

MATRIX FACTORIZATION TECHNIQUES FOR RECOMMENDER SYSTEMS

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As the Netflix Prize competition has demonstrated, **matrix factorization models are superior** to classic nearest-neighbor techniques for producing product recommendations, allowing the incorporation of additional information such as **implicit feedback, temporal effects, and confidence levels.**

Modern consumers are inundated with choices. Electronic retailers and content providers offer a huge selection of products, with unprecedented opportunities to meet a variety of special needs and tastes. Matching consumers with the most appropriate products is key to enhancing user satisfaction and loyalty. Therefore, more retailers have become interested in recommender systems, which analyze patterns of user interest in products to **provide personalized recommendations that suit a user's taste.** Because good personalized recommendations can add another dimension to the user experience, e-commerce leaders like Amazon.com and Netflix have made recommender systems a salient part of their websites.

Such systems are particularly useful for entertainment products such as movies, music, and TV shows. Many customers will view the same movie, and each customer is likely to view numerous different movies. Customers have proven willing to indicate their level of satisfaction with particular movies, so a huge volume of data is available about which movies appeal to which customers. Companies can analyze this data to recommend movies to particular customers.

RECOMMENDER SYSTEM STRATEGIES

Broadly speaking, recommender systems are based on one of two strategies. The **content filtering** approach creates a profile for each user or product **to characterize its nature.** For example, a movie profile could include attributes regarding its genre, the participating actors, its box office popularity, and so forth. User profiles might include demographic information or answers provided on a suitable questionnaire. The profiles allow programs to associate users with matching products. Of course, **content-based strategies require gathering external information that might not be available or easy to collect.**

A known successful realization of content filtering is the Music Genome Project, which is used for the Internet radio service Pandora.com. A trained music analyst scores

each song in the Music Genome Project based on hundreds of distinct musical characteristics. These attributes, or genes, capture not only a song's musical identity but also many significant qualities that are relevant to understanding listeners' musical preferences.

An alternative to content filtering relies only on past user behavior—for example, previous transactions or product ratings—without requiring the creation of explicit profiles. This approach is known as **collaborative filtering**, a term coined by the developers of Tapestry, the first recommender system.¹ Collaborative filtering analyzes relationships between users and interdependencies among products to identify new user-item associations.

A major appeal of collaborative filtering is that **it is domain free**, yet it can address data aspects that are often elusive and difficult to profile using content filtering. While generally more accurate than content-based techniques, collaborative filtering suffers from what is called the **cold start problem**, due to its inability to address the system's new products and users. In this aspect, content filtering is superior.

The two primary areas of collaborative filtering are the **neighborhood methods** and **latent factor models**. Neighborhood methods are centered on computing the relationships between items or, alternatively, between users. The **item-oriented approach** evaluates a user's preference for an item based on ratings of "neighboring" items by the same user. A product's neighbors are other products that tend to get similar ratings when rated by the same user. For example, consider the movie *Saving Private Ryan*. Its neighbors might include war movies, Spielberg movies, and Tom Hanks movies, among others. To predict a particular user's rating for *Saving Private Ryan*, we would look for the movie's nearest neighbors that this user actually rated. As Figure 1 illustrates, the user-oriented approach identifies like-minded users who can complement each other's ratings.

Latent factor models are an alternative approach that tries to explain the ratings by characterizing both items and users on, say, 20 to 100 factors inferred from the ratings patterns. In a sense, such factors comprise a computerized alternative to the aforementioned human-created song genes. For movies, the discovered factors might measure obvious dimensions such as comedy versus drama, amount of action, or orientation to children; less

