



# Introduction to High Performance Computing

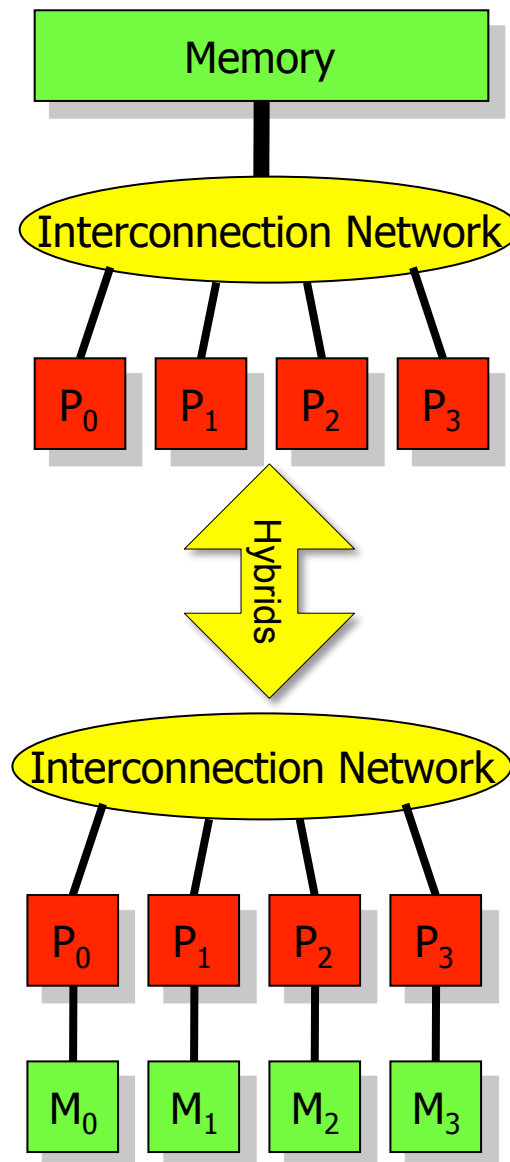
## *Lecture 11 – Basics of Interconnection Networks I*

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# Introduction

- Up to now: Interconnection Network (IN) as a black box
  - Turning into the key component of HPC systems
  - Exact behavior is crucial to overall performance
- INs are found everywhere
  - On-Chip Networks (different modules or cores)
  - Intra-Node (CPU, memory, graphics, devices)
  - Inter-Node (multiple nodes)
    - SAN, LAN, WAN
- Different requirements/workloads!
  - Here: focus on HPC and its demands





# Types of INs in a computer system

Type	Description	Length
Processor or system interconnect	Connections between processors, memory controllers, ... (HyperTransport, QPI, FSB)	10..30cm
Memory network	Connections between memory controller and memory modules	10cm
I/O bus (better: interconnect)	Connection from device to system using connectors (PCI-Express)	30cm..1m
System-Area-Network	Connections within a cluster or parallel computer	5-25m
Storage-Area-Network (SAN)	Connection from processing nodes to storage modules	5-25m
Local-Area-Network (LAN)	Connection between loosely coupled workstations and/or servers (*Ethernet)	25-500m
Metropolitan-Area-Network (MAN)	Connections within the scope of city limits (ATM, FDDI)	~25km
Wide-Area-Network (WAN)	Connections without any length restrictions, worldwide, multiplexing of a large number of connections, typically using fiber optics (SONET)	unlimited



