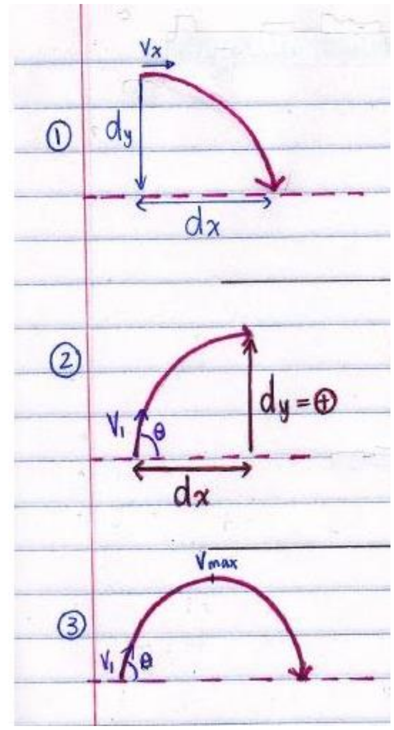
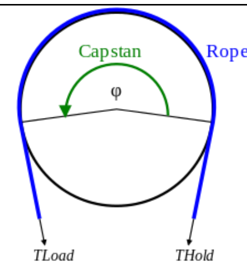
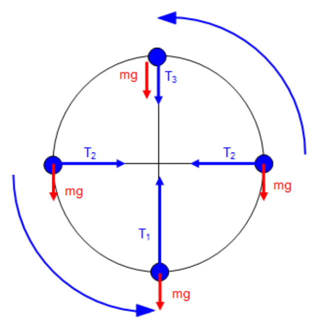
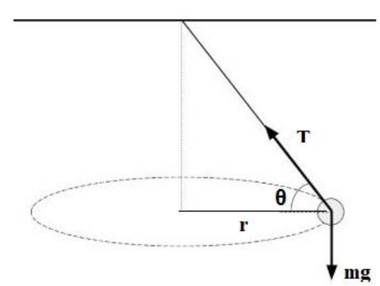
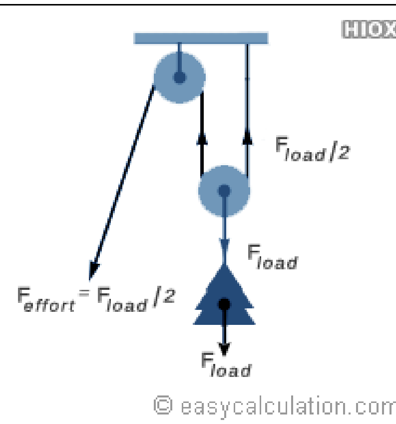
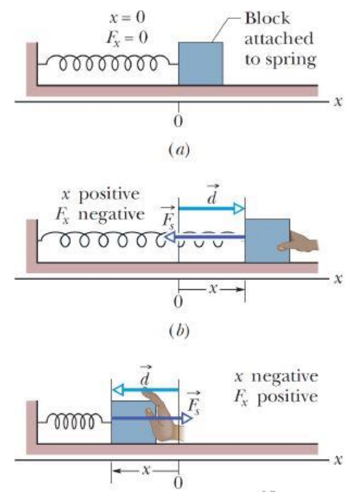
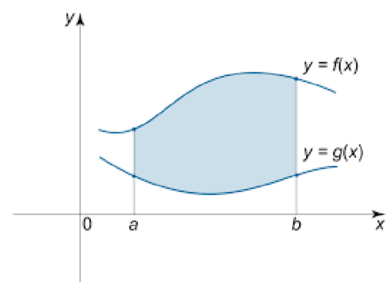
Formula(s)	Description	Notes/diagram
$v_f = v_i + at$ $d = x - x_0 = v_i t + 0.5at^2$ $v_f^2 - v_i^2 = 2ad$ $d = 0.5t(v_i + v_f)$	Translation motion equations ($d = \Delta x$)	Used only with constant acceleration (force), otherwise, we use the equations: $\frac{dx}{dt} = v, \frac{dv}{dt} = a$ $x(t) = \int v(t)dt, v(t) = \int a(t)dt$ *In free-fall: $a = g$
Case 1 $d_x = v_x t,\ d_y = 0.5gt^2$ $v_{f,x} = v_x, v_{f,y} = gt$ $v_f = \sqrt{v_x^2 + g^2t^2}$ **Case 2, 3** $d_x = v_{0,x}t$ $d_y = v_0 t + 0.5gt^2$ $t_{base \to top} = \frac{v_0 \sin\theta}{g}$ $d_y = \frac{v_0^2 \sin^2\theta}{2g}$ $d_x = \frac{v_0^2 \sin 2\theta}{2g}$ **Case 3**: $v_f = v_0$	**Projectile motion equations** In all cases: $a_x = 0, a_y = -g$ $v_{0,x} = v_{f,x} = \text{const}$ $v_y > 0$ when ascending $v_y < 0$ when descending $v_y = 0$ at max-height In cases 2, 3: $v_{0,x} = v_0 \cos\theta$ $v_{0,y} = v_0 \sin\theta$ $v_{top} = v_{0,x}$ $v_0 = \sqrt{v_{0,x}^2 + v_{0,y}^2}$ $\theta = atg\left(\frac{v_{0,y}}{v_{0,x}}\right)$	
$\boldsymbol{F_{net}} = m\frac{dv}{dt} = m\boldsymbol{a}$	F_{net}: summation of all forces acting on a body m: the mass of the body a: acceleration.	When F_{net} depends on velocity, we have a differential equation in v that we can solve to find $v(t)$, then we can find $x(t)$ or $a(t)$ by integration and differentiation
$D = \frac{1}{2}C_d\rho A v^2$	D: Drag force acting on an object C_d: Drag coefficient ρ: Density of fluid causing drag A: Cross-sectional area. v: Velocity of the object moving against the drag.	After a certain amount of time, v reaches v_t: the terminal velocity after which the body moves in a constant max speed, in this case, the force acting on it balances the drag force, and we have $a = 0 \Longrightarrow F = D$ when $F = mg$ we have $v_t = \sqrt{\frac{2mg}{\rho C_d A}}$ During the fall we have $v(t) = \sqrt{\frac{2mg}{\rho C_d A}}\tanh\left(t\sqrt{\frac{g\rho C_d A}{2m}}\right)$
$a_c = \frac{v^2}{R}$ Centripetal acceleration is responsible for keeping the body in a circular motion, it always has a non-zero value if the body is moving, it happens	a_c: Centripetal (radial) acceleration (directed towards the center, perpendicular to velocity) v: velocity of the object moving in a circle (constant, tangent to the trajectory)	This equation is used if the speed v is constant. If it's not, it means there is also linear/tangential acceleration (accelerating the body during its circular motion) $a_t = \frac{dv}{dt}$ and the total acceleration can be calculated as: $a = \sqrt{a_c^2 + a_t^2}$

