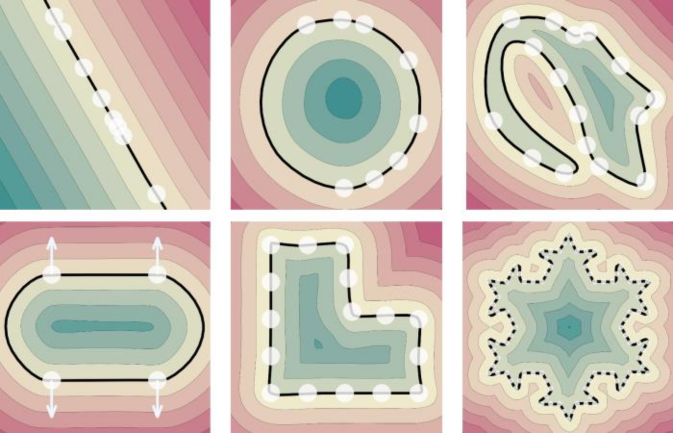
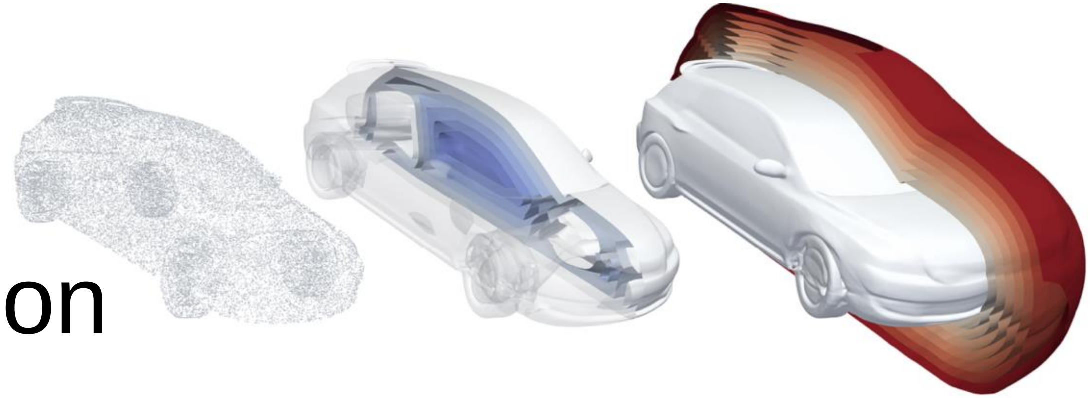




Discussion on

Implicit Geometric Regularization for Learning Shapes

and more

Presented by Yintong Shang

2021.11

Background: Neural Shape Representation

- explicit: a set of charts in an atlas

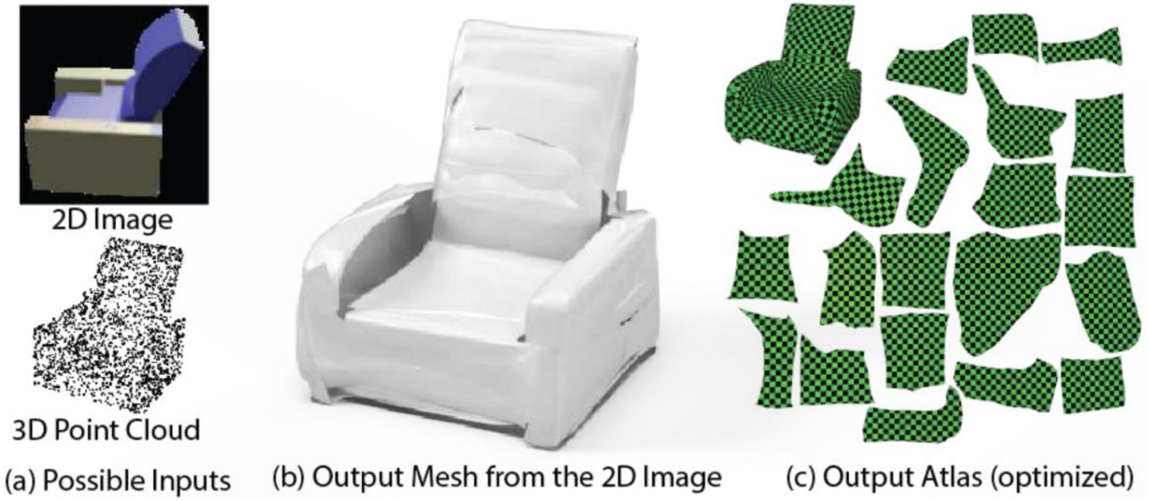
- implicit: level sets

Background: Neural Shape Representation

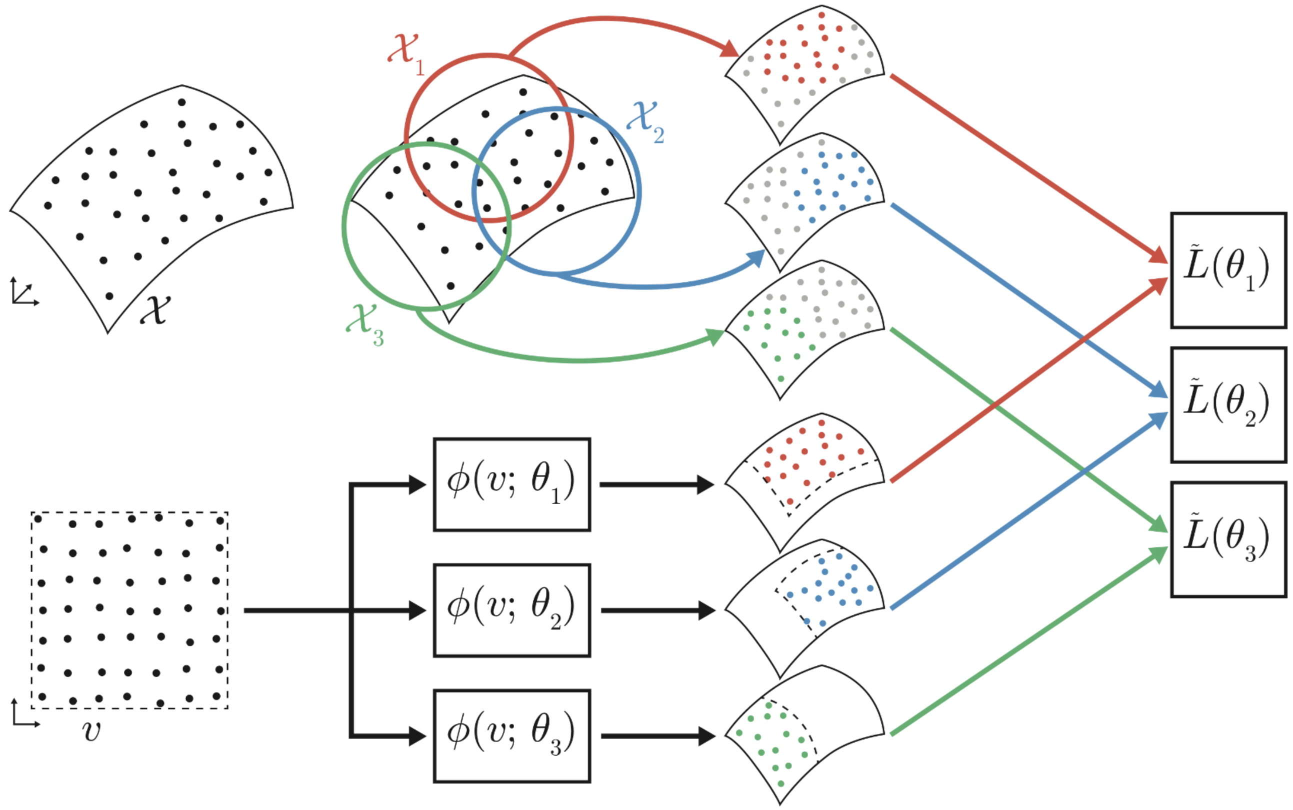
- explicit: a set of charts in an atlas

(chart: a local parametric surface)

Groueix et al. 2018 AtlasNet



Williams et al. 2019 DGP



Background: Neural Shape Representation

- explicit: a set of charts in an atlas

(chart: a local parametric surface)

-> provide a useful global parameterization

-> hard to find a consistent atlas

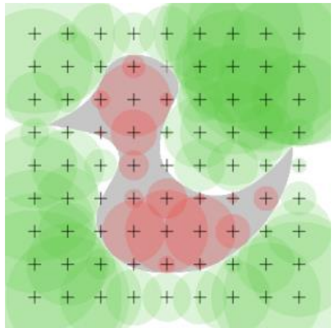
-> hard to produce perfectly overlapping charts

