Stat 330
Homework 7

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1)

(a)

$$\text{PDF} = \begin{cases} 1/5 & for\ 5 \leq x \leq 10 \\ 0 & otherwise \end{cases} \qquad \text{CDF} = \begin{cases} 0 & for\ t \leq 5 \\ (t-5)/5 & for\ 5 \leq t \leq 10 \\ 1 & for\ t \geq 10 \end{cases}$$

(b) E(X) = 15/2 = 9:07.500

(c) P(X<7) = 1 - P(X>7) $\Rightarrow \int_7^{10} \frac{1}{5}/5 = x\Big|_7^{10}/5$ = 10 - 7 / 5 $\Rightarrow$ 1 - 0.6 = 0.4

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2)

(a) E(X) = $\frac{1}{\lambda}$ $\Rightarrow$ There is 1/20 an hour per hit, so E(X) = $\frac{1}{20}$ $\Rightarrow \lambda = 20$

(b) E(X) = $\frac{1}{20}$ = 0.05 an hour

(c) Exponential: X $\sim$ Exp(20)

(d) 20 min is .333 of an hour, so P(X<.333) = 1 - $e^{-20(.333)}$ = 0.9987

(e) $p_{x,y}(1,2) = 0$

(f) 5/20 = 0.25 of an hour

(g) X $\sim$ Pois(20) $\Rightarrow$ P(X$\leq$5) from table = 0.00000027

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3)

(a) X $\sim$ Exp(5)

(b)

$$\text{PDF} = \begin{cases} 5e^{-5x} & for\ x > 0 \\ 0 & otherwise \end{cases} \qquad \text{CDF} = \begin{cases} 0 & for\ t \leq 0 \\ 1 - e^{-5t} & for\ t > 0 \end{cases}$$

(c) P(X < 5) = 1 - e$^{-5(1)}$ = 0.9987

(d) P(hit in 5 min | no hit in first 2 min) = P(X $\leq$ 1 | X > .4) $\Rightarrow \frac{P(X \leq 1 \cap X > .4)}{P(X > .4)} \Rightarrow$

$\frac{P(.4 \leq 1)}{P(X > .4)} \Rightarrow \frac{F_X(1) - F_X(.4)}{1 - F_X(.4)} = \frac{5e^{-5(1)} - 5e^{-5(.4)}}{1 - 5e^{-5(.4)}} = 0.95$

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4)

(a) A computer would require special maintenance every 15 months on average.
P(X < 9) = 1 - e$^{-(1)(.6)}$ = 1 - .5488 = .4512

(b) P(X $\leq$ 1.06 | X > .8) $\Rightarrow \frac{P(X \leq 1.06 \cap X > .8)}{P(X > .8)} \Rightarrow$

$\frac{P(.8 \leq 1.06)}{P(X > .8)} \Rightarrow \frac{F_X(1.06) - F_X(.8)}{1 - F_X(.8)} = \frac{(1 - e^{-15(1.06)}) - (1 - e^{-15(.8)})}{1 - (1 - 5e^{-15(.8)})} = 0.98$

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5)

(a) With X $\sim$ Exp(5), P(X > 15) = 1 - P(X < 15) = 1 - (1 - e$^{-5(3)}$) = 3.05 * 10^{-7}

(b) This can be imagined as waiting 10 minutes for one person $\Rightarrow$
P(X > 10) = 1 - P(X < 10) = 1 - (1 - e$^{-5(2)}$) = 4.53 * 10^{-5}