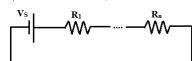
Formula	Description	Notes/Diagram		
$I = \frac{Q}{h}$	I: Electric current Intensity (Ampere A)	Electric charge can be calculated by this		
$I = \frac{1}{t}$	Q: electric charge (Coulomb C)	formula $Q = Ne$, where:		
	t: time (sec)	N: number of carriers		
		e: electron charge = $1.602 * 10^{-19} C$		
R = 0	R: resistance (Ohm Ω)	Electrical conductivity (σ or G) is the inverse of		
$R = \rho \frac{v}{A}$	ρ: resistivity (property of the element)	electrical resistivity.		
	l: length of conductor (e. g. wire) (m)	a = c = 1		
	A: cross sectional area (m^2)	$\sigma = G = \frac{1}{\rho}$ You will use this A LOT :)		
Ohm's law	V: electric potential difference (voltage)	You will use this A LOT :)		
V = IR	between two points (Volt V)			
	I: intensity of current between them.			
	R: resistance between them.			
$V^2 = V^2 dW$	Electric power: the rate at which the	Power is measured in (Watt W)		
$P = IV = I^2 R = \frac{V^2}{R} = \frac{dW}{dt}$	electrical energy is transferred.	Transmitted power over wires $P_T = IV$		
	Energy can then be calculated by	Lost power due to transformation $P_L = I^2 R =$		
$W = \int_{t_0}^{t_1} P(t) dt$	integrating power.	$\frac{V^2}{R}$		
$R_{EQ} = \sum_{k=1}^{n} R_k \tag{1}$	Equivalent resistance for n resistors	R		
$R_{EO} = \sum R_k \tag{1}$	connected in series (1) or in parallel (2).	2		
$R_{EQ} = \sum_{k=1}^{R_E} R_k \tag{1}$	connected in series (1) or in parallel (2).	3		
κ=1		\(\frac{1}{4}\)		
$\left(\frac{n}{n}\right)^{-1}$		(-) < <		
$R_{EQ} = \left(\sum_{k=1}^{n} \frac{1}{R_k}\right)^{-1} \tag{2}$		1 1 1 2		
$\left(\sum_{k=1}^{LQ} R_k\right)$				
n n	Kirchhoff's Current Law (KCL): sum of the	Loop (3): closed path in a circuit		
$\sum_{k=1}^{N} I_k = 0, \sum_{k=1}^{N} V_k = 0$	currents at a node must equal zero.	mesh (2): a loop that has no other loops inside		
k=1 $k=1$	Kirchhoff's Voltage Law (KVL): sum of	it.		
	voltages around a loop must equal zero.	Branch (4): contains an electrical component		
	(going from – to +, we add voltage)	(connects two nodes)		
	(going from + to –, we subtract voltage)	node (3): connecting two or more branches		
		(#wire segments used)		

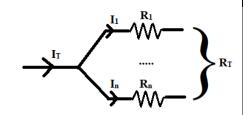
Voltage divider rule: The voltage is divided between resistors connected in series in direct proportion to their resistance (voltage is the same across parallel resistors.)

$$V_X = \frac{R_X}{\Sigma R_i} V_S$$



Current divider rule

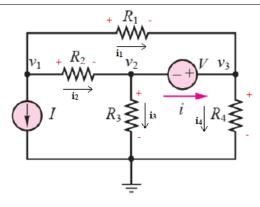
$$I_X = \frac{R_T}{R_X} I_T$$



Node Voltage Method (NVM) (useful for calculating node potential, calculating voltage across circuit elements)

- 1. Choose a reference ground node (has zero potential).
- 2. Identify currents and their direction (from + to across resistors, from to + across voltage sources).
- 3. Write Ohm's law for each pair of nodes $i_{a o b} = rac{v_a v_b}{R_{ab}}$
- For a voltage source going from node a to b, equation will become $v_{source} = v_b - v_a$
- 5. Write KCL equations at each node, then solve the system.

$$\begin{cases} i_1 = \frac{v_1 - v_3}{R_1}, i_2 = \frac{v_1 - v_2}{R_2}, i_3 = \frac{v_2}{R_3}, i_4 = \frac{v_3}{R_4} \\ V = v_3 - v_2 \\ -I - i_1 - i_2 = 0, i_2 - i - i_3 = 0, i_1 + i - i_4 = 0 \end{cases}$$
 Principle of superposition (useful for analyzing circuits with multiple sources)



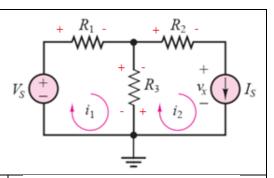
Branch voltage/current is the sum of N voltages/currents, each of which can be computed by setting all but one source to zero and solving the circuit containing that single source.

