

Recitation 14

- Here is a set of additional problems. They range from being very easy to very tough. The best way to learn the material in 310 is to solve problems on your own.
 - Feel free to ask (and answer) questions about this problem set on Piazza.
 - This is an **optional** problem set; do not turn this in for grading.
 - While you don't have to turn this in, be warned that this material **can** appear in a quiz or exam.
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1. *Craps* is a game played in casinos, where players roll a pair of ordinary (6-sided) dice. If the total of the numbers on the rolled dice on the first roll is 2, 3, or 12, the person “craps out”, i.e., they lose the bet. If the total is 7 or 11, they win the bet. In other cases, the game continues (and other rules kick in).
 - a. What is the probability that you roll a 7 on the first roll?
 - b. What is the probability that you **win** the bet on the first roll?
 - c. What is the probability that you **lose** the bet on the first roll?
 - d. What is the probability that you roll a 7 on the second roll?
2. If A_1 and A_2 are arbitrary events of any sample space, then prove that

$$P(A_1 \cup A_2) \leq P(A_1) + P(A_2).$$

In words, the probability of the union of two events can never be greater than the sum of the probabilities of the individual events. This is called the *union bound*, and is one of the foundational results of probability theory. (Hint: use the Principle of Inclusion-Exclusion.)

3. We discuss a curious phenomenon in probability where a certain trend seems to appear in two groups of data but reverses course when the data is combined. Bart and Lisa take a series of 5 courses spread over 2 semesters. They are all CPRE courses (and hence very difficult.) In Semester 1, Lisa does not study at all, and has a 0% probability of passing any given course, while Bart has a marginally higher (25%) probability of passing any course.
 - a. If Bart takes 4 courses and Lisa takes 1 course in Semester 1, how many courses is each student expected to pass?
 - b. In Semester 2, they both improve; Lisa has a 75% probability of passing the course, while Bart has a 100% probability of passing any course. If Bart takes 1 course in Semester 2 and Lisa takes 4 courses, how many courses is each student expected to pass?
 - c. Calculate the total number of courses that each student is expected to pass, and argue that Lisa is the better student (even though Bart shows a higher probability of doing well in *both* semesters.)
 - d. What is the source of this paradox?