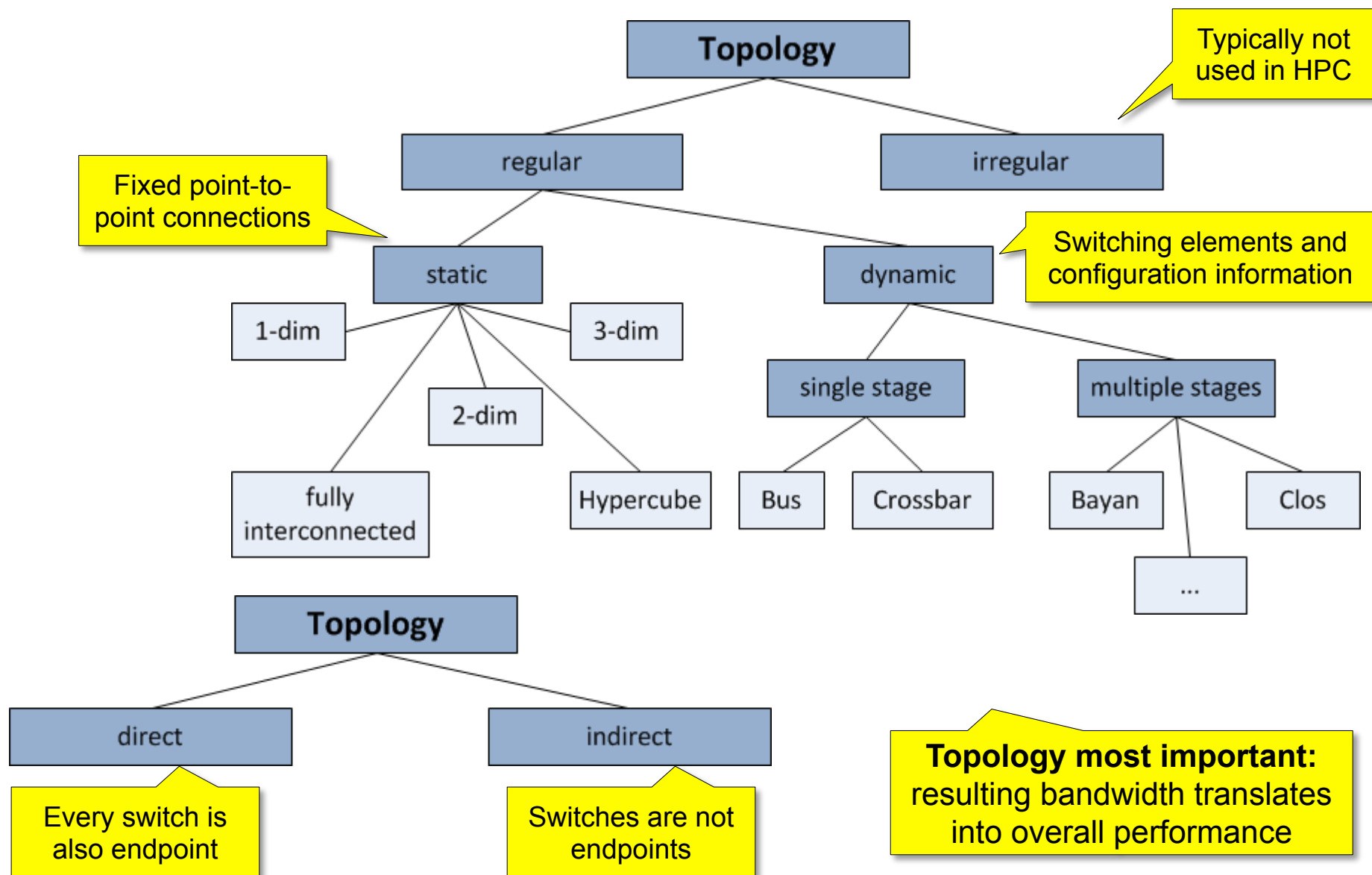
# User requirements

- Costs
- Bandwidth
- Max. supported transmission length
- Scalability
- Latency
- Blocking behaviour
- Lossy/loss-less (reliability)
- In-order/out-of-order delivery

**Order of importance  
application-  
dependent**

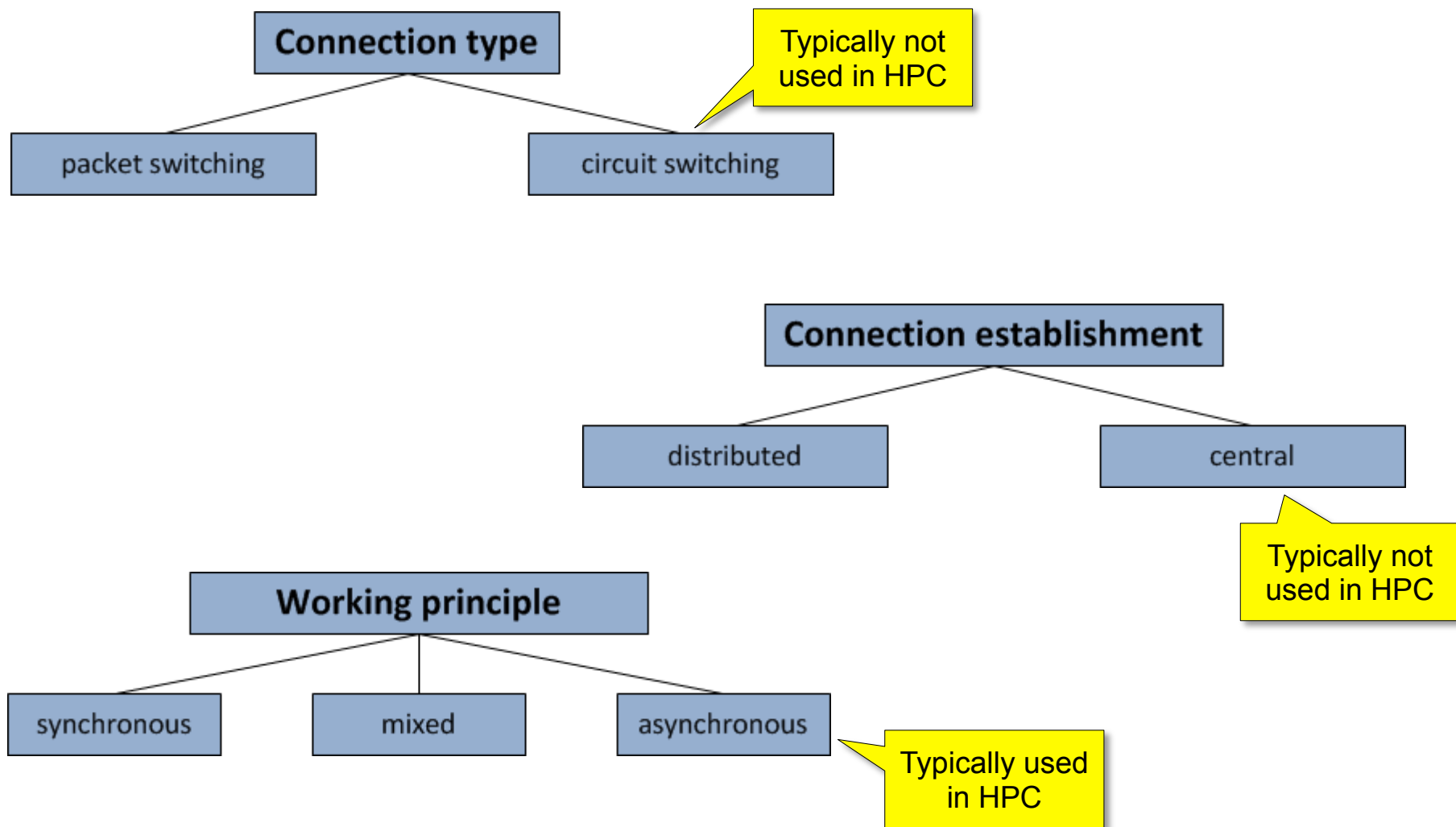


# Classification





# Classification



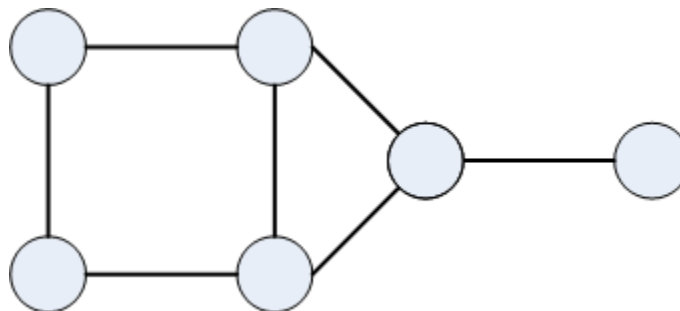


# Static Topologies



# Static Topologies

- **Mainly used in massively parallel processors (MPP)**
  - Fixed communication structure
  - Based on point-to-point connections between processing nodes
    - Node, processor, ...
- **Representation**
  - Node as node, connection as edge
  - Directed or undirected graph



