Quadratic Surfaces:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Hx + Iy + Jz + K = 0,$$

How to determine type of the surface:

1- **If it's rotated(D!=0 or E!=0 or F!=0), unRotate to determine type, then rotate back if needed**
2- **if it's possible to make it in the form a(x-x0)^2 + b(y-y0)^2 + c(z-z0)^2 = d with a, b, c != 0**

1. a, b, c -> All have the same sign

- if it's not the same sign as d -> NO SOLUTION
- if (d=0) -> a dot (0,0,0)
- else
  - a=b=c -> **Sphere**
  - !(a=b=c) -> **Ellipsoid**

2. a, b, c -> One is negative, the other two are positive

- if d>0 -> **Hyperboloid of one sheet**
- if d<0 -> **Hyperboloid of two sheets**
  - Has no intersection with xy plane, vertices at (x0, y0, z0(+/-)c)
- if d=0 -> **Double cone**
  - Both hyperboloid are asymptotic to it
  - If c=1 -> cones has circle base
  - Otherwise -> cones has ellipse base
  - Double cone traces are lines

3. a, b, c -> one positive, two negative -> multiply all equation by -1 and goto 2.

4. a, b, c -> one is zero, the other two have the same sign as d -> Elliptic cylinder

- if they have the same sign but not the same as d -> NO SOULTION
- if the equation has the all-zeros solution (d=0), it's an axis which is not present in the equation

5. a, b, c -> one is zero, the other two have different signs -> Hyperbolic cylinder

- if the equation has the all-zeros solution(d=0), it's two planes intersecting in a line parallel to the axis which is not present in the equation

6. a, b, c -> two are zeros -> two parallel planes (parallel to the yz, xz, xy plane depending on which terms are not present)

3- **if it's not possible (some terms are present but not squared, others are squared) -> it's smth related to parabolas**

1. x, y, z -> one of them is not squared, the other two are squared -> **Paraboloid**
   - if the two squared terms have the same sign -> **Elliptic paraboloid (cup)**
   - if they have different signs -> **Hyperbolic paraboloid (Pringle)**
2. x, y, z -> two of them are not squared, the other one is squared -> **Parabolic cylinder**
3. x, y, z -> one is missing, the other two -> one is squared, one is not -> **Parabolic cylinder**

4. x, y, z -> two are missing, the other one is not squared -> **A plane parallel to (xy, yz, xz planes)**
5. x, y, z -> all not squared -> **A plane.**

General forms:

Sphere $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2.$

Ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$

Hyperboloid of one sheet $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1.$

Hyperboloid of two sheets $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1.$

Double Cone $\frac{x^2}{a^2}+\frac{y^2}{b^2}-z^2=0.$

Elliptic Paraboloids $\frac{x^2}{a^2}+\frac{y^2}{b^2}=z.$

Hyperbolic Paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=z,$

Parabolic Cylinder $x^2=4cy.$

Elliptic Cylinder $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

Hyperbolic Cylinder $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$

Properties of quadratic surfaces:

- **Intercepts:** intersections with x, y, z axis (Solutions when (y=0,z=0), (x=0,z=0), (x=0,y=0) respectively)
- **Traces:** intersections with planes xy, yz, xz
- **Sections**: intersection with any plane parallel to planes xy, yz, xz
- **Centre:** Some Quadratic surfaces doesn't have it
- **Bound:** Some quadratic curves are bounded, others are not
- **Symmetricity:** Some quadratic curves are symmetric about axes or planes

Saddle Point: point where all slopes(derivatives) are zero , but it's NOT a local min or max

ex. Hyperbolic paraboloid center.