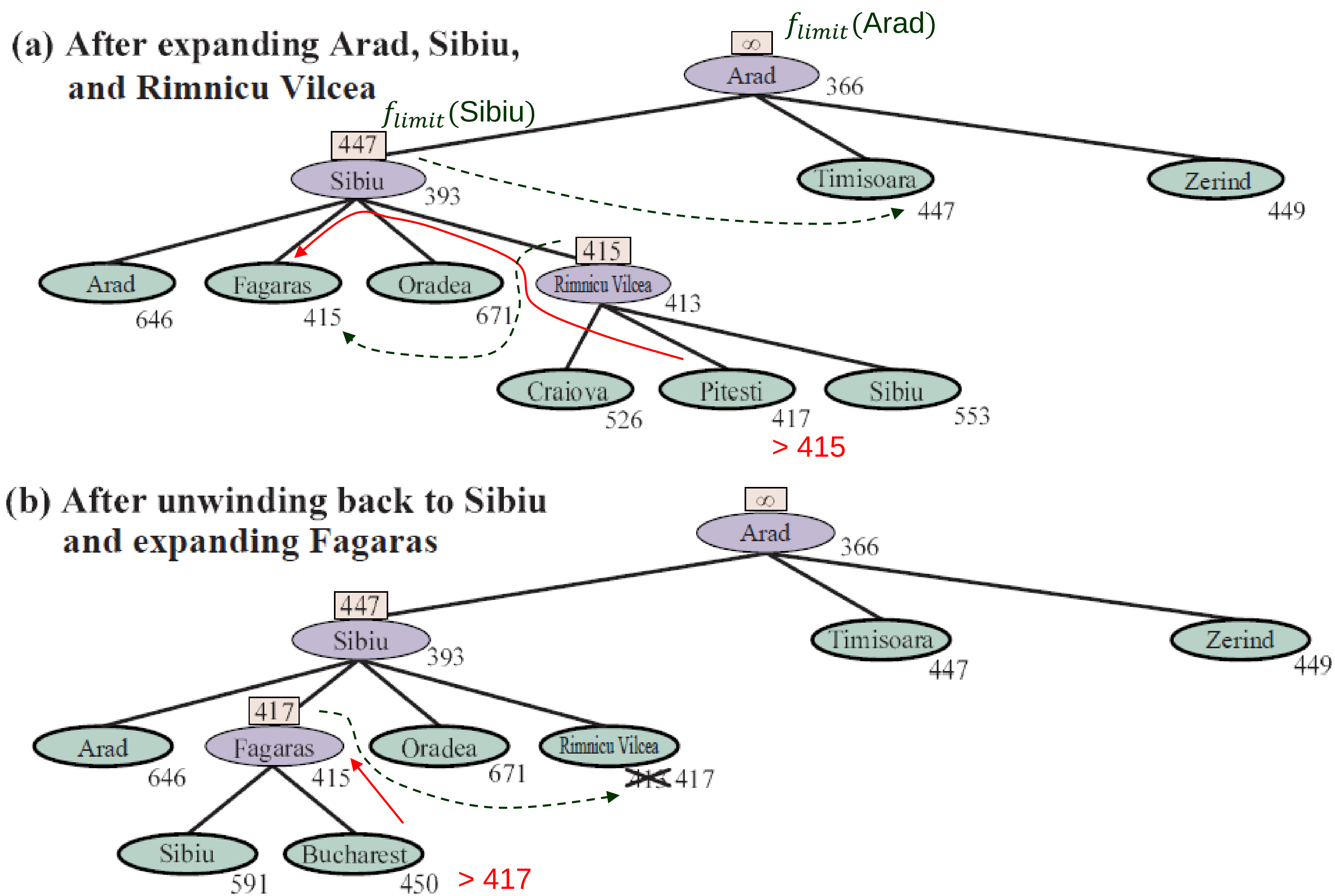

RBFS Example (cont'd)

