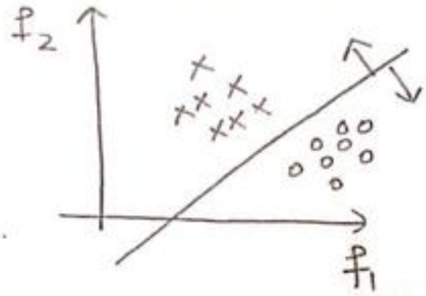
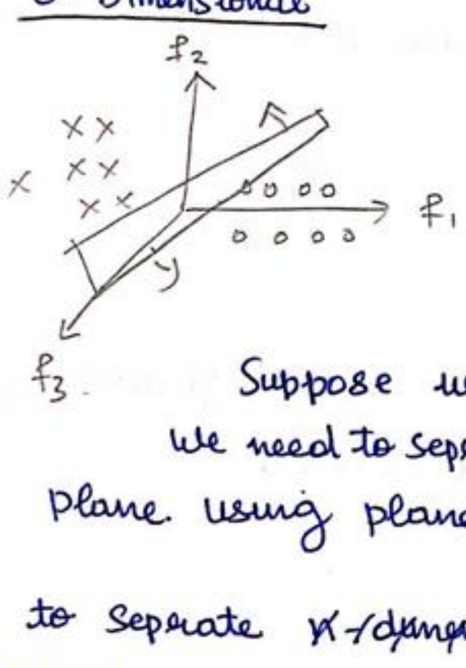
LINEAR ALGEBRA FOR MACHINE LEARNING

1.) 2-Dimensional



Suppose we have 2 features f_1 & f_2
We need to seprate them so we use line to seprate them. (linear seperator).

2.) 3-Dimensional

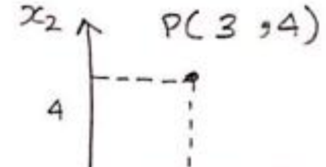


Suppose we have 3 features f_1, f_2, f_3
We need to seprate them by using 3-Dimensional plane using plane.

NOTE:-
So to seprate ~~n-dimen~~ n features we need n Dimensional Hyperplane so we use linear algebra to compute it.

☆ Vector :-

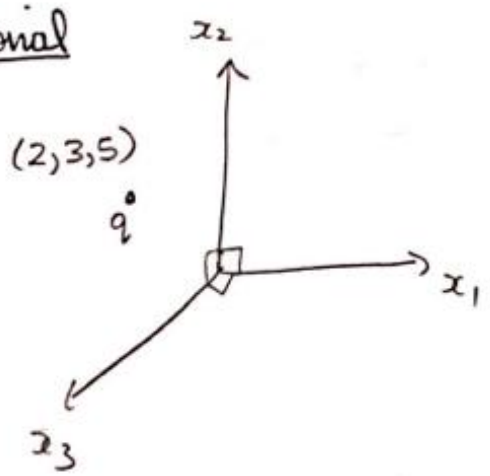
a) 2-Dimensional



? How to store value of point ?
We use vectors to store value of points

$$P = [\underset{x_1}{\overset{\uparrow}{3}}, \underset{x_2}{\overset{\uparrow}{4}}]$$

b.) 3 Dimensional



q = [2, 3, 5]   we store it using vector

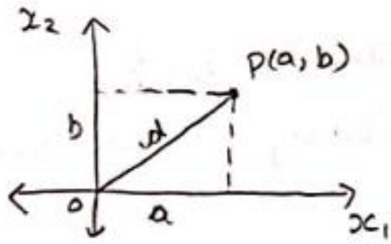
c.) n-dimensional point

$x = [1, 2, 3, 4, 5, 6, \ldots, n]$

we cant visualize n dimensional space but mathematically we can visualize it using vectors.

☆ Distance of a pt. from origin:

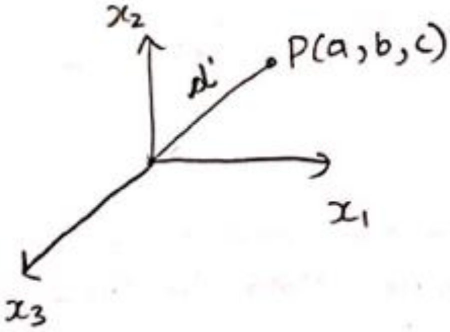
2D



d: distance of P from origin

$d = \sqrt{a^2 + b^2}$

3D



d': distance of P from origin

$d' = \sqrt{a^2 + b^2 + c^2}$

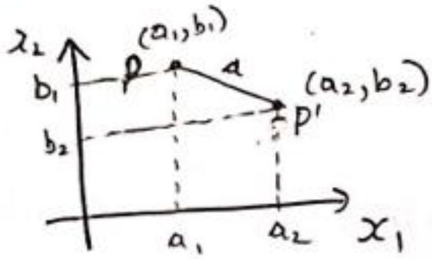
nD :
$$d = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

