## EE 330 Lecture 13

### Devices in Semiconductor Processes

- Diodes
- Capacitors
- MOSFETs

### Exam 1 Friday Sept 27

- Students may bring 1 page of notes
- HW assignment for week of Sept 23 due on Wed Sept 25 at beginning of class
- No 5:00 p.m extension so solutions can be posted
- Scientific calculators will be provided no use of any personal electronic devices of any kind
- Those with special accommodation needs, please send me an email message or contact me so arrangements can be made
- Review session to be determined

# Temperature Coefficients

Used for indicating temperature sensitivity of resistors & capacitors

#### For a resistor:

$$TCR = \left(\frac{1}{R} \frac{dR}{dT}\right)_{\text{op. temp}} \bullet 10^6 \text{ ppm/}^{\circ}\text{C}$$

This diff eqn can easily be solved if TCR is a constant

$$R(T_2) = R(T_1)e^{\frac{T_2-T_1}{10^6}TCR}$$

$$R(T_2) \approx R(T_1) \left[ 1 + (T_2 - T_1) \frac{TCR}{10^6} \right]$$

Identical Expressions for Capacitors

# Voltage Coefficients

Used for indicating voltage sensitivity of resistors & capacitors

#### For a resistor:

$$VCR = \left(\frac{1}{R} \frac{dR}{dV}\right)_{\text{ref voltage}} \bullet 10^6 \text{ ppm/V}$$

This diff eqn can easily be solved if VCR is a constant

$$\mathbf{R}(\mathbf{V_2}) = \mathbf{R}(\mathbf{V_1}) e^{\frac{\mathbf{V_2} - \mathbf{V_1}}{10^6} \mathbf{VCR}}$$

$$R(V_2) \approx R(V_1) \left[ 1 + (V_2 - V_1) \frac{VCR}{10^6} \right]$$

Identical Expressions for Capacitors

### Temperature and Voltage Coefficients

- Temperature and voltage coefficients often quite large for diffused resistors
- Temperature and voltage coefficients often quite small for poly and metal resistors

V١

Type of layer	Sheet Resistance Ω/□	Accuracy (absolute)	Temperature Coefficient ppm/°C	Voltage Coefficient ppm/V
n + diff	30 - 50	20 - 40	200 - 1K	50 - 300
p + diff	50 -150	20 - 40	200 - 1K	50 - 300
n - well	2K - 4K	15 - 30	5K	10K
p - well	3K - 6K	15 - 30	5K	10K
pinched n - well	6K - 10K	25 - 40	10K	20K
pinched p - well	9K - 13K	25 - 40	10K	20K
first poly	20 - 40	25 - 40	500 - 1500	20 - 200
second poly	15 - 40	25 - 40	500 - 1500	20 - 200

(relative accuracy much better and can be controlled by designer)

Example: Determine the percent change in resistance of a 5K Polysilicon resistor as the temperature increases from 30°C to 60°C if the TCR is constant and equal to 1500 ppm/°C