due to the centripetal force (gravity, tension, friction, …) given by $F_c = ma_c$	R: radius of the circular path.	
$\omega = \frac{d\theta}{dt} = \frac{ds}{rdt} = \frac{v}{r}$ $a_c = \frac{v^2}{r} = \omega v = \omega^2 r$ $a_t = \frac{dv}{dt} = \frac{rd\omega}{dt} = \alpha r$ $a = \sqrt{a_c^2 + a_t^2}$	ω: angular velocity for a body moving with a speed v in a circle of radius r. Centripetal (radial) acceleration Tangential (linear) acceleration Net acceleration	Period (T): the time needed for a full revolution. Frequency (f): revolutions per time unit. $T = \frac{1}{f}$ ω is usually measured in rad/s $\pi \text{ rad} = 180°$
$T_{load} = T_{hold} * e^{\mu\phi}$	Capstan equation. Number of rotations = $\frac{\phi}{360}$ $\phi = \frac{\ln\left(\frac{T_{load}}{T_{hold}}\right)}{\mu}$	
$T_{top} = ma_c - mg$ $T_{middle} = ma_c$ $T_{bottom} = ma_c + mg$	For a body with mass m attached to a rope and moving in a **vertical** centripetal motion, the tension in the rope depends on the position of the body.	 Figure 1
$T = \sqrt{T_x^2 + T_y^2}$ $T_x = T\cos(\theta) = ma_c$ $T_y = T\sin(\theta) = mg$	For a horizontal movement with tension making an angle θ. If the body is fast enough, $\theta \cong 0$ and we have $T = T_x = ma_c$	
$F_{load} = F_{effort}$ for any massless, frictionless pulley, for multiple ropes, the load is distributed evenly.		

