NAME: SHAAN SAPRU

ROLL: 20AE30023

COURSE: AE39002

SYSTEMS LAB

EXPERIMENT: 1

MAGNETIC LEVITATION SYSTEM

Aim:-

a) To do a linear modelling of the MAGLEV and design a PID controller for this system.
b) To do a non-linear modelling of the MAGLEV and design a PID controller for this system.

Experimental Setup:-

The experimental setup primarily consists of a metallic ball, and a magnetic coil. There is a IR position sensor to measure Y position of the ball. The sensor and the coil are fitted in a frame. When controlled current is passed through the coil, the magnetic field produced by it, attracts the coil that levitates it. MATLAB based controller algorithm is connected to the system. There is a separate op-amp electric control circuit too.

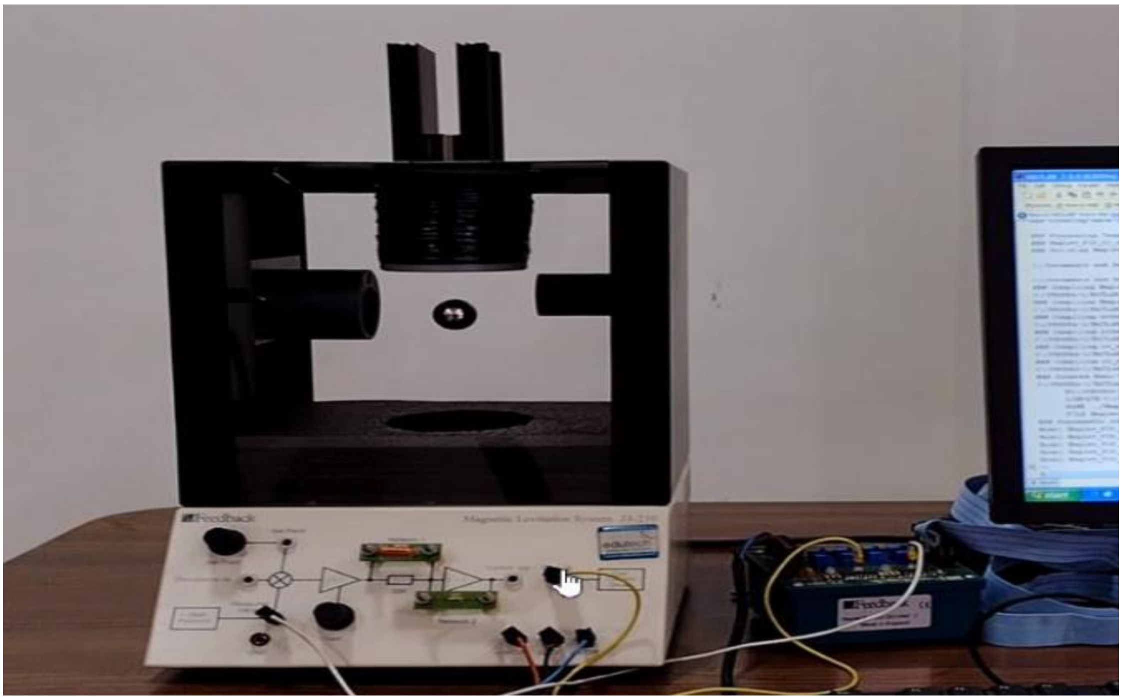


Fig: Experimental setup for Maglev system

Theory

Magnetic levitation system is a non-linear system with fast dynamics. The magnetically levitating ball is under the influence of two counteracting forces:-
a) The magnetic field produced by an inductive coil.
b) The gravitational force acting downwards.

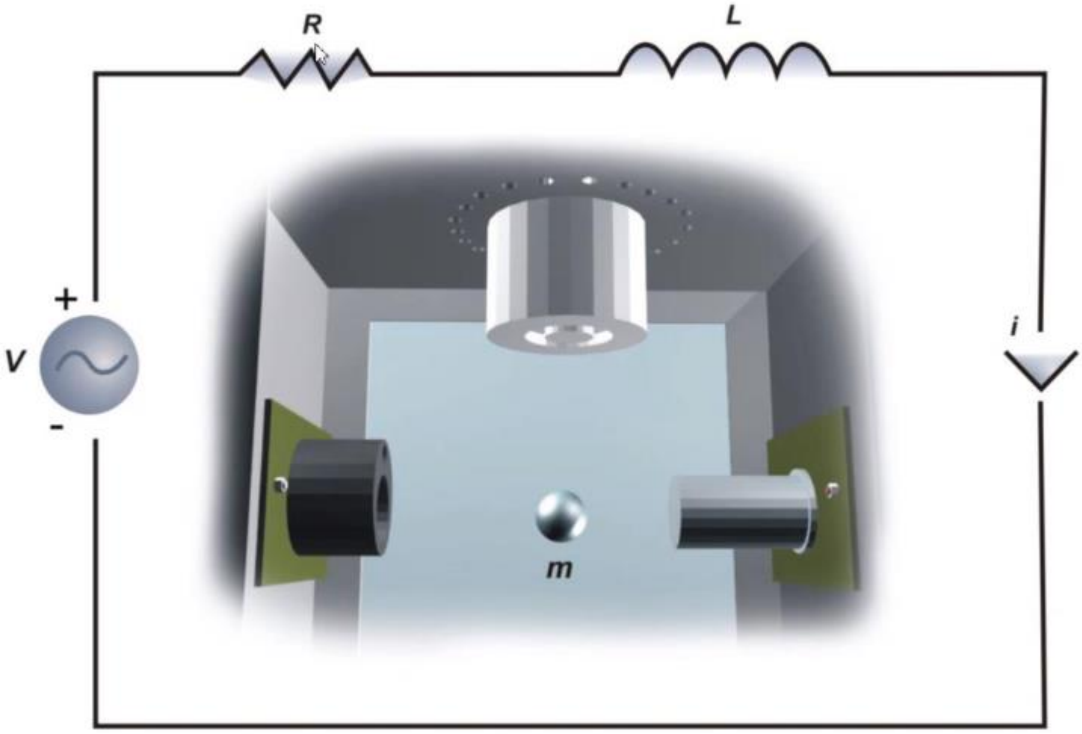


Fig: Electrical schematic of Maglev system

Here, the goal of the control logic should be levitating the metallic ball by means of the electromagnetic field counteracting the force of gravity. The control input is voltage, which is linearly converted into the current ($\boldsymbol{i}$) by the internal circuit within the Maglev system. The current, when passes through the dedicated RL circuit, containing the inductive coil, which creates the corresponding strength magnetic field in its vicinity along its axis. This magnetic force attracts the ball, helping it to levitate. The output of the system is the actual ball position ($\boldsymbol{x}$) in vertical direction. The ball is thus placed along the vertical axis of the electromagnet.