$$R(T_2) \cong R(T_1) \left[ 1 + (T_2 - T_1) \frac{TCR}{10^6} \right]$$

$$R(T_2) \cong R(T_1) \left[ 1 + (30^{\circ}C) \frac{1500}{10^{6}} \right]$$
  
 $R(T_2) \cong R(T_1) \left[ 1 + .045 \right]$ 

$$R(T_2) \cong R(T_1)[1.045]$$

Thus the resistor increases by 4.5%

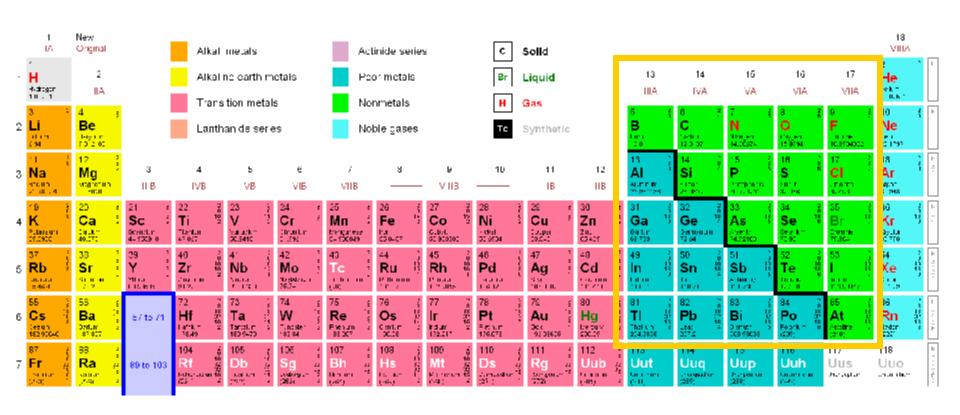
### Basic Devices and Device Models

Resistor

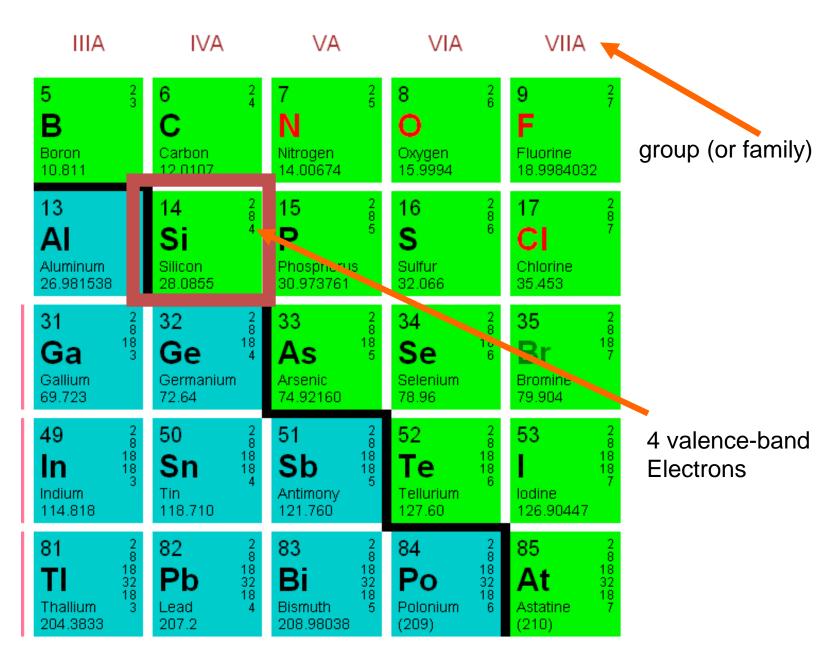


- Capacitor
- MOSFET
- BJT

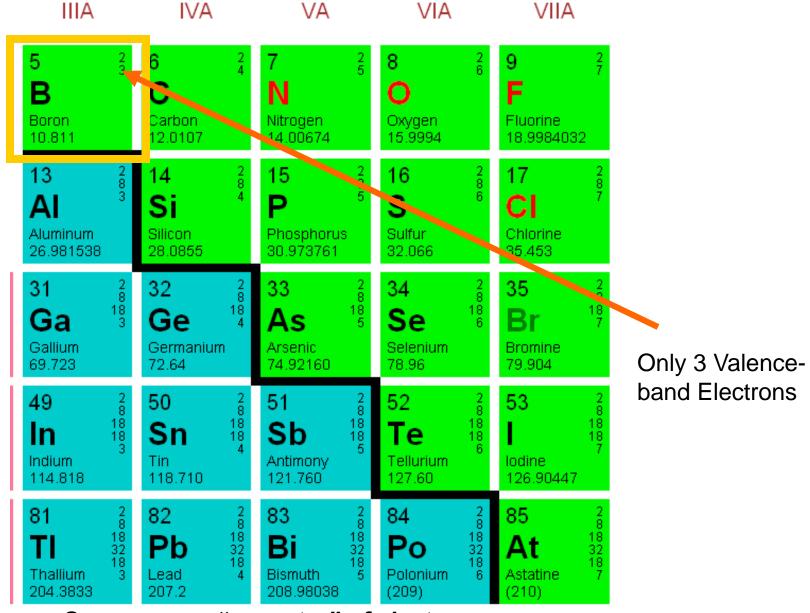
## Periodic Table of the Elements



IIIA	IVA	VA	VIA	VIIA
5 2 8 Boron 10.811	6 2 C Carbon 12.0107	7 2 N Nitrogen 14.00674	8 2 6 Oxygen 15.9994	9 <sup>2</sup> / <sub>7</sub> Fluorine 18.9984032
13 2 8 3 3 Aluminum 26.981538	14 2 Si Silicon 28.0855	15 2 8 5 P Phosphorus 30.973761	16	17 2 8 7 CI Chlorine 35.453
31 2 8 18 3 18 3 Gallium 69.723	32 8 18 18 4 Germanium 72.64	33 <b>As</b> Arsenic 74.92160	34	35 Br Bromine 79.904
49 8 18 18 18 18 18 18 18 18 18 18 18 18 1	50 Sn 18 18 18 18 18 18 18 18 18	51 <b>Sb</b> Antimony 121.760	52 2 8 18 18 18 18 18 18 18 18 18 18 18 18 1	53 2 8 18 18 18 18 7 lodine 126.90447
81 2 8 18 18 32 18 18 32 18 18 32 204.3833	82 2 <b>Pb</b> 18 18 32 18 Lead 4 207.2	83 2 8 Bi 18 32 18 Bismuth 5 208.98038	84 2 8 18 32 18 90 6 (209)	85 2 8 At 18 32 18 Astatine 7 (210)



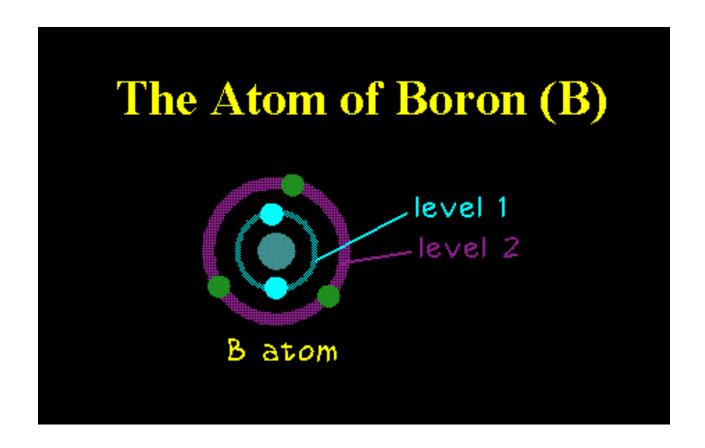
All elements in group IV have 4 valence-band electrons



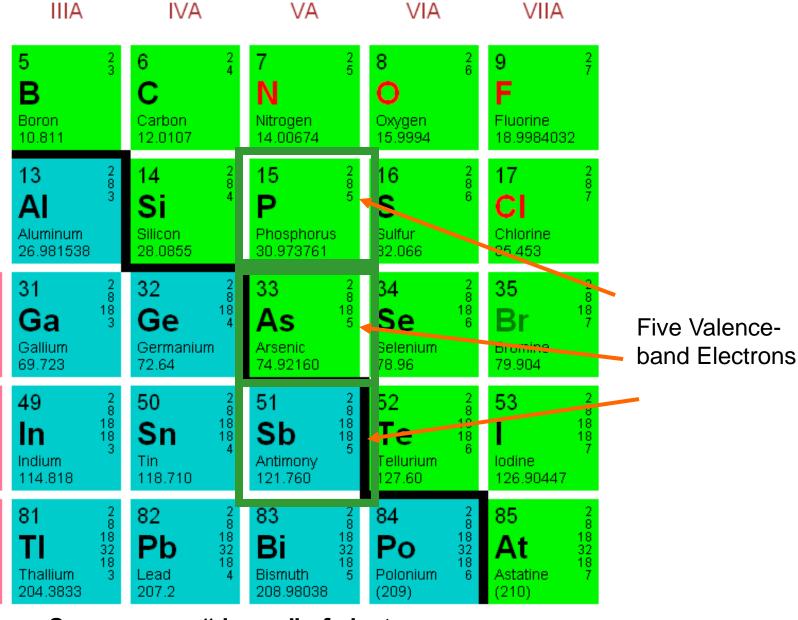
Serves as an "acceptor" of electrons

Acts as a p-type impurity when used as a silicon dopant

All elements in group III have 3 valence-band electrons



http://www.oftc.usyd.edu.au/edweb/devices/semicdev/doping4.html

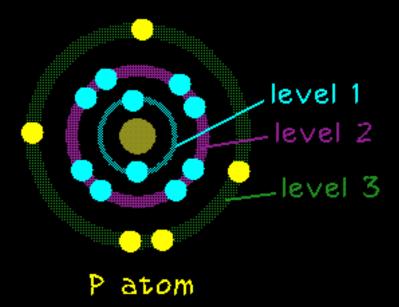


Serves as an "donor" of electrons

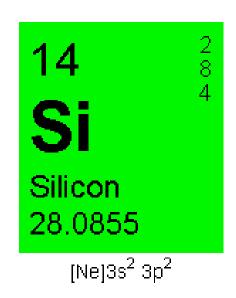
Acts as an n-type impurity when used as a silicon dopant