$W_{net} = W_c + W_{nc} = \Delta KE$ $W_c = -\Delta P$ $W_{nc} = \Delta ME$	W_{net} net work of all the forces acting in a system W_c work done by conservative forces W_{nc} work done by non-conservative forces	$W_{net} = \int_{x_i}^{x_f} F_{net} ds$ if the force is not constant, $W_{net} = F_{net} d$ otherwise Gravitational PE: $PE = mg(y_f - y_i)$ Elastic PE: $\Delta U = 0.5k(x_f^2 - x_i^2)$ In one dimension: $F_x = -\frac{dU}{dx}$, check
$ME = K + U$	Mechanical energy	In an isolated system, where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy is conserved. $K_1 + U_1 = K_2 + U_2$
$E_{total} = \Delta E_{mec} + \Delta E_{int}$	Conservation of energy	The total energy of an isolated system is conserved, E_{int} is the sum of all other internal (thermal, friction, drag) forces acting in the system.
$F_s = -kd$ k is the spring constant, measures its stiffness.	Hooke's law: the resisting force done by a spring is proportional to its displacement. **IMPORTANT:** when you push/pull a spring by some force F, F does work that gives the spring a (spring Elastic PE) which is defined as $U_s = W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} kx\, dx$ $= 0.5k(x_f^2 - x_i^2)$ Remember that the restoring force $F_s = -F$ (newton 3rd law)	
$P = \frac{dW}{dt}$ $P_{avg} = \frac{W}{t} = Fv$	Power (J/s – Watt)	For a particle moving along a straight line and is acted on by a constant force F directed at some constant angle θ to that line. $P = Fv\cos\theta$
$\vec{r}_{com} = (x_{com}, y_{com}, z_{com})$ $\vec{r}_{com} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{M}$ $\vec{v}_{com} = \frac{\sum_{i=1}^{n} m_i \vec{v}_i}{M}$ $\vec{a}_{com} = \frac{\sum_{i=1}^{n} m_i \vec{a}_i}{M}$ $\vec{F}_{net} = M\vec{a}_{com}$	Center of mass (position, velocity, and acceleration) of a system of **n-particles** with masses $m_1, \ldots, m_n$ (constants) And position vectors $\vec{r_1}, \ldots, \vec{r_n}$ M is the total mass of particles $M = \sum_{i=1}^{n} m_i$	If there are no external forces acting on the system (only forces b/w particles), then: $a_{com} = 0 \Rightarrow v_{com} = const$

<table>
<tr>
<td>$$x_{com} = \frac{1}{M}\int_{x_0}^{x_1} x dm$$ $$= \frac{1}{M}\int_{x_0}^{x_1} x\rho dV = \frac{1}{V}\int_{x_0}^{x_1} x dV$$ $$y_{com} = \frac{1}{V}\int_{y_0}^{y_1} y dV$$ $$z_{com} = \frac{1}{V}\int_{z_0}^{z_1} z dV$$
Alternatively, $$\vec{r}_{com} = \frac{(M_x, M_y, M_z)}{M}$$ Where M_x is the first moment of mass around the x-axis (mass of the particle * its distance from x-axis "y value of position")</td>
<td>Center of Mass of a solid body of mass M (constant)
For uniform objects: $$\rho = \frac{dm}{dV} = \frac{M}{V} = const$$ For non-uniform objects, ρ can be a function, in that case, we cannot apply the equalities (in red)</td>
<td>The idea for solid bodies is that we apply the same formula for $\vec{r}_{com}$ as a sum(integral) for every single particle in the object.
Each particle has a position $r(x, y, z)$ and a small mass dm, and the integral simplifies to the one on the left.

To evaluate these integrals, we need to express dV as a function of dx, dy, dz but it's not always that easy to do, so we may use the alternative formula with moments. Keep in mind that dV is the volume of a tiny part of the solid object.</td>
</tr>
<tr>
<td colspan="3">Special cases</td>
</tr>
<tr>
<td>For 1D uniform objects (a rope of length L) we have: $$V = L, dV = dx$$ 

Thus, $x_{com} = \frac{1}{L}\int_0^L v dV = \frac{L}{2}$</td>
<td colspan="2">For 2D uniform objects (a sheet of area A): $$V = A = \int_a^b f(x) - g(x) dx$$ $$dV = (f(x) - g(x))dx$$ Thus, $$x_{com} = \frac{1}{A}\int_a^b x\big(f(x) - g(x)\big)dx$$ $$y_{com} = \frac{1}{2A}\int_a^b y\big(f^2(y) - g^2(y)\big)dy$$ * y_{com} was obtained using the moment formula

