

Algebra

$$f(a) = (5x^3 + 2ax^2 + 17x + 42) \div (x+a)$$

$$x+a=0 \quad x=-a$$

$$5(-a)^3 + 2a(-a)^2 + 17(-a) + 42 = R \quad \text{— remainder theorem}$$

$$x-a=a \quad x=a \quad (a+b) \div 0 \quad x=-b/a$$

absolutely, completely, exactly divided means $R=0$

Q1.) $4x^3 + 13x^2 - 5x + 1 \div x+4$. find remainder

$$x+4=0 \quad x=-4$$

$$4(-4)^3 + 13(-4)^2 - 5(-4) + 1$$

$$101 = R$$

2.) $5x^3 - 3x^2 + cx + 2 \div x+3$ find the value of c if the remainder is 5

$$x=-3$$

$$5(-3)^3 - 3(9) + c(-3) + 2 = 5$$

$$-165 = 3c$$

$$c = -55$$

3.) $(x^{529} - 1) \div (x-1)$ find remainder

$$x=1$$

$$1-1=0$$

4.) $(3x^3 + 6x^2 + 4a) \div (x^2 - 4)$ leaves out remainder $12x$.
find the value of a in expression

$$12x + 24 + 4a = 12x$$

$$4a = -24$$

$$a = -6$$

5.) $x^{1001} + 5x^{999}$
find value

$$-1 + 5(-1) +$$

$$a=$$

6.) if $5^x - 5^{x-1} =$

7.) Express

Take

$$\frac{2}{3}$$

8.)

5.) $x^{1001} + 5x^{999} + a$ is completely divisible by $(x+1)$.
find value of 'a'.

$$-1 + 5(-1) + a = 0$$

$$a = 6$$

6.) if $5^x - 5^{x-1} = 500$. find value of x

$$5^{x-1}(5-1) = 500$$

$$5^{x-1} = \frac{500}{4} = 125$$

$$x = 4$$

$$x-1=3$$

$$x=4$$

See Assume a value that
denominator is not zero

7.) Express the following in rational $\frac{2(x+3)}{3(x-2)} - \frac{2(x+1)}{3(x-3)}$

Take $x=0$

$$\frac{2(3)}{3(-2)} - \frac{2(1)}{3(-3)} = -\frac{1}{3} + \frac{2}{9} = -\frac{1}{9}$$

assume same $x=0$ in all those option
option d

$$\frac{2(-7)}{3(-3)(-2)} = -\frac{7}{9} \checkmark$$

8.) $\frac{a+1}{a-1} + \frac{a-1}{a+1}$ is same as

$$a=1, a=-1$$

$$\frac{1}{-1} + \frac{-1}{1} = -2 \quad \text{iv.) } \frac{2(a)}{-1} = -2$$

9.) find the sum of $\frac{3x^2+4y}{x-4}$ and its reciprocal if any.

$$x+y=1, x=1$$

$$\frac{3+0}{1-4} + \frac{1-4}{3+0} = \underline{\underline{-2}}$$

10.) if $x+5y+3=0$; find the value of $x^3+5y^3+15xy+27$

$$\text{assume } y=0$$

$$x=-3$$

$$x^3+27$$

$$(-3)^3+27 = \underline{\underline{0}}$$

2 11.) if $a+b+c=25$ and $a^2+b^2+c^2=215$. find $4a^2b^2+c(a-b)$

$$a+b+c=25$$

$$a^2+b^2+c^2=215$$

$$a+b=15$$

$$a^2+b^2=215$$

$$c=0$$

$$4a^2b^2$$

$$(a+b)^2 = a^2 + b^2 + 2ab = 625$$

$$225 + 2ab = 625$$

$$2ab = 400$$

$$4a^2b^2 = (400)^2 = 160000$$

12.) $x = (a+2/a); y = (2a-1/a)$. find $\frac{(x^2-y^2)(y^2+3)}{x^2-6}$

$$x = a + \frac{2}{a}, y = 2a - \frac{1}{a}$$

$$a = 1 + \frac{2}{1} = 3 \quad y = 2 \cdot 3 - \frac{1}{3} = 6$$

$$\frac{(x^2-y^2)(y^2+3)}{x^2-6} = \frac{[(9)^2-6](6+3)}{9-6} = 9$$

13.) $x+1/x=a \rightarrow x^2+1/x^2=?$

$$\left(x + \frac{1}{x}\right)^2 = a^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 \cdot \frac{x}{x} = a^2 \Rightarrow x^2 + \frac{1}{x^2} = a^2 - 2$$

$$x + \frac{1}{x} = 7 \Rightarrow 49 - 2 = 47$$

14.) $x^2 + 1/x^2 = a \rightarrow x + 1/x$

as required

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \Rightarrow x + \frac{1}{x} = \sqrt{a+2}$$