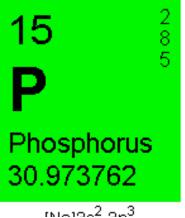
All elements in group V have 5 valence-band electrons

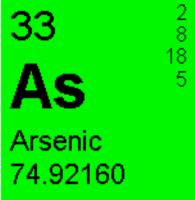
## The Atom of Phosphorus (P)



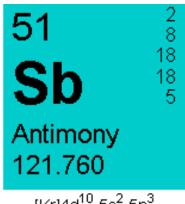








[Ar]3d<sup>10</sup> 4s<sup>2</sup> 4p<sup>3</sup>



 $[Kr]4d^{10} 5s^2 5p^3$ 

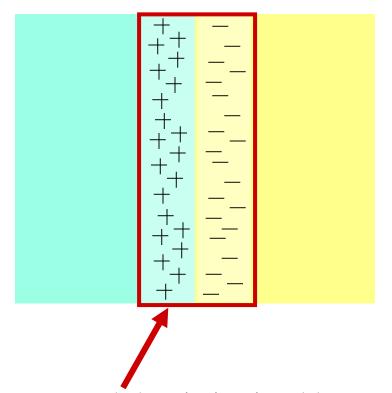
## Silicon Dopants in Semiconductor Processes

**B** (Boron) widely used a dopant for creating p-type regions

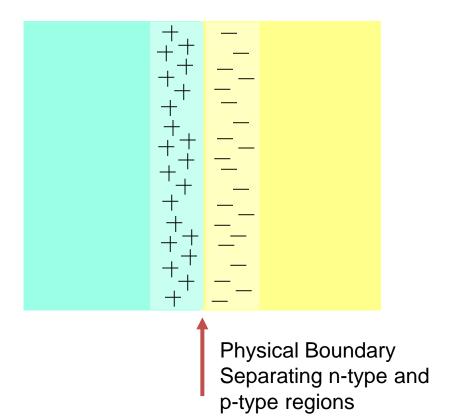
P (Phosphorus) widely used a dopant for creating n-type regions (bulk doping, diffuses fast)

**As** (Arsenic) widely used a dopant for creating n-type regions (Active region doping, diffuses slower)

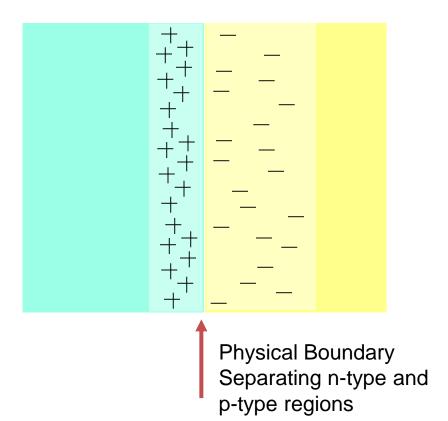
# Diodes (pn junctions)



Depletion region created that is ionized but void of carriers



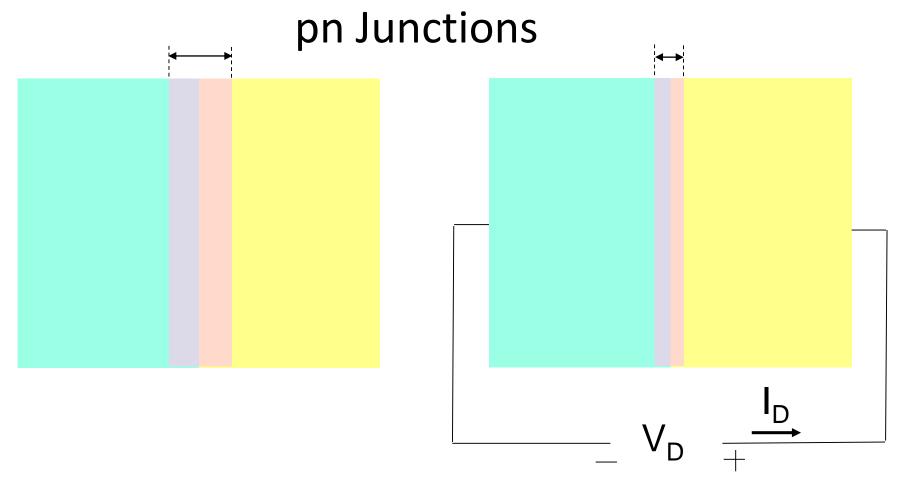
If doping levels identical, depletion region extends equally into n-type and p-type regions



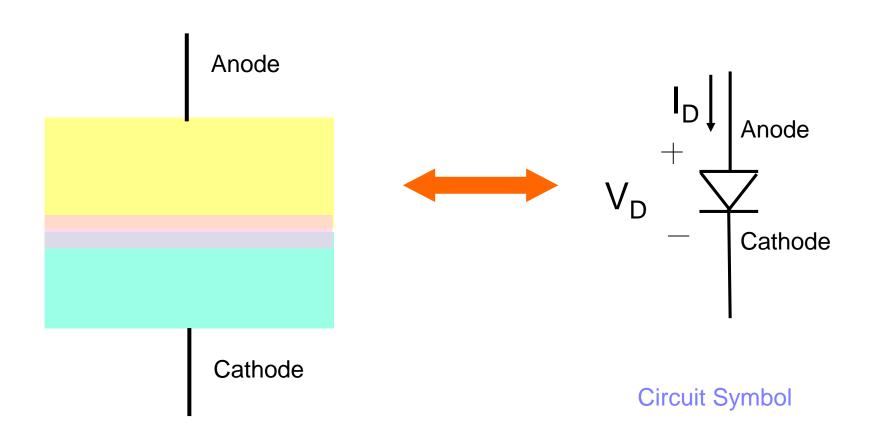
Extends farther into p-type region if p-doping lower than n-doping

