Show that $(P \to (q \to P)) \Rightarrow (\sim P \to (P \to q))$

we have to prove that $(P \to (Q \to P)) \to (\sim P \to (P \to Q)) \equiv T$

$\sim(\sim P \vee (\sim Q \vee P)) \vee (P \vee (\sim P \vee Q))$ [conditional equivalence]

$(P \wedge (Q \wedge \sim P)) \vee (P \vee (\sim P \vee Q))$ [Demorgans law]

$[(P \wedge Q) \wedge (P \wedge \sim P)] \vee [(P \vee \sim P) \vee (P \vee Q)]$ (distributive law)

$[(P \wedge Q) \wedge F] \vee [T \vee (P \vee Q)]$ (complement laws)

$F \vee T$ (Identity laws)

$\equiv T$ (Identity Laws).

---

Establish the validity of following arguments using the rule of contradictions.

$[(p \to q) \wedge (\sim r \vee s) \wedge (p \vee r)] \to [q \to s]$

Given premises : $P \to Q$, $\sim r \to s$, $\sim p \to r$

| Step no | Statement | Reason |
|---|---|---|
| 1. | $\sim(Q \to S)$ | Assume premises |
| 2. | $P \to Q$ | Rule P |
| 3. | $\sim Q \to \sim P$ | Rule T {2} $P \to Q \Leftrightarrow \sim Q \to \sim P$ |
| 4. | $\sim P \to r$ | ~~Rule T {P→Q, Q→R ⇒ P→}~~ Rule P. |
| 5. | $\sim Q \to r$ | Rule T {3,4} chain rule $(P \to Q, Q \to R \Rightarrow P \to R)$ |
| 6. | $\sim r \to s$ | Rule P |
| 7. | ~~$r \to \sim s$~~ $\sim r \to Q$ | Rule T {5} $(P \to Q \Leftrightarrow \sim Q \to \sim P)$ |
| 8. | $Q \to S$ | Rule T {6, 7}. |
| 9. | $(Q \to S) \wedge \sim(Q \to S)$ | contradiction Rule T (8, 1) |
| 10 | $\equiv F$ | |

3) write each of the following in symbolic form by assuming the universal consist of literally everything

i) All men are giants

$M(x)$: $x$ is a Men

$G(x)$: $x$ is giant

$\therefore [M(x) \rightarrow G(x)]$

ii) No men are giants.

If $x$ is a men, then $x$ is not giant

$M(x) \rightarrow \sim G(x)$

iii) Some men are giants.

If $x$ is men, $G(x)$ is giants.

$\exists(x)\,[M(x) \wedge G(x)]$

iv) Some men are not giants.

$x$ is men, $G(x)$ is not Giants

$\exists x\,(M(x) \wedge \sim G(x))$

) prove by mathematical induction, that $2n+1 \leq 2^n$ for $n \geq 3$.

Given: $2n+1 \leq 2^n$ for $n \geq 3$.

1) let check P(3) is true or not

$$p(n) = 2n+1 \leq 2^n$$
$$p(3) = 2(3)+1 \leq 2^3$$
$$= 6+1 \leq 2^3$$
$$\therefore 7 \leq 8$$

P(3) is true

2) P(K) Let assume the statement is true for $n=K$

$$P(K) = 2K+1 \leq 2^K \qquad [2K \leq 2^K - 1] \quad -①$$

3) let us now try to establish that P(K+1) is true

$$P(K+1) = 2(K+1)+1 \leq 2^{K+1}$$
$$= 2K+2+1 \leq 2^K \cdot 2$$

From Equation ①

$$2^K - 1 + 2 + 1 \leq 2^{K+1}$$

$$2^K + 2 \leq 2^{K+1}$$

$2^{K+1}$ will be always ~~be~~ greater or equal than the $2^K+2$

It is proved that $P(K+1)$ holds true, whenever the statement $P(K)$ is true.

Thus, ~~$2K+2$~~ $(2n+1)$ is less or equal than $2^n$ is proved using the principles of mathematical induction