89.Kal递归算法

2024年12月9日 20:06

估计真实数据 ⟶ 取平均值

Hat ← Estimate 估计

$$\hat{x}_k = \frac{1}{k}(z_1+z_2+\cdots+z_k)$$

$$= \frac{1}{k}(z_1+z_2+\cdots+z_{k-1}) + \frac{1}{k}z_k$$

⟶ k-1 次时的平均值 $\hat{x}_{k-1}$

$$= \frac{1}{k}\frac{k-1}{k-1}(z_1+z_2+\cdots+z_{k-1}) + \frac{1}{k}z_k$$

$$= \frac{k-1}{k}\hat{x}_{k-1} + \frac{1}{k}z_k$$

$$= \hat{x}_{k-1} - \frac{1}{k}\hat{x}_{k-1} + \frac{1}{k}z_k$$

$k\uparrow$, $\frac{1}{k}\to 0$, $\hat{x}_k \to \hat{x}_{k-1}$

$$\Rightarrow \boxed{\hat{x}_k = \hat{x}_{k-1} + \frac{1}{k}(z_k - \hat{x}_{k-1})}$$

$$\boxed{\hat{x}_k = \hat{x}_{k-1} + K_k(z_k - \hat{x}_{k-1})}$$

Recursive 递归

当前的估计值 = 上一次的估计值 + 系数 × (当前测量值 − 上一次的估计值)

K_k: Kalman Gain 卡尔曼增益/因数

估计误差: e_{EST} (Error 误差, Estimate 估计)

$$K_k = \frac{e_{EST_{k-1}}}{e_{EST_{k-1}} + e_{MEA_k}}$$

测量误差: e_{MEA} (Measurement 测量)

讨论: 在 k 时刻,

① $e_{EST_{k-1}} \gg e_{MEA_k}$: $K_k \to 1$, $\hat{x}_k = \hat{x}_{k-1} + z_k - \hat{x}_{k-1} = z_k$

k-1时的估计误差　k时的测量误差

更相信这个测量值；

② $e_{EST_{k-1}} \ll e_{MEA_k}$　$K_k \to 0$　$\hat{x}_k = \hat{x}_{k-1}$

下面一种情况的话，更相信我们的估计值

Step 1: 计算 Kalman Gain $K_k = \frac{e_{EST_{k-1}}}{e_{EST_{k-1}} + e_{MEA_k}}$

Step 2: 计算 $\hat{x}_k = \hat{x}_{k-1} + K_k(z_k - \hat{x}_{k-1})$

Step 3: 更新 $e_{EST_k} = (1-K_k)e_{EST_{k-1}}$ ⟶ 后面详细推导

实际长度 $x = 50$mm

$\hat{x}_0 = 40$mm

$e_{EST_0} = 5$mm

$z_1 = 51$mm

$e_{MEA_k} = 3$mm

k	z_k	e_{MEA_k}	$\hat{x}_k$	K_k	e_{EST_k}
0			40		5
1	51	3	46.875	0.625	1.875
2	48	3	47.308	0.3846	1.154
3					

k=1: $K_k = \frac{5}{5+3} = 0.625$

$\hat{x}_k = 40 + 0.625(51-40) = 46.875$

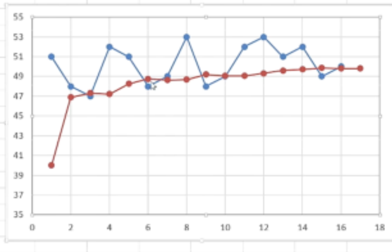
$e_{EST} = (1-0.625)5 = 1.875$

k=2: $K_k = \frac{1.875}{1.875+3} = 0.3846$

$\hat{x}_k = 46.875 + 0.3846(48-46.875) = 47.308$

$e_{EST} = (1-0.3846)1.875 = 1.154$

	A	B	C	D	E	F
1	k	Zk	e(MEAk)	hat(x)k	Kk	e(ESTk)
2	0			40		5
3	1	51	3	46.88	0.625	1.875
4	2	48	3	47.31	0.385	1.154
5	3	47	3	47.22	0.278	0.833
6	4	52	3	48.26	0.217	0.652
7	5	51	3	48.75	0.179	0.536
8	6	48	3	48.64	0.152	0.455
9	7	49	3	48.68	0.132	0.395
10	8	53	3	49.19	0.116	0.349
11	9	48	3	49.06	0.104	0.313
12	10	49	3	49.06	0.094	0.283
13	11	52	3	49.31	0.086	0.259
14	12	53	3	49.6	0.079	0.238
15	13	51	3	49.71	0.074	0.221
16	14	52	3	49.86	0.068	0.205
17	15	49	3	49.81	0.064	0.192
18	16	50	3	49.82	0.06	0.181