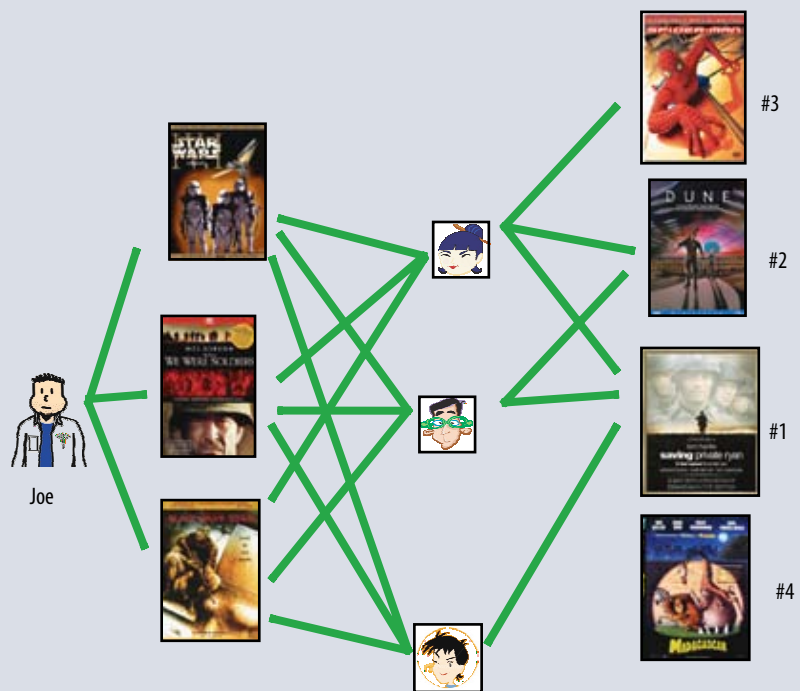


Figure 1. The user-oriented neighborhood method. Joe likes the three movies on the left. To make a prediction for him, the system finds similar users who also liked those movies, and then determines which other movies they liked. In this case, all three liked *Saving Private Ryan*, so that is the first recommendation. Two of them liked *Dune*, so that is next, and so on.

well-defined dimensions such as depth of character development or quirkiness; or completely uninterpretable dimensions. For users, each factor measures how much the user likes movies that score high on the corresponding movie factor.

Figure 2 illustrates this idea for a simplified example in two dimensions. Consider two hypothetical dimensions characterized as female- versus male-oriented and serious versus escapist. The figure shows where several well-known movies and a few fictitious users might fall on these two dimensions. For this model, a user's predicted rating for a movie, relative to the movie's average rating, would equal the dot product of the movie's and user's locations on the graph. For example, we would expect Gus to love *Dumb and Dumber*, to hate *The Color Purple*, and to rate *Braveheart* about average. Note that some movies—for example, *Ocean's 11*—and users—for example, Dave—would be characterized as fairly neutral on these two dimensions.

MATRIX FACTORIZATION METHODS

Some of the most successful realizations of latent factor models are based on **matrix factorization**. In its basic form, matrix factorization characterizes both items and users by vectors of factors inferred from item rating patterns. High correspondence between item and user factors leads to a



Figure 2. A simplified illustration of the latent factor approach, which characterizes both users and movies using two axes—male versus female and serious versus escapist.

recommendation. These methods have become popular in recent years by combining good scalability with predictive accuracy. In addition, they offer much flexibility for modeling various real-life situations.

Recommender systems rely on different types of input data, which are often placed in a matrix with one dimension representing users and the other dimension representing items of interest. The most convenient data is high-quality *explicit feedback*, which includes explicit input by users regarding their interest in products. For example, Netflix collects star ratings for movies, and TiVo users indicate their preferences for TV shows by pressing thumbs-up and thumbs-down buttons. We refer to explicit user feedback as *ratings*. Usually, explicit feedback comprises a *sparse matrix*, since any single user is likely to have rated only a small percentage of possible items.

One strength of matrix factorization is that it allows incorporation of additional information. When explicit feedback is not available, recommender systems can infer user preferences using *implicit feedback*, which indirectly reflects opinion by observing user behavior including purchase history, browsing history, search patterns, or even mouse movements. Implicit feedback usually denotes the presence or absence of an event, so it is typically represented by a densely filled matrix.