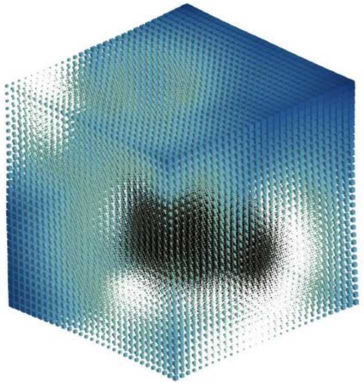
- implicit: level sets
  - RGVF (regular grid volumetric function)

-> most popular

-> memory intensive

-> require interpolation





  - implicit neural representation

-> continuous and smooth

-> differentiable

-> infinite resolution

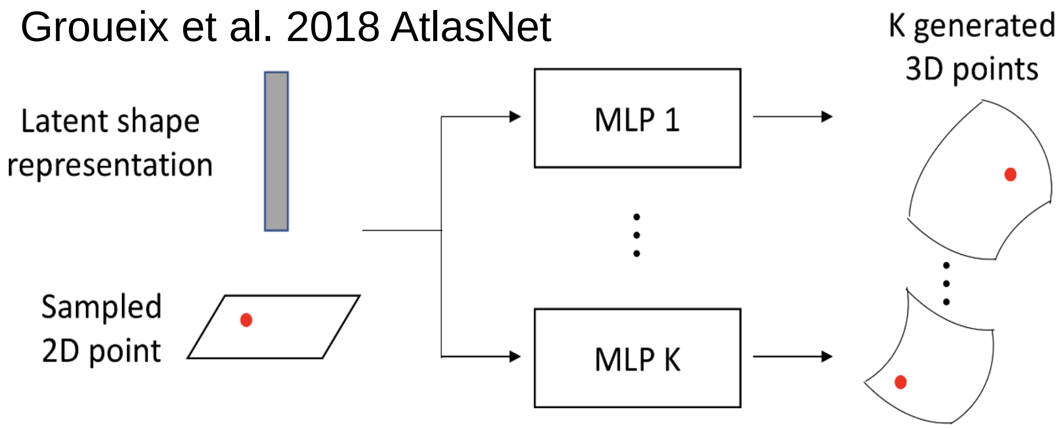
Background: Neural Shape Representation

- explicit: a set of charts in an atlas

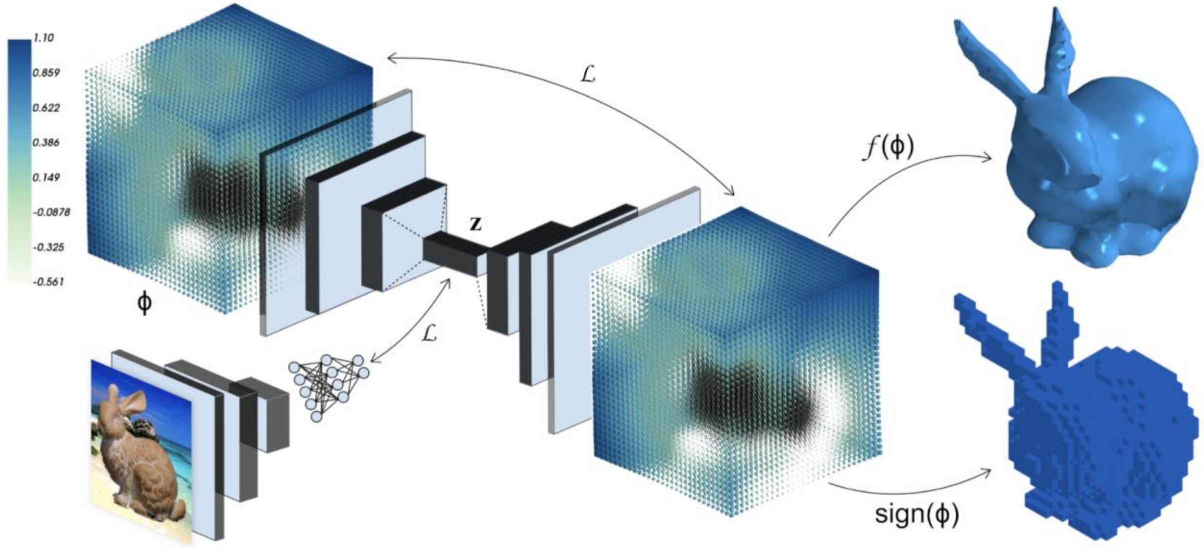
(chart: a local parametric surface)

- implicit: level sets
    - RGVF (regular grid volumetric function)
    - implicit neural representation

Groueix et al. 2018 AtlasNet



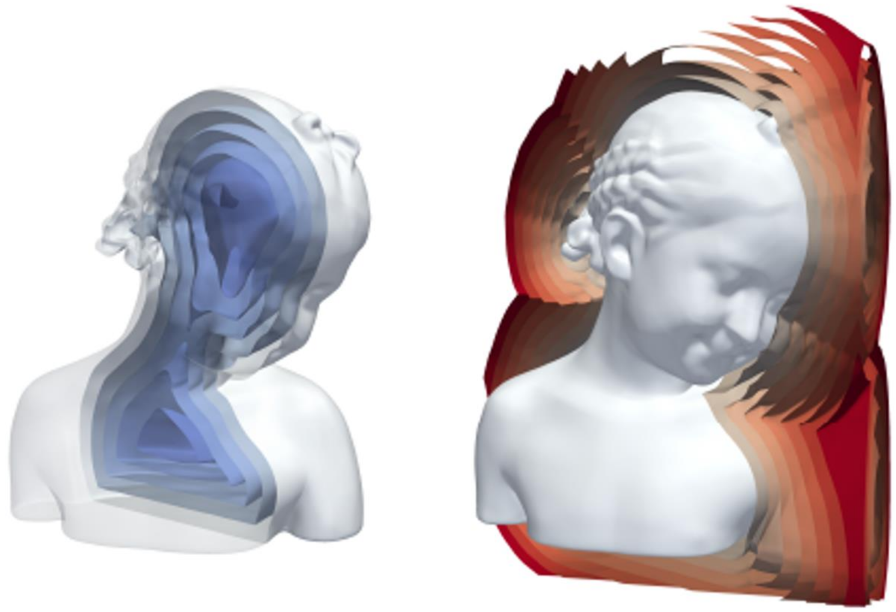
Michalkiewicz et al. 2019 Implicit Surface Representation



Introduction

- Problem 1:

input point cloud $X = \{\boldsymbol{x}_i\}_{i \in I}$ ----(directly)---> signed distance function $f(\boldsymbol{x}; \theta)$



for convenience, cloud X and SDF $f(\boldsymbol{x}; \theta)$

Introduction

- Problem 1: cloud $X = \{\boldsymbol{x}_i\}_{i \in I}$ ----(directly)---> SDF $f(\boldsymbol{x};\theta)$ to shape M

- Previous:

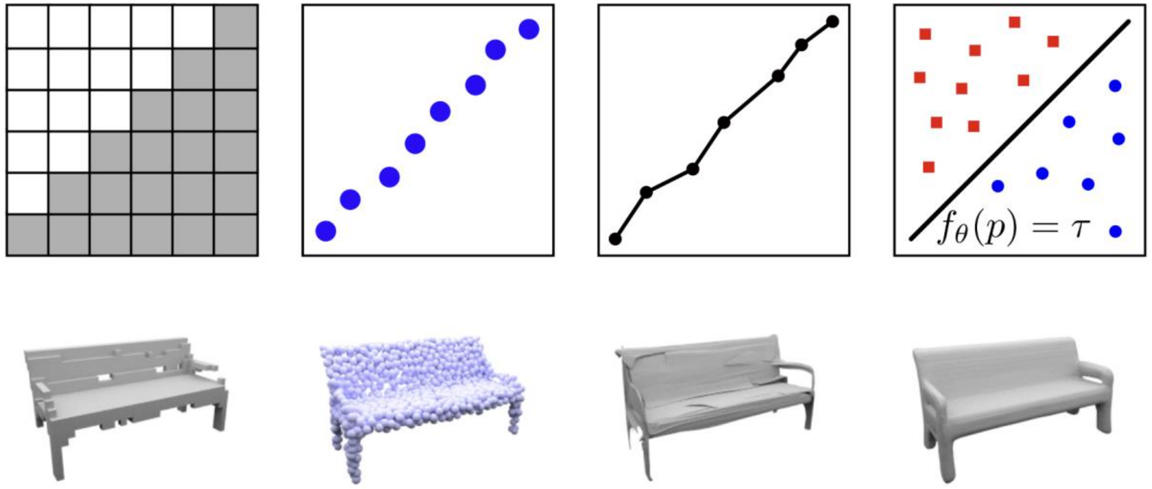
1. not directly, requiring to precompute a shape

Mescheder et al. 2019 Occupancy Networks (680 cites)

Uniformly sample $|B|$ batches, each has K points $\{p_{ij}\}$.
mini-batch loss:

$$\mathcal{L}_{\mathcal{B}}(\theta) = \frac{1}{|\mathcal{B}|} \sum_{i=1}^{|\mathcal{B}|} \sum_{j=1}^{K} \mathcal{L}(f_\theta(p_{ij}, x_i), o_{ij})$$