Removing (setting sources to zero): voltage sources are shortened, current sources are opened.

Mesh Current Method (MCM) (useful for finding current through an element)

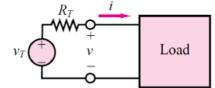
- 1. Identify currents at each mesh, assume clockwise direction.
- 2. Write KVL for each mesh, **mesh current goes from + to –** (we change signs assumptions when considering each mesh).
- 3. Current sources provides the current mesh (trivial).
- 4. Solve the system.

$$\begin{cases} V_S - i_1 R_1 - (i_1 - i_2) R_3 = 0 \\ -v_x - (i_2 - i_1) R_3 - i_2 R_2 = 0 \\ i_2 = I_S \end{cases}$$



Thevenin equilvalent circuit:

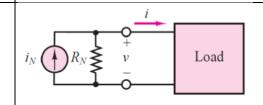
- Works for networks with voltage/current sources and linear resistors
- Assume the load is in between nodes a and b
- Remove (open) the load \rightarrow ($V_{ab} = V_T$)
- Remove (short/open) all sources $\rightarrow R_{a \rightarrow b} = R_T$



Norton equilvalent circuit:

- Works for networks with voltage/current sources and linear resistors
- Assume the load is in between nodes a and b
- Remove (short) the load \Longrightarrow ($I_{ab} = I_N$)
- Remove the source $\Rightarrow R_{a \rightarrow b} = R_N$

Source transform: $V_T = R_T i_N$, $R_T = R_N$



AC sine wave

$$x(t) = A * \cos(\omega t + \phi)$$

In phasor notation

$$x(t) = A \angle \phi$$

A: amplitude (peak value)

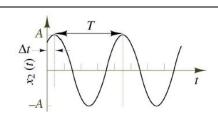
T: period (s) (after which it repeats)

 $\omega = 2\pi f$: radial frequency (rad/s)

f = 1/T: natural frequency (Hz)

 $\phi = \frac{2\pi\Delta t}{T}$: phase shift b/w v and i (rad). $\phi = 90^\circ$ in inductor, $\phi = -90^\circ$ in

 $\phi = 90^{\circ}$ in ir capacitor



$$Q = CV$$

 $Q = It = Ne, e = 1.602 * 10^{-19} C$

C in the second of the secon

- Equivalent capacitance:
- For **parallel** capacitors = ΣC_i
- For **series** capacitors = $1/\Sigma C_i^{-1}$

 \overline{Q} : amount of electric charge stored in the capacitor (columb)

V: voltage b/w capacitor plates

C: capacitance (Farad = coulomb/volt), constant for a certain capacitor.

- Capacitor stores electric charge between its two plates separated by dielectric.
- The higher the capacitance, the more charge the capacitor can store at the same voltage.
- These formulas work only for constant values of Q, V, I

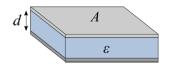
$$C = \varepsilon \frac{A}{d}$$
In vaccum (free space)

(----,

$$\varepsilon_0\approx 8.854\times 10^{-12}\,F.\,m^{-1}$$

A: common area b/w capacitor plates (m^2)

d: distance b/w the plates (m) ε (permittivity): a measure of the electric polarizability of a dielectric.



$$Q(t) = C * V(t)$$

$$I(t) = \frac{dQ}{dt} = C * \frac{dV(t)}{dt}$$

$$P(t) = I(t)V(t) = \frac{dW(t)}{dt}$$

$$W(t) = \int_{-\infty}^{t} I(t')V(t')dt'$$

$$= \frac{C}{2}V^{2}(t) + W(0)$$

- Capacitor equations for varying voltage (and current) in AC circuits.
- Capacitor in DC circuits = Open circuit
- Capacitor in AC circuits = Short circuit
- Capacitor can consume power (P < 0) or deliver power (P > 0) that is the rate of energy consumption per unit time.
- Energy (electric charge) stored in the capacitor is the integral of power
- W(0) and V(0) account for initial values and can be also called $W(-\infty), V(-\infty)$