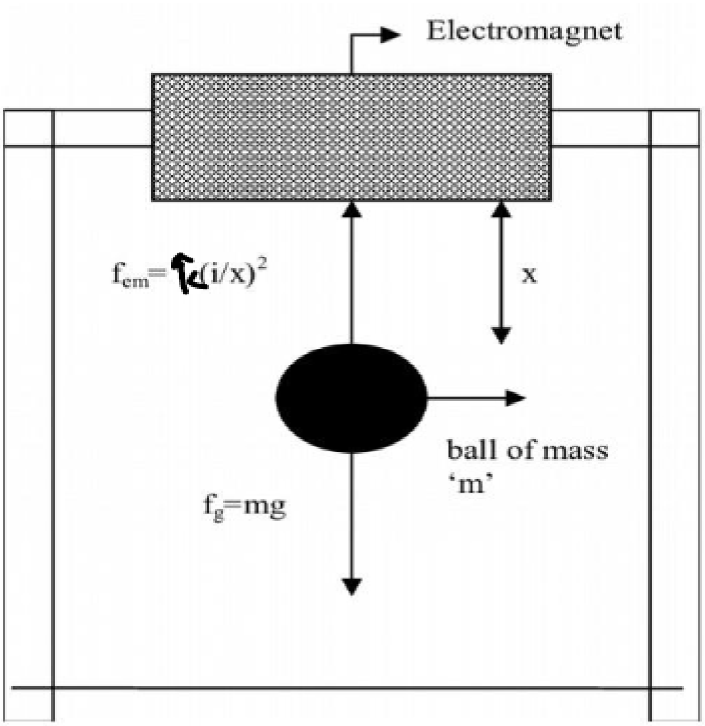


Fig: FBD of Maglev system

The equation of motion of the ball can be written from the FBD as:

$$m\ddot{x} = mg - \hat{k}\frac{i^2}{x^2}$$

Where, $\hat{k}$ is a constant related to the mutual inductance of the ball and coupling coefficients.

The current i, is related to the input voltage, u by: $i = K_1.u$.

Linearization of The non Linear system :- **We Perform the following steps to linearize the non linear system taking the equilibrium position as the operating point about which we linearize the system so that PID can be applied on the system effectively as PID is a control technique designed to be performed for linear systems and the given system to us is a non linear system.**

Given to us is :-

$$\boxed{m\ddot{x} = mg - \frac{\hat{K} i^2}{x^2}} \quad -(1)$$

$$\boxed{i = K_1 u} \quad -(2)$$

$$\boxed{K = \frac{\hat{K}}{M}} \quad -(3)$$

Mass of ball, $m = 0.020\,kg$

$K = 1.24 \times 10^{-3}$

$K_1 = 1.05$

Equilibrium Condition Values:

$x_0 = 0.009m$

$i_0 = 0.8A$

$V_0 = 0.762V$

We know PID Controllers work effectively for Linear systems but from Eqn 1 we can clearly see that eqn given to us is a Non Linear system so we have to linearize it first about the eqb point so we get Linear Approximation for small displacements and then we can apply PID effectively!!!

The Input to system is the voltage and output is the position!!

$$\boxed{m\ddot{x} = mg - \frac{\hat{K} i^2}{x^2}} \quad -(1)$$

Dividing both sides by m we get

$$\ddot{x} = g - \frac{\hat{K}}{m}\left(\frac{i^2}{x^2}\right)$$

As $K = \frac{\hat{K}}{m} = 1.24 \times 10^{-3}$

$$\boxed{\ddot{x} = g - (K)\left(\frac{i^2}{x^2}\right)}$$

As $\boxed{i = K_1(u)}$

$$\boxed{\ddot{x} = g - KK_1^2\left(\frac{u^2}{x^2}\right)} \quad (4)$$

Here u is the voltage and x is the Position

Let $x = x_0$, $u = V_0$ be the eqb values of position and Voltage and Δx and Δu be the small position and small Voltage perturbations about the equilibrium values of x_0, V_0.

At eqb we can write

$$\boxed{\ddot{x} = 0 = g - KK_1\left(\frac{V_0^2}{x_0^2}\right)} \quad (6)$$

Now assumming we give an Input Voltage of $\mu = V_0 + \Delta\mu$, and with this as Input, assumming we get the position $x = x_0 + \Delta x$, substituting this value of x and μ in Expression 4 we get

$$\boxed{\ddot{x} = g - KK_1^2 \left(\frac{V_0 + \Delta\mu}{x_0 + \Delta x}\right)^2}$$

Substituting $x = x_0 + \Delta x$ on LHS as well we get:-

$$(\ddot{x_0 + \Delta x}) = g - KK_1^2 \frac{V_0^2}{x_0^2} \frac{\left(1 + \frac{\Delta\mu}{V_0}\right)^2}{\left(1 + \frac{\Delta x}{x_0}\right)^2}$$

$$\boxed{\ddot{\Delta x} = g - KK_1^2 \left(\frac{V_0^2}{x_0^2}\right)\left(1 + \frac{\Delta\mu}{V_0}\right)^2 \left(1 + \frac{\Delta x}{x_0}\right)^{-2}}$$

Using binomial expansion assuming $\frac{\Delta\mu}{V_0}, \frac{\Delta x}{x_0}$ satisfy $\boxed{\frac{\Delta\mu}{V_0}, \frac{\Delta x}{x_0} << 1}$

We can write

$$\ddot{\Delta x} = g - KK_1^2 \left(\frac{V_0^2}{x_0^2}\right)\left(1 + \frac{2\Delta\mu}{V_0}\right)\left(1 - \frac{2\Delta x}{x_0}\right)$$

Neglecting higher order terms and from eqn Condition 6 the above expression can be Simplified to

$$\ddot{\Delta x} = KK_1^2 \left(\frac{2\Delta x}{x_0} - \frac{2\Delta\mu}{V_0}\right)$$

Taking Laplace transform of both sides and substituting:-

$$\mathcal{L}\{\Delta x\} = x(s)$$

$$\mathcal{L}\{\Delta u\} = u(s)$$

$$s^2 x(s) = \frac{2}{x_0} K K_1^2 (x(s)) - \frac{2 K K_1^2}{V_0} (u(s))$$

$$u(s) \frac{2 K K_1^2}{V_0} = x(s) \left(\frac{2 K K_1^2}{x_0} - s^2 \right)$$

$$\Rightarrow \frac{x(s)}{u(s)} = \frac{2 K K_1^2 / V_0}{\frac{2 K K_1^2}{x_0} - s^2}$$

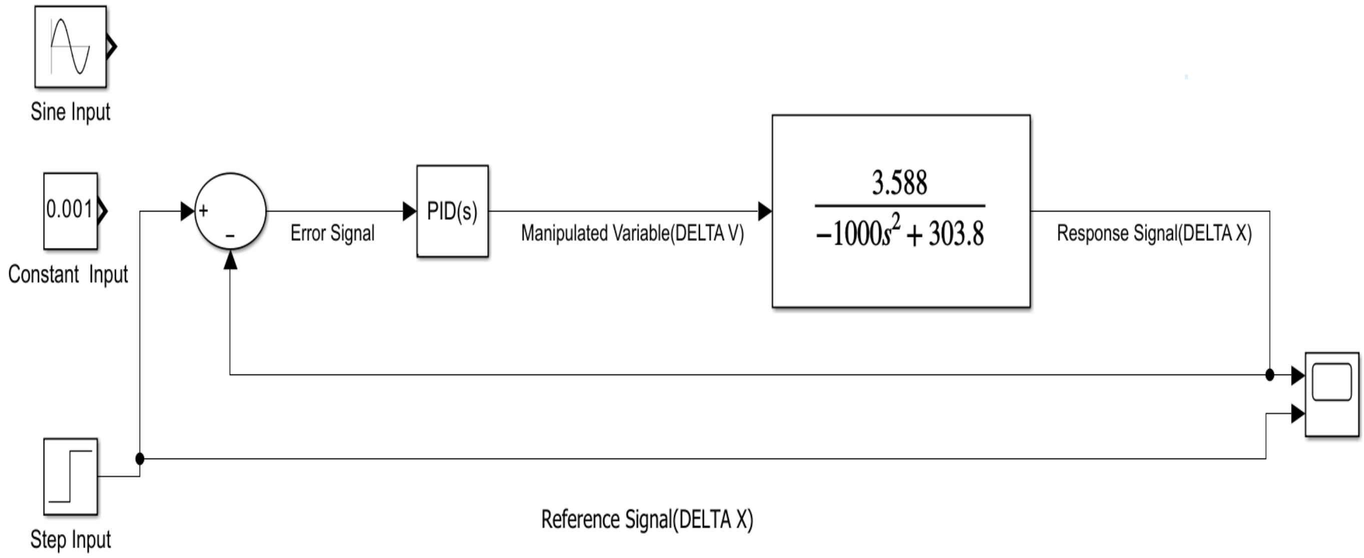
Substituting K, K_1, V, x_0 values we get the transfer function as:-