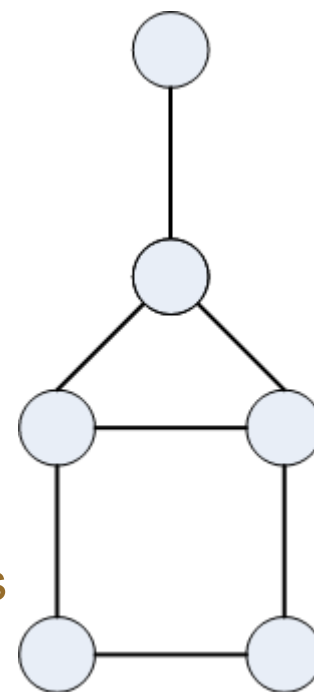
Representation of a static IN as graph





# Properties of (Static) Topologies

- Topological and functional properties
- **Node degree:** Number of connections per node
  - As few as possible due to costs
  - Fixed degree mandatory for scalability
- **Diameter**
  - Maximum distance in hops between any two node pairs
- **Symmetry**
  - IN is symmetric if the view of the IN from each node is identical
- **More:**
  - Scalability, blocking behaviour, costs, latency, fault-tolerance, max. expansion





# Static Topologies - Examples

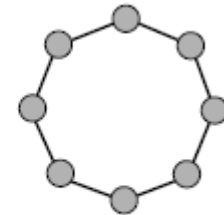
## ■ Trivial ones

- Chain
- Ring
- Star
- (binary) Tree

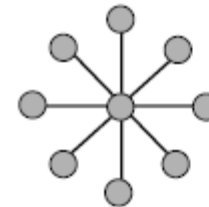
**Node degree?  
Diameter?  
Symmetry?**



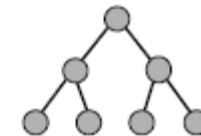
**Chain**



**Ring**



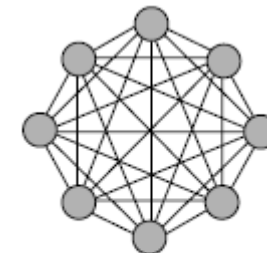
**Star**



**(Binary) Tree**

## ■ Completely interconnected

- Every node is connected to every other node
- Not used in practise
- Max. distance from one node to another = 1
  - „1 hop“



**Completely interconnected**



# Static Topologies - Examples

## ■ Grid or Mesh

- Nearest neighbor mesh
- N-dimensional mesh suitable for n-dimensional problems

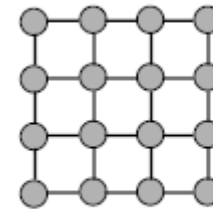
## ■ Hexagonal grid

- 2D, but maps nicely to 3D problems
- Systolic algorithms

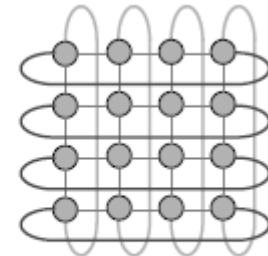
## ■ Torus

- Based on grid with wrap-around links
- Better connectivity

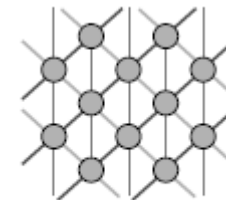
**Node degree?**  
**Diameter?**  
**Symmetry?**



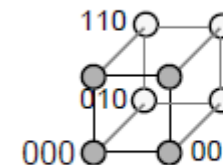
**Grid**



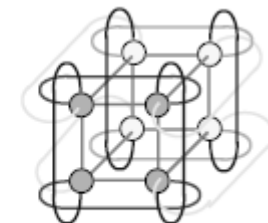
**2D Torus**



**Hexagonal grid**



**Cube**



**3D Torus**

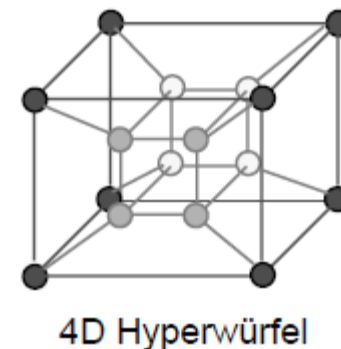
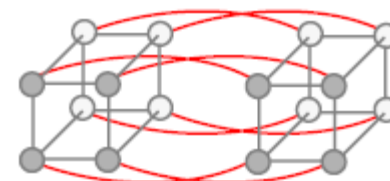
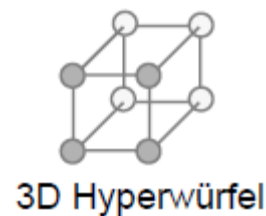
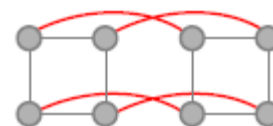
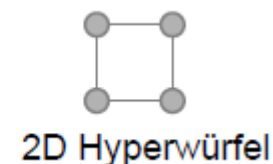
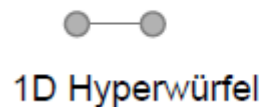


# Static Topologies - Examples

## ■ Hypercube

- Given:  $n$  dimensions
- $2^n$  nodes,  
 $n \cdot 2^{n-1}$  connections,  
 $n$  connections per node
- diameter =  $n$
- Better properties than a grid
  - Limited scalability (node degree)
- Mainly used in the beginning of MIMD-based parallel computing: nCube

Construction:  
double each  
node per  
additional  
dimension





# Properties of Static Topologies

Translates  
into scalability

Only scalability with  
regard to topology

Topology	Node degree	Diameter	Number of connections	Scalable	Symmetric
1D grid (chain)	2	$N-1$	$N-1$	Yes	No
1D torus (ring)	2	$(N-1)/2$	$N$	Yes	Yes
2D grid	4	$2(N^{1/2}-1)$	$2N-2N^{1/2}$	Yes	No
2D torus	4	$N^{1/2}-1$	$2N$	Yes	Yes
3D grid	6	$3(N^{1/3}-1)$	$3N-3N^{1/3}$	Yes	No
3D torus	6	$3/2(N^{1/3}-1)$	$3N$	Yes	Yes
Hypercube	$\log_2 N$	$\log_2 N$	$N \log_2(N/2)$	No	Yes
Binary Tree	3	$2(\log_2 N-1)$	$N-1$	Yes	No
Completely interconnected	$N-1$	1	$N(N-1)/2$	No	Yes