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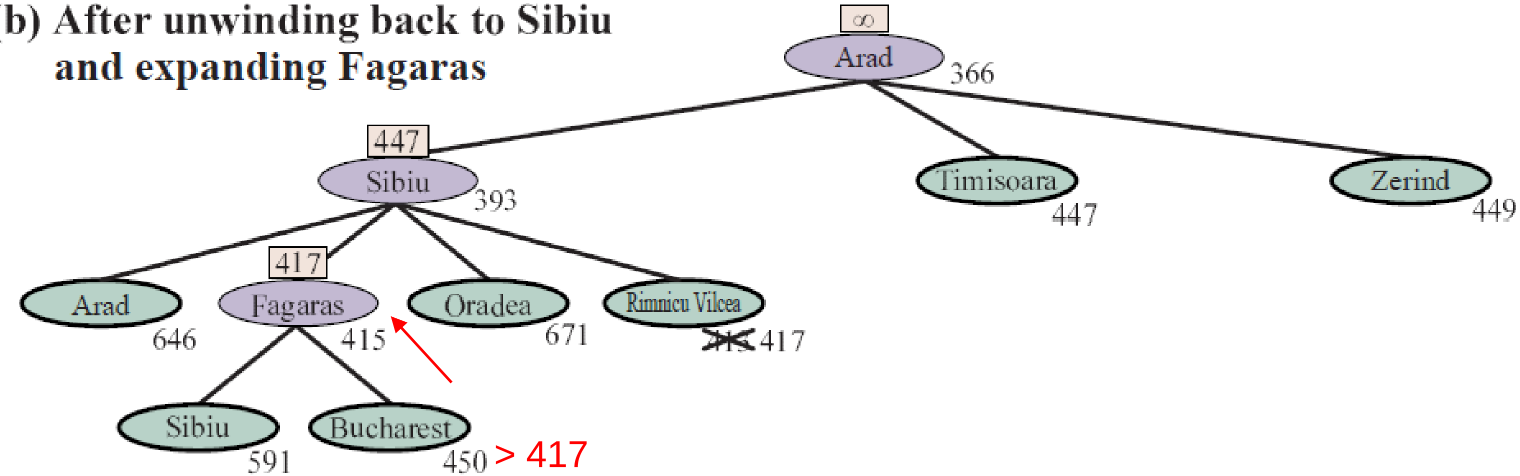
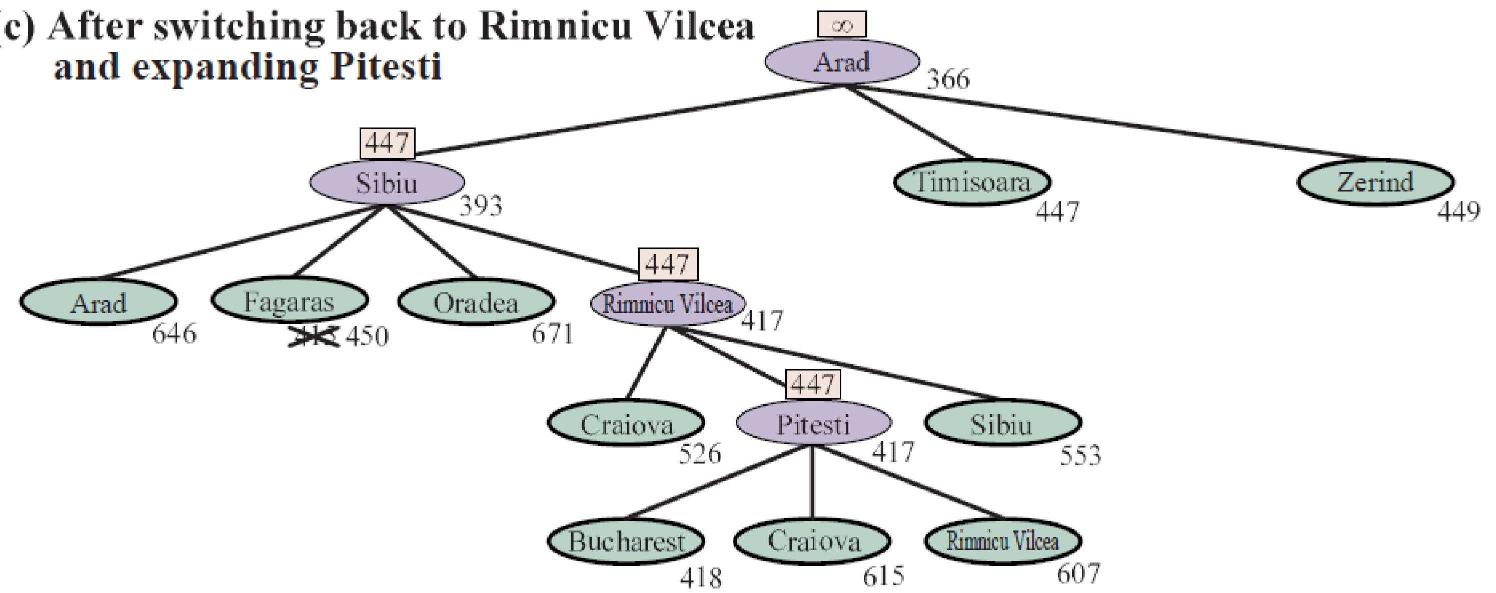