$$\boxed{\frac{x(s)}{u(s)} = \frac{3.588 \times 10^{-3}}{0.3038 - s^2}}$$

$$\boxed{\frac{x(s)}{u(s)} = \frac{3.588}{303.8 - 1000 s^2}}$$

Note:- **Here just to be clear x(s) is the Laplace transform of position about equilibrium location 0.009m and u(s) is the Laplace Transform of voltage about equilibrium voltage 0.762V .**

Simulink model with PID control:
a) Linear model and PID control



The PID parameters that we used for the PID controller after tuning the PID Parameters to get a suitable response are shown below:-

PID PARAMETERS	**VALUE**
Kp	**-25266.8647882696**
Ki	**-8662.80103423263**
Kd	**-16374.3980732966**
N	**6716.24786112542**

A step input of 0.001 m is provided, which means a displacement of 0.001 m from the equilibrium position/operating point of 0.009m and we can clearly see that the response signal was closely able to follow the reference signal .

On the y axis is the Δx (or position with respect to equilibrium position in m) and on x axis we have time(sec). Yellow curve is the reference signal and blue curve is the response signal.

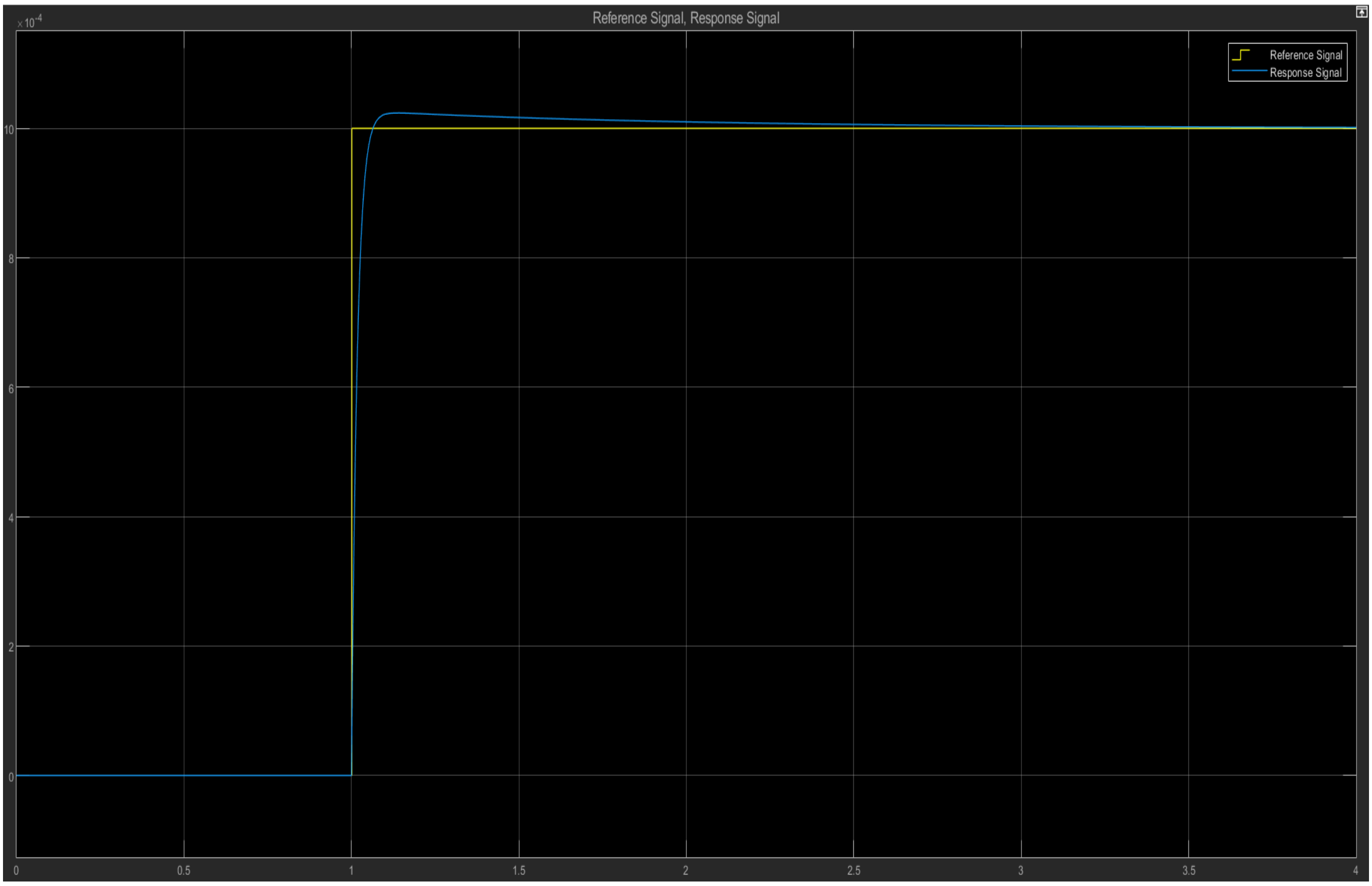


Fig: Controlled response of linear model to Step reference Input

A constant input of 0.001 m is provided, which means a displacement of 0.001 m from the equilibrium position/operating point of 0.009m and we can clearly see that the response signal was closely able to follow the reference signal .

On the y axis is the $\Delta \mathbf{x}$ (or position with respect to equilibrium position in m) and on x axis we have time(sec) . Yellow curve is the reference signal and blue curve is the response signal.

