Weak Lensing for Precision Cosmology

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1 Introduction

The most revolutionary discovery in cosmology since Hubble observed that the Universe is expanding is that this expansion is accelerating. A revelation that was awarded the 2011 Nobel Prize for its profound implications. [1]. An accelerating universe implies that either our understanding of gravity is flawed or that a mysterious pressure known as Dark Energy is driving the expansion [10]. This Dark Energy accounts for most (over 68%) of the energy density in the observable universe, however its origin and physics are presently unknown [2]. As a result, the nature of Dark Energy is considered one of the greatest mysteries of modern science [3].

One of the most powerful techniques to probe Dark Energy and modified theories of gravity is weak lensing. Insert discription of weak lensing here [5].

2 Background Theory

2.1 Standard Model of Cosmology

The fundamental assumption in cosmology, known as the cosmological principle, is that we live in a homogenous and isotropic universe [12].

2.2 Bending of Light

The fundamental concept on which weak lensing is built is gravity's ability to alter the path of a photon. In this section we review the theory behind the bending of light necessary to develop the weak lensing formalism.

2.2.1 Newtonian Lens

It is a common misconception that the gravitational bending of light is an exclusive property of GR. However, gravity induced alterations to a photon's path are predicted by newtonian mechanics [9]. To illustrate this consider a mass M located at the origin of the cartesian plane and a corpuscle(newtonian photon) propagating along the $x = b$ line (in this context b is known as the impact parameter). Newton's second law predicts that the presence of the point mass will result in a momentum transfer between the two objects. If the corpuscle starts with momentum $(p, 0)$ then it will end up with momentum (p_x, p_y). Therefore, the particle path is deflected by some angle $\Delta\theta$. The deflection angle is simply given by

$$\sin(\Delta\theta) = \frac{p_y}{\sqrt{p_x^2 + p_y^2}} \tag{1}$$

For very small deflections we have $p \approx p_x >> p_y$ and $\Delta\theta << 1$. Therefore Equation 1 simplifies to $\Delta\theta \approx \frac{p_y}{p_x}$. We now consider the infinitesimal deflection along the entire path of the photon with $d\Delta\theta = \frac{dp_y}{p_x} = \frac{1}{px} dx \frac{dp_y}{dx}$. Therefore, we can find the deflection angle by

$$\begin{aligned}
\Delta\theta_N &= -\frac{1}{p_x} \int dx \frac{dp_y}{dx} \\
&= -\frac{1}{cp_x} \int dx \frac{dp_y}{dt} = -\frac{1}{cp_x} \int F_y dx \\
&= \frac{2GM}{c^2 b}
\end{aligned} \tag{2}$$

We note that the mass of the corpuscle cancels out of the deflection equation. Therefore this equation applies for massless particles i.e. photons. Therefore Equation 2 provides a newtonian description for the bending of light [9].

2.2.2 General Relativistic Bending of Light

In this subsubsection I give a quick sketch of the bending of light in the context of general relativity, for a more detailed calculation please consult [11].

The Einstein's field equations in the presence of a charge free static point mass is uniquely solved by the Schwarzchild metric [11]. The Schwarzchild metric is

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{3}$$

Where r_s is the Shcwarzchild radius of the system given by $r_s = 2\mu = 2GM/c^2$. We can analyze the path of the photon from subsubsection 2.2.1 by studying the geodesic equations of the metric and finding the conserved quantities of the system. We can then combine the conservation equations with the tangent vector norm condition for a null path to get the shape equation of the system as

$$\frac{d\phi}{dr} = \frac{1}{r^2} \left(\frac{1}{b^2} - \frac{1}{r^2} \left(1 - \frac{2\mu}{r} \right) \right)^{-1/2} \tag{4}$$

where (r, ϕ) are the photons position in 2D polar coordinates and b is the impact parameter. Rewriting this equation under the transformation of $r = 1/u$ and working pertrubatively around $u(\mu = 0) = \frac{1}{b} \sin\phi$ we get

$$u(\phi) \approx \frac{1}{b}\sin\phi + \frac{3\mu}{2b^2}\left(1 + \frac{1}{3}\cos 2\phi\right) \tag{5}$$

in the limit were $\phi << 1$ and $u \to 0$ Equation 5 simplifies to $\phi = \Delta\theta_N = \frac{2GM}{c^2 b}$. Geometrically the deflection is given by $\Delta\theta = 2\phi$ and therefore the deflection angle is

$$\Delta\theta = 2\Delta\theta_N = \frac{4GM}{c^2 b} \tag{6}$$

We conclude that general relativity predicts a factor of 2 greater deflection form a point mass than is predicted by newtonian mechanics. This relationship greatly simplifies the formalism developed for weak lensing.

2.3 Weak Lensing Formalism

[6]

3 Measuring Shear

4 Cosmic Shear

[9] [7] [5]

5 Problems With Weak Lensing

[8]

6 Weak Lensing Results

[4]

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