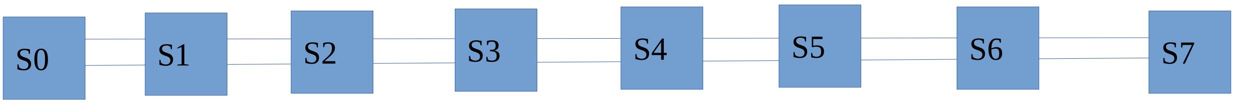
Report: Assignment 1

Task description

On average, 3 cars arrive at the warehouse per hour. Unloading is carried out by 3 teams of loaders. The average unloading time of the machine is 1 hour. There can be no more than 4 cars in the queue waiting for unloading. Determine the performance of the queuing system.

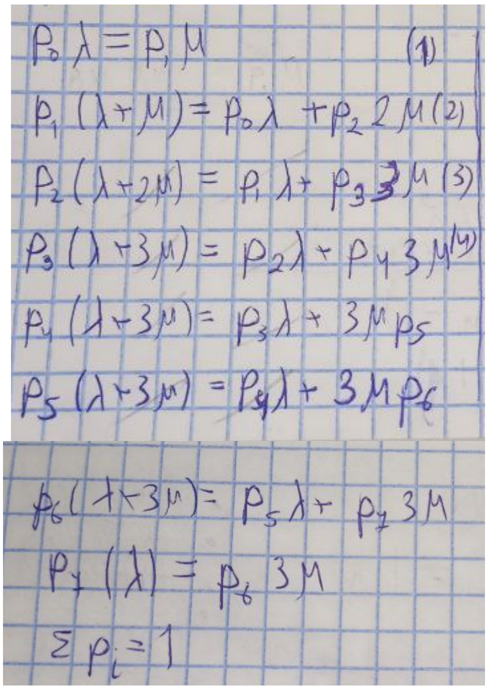
Analitical solution

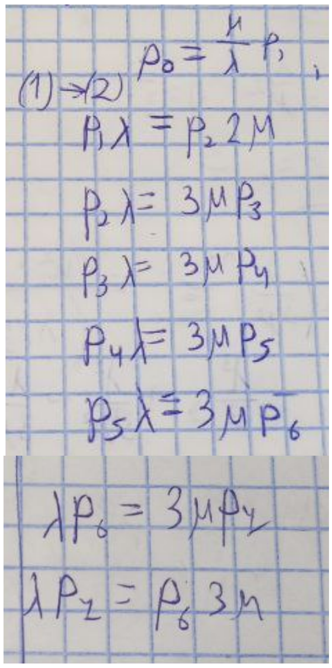
First consider system that was described in task description as queue system with 7 states. Transition from S_i to S_(i+1) will happend with intensity λ = 3. Tranistion from S_(i+1) to S_i will happend with intensity μ = 1



Picture 1 Queue system.

Consider steady-state distribution (Вероятности в предельном стационарном состоянии) of given system. With use of Kolmogorov`s equations following system was optained. Transform it to system on picture 2 right.





Picture 2 System of equations

This system can be transformed to this system.

$$(1) \rightarrow p_1 = \frac{\lambda}{\mu} p_0$$

$$(2) \quad p_2 = \frac{\lambda}{2\mu} p_1 = \frac{\lambda\lambda}{2\mu^2} p_0$$

$$(3) \rightarrow p_3 = \frac{\lambda}{3\mu} p_2 = \frac{\lambda^3}{6\mu^3} p_0$$

$$(4) \quad p_4 = \frac{\lambda}{3\mu} p_3 = \frac{\lambda^4}{3^2 2 \cdot \mu^4} p_0$$

$$(5) \quad p_5 = \frac{\lambda^5}{3^3 2 \mu^5} p_0$$

$$(6) \rightarrow p_6 = \frac{\lambda^6}{3^4 2 \mu^6} p_0, \quad p_7 = \frac{\lambda^7}{3^5 2 \mu^7} p_0$$

$$\sum_{i=0}^{7} p_i = 1 \Rightarrow \left(p_0 + \sum_{n=1}^{2} \frac{\lambda^n}{n!\,\mu^n} p_0 + \sum_{n=3}^{7} \frac{\lambda^n}{3^{n-3}\, 2 \cdot 3\, \mu^n} p_0\right) = 1$$

$$\Rightarrow p_0 = \left(1 + \sum_{n=1}^{2} \frac{\lambda^n}{n!\,\mu^n} + \sum_{n=3}^{7} \frac{\lambda^n}{3^{n-3}\, 2 \cdot 3\, \mu^n}\right)^{-1}$$

