

Bayes' Rule

Outline

I. Bayes' rule

II. Conditional independence

III. Naïve Bayes model

I. Bayes' Rule

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It tells us how often b happens given that a happens,
when we know:

- how often a happens given that b happens, and
- how likely a is on its own, and
- how likely b is on its own.

Bayes' Rule for Multivalued Variables

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A more generalized version conditionalized on some evidence e :

$$P(Y | X, e) = \frac{P(X | Y, e)P(Y | e)}{P(X | e)}$$

Applying Bayes' Rule

- Perceive as the evidence the *effect* of some unknown *cause*.
- Determine the cause.

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The doctor knows $P(\text{symptoms} \mid \text{disease})$ and wants to derive a diagnosis $P(\text{disease} \mid \text{symptoms})$.

Example

The doctor knows:

- The disease meningitis causes a patient to have a stiff neck 70% of time.
- The prior probability that any patient has meningitis is $1/50,000$.
- The prior probability that any patient has a stiff neck is $1/100$.

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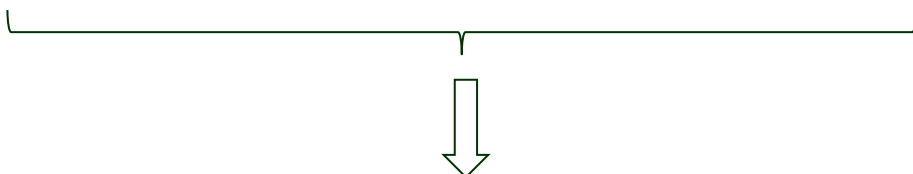
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normalization constant to make
the entries in $P(Y | X)$ sum to 1.

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What happens when we have two or more pieces of evidences?

- Suppose we know the full joint distribution.

	<i>toothache</i>		\neg <i>toothache</i>	
	<i>catch</i>	\neg <i>catch</i>	<i>catch</i>	\neg <i>catch</i>
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$\implies 2^n$ possible combinations of observed values needed to determine $P(\text{toothache} \wedge \text{catch} \wedge \dots \mid \text{Cavity})!$

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Decomposition into smaller conditional assertions.

Decomposition of Joint Distribution

$P(\text{Toothache}, \text{Catch}, \text{Cavity})$

Decomposition of Joint Distribution

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3 tables of dimensions 2×2 , 2×2 , and 2×1
 with a total of $2 + 2 + 1 = 5$ independent numbers
 (which appear in the first row of every table).

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- ◆ Conditional independence assertions allow probabilistic systems to scale up.
- ◆ They are more commonly available than absolute independence assertions.
- ◆ Decomposition of large probabilistic domains through conditional independence is one of the most important recent developments in AI.

III. Naive Bayes Model

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Classify each sentence into a Category.

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$P(\text{HasWord}_6 = \text{true} \mid \text{Category} = \text{business}) \approx 0.37$
// 37% of articles about business contain word 6, “stocks”.

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appearances/disappearances of the key words.

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- ◆ Language determination (to detect the language a text is written in)
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- ◆ Sentiment analysis (to identify positive and negative customer sentiments in social media)
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Naïve Bayes models are not used in

- ♠ Medical diagnosis (which requires more sophisticated models)