מתמטיקה B ~ תרגיל בית 4

פרץ שחר

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(1)

.1

$$\int \cos^3 x \sin x \,\mathrm{d}x = \begin{bmatrix} u = \cos x & u' = -\sin x \\ \mathrm{d}u = -\sin x \,\mathrm{d}x \end{bmatrix} = \int -u^3 = -\frac{1}{4}u^4 = \boldsymbol{-\frac{\cos^4 x}{4} + C}$$

.2

$$\int \sqrt{\frac{\arcsin x}{1-x^2}} \,\mathrm{d}x = \int \sqrt{\arcsin x} \arcsin' = \begin{bmatrix} \theta = \arcsin x & \theta' = \arcsin' \\ \mathrm{d}\theta = \arcsin' \,\mathrm{d}x \end{bmatrix} = \int \sqrt{\theta} \,\mathrm{d}\theta = \frac{2}{3}\theta^{1.5} = \boldsymbol{\frac{\arcsin^{1.5} x}{1.5} + C}$$

.3

$$\int \frac{\ln^2 x}{x} \,\mathrm{d}x = \begin{bmatrix} u = \ln x & u' = \frac{1}{x} \\ \mathrm{d}u = \frac{1}{x} \,\mathrm{d}x \end{bmatrix} = \int u^2 \,\mathrm{d}u = \frac{1}{3}u^3 = \boldsymbol{\frac{\ln^3 x}{3} + C}$$

.4

$$\int \frac{\mathrm{d}x}{\sqrt{x} + \sqrt[3]{x}} = \begin{bmatrix} u = x^{\frac{1}{6}} & u' = \frac{1}{6}x^{-\frac{5}{6}} \\ \mathrm{d}u = \frac{1}{6}x^{-\frac{5}{6}} \,\mathrm{d}x & \mathrm{d}x = 6u^5 \,\mathrm{d}u \end{bmatrix} = \int \frac{6u^5 \,\mathrm{d}u}{u^3 + u^2} = \int \frac{\cancel{u^2}6u^3 \,\mathrm{d}u}{\cancel{u^2}(1+u)} = \begin{bmatrix} t = u + 1 & t' = u \\ \mathrm{d}t = u \,\mathrm{d}u \end{bmatrix}$$

$$= \frac{6t^2 \,\mathrm{d}t}{t} = 6\int t \,\mathrm{d}t = 3t^2 = 3(u+1)^2 = 3u^2 + 6u + 1 = \boldsymbol{3\sqrt[3]{x} + 6\sqrt[6]{x} + 1 + C}$$

.5

$$\int \frac{x}{(x+3)^{\frac{1}{5}}} \,\mathrm{d}x = \int x(x+3)^{-\frac{1}{5}} = \int \sum_{i=0}^{5} \binom{5}{i} x^{5-i+1} 3^i \,\mathrm{d}x = \sum_{i==0}^{5} \left[\binom{5}{i} 3^i \int \left(x^{6-i}\right) \,\mathrm{d}x \right] = \sum_{i=0}^{5} \binom{5}{i} \frac{3^i}{6-i} x^{6-i}$$

$$= \boldsymbol{\tfrac{1}{6}x^6 + 3x^5 + 22.5x^4 + 90x^3 + 202.5x^2 + 243x + C}$$

(2)

(3)

.1

$$\int \frac{\sqrt{25x^2 - 4}}{x} \,\mathrm{d}x = \begin{bmatrix} x = 0.4\sinh x & x' = 0.4\cosh x \\ \mathrm{d}x = 0.4\cosh x \,\mathrm{d}\theta \end{bmatrix} = \int \frac{\sqrt{4(6.25 \cdot 0.4^2 \sinh^2 x - 1)}}{0.4\sinh x} 0.4\cosh x \,\mathrm{d}\theta$$

$$= \int \frac{\cancel{0.4}\sqrt{2}\sqrt{\sinh^2 x - 1}}{\cancel{0.4}\sinh x} \cosh x = \sqrt{2}\cosh x \frac{\cosh x}{\sinh x} = \sqrt{2}\cosh x \coth x$$

.2 $\ell et\ a = 1 + \frac{3}{\sqrt{2}}$

$$\int \frac{x}{\sqrt{2x^2 - 4x - 7}} \,\mathrm{d}x = \frac{x}{\sqrt{\left(x - 1 - \frac{3}{\sqrt{2}}\right)\left(x - 1 + \frac{3}{\sqrt{2}}\right)}} \,\mathrm{d}t = \begin{bmatrix} t = x - 1 & t' = 1 \\ \mathrm{d}t = 1 \,\mathrm{d}x \end{bmatrix} = \int \frac{t+1}{t^2 + a^2} \,\mathrm{d}t$$

$$= \int \frac{1}{x^2 + a^2} + \int \frac{t}{den}$$