## Homework 2

Please scan and upload your assignments to Canvas on or before February 6, 2019.

- You must do your work independently and on your own. That means no collaborations!
- However, you *can* ask questions about the homework on Piazza. You can also answer others' questions. It is possible that your question is already answered there, so check Piazza regularly.
- Scores on late submissions will be penalized by 50% for every day submitted late. Be on time!
- 1. (10 points) Translate the following arguments into symbolic logic. Give your reasoning whether each argument is valid or not. If the logical expressions corresponds to a rule of inference discussed in lecture or in notes, identify the name of that rule.
  - a. If I go to the movies, I won't finish my homework. If I don't finish my homework, I won't do well on the exam. Therefore, if I go to the movies, I won't do well on the exam.
  - b. If this number is larger than 2, then its square is larger than 4. This number is not larger than 2. Therefore, the square of this number is not larger than 4.
  - c. Mary knows C and Mary knows Python. Therefore, Mary knows Python.
- 2. **(10 points)** Recall that a number is called *rational* if it can be written as the ratio of two integers. Numbers that are not rational are called *irrational*. Prove **by contraposition** the following statement:
  - "If r is irrational, then  $\sqrt{r}$  is irrational."
- 3. (10 points) Most geometry that we learn in high-school is an instance of *Euclidean* geometry. A central axiom of Euclidean geometry is that the sum of angles in a triangle equals 180°. Using this axiom, prove by contradiction that in Euclidean geometry, if two lines are each perpendicular to a given line segment, then the two lines have to be parallel.
- 4. (10 points) An integer n is called *frumpy* if  $n^3 + 2n^2 + 4n + 6$  is an odd number. Prove that all frumpy numbers are themselves odd numbers. (Clearly state your method of proof in the beginning.)
- 5. (10 points) Given n arbitrary real numbers  $a_1, a_2, \ldots, a_n$ , prove that at least one of these numbers is less than or equal to their average.