CPRE 310

Sample Midterm Exam 1

- Total duration: 60 minutes.
- Please **write your name, netid, and recitation section** on the top of this page.
- You **can** use two pages as cheat sheets.
- You **cannot** consult your notes, textbook, your neighbor, or Google.
- Maximum points: 50 (any score above 50 will be rounded to 50).

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1. **(6 points)** Consider the propositional variables:

   q: You can ride the rollercoaster.

   r: You are under 4 feet tall.

   s: You are older than 16.

Express the following natural language statements in terms of these variables and appropriate logical connectives.

(a) You are under 4 feet tall and you are not older than 16.

(b) If you are under 4 feet tall, then you cannot ride the rollercoaster.

(c) You can ride a rollercoaster only if you are over 4 ft tall or you are older than 16.

2. **(4 points)** For each of the below propositions:

   (i) $p : \forall x, \exists y, 2x - y = 0$

   (ii) $q : \forall x, \exists y, 2y - x = 0$

indicate (no explanations necessary) which propositions are true when:

(a) the domain of discourse is the nonnegative integers.

(b) the domain of discourse is the real numbers.

3. **(10 points)** This is a 3-part question.

(a) (4 points) Express each of the three sentences below using propositional variables: "If the shoe does not fit, then he is acquitted. He isn't acquitted. Therefore, the shoe fits."

(b) (1 point) Identify the rule of inference implicitly used in this argument.

(c) (5 points) Prove that this particular rule of inference is *valid* in general using a truth table.

4. **(10 points)** Prove that $\sqrt{\frac{1}{5}}$ is irrational.

Points will be given for (a) clearly stating your proof technique in the beginning, (b) listing assumptions, if any, and (c) finishing your proof with a concluding statement. You are free to use any familiar facts that you know about integer arithmetic.

5. **(10 points)** The following table lists driving distances between some California cities (in miles):

- Barstow to Fresno: 245 miles
- Eureka to Fresno: 450 miles
- Barstow to LA: 115 miles
- Eureka to LA: 645 miles
- Fresno to LA: 220 miles
- San Diego to Barstow: 175 miles
- San Diego to LA: 125 miles

Draw the above information in the form of a graph. Clearly label the nodes, define what you mean by an edge, and draw all relevant edges. Assign "weights" to the edges according to the driving distance information provided.

Using the graph, figure out a round trip (no proof needed) starting from San Diego and visiting all other cities using as few miles as possible.

6. **(5 points)** Which of the following is true and which is false? No explanations or proofs necessary.
   (a) $3 \in \{1, 2, 3\}$.
   (b) $\{2\} \in \{1, 2\}$.
   (c) $1 \in \{1\}$.
   (d) $\{1\} \subseteq \{1, 2\}$.
   (e) $\{1\} \subseteq \{1, \{2\}\}$.
   (f) $\{1\} \subseteq \{1\}$.
   (g) $\mathbb{Z}^+ \subseteq \mathbb{Q}$.
   (h) $\mathbb{Q} \subseteq \mathbb{Z}$.
   (i) $\emptyset \subset \mathbb{R}$
   (j) $0 \in \mathbb{Z}^+$

7. **(10 points)** Let a, b, c be three arbitrary positive real numbers. Define their *arithmetic mean* as $\frac{a+b+c}{3}$ and their *geometric mean* as $\sqrt[3]{abc}$. Prove that if the arithmetic mean is different from the geometric mean, then the three numbers cannot all be equal to each other.