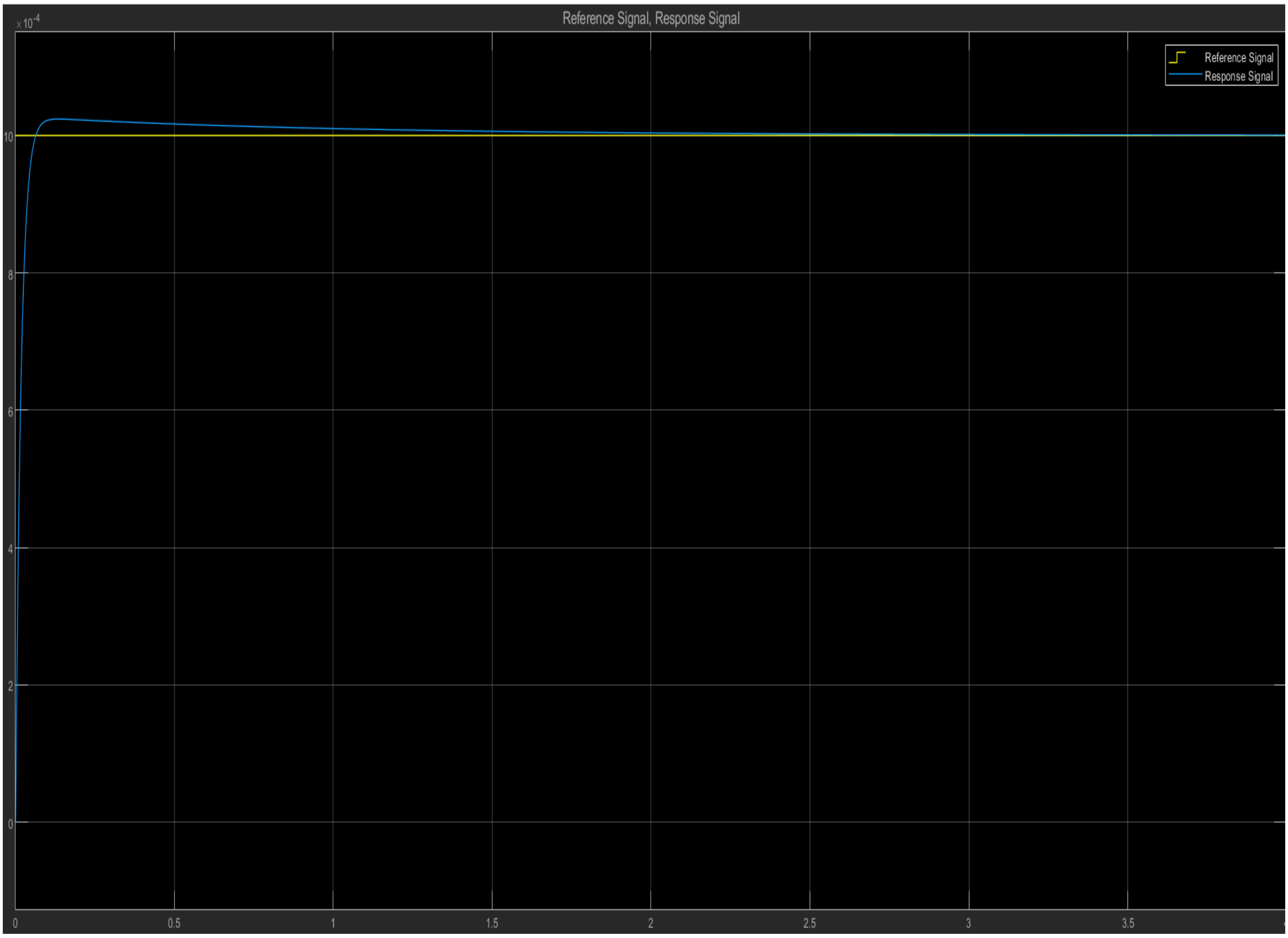


Fig: Controlled response of linear model to constant reference Input

A sine input of $\Delta \mathbf{x} = \mathbf{0.001} \boldsymbol{sin}(0.5\boldsymbol{t})\ \boldsymbol{m}$ is provided, which means the position should vary like a sine wave of 0.001 m amplitude and angular frequency 0.5 as this is the desired response about the equilibrium position/operating point of 0.009m and we can clearly see that the response signal was closely able to follow the reference signal .

On the y axis is the $\Delta \mathbf{x}$ (or position with respect to equilibrium position in m) and on x axis we have time. Yellow curve is the reference signal and blue curve is the response signal.

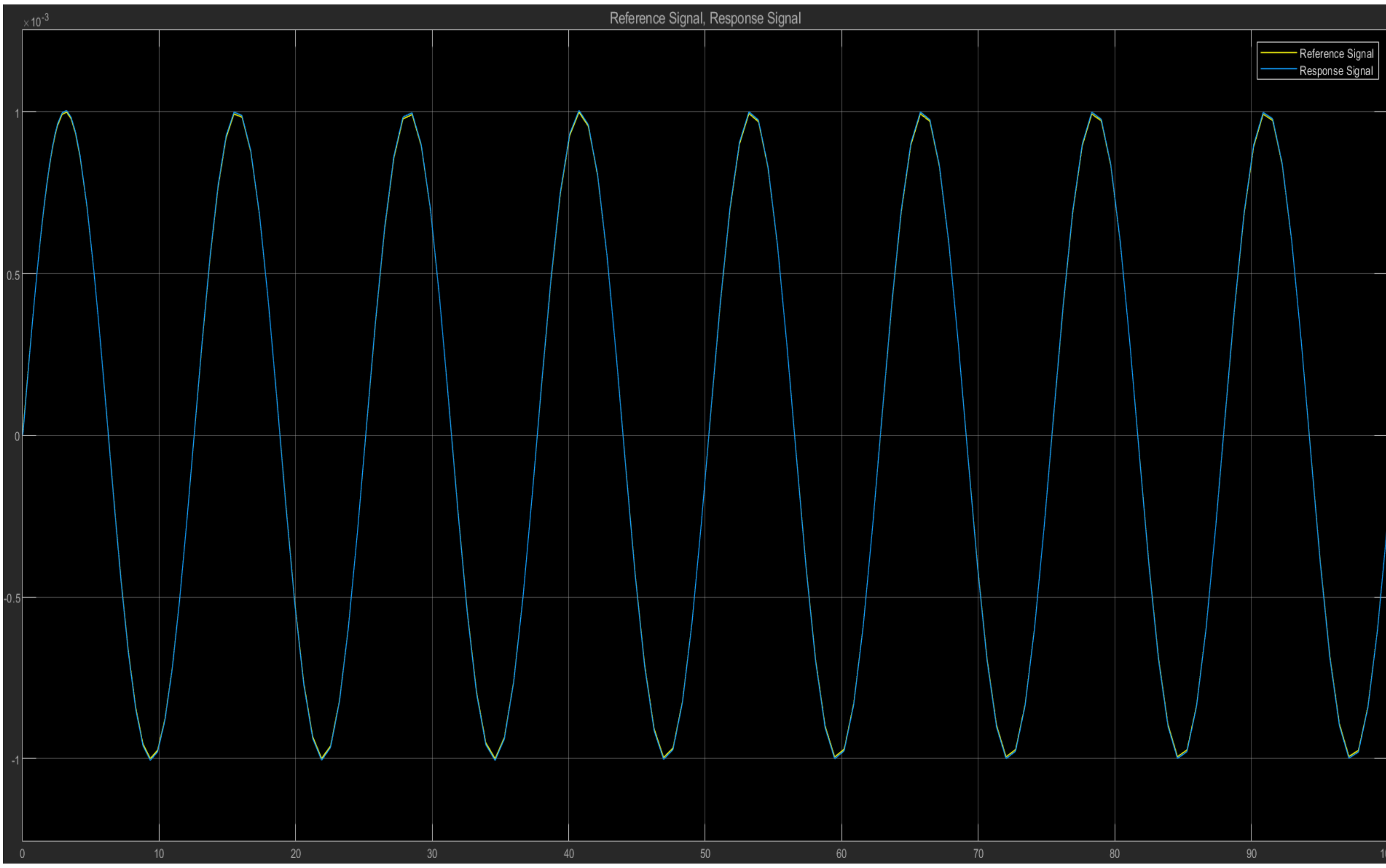


Fig: Controlled response of linear model to Sine reference Input

b) Non-linear model and PID control:

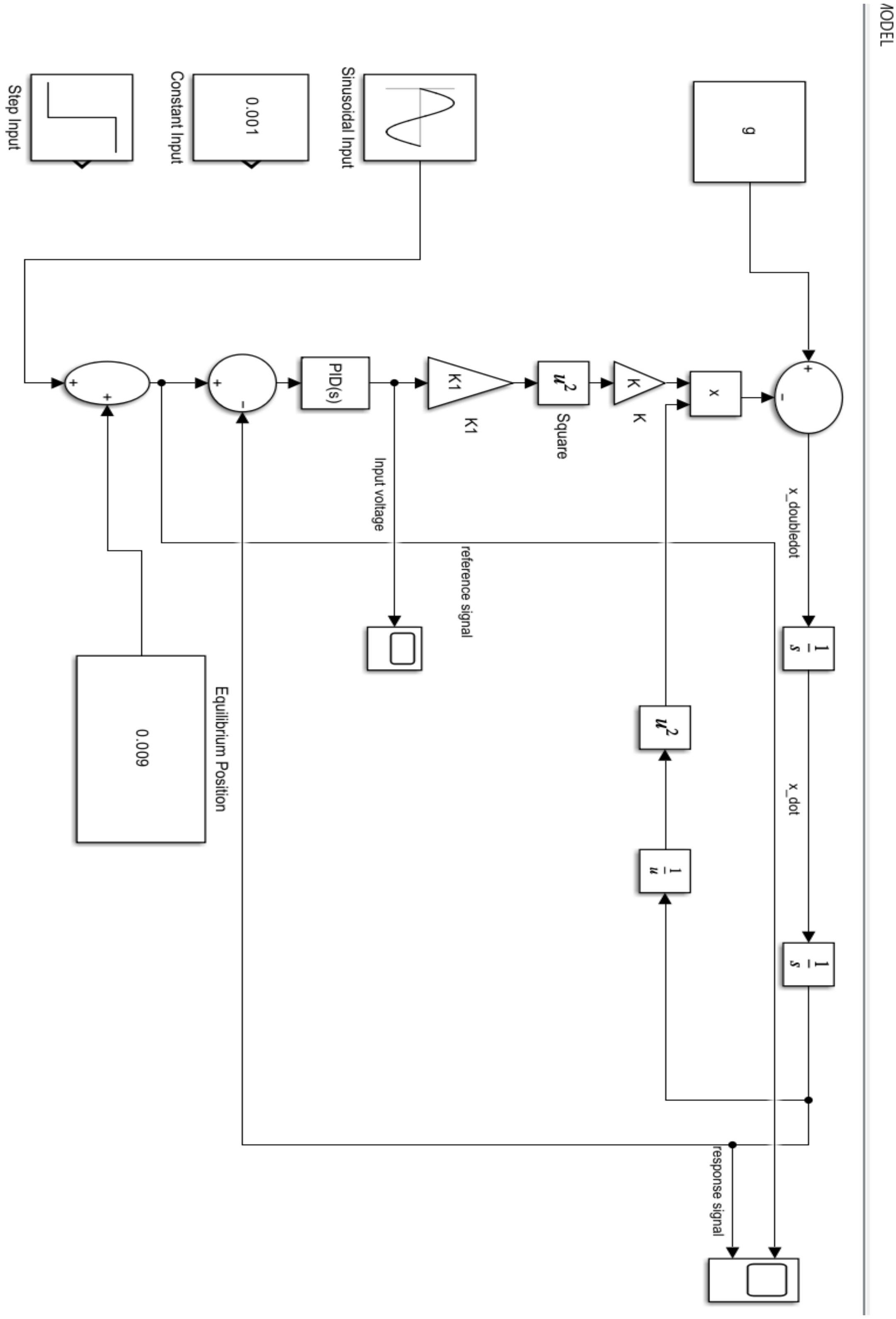


Fig: Non-Linear Maglev complete system with PID control

The appropriately tuned PID parameters used are shown below:-

PID PARAMETERS	**VALUE**
Kp	**-2000**
Ki	**-42000**
Kd	**-20**
N	**100**

A step input of 0.001m is provided , which means a displacement of 0.001 m from the equilibrium position of 0.009m and we can clearly see that the response signal was not able to follow the reference signal and this is because PID controllers are not really designed to work for non linear systems and hence they do a poor job in controlling a non linear system just as shown below .

On the y axis is the X(Position in metre) and on x axis we have time(sec). Yellow curve is the reference signal and blue curve is the response signal.