Kal数据融合

2024年12月9日 20:22

Kalman Filter Part 2. Data Fusion, Covarince Matrix, State Space, Observation

卡尔曼滤波器，数据融合，协方差矩阵，状态空间方程，观测器

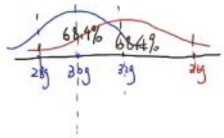
Data Fusion 数据融合

标准差 Standard Deviation

$z_1 = 30g$ $\quad \sigma_1 = 2g$

$z_2 = 32g$ $\quad \sigma_2 = 4g$

> 正态分布 Natural Distribution Gaussian



估计真实值 $\hat{z} = ?$

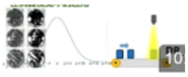
Kalman Gain

$$\hat{z} = z_1 + k(z_2 - z_1)$$

$k \in [0,1]$

$k=0$: $\hat{z} = z_1$

$k=1$: $\hat{z} = z_2$



求 K 使得 $\sigma_{\hat{z}}$ 最小 $\Rightarrow$ 方差 $Var(\hat{z})$ 最小

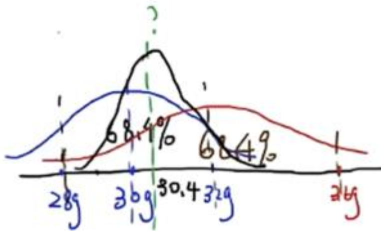
相互独立

$$\sigma_{\hat{z}}^2 = Var(z_1 + k(z_2 - z_1)) = Var(z_1 - kz_1 + kz_2) = Var((1-k)z_1 + kz_2)$$

$$= Var((1-k)z_1) + Var(kz_2) = (1-k)^2 \underbrace{Var(z_1)}_{\sigma_1^2} + k^2 \underbrace{Var(z_2)}_{\sigma_2^2} = (1-k)^2\sigma_1^2 + k^2\sigma_2^2$$

$$\frac{d\sigma_{\hat{z}}^2}{dk} = 0 \qquad -\cancel{2}(1-k)\sigma_1^2 + \cancel{2}k\sigma_2^2 = 0 \qquad -\sigma_1^2 + k\sigma_1^2 + k\sigma_2^2 = 0 \qquad k(\sigma_1^2 + \sigma_2^2) = \sigma_1^2$$

$$K = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} = \frac{2^2}{2^2+4^2} = \frac{4}{4+16} = 0.2$$



我吼，这协方差形式和线代中的旋转矩阵的形式不如出一辄嘛

$$P = \begin{bmatrix} \sigma_x^2 & \sigma_x\sigma_y & \sigma_x\sigma_z \\ \sigma_y\sigma_x & \sigma_y^2 & \sigma_y\sigma_z \\ \sigma_z\sigma_x & \sigma_z\sigma_y & \sigma_z^2 \end{bmatrix} = \begin{bmatrix} 24.89 & 18.7 & 4.4 \\ 18.7 & 14 & 3.3 \\ 4.4 & 3.3 & 4.22 \end{bmatrix}$$

例：

球员	身高 x	体重 y	年龄 z
瓦尔迪	179	74	33
奥巴梅扬	187	80	31
萨拉赫	175	71	28
平均	180.3	75	30.7

平均值

$$a = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

$$P = \frac{1}{3}a^T a$$

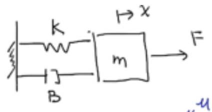
方差 $\sigma_x^2 = \frac{1}{3}((179-180.3)^2 + (187-180.3)^2 + (175-180.3)^2) = 24.89$

$\sigma_y^2 = 14$

$\sigma_z^2 = 4.22$

协方差 $\sigma_x\sigma_y = \frac{1}{3}(\overset{<0}{(179-180.3)}\overset{<0}{(74-75)} + \overset{>0}{(187-180.3)}\overset{>0}{(80-75)} + (175-180.3)(71-75)) = 18.7 = \sigma_y\sigma_x$ >0 正相关