- Filtering for scalability and symmetry:  
only **n-dimensional tori**
- Torus vs. Mesh
  - Basically only advantages for tori: highly reduced diameter, symmetric
  - Slightly higher connection count is not relevant in practise
- Side note: Binary tree
  - Disadvantage: root is bottleneck
  - Typically only used for specialized tasks: synchronization and collective communication (barrier, multi-/broad-cast)



# Dynamic Topologies



# Dynamic Topologies

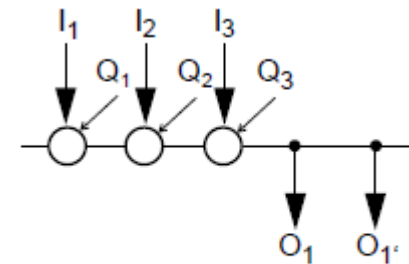
- Dynamic INs are based on **configurable switching elements**

- Different number of stages

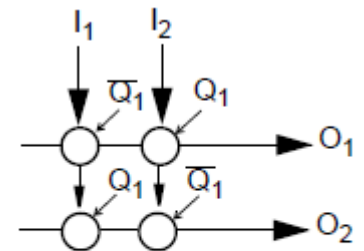
- Single stage

- Building blocks
- Shuffle, crossbar, bus
- Representation as graph: switching elements are nodes, connections are edges
- Control signals  $Q_i$
- Inputs  $I_i$
- Outputs  $O_i$

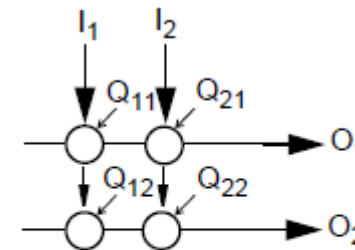
- Today basically only crossbar used



Bus



shuffle



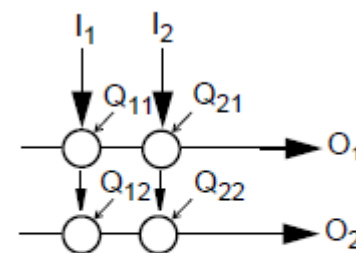
2 x 2 Crossbar





# Dynamic Topologies: Crossbar

- Most universal element
- Can connect arbitrary combinations of inputs and outputs
  - Broadcast
- Conflicts avoided by arbiter
  - For all  $i$ : only one  $Q(i,y)=1$
- Logical complexity is  $O(N^2)$ 
  - For  $N$  inputs and  $N$  outputs
  - Due to VLSI technology basically no limitation
  - Most limiting today is pin count
    - Number of pins for a certain package
    - Pin is several orders of magnitude larger than a transistor!
      - Micrometers vs. nanometers
  - Pin limitation

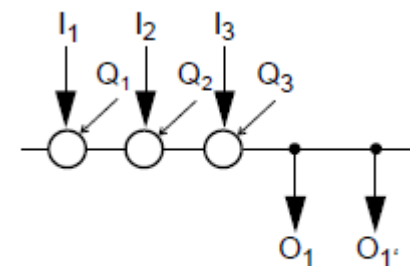


2 x 2 Crossbar



# Dynamic Topologies: Bus

- A bus is basically a crossbar with a **1 x m** configuration
  - Only one driver at a time
    - High blocking potential
    - Arbiter required
  - Limited operation frequency
    - Length of connection, capacities, signal levels
  - Limited number of nodes
- Advantages: implicit broadcasts
  - Simplicity
  - Snooping protocols for cache coherency
- Today: almost vanished
  - Except human I/O and other peripherals



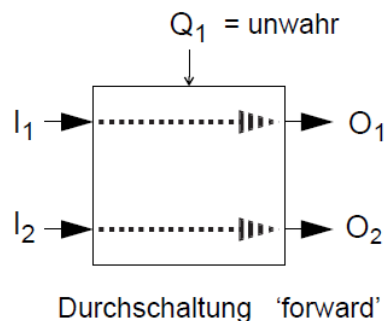
Bus



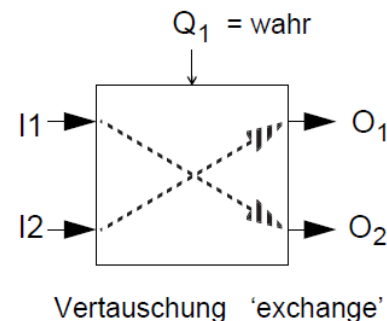
# Dynamic Topologies: Shuffle

- A shuffle is basically a crossbar with a restricted set of configurations
  - „Forward“ or „exchange“
  - Possible extensions: upper and lower broadcast

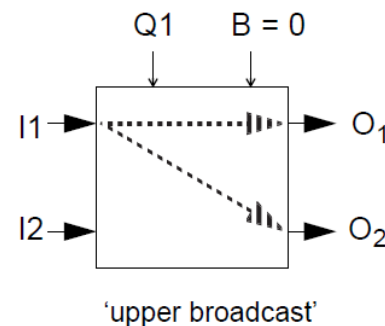
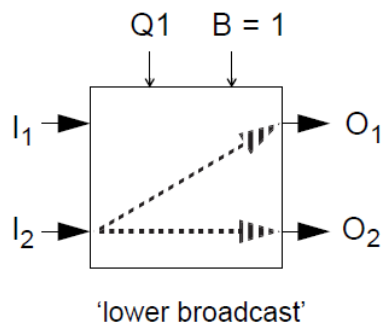
Grundsaltung



- Shuffle only as 2x2 element available
  - Larger structures based on shuffle as building block



Erweiterung  
um 'broadcast'





