Weak Lensing for Precision Cosmology

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1 Introduction

The most revolutionary discovery in cosmology since Hubble observed that the Universe is expanding is that this expansion is accelerating. A revelation that was awarded the 2011 Nobel Prize for its profound implications. [1]. An accelerating universe implies that either our understanding of gravity is flawed or that a mysterious negative pressure known as Dark Energy is driving the expansion [13]. This Dark Energy accounts for most (over 68%) of the energy density in the observable universe, however its origin and physics are presently unknown [2]. As a result, the nature of Dark Energy is considered one of the greatest mysteries of modern science [3].

One of the most powerful ways to probe Dark Energy, as well as modified theories of gravity, is a technique known as weak gravitational lensing, or weak lensing for short [6, 9]. Gravitational lensing is the phenomenon of light ray deflection by intervening mass. When the deflection is sufficiently weak, this phenomenon manifests in images of galaxies as a shearing effect due to the differential deflection of neighboring light rays [17, 6]. This shearing induces a subtle (sub 1%) change in the ellipticity of the images. Although such a change is negligible in comparison to the 30% dispersion in intrinsic galaxy ellipticities, it can be statistically measured by using the coherence of the lensing shear over the sky [17]. The reason weak lensing is considered very powerful is because it provides a direct measurement of the matter distribution in the universe as a function of redshift independent of any cosmological assumptions. Thus, allowing us to directly probe the growth of cosmic structure with time [6].

In this report I present a general overview of weak lensing in the context of cosmology. I begin by presenting the theory behind the Cosmology we would like to probe, as well as the general theoretical framework on which weak lensing operates. I then present the weak lensing measurment procedure and how cosmological data can be extracted from such measurements. Finally, I end this report by presenting some results from an active experiment and a short discussion on potentially concerning systematics.

2 Background Theory

2.1 Standard Model of Cosmology

The fundamental assumption in cosmology, known as the cosmological principle, is that we live in a homogenous (independent of position) and isotropic (independent of direction) universe [17, 15]. Solving Einstein's equations under the geometric symmetries provided by the cosmological principle and taking into account the expansion of the universe one finds that the dynamics of the universe are governed by the Friedman equation [15, 17],

$$H^2(z) = H_0^2 \left(\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda \frac{\rho_\Lambda(z)}{\rho_\Lambda(0)} \right) \tag{1}$$

Remeber to show the Standard Model parameters! can be found in wikipedia.

Angular diameter distance is the one of relvance in lensing (distance measures in cosmology) Distances in cosmo [7] lol

For current and future surveys, one goal is to use the redshifts of the background galaxies (often approximated using photometric redshifts) to divide the survey into multiple redshift bins. The low-redshift bins will only be lensed by structures very near to us, while the high-redshift bins will be lensed by structures over a wide range of redshift. This technique, dubbed "cosmic tomography", makes it possible to map out the 3D distribution of mass. Because the third dimension involves not only distance but cosmic time, tomographic weak lensing is sensitive not only to the matter power spectrum today, but also to its evolution over the history of the universe, and the expansion history of the universe during that time. This is a much more valuable cosmological probe, and many proposed experiments to measure the properties of dark energy and dark matter have focused on weak lensing, such as the Dark Energy Survey, Pan-STARRS, and Large Synoptic Survey Telescope.

2.1.1 Matter Power Spectrum

[16]

2.2 Bending of Light

The fundamental concept on which weak lensing is built is gravity's ability to alter the path of a photon. This phenomenon is explored in full detail in [14, 8, 11]. In this subsection I present a simple overview of the theory behind the bending of light necessary to develop the weak lensing formalism. For more detailed calculations consult [14, 8, 11].