$$P = [a_1, a_2, a_3, \dots, a_n]$$

$$d = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$$

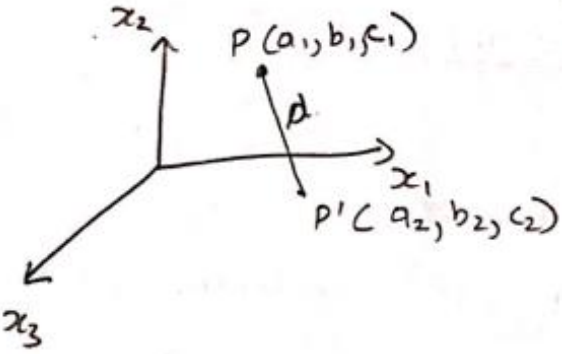
Distance between 2 Points

i) 2D



$$d = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$$

ii) 3D



$$d = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2}$$

iii) nD $P(a_1, a_2, \dots, a_n)$
$P'(b_1, b_2, \dots, b_n)$

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \dots + (a_n - b_n)^2}$$

Row vector $A = [a_1, a_2, a_3, \ldots, a_n]_{1 \times n}$

Column → Row

Column vector $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n \times 1}$ — row

Column

Dot Product

$a = [a_1, a_2, \ldots, a_n]$

$b = [b_1, b_2, \ldots, b_n]$

Addition of a and b ⇒ $C = a + b = [a_1+b_1, a_2+b_2, \ldots, a_n+b_n)$

Concept of multiplication of 2 vectors

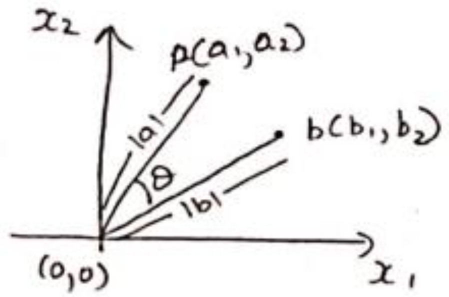
Dot product

$a \cdot b = a_1b_1 + a_2b_2 + \ldots + a_nb_n$

$= [a_1, a_2, a_3, \ldots, a_n] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

$= a_1b_1 + a_2b_2 + \ldots + a_nb_n$

Dot product significance (Geometrically)



$$a \cdot b = \|a\| \, \|b\| \cos\theta = a_1 b_1 + a_2 b_2$$

$\|a\|$: length of a; $\|b\|$: length of b

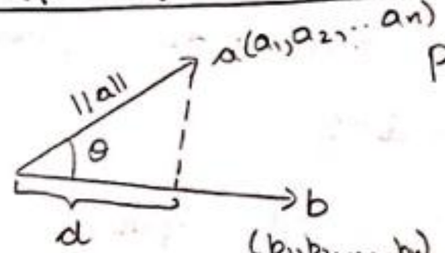
$$\theta = \cos^{-1}\left\{ \frac{a_1 b_1 + a_2 b_2}{\|a\| \, \|b\|} \right\}$$

It is for 2 dimensional vectors

But we can easily compute θ for n-dimensional vectors

$$a \cdot b = \sum_{i=1}^{n} a_i b_i$$

Projection and Unit Vector



Projection of $\vec{a}$ onto $\vec{b}$ is d

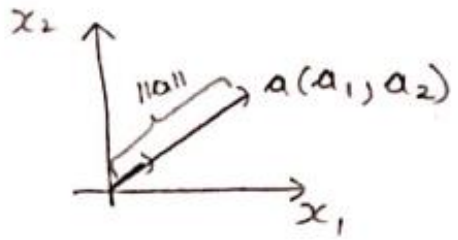
$$d = \|a\| \cos\theta$$

$$a \cdot b = \sum_{i=1}^{n} a_i b_i = \|a\| \, \|b\| \cos\theta$$

$$\boxed{d = \frac{a \cdot b}{\|b\|}} = \frac{\|a\| \, \|b\| \cos\theta}{\|b\|}$$

Unit Vector : vector in same direction as my original direction.

$$\hat{a} = \frac{a}{\|a\|}$$



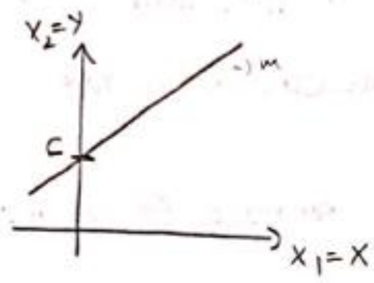
length of unit vector is 1.