# Dynamic Topologies: multi-stage

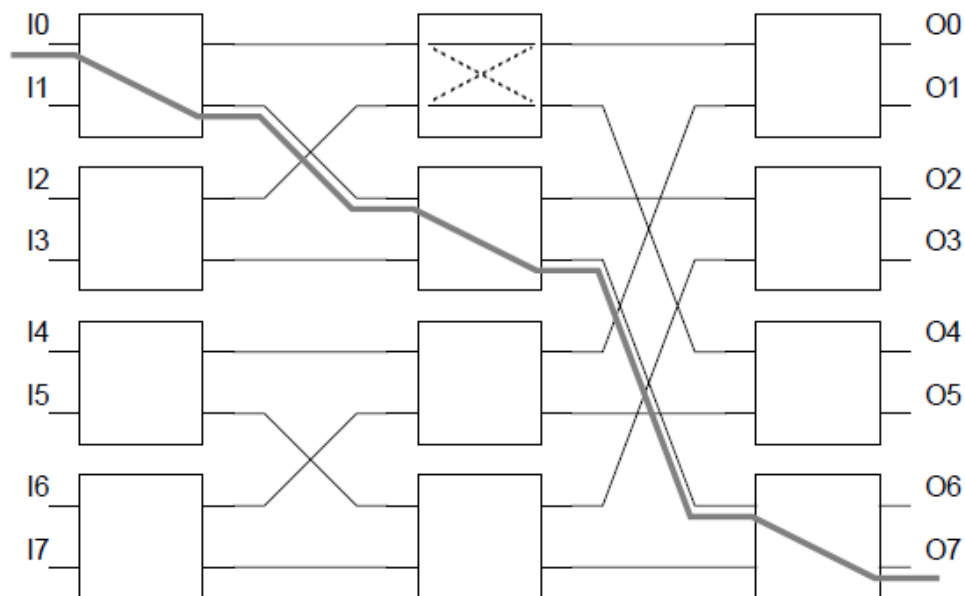
- **No single-stage element scales!**

- Multiple stages with one-staged elements as building blocks
  - (Bus,) crossbar, **shuffle**

- **Examples**

- Banyan, Baseline, Cube, Delta, Flip, Indirect Cube, Omega

- Basically identical, only connectivity differs



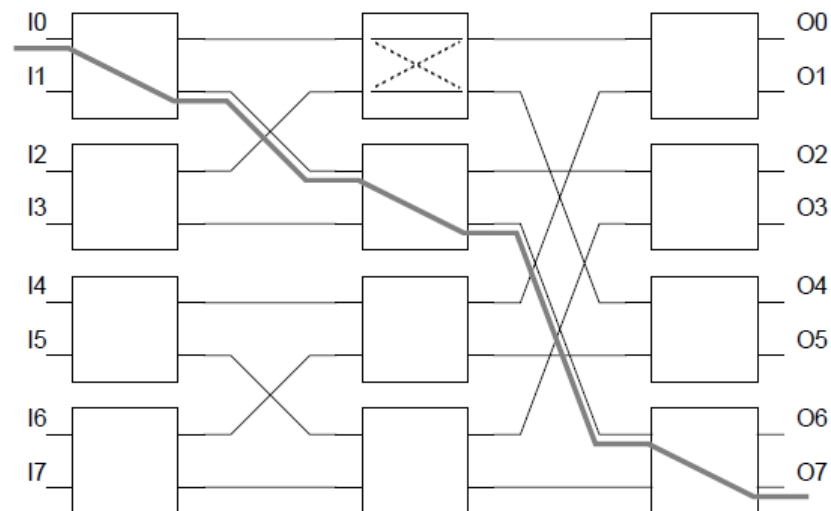
**Banyan Network**

Note that the shaded connection may block other connections

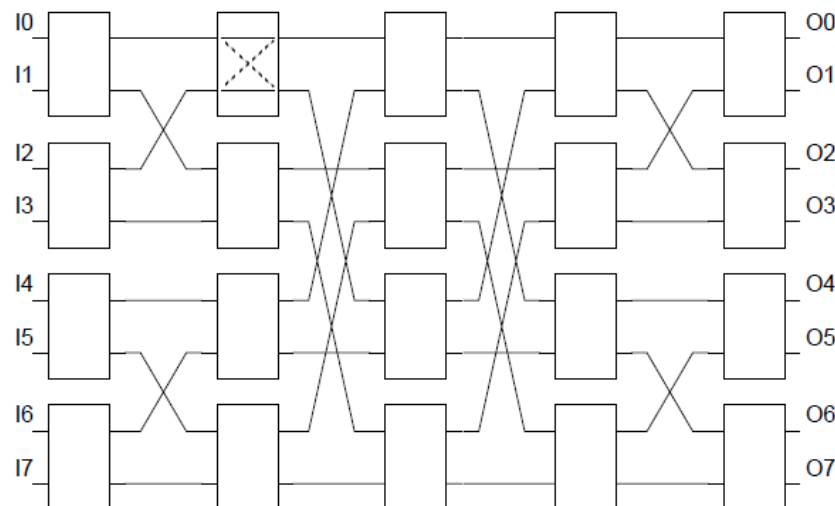


# Dynamic Topologies: multi-stage

- **Unidirectional**
    - N inputs and N outputs
  - **Properties**
    - $\log_2 N$  stages
    - $N/2 * \log_2 N$  shuffle elements
  - **Blocking!**
    - Unlike crossbar
  - **Improved blocking behaviour**
    - Additional stages
    - Benes network, composed of two banyan networks
- ➔ Nonblocking