$$x^2 + \frac{1}{x^2} = 62; x + \frac{1}{x} = \sqrt{62+2} = 8$$

$$\text{for } x^2 + 1/x^2 = a \Rightarrow x + 1/x = \sqrt{a+2}$$

15.) $x^8 + \frac{1}{x^8}$; if $x + 1/x = \sqrt{6}$

$$x + \frac{1}{x} = \sqrt{6} \Rightarrow x^2 + \frac{1}{x^2} = 6 - 2 = 4 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 16 \Rightarrow x^4 + \frac{1}{x^4} = 16 - 2 = 14$$

$$\left(x^4 + \frac{1}{x^4}\right)^2 = 196 \quad x^8 + \frac{1}{x^8} = 194$$

14.) $x + \frac{1}{x} = 7$ find $\sqrt{x} + \frac{1}{\sqrt{x}}$

$\sqrt{x} = a; x = a^2$

$a^2 + \frac{1}{a^2} = 7; a + \frac{1}{a} = ?$

$\left(a + \frac{1}{a}\right)^2 = 7 + 2$

$a + \frac{1}{a} = \sqrt{9} = 3$

$\sqrt{x} + \frac{1}{\sqrt{x}} = 3$

$x^2 + \frac{1}{x^2} = 23$ find $\sqrt{x} + \frac{1}{\sqrt{x}}$

$\sqrt{x} = a; x = a^2$

$a^4 + \frac{1}{a^4} = 23; a + \frac{1}{a} = ?$

$\left(a^2 + \frac{1}{a^2}\right)^2 = a^4 + \frac{1}{a^4} + 2$

$a^2 + \frac{1}{a^2} = 5$ $23 + 2 = 25$

$\left(a + \frac{1}{a}\right)^2 = 5 + 2 = 7; a + \frac{1}{a} = \sqrt{7}$

15.) if $x - \frac{1}{x} = 5$ find $x^2 + \frac{1}{x^2}$

$\left(x - \frac{1}{x}\right)^2 = 25 = x^2 + \frac{1}{x^2} - 2$

$x^2 + \frac{1}{x^2} = 27$

if $x - \frac{1}{x} = 3$ find $x^4 + \frac{1}{x^4}$

$x - \frac{1}{x} = 3 \Rightarrow x^2 + \frac{1}{x^2} = 9 + 2 = 11$

$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$

$121 - 2 = x^4 + \frac{1}{x^4}$

(-)
but

119

16.) $x + \frac{1}{x}, x^3 + \frac{1}{x^3}$

$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right)$

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$$\boxed{x^3 + \frac{1}{x^3} = a^3 - 3} \quad \left| - \left(x + \frac{1}{x} \right)^3 - 3 \left(x + \frac{1}{x} \right) \right.$$

if $x + 1/x = 3\sqrt{2}$ find $x^3 + 1/x^3$

$$(3\sqrt{2})^3 - 3(3\sqrt{2})$$

$$45\sqrt{2}$$

if value diff = 1 change the sign

a) if $x = 5 + 2\sqrt{6}$ find $1/x$

$$x = 5 + 2\sqrt{6} \quad ; \quad \frac{1}{x} = \frac{1}{5 + 2\sqrt{6}}$$

$$\frac{1}{5 + 2\sqrt{6}} \times \frac{5 - 2\sqrt{6}}{5 - 2\sqrt{6}} = \frac{5 - 2\sqrt{6}}{1} \quad x = 5 + 2\sqrt{6} \quad \frac{1}{x} = 5 - 2\sqrt{6}$$

only if diff = 1

b) if $x^{1/3} = 4 + \sqrt{15}$ find $1/x^{1/3}$

$$y = 4 + \sqrt{15} \quad , \quad \frac{1}{y} \quad x^{1/3} = 4 + \sqrt{15} \quad \frac{1}{x^{1/3}} = \frac{1}{2}$$

$$\frac{1}{y} = 4 - \sqrt{15}$$

18.) if $x = 5 + 2\sqrt{6}$ find $x + 1/x$

$$\frac{1}{x} = 5 - 2\sqrt{6}$$

$$x + 1/x = 10$$

b) if $x = 5 + 2\sqrt{6}$ find x^{-1}/x

$$\frac{1}{x} = 5 - 2\sqrt{6}$$

$$x - \frac{1}{x} = 4\sqrt{6} \quad 2 \text{ (smaller term)}$$

19.) if $x = 13 + 3\sqrt{2}$ find $1/x$

$$\frac{1}{x} = \frac{1}{13+3\sqrt{2}} \times \frac{13-3\sqrt{2}}{13-3\sqrt{2}}$$

$$\frac{13-3\sqrt{2}}{151} = \frac{1}{x} = \frac{13-3\sqrt{2}}{13^2 - (3\sqrt{2})^2}$$

b.) if $x^{45} = 12 + 3\sqrt{7}$ find $1/x^{45}$

$$144 - 63 = 81$$

$$\frac{1}{y} = \frac{12-3\sqrt{7}}{144-63} = \frac{12-3\sqrt{7}}{81}$$