2.2.1 Newtonian Lens

It is a common misconception that the gravitational bending of light is an exclusive property of GR. However, gravity induced alterations to a photon's path are predicted by newtonian mechanics [12]. To illustrate this consider a mass M located at the origin of the cartesian plane and a corpuscle(newtonian photon) propagating along the $x = b$ line (in this context b is known as the impact parameter). Newton's second law predicts that the presence of the point mass will result in a momentum transfer between the two objects. If the corpuscle starts with momentum $(p, 0)$ then it will end up with momentum (p_x, p_y). Therefore, the particle path is deflected by some angle $\hat{\alpha}$. The deflection angle is simply given by

$$\sin(\hat{\alpha}) = \frac{p_y}{\sqrt{p_x^2 + p_y^2}} \tag{2}$$

For very small deflections we have $p \approx p_x >> p_y$ and $\hat{\alpha} << 1$. Therefore Equation 2 simplifies to $\hat{\alpha} \approx \frac{p_y}{p_x}$. We now consider the infinitesimal deflection along the entire path of the photon with $d\hat{\alpha} = \frac{dp_y}{p_x} = \frac{1}{px} dx \frac{dp_y}{dx}$. Therefore, we can find the deflection angle by

$$\begin{aligned} \hat{\alpha}_N &= -\frac{1}{p_x} \int dx \frac{dp_y}{dx} \\ &= -\frac{1}{cp_x} \int dx \frac{dp_y}{dt} \\ &= \frac{2GM}{c^2 b} \end{aligned} \tag{3}$$

We note that the mass of the corpuscle cancels out of the deflection equation. Therefore this equation applies for massless particles i.e. photons. Therefore Equation 3 provides a newtonian description for the bending of light [12].

2.2.2 General Relativistic Bending of Light

The Einstein's field equations in the presence of a charge free static point mass is uniquely solved by the Schwarzchild metric [14]. The Schwarzchild metric is

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{4}$$

where r_s is the Shcwarzchild radius of the system given by $r_s = 2\mu = 2GM/c^2$ and (t, r, Ω) are the standard parameters for 4D space-time in polar coordinates. We can analyze the path of the photon from subsubsection 2.2.1 by studying the geodesic equations of the metric and finding the conserved quantities of the system. We can then combine the conservation equations with the tangent vector norm condition for a null path to get the shape equation of the system as

$$\frac{d\phi}{dr} = \frac{1}{r^2} \left(\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{2\mu}{r} \right) \right)^{-1/2} \tag{5}$$

where (r, ϕ) are the photons position in 2D polar coordinates and b is the impact parameter. Rewriting this equation under the transformation of $r = 1/u$ and working pertrubatively around $u(\mu = 0) = \frac{1}{b}\sin\phi$ we get

$$u(\phi) \approx \frac{1}{b}\sin\phi + \frac{3\mu}{2b^2}\left(1 + \frac{1}{3}\cos 2\phi\right) \tag{6}$$

in the limit were $\phi << 1$ and $u \to 0$ Equation 6 simplifies to $\phi = \hat{\alpha}_N = \frac{2GM}{c^2 b}$. Geometrically the deflection is given by $\hat{\alpha} = 2\phi$ and therefore the deflection angle is

$$\hat{\alpha} = 2\hat{\alpha}_N = \frac{4GM}{c^2 b} \tag{7}$$

We conclude that general relativity predicts a factor of 2 greater deflection form a point mass than is predicted by newtonian mechanics. This relationship greatly simplifies the formalism developed for weak lensing.

2.3 Weak Lensing Formalism

Now that we have a theoretical understanding of the Cosmological parameters we would like to measure, as well as an understanding of how gravitational fields impact the trajectory of light. We are ready to develop the theoretical formalism on which all weak lensing applications are built. This formalism is based on a combination of the frameworks developed in these sources [8, 17, 9, 6, 10, 11, 5].

2.3.1 Weak thin lens

In order to develop our formalism let us consider a general lensing system as seen in Figure 1. For astronomical applications on cosmological scales the distance from the observer to the lens D_L, the distance from the lens to the source D_{LS}, and the distance from the observer to the source D_S are much greater than the thickness of the lens along the optical axis. Therefore we can treat the lens as a "thin lens", i.e. it lives on a planar slice (lensing plane) along the line of sight. We project the mass and potential of the lens onto the lensing plane by defining the projected surface density Σ and projected potential Φ as

$$\begin{aligned} \Sigma(x, y) &= \int \rho(x, y, z) dz \\ \Phi &= \int \phi dz \end{aligned} \tag{8}$$