例：



$$m\ddot{x} + B\dot{x} + kx = F \quad (F = u\text{: Input})$$

$$m\ddot{x} = u - B\dot{x} - kx$$

State 状态变量 $x_1 = x$

$x_2 = \dot{x}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = \ddot{x} = \frac{1}{m}u - \frac{B}{m}\dot{x} - \frac{k}{m}x$

$= \frac{1}{m}u - \frac{B}{m}x_2 - \frac{k}{m}x_1$

测量 Measurement $z_1 = x = x_1$ 位置

$z_2 = \dot{x} = x_2$ 速度

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{B}{m} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}u$$

随时间的变化 $\dot{X}(t) = A\,X(t) + B\,u(t)$ State Space

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ 连续

$Z(t) = H\,X(t)$ 不确定性

离散：

$X_k = AX_{k-1} + Bu_k + w_{k-1}$ 下标 $k, k-1, k+1$ 时间单位 Sample time 采样

w_{k-1}: 过程噪音 Process Noise

$Z_k = HX_k + v_k$

v_k: 测量噪音 Measurement Noise

问题：估计 $\hat{X}_k = ?$

Kalman Filter

Kal增益

2025年1月16日 19:13

Kalman Filter Part 3. Step by Step Derivation of Kalman Gain

卡尔曼滤波器 卡尔曼增益/超详细推导

状态空间方程：

$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$ 过程噪声

$z_k = Hx_k + v_k$ 测量噪声

期望 协方差矩阵

$P(w) \sim (0, Q)$

$Q = E[ww^T]$

$VAR(x) = E(x^2) - E^2(x)$

$E\left[\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} [w_1 \ w_2]\right] = \begin{bmatrix} Ew_1^2 & Ew_1w_2 \\ Ew_2w_1 & Ew_2^2 \end{bmatrix}$

$\sigma_{w_1}^2 \quad \sigma_{w_1}\sigma_{w_2}$

$\sigma_{w_2}\sigma_{w_1} \quad \sigma_{w_2}^2$

$P(v) \sim (0, R)$

$E[vv^T] = R$

先验

$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$ ① 算出来的

$z_k = Hx_k \rightarrow \hat{x}_{k_{mea}} = H^- z_k$ 测出来的

后验

$\hat{x}_k = \hat{x}_k^- + G(H^- z_k - \hat{x}_k^-)$

$G = K_kH$

卡尔曼滤波 $\boxed{\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)}$

$G = 0$

$\hat{x}_k = \hat{x}_k^-$

$G = 1$

$\hat{x}_k = H^- z_k$

$K_k \in [0, H^-]$

$K_k = 0$

$\hat{x}_k = \hat{x}_k^-$

$K_k = H^-$

目标：去寻找 K_k 使得 $\hat{x}_k \rightarrow x_k$ 实际值

$\boxed{e_k = x_k - \hat{x}_k}$

$P = E[ee^T]$

$= E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T]$

$p(e_k) \sim (0, P)$

$P = E[ee^T] = \begin{bmatrix} \sigma_{e_1}^2 & \sigma_{e_1}\sigma_{e_2} \\ \sigma_{e_2}\sigma_{e_1} & \sigma_{e_2}^2 \end{bmatrix}$

$tr(P) = \sigma_{e_1}^2 + \sigma_{e_2}^2$ 方差和

迹

$x_k - \hat{x}_k = x_k - (\hat{x}_k^- + K_k(z_k - H\hat{x}_k^-))$

$= x_k - \hat{x}_k^- - K_kz_k + K_kH\hat{x}_k^-$ （$z_k = Hx_k + v_k$）

$= x_k - \hat{x}_k^- - K_kHx_k - K_kv_k + K_kH\hat{x}_k^-$

$= (x_k - \hat{x}_k^-) - K_kH(x_k - \hat{x}_k^-) - K_kv_k$

$= (I - K_kH)\underbrace{(x_k - \hat{x}_k^-)}_{e_k^-} - K_kv_k$

$P_k = P_k^- - K_kHP_k^- - P_k^-H^TK_k^T + K_kHP_k^-H^TK_k^T + K_kRK_k^T$

$tr(P_k) = tr(P_k^-) - 2tr(K_kHP_k^-) + tr(K_kHP_k^-H^TK_k^T)$

$+ tr(K_kRK_k^T)$

$\frac{d\,tr(P_k)}{dK_k} = 0$

$\frac{d\,tr(P_k)}{dK_k} = 0$

$K_k = H^-$

$\hat{x}_k = H^- z_k$

$(P_k^-H^TK_k^T)^T = K_k(P_k^-H^T)^T$

$= K_kHP_k^{-T}$

$\frac{d\,tr(AB)}{dA} = B^T$

$$tr\left(\overset{A}{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}\overset{B}{\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}}\right)$$