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<tr>
<td>$$\mathbf{P} = \mathrm{m}\mathbf{v}$$ $$\mathbf{P} = \mathrm{M}\mathbf{v_{com}}$$ $$\mathbf{F_{net}} = \frac{\mathrm{d}\mathbf{P}}{\mathrm{dt}} = \mathrm{M}\mathbf{a_{com}}$$ * the red equality holds only when the system has a constant mass</td>
<td>Linear momentum of a particle
Linear momentum of a system of particles.
Newton's second law (in terms of momentum): the net force acting on a system is equal to the time rate of change of its linear momentum</td>
<td>Linear momentum is a vector quantity
Measured in kg.m/s</td>
</tr>
<tr>
<td>$$P_1 = P_2$$</td>
<td>Law of conservation of linear momentum: the linear momentum of an isolated system remains constant
This helps to solve exercises involving collisions</td>
<td>In an isolated system: no external forces acting ($F_{net} = 0$), particles are not entering/leaving the system and have constant masses ($M = const$).

Collisions b/w two objects: $$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$ $$v_{com,i} = v_{com,f}$$ 1. Elastic (no energy losses): $$(KE)_i = (KE)_f$$ $$m_1v_{1i}^2 + m_2v_{2i}^2 = m_1v_{1f}^2 + m_2v_{2f}^2$$</td>
</tr>
</table>

		2. Completely inelastic (bodies fused together) $$v_{f1} = v_{f2} = v_f$$ **3. Inelastic** (some energy losses) Law of conservation of energy might help
$$\boldsymbol{J} = \int_{t_i}^{t_f} \boldsymbol{F_{net}} dt = \Delta \mathbf{P}$$ $$J = F_{avg} \Delta t = ma\Delta t$$	**Impulse** is the change in **linear momentum** caused by a force that acts on a particle (or a system) during a short time (from t_i to t_f) Impulse can also be found if the average force acting during the time $\Delta t = t_f - t_i$ is known	Impulse is a vector quantity Measured in N.s = kg.m/s
$$\theta = \frac{s}{r}$$ $$\omega = \frac{d\theta}{dt} = \frac{ds}{rdt} = \frac{v}{r}$$ $$\alpha = \frac{d\omega}{dt} = \frac{d^2 s}{rdt^2} = \frac{a_t}{r}$$	Angular displacement Angular velocity (to linear) $$v = \omega r$$ Angular acceleration (to tangential) $$a_t = \alpha r$$	θ is measured in radians $$\pi \text{ rad } = 180°$$ CCW direction is positive, CW is negative Revolution: $1 \text{ rev} = 360° = 2\pi \text{ rad}$ The direction of ω can be found using the right-hand rule Δθ is **not** a vector quantity (addition is not commutative)
$$\omega = \omega_0 + \alpha t$$ $$\Delta\theta = \omega_0 t + 0.5\alpha t^2$$ $$\omega^2 - \omega_0^2 = 2\alpha\Delta\theta$$ $$\Delta\theta = 0.5t(\omega + \omega_0)$$	Rotational motion equations	Applied only when the body has a constant angular acceleration α
$$I = \sum m_i r_i^2$$ $$I = \int r^2 dm$$	The moment of inertia (rotational inertia, angular mass) is the 2nd moment of mass; it describes how the mass of the body is distributed around the axis of rotation.	Moment of inertia in rotational motion plays the role of mass in translational motion, in the sense that: For two bodies $m, M: M > m$: To give both bodies the same acceleration. One needs a bigger force in case of M since $F = ma$ If both bodies are moving with the same constant speed, M will have bigger kinetic energy since $KE = 0.5mv^2$

