Geometric
Progression

$a_1, a_2, a_3 \cdots a_n$ - we need common ratio to be same

$$a, ar, ar^2 \cdots a_n$$

$$a_n = ar^{(n-1)}$$

2, 4, 6, 8, 16, 32 ...

9^{th} term

$$a_n = ar^{n-1}$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = r$$

$$a_9 = 2 \times r^8$$

$$\frac{4}{2} = 2 = r$$

$$2(2)^8 = 512$$

$$S_n = a + ar + ar^2 \cdots + ar^{(n-1)} \quad -①$$

$$rS = ar + ar^2 + \cdots + ar^{(n-1)} + ar^n \quad -② \times r$$

$$(1-r)S = a - ar^n$$

$$S_n = \frac{a(1-r^n)}{1-r} \; ; \; r<1$$

or

$$S_n = \frac{a(r^n-1)}{r-1} \; ; \; r>1$$

$$S_n = \frac{a(1-r^n)}{1-r} \Rightarrow S_n = \frac{[(a) - (ar^{n-1}) * r]}{1-r}$$

$$S_n = \frac{[\text{first term} - (\text{last term} * r)]}{1-r}$$

for $r>1$

$$S_n = \frac{a(r^n-1)}{r-1} \Rightarrow S_n = \frac{[(ar^{n-1})*r - a]}{r-1} \quad r \neq 0$$

$$S_n = \frac{[(\text{last term} * r) - \text{first term}]}{r-1}$$

Geometric Mean

$a, b, c \Rightarrow \frac{b}{a} = \frac{c}{b} = b^2 = ac = b = \sqrt{ac}$

odd numbers of terms in GP = Middle term

Even numbers : $(a_m \times a_1)^{1/2} = \sqrt{a_m \cdot a_1}$

Inserting terms

$a, m_1, m_2 \ldots, m_n, b$

$r = \left(\frac{b}{a}\right)^{1/n+n}$ $\quad m_1 = a \times r \quad m_3 = ar^3$

$m_2 = ar^2$

Multiply or Divide each term of GP by same number

$\rightarrow$ resultant GP

2, 4, 6, 8

$\times 3 \quad \times 2$

$\frac{6, 12\ 24}{2}$

Raise each term by same Power

$2^2\ 4^2\ 6^2\ 16^2$

$\frac{4}{4}\ 16\ \frac{144\ 256}{4}$

Multiplying 2 gp results in GP

$2, 4, 8 \ldots$

$3, 9, 27 \ldots$

$\underline{6}, 36, \underline{216}$

log of each term of GP : gives AP

$2, 4, 8$

$\log 2, \log 2^2, \log 2^3$

$\log 2 = x$

$x, 2x, 3x$

AP

1.) find the 7^{th} term of series: $\frac{3}{8}, \frac{3}{4}, \frac{3}{2}, 3$. what is GM of this GP series of 7 term

$$r = \frac{3}{4} \times \frac{8}{3} = 2$$

$$a_7 = \frac{3}{8}(2)^{7-1} = 24$$

2.) If 3^{rd}, 5^{th}, 7^{th} term of a GP are p, q and r respectively, then which one is true

a.) $3q = (pr)^{1/3}$ b.) $q^2 = pr$ c.) $p + q - r = 1$ d.) $p + q + r = 0$

$$\frac{a}{R^3}, \frac{a}{R^2}, \frac{a}{R}, a, aR, aR^2, aR^3$$

for our convenience

$p = a/R$

$q = aR$

$r = aR^3$

ii.) $q^2 = pr$

$$aR^2 = \frac{a}{R} \times aR^3$$

$$a^2R^2 = a^2R^2$$

3) Geometric mean of two numbers is 16 and their AM is 20. find 2 numbers

$$AM = \frac{a+b}{2} = 20 \quad AM = a+b = 40 \quad -①$$

$$GM = \sqrt{ab} = 16 \quad ab = 256$$

$$\frac{256}{b} + b = 40$$

$a = 32$
$b = 8$

$$b^2 - 40b + 256 = 0$$

$$b = 32 \; ; \; b = 8$$

4.) Third term of a GP is 17. what is the product of first five terms of this GP

a.) $(17)^5$ b.) 4913 c.) 17^{*5}

$$\frac{a}{r^2} \quad \frac{a}{r} \quad a \quad ar \quad ar^2$$

$$a^5 = 17^5$$

5.) Three consecutive terms of GP have their sum as 39 and product as 729. find value of smallest