$\begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & \\ & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$

$tr(AB) = a_{11}b_{11} + a_{12}b_{21} + a_{21}b_{12} + a_{22}b_{22}$

$\frac{d\,tr(AB)}{dA} = \begin{bmatrix} \frac{\partial tr(AB)}{\partial a_{11}} & \frac{\partial tr(AB)}{\partial a_{12}} \\ \frac{\partial tr(AB)}{\partial a_{21}} & \frac{\partial tr(AB)}{\partial a_{22}} \end{bmatrix}$

$= \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} = B^T$

$\frac{d(ABA^T)}{dA} = 2AB$

协方差矩阵的转置就等于它本身！

$\frac{d\,tr(P_k)}{dK_k} = 0$

$\frac{d\,tr(P_k)}{dK_k} = 0 - 2(HP_k^-)^T + 2K_kHP_k^-H^T$

$+ 2K_kR = 0$

协方差

$-P_k^-H^T + K_k(HP_k^-H^T + R) = 0$

$K_k(HP_k^-H^T + R) = P_k^-H^T$

Kalman Gain.

$\boxed{K_k = \frac{P_k^-H^T}{HP_k^-H^T + R}}$

$R \uparrow \ K_k \rightarrow 0$

$R \downarrow$

$K_k = \frac{P_k^-H^T}{HP_k^-H^T}$

$= H^-$

kal协方差

2025年2月8日 14:35

Kalman Filter Part 4. Priori/Posteriori Error Covariance Matrix

卡尔曼滤波器 误差协方差矩阵

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \qquad w \sim p(0, Q)$$

$$z_k = Hx_k + v_k \qquad v \sim p(0, R)$$

先验估计

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$$

后验估计

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$$

卡尔曼增益

$$K_k = \frac{P_k^- H^T}{HP_k^- H^T + R}$$

预测			校正		
先验	$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$	①	卡尔曼增益	$K_k = \frac{P_k^- H^T}{HP_k^- H^T + R}$	③
先验误差协方差	$P_k^- = AP_{k-1}A^T + Q$ (上一次的误差协方差)	②	后验估计	$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$	④
			更新误差协方差	$P_k = (I - K_kH)P_k^-$	⑤

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \qquad w \sim p(0, Q)$$

$$z_k = Hx_k + v_k \qquad v \sim p(0, R)$$

先验估计

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$$

后验估计

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$$

卡尔曼增益

$$K_k = \frac{P_k^- H^T}{HP_k^- H^T + R} \quad ?$$

$$e_k^- = x_k - \hat{x}_k^-$$

$$= Ax_{k-1} + \cancel{Bu_{k-1}} + w_{k-1} - A\hat{x}_{k-1} - \cancel{Bu_{k-1}}$$

$$= A\underbrace{(x_{k-1} - \hat{x}_{k-1})}_{= e_{k-1}} + w_{k-1}$$

$$= Ae_{k-1} + w_{k-1}$$

求 P_k^-

$$P_k^- = E[e_k^- e_k^{-T}]$$

$$= E[(Ae_{k-1} + w_{k-1})((Ae_{k-1})^T + (w_{k-1})^T)]$$

$$= E[(Ae_{k-1} + w_{k-1})(e_{k-1}^T A^T + w_{k-1}^T)]$$

$$= E[Ae_{k-1}e_{k-1}^T A^T] + \cancel{E[Ae_{k-1}w_{k-1}^T]}^{\,0} + \cancel{E[w_{k-1}e_{k-1}^T A^T]}^{\,0} + E[w_{k-1}w_{k-1}^T]$$

