

Algorithm	Worst-case running time	Average-case/expected running time
Insertion sort	$\Theta(n^2)$	$\Theta(n^2)$
Merge sort	$\Theta(n \lg n)$	$\Theta(n \lg n)$
Heapsort	$O(n \lg n)$	—
Quicksort	$\Theta(n^2)$	$\Theta(n \lg n)$ (expected)
Counting sort	$\Theta(k + n)$	$\Theta(k + n)$
Radix sort	$\Theta(d(n + k))$	$\Theta(d(n + k))$
Bucket sort	$\Theta(n^2)$	$\Theta(n)$ (average-case)

<ul style="list-style-type: none"> ▪ n vertices, m edges ▪ no parallel edges ▪ no self-loops 	Edge List	Adjacency List	Adjacency Matrix
Space	$n + m$	$n + m$	n^2
incidentEdges(v)	m	deg(v)	n
areAdjacent(v, w)	m	min(deg(v), deg(w))	1
insertVertex(o)	1	1	n^2
insertEdge(v, w, o)	1	1	1
removeVertex(v)	m	deg(v)	n^2
removeEdge(e)	1	1	1

Remember: $m = n(n-1)/2$ in worst case

```

MAX-SUBARRAY-SUM(Array A)
    sum = sum' = A[0]
    for i = 1 to A.length - 1
        sum' = max(A[i], sum' + A[i])
        sum = max(sum, sum')
    return sum

```

```

LONGEST-COMMON-SUBSEQUENCE(String A, String B) // O(nm)
    return LCS(A.length-1, B.length-1)

LCS(Integer i, Integer j)
    if i = -1 OR j = -1 then return 0
    if mem[i][j] then return mem[i][j]
    if a[i] == b[j] return mem[i][j] = 1 + LCS(i-1, j-1)
    else return mem[i][j] = max(LCS(i, j-1), LCS(i-1, j))

```

```

LONGEST-COMMON-SUBSEQUENCE(String A, String B)
    N = A.length, M = B.length;
    Array dp[A.length][B.length]
    for i = 0 to N
        for j = 0 to M
            if i = 0 OR j = 0 then dp[i][j] = 0;
            else if A[i-1] = B[j-1] then dp[i][j] = 1 + dp[i-1][j-1]
            else dp[i][j] = max(dp[i][j-1], dp[i-1][j])
    return dp[n][m]

```

```
// EZ, Check Slides.  
- BST-INSERT  
- BST-SEARCH  
- BST-REMOVE  
- BST-PREORDER  
- BST-INORDER  
- BST-POSTORDER  
- BST-MAX  
- BST-MIN
```

```
BST-SUCCESSOR(Node x)  
    if x.right ≠ NIL  
        return BST-MIN(x.right)  
    y = x.p  
    while y ≠ NIL AND x == y.right  
        x = y  
        y = y.p  
    return y
```

```
BST-PREDECESSOR(Node x)  
    if x.left ≠ NIL  
        return BST-MAX(x.left)  
    y = x.p  
    while y ≠ NIL AND x == y.left  
        x = y  
        y = y.p  
    return y;
```

```
BUBBLE-SORT(Array A) // Stable, In-Place
  for i = 1 to A.length-1
    for j = A.length downto i+1
      if A[j] < A[j-1] then
        exchange A[j] with A[j-1]
```

```
SELECTION-SORT(Array A) // Not Stable, In-Place
  for i = 1 to A.length-1
    m = i
    for j = i to n
      if A[j] < A[m] then m = j
    exchange A[i] with A[m]
```

```
INSERTION-SORT(Array A) // Stable, In-Place
  for j = 2 to A.length
    k = A[j]
    i = j - 1
    while i > 0 and A[i] > key
      A[i + 1] = A[i]
      i = i - 1
    A[i+1] = k
```

```
MERGE-SORT(List A, Int l, Int r) // Not stable, Out-Of-Place
  if l = r then
    return A[l]
  m = floor((l+r)/2)
  return MERGE(MERGE-SORT(A, l, m), MERGE-SORT(A, m+1, r))

MERGE(List A, List B)
  List C =  $\emptyset$ 
  While A  $\neq \emptyset$  AND B  $\neq \emptyset$ 
    if a.getFirst() < b.getFirst()
      C.Append(A.getFirst())
      A.removeFirst()
    else
      C.Append(B.getFirst())
      B.removeFirst()

  if a =  $\emptyset$  then C.append(B)
  else if b =  $\emptyset$  then C.append(A)
  return C
```

```
RADIX-SORT(Array A, Integer d) // Non-Comparison, Stable, In-place
  for i = 1 to d
    StableSortOnDigit(A, i)
```

```
BUCKET-SORT(Array A) // Out-of-place, stability depends
  n = A.length
  Array_of_Lists B[n] =  $\emptyset$ 
  for i = 1 to n
    insert A[i] into list B[floor(n*A[i])]
  for i = 0 to n - 1
    INSERTION-SORT(B[i])
  concatenate all lists B[0]..B[n-1]
```

```

QUICK-SORT(Array A, Int l, Int r) // Not stable, can be in-place
    if l < r
        m = PARTITION(A, l, r)
        QUICK-SORT(A, l, m-1)
        QUICK-SORT(A, m+1, r)

PARTITION(Array A, Integer l, Integer r)
    x = A[r]
    i = l-1
    for j = l to r-1
        if A[j] <= x
            i = i+1
            exchange A[i] with A[j]
    exchange A[i+1] with A[r]
    return i+1