A BASIC MATRIX FACTORIZATION MODEL

Matrix factorization models map both users and items to a joint latent factor space of dimensionality f , such that user-item interactions are modeled as inner products in that space. Accordingly, each item i is associated with a

vector $q_i \in \mathbb{R}^f$, and each user u is associated with a vector $p_u \in \mathbb{R}^f$. For a given item i , the elements of q_i measure the extent to which the item possesses those factors, positive or negative. For a given user u , the elements of p_u measure the extent of interest the user has in items that are high on the corresponding factors, again, positive or negative. The resulting dot product, $q_i^T p_u$, captures the interaction between user u and item i —the user's overall interest in the item's characteristics. This approximates user u 's rating of item i , which is denoted by r_{ui} , leading to the estimate

$$\hat{r}_{ui} = q_i^T p_u. \quad (1)$$

The major challenge is computing the mapping of each item and user to factor vectors $q_i, p_u \in \mathbb{R}^f$. After the recommender system completes this mapping, it can easily estimate the rating a user will give to any item by using Equation 1.

Such a model is closely related to *singular value decomposition* (SVD), a well-established technique for identifying latent semantic factors in information retrieval. Applying SVD in the collaborative filtering domain requires factoring the user-item rating matrix. This often raises difficulties due to the high portion of missing values caused by sparseness in the user-item ratings matrix. Conventional SVD is undefined when knowledge about the matrix is incomplete. Moreover, carelessly addressing only the relatively few known entries is highly prone to overfitting.

Earlier systems relied on imputation to fill in missing ratings and make the rating matrix dense.² However, imputation can be very expensive as it significantly increases the amount of data. In addition, inaccurate imputation might distort the data considerably. Hence, more recent works³⁻⁶ suggested modeling directly the observed ratings only, while avoiding overfitting through a regularized model. To learn the factor vectors (p_u and q_i), the system minimizes the regularized squared error on the set of known ratings:

$$\min_{q^*, p^*} \sum_{(u,i) \in \kappa} (r_{ui} - q_i^T p_u)^2 + \lambda (\|q_i\|^2 + \|p_u\|^2) \quad (2)$$

Here, κ is the set of the (u,i) pairs for which r_{ui} is known (the training set).

The system learns the model by fitting the previously observed ratings. However, the goal is to generalize those previous ratings in a way that predicts future, unknown ratings. Thus, the system should avoid overfitting the observed data by regularizing the learned parameters, whose magnitudes are penalized. The constant λ controls

the extent of regularization and is usually determined by cross-validation. Ruslan Salakhutdinov and Andriy Mnih's "Probabilistic Matrix Factorization"⁷ offers a probabilistic foundation for regularization.

LEARNING ALGORITHMS

Two approaches to minimizing Equation 2 are *stochastic gradient descent* and *alternating least squares* (ALS).

Stochastic gradient descent

Simon Funk popularized a stochastic gradient descent optimization of Equation 2 (<http://sifter.org/~simon/journal/20061211.html>) wherein the algorithm **loops through all ratings in the training set**. For each given training case, the system predicts r_{ui} and computes the associated prediction error

$$e_{ui} \stackrel{\text{def}}{=} r_{ui} - q_i^T p_u.$$

Then it modifies the parameters by a magnitude proportional to γ in the opposite direction of the gradient, yielding:

- $q_i \leftarrow q_i + \gamma \cdot (e_{ui} \cdot p_u - \lambda \cdot q_i)$
- $p_u \leftarrow p_u + \gamma \cdot (e_{ui} \cdot q_i - \lambda \cdot p_u)$

This popular approach⁴⁻⁶ combines implementation ease with a relatively fast running time. Yet, in some cases, it is beneficial to use ALS optimization.

Alternating least squares

Because both q_i and p_u are unknowns, Equation 2 is not convex. However, **if we fix one of the unknowns, the optimization problem becomes quadratic and can be solved optimally**. Thus, ALS techniques rotate between fixing the q_i 's and fixing the p_u 's. **When all p_u 's are fixed, the system recomputes the q_i 's by solving a least-squares problem, and vice versa**. This ensures that each step decreases Equation 2 until convergence.⁸

While in general stochastic gradient descent is easier and faster than ALS, ALS is favorable in at least two cases. The first is when the system can use parallelization. In ALS, the system computes each q_i independently of the other item factors and computes each p_u independently of the other user factors. This gives rise to potentially massive parallelization of the algorithm.⁹ The second case is for systems centered on implicit data. Because the training set cannot be considered sparse, looping over each single training case—as gradient descent does—would not be practical. ALS can efficiently handle such cases.¹⁰