(c) After switching back to Rimnicu Vilcea and expanding Pitesti



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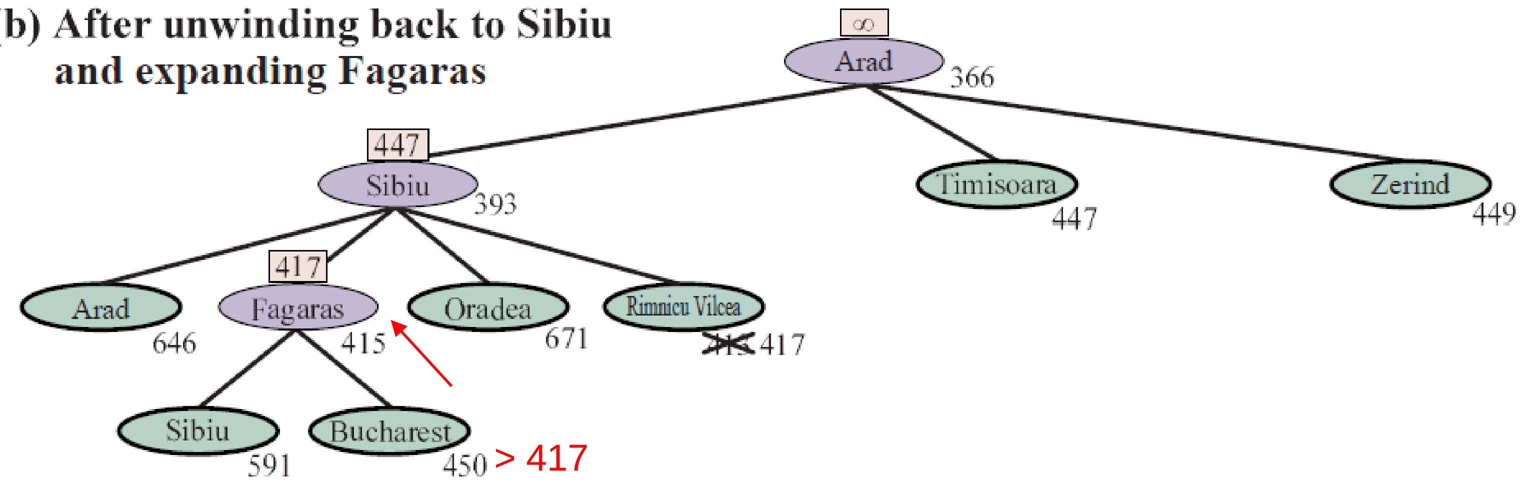

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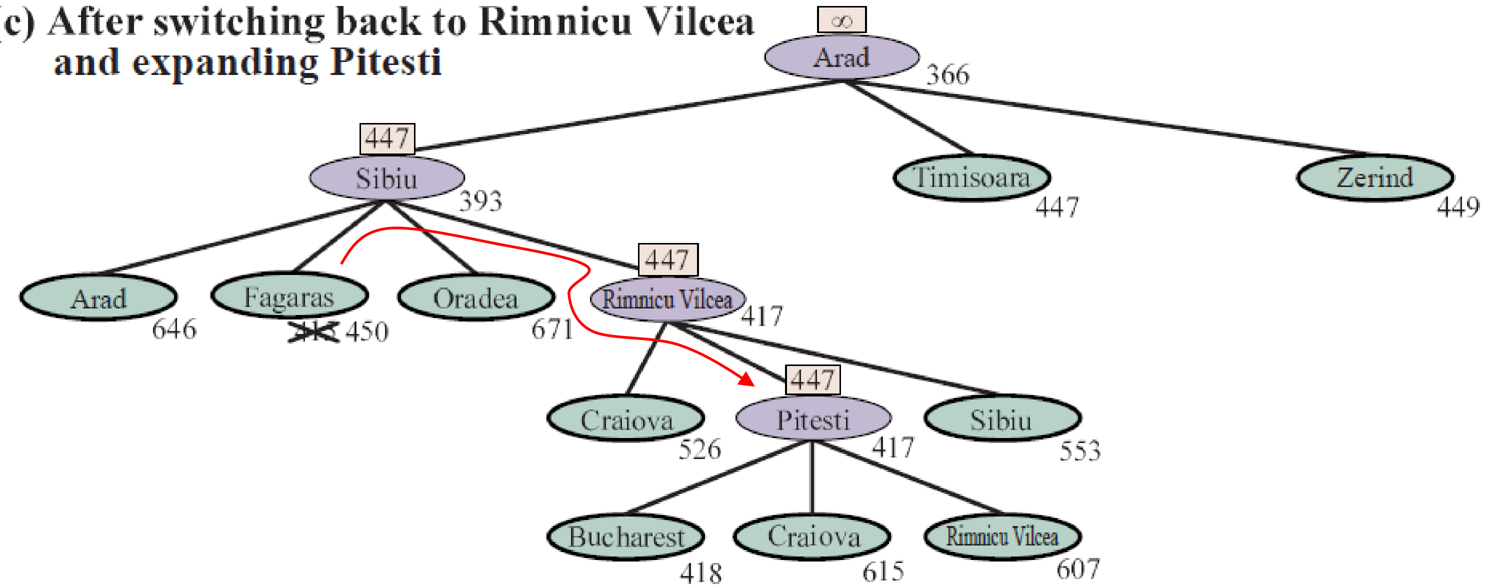

RBFS Summary

- Optimal with admissible heuristic function $h(n)$.

- Space complexity $O(bd)$.

  branching factor depth

- Time complexity difficult to analyze, depending on

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  - how often the best path changes

- Slightly more efficient than IDA*.