Given $i_c(t)$ we can get an expression for voltage on the capacitor

$$V_c(t) = \frac{1}{C} \int_{-\infty}^{t} I_c(t') dt'$$

If $I_c(t)$ is a piece-wise defined function, $V_c(t)$ will also be so. **Example**

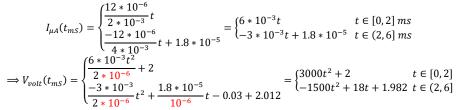
$$I(t) = \begin{cases} 1 & t \in (-\infty, 5] \\ t & t \in (5, +\infty) \end{cases} \Rightarrow$$

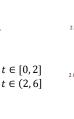
$$v(t) = \begin{cases} \frac{t}{C} + v(0) & t \in (-\infty, 5] \\ \frac{t^2}{C} - \frac{5^2}{2} + v(5) & t \in (5, +\infty) \end{cases}$$

Graphing integration/differentiation relations.

- 1. Extract the defintion for the piecewise function from the given graph.
 - a. Convert the graph and everything to standard units.
- 2. Integrate/differentiate functions for each case separately

Example integration problem: graph $V_c(t)=rac{1}{C}\int_0^t I_c(t')dt'+V_0$ given the graph of I(t) and $V_0 = 2 V$, $C = 1 \mu F = 10^{-6} F$





More curves for Power (P(t)=I(t)V(t)) or Energy $W_L(t)=L\int_{-\infty}^t IdI$ or $W_C=$ $C\int_{-\infty}^{t}VdV$ can be constructed and drawn in the same manner.

Note: integration graph for $0 \Rightarrow$ constant, constant \Rightarrow line, line \Rightarrow parabola, etc.

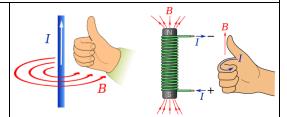
Faraday law

$$\varepsilon = -\frac{d\Phi}{dt}$$

- Magnetic flux is measured in weber (Wb = V.S)

Induced EMF is given by the rate of change of the magnetic flux.

Inductance (Henry = V/A): a propery of incuctor, measures the amount of induced EMF per unit change in currect



$$V_L(t) = L \frac{dI}{dt}$$

$$I_L = \frac{1}{L} \int_{-t}^{t} V_L(t') dt'$$

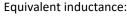
$$P(t) = I(t)V(t) = \frac{dW(t)}{dt}$$

Voltage on an inductor

Current flowing through inductor

- Inductor in DC circuits = Short circuit
- Inductor in AC circuits = Open circuit

Same equations for power and energy apply



i(t) (μA)

V(volt)

Integrate

Differentiate

- For **series** inductors = ΣL_i
- For **parallel** inductors = $1/\Sigma L_i^{-1}$

$$W(t) = \int_{-\infty}^{t} I(t')V(t')dt'$$
$$= L \int_{-\infty}^{t} IdI = \frac{L}{2}I^{2}(t) + W(0)$$

 $A + Bj = L(\cos(\theta) + j\sin(\theta)) = Le^{j\theta}$ $= L \angle \theta$

$$= L \angle \theta$$

$$L = \sqrt{a^2 + b^2}, \theta = \arctan\left(\frac{b}{a}\right)$$

$$A = L * \cos(\theta), B = L * \sin(\theta)$$

$$\frac{L_1 \angle \theta_1}{L_2 \angle \theta_2} = \frac{L_1}{L_2} \angle (\theta_1 - \theta_2)$$

$$L_1 \angle \theta_1 * L_2 \angle \theta_2 = L_1 L_2 \angle (\theta_1 + \theta_2)$$

Complex number formulas for phasor calculations.

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

Recall

 $sin(A \pm B) = sin A cos B \pm cos A sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ $\tan A \pm B = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ $\sin A \cos B = 0.5(\sin(A+B) + \sin(A-B))$ $\cos A \cos B = 0.5(\cos(A+B) + \cos(A-B))$ $\sin A \sin B = 0.5(\cos(A - B) - \cos(A + B))$

$$Z_R = R$$

$$Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

$$Z_L = j\omega L$$

$$Z = R + jX$$

- Re(Z) = Resistance
- Im(Z) = Reactance

- Impedance of Resistor, Capacitor, and Inductor.