ADDING BIASES

One benefit of the matrix factorization approach to collaborative filtering is its flexibility in dealing with various

data aspects and other application-specific requirements. This requires accommodations to Equation 1 while staying within the same learning framework. **Equation 1 tries to capture the interactions between users and items that produce the different rating values**. However, much of the observed variation in rating values is due to effects associated with either users or items, known as *biases* or *intercepts*, independent of any interactions. For example, **typical collaborative filtering data exhibits large systematic tendencies for some users to give higher ratings than others, and for some items to receive higher ratings than others**. After all, some products are widely perceived as better (or worse) than others.

Thus, **it would be unwise to explain the full rating value by an interaction of the form $q_i^T p_u$** . Instead, the system tries **to identify the portion of these values that individual user or item biases can explain**, subjecting only the true interaction portion of the data to factor modeling. A first-order approximation of the bias involved in rating r_{ui} is as follows:

$$b_{ui} = \mu + b_i + b_u \quad (3)$$

The bias involved in rating r_{ui} is denoted by b_{ui} and accounts for the user and item effects. The overall average rating is denoted by μ ; the parameters b_u and b_i indicate the observed deviations of user u and item i , respectively, from the average. **For example, suppose that you want a first-order estimate for user Joe's rating of the movie *Titanic*. Now, say that the average rating over all movies, μ , is 3.7 stars. Furthermore, *Titanic* is better than an average movie, so it tends to be rated 0.5 stars above the average. On the other hand, Joe is a critical user, who tends to rate 0.3 stars lower than the average. Thus, the estimate for *Titanic*'s rating by Joe would be 3.9 stars ($3.7 + 0.5 - 0.3$).** Biases extend Equation 1 as follows:

$$\hat{r}_{ui} = \mu + b_i + b_u + q_i^T p_u \quad (4)$$

Here, the observed rating is broken down into its four components: **global average, item bias, user bias, and user-item interaction**. This allows each component to explain only the part of a signal relevant to it. The system learns by minimizing the squared error function:^{4,5}

$$\min_{p^*, q^*, b^*} \sum_{(u,i) \in K} (r_{ui} - \mu - b_u - b_i - p_u^T q_i)^2 + \lambda (\|p_u\|^2 + \|q_i\|^2 + b_u^2 + b_i^2) \quad (5)$$

Since biases tend to capture much of the observed signal, their accurate modeling is vital. Hence, other works offer more elaborate bias models.¹¹

ADDITIONAL INPUT SOURCES

Often a system must deal with the cold start problem, wherein many users supply very few ratings, making it

difficult to reach general conclusions on their taste. A way to relieve this problem is to incorporate additional sources of information about the users. Recommender systems can use implicit feedback to gain insight into user preferences. Indeed, they can gather behavioral information regardless of the user's willingness to provide explicit ratings. A retailer can use its customers' purchases or browsing history to learn their tendencies, in addition to the ratings those customers might supply.

For simplicity, consider a case with a Boolean implicit feedback. $N(u)$ denotes the set of items for which user u expressed an implicit preference. This way, the system profiles users through the items they implicitly preferred. Here, a new set of item factors are necessary, where item i is associated with $x_i \in \mathbb{R}^f$. Accordingly, a user who showed a preference for items in $N(u)$ is characterized by the vector

$$\sum_{i \in N(u)} x_i.$$

Normalizing the sum is often beneficial, for example, working with

$$|N(u)|^{-0.5} \sum_{i \in N(u)} x_i^{4.5}$$

Another information source is known user attributes, for example, demographics. Again, for simplicity consider Boolean attributes where user u corresponds to the set of attributes $A(u)$, which can describe gender, age group, Zip code, income level, and so on. A distinct factor vector $y_a \in \mathbb{R}^f$ corresponds to each attribute to describe a user through the set of user-associated attributes:

$$\sum_{a \in A(u)} y_a$$

The matrix factorization model should integrate all signal sources, with enhanced user representation:

$$\hat{r}_{ui} = \mu + b_i + b_u + q_i^T [p_u + |N(u)|^{-0.5} \sum_{i \in N(u)} x_i + \sum_{a \in A(u)} y_a] \quad (6)$$

While the previous examples deal with enhancing user representation—where lack of data is more common—items can get a similar treatment when necessary.