$x+y+z=39$ — ①

$xyz=729$

$y=\sqrt{xz}$

$y^3=729 \quad y=9$

$x+y+z=39$

$xz=\frac{729}{9}=81$

$x+z=30$

$xz=81$

$z=\frac{81}{x}$

$x+\frac{81}{x}=30 \quad x^2-30x+81$

$x+y=30 \quad x=27, x=3$

$y=3, y=27$

3, 9, 27

27, 9, 3

7.) find

6.) find the sum of 11 + 103 + 1005 + 10007 + ... upto n terms

8.)

$(10+1)+(100+3)+(1000+5)+\ldots n$ terms

$(10+100+1000\cdots n)+(1+3+5+7\ldots n$ terms$)$

$(10+10^2+10^3+\ldots 10\times10^{n-1})+(1+3+5+7\ldots n$ terms$)$

Sum of odd term

$\frac{10\times(10)^{n-1}\times10-10}{10-1}+n^2=\frac{10(10^n-1)}{9}+n^2$

7.) find the sum of the series : 3+33+333+3333+...

$3(1+11+111+1111+\dots)$

$\frac{3}{9}(9+99+999+\dots)$

$\frac{1}{3}[(10+100+1000\dots)+(-1-1+\dots n)]$

$\frac{1}{3}[(10+10^2+10^3+\dots(10\times10^{n-1}))-n]$

$\frac{1}{3}\left[\left(\frac{(10^n\times10)-10}{9}-n\right)\right]$

$\frac{1}{27}\left[\frac{10(10^n-1)}{\cancel{9}}-9n\right]$

8.) find the sum of $\left(1+\frac{2}{3}+\frac{3}{3\times3}+\frac{4}{3\times3\times3}+\dots n\text{ terms}\right)$

i.) $\frac{3}{4}-\frac{3+3n}{4(3^n-1)}$ ii.) $\frac{3}{4}\left[3-\frac{(3+2n)}{3^n}\right]$ iii.) none of them

$n=2$, $1+\frac{2}{3}=\frac{5}{3}$ Taking only first 2 term and substituting in eqn

ii.) $\frac{3}{4}\left[3-\frac{(3+4)}{9}\right]=\frac{5}{3}=\frac{5}{3}$

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$\frac{1}{3}\left[\left((10+10^2+10^3+\dots(10\times10^{n-1})\right)-n\right]$

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9) find the sum of an infinite GP whose first term is 25 and second term is 5

25, 5, ... ∞

$S_\infty = \frac{a}{1-r}$, $|r| < 1$

$r = \frac{5}{25} = \frac{1}{5}$

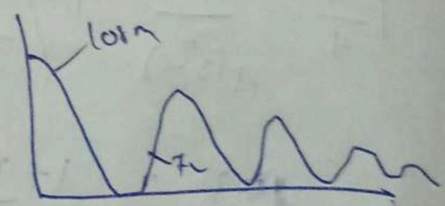
$S_\infty = \frac{25}{1-\frac{1}{5}} = \frac{125}{4}$

10) A ball falls from a height of 108 meter. it rebounces 2/3 times its high every time it touches. if it continues to fall and rebounce, what is the total distance covered

$T_A = 108 + 72 = 180m$

$T_B = 72 + 48 = 120m$

$T_P = 48 + 32 = 80$



$\frac{2}{3} \times 108 = 72$

$\frac{120}{180} = \frac{80}{120} = \frac{2}{3}$

180, 120, ... ∞

$r = \frac{2}{3}$

$S_\infty = \frac{180}{1-2/3} = \frac{180 \times 3}{1} = 540m$

$r = \frac{2}{3}$ $t_1 = ~~t~~$

$108 + \frac{2}{3}(108)$

180 m

$S_\infty = \frac{a}{1-r}$

$\frac{180}{1-2/3} = 540m$