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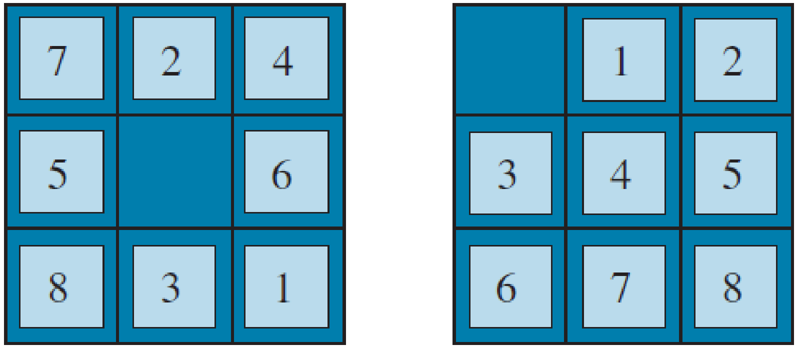

Start State   Goal State

Order: Every tile with a smaller number should appear either *above* or to the *left* of any tile with a larger number.

Inversion: One violation of this order.

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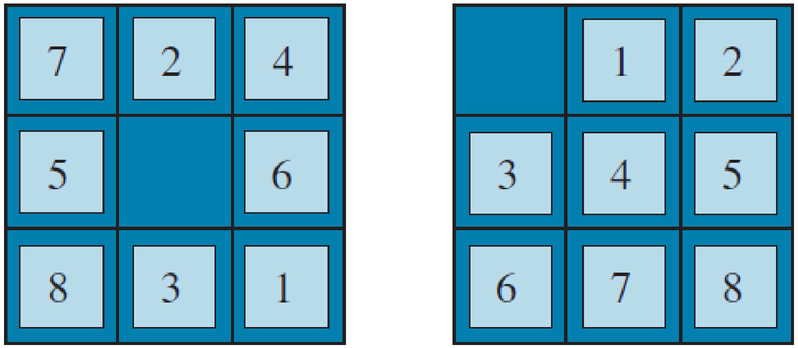

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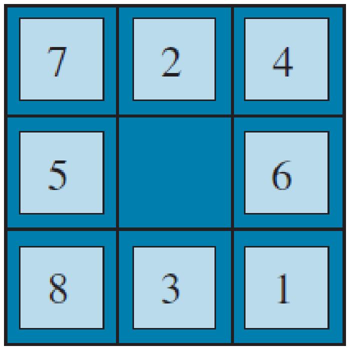

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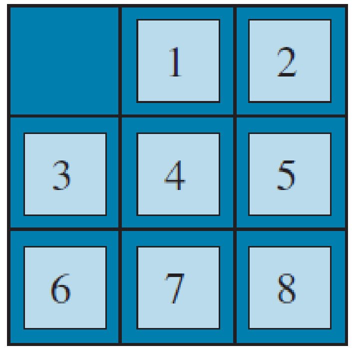

Goal State

0 inversion

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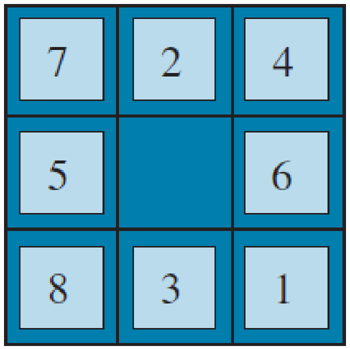

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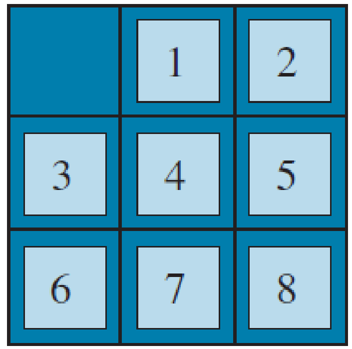

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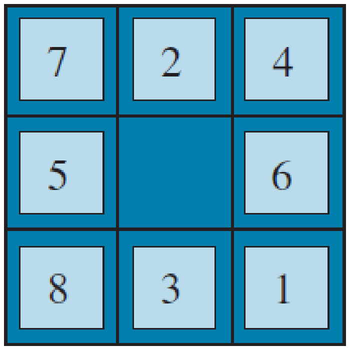

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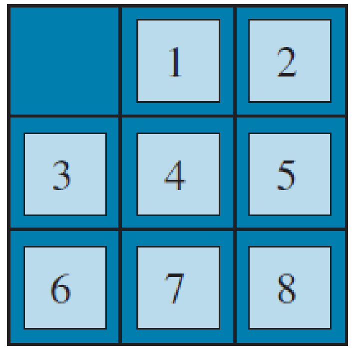

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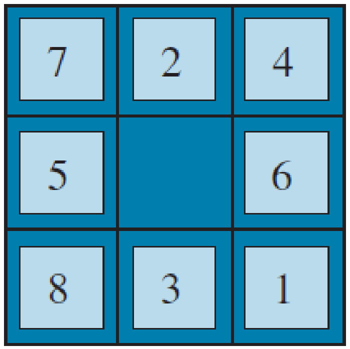

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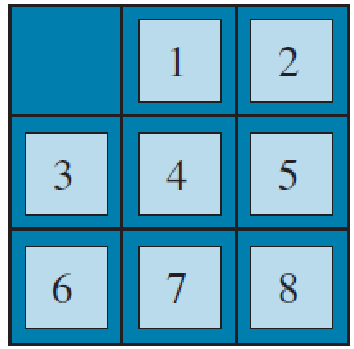