$o_{ij} \in \{0,1\}$is true occupancy at point p_{ij}
f_θ: $\mathbb{R}^3 \times X \rightarrow [0,1]$ is a binary classification network
x_i is the observation point
L is cross-enthropy loss



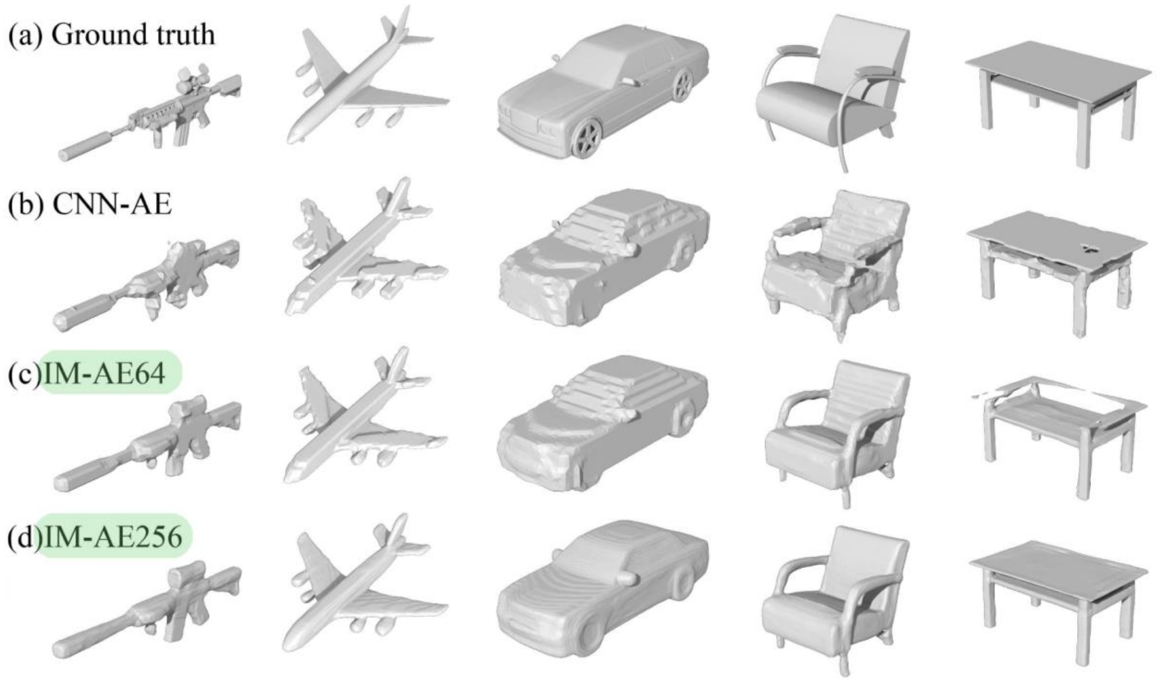
(a) Voxel (b) Point (c) Mesh (d) Ours

Introduction

- Problem 1: cloud $X = \{\boldsymbol{x}_i\}_{i \in I}$ ----(directly)---> SDF $f(\boldsymbol{x}; \theta)$ to shape M

- Previous:

1. not directly, requiring to precompute a shape



Chen & Zhang 2019 IM-NET (402 cites)

Sample on different resolutions($32^3, 64^3, 128^3, 256^3$).
Sample more near the surface (with smaller weight ω_p).

$$\mathcal{L}(\theta) = \frac{\sum_{p \in S} |f_\theta(p) - \mathcal{F}(p)|^2 \cdot w_p}{\sum_{p \in S} w_p}$$

$F(p) \in \{1, 0\}$ is inside/outside field at point p
$f_\theta : p \to [0, 1]$ is a binary classification network

Introduction

- Problem 1: cloud $X = \{\boldsymbol{x}_i\}_{i \in I}$ ----(directly)---> SDF $f(\boldsymbol{x}; \theta)$ to shape M

- Previous:

1. not directly, requiring to precompute a shape
2. directly, but explicitly enforce regularizations

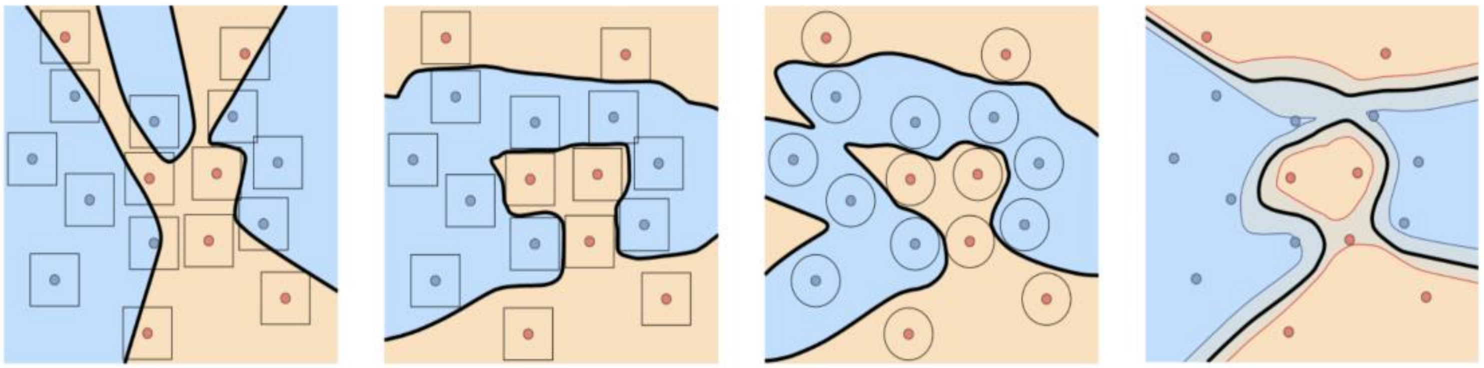
Atzmon et al. 2019

Controlling Neural Level Sets

- Sample the level set.
- Use different loss to regularize

the decision boundary

cross-entrophy | ε from L_∞ | ε from L_2 | SVM



(a) (b) (c) (d)

Introduction

- Problem 1: cloud $X = \{\boldsymbol{x}_i\}_{i \in I}$ ----(directly)---> SDF $f(\boldsymbol{x}; \theta)$ to shape M

- Previous:

1. not directly, requiring to precompute a shape
2. directly, but explicitly enforce regularizations

Atzmon & Lipman 2020 SAL

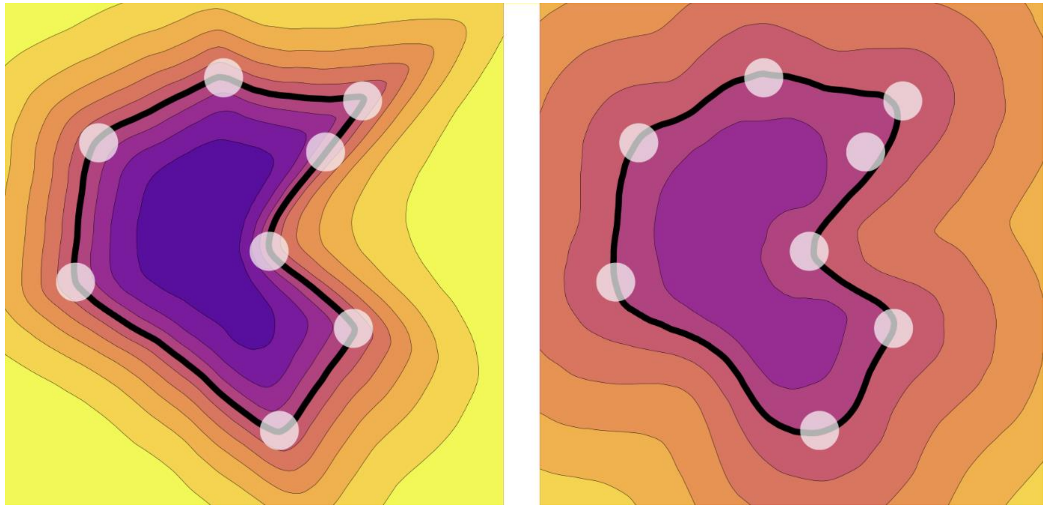
Sign Agnostic Learning

$$\mathrm{loss}(\boldsymbol{\theta}) = \mathbb{E}_{\boldsymbol{x} \sim D_{\mathcal{X}}}\, \tau\Big(f(\boldsymbol{x};\boldsymbol{\theta}), h_{\mathcal{X}}(\boldsymbol{x})\Big)$$

(i) *Sign agnostic:* $\tau(-a, b) = \tau(a, b), \forall a \in \mathbb{R}, b \in \mathbb{R}_+.$

(ii) *Monotonic:* $\frac{\partial \tau}{\partial a}(a, b) = \rho(a - b), \forall a, b \in \mathbb{R}_+,$