Hyperplane

1) Line

2-Dimensional

$y = mx + c$
equation of line



For machine learning

- $w_1 x_1 + w_2 x_2 + w_0 = 0 \longrightarrow$ 2D (line)
- $w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0 \rightarrow$ 3D (plane)

As equation of plane $ax + by + cz + d = 0$

- For n dimensional space $\rightarrow$ hyperplane

$w_1 x_1 + w_2 x_2 + \dots + w_n x_n + w_0 = 0$ (hyperplane)

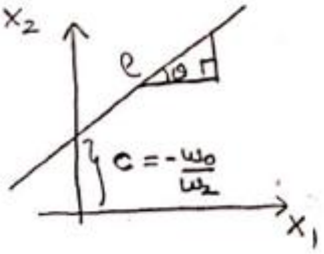
$$\Rightarrow w_0 + \sum_{i=1}^{n} w_i x_i = 0$$

$$\Rightarrow w_0 + [w_1\ w_2\ w_3\ \dots\ w_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = 0 \quad \text{(Vector Notation)}$$

plane denoted by π: $\boxed{w_0 + w^T x = 0}$

Q What is w_0 ?

In 2D →



$y = mx + c$

$c \downarrow$ intercept

$w_1 x_1 + w_2 x_2 + w_0 = 0$

$$x_2 = -\frac{w_0}{w_2} - \frac{w_1}{w_2} x_1$$

$$\underset{y}{\downarrow} = \underset{c}{\downarrow} + \underset{mx}{\downarrow}$$

If $c = 0$ $\left(-\frac{w_0}{w_2} = 0\right)$ that means line is passing through origin.

NOTE: If $w_0 = 0$ that means line / plane / hyperplane is passing through origin.

π: $w^T x = 0$ → eqn. of plane passing through origin

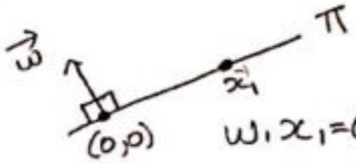
π: $w_0 + w^T x = 0$ → eqn. of plane not passing through origin

◇ Special Case:

$w \cdot x = 0$

if $w \perp x$ ⇒ $\theta = 90°$

As $w \cdot x = \|w\| \|x\| \cos 90° = 0$

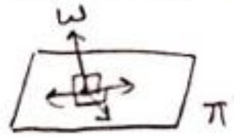


$w_1 \cdot x_1 = 0$ As they are ⊥

If $\omega \perp \pi$

then $\omega \cdot x_i = 0 \quad \forall x_i \in \pi$

Geometrically ω means that it is vector that is perpendicular to plane.



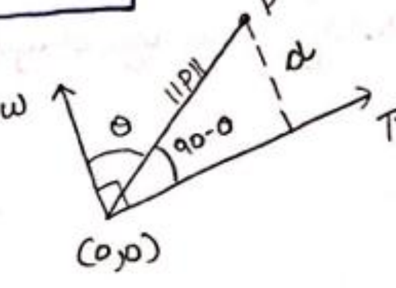
~~Distance of a plane~~.

Distance of point from a plane / Hyperplane

$\omega^T x = 0 \quad \Rightarrow \quad \omega \perp x$

$$\boxed{d = \frac{\omega^T p}{\|\omega\|}}$$

Proof:



As $\Rightarrow \sin(90 - \theta) = \frac{d}{\|P\|}$

$$\cos\theta = \frac{d}{\|P\|}$$

$$d = \|P\| \cos\theta \quad \text{— I}$$

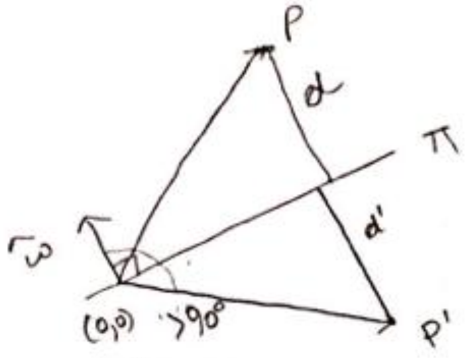
As $\Rightarrow \omega \cdot P = \|\omega\| \, \|P\| \cos\theta$

Using I

$$\omega \cdot P = \|\omega\| \, d$$

$$d = \frac{\omega \cdot P}{\|\omega\|}$$

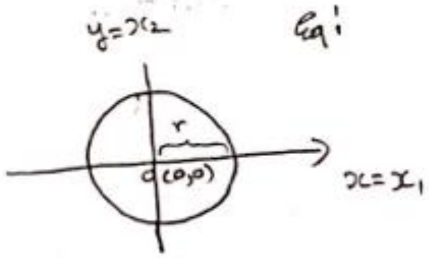
◇ Distance from half space



$d' = \dfrac{w \cdot p'}{\|w\|} = -ve$ As direction of p' is opposite of w

$d = \dfrac{w \cdot p}{\|w\|} = +ve$ As direction of p is same as of w

◇ Circles

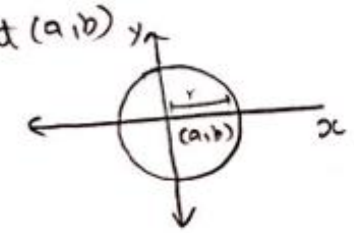


Eq: $x^2+y^2=r^2$ if origin

$x^2+y^2 \leq r^2$ than point lies inside or on circle

$x^2+y^2 > r^2$ outside circle

Center not origin but (a,b)



$(x-a)^2+(y-b)^2 = r^2$ not origin