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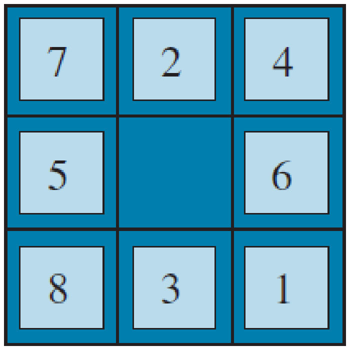

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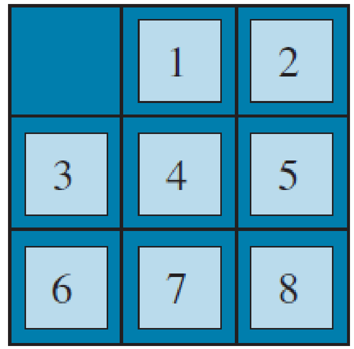

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7 appears before 1, 2, 3, 4, 5, 6

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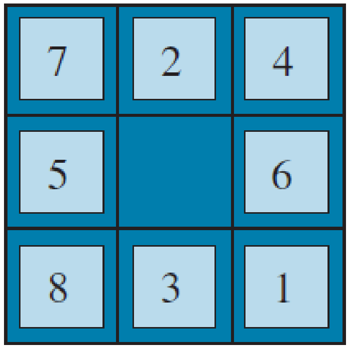

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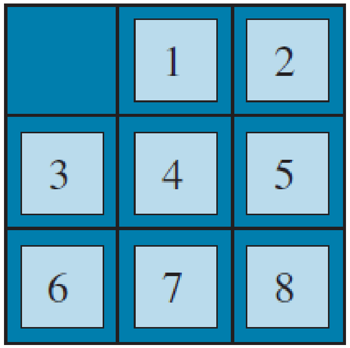

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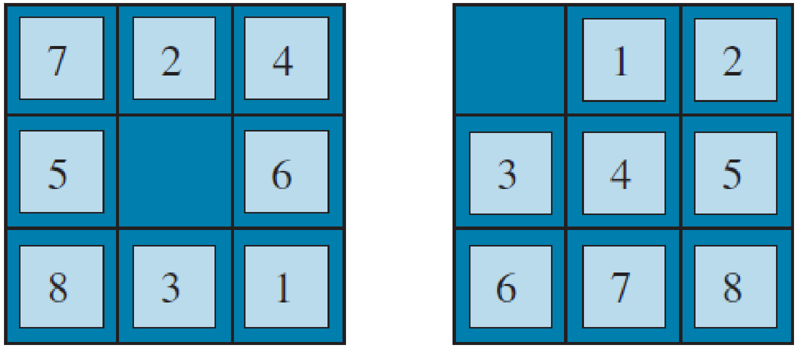

Start State

Goal State

16 inversions

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II. 8-Puzzle: Solvability

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

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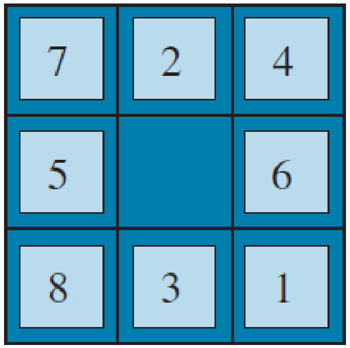
Theorem An 8-puzzle is solvable if and only if the numbers of inversions in the start and goal states differ by an even integer.

II. 8-Puzzle: Solvability



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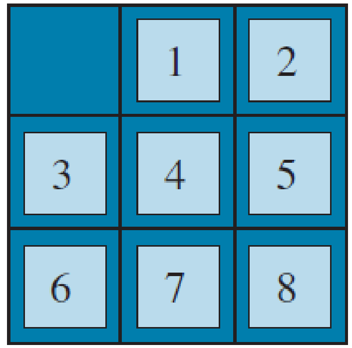

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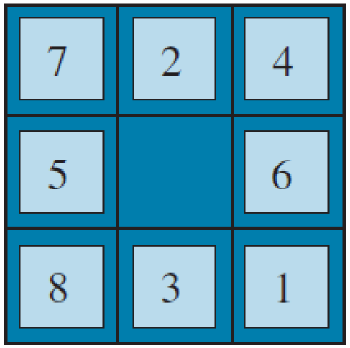
Theorem An 8-puzzle is solvable if and only if the numbers of inversions in the start and goal states differ by an even integer.

$\frac{9!}{2} = 181{,}400$ reachable states from start.