Physical Boundary Separating n-type and p-type regions

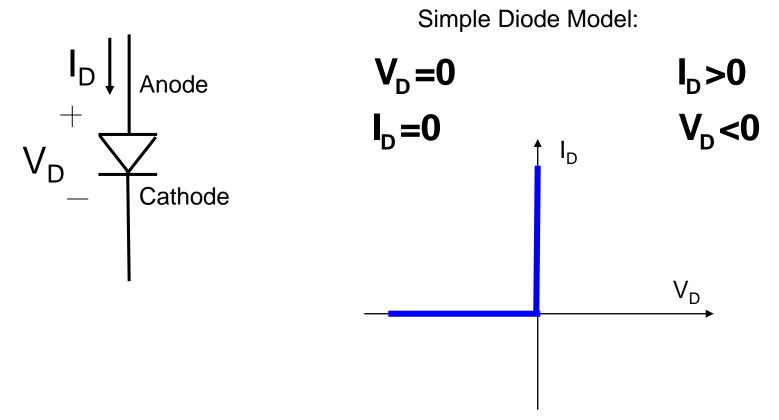
Extends farther into n-type region if n-doping lower than p-doping



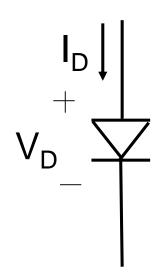
- Positive voltages across the p to n junction are referred to forward bias
- Negative voltages across the p to n junction are referred to reverse bias
- As forward bias increases, depletion region thins and current starts to flow
- Current grows <u>very rapidly</u> as forward bias increases
- Current is very small under revere bias

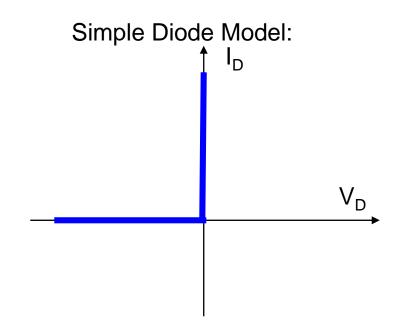


- As forward bias increases, depletion region thins and current starts to flow
- Current grows very rapidly as forward bias increases



Simple model often referred to as the "Ideal" diode model

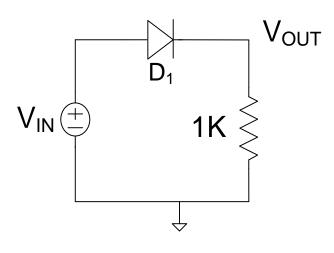


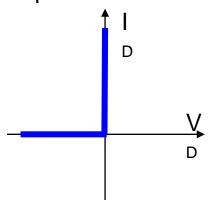


pn junction serves as a "rectifier" passing current in one direction and blocking it in the other direction

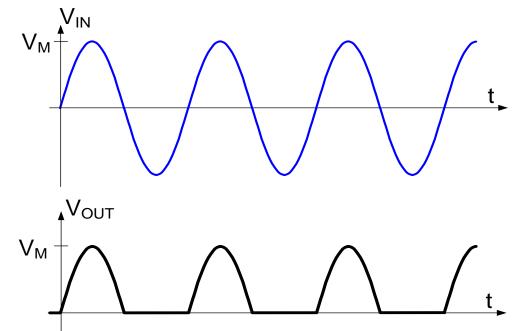
### **Rectifier Application:**

#### Simple Diode Model:





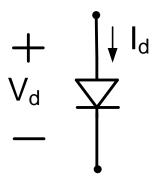




## I-V characteristics of pn junction

(signal or rectifier diode)

#### Improved Diode Model:



#### **Diode Equation**

$$I_{D} = I_{S} \left( e^{\frac{V_{d}}{V_{t}}} - 1 \right)$$

What is V<sub>t</sub> at room temp?

V<sub>t</sub> is about 26mV at room temp

I<sub>S</sub> in the 10fA to 100fA range

I<sub>S</sub> proportional to junction area

$$V_t = \frac{kT}{q}$$

 $k = 1.38064852 \times 10^{-23} \text{JK}^{-1}$ 

 $q = -1.60217662 \times 10^{-19} C$ 

$$k/q=8.62 \times 10^{-5} VK^{-1}$$

Diode equation due to William Shockley, inventor of BJT

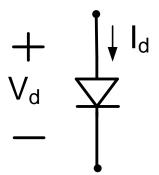
In 1919, William Henry Eccles coined the term *diode* 

In 1940, Russell Ohl "stumbled upon" the p-n junction diode

## I-V characteristics of pn junction

(signal or rectifier diode)

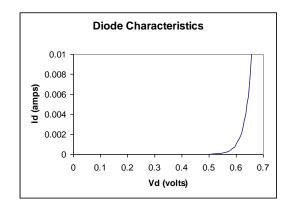
Improved Diode Model:



Diode Equation 
$$I_D = I_S \left( e^{\frac{V_d}{V_t}} - 1 \right)$$

#### **Simplification of Diode Equation:**

Under reverse bias ( $V_d$ <0),  $I_D \cong -I_S$ Under forward bias ( $V_d$ >0),  $I_D = I_S e^{\frac{V_d}{V_t}}$ 



$$I_{S}$$
 in 10fA -100fA range (for signal diodes) 
$$V_{t} = \frac{kT}{q}$$

k= 
$$1.380 6504(24) \times 10^{-23} \text{JK}^{-1}$$
  
q =  $-1.602176487(40) \times 10^{-19} \text{ C}$   
k/q= $8.62 \times 10^{-5} \text{ VK}^{-1}$ 

 $V_t$  is about 26mV at room temp

Simplification essentially identical model except for V<sub>d</sub> very close to 0

Diode Equation or forward bias simplification is unwieldy to work with analytically

## I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:

**Diode Equation** 

$$I_D = I_S$$
  $\left( \begin{array}{c} \frac{V_d}{V_t} \\ e \end{array} \right)$   $\left( \begin{array}{c} I_S \\ I_S \end{array} \right)$  often in the 10fA to 100fA range  $I_S$  proportional to junction area  $I_S$  by the sum of the sum

#### **Simplification of Diode Equation:**

Under reverse bias,

Under forward bias,

$$I_{D} = I_{S}e^{\overline{V_{t}}}$$

How much error is introduced using the simplification for  $V_d > 0.5V$ ?

$$\varepsilon = \frac{I_{s}\left(e^{\frac{V_{d}}{V_{t}}}-1\right)-I_{s}e^{\frac{V_{d}}{V_{t}}}}{I_{s}\left(e^{\frac{V_{d}}{V_{t}}}-1\right)} \qquad \varepsilon < \frac{1}{e^{\frac{0.5}{0.026}}} = 4.4 \bullet 10^{-9}$$

How much error is introduced using the simplification for  $V_d < -0.5V$ ?

