Logarithms

Exponent form: $a^m = x$ $\quad 3^2 = 9 \quad 4^x = 16$

log form: $\log_a(x) = m$

exp ⇄ log

base doesn't change

x = ans, ans = x) swapped

Exp to log conversion.

$10^2 = 1000 \Rightarrow \log_{10}(1000) = 3$

$2^3 = 8 \Rightarrow \log_2(8) = 3$

$3^{-2} = 1/9 \Rightarrow \log_3(9) = -1/2$

$\log 1 = 0$

$\log_a(a) = 1$

Product rule: $\log_a(xy) = \log_a(x) + \log_a(y) \neq \log(x+y)$

quotient rule: $\log_a(x/y) = \log_a(x) - \log_a(y)$

log Power: $\log_a(x^y) = y * \log_a(x)$

Power is to base $\log_{a^b}(x) = (1/b) \log_a(x)$

Switch base: $\log_a(x) = 1/\log_x(a)$

change base: $\log_a(x) = \log_c(x) / \log_c(a) =$

$\log(x) / \log(a)$

find the value of

i) $\log_5(25)$

$\log_5(5)^2 = 2 \times \log_5(5)$

2×1

2

ii) $\log_{81} 3$

$81 = 3^4$

$\log_{3^4}(3) = \frac{1}{4} \times \log(3)$

$\frac{1}{4} \times 1$

$\frac{1}{4}$

find the value of

$\log_{\sqrt{7}}(1/243)$

$\frac{1}{243} = \frac{1}{7^3} = 7^{-3}$ —①

$\sqrt{7} = 7^{1/2}$

$\log_{7^{1/2}}(7^{-3}) = -3 \times \log_{7^{1/2}}(7)$

$-3 \times \frac{1}{1/2} \log_7 7$

$-3 \times 2 \times 1 = -6$

$\log_{.0001}(1000)$

$1000 = 10^3$

$.0001 = \frac{1}{10^4} = 10^{-4}$

$\log_{10^{-4}} 10^3 = 3 \times \frac{1}{-4} \log_{10} 10$

$-3/4$

v) $\log_2\left(\frac{512 * 256}{32}\right)$

$\log_2\left(\frac{2^9 \times 2^8}{2^5}\right)$

$\log_2 \times 2^{12}$

$12 \times \log_2(12)$

12

vi) $\log_2(0.0625)$

$.0625 = \frac{1}{2^4}$

$\log_2\left(\frac{1}{2^4}\right) = \log_2(2^{-4})$

$-4 \times \log_2 = -4 \times 1$

-4

2.) find the value of y, if $\log_y(25/9) = -2$

$y^{-2} = \frac{25}{9}$ $\quad \frac{1}{y^2} = \frac{25}{9}$ $\quad y = \frac{3}{5}$

3.) If $\log_9 v = 36$ and $\log_3 v = b$, what is relation between v and b.

$9^{36} = v$ $\quad 3^b = v$

$v = 3^{72}$ $\quad v = 3^b$

$(3^b)^{12} = v^{12} = v$

when no base given take it as 10

4.) find $\frac{10\log 10000}{5\log 100}$

$\frac{10\log(10)^4}{5\log(10)^2} = \frac{10\times 4\times \log_{10}(10)}{5\times 2\times \log_{10}(10)} = \frac{4\times 1}{1} = 4$

5.) $\log_2(2^1 * 2^2 * 2^3 \cdots 2^9)$

$\log_2(1+2+\cdots+9)$ $\quad \frac{n(n+1)}{2}$

$\log_2(2)^{45} = 45\times \log_2(2)$ $\quad \frac{9(10)}{2}$

45

6.) $\log_2 \log_2 \log_3 \log_5(125) = ?$ $\quad \log_5(125)$

$\log_5(5^3)$

$\log_2 \log_2 \log_3(3)$ $\quad 3$

$\log_2 \log_2(1) = 0$ $\quad \log_2(0) = \infty$

$\log_2$

7.) find the value of $\log(15/16) - \log(27/45) + \log(48/75)$

by Product

$$\log\left(\frac{15}{16} \times \frac{45}{27} \times \frac{48}{75}\right)$$

$\log_a(xy) = \log_a(x) + \log_a$

$\log 1 = 0$

8.) $\log_{10} 2 = p$; $\log_{10} 7 = q$. find the value of $\log_5 14$

use change base

$\log_a(x) = \log_c(x) / \log_c$

and Product rule

$\log_5(4 \times 7) = \log_5 4 + \log_5 7$

$\log_5(2)^2 + \log_5 7 = 2\log_5 2 + \log_5 7$

change base rule

$$\frac{2\log_{10} 2}{\log_{10} 5} + \frac{\log_{10} 7}{\log_{10} 5} = \frac{2p}{\log_{10}(10/2)} + \frac{q}{\log_{10}(10/2)}$$

$$\frac{2p}{\log_{10} 10 - \log_{10} 2} + \frac{q}{\log_{10} 10 - \log_{10} 2} = \frac{2p}{1-p} + \frac{q}{1-p} = \frac{2p+q}{1-p}$$

9.) $\log(x^2 - 6x + 10) = 0$ find the value of x

$$\log^{(x^2-6x+10)} = 0 = \log^1$$

$x^2 - 6x + 10 = 0$

$x^2 - 6x + 9 = 0$

$x = 3$

10.) If $\log$

$(\log_{10}$

$\log_{100}$

$\frac{\log(a)}{\log 100}$

11.) $\log$

10.) If $\log(105) = p$. find $(\log_{100} 101)(\log_{101} 102)$ $(\log_{103} 104)$.

$\log_a(x) = \frac{\log(x)}{\log(a)}$

$\log_{100}(101) = \frac{\log 101}{\log_{10} 100}$

$\frac{\log 101}{\log 100} \times \frac{\log 102}{\log 101} \times \cdots \times \frac{\log 105}{\log 102}$

$\frac{\log 105}{\log 100} = \log_{100}(105) = \log_{10^2}(105)$

$\frac{1}{2} \times \log_{10} 105 = \frac{1}{2} \times p = p/2$

11.) $\log 11 = a$. find $\log(1/110)$.

$1/a = a^{-1}$

Product rule

$\log \frac{1}{110} = \log(110)^{-1}$

$-1 \times \log 110$

$-\log(1 \times 10)$

$-[\log_{10} 11 \times \log_{10} 10]$

$-[a \times 1] = -a \times 1 = \log \frac{1}{110}$

12) if $\log 2 = 0.3010$, find the value of $\log \log 4$

$$\log \log 4 = \log (2)^{10}$$

$$10 \log 2$$

$$10 \times (.3010)$$

$$3.010$$

//

Characteristic

&

mantissa

Characteristic
we can calculate

mantissa
needs log table
Always five

No 1: (No. of digits in integral part - 1)

No < 1: - [No. of zeroes immediately after decimal point]

applying those rule

before decimal

$\log 7.23 = > 1 = 1 - 1 = 0$

$\log 0.0723 < 1 \Rightarrow$

$\log 71.3271 \quad 2 - 1 = 1$

$-[+1] = -2$