**Banyan Network**

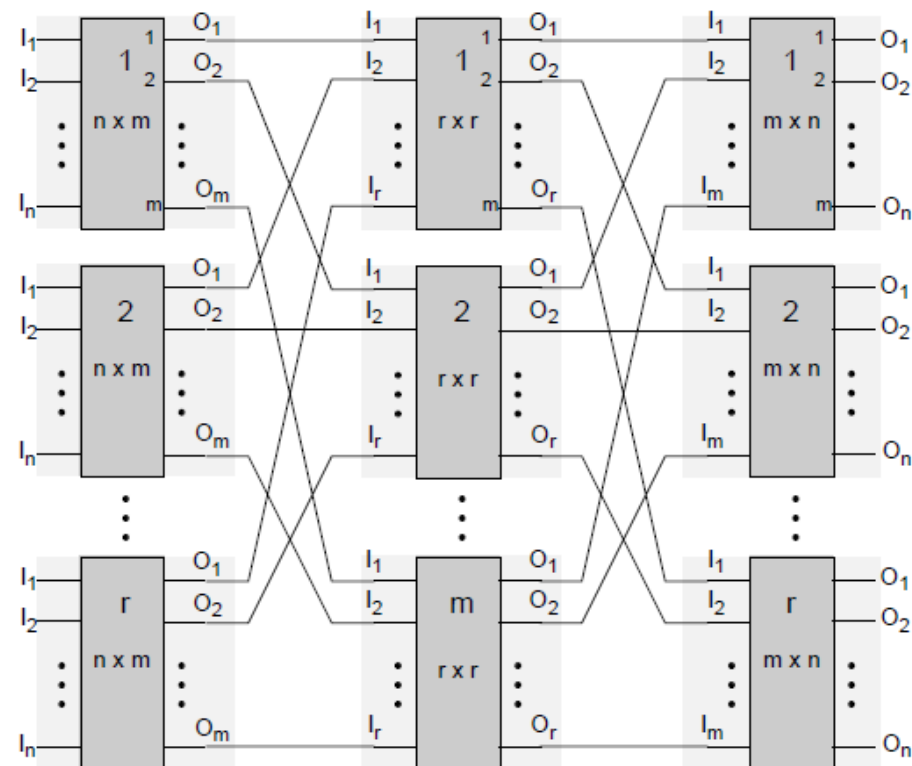


**Benes Network**



# Dynamic Topologies: multi-stage

- Use of crossbars instead of shuffles: CLOS network
  - Advantages of crossbars (no blocking) and of multi-staged INs (reduced complexity)
- 1-stage CLOS: identical to crossbar
- 2-stage CLOS: blocking
- 3-stage CLOS: nonblocking (see Banyan-Benes)
- Each CLOS can be seen as a crossbar with higher degree



## CLOS Network

Notice the 3 different types of XBARs used  
Assuming  $n=m=r$  and a 16x16 building block:  
256x256 CLOS



# Dynamic Topologies: BMIN

## ■ BMIN: bi-directional multi-stage IN

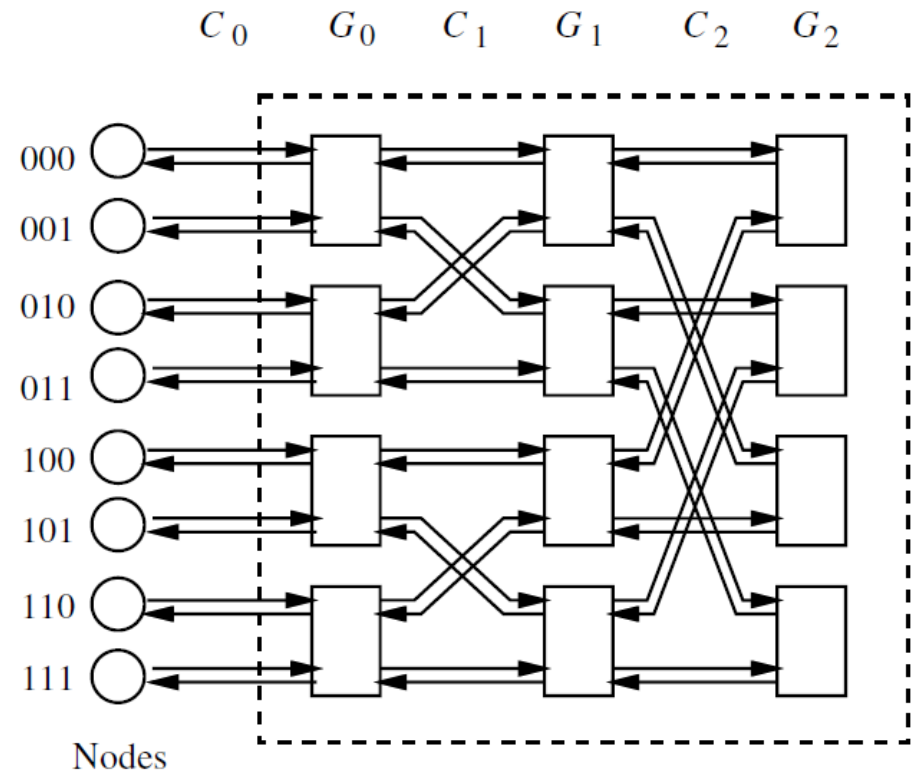
- Similar to before, but inputs/outputs all on the left

## ■ Switching elements are extended

- Forward
- Backward
- Turnaround

## ■ Alternate paths

- Path diversity



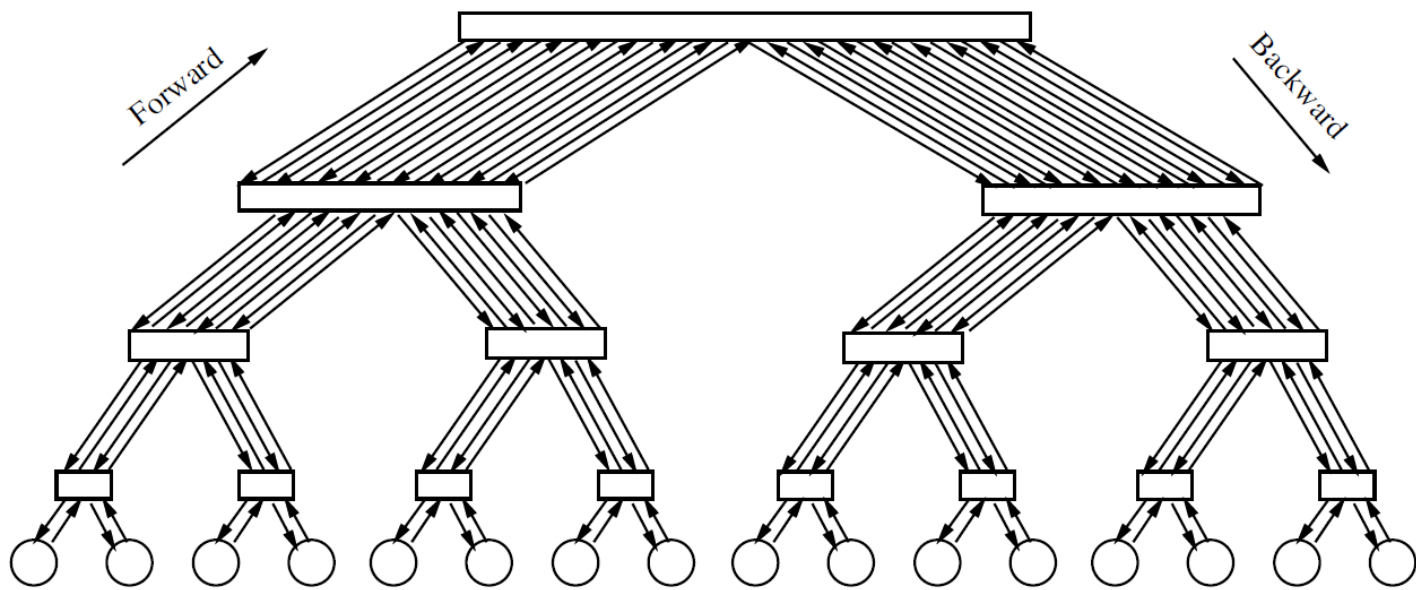
**8-node butterfly BMIN**

[Duato et al., Interconnection Networks, 2003]



