Summary of Phasors

A phasor is like a vector, but in the complex plane. It's used to visualize the magnitude and phase shift associated with a sinusoidal function.

For example, in steady-state we often need to work with voltages of the form:

$$v(t) = v_m \cos(\omega t + \theta_v)$$

We can visualize this as a vector in the complex plane, in which the real part corresponds to the above form. The imaginary part can be found using Euler's identity, or

$$\cos(\theta) = Re\{e^{j\theta}\} = Re\{\cos\theta + j\sin\theta\}$$

Note that we always use cosines for our steady-state functions! If the function is a sine, we need to convert to a cosine to make sure everything works out right.

Phasors, Example

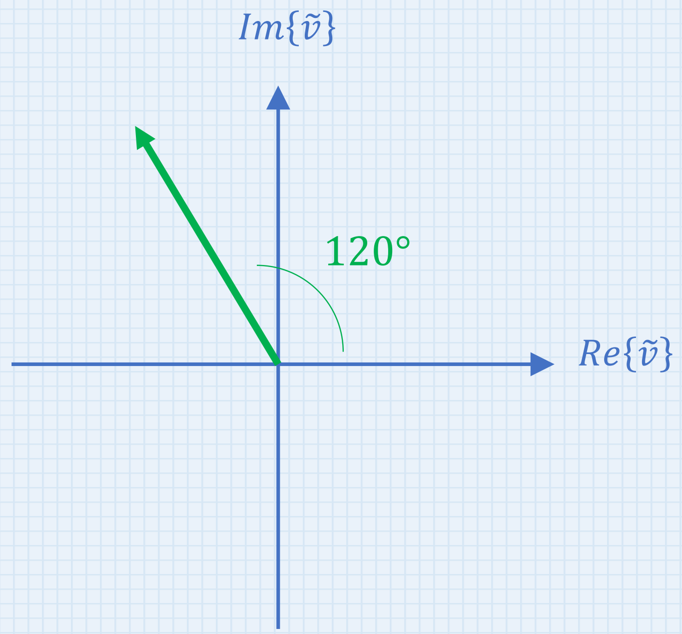
Let's take a specific example to show how this might work. Consider the following function:

$$v(t) = 12\cos(300t + 120°)\ V$$

How do we represent this as a phasor? We note that the phasor representation is

$$\tilde{v} = 12\ V\ \angle 120°$$

This form leads to the plot shown on the right. Note that we don't need to consider the 300t part of the cosine, because we assume everything in the circuit has the exact same frequency dependence. Including the 300t term would cause the phasor to rotate around the complex plane with an angular frequency of 300; however, we can just make the plot at t=0 and it has all of the information we need.



Adding Phasors, Example

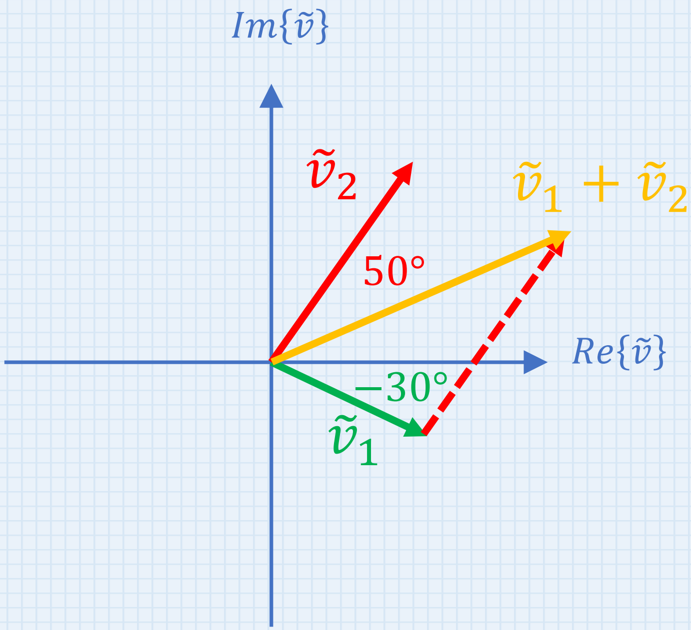
It's often the case you will have two or more voltages that need to be added or subtracted. This can be done in phasor form more easily than trying to use trig identities. Consider the following:

$$v_1(t) = 5\cos(200t - 30°)\ V \qquad v_2(t) = 7\cos(200t + 50°)\ V$$

We want the sum of these. Let's write these in phasor form:

$$\tilde{v}_1 = 5\ V \angle -30° \qquad \tilde{v}_2 = 7\ V \angle +50°$$

On a phasor plot, we can add these like vectors using the "head to tail" method, as shown on the right. Here, we have added the two, sliding $\tilde{v}_2$ over.



How can we quantitatively add these together? We add the real and imaginary components separately.

