Juarez Disciplina Ricardo

Encuentre la respuesta de estado cero, $y[n]$ del Sistema descrito por:

$$y[n+2]+4y[n+1]+4y[n]=x[n+2]-x[n+1]$$

Cuando la secuencia de entrada es $x[n]=n\,u[n]$

Considere $y[-1]=y[-2]=0$

$$Y(z)=\frac{y[0]z^2+y[1]z+4y[0]z}{z^2+4z+4}+\frac{(z^2-z)X(z)}{z^2+4z+4}$$

$$-\frac{x[0]z^2+x[1]z+x[0]z}{z^2+4z+4}$$

→ $x[0]$, $x[1]$ se obtienen

$x[n]=n\,u[n]$

$x[0]=0\quad u[0]=0$

$x[1]=1\quad u[1]=1$

$y[0]$, $y[1]$ se obtienen

$y[n]=-4y[n-1]-4y[n-2]+x[n]-x[n-1]$

$y[1]=-4y[1-1]-4y[1-2]+x[1]-x[1-1]$

$y[0]=-4y[0-1]-4y[0-2]+x[0]-x[0-1]$

Sostituyendo

$$y(z) = \frac{1z^2 - 3z + 4z}{z^2+4z+4} + \frac{(z^2-z)X(z)}{z^2+4z+4} - \ldots$$

$$\ldots \frac{\cancel{0z} + z + \cancel{6z}}{z^2+4z+4}$$

$$y(z) = \frac{z^2}{z^2+4z+4} + \frac{z}{z^2+4z+4} + \frac{(z^2-z)X(z)}{z^2+4z+4}$$

$$-\frac{z}{(z^2-z)X(z)}$$

$$y(z) = \frac{z^2}{z^2+4z+4} + \frac{(z^2-z)X(z)}{z^2+4z+4}$$

$$X(z) = z/(z-1)^2$$

Simplificando $y(z)$

$$y(z) = \frac{z^2}{z^2+4z+4} + \frac{(z^2-z)X(z)}{z^2+4z+4}$$

$$y(z) = \frac{z^2}{z^2+4z+4} + \frac{(z^2-z)(z/(z-1)^2)}{z^2+4z+4}$$

$$y(z) = \frac{z^2 + (z^2 - z)\left(z/(z-1)^2\right)}{z^2+4z+4} = \frac{z^2 + \frac{z^2}{z-1}}{z^2+4z+4}$$

$$y(z) = \frac{\frac{z^2(z-1)+z^2}{z-1}}{z^2+4z+4} = \frac{z^2(z-1)+z^2}{(z-1)(z^2+4z+4)}$$

$$y(z) = \frac{z^2}{(z-1)(z^2+4z+4)}$$

Mediante fracciones parciales y ayuda de matlab

$$y[n] = \frac{2(-2)^n n}{3} + \frac{8(-2)^n}{9} = \frac{1}{9}$$

$$y[n] = \frac{2}{3}(-2)^n\left[n + \frac{4}{3}\right] + \frac{1}{9}\, u[n]$$