Picture 3 System of equations

On picture values of probalbilites to be in state S in steady-state position.
Let“s calculate values of above probobilities:
p0 = 1/31, p1 = 3/31, p2 = 4.5/31, p3 = 4.5/31, p4 = 4.5/31, p5 = 4.5/31, p6 = 4.5/31, p7 = 4.5/31
Let“s calculate performance metrics:

Относительная пропускная способность

Абсолютная пропускная способность

Средняя продолжительность периода занятости

Коэфициент использования

Среднее время ожидания в очереди

Расспределение
Расспределение

Среднее время пребывания заявки
Вероятность отказа
Вероятность немедленного принятия
Средняя длинна очереди
Среднее число заявок в Системе массового обслуживания

Относительная пропускная способность
$1 — p_7 = 26{,}5/31 = 53/62$
Абсолютная пропускная способность
$\lambda\ (1 — p_7) = 3\ (1 — p_7) = 2{,}564516129$
Средняя продолжительность периода занятости
System utilisation
System utilisation is time during which system is not idle.
$1 — p_0 = (31\text{-}1)/31 = 30/31 = 0{,}967741935$
Mean busy period of the system
Ratio of busy period and idle period
$(1 — p_0)/(\lambda p_0) = (1 — p_0)/(3p_0) = 3{,}111111111$

Avarage time in queue
Make 8 hyposesis H_i mean system in state S_i.
For states S0..S3 car doesn"t need to wait in queue as some unload teams are free.
For states S4..S6 car should wait until system serve clients on queue
For states S7 car will not come to queue

Hyposeses	H0	H1	H2	H3	H4	H5	H6	H7
Probability	P0	P1	P2	P3	P4	P5	P6	P7
E(T|H)	0	0	0	0	$1/\mu$	$2/\mu$	$3/\mu$	0

To calcualte avarage time in queue. Use formula bellow:
$E(T) = \sum p*E(T|H) = 27/31 = 0{,}870967742$

Среднее время пребывания заявки
Avarage time car spend in warehouse system
Make 8 hyposesis H_i mean system in state S_i.
For states S0..S3 car doesn"t need to wait in queue as some unload teams are free and wait unload time.
For states S4..S6 car should wait until system serve clients on queue and wait unload time
For states S7 car will not come to queue

Hyposeses	H0	H1	H2	H3	H4	H5	H6	H7
Probability	P0	P1	P2	P3	P4	P5	P6	P7

E(T|H)	0+1/μ	0+1/μ	0+1/μ	0+1/μ	1/μ+1/μ	2/μ+1/μ	3/μ+1/μ	0

To calcualte avarage time in queue. Use formula bellow:
$E(T) = \sum p*E(T|H) = (p0+..+p3) + p4*2 + p5*3 + p6*4 = 53.5/31 = 1,725806452$
Вероятность отказа
$p_7 = 9/62 = 0,14516129$
Вероятность немедленного принятия
The request will be served immidiately if there are exist free servers (on strates S0,S1,S3)
$p_0 + p_1 + p_2 + p_3 = 16,5/31 = 33/62 = 0,532258065$
Avarage queue length

Queue length (k)	0	1	2	3	4
Probability (p)	$p_0 + p_1 + p_2 + p_3$	P_4	P_5	P_6	P_7

Avarage queue length is expacted value of variable described in table above above. Queue length depends on state thus probability to have queue length k is equal to probability to be in state or states S.
$E(k) = (p_0 + p_1 + p_2 + p_3)*0 + P_4*1 + P_5*2 + P_6*3 + P_7*4 = P_4*1 + P_5*2 + P_6*3 + P_7*4 = (4,5*10)/31 = 1,451612903$

Среднее число заявок в Системе массового обслуживания
Avarage number of cars in system
Avarage number of cars in system can be calculated as expected value of variable k described in table bellow.

# of requests (k)	0	1	2	3	i	7
Probability (p)	p_0	P_1	P_2	P_3	P_i	P_7

$E(k) = \Sigma i*P_i$ (i is number from 0 to 7)
$E(k) = \Sigma i*P_i = 4,064516129$