An example is $\tau(a, b) = ||a| - b|.$



$$h_0(\boldsymbol{z}) = \begin{cases} 0 & \boldsymbol{z} \in \mathcal{X} \\ 1 & \boldsymbol{z} \notin \mathcal{X} \end{cases} \qquad h_2(\boldsymbol{z}) = \min_{\boldsymbol{x} \in \mathcal{X}} \|\boldsymbol{z} - \boldsymbol{x}\|_2$$

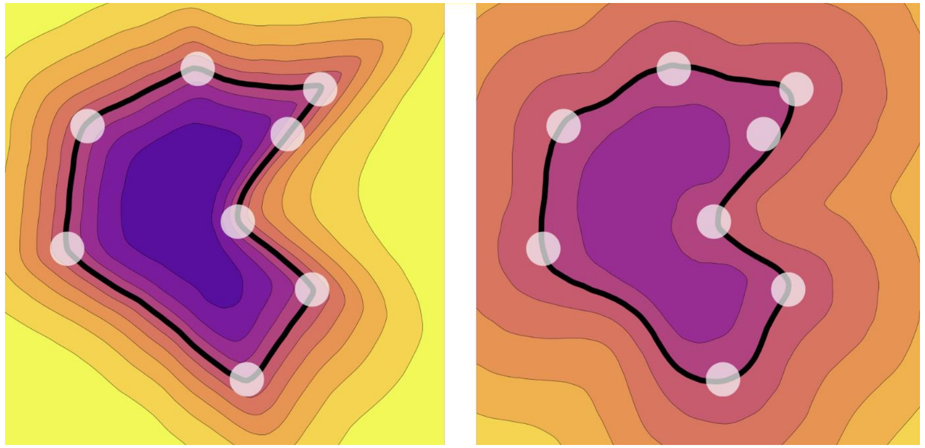
Introduction

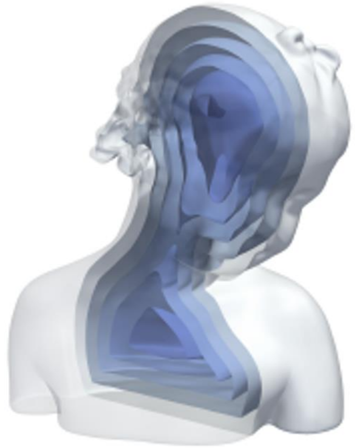
- Problem 2: how to get SDFs? how to get the best SDF?

(why SDFs?) --> SDF is the best among Level Set Functions (LSF) whose regularized distance is desirable.

(Li et al. 2010 Distance Reularized Level Set Evolution)

$\|\nabla_x f\| - 1 = 0$





Introduction

- Problem 2: how to get SDFs? how to get the best SDF?

- Previous:

3. explicit regularization + normalization (as Eikonal implicit regularization)

Michalkiewicz et al. 2019 Implicit Surface Representation

$$L(\theta) = \sum_{j\in\mathcal{D}} E_{\mathcal{X}^j}(\tilde{\Gamma}(I^j;\theta)) + \alpha_1 \sum_{j\in\mathcal{D}} E_{\mathcal{N}^j}(\tilde{\Gamma}(I^j;\theta)) + \alpha_2 \sum_{j\in\mathcal{D}} E_{sdf}(\tilde{\phi}(I^j;\theta)) + \alpha_3 \sum_{j\in\mathcal{D}} E_{area}(\tilde{\Gamma}(I^j;\theta)) + \alpha_4 \sum_{j\in\mathcal{D}} E_{vol}(\tilde{\Gamma}(I^j;\theta)). \quad (12)$$

$$\sum_{j\in\mathcal{D}} E_{sdf}(\tilde{\phi}(I^j;\theta)) = \sum_{j\in\mathcal{D}} \int_{\mathbb{R}^3} (\|\nabla\tilde{\phi}(x,I^j;\theta)\| - 1)^2 dx,$$

$$L_\epsilon(\theta) = \sum_{j\in\mathcal{D}} \left(\sum_{x\in\Omega} \delta_\epsilon(\tilde{\phi}^j(x)) d^j(x)^p \right)^{1/p} + \alpha_1 \sum_{j\in\mathcal{D}} \left(\sum_{x\in\Omega} \delta_\epsilon(\tilde{\phi}^j(x)) \left(1 - \left| N^j(x) \cdot \frac{\nabla\tilde{\phi}^j(x)}{\|\nabla\tilde{\phi}^j(x)\|} \right| \right)^p \right)^{1/p} + \alpha_2 \sum_{j\in\mathcal{D}} \sum_{x\in\Omega} (\|\nabla\tilde{\phi}^j(x)\| - 1)^2 + \alpha_3 \sum_{j\in\mathcal{D}} \sum_{x\in\Omega} \delta_\epsilon(\tilde{\phi}^j(x)) + \alpha_4 \sum_{j\in\mathcal{D}} \sum_{x\in\Omega} H_\epsilon(\tilde{\phi}^j(x)). \quad (20)$$

Introduction

- Problem 2: how to get SDFs? how to get the best SDF?

- Previous:

3. explicit regularization + normalization (as Eikonal implicit regularization)
4. implicit regularization phenomenon in n.n. optimization

Neyshabur et al. 2014 Implicit Regularization

- Gradient Descent prefers less complex solutions, which generalize well.

Introduction

- Problems: how to get SDFs ? -> solve Eikonal equation

  ...directly from point cloud? -> add Eikonal regularization into loss

  how to get the best SDF? -> plane reproduction property

Previous:

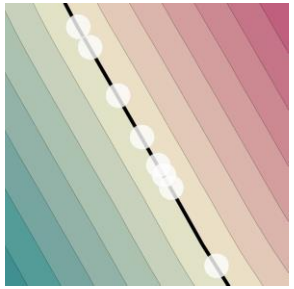
1. not directly: requiring to precompute shape
2. directly, but enforce explicit regularizations
3. explicit regularization + normalization (as Eikonal implicit regularization)
4. implicit regularization phenomenon in n.n. optimization

- Answer: implicit geometric regularization *multilayer perceptron

Method

Given cloud $X = \{\boldsymbol{x}_i\}_{i \in I} \subset \mathbb{R}^3$, with or without normal $N = \{\boldsymbol{n}_i\}_{i \in I} \subset \mathbb{R}^3$,
compute parameters θ of an MLP* $f(\boldsymbol{x}; \theta)$: $\mathbb{R}^3 \times \mathbb{R}^m \to \mathbb{R}$
so that it approximates a SDF to a plausible surface M

Consider a loss $l(\theta) = \lambda E_{\boldsymbol{x}}(\|\nabla_{\boldsymbol{x}} f\| - 1)^2 + \frac{1}{|I|} \sum_{i \in I} (|f(\boldsymbol{x}_i; \theta)| + \tau \|\nabla_{\boldsymbol{x}} f(\boldsymbol{x}_i; \theta) - \boldsymbol{n}_i\|)$



Eikonal regularization term (making f an SDF) boundary term boundary normal term

Optimizing the loss is solving Eikonal equation with insufficient boundary condition.

$$\begin{cases} \|\nabla_{\boldsymbol{x}} f\| - 1 = 0 \\ |f(x_i; \theta)| = 0 \\ \|\nabla_{x} f(x_i; \theta) - n_i\| = 0 \end{cases}$$

Method

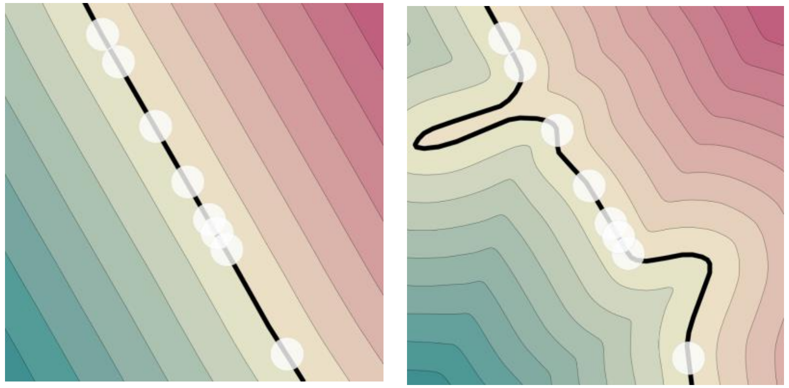
$$l(\theta) = \lambda E_{\boldsymbol{x}}(\|\nabla_{\boldsymbol{x}} f\| - 1)^2 + \frac{1}{|I|}\sum_{i \in I}(|f(\boldsymbol{x}_i;\theta)| + \tau\|\nabla_{\boldsymbol{x}} f(\boldsymbol{x}_i;\theta) - \boldsymbol{n}_i\|)$$

$$\begin{cases} \|\nabla_{\boldsymbol{x}} f\| - 1 = 0 \\ |f(x_i;\theta)| = 0 \\ \|\nabla_x f(x_i;\theta) - n_i\| = 0 \end{cases}$$

for simplicity, let $\tau = 0$ (omit boundary normal)

