Weak Lensing for Precision Cosmology

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1 General Introduction

Layman friendly descirption of cosmology + weak lensing and its purpose.

2 Theory

2.1 Bending of Light

The fundamental concept on which weak lensing is built is gravity's ability to alter the path of a photon. In this section we review the theory behind the bending of light necessary to develop the weak lensing formalism.

2.1.1 Newtonian Lens

It is a common misconception that the gravitational bending of light is an exclusive property of GR. However, gravity induced alterations to a photon's path are predicted by newtonian mechanics [5]. To illustrate this consider a mass M located at the origin of the cartesian plane and a corpuscle(newtonian photon) propagating along the $x = b$ line (in this context b is known as the impact parameter). Newton's second law predicts that the presence of the point mass will result in a momentum transfer between the two objects. If the corpuscle starts with momentum $(p, 0)$ then it will end up with momentum (p_x, p_y). Therefore, the particle path is deflected by some angle $\Delta\theta$. The deflection angle is simply given by

$$\sin(\Delta\theta) = \frac{p_y}{\sqrt{p_x^2 + p_y^2}} \tag{1}$$

For very small deflections we have $p \approx p_x >> p_y$ and $\Delta\theta << 1$. Therefore Equation 1 simplifies to $\Delta\theta \approx \frac{p_y}{p_x}$. We now consider the infinitesimal deflection along the entire path of the photon with $d\Delta\theta = \frac{dp_y}{p_x} = \frac{1}{px}dx\frac{dp_y}{dx}$. Therefore, we can find the deflection angle by

$$\begin{aligned}\Delta\theta_N &= -\frac{1}{p_x}\int dx \frac{dp_y}{dx} \\ &= -\frac{1}{cp_x}\int dx \frac{dp_y}{dt} = -\frac{1}{cp_x}\int F_y dx \\ &= \frac{2GM}{c^2 b}\end{aligned} \tag{2}$$

We note that the mass of the corpuscle cancels out of the deflection equation. Therefore this equation applies for massless particles i.e. photons. Therefore Equation 2 provides a newtonian description for the bending of light [5].

2.1.2 General Relativistic Bending of Light

In this subsubsection I give a quick sketch of the bending of light in the context of general relativity, for a more detailed calculation please consult [6].

The Einstein's field equations in the presence of a charge free static point mass is uniquely solved by the Schwarzchild metric [6]. The Schwarzchild metric is

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{3}$$

Where r_s is the Shcwarzchild radius of the system given by $r_s = 2\mu = 2GM/c^2$. We can analyze the path of the photon from subsubsection 2.1.1 by studying the geodesic equations of the metric and finding the conserved quantities of the system. We can then combine the conservation equations with the tangent vector norm condition for a null path to get the shape equation of the system as

$$\frac{d\phi}{dr} = \frac{1}{r^2}\left(\frac{1}{b^2} - \frac{1}{r^2}\left(1 - \frac{2\mu}{r}\right)\right)^{-1/2} \tag{4}$$

where (r, ϕ) are the photons position in 2D polar coordinates and b is the impact parameter. Rewriting this equation under the transformation of $r = 1/u$ and working pertrubatively around $u(\mu = 0) = \frac{1}{b}\sin\phi$ we get

$$u(\phi) \approx \frac{1}{b}\sin\phi + \frac{3\mu}{2b^2}\left(1 + \frac{1}{3}\cos 2\phi\right) \tag{5}$$

$$\Delta\theta = 2\Delta\theta_N = \frac{4GM}{c^2 b} \tag{6}$$

2.2 Lensing Formalism

[2] Cosmology standard model and lensing formalism

3 Cosmological Observables

Discussion about cosmology in general and observable in the framework [7].

4 Weak Gravitational Lensing

Discussion about lensing as a whole then specifics [3] [4] [1]

5 Problems With Weak Lensing

6 Weak Lensing Results

References

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