Weak Lensing for Precision Cosmology

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1 Introduction

The most revolutionary discovery in cosmology since Hubble observed that the Universe is expanding is that this expansion is accelerating. A revelation that was awarded the 2011 Nobel Prize for its profound implications. [1]. An accelerating universe implies that either our understanding of gravity is flawed or that a mysterious negative pressure known as Dark Energy is driving the expansion [11]. This Dark Energy accounts for most (over 68%) of the energy density in the observable universe, however its origin and physics are presently unknown [2]. As a result, the nature of Dark Energy is considered one of the greatest mysteries of modern science [3].

One of the most powerful ways to probe Dark Energy as well as modified theories of gravity is a technique known as weak gravitational lensing, or weak lensing for short [5, 8]. Gravitational lensing is the phenomenon of light ray deflection by intervening mass. When the deflection is sufficiently weak this phenomenon manifests in images of stars and galaxies as a shearing effect due to the differential deflection of neighboring light rays [14, 5]. This shearing induces a subtle (sub 1%) change in the ellipticity of the images which is insignificant in comparison to the 30% dispersion in intrinsic galaxy ellipticities [14].

In this report I present a general overview of weak lensing in the context of cosmology. I begin by presenting the theory behind the Cosmology we would like to probe and the general theoretical framework on which weak lensing operates. I then present the process oh how weak lensing measurements are made and how those measurements can be used to extract cosmological data. Finally, I end this report by presenting real weak lensing cosmology results from an active experiment and a short discussion on the general systematics in the method.

2 Background Theory

2.1 Standard Model of Cosmology

1. Motivation (modified gravity vs Dark energy) is DE constant in space ? and time ?

2. FRW Standard Cosmology

3. Distance Measures In Cosmo
4. Gravitational Evolution of Dark matter
5. Observables and Parameters

The fundamental assumption in cosmology, known as the cosmological principle, is that we live in a homogenous and isotropic universe [14]. Remeber to show the Standard Model parameters! can be found in wikipedia.

Angular diameter distance is the one of relvance in lensing (distance measures in cosmology) Distances in cosmo [6] lol

For current and future surveys, one goal is to use the redshifts of the background galaxies (often approximated using photometric redshifts) to divide the survey into multiple redshift bins. The low-redshift bins will only be lensed by structures very near to us, while the high-redshift bins will be lensed by structures over a wide range of redshift. This technique, dubbed "cosmic tomography", makes it possible to map out the 3D distribution of mass. Because the third dimension involves not only distance but cosmic time, tomographic weak lensing is sensitive not only to the matter power spectrum today, but also to its evolution over the history of the universe, and the expansion history of the universe during that time. This is a much more valuable cosmological probe, and many proposed experiments to measure the properties of dark energy and dark matter have focused on weak lensing, such as the Dark Energy Survey, Pan-STARRS, and Large Synoptic Survey Telescope.

2.1.1 Matter Power Spectrum

[13]

2.2 Bending of Light

The fundamental concept on which weak lensing is built is gravity's ability to alter the path of a photon. In this section we review the theory behind the bending of light necessary to develop the weak lensing formalism.

2.2.1 Newtonian Lens

It is a common misconception that the gravitational bending of light is an exclusive property of GR. However, gravity induced alterations to a photon's path are predicted by newtonian mechanics [10]. To illustrate this consider a mass M located at the origin of the cartesian plane and a corpuscle(newtonian photon) propagating along the $x = b$ line (in this context b is known as the impact parameter). Newton's second law predicts that the presence of the point mass will result in a momentum transfer between the two objects. If the corpuscle starts with momentum $(p, 0)$ then it will end up with momentum (p_x, p_y). Therefore, the particle path is deflected by some angle $\Delta\theta$. The deflection angle is simply given by

$$\sin(\Delta\theta) = \frac{p_y}{\sqrt{p_x^2 + p_y^2}} \tag{1}$$

For very small deflections we have $p \approx p_x >> p_y$ and $\Delta\theta << 1$. Therefore Equation 1 simplifies to $\Delta\theta \approx \frac{p_y}{p_x}$. We now consider the infinitesimal deflection along the entire path of the photon with $d\Delta\theta = \frac{dp_y}{p_x} = \frac{1}{px} dx \frac{dp_y}{dx}$. Therefore, we can find the deflection angle by

$$\begin{aligned}\Delta\theta_N &= -\frac{1}{p_x}\int dx \frac{dp_y}{dx} \\ &= -\frac{1}{cp_x}\int dx \frac{dp_y}{dt} = -\frac{1}{cp_x}\int F_y dx \\ &= \frac{2GM}{c^2 b}\end{aligned} \tag{2}$$

We note that the mass of the corpuscle cancels out of the deflection equation. Therefore this equation applies for massless particles i.e. photons. Therefore Equation 2 provides a newtonian description for the bending of light [10].

2.2.2 General Relativistic Bending of Light

In this subsubsection I give a quick sketch of the bending of light in the context of general relativity, for a more detailed calculation please consult [12].

The Einstein's field equations in the presence of a charge free static point mass is uniquely solved by the Schwarzchild metric [12]. The Schwarzchild metric is

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{3}$$

Where r_s is the Shcwarzchild radius of the system given by $r_s = 2\mu = 2GM/c^2$. We can analyze the path of the photon from subsubsection 2.2.1 by studying the geodesic equations of the metric and finding the conserved quantities of the system. We can then combine the conservation equations with the tangent vector norm condition for a null path to get the shape equation of the system as

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2\mu}{r}\right)\right)^{-1/2} \tag{4}$$

where (r, ϕ) are the photons position in 2D polar coordinates and b is the impact parameter. Rewriting this equation under the transformation of $r = 1/u$ and working pertrubatively around $u(\mu = 0) = \frac{1}{b}\sin\phi$ we get

$$u(\phi) \approx \frac{1}{b}\sin\phi + \frac{3\mu}{2b^2}\left(1 + \frac{1}{3}\cos 2\phi\right) \tag{5}$$

in the limit were $\phi << 1$ and $u \to 0$ Equation 5 simplifies to $\phi = \Delta\theta_N = \frac{2GM}{c^2 b}$. Geometrically the deflection is given by $\Delta\theta = 2\phi$ and therefore the deflection angle is

$$\Delta\theta = 2\Delta\theta_N = \frac{4GM}{c^2 b} \tag{6}$$

We conclude that general relativity predicts a factor of 2 greater deflection form a point mass than is predicted by newtonian mechanics. This relationship greatly simplifies the formalism developed for weak lensing.

2.3 Weak Lensing Formalism

Now that we have a theoratical understanding of the Cosmological parameters we would like to measure as well as an understanding of how gravitational fields impact the path of light we are ready to develop the weak lensing formalism on which all weak lensing applications are built. Talk about thin lens and how the formalism revloves around projecting newtonian potential onto the lens 2D surface.[7]

3 Measuring Shear

4 Cosmic Shear

[10] [8] [5]

5 Problems With Weak Lensing

[9]

6 Weak Lensing Results

[4]

1. Current observations imply w=-1 however that is no constraint on its evolution
2. The inital fluctation ins matter density produced by random quantum events therefore CLT tells us that is a gaussiam random field therefore power spectrum is enough to get all statistics

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