Method

two of assumably possible solutions →



$$l(\theta) = \lambda E_{\boldsymbol{x}}(\|\nabla_{\boldsymbol{x}} f\| - 1)^2 + \frac{1}{|I|}\sum_{i\in I}(|f(\boldsymbol{x}_i;\theta)|)$$

$$\begin{cases} \|\nabla_{\boldsymbol{x}} f\| - 1 = 0 \\ |f(x_i;\theta)| = 0 \end{cases}$$

Analysis of linear case:

$f(\boldsymbol{x};\boldsymbol{\omega}) = \boldsymbol{\omega}^T\boldsymbol{x}$

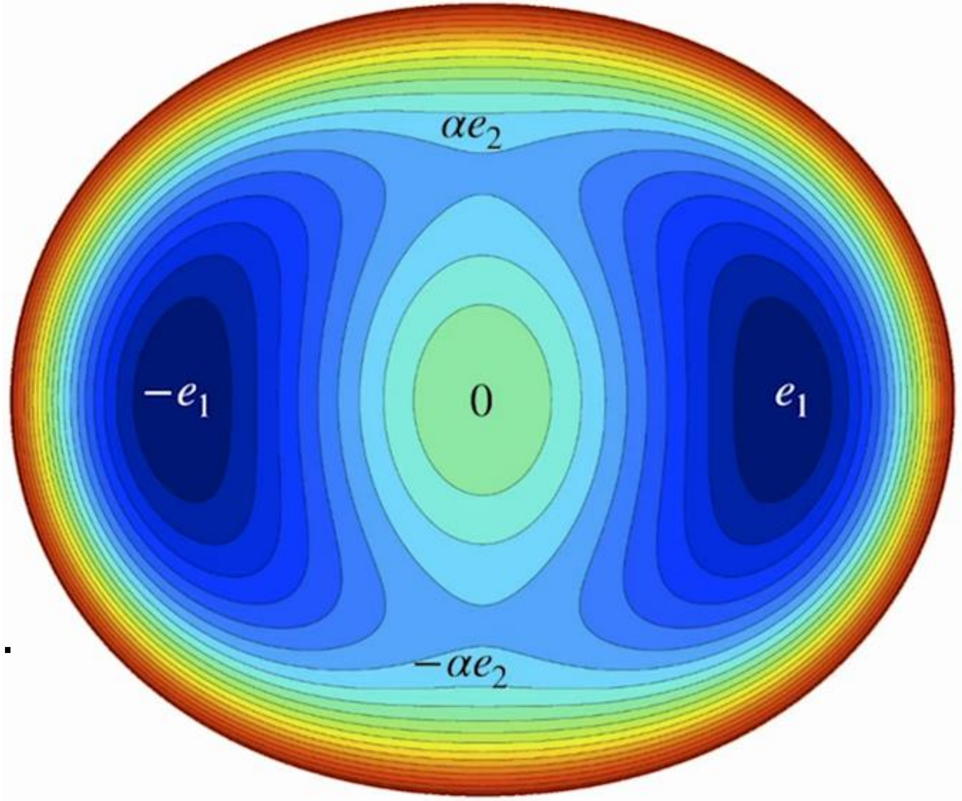
$$l(\boldsymbol{\omega}) = \lambda(\|\boldsymbol{\omega}\|^2 - 1)^2 + \sum_{i\in I}(\boldsymbol{\omega}^T\boldsymbol{x}_i)^2$$

$$\boldsymbol{x}_i\boldsymbol{x}_i^T = \boldsymbol{U}D\boldsymbol{U}^T$$

$$\boldsymbol{q} = \boldsymbol{U}^T\boldsymbol{\omega}$$

$$= \lambda(\|\boldsymbol{q}\|^2 - 1)^2 + \sum_{i\in I}\boldsymbol{q}^T D\,\boldsymbol{q}$$

......(see section 4)



Theorems:

1. At least 3 critical points: maxima $\boldsymbol{\omega} = \boldsymbol{0}$ and two SDF solutions.
2. Other critical points are maxima or strict saddle points, which GD will evade (proven by Lee et al. 2016).

Method

$$l(\theta) = \lambda E_{x}(\|\nabla_{x} f\| - 1)^2 + \frac{1}{|I|}\sum_{i\in I}(|f(\boldsymbol{x}_i;\theta)|)$$

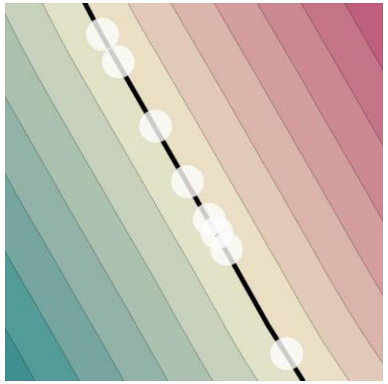
$$\begin{cases} \|\nabla_{x} f\| - 1 = 0 \\ |f(x_i;\theta)| = 0 \end{cases}$$

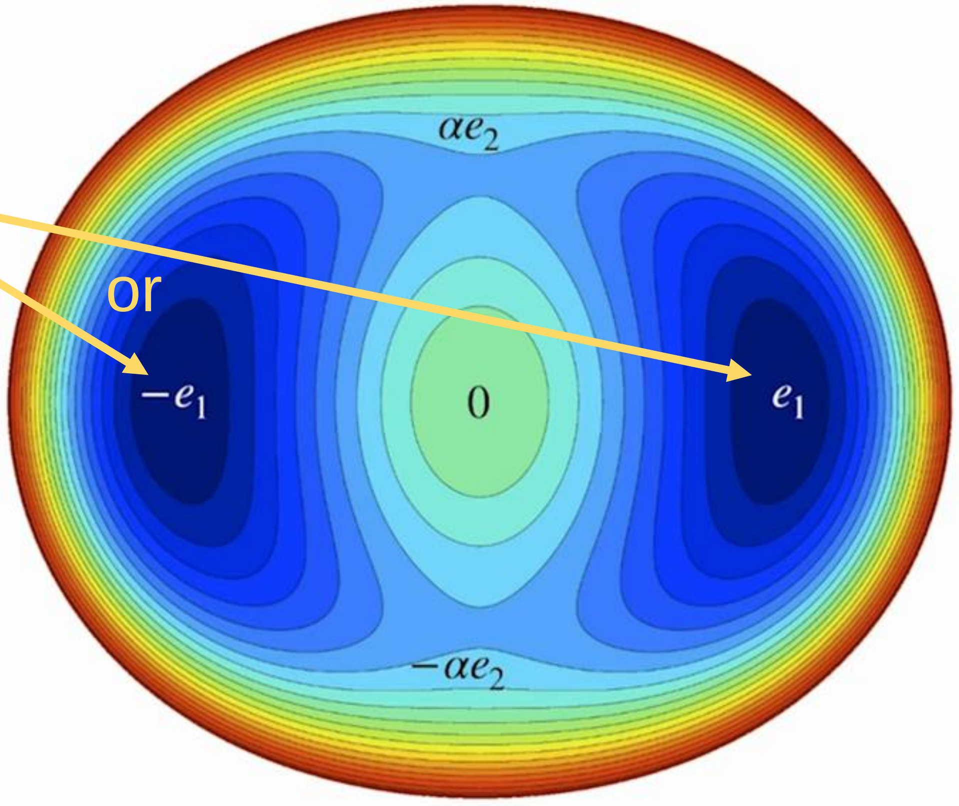
conclusion:

In linear case, this loss can produce the best SDF.

And it is called the plane reproduction property.