TEMPORAL DYNAMICS

So far, the presented models have been static. In reality, product perception and popularity constantly change as new selections emerge. Similarly, customers' inclinations evolve, leading them to redefine their taste. Thus, the system should account for the temporal effects reflecting the dynamic, time-drifting nature of user-item interactions.

The matrix factorization approach lends itself well to modeling temporal effects, which can significantly im-

prove accuracy. Decomposing ratings into distinct terms allows the system to treat different temporal aspects separately. Specifically, the following terms vary over time: item biases, $b_i(t)$; user biases, $b_u(t)$; and user preferences, $p_u(t)$.

The first temporal effect addresses the fact that an item's popularity might change over time. For example, movies can go in and out of popularity as triggered by external events such as an actor's appearance in a new movie. Therefore, these models treat the item bias b_i as a function of time. The second temporal effect allows users to change their baseline ratings over time. For example, a user who tended to rate an average movie "4 stars" might now rate such a movie "3 stars." This might reflect several factors including a natural drift in a user's rating scale, the fact that users assign ratings relative to other recent ratings, and the fact that the rater's identity within a household can change over time. Hence, in these models, the parameter b_u is a function of time.

Temporal dynamics go beyond this; they also affect user preferences and therefore the interaction between users and items. Users change their preferences over time. For example, a fan of the psychological thrillers genre might become a fan of crime dramas a year later. Similarly, humans change their perception of certain actors and directors. The model accounts for this effect by taking the user factors (the vector p_u) as a function of time. On the other hand, it specifies static item characteristics, q_i , because, unlike humans, items are static in nature.

Exact parameterizations of time-varying parameters¹¹ lead to replacing Equation 4 with the dynamic prediction rule for a rating at time t :

$$\hat{r}_{ui}(t) = \mu + b_i(t) + b_u(t) + q_i^T p_u(t) \quad (7)$$

INPUTS WITH VARYING CONFIDENCE LEVELS

In several setups, not all observed ratings deserve the same weight or confidence. For example, massive advertising might influence votes for certain items, which do not aptly reflect longer-term characteristics. Similarly, a system might face adversarial users that try to tilt the ratings of certain items.

Another example is systems built around implicit feedback. In such systems, which interpret ongoing user behavior, a user's exact preference level is hard to quantify. Thus, the system works with a cruder binary representation, stating either "probably likes the product" or "probably not interested in the product." In such cases, it is valuable to attach confidence scores with the estimated preferences. Confidence can stem from available numerical values that describe the frequency of actions, for example, how much time the user watched a certain show or how frequently a user bought a certain item. These numerical values indicate the confidence in each observation. Various factors that have nothing to do with user

preferences might cause a one-time event; however, **a recurring event is more likely to reflect user opinion.**

The matrix factorization model can readily accept varying confidence levels, which let it give less weight to less meaningful observations. If confidence in observing r_{ui} is denoted as c_{ui} , then the model enhances the cost function (Equation 5) to account for confidence as follows:

$$\min_{p^*, q^*, b^*} \sum_{(u,i) \in K} c_{ui} (r_{ui} - \mu - b_u - b_i - p_u^T q_i)^2 + \lambda (\|p_u\|^2 + \|q_i\|^2 + b_u^2 + b_i^2) \quad (8)$$

For information on a real-life application involving such schemes, refer to "Collaborative Filtering for Implicit Feedback Datasets."¹⁰

NETFLIX PRIZE COMPETITION

In 2006, the online DVD rental company Netflix announced a contest to **improve the state of its recommender system.**¹² To enable this, the company released a training set of more than 100 million ratings spanning about 500,000 anonymous customers and their ratings on more than 17,000 movies, each movie being rated on a scale of 1 to 5 stars. **Participating teams submit predicted ratings for a test set of approximately 3 million ratings,** and Netflix calculates a root-mean-square error (RMSE) based on the held-out truth. The first team that can improve on the Netflix algorithm's RMSE performance by 10 percent or more wins a \$1 million prize. If no team reaches the 10 percent goal, Netflix gives a \$50,000 Progress Prize to the team in first place after each year of the competition.