- Equivalent impenance can be found in the same way as equivalent resistance.

 $X > 0 \Rightarrow$ Inductive impedance $X < 0 \Rightarrow$ Capacitive Impedance

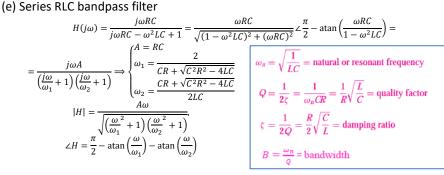
Recall

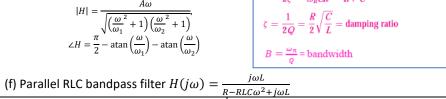
Angular frequency (rad/s) = $2\pi * natural$ frequency (Hz)

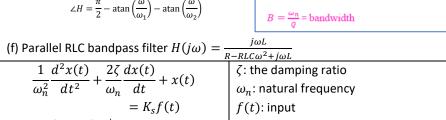
$$\omega = 2\pi f = \frac{2\pi}{T}$$

$\tau \frac{dx}{dt} + x = K_s f(t)$ General form of a differential equation for a 1 st order circuit (with 1 capacitor or 1 inductor)	x : some variable, current/voltage through/on the capacitor/inductor τ : transient time (growth/decay ur steady state) $f(t)$: input to the system (from current/volatage source) K_s : DC gain, final output at the steat state (when $t \to \infty$ and the input became constant)
Solution: $x(t) = x(\infty) + (x(0^{+}) - x(\infty))e^{-t/\tau}$ $x(t) = x_F + x_N$ Transient response = Forced (steady state) response + Natural response	Recall: capacitor discharging acts a voltage source with voltage $V=V_C(0)$, inductor discharging acts as current source with $I_S=I_L(0)$ $x_F=$ particular solution to the ODI $x_N=$ homogeneous solution
$H(j\omega) = \frac{V_{out}}{V_{in}} = H \angle - \operatorname{atan}\left(\frac{\omega}{\omega_0}\right)$ Frequency response: how the output changes in response to a sinusoidal input. Recall: $ H = \sqrt{\left(Re(H)\right)^2 + \left(Im(H)\right)^2}, \angle H = \operatorname{atan}\left(\frac{Im(H)}{Re(H)}\right)$	Cutoff angular frequency of the filt ω_0 : the value after which the filter effect begins to be visible. RC filters $\omega_0=1/RC$ RL filters $\omega_0=R/L$
Frequency response for filters (a) Low-pass RC filter $H(j\omega) = \frac{1}{1+j\omega RC} = \frac{1}{1+j\omega RC}$ (b) High-pass RL filter $H(j\omega) = \frac{j\omega L}{R+j\omega L} = \frac{1}{1+j\omega RC}$	$\frac{1}{\sqrt{1 + (\omega RC)^2}} \angle \operatorname{atan}(-\omega RC)$ $\frac{1}{\sqrt{1 + \left(\frac{R}{ct}\right)^2}} \angle \operatorname{atan}\left(\frac{R}{\omega L}\right)$

