Homework 4
Com S 311
Due: October 25 11:59 pm

5% bonus for submission by October 24, 11:59 pm

10% penalty for late submission by October 26, 11:59 pm

No submissions accepted after October 26

Note: The "Do not grade" option is not available for this HW

For all algorithm design problems, part of the grade depends on efficiency. For every graph $G = (V, E)$, the vertex set is $\{1, 2, \cdots, n\}$. We use n to denote the number of vertices and m to denote number of edges. Unless otherwise stated, all graphs are represented as adjacency lists. Each problem is worth 40 points.

1. Let $G = (V, E)$ be a directed graph. Define a graph $G^2 = (V', E')$ as follows: $V' = V$; $\langle u, v\rangle \in E'$ if $u \neq v$ and there is a a path of length exactly 2 between u and v in G. Suppose that a directed graph G is given as adjacency list. Give an algorithm to compute G^2. Derive its runtime.

2. Give an algorithm that, given an undirected graph G and node s, creates an array `ShortestCount` in which `ShortestCount[i]` is the *number* of shortest paths from s to vertex i. Provide a proof by induction that your algorithm is correct. Derive its runtime.

   (*Tip:* Start with the BFS algorithm as given in the text, in which nodes are organized into layers L_i based on distance from s, and update the counts as you build the tree.)

3. Given a directed graph $G = (V, E)$, a vertex v is called a *sink* if there are no outgoing edges from v.

   - Prove that every DAG (Directed Acyclic Graph) has a sink.
   - Use the above to define at an algorithm (that is different from the one described in the text) that computes topological ordering of a DAG

4. Let $G = (V, E)$ be a directed graph and let $S_1, \cdots, S_k$ be its strongly connected components. Define a new graph $G' = (V', E')$ as follows. $V' = \{1, \cdots, k\}$. Place an edge from i to j in G' if there is a vertex $u \in S_i$ and a vertex $v \in S_j$ such that $\langle u, v\rangle \in E$. Prove that G' is a DAG.

GUIDE LINES:

- Please write your recitation number, time and TA name.
- You must work on the homework problems on your own. You should write the final solutions alone, without consulting any one. Your writing should demonstrate that you understand the proofs completely.
- When proofs are required, you should make them clear and rigorous.
- Any concerns about grading should be made within one week of returning the homework.
- **Please submit your HW via Canvas. If you type your solutions, then please submit pdf version. If you hand-write your solutions, then please scan your solutions and submit a pdf version. Please make sure that the quality of the scan is good, and your hand writing is legible. HW's submitted in incorrect format (non pdf) may incur a penalty of 20%**
- If you hand writing is not legible or the quality of the scan is poor, your homework will not be graded.