were ρ and ϕ are the spatial mass density and the newtonian potential respectively. We can now use the results from the previous section to find the deflection angle $\hat{\alpha}$ due to the extended thin lens. The deflection is given by

$$\hat{\alpha} = \frac{2}{c^2}\nabla\Phi(x, y) \tag{9}$$

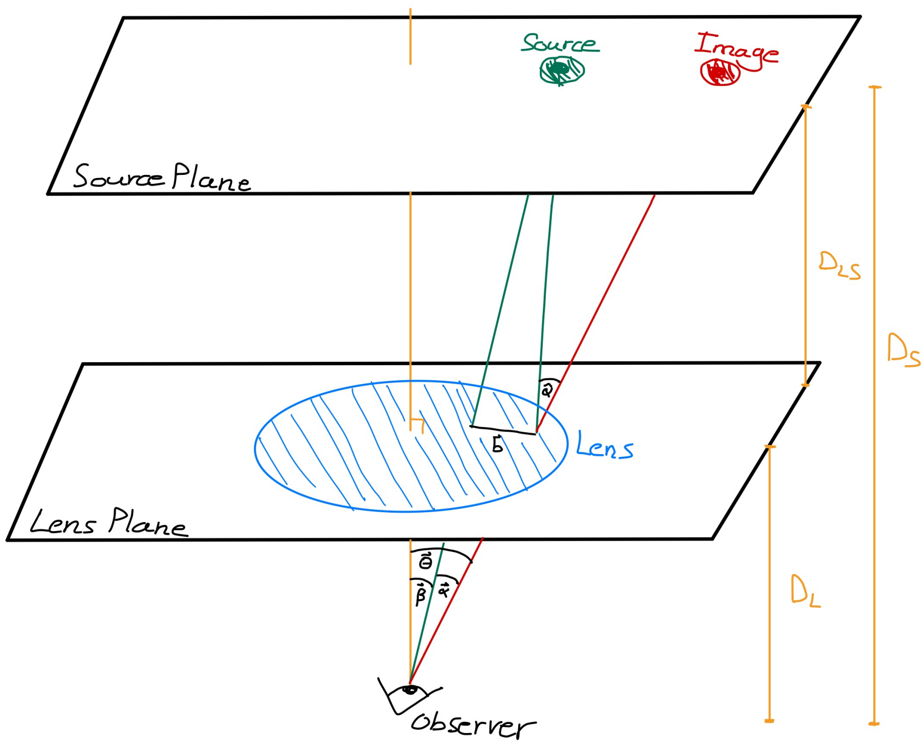

Figure 1: Sketch of a thin lens system highlighting the parameters of relevance in the weak lensing formalism. It is conventional to assume the planes are orthogonal to the z axis.

were the factor of 2 comes from Equation 7 and ∇ is the two dimensional gradient. Note that this equation is equivalent to that describing the deflection of light by an optical lens with refractive index $n = 1 - 2\phi/c^2$, hence the name lensing. Geometrically we have $\beta = \theta - \alpha$ and $\hat{\alpha} = \frac{D_{LS}}{D_S}\alpha$ from Figure 1. This leads us to the ray trace equation

$$\beta = \theta - \frac{D_{LS}}{D_S}\hat{\alpha}(\theta) \tag{10}$$

The ray trace equation is the fundamental equation in weak lensing relating all the geometric properties of the system to one another.

2.3.2 Differential deflection of adjacent light rays

In the weak limit the actual deflections are not observable because the true position of the source is unknown. As a result, only the effects of differential deflection can be measured. Two adjacent light rays from the source pass through the lens at slightly different positions and will therefore be deflected differently. This effect results in a remapping of the observed surface brightness of the source f_s to the observed surface brightness f_{obs}. This mapping can be linearized and is therefore given by

$$f_{obs}(\vec{\theta}) \approx f_s(A\vec{\theta}) \tag{11}$$

were A is the Jacobian of the transformation. A convenient convention is to rewrite the 2D Jacobian as

$$A = \delta_{ij} - \frac{\partial^2\Phi}{\partial\theta_i\partial\theta_j} = \begin{pmatrix} 1-\kappa-\gamma_+ & -\gamma_\times \\ -\gamma_\times & 1-\kappa+\gamma_+ \end{pmatrix} \tag{12}$$

were we have defined the convergence κ shear $\gamma = \gamma_+ + i\gamma_\times$ as

$$\begin{aligned} \kappa &= \frac{1}{2}(\partial_x^2\Phi + \partial_y^2\Phi) \\ \gamma_+ &= \frac{1}{2}(\partial_x^2\Phi - \partial_y^2\Phi) \\ \gamma_\times &= \partial_x\partial_y\Phi \end{aligned} \tag{13}$$