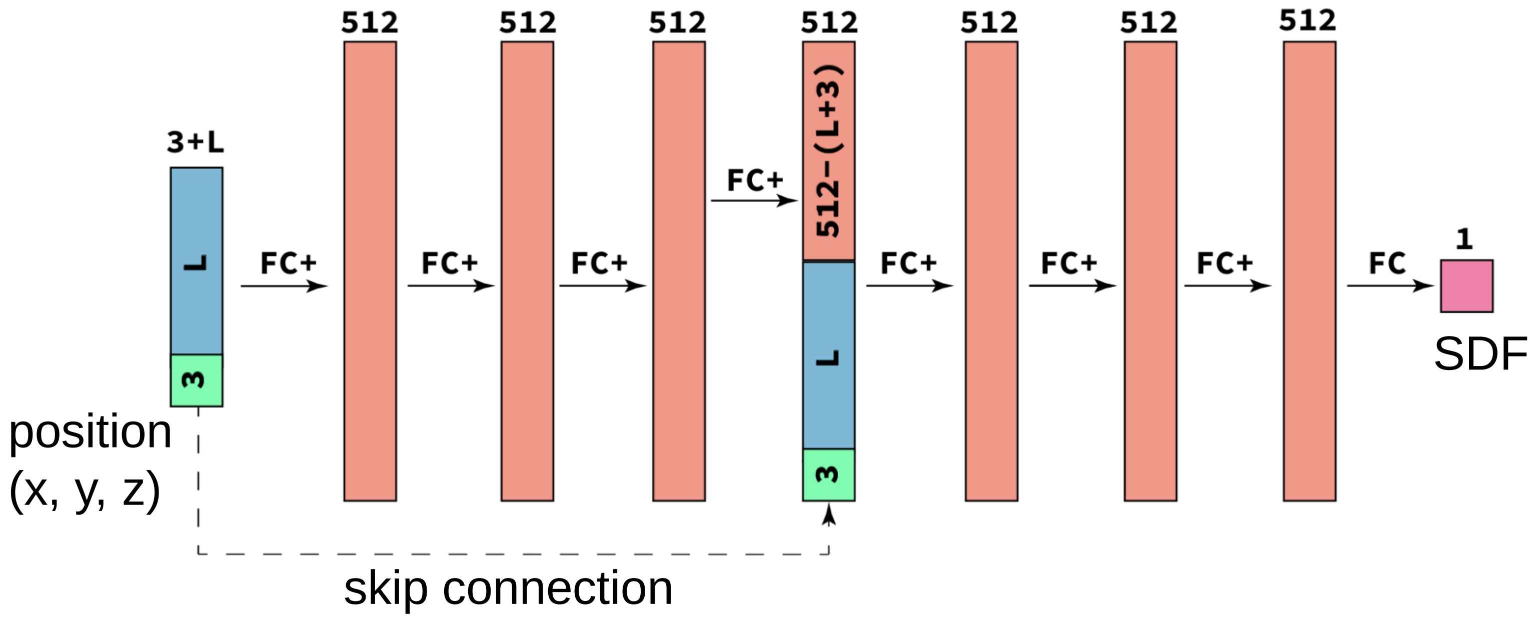
explanation: (by Atzmon et al. 2020 SAL)





Method

- network: auto-decoder (Park et al. 2019 DeepSDF)



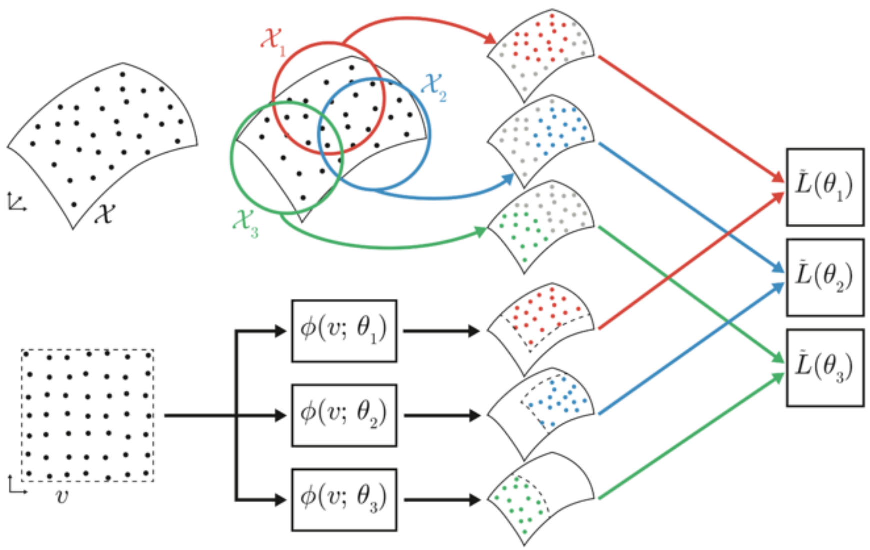
- activation: softplus
- optimizer: Adam
- $L = 0$ for shape reconstruction, $L = 256$ for shape space learnig

Result

surface reconstruction from point cloud

- better than DGP

Williams et al. 2019 DGP



- explicit representation
- another treatment of IGR
- no training data
- no explicit regularization

space learning

- better than SAL

2. directly, but explicitly enforce regularizations

Atzmon & Lipman 2020 SAL

Sign Agnostic Learning

$$\text{loss}(\boldsymbol{\theta}) = \mathbb{E}_{\boldsymbol{x}\sim D_{\mathcal{X}}}\, \tau\Big(f(\boldsymbol{x};\boldsymbol{\theta}), h_{\mathcal{X}}(\boldsymbol{x})\Big)$$

(i) *Sign agnostic:* $\tau(-a,b) = \tau(a,b), \forall a \in \mathbb{R}, b \in \mathbb{R}_+.$

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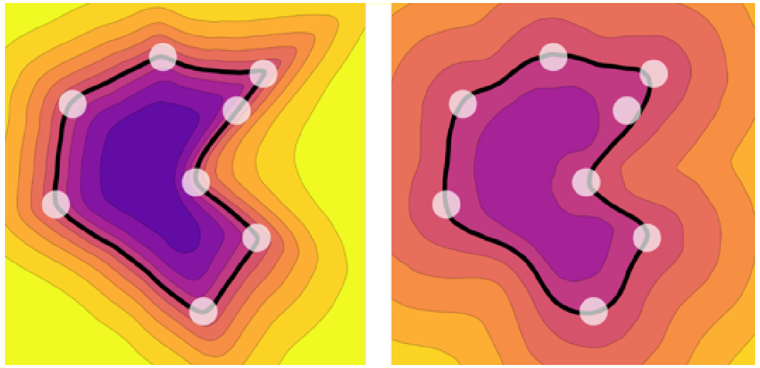
An example is $\tau(a,b) = ||a| - b|.$



$$h_0(\boldsymbol{z}) = \begin{cases} 0 & \boldsymbol{z} \in \mathcal{X} \\ 1 & \boldsymbol{z} \notin \mathcal{X} \end{cases} \qquad h_2(\boldsymbol{z}) = \min_{\boldsymbol{x}\in\mathcal{X}} \|\boldsymbol{z} - \boldsymbol{x}\|_2$$

Contribution

- find out the most simple loss funtion
- $l(\theta) = \lambda E_{\boldsymbol{x}}(\|\nabla_{\boldsymbol{x}} f\| - 1)^2 + \frac{1}{|I|}\sum_{i\in I}(|f(\boldsymbol{x}_i;\theta)| + \tau\|\nabla_{\boldsymbol{x}} f(\boldsymbol{x}_i;\theta) - \boldsymbol{n}_i\|)$

- directly get SDF from raw data without explicit regularization?
- yes

- prove convergence to the best SDF?
- yes, but limited to the linear case (future work: nonlinear analysis)

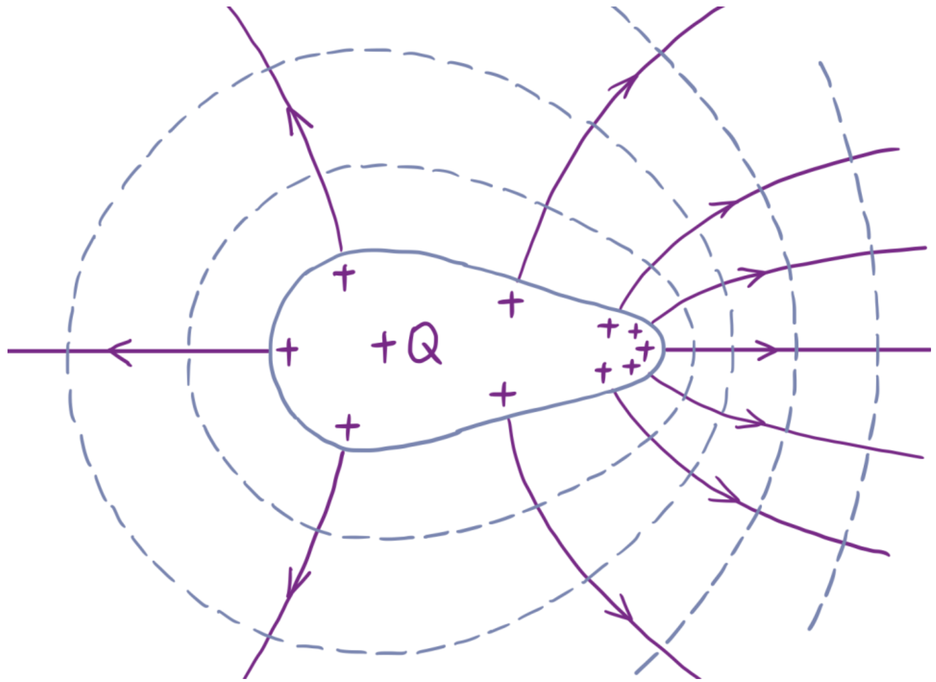
- state-of-the-art

Comment: thoughts on Eikonal equation

- Eikonal equation $\|\nabla_{\boldsymbol{x}} f\| - 1/\mathrm{g} = 0$, special case: $\|\nabla_{\boldsymbol{x}} f\| - 1 = 0$
- Poisson equation $\nabla_{\boldsymbol{x}}^2 f - 1/\mathrm{g} = 0$, Laplace equation $\nabla_{\boldsymbol{x}}^2 f - 1 = 0$

f is electric potential,

$E = -\nabla f$ is electric field (constrained by normals $\boldsymbol{n}$ on surface)



$$l(\theta) = \lambda E_{\boldsymbol{x}}(\|\nabla_{\boldsymbol{x}} f\| - 1)^2 + \frac{1}{|I|}\sum_{i\in I}|f(\boldsymbol{x}_i;\theta)|$$

A more natural contour map:

- dense <u>near</u> surface, and sparse <u>far</u> away.
- dense when <u>sharp</u>, and sparse when <u>flat</u>.