The contest created a buzz within the collaborative filtering field. Until this point, the only publicly available data for collaborative filtering research was orders of magnitude smaller. The release of this data and the competition's allure spurred a burst of energy and activity. According to the contest website (www.netflixprize.com), more than 48,000 teams from 182 different countries have downloaded the data.

Our team's entry, originally called BellKor, took over the top spot in the competition in the summer of 2007, and won the 2007 Progress Prize with the best score at the time: 8.43 percent better than Netflix. Later, we aligned with team Big Chaos to win the 2008 Progress Prize with a score of 9.46 percent. **At the time of this writing, we are still in first place, inching toward the 10 percent landmark.**

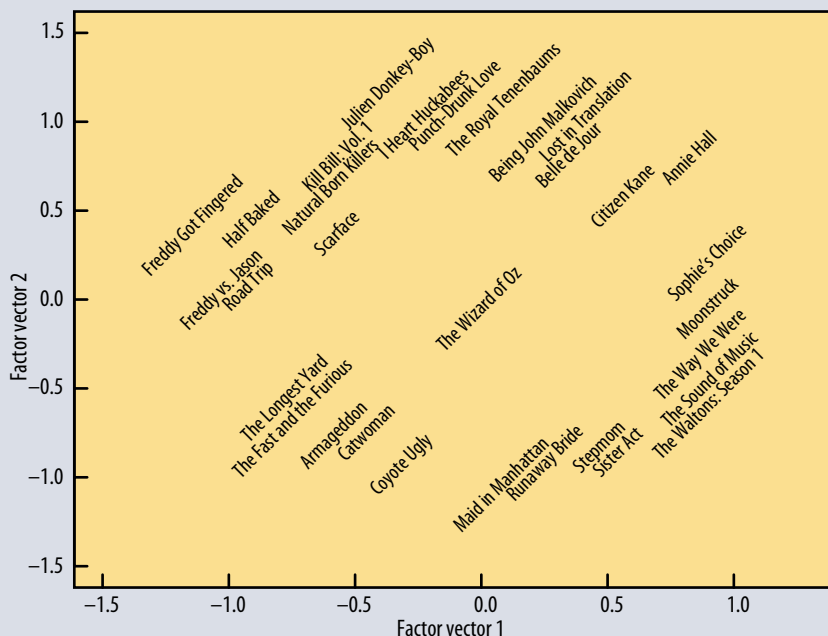


Figure 3. The first two vectors from a matrix decomposition of the Netflix Prize data. Selected movies are placed at the appropriate spot based on their factor vectors in two dimensions. The plot reveals distinct genres, including clusters of movies with strong female leads, fraternity humor, and quirky independent films.

Our winning entries consist of more than 100 different predictor sets, the majority of which are factorization models using some variants of the methods described here. Our discussions with other top teams and postings on the public contest forum indicate that these are the most popular and successful methods for predicting ratings.

Factorizing the Netflix user-movie matrix allows us to discover the most descriptive dimensions for predicting movie preferences. **We can identify the first few most important dimensions from a matrix decomposition and explore the movies' location in this new space.** Figure 3 shows the first two factors from the Netflix data matrix factorization. Movies are placed according to their factor vectors. Someone familiar with the movies shown can see clear meaning in the latent factors. The first factor vector (x-axis) has on one side **lowbrow comedies and horror movies**, aimed at a male or adolescent audience (*Half Baked*, *Freddie vs. Jason*), while the other side contains drama or comedy with serious undertones and **strong female leads** (*Sophie's Choice*, *Moonstruck*). The second factorization axis (y-axis) has independent, critically acclaimed, quirky films (*Punch-Drunk Love*, *I Heart Huckabees*) on the top, and on the bottom, mainstream formulaic films (*Armageddon*, *Runaway Bride*). There are interesting intersections between these boundaries: On the top left corner, where indie meets lowbrow, are *Kill Bill* and *Natural Born Killers*, both arty movies that play off violent themes. On the bottom right, where the serious female-driven movies meet