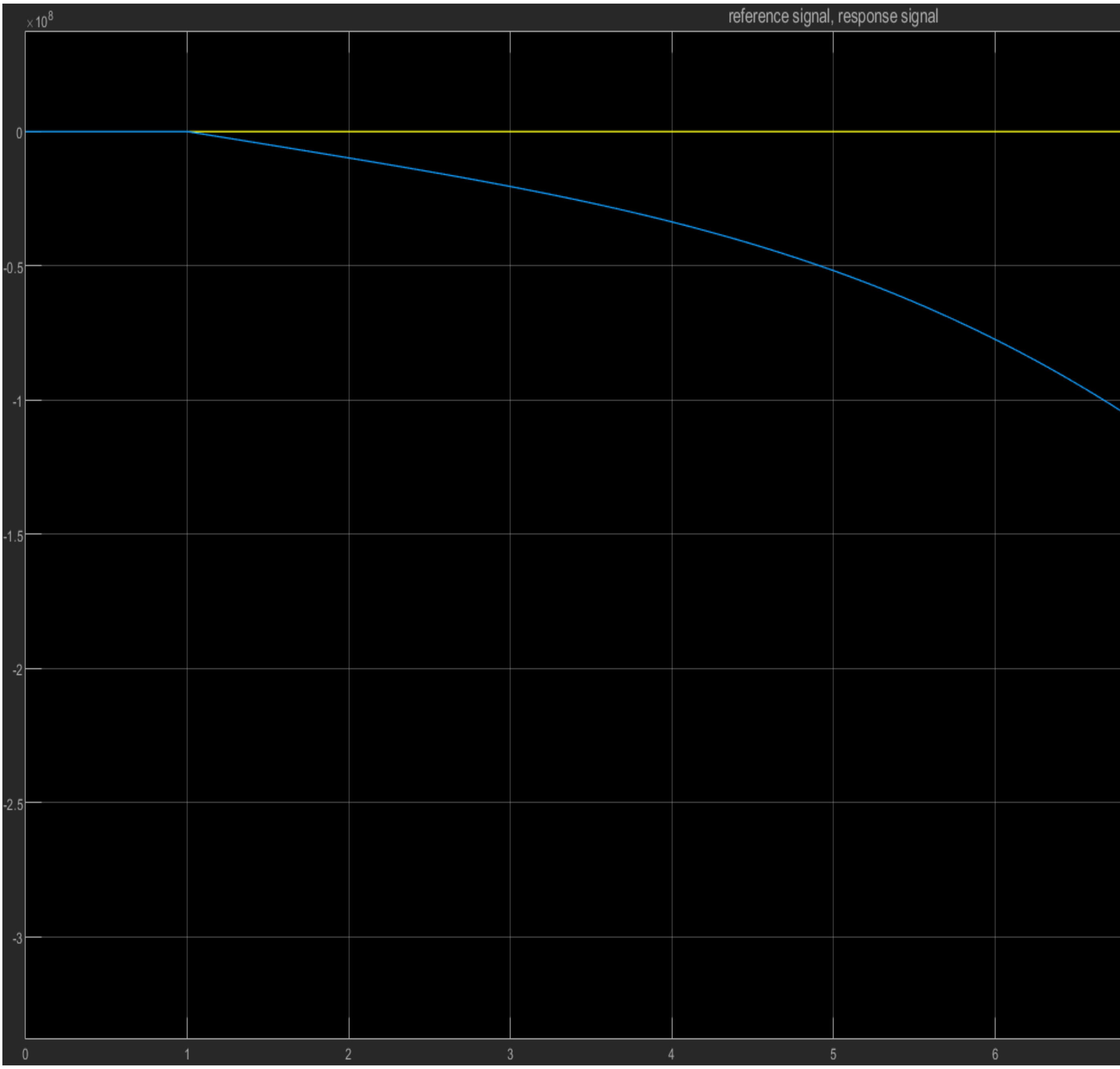
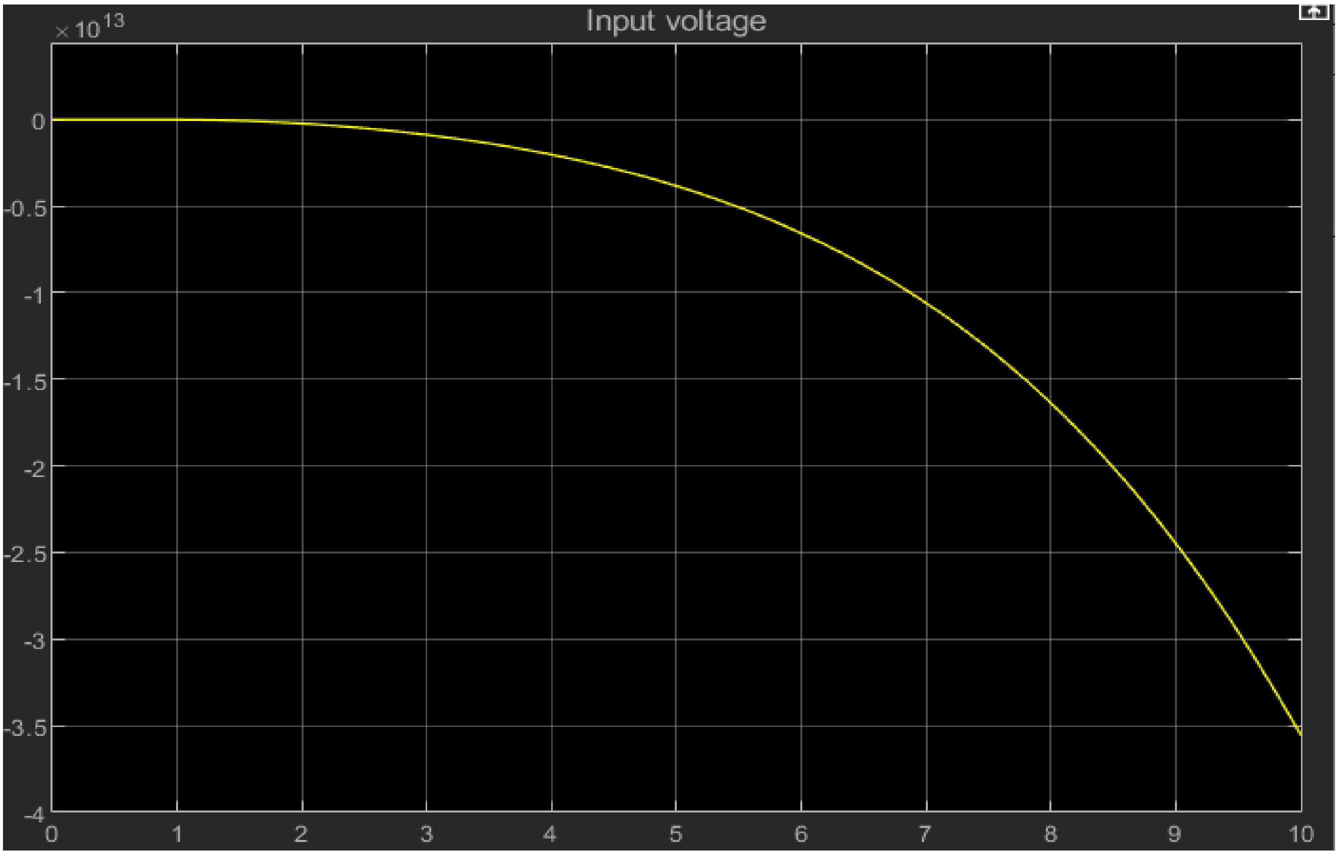


Fig: Controlled response of non-linear model to Step reference Input

Input Voltage Vs Time curve is shown below when step reference input is applied to non linear model shown above:-



A constant input of 0.001m is provided , which means a displacement of 0.001 m from the equilibrium position of 0.009m and we can clearly see that the response signal was not able to follow the reference signal and this is because PID controllers are not really designed to work for non linear systems and hence they do a poor job in controlling a non linear system just as shown below .

On the y axis is the X(Position in metre) and on x axis we have time(sec). Yellow curve is the reference signal and blue curve is the response signal.