e capacitor/inductor. inductor cannot change immediately because me (growth/decay until this will require infinite power. $V_{C|L}(0^{-}) = V_{C|L}(0^{+}) = V_{C|L}(0)$ $I_{C|L}(0^-) = I_{C|L}(0^+) = I_{C|L}(0)$ the system (from age source) nal output at the steady Recall: in DC circuits (in steady state, circuit $\rightarrow \infty$ and the input has been closed for a long time), capacitor acts ant) as an open circuit, inductor - closed. $\tau = R_T C$ for single capacitor circuit tor discharging acts as a $\tau = \frac{L}{R_T}$ for single inductor circuit e with voltage V =or discharging acts as a - R_T is the thevenin equivalent resistance e with $I_S = I_L(0)$ looking from capacitor(inductor) terminals. Thevenin ODE: $V_T = CR_T \frac{dV_c}{dt} + V_c$ ar solution to the ODE Norton ODE: $i_N = \frac{L}{R_N} \frac{di_L}{dt} + i_L$ neous solution r frequency of the filter Low(high)-pass filter: passes low(high)after which the filter frequency singals only and block other freqs o be visible. (low=below cutoff, high=above cutoff) = 1/RCBandpass filter: passes signals with = R/Lfrequencies only in some range $[\omega_1, \omega_2]$ **Bode-plot for filters:** two plots, first $(\omega, |H|)$, second $(\omega, \angle H)$, usually logarithmic scaled. $n(-\omega RC)$ (c) High-pass RC filter $H(j\omega) = \frac{j\omega RC}{j\omega RC + 1} = \frac{1}{\sqrt{1 + (\frac{1}{\omega})^2}} \angle \operatorname{atan}\left(\frac{1}{\omega RC}\right)$ (d) Low-pass RL filter $H(j\omega) = \frac{R}{R+j\omega L} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \angle \operatorname{atan}\left(-\frac{\omega L}{R}\right)$





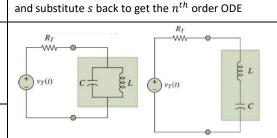


General form of 2nd order circuit ODE

 K_s : DC gain

$$\begin{aligned} & Solution & x(t) = x_F + x_N = K_s f + (\alpha_1 e^{s_1 t} + \alpha_2 e^{s_2 t}) \\ & s_{1,2} = \begin{cases} -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} & \zeta > 1 & \text{Overdamped response} \\ -\omega_n & \zeta = 1 & \text{Critically damped response} \\ -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2} & 0 < \zeta < 1 & \text{Underdamped response} \end{cases}$$

Analogies b/w Series RLC circuit and Mass-Spring-Damper mechanical system $Lv_c^{\prime\prime} + Rv_c^{\prime} + 1/C v_c = V_T$ mv'' + bv' + kv = u



 $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases} \Rightarrow \begin{cases} x = \frac{c_1b_2 - b_1c_2}{a_1b_2 - b_1a_2} \\ y = \frac{a_1c_2 - c_1a_2}{a_1b_2 - b_1a_2} \end{cases}$

substitute $s = \frac{d}{dt}$ then solve for any of the variables

Operator method: write n 1st order ODEs

Voltage and current on/through capacitor or

Parallel RLC circuit: $LCi''_L + \frac{L}{R}i'_L + i_L = \frac{1}{R}V_T$ Series RLC circuit: $LCv_c^{\prime\prime} + RCv_c^{\prime} + v_c = V_T$

Electrical System	Mechanical System		
Inductance L	Mass m		
Resistance R	Damping coeff. b		
Capacitance c	Compliance 1/k		

