Recitation 7

- Here is a set of additional problems. They range from being very easy to very tough. The best way to learn the material in 310 is to solve problems on your own.
- Feel free to ask (and answer) questions about this problem set on Piazza.
- This is an **optional** problem set; do not turn this in for grading.
- While you don't have to turn this in, be warned that this material **can** appear in a quiz or exam.

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1. Define a binary relation R from $\mathbb{N}$ to $\mathbb{N}$ as follows: for all $x, y \in \mathbb{N}, xRy \leftrightarrow x \geq y$.
   (a) Is $(2, 1) \in R$? Is $(2, 2) \in R$? Is $2R3$?
   (b) Draw (or describe) the graph representation of R. (i.e., represent elements as nodes and relations between elements as directed edges.)

2. In the following, determine whether or not the given binary relation is reflexive, symmetric, transitive, or none of these. Justify your answer.
   (a) C is the circle relation on the set of real numbers $\mathbb{R}$: for all $x, y \in \mathbb{R}, xCy$ if $x^2 + y^2 = 1$.
   (b) O is the binary relation on the set of integers $\mathbb{Z}$: for all $m, n \in \mathbb{Z}, mOn$ if $m - n$ is odd.

3. Recall from lecture that every *equivalence relation* induces a *partition* on the domain of discourse. In this problem, we will go backwards. Given that $\{\{a, c\}, \{b, d\}, \{e, f\}\}$ is a partition of the set $A = \{a, b, c, d, e, f\}$, and elements with each piece of the partition are equivalent, then draw a graph representation of the corresponding equivalence relation R.

4. Let $S = \{1, 2, 3, 4, 5\}$ and let $A = S \times S$. Define the following relation R from A to A as follows: $(a, b)R(a', b')$ if and only if $ab' = a'b$. Prove that R is an equivalence relation.

5. Let $A = \{2, 4, 10, 16, 20, 70\}$ and R be the "divides" relation. (for example, 2R4, 4R16, 10R20, etc.)
   (a) Draw the Hasse diagram representation of the above relation.
   (b) List all minimal and maximal elements. Minimal elements are those that can be found at the "bottom level" of the Hasse diagram, and maximal elements are those that are at the top.

(c) Run topological sort on the Hasse diagram to obtain a compatible total ordering of the elements.