We can now study the geometric implications of the remapping. If we consider a circular source we see that γ_+ and $\gamma_\times$ correspond to a stretching of the circle along the x/y axis and the x=y line respectively, κ corresponds to isotropic enlargement of the source's profile, and since the mapping conserves surface brightness we observe an increase of the total flux by a magnification factor

$$\mu = \frac{1}{\det A} = \frac{1}{(1-\kappa)^2 - \gamma_+^2 - \gamma_\times^2} \tag{14}$$

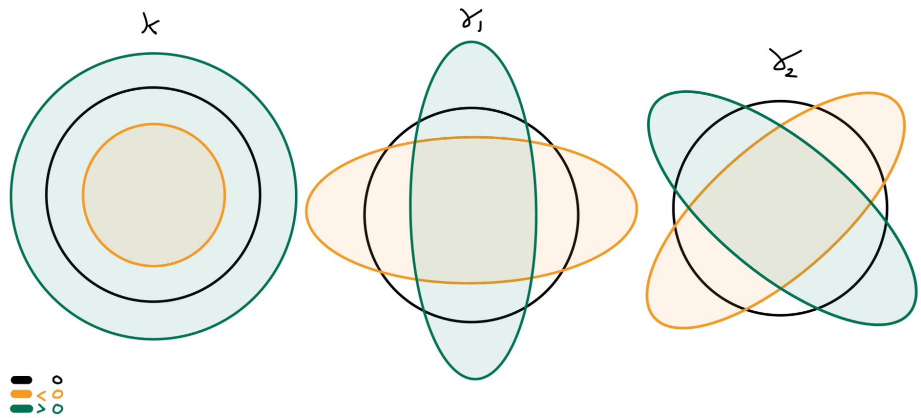

Figure 2: The effects of the convergence κ and the shear γ on a circular image of a galaxy. The black line represents the nominal image, orange represents positive parameter values, and green represents negative parameter values.

these geometric effects are illustrated in Figure 2. To be more specific a circular source is mapped to an ellipse with major axis $a = (1 - \kappa - |\gamma|)^{-1}$ and minor axis $b = (1 - \kappa + |\gamma|)^{-1}$ [10]. If we define the reduced shear as $g = \gamma/(1 - \kappa)$ then an ellipse with ellipticity ϵ_{orig} is mapped to an ellipse with ellipticity ϵ_{obs} given by

$$\epsilon_{obs} = \frac{\epsilon_{orig} + g}{1 + g^{*}\epsilon_{orig}} \approx \epsilon_{orig} + \gamma \tag{15}$$

the approximate relationship is a result of the weak limit ($\kappa << 1$). Equation 15 and Equation 13 demonstrate that by measuring the apparent shapes of lensed objects we are measuring information about the lensing potential and hence the matter overdensity [9, 5, 6]. In the next section we talk about how such measurements are made.

3 Measuring Shear (Images to Catalogs)

The weak lensing analysis process can be conceptually split into two parts; 1) converting images to catalogs of galaxy shapes and 2) extracting scientific results from shape catalogs. In this section we present a sample image to catalog pipeline.

Equation 15 indicates that we can measure the weak lensing effect A population of intrinsically round sources would therefore be ideal, but unfortunatelyreal galaxies have an average intrinsic ellipticity of 0.25 per component.Instead the lensing signal is inferred by averaging over an ensemble of sources, under theassumption that the unlensed orientations are random.

3.0.1 Point Spread Function

3.0.2 Object Detection and Deblending

4 Catalogs to Science

[12] [9] [6]

5 Weak Lensing Results

[4]

6 Problems With Weak Lensing

[10]

1. PSF / detector effects
2. Blending
3. Selection Bias
4. Alignment

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