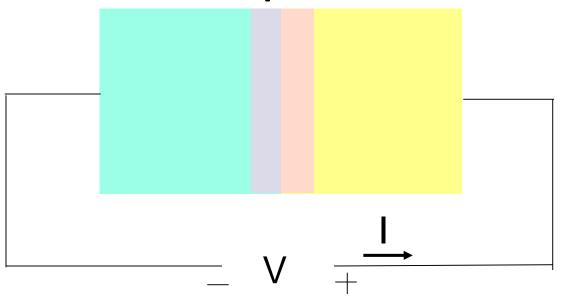
$$\varepsilon < e^{\frac{-0.5}{.026}} = 4.4 \cdot 10^{-9}$$

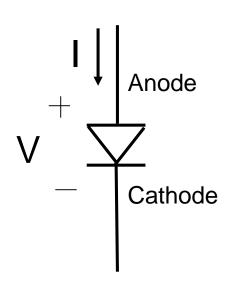
Simplification almost never introduces any significant error

Will you impress your colleagues or your boss if you use the more exact diode equation when  $V_d < -0.5V$  or  $V_d > +0.5V$ ?



Will your colleagues or your boss be unimpressed if you use the more exact diode equation when  $V_d < -0.5V$  or  $V_d > +0.5V$ ?





Diode Equation:  $I = \begin{cases} J_s A e^{\frac{1}{nV_T}} \end{cases}$ 

$$\begin{array}{ll} \text{Diode Equation:} & I = \begin{cases} J_{s}Ae^{\frac{V}{nV_{T}}} \\ 0 \end{cases}$$

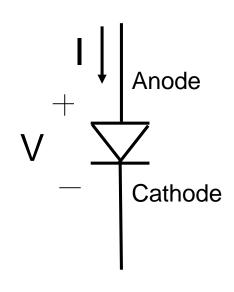
J<sub>S</sub>= Sat Current Density (in the 1aA/u² to 1fA/u² range) A= Junction Cross Section Area  $V_T = kT/q$  (k/q=1.381x10<sup>-23</sup>V•C/°K/1.6x10<sup>-19</sup>C=8.62x10<sup>-5</sup>V/°K) n is approximately 1

Diode Equation: 
$$I = \begin{cases} J_s A e^{\frac{V}{NV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

J<sub>s</sub> is strongly temperature dependent

With n=1, for V>0,

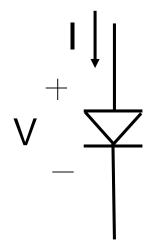
$$I(T) = \left(J_{SX} \left[T^{m} e^{\frac{-V_{GO}}{V_{t}}}\right]\right) A e^{\frac{V_{D}}{V_{t}}}$$



Typical values for key parameters:  $J_{SX}=0.5A/\mu^2$ ,  $V_{G0}=1.17V$ , m=2.3

Example:

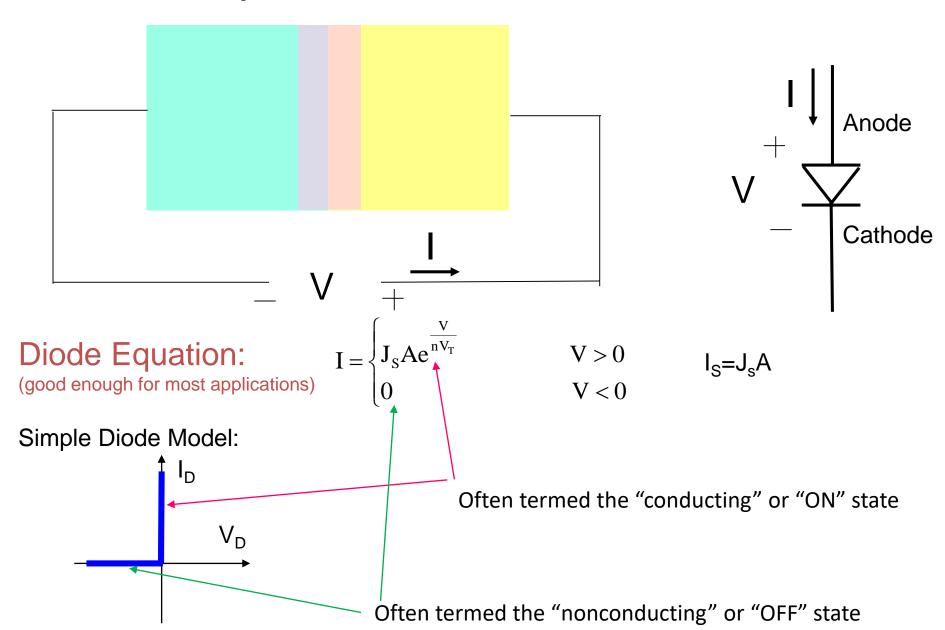
$$I(T) = \left(J_{sx} \left[T^m e^{\frac{-V_{go}}{V_t}}\right]\right) A e^{\frac{V_D}{V_t}}$$



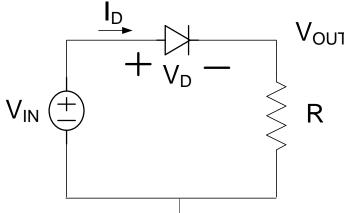
What percent change in I<sub>S</sub> will occur for a 1°C change in temperature at room temperature?