Rotational inertia for special solids

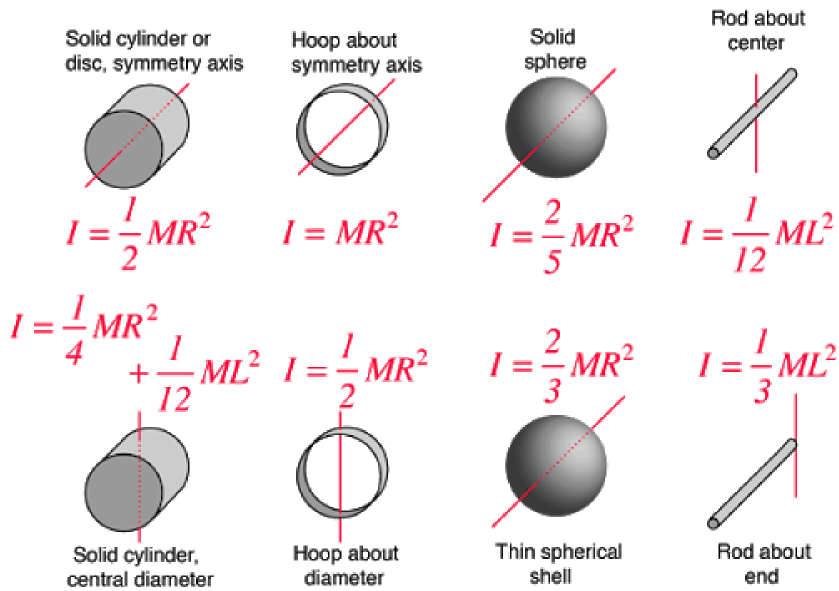
Parallel axis theorem

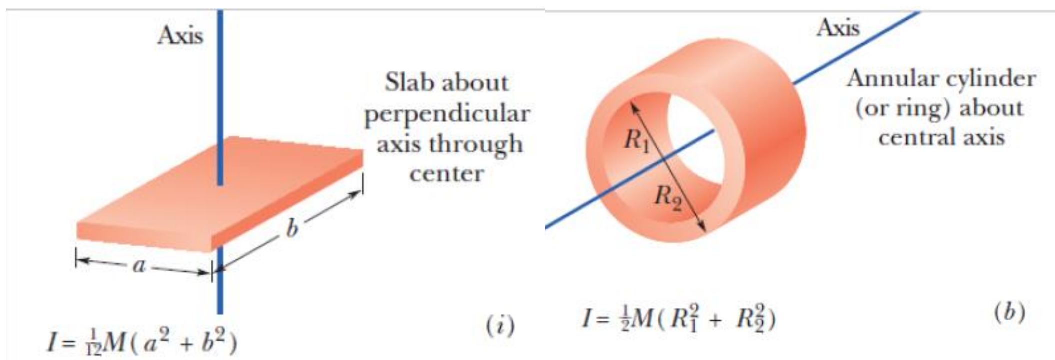
$$I = I_{com} + Mh^2$$

Used to find the rotational inertia of a body around an axis different from the one passing through the center of gravity (red axis)

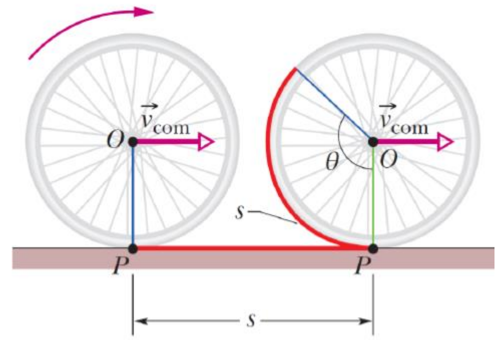
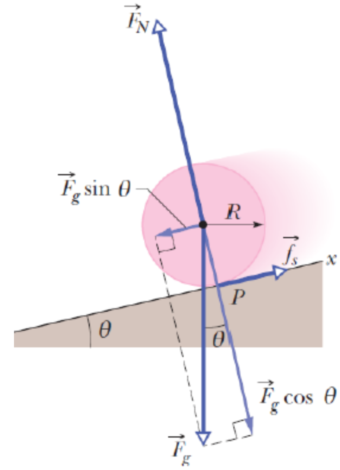
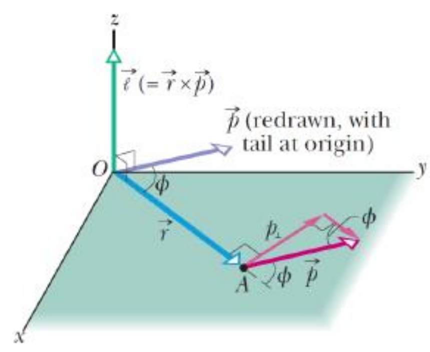
h is the distance **from** the axis of rotation of the center of mass **to** the given axis around which we are measuring the moment of inertia.