$n = p = n_i$	Intrinsic concentration of electrons holes for some pure material are eq to the intrinsic carrier concentration	ual meters) for silic	$^{5}m^{-3}$ (electrons per cubic con at room temperature.		
n n -n n -n ²			ns con * holos con romains		
$n_{ exttt{p_side}}p_{ exttt{p_side}}=n_{ exttt{n_side}}p_{ exttt{n_side}}=n_i^2$	Mass action law: for any (doped) semiconductor: electrons con. * holes con. remains constant and is equal to the carrier con. squared, this can be used with p-side, or n-side.				
n -type: $n > n_i$, $p < p_i$	The number of electrons for an n-ty		with an element of group 15		
$\text{p-type: } n < n_i, p > p_i$	semiconductor is greater than the intrinsic concentration for pure material.	(e.g., Arsenic) p-type: doped (e.g., Indium)	with an element of group 13		
$V_0 = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$	N_A = acceptor concentration.		W ₀ ++		
	N_D = donor concentration V_0 = Built-in potential for the semi-cond	uctor	<i>p</i> -type + <i>n</i> -type		
$x_n = \sqrt{\frac{2\epsilon_s}{q} \frac{N_a}{N_d} \frac{1}{N_a + N_d} V_0}$	x_n = n-side width of depletion region.	uctor.	+ +		
` <u> </u>	x_p = p-side width of depletion region.		$-x_{p0}$ 0 x_{n0} x $Q(x)$		
$x_p = \sqrt{\frac{2\epsilon_s}{q} \frac{N_d}{N_a} \frac{1}{N_a + N_d} V_0}$	W_0 = total width of depletion region. $Q(x)$ = electric charge		+aN-		
	E(x) = strength of electric field		++ +		
$W_0 = x_n + x_p, Q(x) = -qN_A$ $dV(x) dF(x) Q(x) = 0$	ε = permittivity of substance	_	-x _{p0} ++		
$E(x) = -\frac{dV(x)}{dx}, \frac{dE(x)}{dx} = \frac{Q(x)}{\varepsilon}, \kappa = \frac{\varepsilon}{\varepsilon_0}$	ε_0 = permittivity of vaccum		x _{n0} x		
	κ = dielectric constant.		$-qN_A$		
$i_D = -I_0 = I_S$	$q \approx 1.602 * 10^{-19} C$ electron charg		0 9 9		
$i_D = I_0 e^{\frac{qv_D}{kT}} - I_0 \cong I_0 e^{38.67v_D}$	$k \cong 1.381 * 10^{-23} \frac{J}{K}$ boltzmann con	st. + I	\overline{v}_{α}		
Diffusion current for reverse-biased and	T: material temprature in kelvins	<i>v</i> _D	+ 1 9		
forward-biased diode.	(room temp \cong 300 K) I_s reverse saturation current:	6	8 0 0		
Diffusion current i_D : holes moving from p to n (conventional direction)	very small current (10^{-12} to 10^{-15} A) Circuit mode $v_D \ge 0$ (short				
Drift current: electrons flowing from n to p.	resulting from minority charge carri		<u> </u>		
	flowing through the depletion regio	n.	1		
Two methods to determine whether th	_	al a manufactura	Diode off = open circuit		
Assume it's not conducting, replace terminals using NVM (V V) if	Diode on = short circuit VD1 O O				
terminals using NVM (V_{D1} , V_{D2}). If $V_{D1} < V_{D2} \Rightarrow V_D < 0$, then our assumption was correct. • Assume it's conducting, replace it with short circuit, calculate current through the short					
circuit using MCM (i_d). If $i_d > 0$ th	i _d 文 = , ⊥				
* Non-ideal diode is equivalent to an id	,	υ ,	↓		
diodes, $v_{\gamma}=0.6V$, germanium $v_{\gamma}=0.6V$			V _{D2} 6		
	cent/operating/Q) point represents the		R_T		
	le diode and can be calculated by solvir multaneously for $v_{\scriptscriptstyle D}$, $i_{\scriptscriptstyle D}$ (approx. graphi				
The first equ	ation comes from the Thevenin equiva		$V_T = V_D$		
	ed with the diode treated as the load.				
Half-wave rectifier (using diode and	resistor): ∪ _↑	U _A	Adding a capacitor in		
$v_{peak} = 2v_{rms}^{rectifier} = \sqrt{2} v_{rms}^{source},$	$v_{ij}^{avg} \equiv \frac{v_{peak}}{\sqrt{1 - v_{peak}}}$		parallel with R_{Load} results		
- peak - rms - V 2 rms ,	π	t	in a peak-rectified voltage.		
Full-wave rectifier (using bridge rec	tifier):	[∪] ↑	$v_{rms} = \sqrt{\frac{1}{T}} \int_0^T v_s^2(t) dt$		
$v_{peak} = \sqrt{2} v_{rms}, \qquad v_{dc}^{avg}$	$=\frac{2v_{peak}}{\pi}$	t	$1 Hz = 2\pi rad/sec$		
	+5 V ₀		Logic gates using diadas		
+5 V (+ °		Logic gates using diodes (from left to right):		
	$\stackrel{Vout}{\longleftarrow}$ $\stackrel{R}{\geqslant}$ $\stackrel{R}{\geqslant}$ $\stackrel{Vout}{\triangleright}$	• v _{out}	AND, NOR, NAND, OR		
	+5 V	, \{	R C		
	, , , , , , , , , , , , , , , , , , ,	>	- W		
v_A + v_{out} -	¹⁸ 2K ₹]			
<i>v_B</i> =	<u>_</u>		$v_{S}(t)$		
Diode clamp with forward diode (in picture, -) or reverse diode (+) shifted by v_{DC}					
$v_{out} = v_S \pm V_{peak}, \qquad v_{out} = v_S \pm V_{peak} - v_{DC}$					