```

```

HEAP-SORT(Array A) // Not Stable, in-Place, A is 1-indexed
    // n is a pointer to the last element in heap
    int n = A.length - 1

    // build a max heap from A
    for i = n downto 0 do SWIM(i)

    // Repeatedly extract maximum from heap
    while n > 0
        exchange A[0] with A[n]
        n = n - 1
        SINK(0)

SWIM(Integer k) // Swims A[k] up to its correct position
    if k = 0 then exit
    if x[k] > x[k/2] then exchange x[k] with x[k/2]
    SWIM(k/2)

SINK(Integer k) // Sinks A[k] down to its correct position
    if 2k = n AND A[k] < A[2k] then
        exchange A[k] with A[2k]
    else if 2k < n then
        g = Index of max(A[2k], A[2k+1])
        exchange A[k] with A[g]
        SINK(g)

```

```

COUNTING-SORT(Array A) // Non-Comparison, Stable, Out-of-Place
    Array B[A.length] = ∅
    Array C[A.max_element()+1]
    for i = 0 to A.length
        C[A[i]] = C[A[i]] + 1
    for i = 1 to A.max_element()
        C[i] = C[i] + C[i-1]
    for i = 0 to n-1
        B[C[A[i]]-1] = A[i]
        C[A[i]] = C[A[i]] - 1
    return B

```

```

BFS(Graph G, vertex s) // O(|V| + |E|)
    foreach vertex u ∈ G.V
        u.distance = ∞
        u.color = WHITE

    s.color = GRAY
    d.distance = 0
    Queue q = ∅
    q.enqueue(s)
    while q ≠ ∅
        u = q.dequeue()
        foreach vertex v ∈ G.Adj[u]
            if v.color = WHITE
                v.color = GRAY
                v.distance = u.distance + 1
                v.parent = s
                q.enqueue(v)
        u.color = BLACK

```

```

DFS(Graph G) // O(|V| + |E|)
    foreach vertex u ∈ G.V
        u.color = WHITE
    foreach vertex u ∈ G.V
        if u.color == WHITE
            DFS-VISIT(G, u)

DFS-VISIT(vertex s)
    s.color = GRAY
    foreach vertex u ∈ G.Adj[s]
        if u.color = WHITE
            u.parent = s
            DFS-VISIT(u)
    s.color = BLACK

```

```

TOPOLOGICAL-SORT(Graph G) // O(|V| + |E|)
    Stack S = ∅
    List order = ∅
    Foreach vertex v ∈ G.V
        if visited[v] = false
            DFS-VISIT(v)
    While S ≠ ∅
        order.add(S.pop())

    return order

DFS-VISIT(vertex s, Stack S)
    visited[s] = true
    foreach vertex u ∈ G.Adj[s]
        if visited[u] = false
            DFS-VISIT(u)
    S.push(s)

```

```

PRIM-MST(Graph G, vertex s) //  $O(E \log(V))$ 
    List<Edge> MST =  $\emptyset$ 
    MIN-PQ<Edge> Q =  $\emptyset$ 
    foreach vertex v  $\in$  G.Adj[s]
        Q.push(Edge(s, v))

    while Q  $\neq \emptyset$ 
        Edge e = Q.EXTRACT-MIN()
        if e not in mst
            MST.add(e)
            vertex u = e.to()
            foreach vertex v  $\in$  G.Adj[u]
                if v  $\notin$  MST
                    Q.push(Edge(u, v))

    return MST

```

```

KRUSKAL-MST(Graph G) //  $O(E \log(E))$  or  $O(E \log(V))$ 
    List<Edge> MST =  $\emptyset$ 
    List E = SORT(G.E) // Sort all edges by weight in ascending order
    foreach edge in E
        if e doesn't create a cycle
            MST.add(e)

    return MST

```

```

SINGLE-SOURCE-DIJKSTRA(Graph G, vertex s) //  $O(|E| \log(V))$ 
    foreach vertex u  $\in$  G.V
        u.distance =  $\infty$ 
        u.marked = false
    s.distance = 0
    s.marked = true

    MIN-PQ<Edge> Q =  $\emptyset$ 
    foreach vertex v  $\in$  G.Adj[s]
        v.distance = weight(s, v)
        Q.push(Edge(s, v))

    while Q  $\neq \emptyset$ 
        vertex u = Q.EXTRACT-MIN().to
        u.marked = true
        foreach vertex v  $\in$  G.Adj[u]
            v.distance = min(v.distance, u.distance + weight(u, v))
            if v.marked = false
                Q.push(Edge(u, v))

```

```

FORD-FULKERSON(Network N) //  $O(E \cdot F^*)$ , Check MaxFlow.docx

    Begin with flow = 0.

    While there is an augmenting path in the residual network
        Augment the flow through that path.

```