I_{com} can be found from the pictures:





$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$ $\lVert\boldsymbol{\tau}\rVert = Fr$ $\tau_{net} = I\alpha$	**Torque**: the rotational equivalent of linear force, describes the force causing rotation. Newton's second law for rotational motion.	- Torque is a vector quantity, is measured in N.m - Positive torque causes CCW rotation. - Negative torque causes CW rotation. - For cases when only one force is acting, $\tau_{net} = \tau \Longrightarrow Fr = I\alpha = I\frac{a_t}{r}$ this is useful for solving problems.
$KE = 0.5I\omega^2$ $W = \Delta KE = \int_{\theta_i}^{\theta_f} \tau d\theta = \tau\Delta\theta$ $P = \frac{dW}{dt} = \tau\omega$	(Rotational) Kinetic Energy for a body rotating with an angular velocity ω. The work done by torque rotating a body from θ_i to θ_f. The mechanical power of rotation	Red equalities hold only when τ is constant All laws for translational motion (conservation of energy, conservation of linear momentum, …) can be applied to rotational motion using the equivalence: $x \leftrightarrow \theta, v \leftrightarrow \omega, a \leftrightarrow r, F \leftrightarrow \tau, m \leftrightarrow I$

$v_{com} = \omega R$ $a_{com} = \alpha R$ $KE = 0.5 I_{com}\omega^2 + 0.5 M v_{com}^2$	Equations for a circular body of radius R rolling smoothly on some surface (no slipping/sliding/bouncing).	
$f_s - mg\sin\theta = ma_{com}$ $f_s R = I_{com}\alpha = I_{com}\frac{-a_{com}}{R}$ $a_{com} = \frac{-g\sin\theta}{1 + I_{com}/MR^2}$	Equations for a circular object of mass M and radius R rolling smoothly down a ramp inclined at angle θ Assuming the positive axis is pointing upward the ramp, α is **positive** and a_{com} is **negative** Equations used $F_{net} = ma_{com}$ $\tau_{net} = I\alpha$ $\alpha = \frac{a_{com}}{R}$ for smooth rolling Notice that $f_s \neq f_s^{max} = \mu N$ since the body is not slipping (smooth-rolling)	
$\boldsymbol{l} = \boldsymbol{r} \times \boldsymbol{p} = m\,(\boldsymbol{r} \times \boldsymbol{v})$ $\lvert\lvert \boldsymbol{l} \rvert\rvert = rp = mvr\sin\theta$ $L = I\omega$ $\tau_{net} = \frac{dL}{dt}$	Angular momentum for a particle of mass m, located at some point with position vector $\boldsymbol{r}$. The particle is moving with velocity $\boldsymbol{v}$, and has a linear momentum $\boldsymbol{p} = m\boldsymbol{v}$ θ is the angle between $\boldsymbol{p}$ and $\boldsymbol{r}$ Angular momentum of a body with a moment of Inertia I rotating with an angular velocity ω (Rotational) Newton's second law in terms of angular momentum (net torque equals the time derivative of total angular momentum)	Angular momentum is measured in J.s  $L_i = L_f \Rightarrow I_i\omega_i = I_f\omega_f$ The Law of conservation of angular momentum holds when there is no external torque acting on the system.

What is “moment”?

“Moment” in general describes how a physical quantity is distributed over some region

$$\mu_n = r^n Q$$

Where

- μ is the moment (vector quantity)
- n is a parameter of the moment (0th, 1st, 2nd moment, ...)
- Q is a physical quantity (force, mass, momentum, …)
- r is the distance **from** the point at which the physical quantity is located (acting) **to** some reference point.
    - We take the moment with respect to that reference point.
    - When the quantity is not concentrated at a single point, the moment is the summation of moments of all its particles (Σ, ∫ detected, for discrete and continuous cases).

Examples:

- The first moment of **force** = Torque: the rotational equivalent of linear force, describes the force causing rotation: $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$
- The first moment of **momentum** = angular momentum: $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$
- Moments of **mass** = $\sum m_i r_i^n$
    - The 0th moment is the total mass of all the particles (M)
    - The 1st moment is the center of mass (multiplied by M) of the particles (Mr_{com})
    - The 2nd moment is the moment of inertia (angular mass, rotational inertia) (I)
        - Determines the torque needed for a desired angular acceleration
        - Depends on how the mass is distributed around the axis of rotation
        - Plays the role of mass in translational motion