...possibly better details?

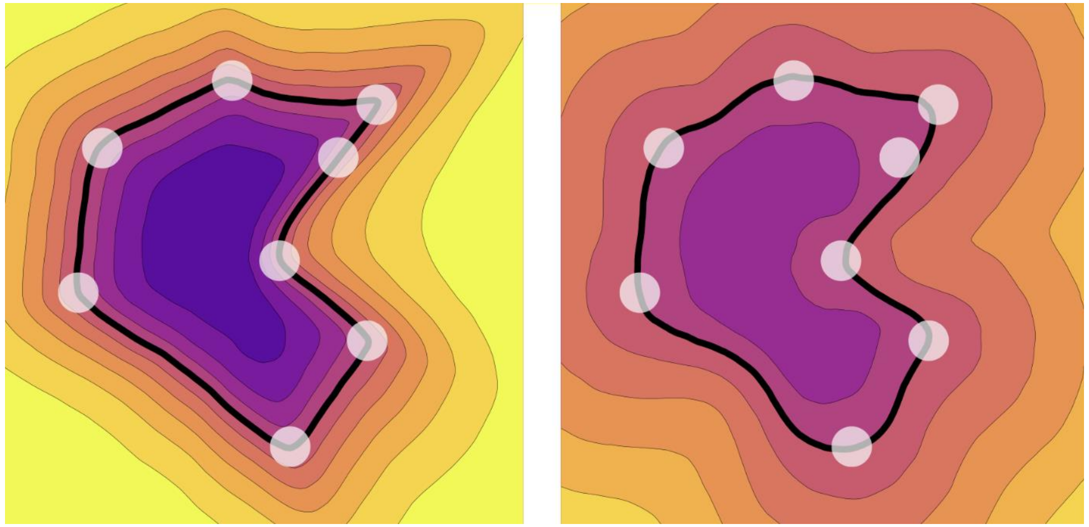
Comment: more ‘SDFs’

- SDF: signed version of L^2 norm.
- L^2 norm (Euclidean distance, also the solution to Eikonal)
- L^0 norm (in/out indicator)
- L^1 norm (Manhattan Norm)
- L^3 norm (?)

- definition: $\|x\|_p = (\sum_{i=1}^{n} |x_i|^p)^{\frac{1}{p}}$

$p=\infty$ $p=2$ $p=1$ $0<p<1$ $p=0$

Atzmon & Lipman 2020 SAL

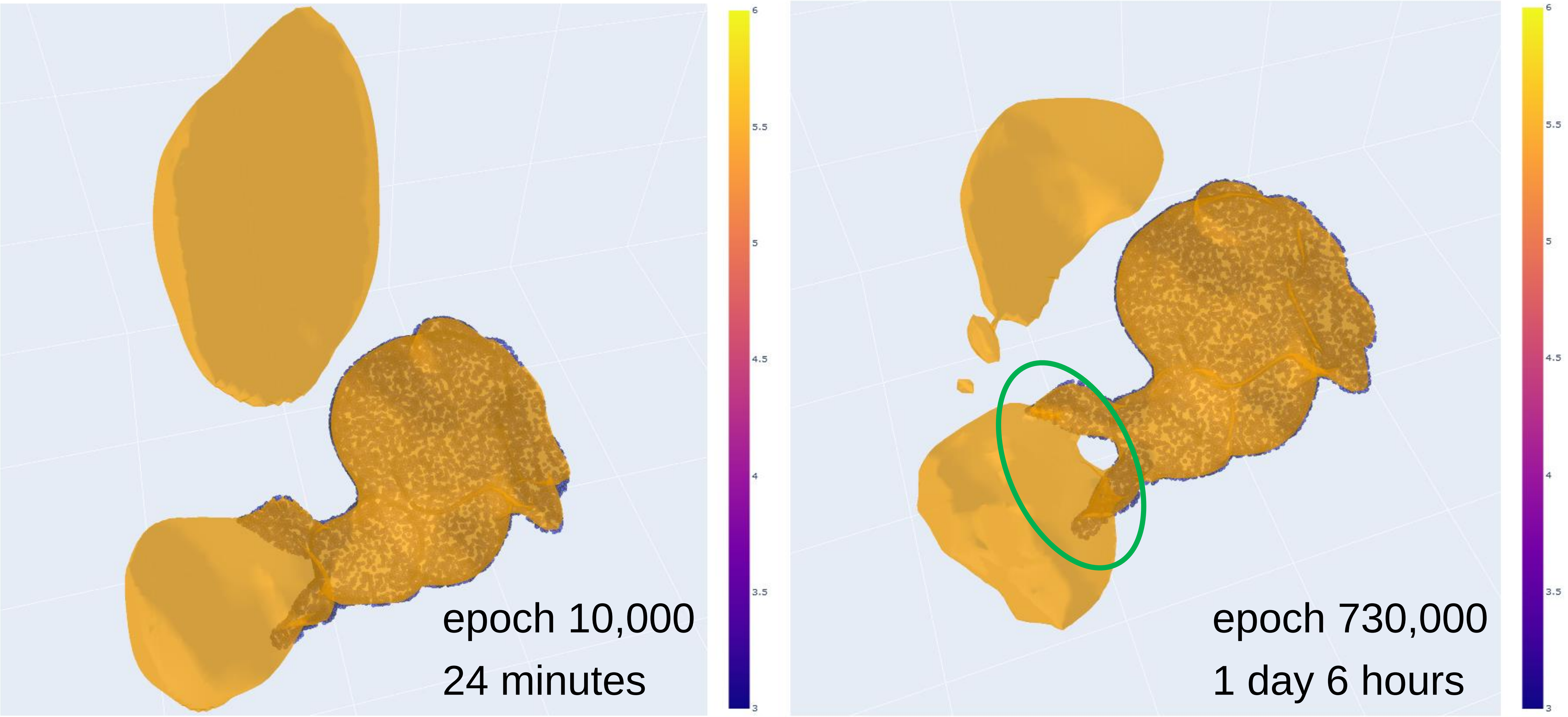


$$h_0(\boldsymbol{z}) = \begin{cases} 0 & \boldsymbol{z} \in \mathcal{X} \\ 1 & \boldsymbol{z} \notin \mathcal{X} \end{cases}$$

$$h_2(\boldsymbol{z}) = \min_{\boldsymbol{x} \in \mathcal{X}} \|\boldsymbol{z} - \boldsymbol{x}\|_2$$

Comment:

- slow change
- difficulties at sharp surfaces



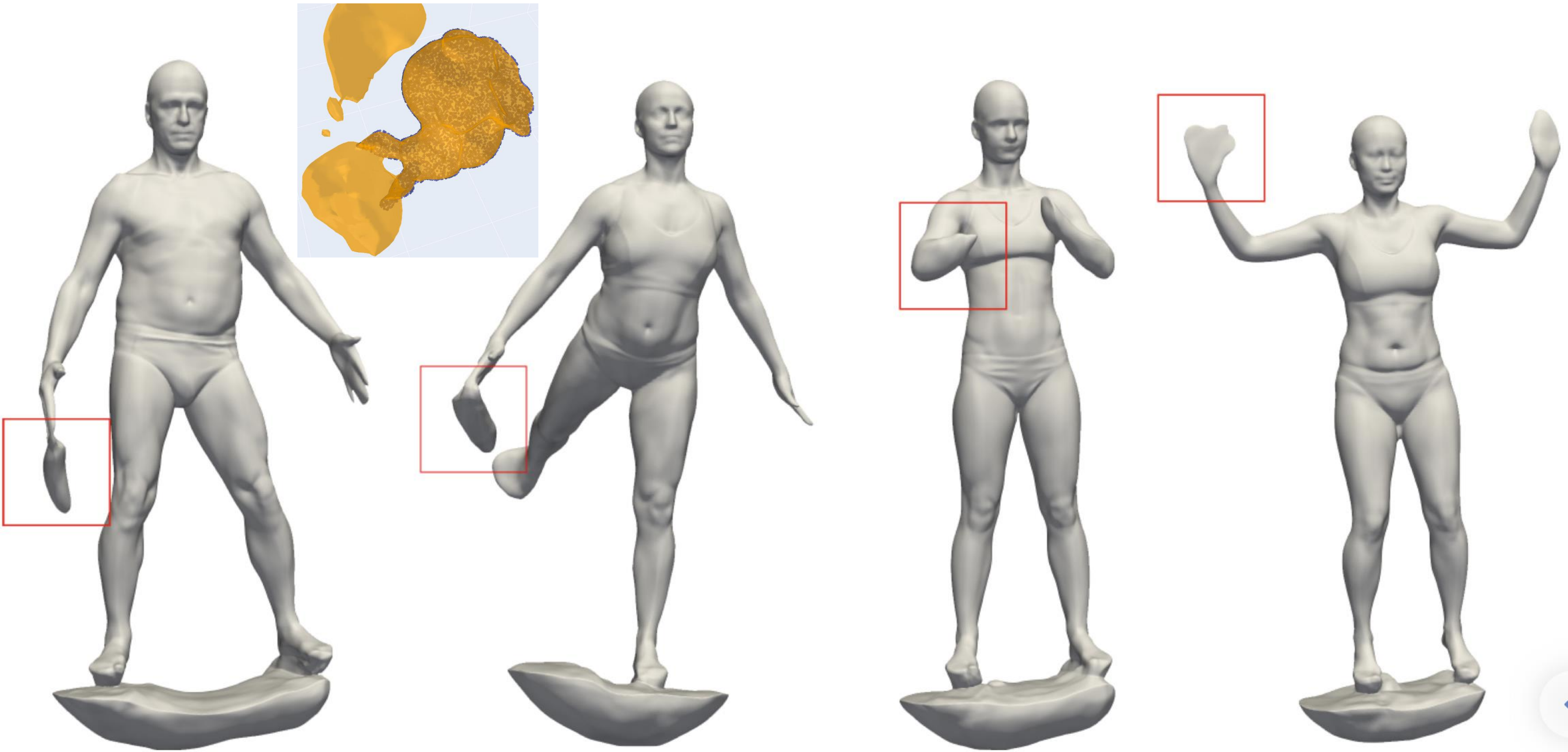


Figure 9. Failures of our method on D-Faust.

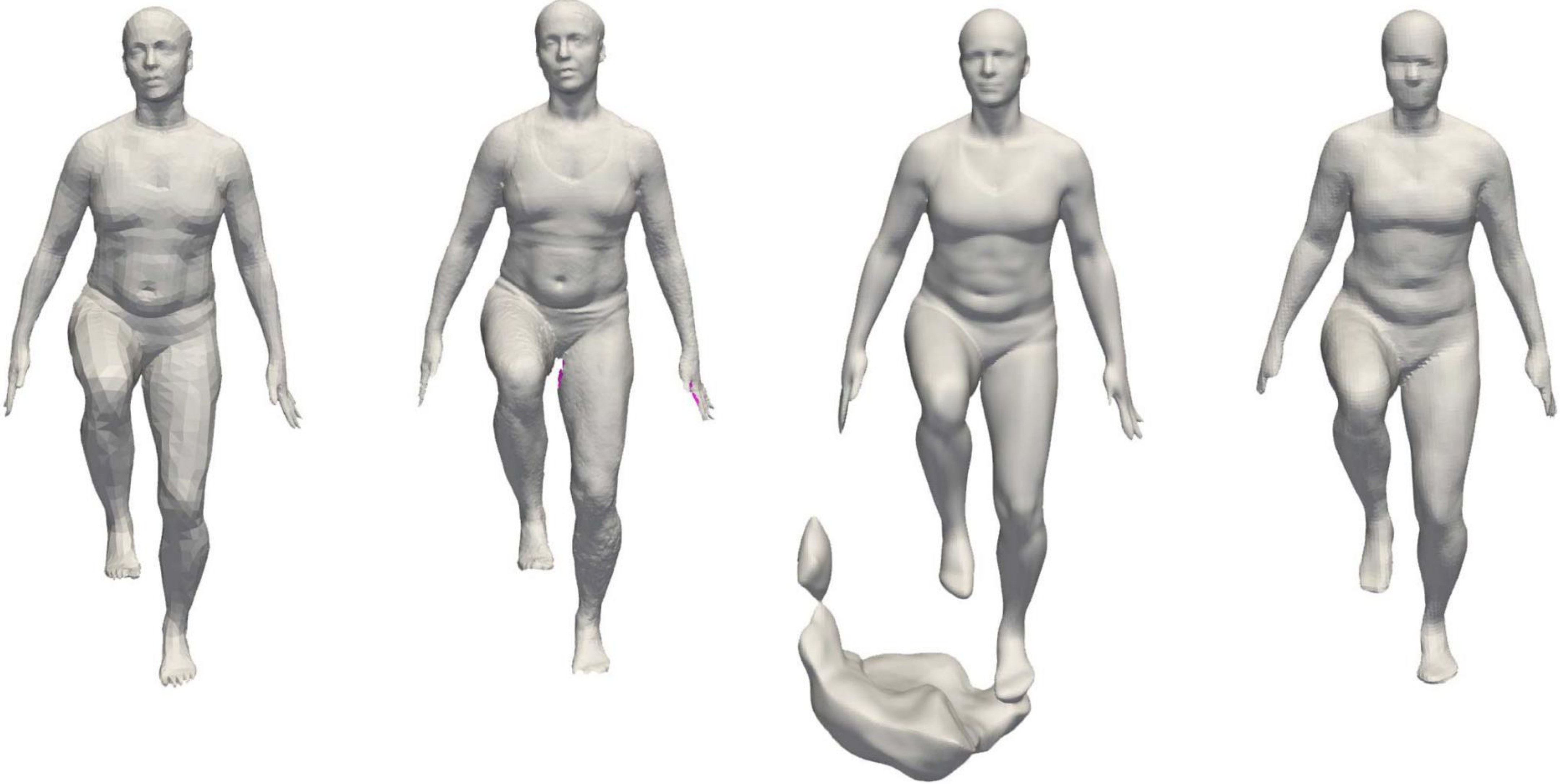


Figure 11. A test result on D-Faust with unseen humans split. Left to right: registrations, scans, our results, SAL.

(from the talk of IGR's auther)

Future Directions

- High frequency details
  - NeRF [Mildenhall etal. 2020]
  - Fourier features [Tancik et al. 2020] https://arxiv.org/abs/2006.10739

generalization: $F\left(\mathbf{x}, \Phi, \nabla_{\mathbf{x}} \Phi, \nabla_{\mathbf{x}}^2 \Phi, \ldots\right)=0, \quad \Phi: \mathbf{x} \mapsto \Phi(\mathbf{x}).$

(SIREN)

  - Sinus activation functions [Sitzmann et al. 2020] https://arxiv.org/abs/2006.09661

- Robustness to noisy inputs

- Incorporate IGR in other 3D geometry learning methods
  - Implicit Differentiable Rendering [Yariv et al. 2020] https://arxiv.org/abs/2003.09852
  - 3D Generative Models

Comment: Future Directions

- SDF actually provides more than surface (non-zero level sets?)
- accelerating rendering using SDF?

- solution to $||\nabla_{\boldsymbol{x}} f|| - 1 = 0$: regular, but lack physical meanings?

- Poisson equation $\nabla_{\boldsymbol{x}}^2 f - 1/\mathrm{g} = 0$? L^p norm?
- volume rendering?

xyz -> SDF

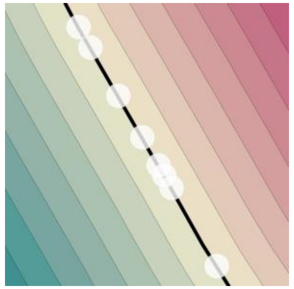
xyz,θφ -> σ,RGB (NeRF-Mildenhall-2020)

SDF <- RGB (NeuS-Wang-2021)

Reminder

Given cloud $X = \{\boldsymbol{x}_i\}_{i \in I} \subset \mathbb{R}^3$, with or without normal $N = \{\boldsymbol{n}_i\}_{i \in I} \subset \mathbb{R}^3$,
compute parameters θ of an MLP* $f(\boldsymbol{x}; \theta)$: $\mathbb{R}^3 \times \mathbb{R}^m \to \mathbb{R}$
so that it approximates a SDF to a plausible surface M

Consider a loss $l(\theta) = \lambda E_{\boldsymbol{x}}(\|\nabla_{\boldsymbol{x}} f\| - 1)^2 + \frac{1}{|I|} \sum_{i \in I} (|f(\boldsymbol{x}_i; \theta)| + \tau \|\nabla_{\boldsymbol{x}} f(\boldsymbol{x}_i; \theta) - \boldsymbol{n}_i\|)$



Eikonal regularization term (making f an SDF) — boundary term — boundary normal term

Optimizing the loss is solving Eikonal equation
with insufficient boundary condition
+ implicit geometric regularization

$$\begin{cases} \|\nabla_{\boldsymbol{x}} f\| - 1 = 0 \\ |f(x_i; \theta)| = 0 \\ \|\nabla_x f(x_i; \theta) - n_i\| = 0 \end{cases}$$