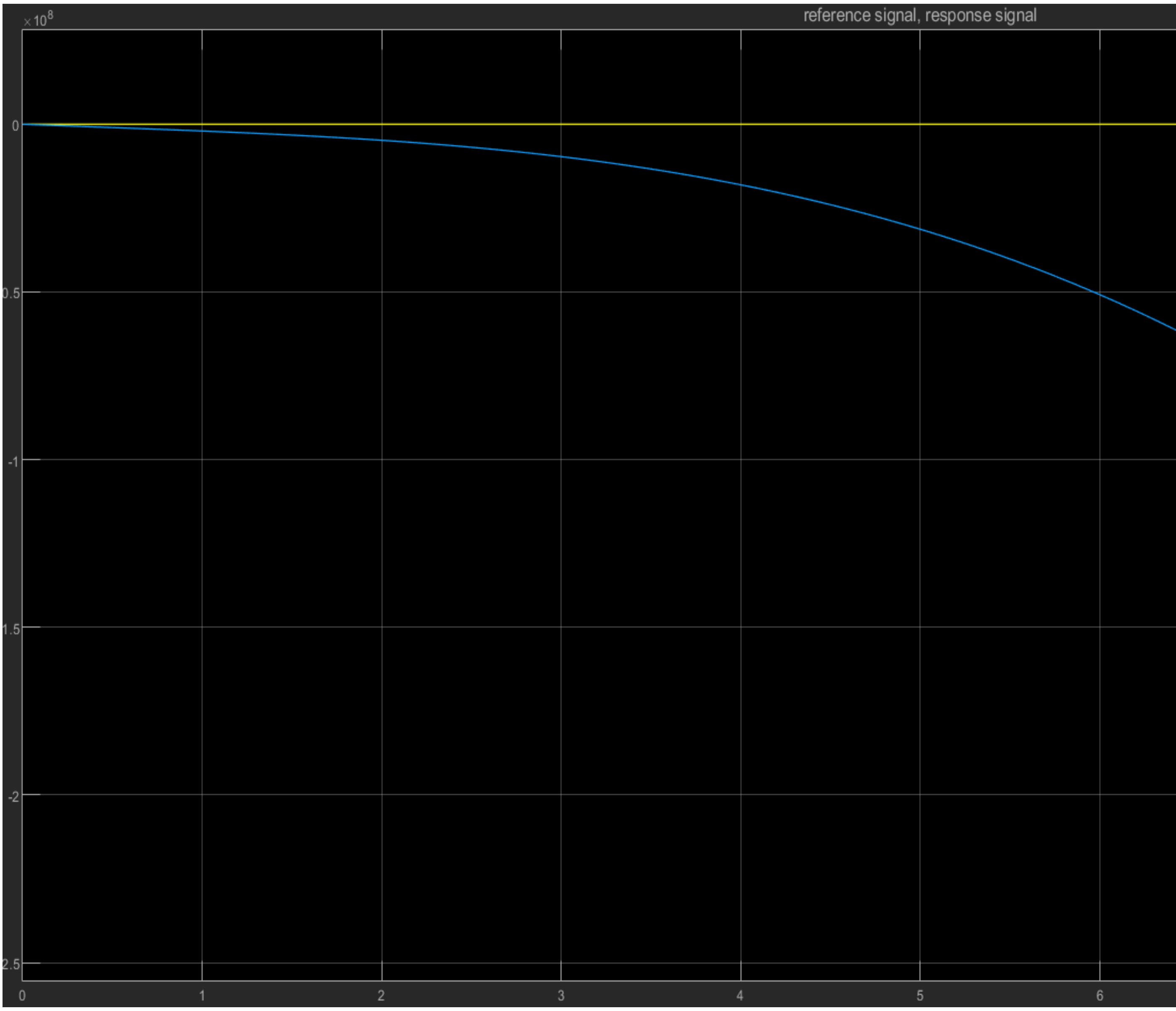
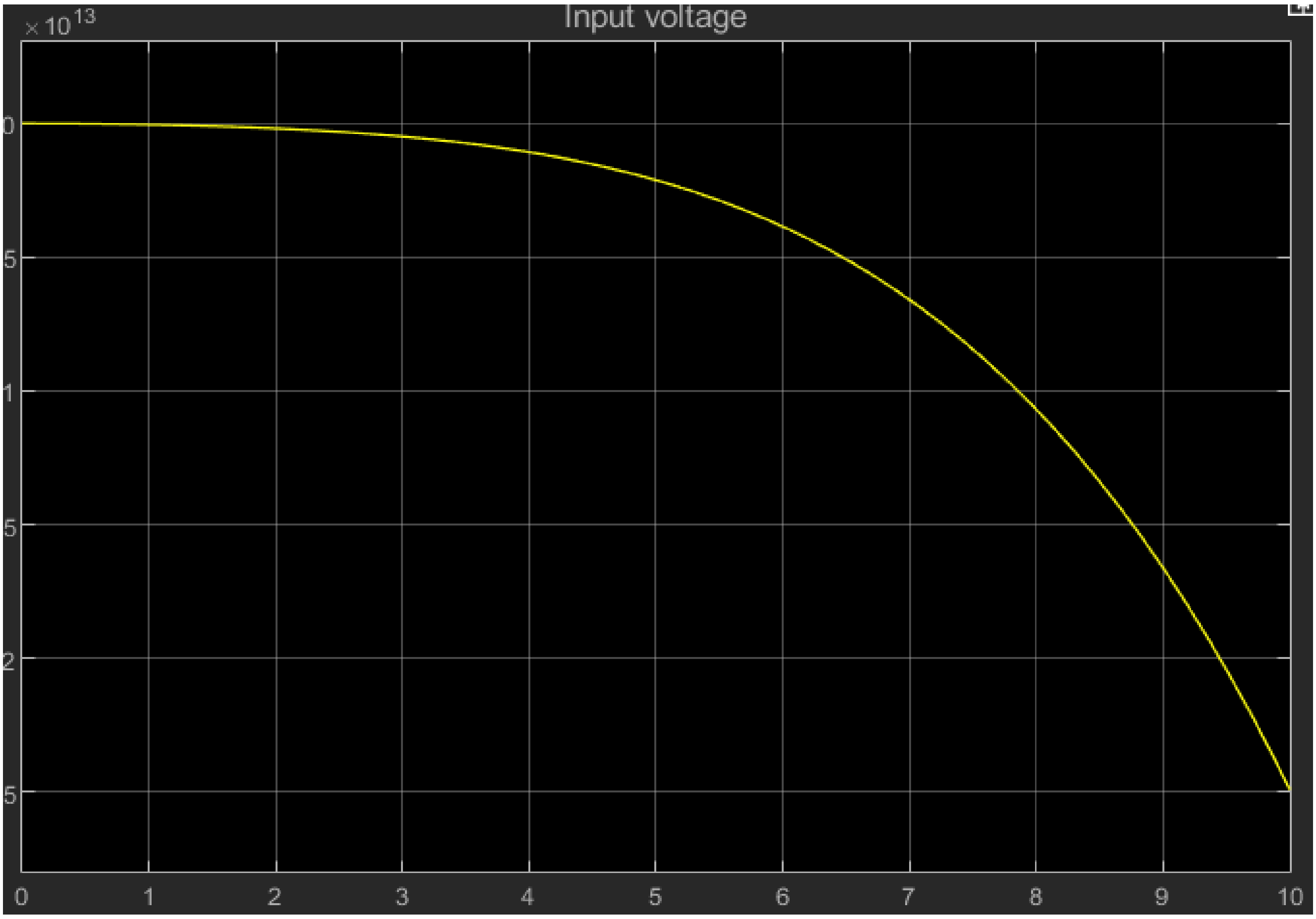


Fig: Controlled response of non-linear model to constant reference Input

Input Voltage Vs Time curve is shown below when constant reference input is applied to the non linear model shown above:-



A sine input of $\Delta x = \mathbf{0.001sin(0.5}t\mathbf{)}\ m$ is provided, which means the position should vary like a sine wave of 0.001 m amplitude and angular frequency 0.5 as this is the desired response about the equilibrium position of 0.009m and we can clearly see that the response signal was somewhat able to follow the reference signal but not as well as the linearized system followed the sinusoidal input and this is because PID controllers are not really designed to work for non linear systems and hence they do a poor job in controlling a non linear system just as shown below .

On the y axis is the X (Position in metre) and on x axis we have time(sec). Yellow curve is the reference signal and blue curve is the response signal.