相互独立 $AE(e_{k-1})E(w_{k-1}^T)$ $0 \quad 0$

$$= A\underbrace{E[e_{k-1}e_{k-1}^T]}_{= P_{k-1}}A^T + \underbrace{E[w_{k-1}w_{k-1}^T]}_{= Q}$$

$$\boxed{P_k^- = AP_{k-1}A^T + Q}$$

使用卡尔曼滤波器估计状态变量的值

预测		校正	
先验	$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$	卡尔曼增益	$K_k = \frac{P_k^- H^T}{HP_k^- H^T + R}$
先验误差协方差	$P_k^- = A\underbrace{P_{k-1}}_{\text{上一次的误差协方差}}A^T + Q$	后验估计	$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$
		更新误差协方差	$P_k = (I - K_kH)P_k^-$

$$P_k = P_k^- - K_kHP_k^- - \cancel{P_k^- H^T K_k^T} + \underbrace{K_kHP_k^-H^TK_k^T + K_kRK_k^T}$$

$$K_k(HP_k^-H^T + R)K_k^T$$

$$\cancel{\frac{P_k^-H^T K_k^T}{HP_k^-H^T + R}(HP_k^-H^T + R)K_k^T}$$

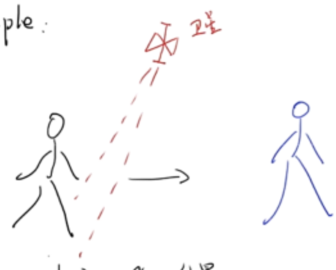
$$= P_k^- - K_kHP_k^-$$

$$P_k = (I - K_kH)P_k^-$$

105.kal二维实例

2025年2月8日 14:52

Example:



匀速：

$\Delta T = 1$ 不确定

位置： $x_{1,k} = x_{1,k-1} + x_{2,k-1} + w_{1,k-1}$

速度： $x_{2,k} = x_{2,k-1} + w_{2,k-1}$

采样时间 ΔT k 时刻与 $k-1$ 时刻的间隔

w：过程噪声 Process Noise

$p(w) \sim N(0, Q)$

期望 协方差矩阵

States: $\mapsto$ x_1：位置.
x_2：速度

不确定性

$z_{1,k} = x_{1,k} + v_{1,k}$ $p(v) \sim N(0, R)$

$z_{2,k} = x_{2,k} + v_{2,k}$

Kalman Filter Part 5. An Example

卡尔曼滤波器：2D 例子.

宏码. Data Fusion.

$z_1 = 6.5\,mm$ $\sigma_1 = 0.2\,mm$ 最优估计 $\hat{z} = ?$

$z_2 = 7.3\,mm$ $\sigma_2 = 0.4\,mm$

$$X_k = A X_{k-1} + W_{k-1} \quad \text{不准确}$$

$$\begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,k-1} \\ x_{2,k-1} \end{bmatrix} + \begin{bmatrix} w_{1,k-1} \\ w_{2,k-1} \end{bmatrix} \Rightarrow \hat{x}_k \text{ 最优}$$

$$Z_k = H X_k + V_k \quad \text{也不准确}$$

$$\begin{bmatrix} z_{1,k} \\ z_{2,k} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} v_{1,k} \\ v_{2,k} \end{bmatrix}$$

Q矩阵		R矩阵		A矩阵		H矩阵		单位矩阵	
1.00	0	0.10	0	1	1	1	0	1	0
0	1.00	0	0.10	0	1	0	1	0	1

Kalman Filter

预测

$$\hat{x}_k^- = A \hat{x}_{k-1}$$

$$P_k^- = A P_{k-1} A^T + Q$$

校正

$$K_k = \frac{P_k^- H^T}{H P_k^- H^T + R}$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H \hat{x}_k^-)$$

$$P_k = (I - K_k H) P_k^-$$

kal扩展滤波器

2025年2月8日 16:21

Kalman Filter Part6 Extended Kalman Filter (EKF)

扩展卡尔曼滤波器

Revisit Linear System

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \qquad P(w) \sim N(0, Q)$$
$$z_k = Hx_{k} + v_k \qquad P(v) \sim N(0, R)$$

预测

先验 $\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$

$P_k^- = AP_{k-1}A^T + Q$

校正

$K_k = \dfrac{P_k^- H^T}{HP_k^- H^T + R}$

后验 $\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$

$P_k = (I - K_k H)P_k^-$

此前的案例是符合线性的

Data Fusion

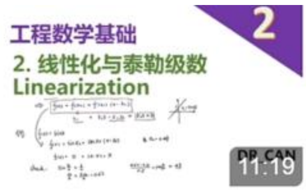
Nonlinear

非线性.

