ComS 311
Recitation 3, 2:00 Monday
Homework 3

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1a) $T(n) \leq 3T(\frac{n}{2}) + Cn^2, \ \ T(2) \leq c$

$3[3T(\frac{n}{4}) + c(\frac{n}{2})^2] + cn^2$
$9T(\frac{n}{4}) + 3c(\frac{n}{2})^2 + cn^2$

$9[3T(\frac{n}{8}) + c(\frac{n}{4})^2] + 3c(\frac{n}{2})^2 + cn^2$
$27T(\frac{n}{8}) + 9c(\frac{n}{4})^2 + 3c(\frac{n}{2})^2 + cn^2$

$27[3T(\frac{n}{16}) + c(\frac{n}{8})^2] + 9c(\frac{n}{4})^2 + 3c(\frac{n}{2})^2 + cn^2$
$81T(\frac{n}{16}) + 27c(\frac{n}{8})^2 + 9c(\frac{n}{4})^2 + 3c(\frac{n}{2})^2 + cn^2$

$3^4T(\frac{n}{2^4}) + 3^3c(\frac{n}{2^3})^2 + 3^2c(\frac{n}{2^2})^2 + 3^1c(\frac{n}{2^1})^2 + 3^0c(\frac{n}{2^0})^2$
$3^4T(\frac{n}{2^4}) + \frac{3^3}{2^{3*2}}cn^2 + \frac{3^2}{2^{2*2}}cn^2 + \frac{3^1}{2^{1*2}}cn^2 + \frac{3^0}{2^{0*2}}cn^2$

Final term is $3^kT(\frac{n}{2^k})$. Assuming that $n/2^k = 2$ so that T(2) = c
$\frac{n}{2^k} = 2 \quad \Rightarrow \quad n = 2^k * 2 \quad \Rightarrow \quad n = 2^{k+1} \quad \Rightarrow$
$log(n) = k + 1 \quad \Rightarrow \ k = log(n) - 1$
$\therefore$ end term $== 3^{log(n)-1} * c$

Full: $3^{log(n)-1} * c + cn^2 \sum_{k=0}^{log(n)-1} \frac{3^k}{2^{k*2}}$
However, $\lim_{k\to\infty} \frac{3^k}{2^{k*2}} = 0$
$\Rightarrow 3^{log(n)-1} * c + cn^2 * (0) \quad \Rightarrow$
$c * 3^{log(n)-1}$

1b) $T(n) \leq 2T(\frac{n}{2}) + Cnlog(n), \quad T(2) \leq c$

$2[2T(\frac{n}{4}) + c(\frac{n}{2})log(\frac{n}{2})] + cnlog(n)$
$4T(\frac{n}{4}) + 2c(\frac{n}{2})log(\frac{n}{2}) + cnlog(n)$

$4[2T(\frac{n}{8}) + c(\frac{n}{4})log(\frac{n}{4})] + 2c(\frac{n}{2})log(\frac{n}{2}) + cnlog(n)$
$8T(\frac{n}{8}) + 4c(\frac{n}{4})log(\frac{n}{4}) + 2c(\frac{n}{2})log(\frac{n}{2}) + cnlog(n)$

$8[2T(\frac{n}{16}) + c(\frac{n}{8})log(\frac{n}{8})] + 4c(\frac{n}{4})log(\frac{n}{4}) + 2c(\frac{n}{2})log(\frac{n}{2}) + cnlog(n)$
$16T(\frac{n}{16}) + 8c(\frac{n}{8})log(\frac{n}{8}) + 4c(\frac{n}{4})log(\frac{n}{4}) + 2c(\frac{n}{2})log(\frac{n}{2}) + cnlog(n)$

End term: $2^k T(\frac{n}{2^k})$
Assuming $\frac{n}{2^k} = 2$ so that $T(\frac{n}{2^k}) = T(2) = c$

$\frac{n}{2^k} = 2 \quad \Rightarrow \quad n = 2^k * 2 = 2^{k+1} \quad \Rightarrow$
$log(n) = k + 1 \quad \Rightarrow \quad k = log(n) - 1$

Full: $2^{log(n)-1} * c + cn \sum_{k=0}^{log(n)-1} log(\frac{n}{2^k})$
As n increases, the summed term $\rightarrow \frac{n}{2^{log(n)-1}}$
$\therefore$ the solution $= c * 2^{log(n)-1} + cn\frac{n}{2^{log(n)-1}}$