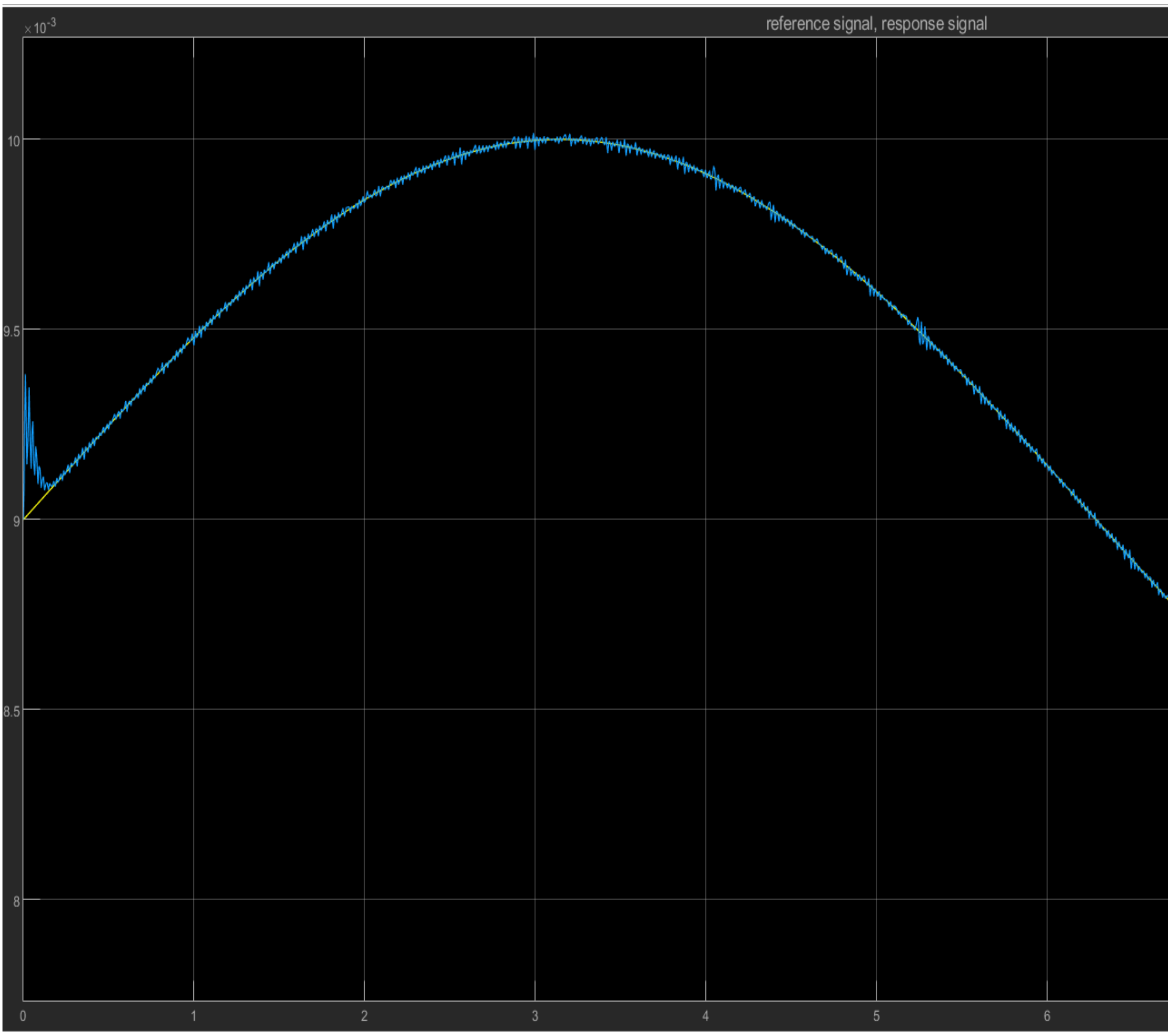
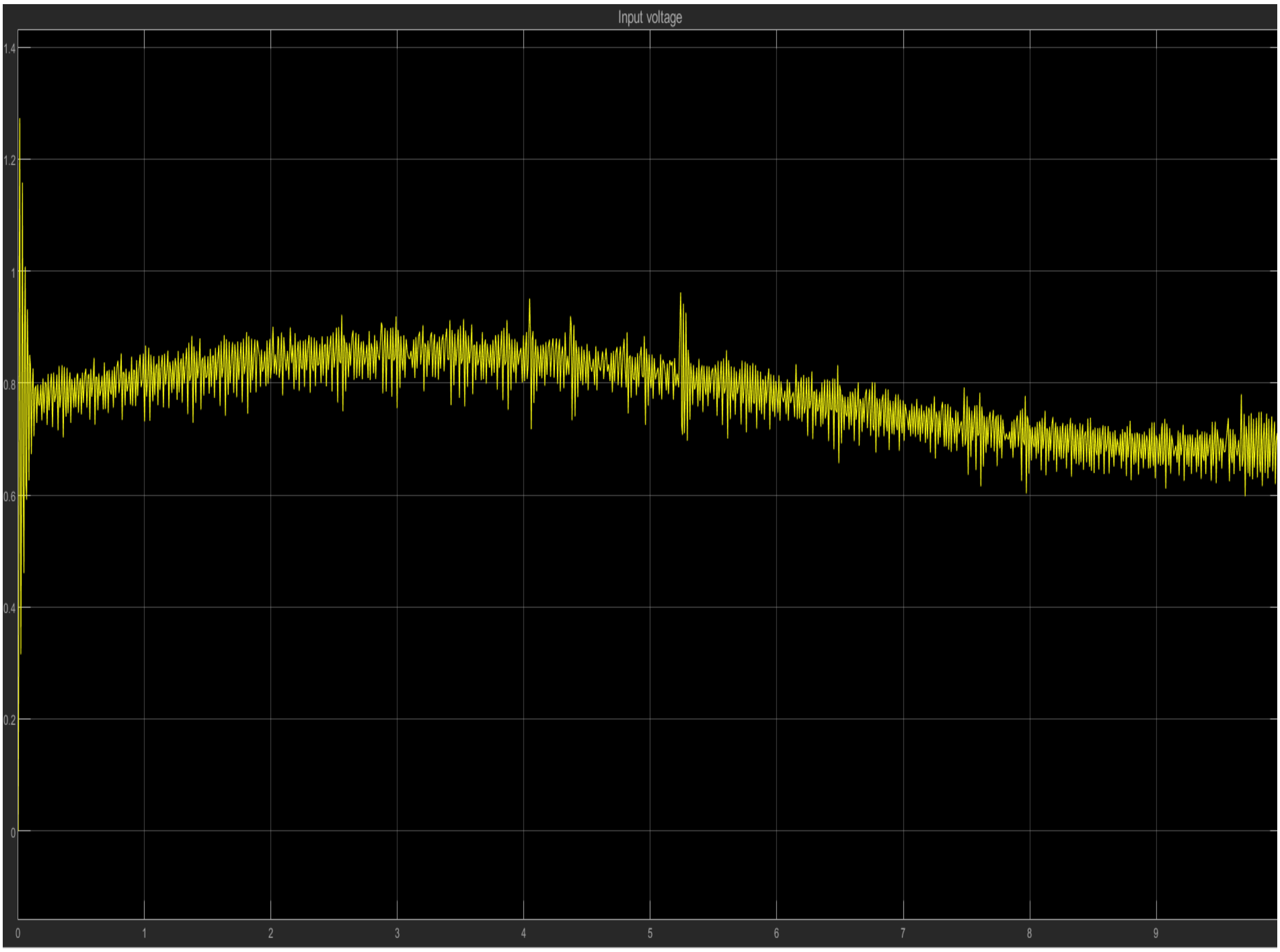


Fig: Controlled response of non-linear model to Sinusoidal reference Input

Input Voltage(V) Vs Time(sec) curve is shown below when Sinusoidal reference input is applied to non linear model shown above :-



Discussions/Conclusions

The 3 sets of inputs used to test were same for both linearized approximation of the Magnetic levitation system and the non linear system which the magnetic levitation system actually is , we can clearly see that PID does a poor job in controlling when applied to the non linear system but works fantastically when applied to the linearized approximation of the magnetic levitation system for small disturbances about the operating point which is the point about which we have linearized the system . The final conclusions can be summarized as follows:-

a) The linear model with PID controller is perfectly able to control the system towards step, sinusoidal or constant input.

b) The non-linear model with PID controller is not really able to control the system towards step and constant response and is barely able to control the system towards sinusoidal input.