II. 8-Puzzle: Solvability



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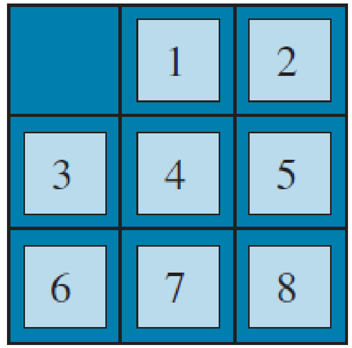

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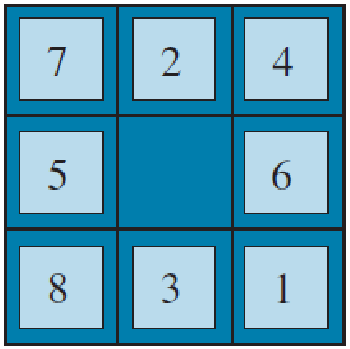
7 appears before 1, 2, 3, 4, 5, 6

8 appears before 1, 3

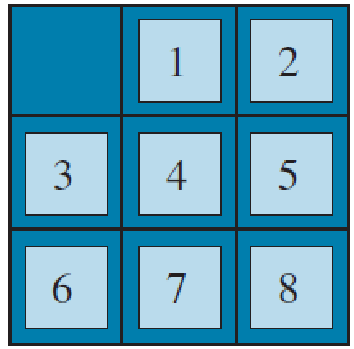
$\frac{9!}{2} = 181{,}400$ reachable states from start.

Theorem An 8-puzzle is solvable if and only if the numbers of inversions in the start and goal states differ by an even integer.

II. 8-Puzzle: Solvability



Start State



Goal State

Order: Every tile with a smaller number should appear either *above* or to the *left* of any tile with a larger number.

Inversion: One violation of this order.

16 inversions

0 inversion

2 appears before 1

3 appears before 1

4 appears before 1, 3

5 appears before 1, 3

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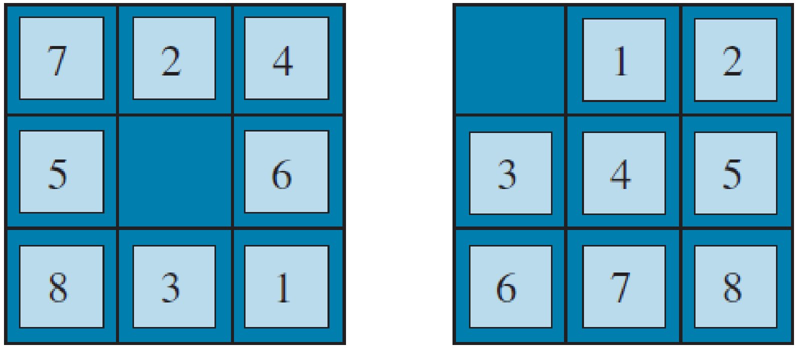
Theorem An 8-puzzle is solvable if and only if the numbers of inversions in the start and goal states differ by an even integer.

$\frac{9!}{2} = 181{,}400$ reachable states from start.

$\frac{16!}{2} > 10^{13}$ reachable states for the 15-puzzle!

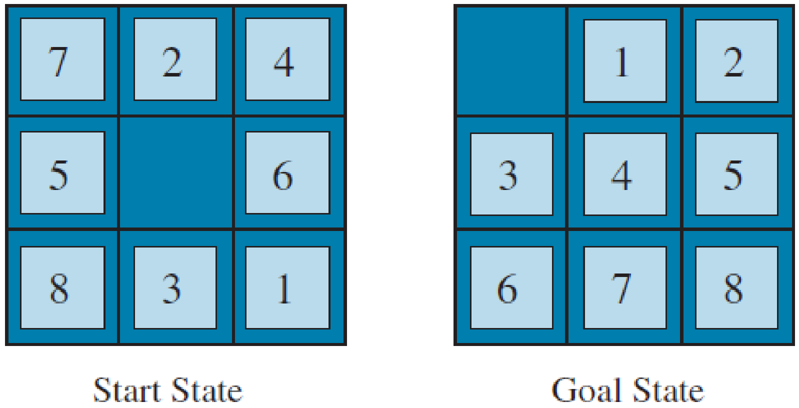
Two Heuristics



Start State   Goal State

Heuristics are needed for searching the vast state space.

Two Heuristics



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- h_1 = # tiles misplaced

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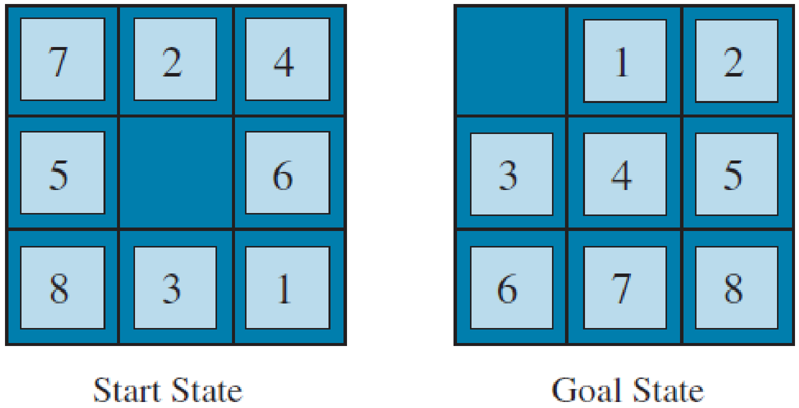