$$\frac{\Delta I_{s}}{I_{s}} = \frac{\left(J_{sx}\left[T_{T_{s}}^{m}e^{\frac{-V_{os}}{V_{t}(T_{s})}}\right]\right)Ae^{\frac{V_{o}}{V_{t}}} - \left(J_{sx}\left[T_{T_{s}}^{m}e^{\frac{-V_{os}}{V_{t}(T_{s})}}\right]\right)Ae^{\frac{-V_{os}}{V_{t}(T_{s})}} = \frac{\left(\left[T_{T_{s}}^{m}e^{\frac{-V_{os}}{V_{t}(T_{s})}}\right]\right) - \left(\left[T_{T_{s}}^{m}e^{\frac{-V_{os}}{V_{t}(T_{s})}}\right]\right)}{\left(\left[T_{T_{s}}^{m}e^{\frac{-V_{os}}{V_{t}(T_{s})}}\right]\right)Ae^{\frac{-V_{os}}{V_{t}(T_{s})}}} = \frac{\left(\left[T_{T_{s}}^{m}e^{\frac{-V_{os}}{V_{t}(T_{s})}}\right]\right) - \left(\left[T_{T_{s}}^{m}e^{\frac{-V_{os}}{V_{t}(T_{s})}}\right]\right)}{\left(\left[T_{T_{s}}^{m}e^{\frac{-V_{os}}{V_{t}(T_{s})}}\right]\right)Ae^{\frac{-V_{os}}{V_{t}(T_{s})}}} = \frac{\Delta I_{s}}{I_{s}} = \frac{\left(1.240x10^{-15}\right) - \left(1.025x10^{-15}\right)}{\left(1.025x10^{-15}\right)}100\% = 21\%$$

- Attempts to measure I<sub>s</sub> in out laboratories can result in large errors!
- Most circuits whose performance depends upon precise value for I<sub>s</sub> are not practical



### Consider again the basic rectifier circuit



- Previously considered sinusoidal excitation  $\downarrow$
- Previously gave "qualitative" analysis
- Rigorous analysis method is essential

$$V_{OUT} = ?$$

## Analysis of Nonlinear Circuits

(Circuits with one or more nonlinear devices)

What analysis tools or methods can be used?

KCL? Nodal Analysis?

KVL? Mesh Analysis?

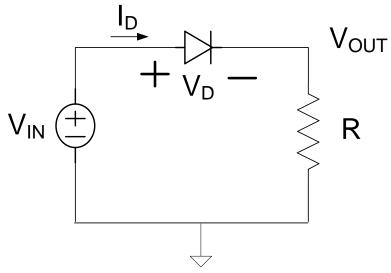
Superposition? Two-Port Subcircuits?

Voltage Divider?

Current Divider?

Thevenin and Norton Equivalent Circuits?

## Consider again the basic rectifier circuit



$$\begin{aligned} V_{IN} &= V_D + I_D R \\ V_{OUT} &= I_D R \\ I_D &= I_S \left( e^{\frac{V_d}{V_t}} - 1 \right) \end{aligned}$$

$$V_{IN} = V_D + I_D R$$

$$V_{OUT} = I_D R$$

$$I_D = I_S \left( e^{\frac{V_d}{V_t}} - 1 \right)$$

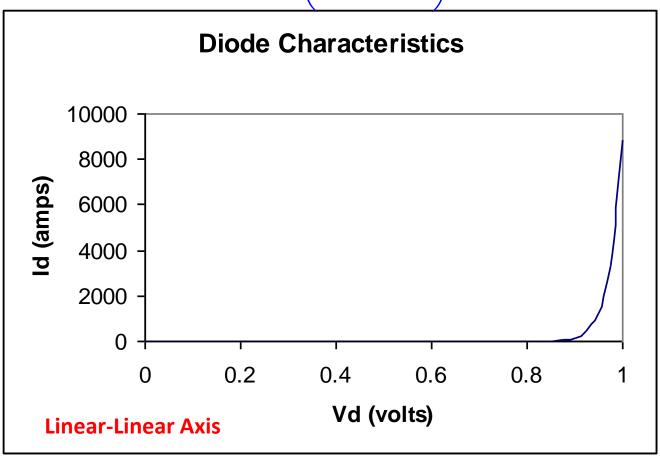
$$V_{OUT} = I_S R \left( e^{\frac{V_{IN} - V_{OUT}}{V_t}} - 1 \right)$$

Even the simplest diode circuit does not have a closed-form solution when diode equation is used to model the diode!!

Due to the nonlinear nature of the diode equation

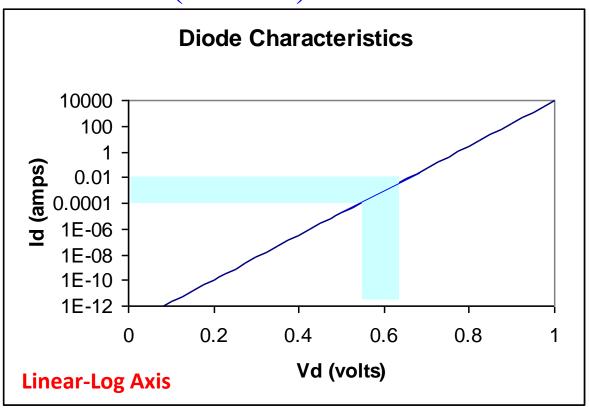
Simplifications are essential if analytical results are to be obtained

$$I_d = I_S \left( e^{\frac{V_d}{V_t}} - 1 \right)$$



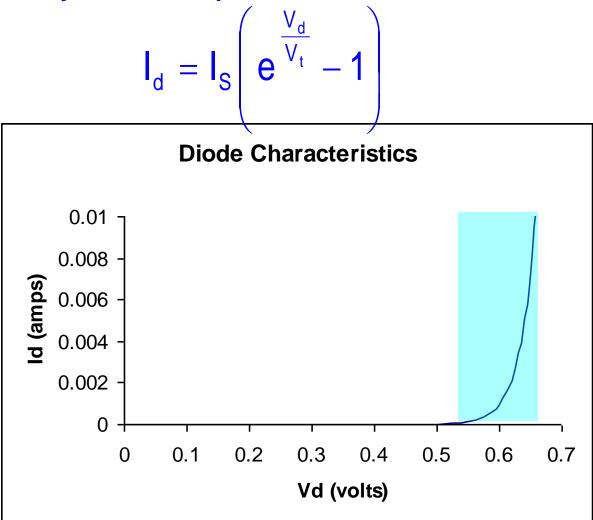
Power Dissipation Becomes Destructive if Vd > 0.85V (actually less)