$x_k = f(x_{k-1}, u_{k-1}, w_{k-1})$ → $P(w) \sim N(0, Q)$ f, h 非线性

$z_k = h(x_k, v_k)$ → $P(v) \sim N(0, R)$

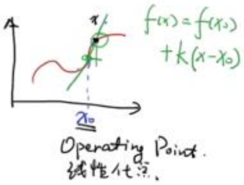
正态分布的随机变量通过非线性系统后就不再是正态的了



线性化: Linearization

Taylor Series. 泰勒级数

$$f(x) = f(x_0) + \frac{\partial f}{\partial x}(x - x_0)$$



系统有误差, 无法在真实值线性化.

$f(x_k)$ 在 $\hat{x}_{k-1}$ 处线性化 → $k-1$ 时的后验估计 (上一次)

$f(\hat{x}_{k-1}, u_{k-1}, 0) = \tilde{x}_k$

$$x_k = f(\hat{x}_{k-1}, u_{k-1}, w_{k-1}) + A(x_k - \hat{x}_{k-1}) + W_k w_{k-1}$$

误差 假设为0

$$A = \frac{\partial f}{\partial x}\Big|_{\hat{x}_{k-1}, u_{k-1}}$$

例

$x_1 = x_1 + \sin x_2 = f_1$

$x_2 = x_1^2 = f_2$

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \cos x_2 \\ 2x_1 & 0 \end{bmatrix}\Big|_{\hat{x}_{1,k-1}, \hat{x}_{2,k-1}}$$

$$A_k = \begin{bmatrix} 1 & \cos\hat{x}_{2,k-1} \\ 2\hat{x}_{1,k-1} & 0 \end{bmatrix}$$

A_k 随着 k 变化.

系统有误差, 无法在真实值线性化.

$f(x_k)$ 在 $\hat{x}_{k-1}$ 处线性化 → $k-1$ 时的后验估计 (上一次)

$f(\hat{x}_{k-1}, u_{k-1}, 0) = \tilde{x}_k$

$$x_k = f(\hat{x}_{k-1}, u_{k-1}, w_{k-1}) + A(x_k - \hat{x}_{k-1}) + W_k w_{k-1}$$

误差 假设为0

$$A = \frac{\partial f}{\partial x}\Big|_{\hat{x}_{k-1}, u_{k-1}}$$

$$W_k = \frac{\partial f}{\partial w}\Big|_{\hat{x}_{k-1}, u_{k-1}}$$

z_k 在 $\tilde{x}_k$ 线性化.

误差 假设 0 $h(\tilde{x}_k, 0) = \tilde{z}_k$

$$z_k = h(\tilde{x}_k, v_k) + H(x_k - \tilde{x}_k) + V v_k$$

$$H = \frac{\partial h}{\partial x}\Big|_{\tilde{x}_k}$$

$$V = \frac{\partial h}{\partial v}\Big|_{\tilde{x}_k}$$

$$x_k = \tilde{x}_k + A(x_{k-1} - \hat{x}_{k-1}) + W w_{k-1}$$

$$z_k = \tilde{z}_k + H(x_k - \tilde{x}_k) + V v_k$$

Operating Point.
线性化点.

例

$x_1 = x_1 + \sin x_2 = f_1$

$x_2 = x_1^2 = f_2$

$$A = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

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$$A_k = \begin{bmatrix} 1 & \cos\hat{x}_{2,k-1} \\ 2\hat{x}_{1,k-1} & 0 \end{bmatrix}$$

A_k 随着 k 变化.

$$P(w) \sim N(0, Q)$$

$$P(Ww) \sim N(0, \underline{WQW^T})$$

$$P(Vv_k) \sim N(0, VRV^T)$$

$$E(ax) = aE(x) = 0$$

$$Var(ax) = a^2 Var(x)$$

预测

先验 $\hat{x}_k^- = f(\hat{x}_{k-1}, u_{k-1}, 0)$

$$P_k^- = AP_{k-1}A^T + WQW^T$$

校正

$$K_k = \frac{P_k^- H^T}{HP_k^- H^T + VRV^T}$$

后验 $\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0))$

$$P_k = (I - K_k H) P_k^-$$