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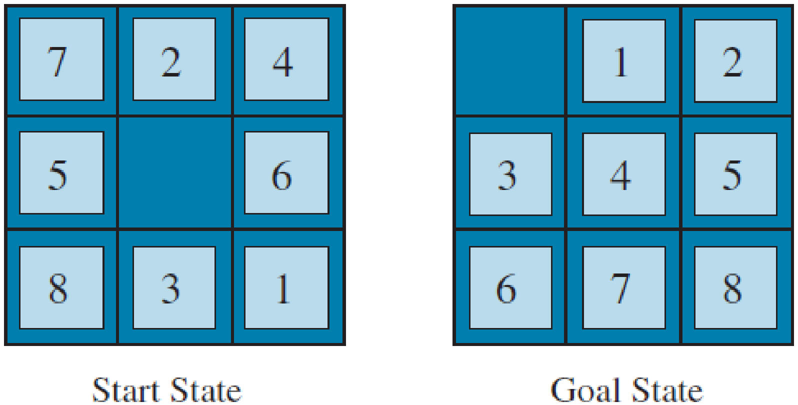

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Misplaced tiles:
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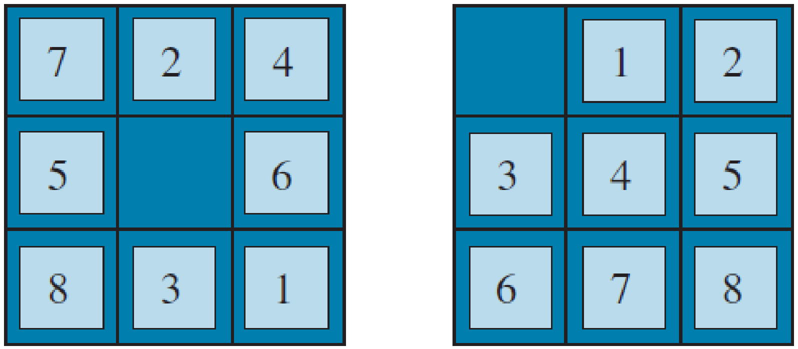

Start State Goal State

Heuristics are needed for searching the vast state space.

♦ h_1 = # tiles misplaced

Admissible: any tile out of place will require ≥ 1 move to fix.

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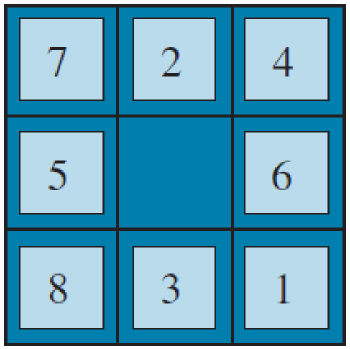

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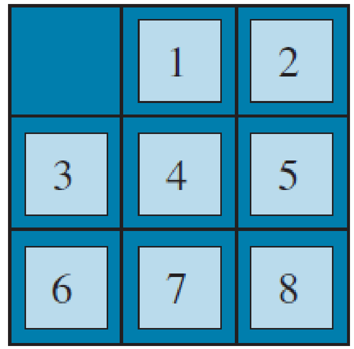

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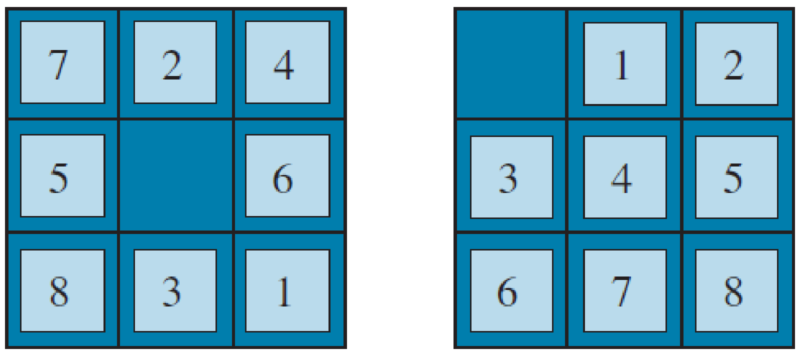

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Manhattan distance	3	1	2	2	2	3	3	2

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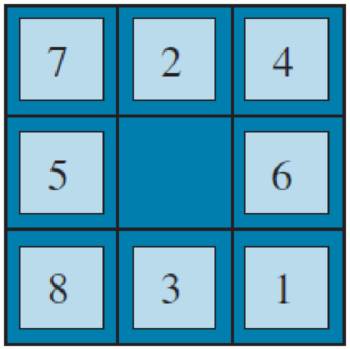

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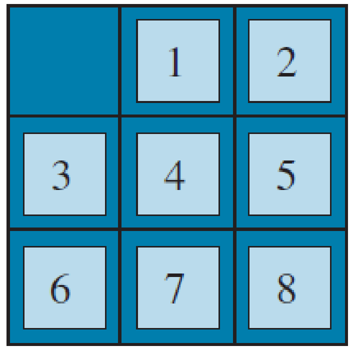

