תרגיל הכנה – לקראת קורס ב' – חופשת סמסטר

שחר פרץ

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1 Complex Numbers' Operations

$\ell et\ z = 6 + 3i,\ w = 2 - 5i$

- $\Re(z) = \mathbf{6}$
- $\Im(z) = \mathbf{3}$
- $\bar{z} = \Re(z) - \Im(z)i = \mathbf{6 - 3i}$
- $\Re(w) = \mathbf{2}$
- $\Im(w) = -\mathbf{5}$
- $\bar{w} = \Re(z) - \Im(z)i = \mathbf{3i + 5i}$
- $|w| = \sqrt{2^2 + 5^2} = \sqrt{\mathbf{29}} \approx 5.385$
- $z + w = 6 + 2 + 3i - 5i = \mathbf{8 - 2i}$
- $z - w = 6 - 2 + 3i + 5i = \mathbf{4 + 8i}$
- $z \cdot w = 6 \cdot 2 - 6 \cdot 5i + 3i \cdot 2 - 3i \cdot 5i = 12 - 30i + 6i + 15 = \mathbf{27 - 24i}$
- $\frac{z}{w} = \frac{6+3i}{2-5i} = \frac{12-15}{4+25} + \frac{6+30}{4+25}i = -\mathbf{\frac{3}{29}} + \mathbf{\frac{36}{29}i}$
- $\frac{w}{z} = \frac{2-5i}{6+3i} = \frac{12-15}{36+9} + \frac{-30-6}{36+9}i = -\mathbf{\frac{1}{15}} + -\mathbf{\frac{4}{5}i}$
- $z^2 = (6 + 3i)^2 = (6^2 - 3^2) + 2 \cdot 3 \cdot 6i = \mathbf{27 + 36i}$
- $w^3 = (2 - 5i)^3 = (2 - 5i)^2 \cdot (2 - 5i) = [(2^2 - 5^2) - 4 \cdot 5i](2 - 5i) = (-21 - 20i)(2 - 5i) = -\mathbf{142 + 65i}$

2 An Equation

$$(1 - 5i)x^2 + 2 + 10i = 12x \tag{1}$$
$$(1 - 5i)x^2 - 12x + (2 + 10i) = 0 \tag{2}$$
$$x_{1,2} = \frac{12 \pm \sqrt{12^2 - 4(1 - 5i)(2 + 10i)}}{2(1 - 5i)} \tag{3}$$
$$x_{1,2} = \frac{12 \pm \sqrt{-64}}{2 - 10i} = \frac{12 \mp \frac{1+i}{\sqrt{2}}}{(2 - 10i)} \tag{4}$$
$$x_{1,2} = \frac{12 \pm \sqrt{2}/2 + \sqrt{2}/2i}{2 - 10i} \tag{5}$$
$$x_{1,2} \approx \mathbf{0.1764 + 1.235i},\ \mathbf{0.2852 + 1.0723i} \tag{6}$$

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"There is nothing to do" $\sim$ pacman

4 Differentiation and Integration

a.

$$\begin{aligned} & [9x\cos x - 8x\sin x + 15x + 4]' && (7)\\ &= 9\cos x - 9x\sin x - 8\sin x - 8x\cos x + 15 && (8)\end{aligned}$$

b.

$$\begin{aligned} & [(x^3 - 8x + 1)^{20} + \sin(\sin(x))]' && (9)\\ &= 20(x^3 - 8x + 1)^{19} \cdot (3x^2 - 8) + \cos(\sin(x)) \cdot (-\cos x) && (10)\\ &= 20(x^3 - 8x + 1)^{19}(3x^2 - 8) - \cos(\sin x)\cos x && (11)\end{aligned}$$

c.

$$\begin{aligned} & \left[\frac{2x+6}{2-3x^2}\right]' && (12)\\ &= \frac{x(2-3x^2) - (-6x)(2x+6)}{(2-3x)^2} && (13)\\ &= \frac{2x - 3x^3 + 12x^2 + 36x}{4 - 6x + 9x^2} = \frac{-3x^3 + 12^2 + 38x}{9x^2 - 6x + 4} && (14)\end{aligned}$$

d.

$$\tan x' = \left[\frac{\sin x}{\cos x}\right]' = \frac{\sin x' \cos x - \cos x' \sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^x} = \cos^{-2} x \qquad (15)$$

hence

$$\begin{aligned} & [(2x)^{50}\tan 3x]' && (16)\\ &= (50(2x)^{49} \cdot \tan 3x) + (-\cos^{-2} 3x \cdot (2x)^{50}) && (17)\\ &= (2x)^{49}(50\tan 3x - 2x\cos^{-2} 3x) && (18)\end{aligned}$$

e.

$$\begin{aligned} & \int \frac{5}{x^2} - \frac{3}{\cos^2 x} + 9 \,\mathrm{d}x && (19)\\ &= 5\int x^{-2}\,\mathrm{d}x + 3\int \cos^{-2} x\,\mathrm{d}x + \int 9\,\mathrm{d}x && (20)\\ &= -5 \cdot x^{-1} + 3\tan x + 9x + c \qquad \text{(since was proven } \tan x' = \cos^{-2} x \text{ before)} && (21)\\ & && (22)\end{aligned}$$

f.

$$\begin{aligned} & \int \cos(7x+7) - 5x^4 \,\mathrm{d}x && (23)\\ &= \int \cos(7x+7) - 5\int x^4 && (24)\\ &= \frac{\sin(7x+7)}{7} - 5 \cdot \frac{1}{5}x^5 && (25)\\ &= \frac{1}{7}\sin(7x+7) - x^5 && (26)\end{aligned}$$