Recall:

$$|\tilde{v}| = \sqrt{Re\{\tilde{v}\}^2 + Im\{\tilde{v}\}^2} \qquad \tan\theta = \frac{Im\{\tilde{v}\}}{Re\{\tilde{v}\}}$$

Adding Phasors, Example

$$\tilde{v} = \tilde{v}_1 + \tilde{v}_2 = Re\{\tilde{v}_1 + \tilde{v}_2\} + j\,Im\{\tilde{v}_1 + \tilde{v}_2\}$$

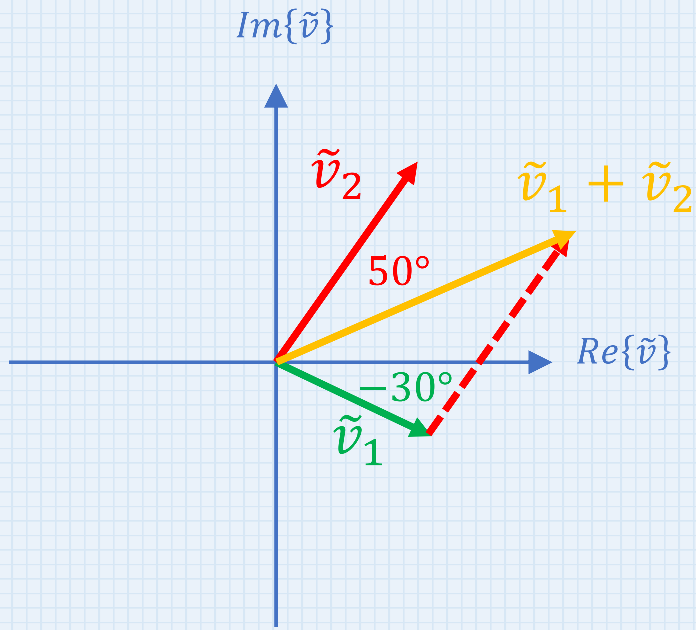
$$Re\{\tilde{v}\} = Re\{\tilde{v}_1\} + Re\{\tilde{v}_2\} = 5\cos(-30°) + 7\cos(50°) = 4.33 + 0.64 = 8.83$$

$$Im\{\tilde{v}\} = Im\{\tilde{v}_1\} + Im\{\tilde{v}_2\} = 5\sin(-30°) + 7\sin(50°) = -2.5 + 5.36 = 2.86$$

$$|\tilde{v}| = \sqrt{8.83^2 + 2.86^2} = 9.28$$

$$\theta = \tan^{-1}\left(\frac{2.86}{8.83}\right) = 17.95°$$

$$\tilde{v} = 9.28\,V \angle 17.95°$$



Summary of Adding Phasors

It's easiest usually to add and subtract phasors by writing the components, which is the same as writing complex numbers, and adding or subtracting term by term.

A phasor is a complex number, so

$$\tilde{v}_1 = a + jb \qquad \tilde{v}_2 = c + jd$$

$$\tilde{v}_1 + \tilde{v}_2 = (a + c) + j(b + d)$$

$$\tilde{v}_1 - \tilde{v}_2 = (a - c) + j(b - d)$$

Multiplying Phasors

Often we need to multiply two complex quantities. Consider, for example, finding the voltage from a current and impedance in phasor form:

$$\tilde{v} = \tilde{Z}\,\tilde{\imath}$$

We can write any complex number in exponential form using Euler's relation:

$$\tilde{z} = a + jb = re^{j\theta} = r\cos\theta + jr\sin\theta = r\,\angle\theta$$

So if we want to multiply two numbers, it's easiest to use exponential or phasor notation:

$$\tilde{z}_1 = r_1 e^{j\theta_1} \qquad\qquad \tilde{z}_2 = r_2 e^{j\theta_2}$$

$$\tilde{z}_1\tilde{z}_2 = r_1 r_2 e^{j(\theta_1+\theta_2)} = r_1 r_2\,\angle(\theta_1 + \theta_2)$$

$$\tilde{z}_1/\tilde{z}_2 = (r_1/r_2)e^{j(\theta_1-\theta_2)} = (r_1/r_2)\angle(\theta_1 - \theta_2)$$

Problem 10.3-5 in Svoboda and Dorf:

Determine the polar and rectangular forms of the expression:

$$\frac{(60\angle 120°)(-16 + j12 + 20\angle 15°)}{5\angle -75°}$$

In this question, the polar and rectangular forms of complex numbers have been mixed. We have to find a way to simplify things. First, we need to add the two complex numbers in the second set of parentheses. Adding is easier in rectangular, so convert the second term to rectangular:

$$20\angle 15° = 20\cos 15° + j20\sin 15° = 19.32 + j5.18$$

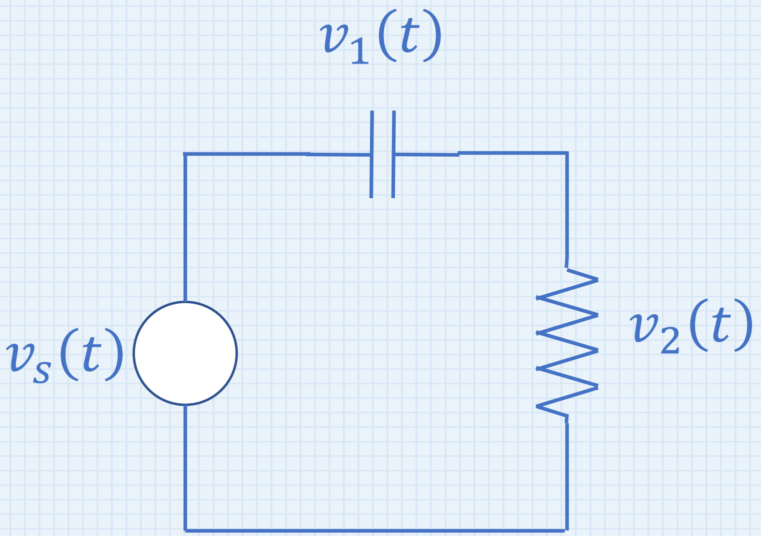
Then, the term in parentheses is:

$$(-16 + j12 + 20\angle 15°) = 3.32 + j17.18 = 17.50\angle 79°$$

$$\frac{(60\angle 120°)(17.50\angle 79°)}{5\angle -75°} = \frac{(1050\angle 199°)}{5\angle -75°} = 210\angle 274° = 14.65 - j209.50$$

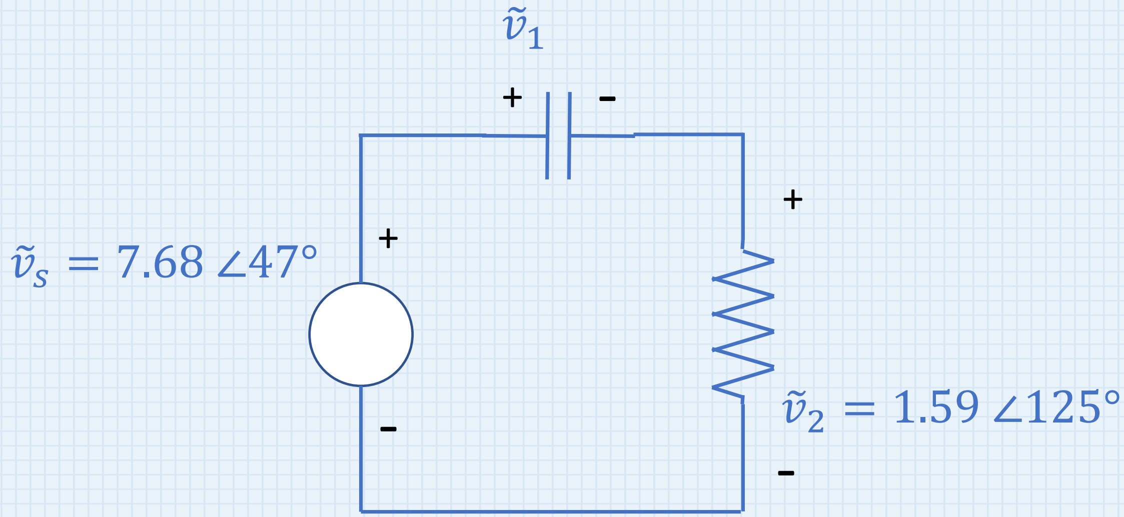
Problem 10.3-10 in Svoboda and Dorf

Note that KVL and KCL both work for phasors, so that all of the circuit theorems learned so far apply. Consider the circuit below. We know the steady state time dependence of two of the voltages. Find the third (v_1)



$$v_s(t) = 7.68\cos(2t + 47°)$$

$$v_2(t) = 1.59\cos(2t + 125°)$$



Using KVL, the sum of complex voltages around the loop must sum to zero (note that the passive/active convention still holds):

$$\tilde{v}_1 = \tilde{v}_s - \tilde{v}_2$$

Problem 10.3-10 in Svoboda and Dorf (continued)

$$\tilde{v}_1 = \tilde{v}_s - \tilde{v}_2 = 7.68\ \angle 47° - 1.59\ \angle 125°$$

$$\tilde{v}_1 = 7.68\ \angle 47° - 1.59\ \angle 125° = 5.24 + j5.62 + 0.91 - j1.3$$

$$\tilde{v}_1 = 6.15 + j4.32 = 7.51\ \angle 35°$$

$$v_1(t) = 7.51 \cos(2t + 35°)$$