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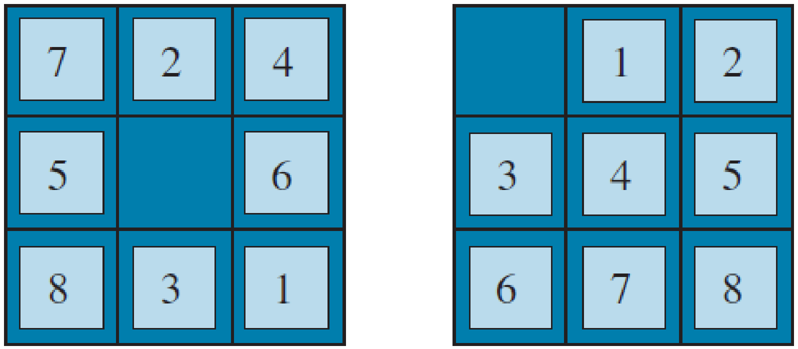
$$h_2 = 18$$

Two Heuristics



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  Admissible: every move reduces the Manhattan distance of only one tile by ≤ 1.

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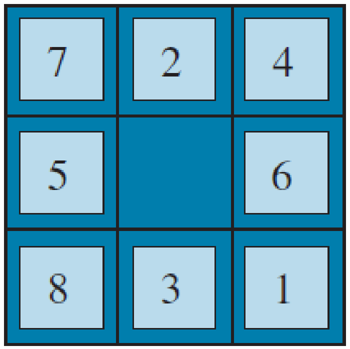

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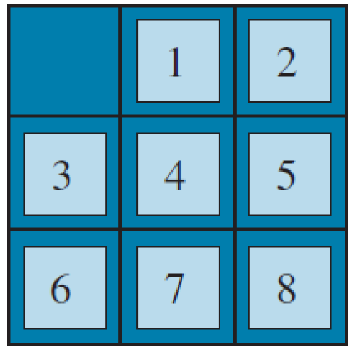

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Tile	1	2	3	4	5	6	7	8
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$\Downarrow$

$$h_2 = 18$$

Neither heuristic overestimates the shortest solution (26 actions for the problem instance).

Heuristic Accuracy on Performance

Quality of a heuristic is often measured by the effective branching factor.

If A* generates N nodes to find a solution at depth d, then its *effective branching factor* b^* is the root of the following equation:

$$N + 1 = 1 + b^* + (b^*)^2 + \cdots + (b^*)^d$$

Intuitively, the $N + 1$ nodes handled by A* would fill a tree of height d in which every node at depth $< d$ has exactly b^*children.

e.g. A* finds a solution at depth 5 using 52 nodes has $b^* = 1.92$.

Performance Comparison on 8-Puzzle

	Search Cost (nodes generated)			Effective Branching Factor		
d	BFS	$A^*(h_1)$	$A^*(h_2)$	BFS	$A^*(h_1)$	$A^*(h_2)$
6	128	24	19	2.01	1.42	1.34
8	368	48	31	1.91	1.40	1.30
10	1033	116	48	1.85	1.43	1.27
12	2672	279	84	1.80	1.45	1.28
14	6783	678	174	1.77	1.47	1.31
16	17270	1683	364	1.74	1.48	1.32
18	41558	4102	751	1.72	1.49	1.34
20	91493	9905	1318	1.69	1.50	1.34
22	175921	22955	2548	1.66	1.50	1.34
24	290082	53039	5733	1.62	1.50	1.36
26	395355	110372	10080	1.58	1.50	1.35
28	463234	202565	22055	1.53	1.49	1.36

Figure 3.26 Comparison of the search costs and effective branching factors for 8-puzzle problems using breadth-first search, A* with h_1 (misplaced tiles), and A* with h_2 (Manhattan distance). Data are averaged over 100 puzzles for each solution length d from 6 to 28.

#Misplaced Tiles vs Manhattan Distance

Given two heuristic functions h_1 and h_2, we say h_2 *dominates* h_1 if $h_2(n) \geq h_1(n)$ at every node n.

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$$*\ \ 2\ 2\ 2\ 1\ 1\ 1\ (n) \leq h_2(n) < C^* - g(n)$$

$\Downarrow$

The node n will be expanded by A* with h_1.

♣ h_1 might cause other nodes to be expanded as well.

Generating Heuristics by Relaxation

- An admissible heuristic can be derived from exact solution cost of a *relaxed problem*.

with fewer restrictions on the actions

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h_2 would give the length of the shortest solution.

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The heuristic is consistent.

Heuristics from Formal Specification

Formal specification of a problem (8-puzzle):

A tile can move from square X to square Y if X is adjacent to Y and Y is blank.

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- Program ABSOLVER generates heuristics automatically from problem definitions, including the best one for the 8-puzzle and the first one for the Rubik's Cube puzzle.

Multiple Heuristics Available

Admissible heuristics $h_1, h_2, \cdots, h_k$ are available but none is clearly better than the others.

Use a composite heuristic:

$$h(n) = \max\{h_1(n), h_2(n), \cdots, h_k(n)\}$$

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