$$I_{d} = I_{S} \left( e^{\frac{V_{d}}{V_{t}}} - 1 \right)$$



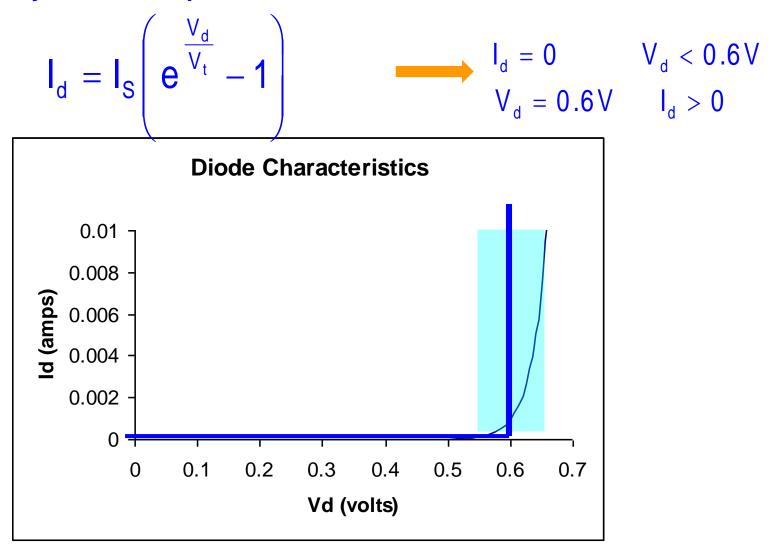
For two decades of current change, Vd is close to 0.6V

This is the most useful conducting current range for many applications



For two decades of current change, Vd is close to 0.6V

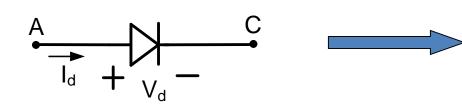
This is the most useful current range when conducting for many applications

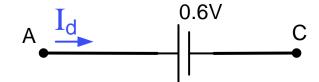


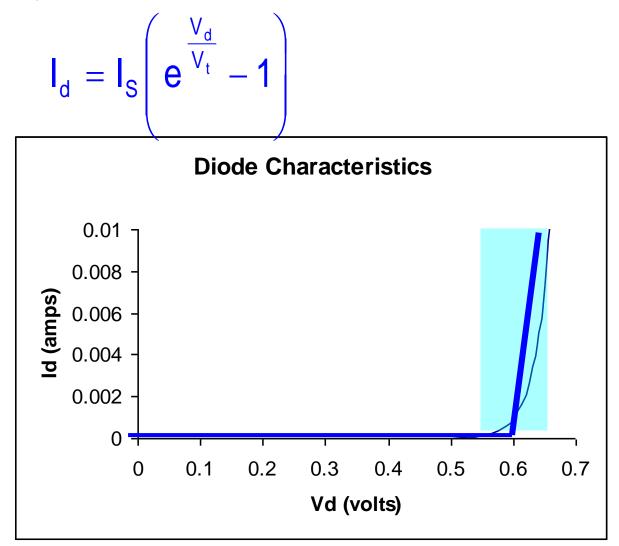
Widely Used Piecewise Linear Model

$$I_{d} = I_{S} \begin{pmatrix} e^{\frac{V_{d}}{V_{t}}} - 1 \\ 0.008 \\ 0.008 \\ 0.004 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.001 \\ 0.002 \\ 0.001 \\ 0.002 \\ 0.001 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.002 \\ 0.003 \\ 0.004 \\ 0.002 \\ 0.004 \\ 0.004 \\ 0.004 \\ 0.002 \\ 0.004 \\ 0.$$

### **Equivalent Circuit**







Better model in "ON" state though often not needed Includes Diode "ON" resistance

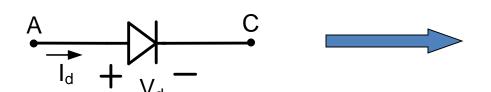
$$I_{d} = I_{S} \left( e^{\frac{V_{d}}{V_{t}}} - 1 \right)$$

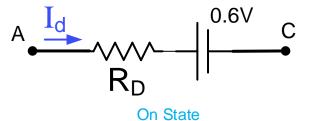
#### **Piecewise Linear Model with Diode Resistance**

$$I_d = 0$$
 if  $V_d < 0.6V$   
 $V_d = 0.6V + I_d R_D$  if  $I_d > 0$ 

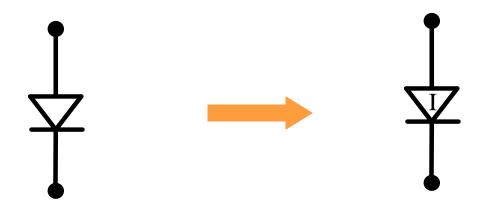
 $(R_D \text{ is rather small: often in the } 20\Omega \text{ to } 100\Omega \text{ range})$ :