# Dynamic Topologies: BMIN

- Remember the nice scalability of binary trees
  - Replace graph nodes with switching elements and increase number of connections accordingly
- Fat Tree
  - Bottleneck at root is avoided by appropriate provisioning
  - Typically uses crossbars
  - Main disadvantage: heterogeneity, different elements required



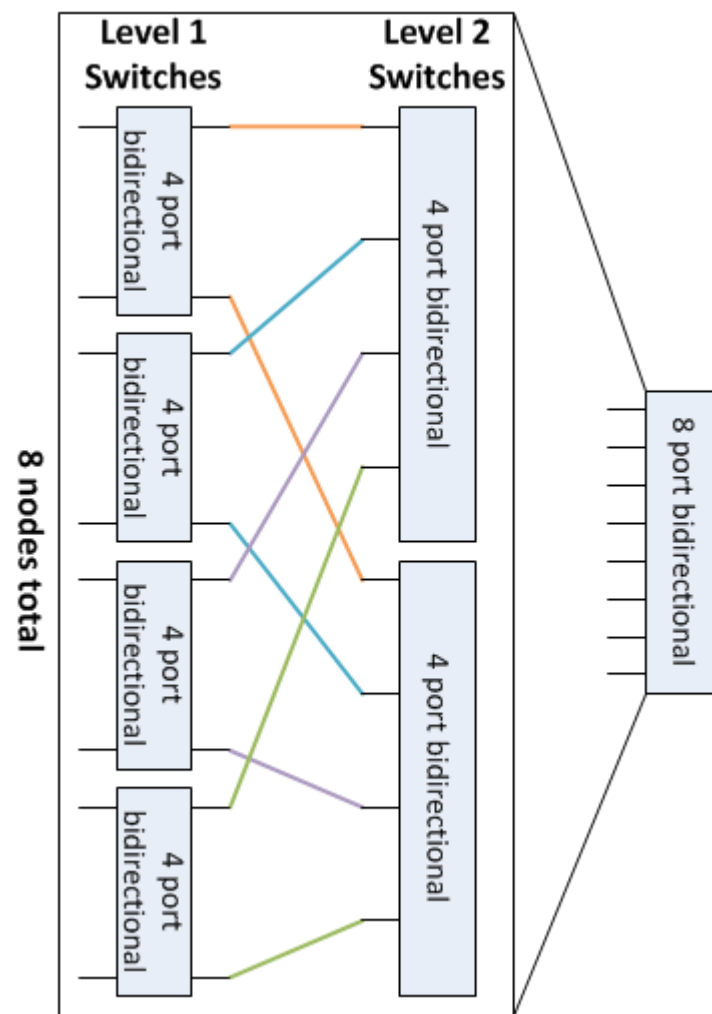
BMIN with turnaround viewed as **Fat Tree** – switching elements are typically crossbars or CLOS





# Dynamic Topologies: BMIN

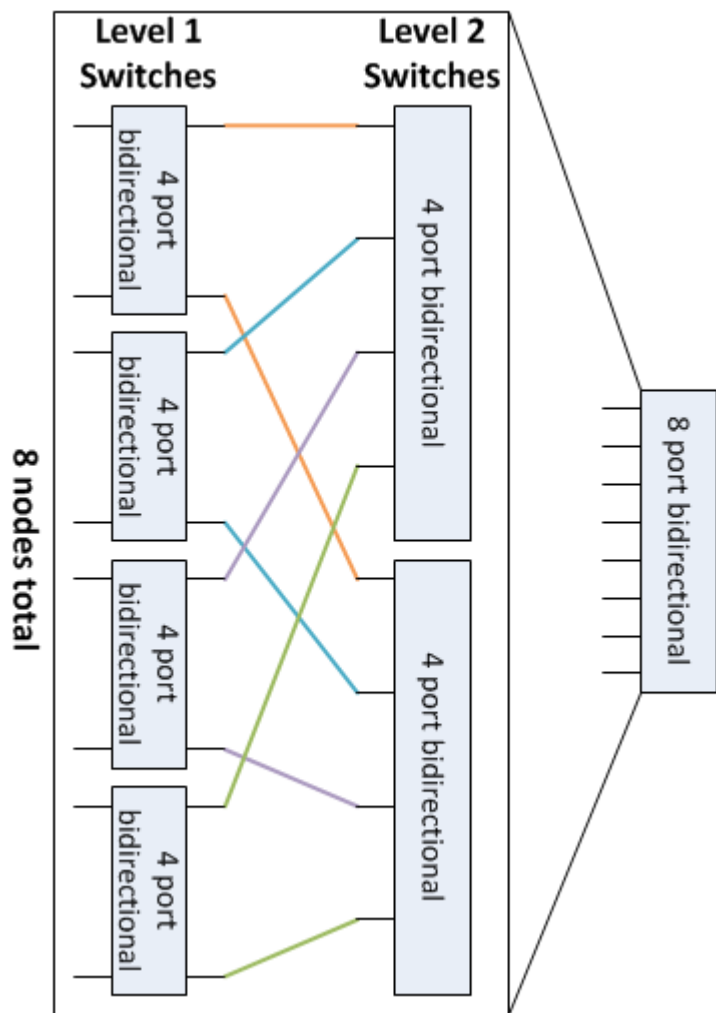
- Typically complete Fat Tree is based on one building block
- $n$ -port crossbar switch (single chip)
- „Fatter“ switches constructed out of this switch in a CLOS fashion
  - 2 stages: max.  $(n^2/2)$  end points
  - 3 stages: max.  $(n^3/4)$  end points
- User point of view:
  - Non-blocking fat crossbar
  - But number of internal stages may increase → hop latency increases!