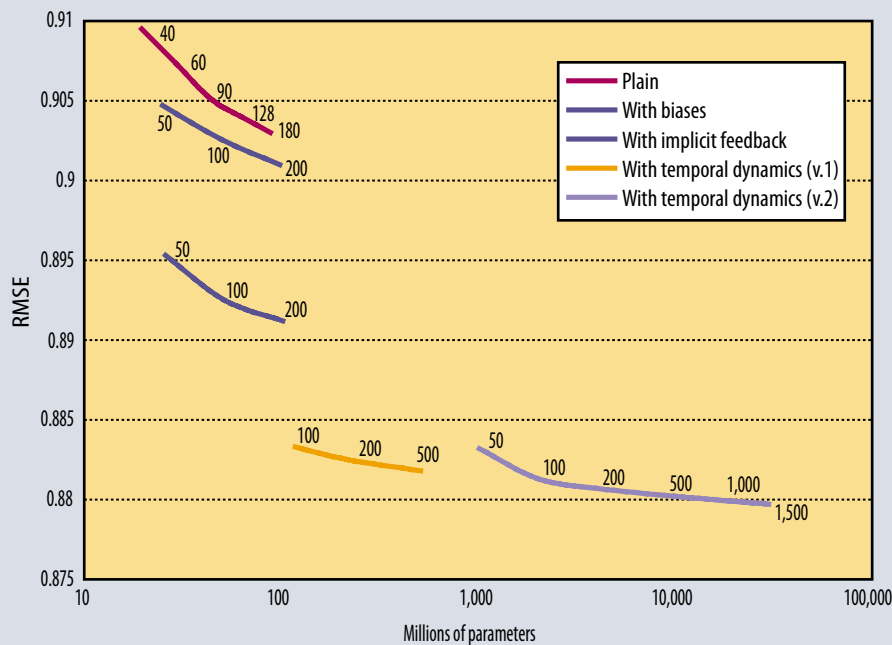


Figure 4. Matrix factorization models' accuracy. The plots show the root-mean-square error of each of four individual factor models (lower is better). Accuracy improves when the factor model's dimensionality (denoted by numbers on the charts) increases. In addition, the more refined factor models, whose descriptions involve more distinct sets of parameters, are more accurate. For comparison, the Netflix system achieves $RMSE = 0.9514$ on the same dataset, while the grand prize's required accuracy is $RMSE = 0.8563$.

the mainstream crowd-pleasers, is *The Sound of Music*. And smack in the middle, appealing to all types, is *The Wizard of Oz*.

In this plot, some movies neighboring one another typically would not be put together. For example, *Annie Hall* and *Citizen Kane* are next to each other. Although they are stylistically very different, they have a lot in common as highly regarded classic movies by famous directors. Indeed, the third dimension in the factorization does end up separating these two.

We tried many different implementations and parameterizations for factorization. Figure 4 shows how different models and numbers of parameters affect the RMSE as well as the performance of the factorization's evolving implementations—plain factorization, adding biases, enhancing user profile with implicit feedback, and two variants adding temporal components. The accuracy of each of the factor models improves by increasing the number of involved parameters, which is equivalent to increasing the factor model's dimensionality, denoted by numbers on the charts.

The more complex factor models, whose descriptions involve more distinct sets of parameters, are more accurate. In fact, the temporal components are particularly important to model as there are significant temporal effects in the data.

Matrix factorization techniques have become a dominant methodology within collaborative filtering recommenders. Experience with datasets such as the Netflix Prize data has shown that they deliver accuracy superior to classical nearest-neighbor techniques. At the same time, they offer a compact memory-efficient model that systems can learn relatively easily. What makes these techniques even more convenient is that models can integrate naturally many crucial aspects of the data, such as multiple forms of feedback, temporal dynamics, and confidence levels. ■

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