Name_____ Score(100)_____

Final Exam

Algorithms (CS 322 Spring 2020)

1. (3 points) Which of the following functions has the lowest order of growth? _____
   - $5\ln(n+100)^{10}$
   - $0.001n^4 + 3n^3 + 1$
   - $\ln^2 n$
   - $(n-2)!$
2. (3 points) Which of the following is false? _____
   - $n(n+1)/2 \in O(n^3)$
   - $n(n+1)/2 \in O(n^2)$
   - $n(n+1)/2 \in \Theta(n^3)$
   - $n(n+1)/2 \in \Omega(n)$
3. (3 points) Which of the following is the array representation of the heap structure for $1, 8, 6, 5, 3, 9, 4$? _____
   - $1, 8, 6, 5, 3, 9, 4$
   - $9, 6, 8, 5, 4, 3, 1$
   - $9, 8, 6, 5, 3, 1, 4$
   - $9, 6, 8, 1, 4, 5, 3$
4. (3 points) Which sorting algorithm is unstable? _____
   - heap sort
   - merge sort
   - bubble sort
   - insertion sort
5. (3 points) Which permutation succeeds 362541 in lexicographic order? _____
   - 361245
   - 412356
   - 364125
   - 364512
6. (3 points) Suppose A can be reduced to B in polynomial time, which of following statement is **NOT** confirmed yet? _____
   - if $B \in P$, then $A \in P$
   - if $B \in P$, then $A \in NP$
   - if $B \in NP$, then $A \in NP$
   - if $B \in NP$, then $A \in P$

7. Master Theorem.

   (a) (5 points) Please write the binary search recurrence relationship and use the Master Theorem method to analyze the asymptotic bound.

   (b) (5 points) Use the Master Theorem to give a tight asymptotic bound for $T(n) = 2T(n/4) + \sqrt{n}$.

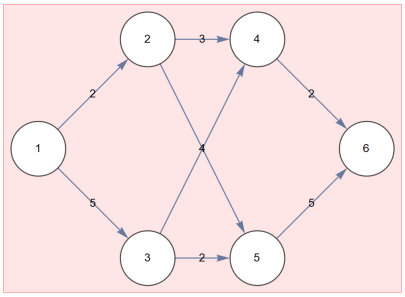
8. Answer the following questions related to the heap whose array representation is 10, 7, 6, 1, 3, 2, 5.

   (a) (5 points) Insert 8 into the heap.

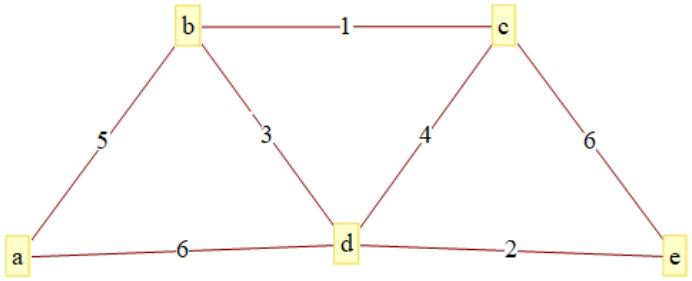
   (b) (5 points) Delete the largest element in the heap you created in (a).

9. Graph

(a) (5 points) Please find the maximum flow for the following flow network. Make sure to list each augmenting path and the flow moving through it.



(b) (5 points) Apply Kruskal's algorithm to find a minimum spanning tree for the following graph. Label the edges in the order that they are added into the spanning tree.



10. Dynamic Programming

(a) (5 points) Use dynamic programming to solve the coin-row problem for the coin row $5, 1, 2, 10, 6$.

(b) (5 points) Use dynamic programming to solve the change-making problem for the denominations $1, 3, 7$ and the amount $n = 11$.

(c) (5 points) Consider the knapsack problem with 4 items whose weights are $3, 1, 3, 4$ and whose values are $12, 20, 15, 10$, respectively. The capacity for the knapsack is 8. Please construct the dynamic table and determine the maximal value.

11. Huffman tree and code

(a) (5 points) Construct a Huffman code for the following data:

symbol	A	B	C	D	_
frequency	0.4	0.1	0.2	0.15	0.15

(b) (5 points) Encode ABACABAD using the code you developed in (a).

(c) (5 points) How many bits are needed for a symbol in the fixed-lengh encoding for the same alphabet? How many total bits are needed for the example in (b)?

12. (5 points) Please fill in the blank for the Euclidean algorithm that computes the greatest common divisor of m and n.

```
Euclid(m, n)
//Input:  Two nonnegative, not-both-zero integers m and n
//Output:  Greatest common divisor of m and n

while _____________ do

    r ← _____________

    m ← _____________

    _________________

return m
```

13. (5 points) Write a program to compute the nth Fibonacci number in linear time.

14. Given the following BubbleSort algorithm.

```
BubbleSort(A[0..n-1])
//Sorts a given array by bubble sort
//Input:  An array A[0..n-1] of orderable elements
//Output:  Array A[0..n-1] sorted in nondecreasing order
for i ← 0 to n - 2 do
   for j ← 0 to n -2 - i do
        if A[j+1] < A[j] do
           swap A[j] and A[j+1]
```

(a) (2 points) What is the basic operation for this algorithm?

(b) (3 points) Write the total number of basic operations in terms of the input size n in the worst case.

(c) (3 points) What is the order of growth using the most appropriate notation (O, Ω or Θ) for the total number of basic operations in (b)? Please prove it.

15. What is the result of the following two summation formulas?

(a) (2 points) $\sum_{i=l}^{u} 1$ where $l \leq u$ are known lower and upper integers.

(b) (2 points) $\sum_{i=0}^{n} i$ where $n > 0$ is a known integer.