Constructing a 8 port BMIN with 4-port building blocks

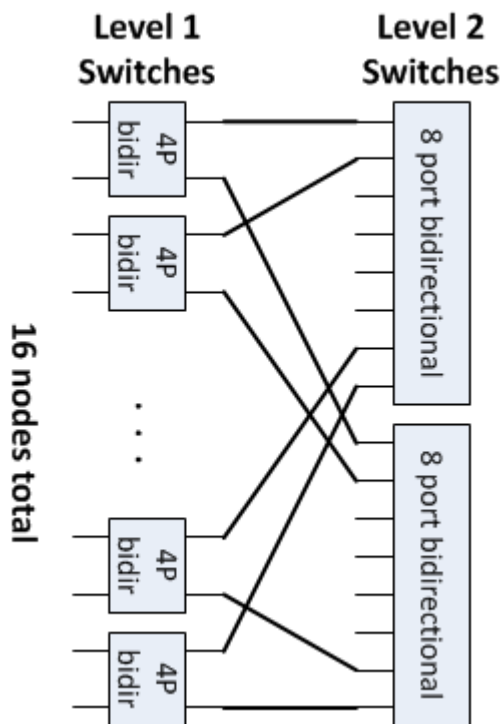


# Dynamic Topologies: BMIN



Constructing a 8 port BMIN with 4-port building blocks

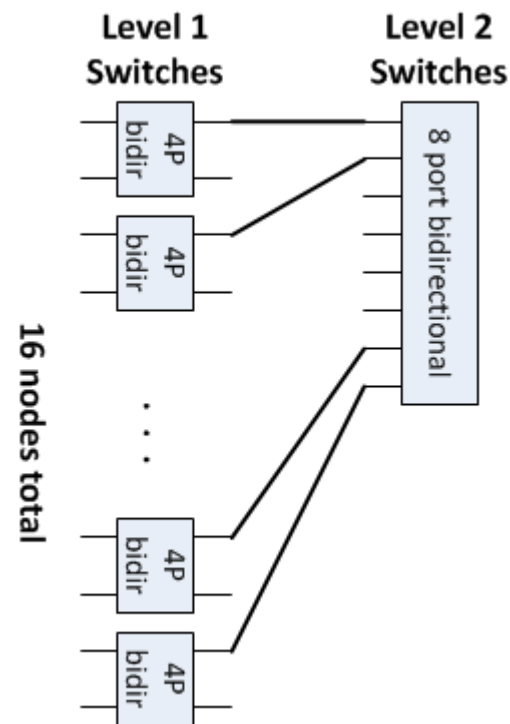
3 stages -  
nonblocking



Larger configuration based on element on the left

**Full bisection configuration**

3 stages -  
blocking



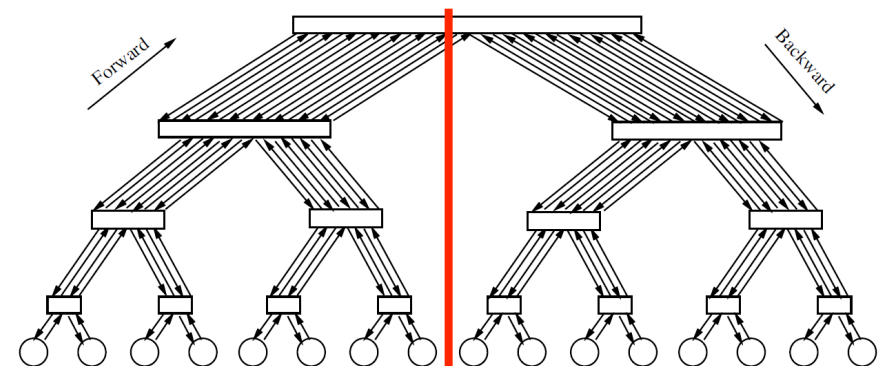
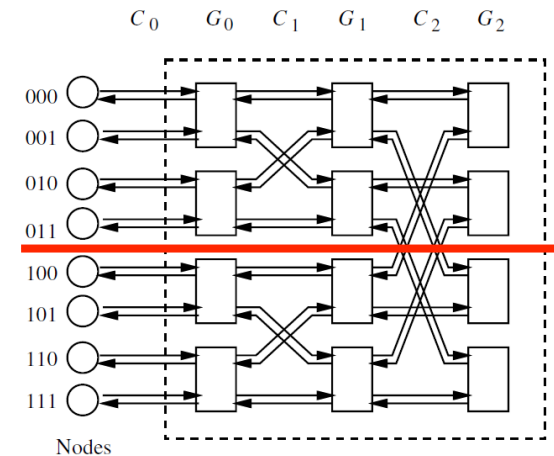
Larger configuration based on element on the left

**Reduced bisection configuration**



# Bisection bandwidth

- **Bisection:** Segmentation of an IN into two equal parts
  - As few cuts as possible
- **Bisection BW:** sum of the data rate of all cutted links
- The higher the bisection BW is, the lower is the blocking potential
  - Uniform traffic:  $\frac{1}{2}$  of traffic crosses bisection





# Overview of Properties

Topology	Node degree	Diameter	Number of connections	Scalable	Symmetric	Bisection
2D grid	4	$2(N^{1/2}-1)$	$2N-2N^{1/2}$	Yes	No	$N^{1/2}$
2D torus	4	$N^{1/2}-1$	$2N$	Yes	Yes	$2N^{1/2}$
3D grid	6	$3(N^{1/3}-1)$	$3N-3N^{1/3}$	Yes	No	$N^{2/3}$
3D torus	6	$3/2(N^{1/3}-1)$	$3N$	Yes	Yes	$2N^{2/3}$
Hypercube	$\log_2 N$	$\log_2 N$	$N \log_2(N/2)$	No	Yes	$2^{(\log_2 N)-1}$
Crossbar	1	1	$N^2$	No	Yes	$N/2$
CLOS	1	3	$r(2n+2m)$ ( $4N^2$ for $r=n=m$ )	Yes	Yes	$N/2$
Fat Tree, $S$ <i>is number of stages</i>	1	$2(S-1);$ $S=O(\log N)$	$N*S$	Yes	Yes	$N/2$



- **Interconnection networks as key in HPC**
  - INs are pervasive today: from smartphones to microcontrollers to large computing facilities
- **Topologies and their properties**
  - Direct vs. indirect
  - Static vs. dynamic
  - Node degree, diameter, number of connections, symmetry, scalable, (non-)blocking
- **Bisection bandwidth**
- **Many more topologies possible**
  - Regular but hierarchical
  - Irregular



# Credits & Further Reading

- Duato, Yalamanchili, Ni:  
Interconnection Networks -  
An Engineering Approach.  
2002
- Dally, Towles: Principles  
and Practices of  
Interconnection Networks.  
2003