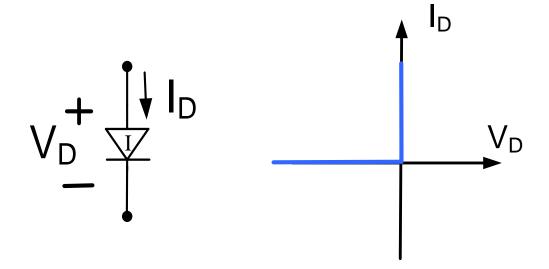
## **Equivalent Circuit**





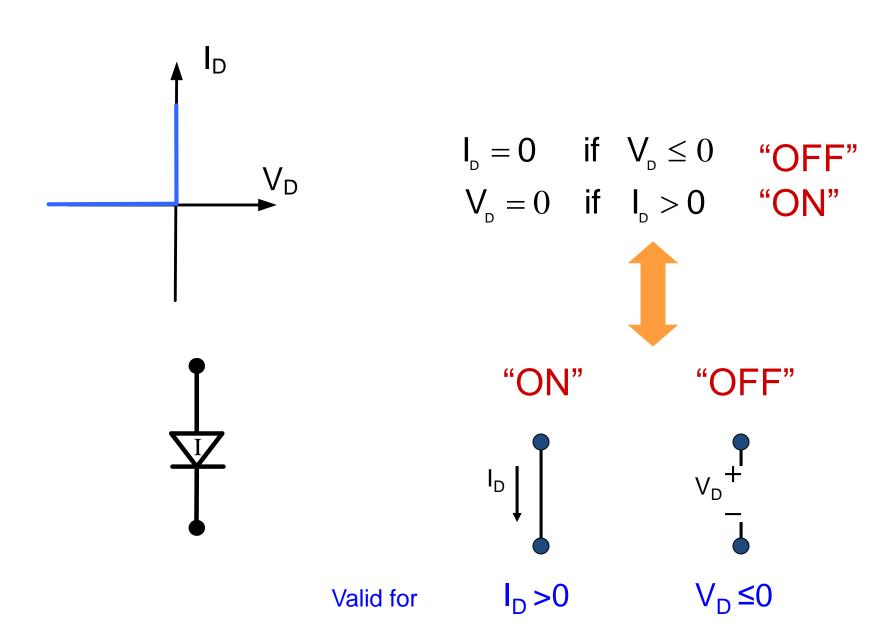
# The Ideal Diode



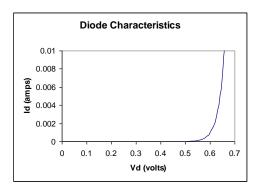


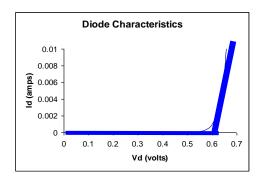
$$I_D = 0$$
 if  $V_D \le 0$   
 $V_D = 0$  if  $I_D > 0$ 

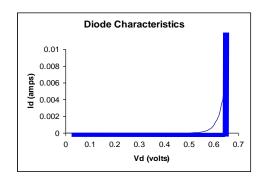
# The Ideal Diode

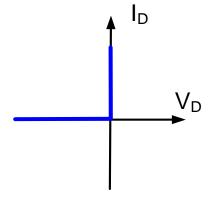


## **Diode Models**









Which model should be used?

The simplest model that will give acceptable results in the analysis of a circuit

## Diode Model Summary

#### Piecewise Linear Models

$$I_d = 0$$

if 
$$V_d < 0$$

$$V_d = 0$$

$$V_d = 0$$
 if  $I_d > 0$ 

$$I_d = 0$$

$$I_d = 0$$
 if  $V_d < 0.6V$ 

$$V_d = 0.6V$$
 if  $I_d > 0$ 

if 
$$I_d > 0$$

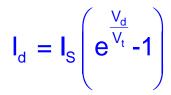
$$I_{d} = 0$$

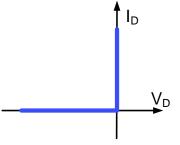
if 
$$V_d < 0.6$$

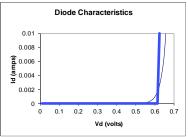
$$V_d = 0.6 + I_d R_d$$

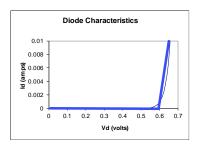
if 
$$I_d > 0$$

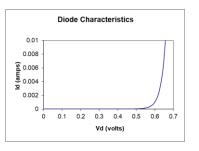
### **Diode Equation**











## **Diode Model Summary**

#### Piecewise Linear Models

$$\begin{split} I_{d} &= 0 & \text{if } V_{d} < 0 \\ V_{d} &= 0 & \text{if } I_{d} > 0 \\ \\ I_{d} &= 0 & \text{if } V_{d} < 0.6V \\ V_{d} &= 0.6V & \text{if } I_{d} > 0 \\ \\ I_{d} &= 0 & \text{if } V_{d} < 0.6 \\ V_{d} &= 0.6 + I_{d}R_{d} & \text{if } I_{d} > 0 \\ \end{split}$$

**Diode Equation** 

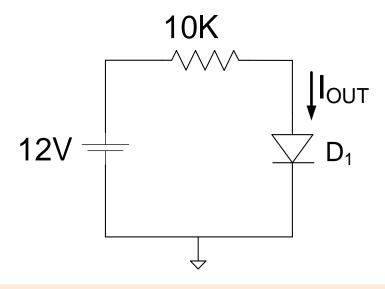
$$I_{d} = I_{S} \left( e^{\frac{V_{d}}{V_{t}}} - 1 \right)$$

When is the ideal model adequate?

When it doesn't make much difference whether  $V_d$ =0V or  $V_d$ =0.6V When is the second piecewise-linear model adequate? When it doesn't make much difference whether  $V_d$ =0.6V or  $V_d$ =0.7V

Example:

Determine I<sub>OUT</sub> for the following circuit



Solution:

Strategy:

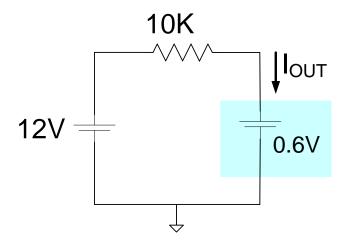
- 1. Assume PWL model with  $V_D=0.6V$ ,  $R_D=0$
- 2. Guess state of diode (ON)
- 3. Analyze circuit with model
- 4. Validate state of guess in step 2 (verify the "if" condition in model)
- 5. Assume PWL with  $V_D=0.7V$
- 6. Guess state of diode (ON)
- 7. Analyze circuit with model
- 8. Validate state of guess in step 6 (verify the "if" condition in model)
- 9. Show difference between results using these two models is small
- 10. If difference is not small, must use a different model

Select Model

Validate Model

### Solution:

- 1. Assume PWL model with  $V_D=0.6V$ ,  $R_D=0$
- 2. Guess state of diode (ON)



3. Analyze circuit with model

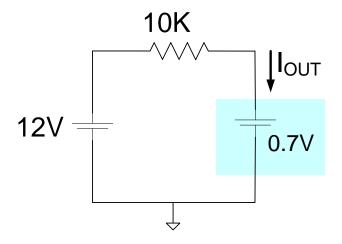
$$I_{OUT} = \frac{12V - 0.6V}{10K} = 1.14mA$$

4. Validate state of guess in step 2  $\text{To validate state, must show I}_{\text{D}} \!\!>\!\! 0$ 

$$I_{D} = I_{OUT} = 1.14 \text{ mA} > 0$$

### Solution:

- 5. Assume PWL model with  $V_D=0.7V$ ,  $R_D=0$
- 6. Guess state of diode (ON)



7. Analyze circuit with model

$$I_{OUT} = \frac{12V - 0.7V}{10K} = 1.13mA$$

8. Validate state of guess in step 6 To validate state, must show  $I_D>0$ 

$$I_{D} = I_{OUT} = 1.13 \text{mA} > 0$$

Solution:

Show difference between results using these two models is small 9.

$$I_{OUT}$$
=1.14mA and  $I_{OUT}$ =1.13 mA are close

Thus, can conclude 
$$I_{\text{OUT}} \cong 1.14\text{mA}$$

# End of Lecture 13