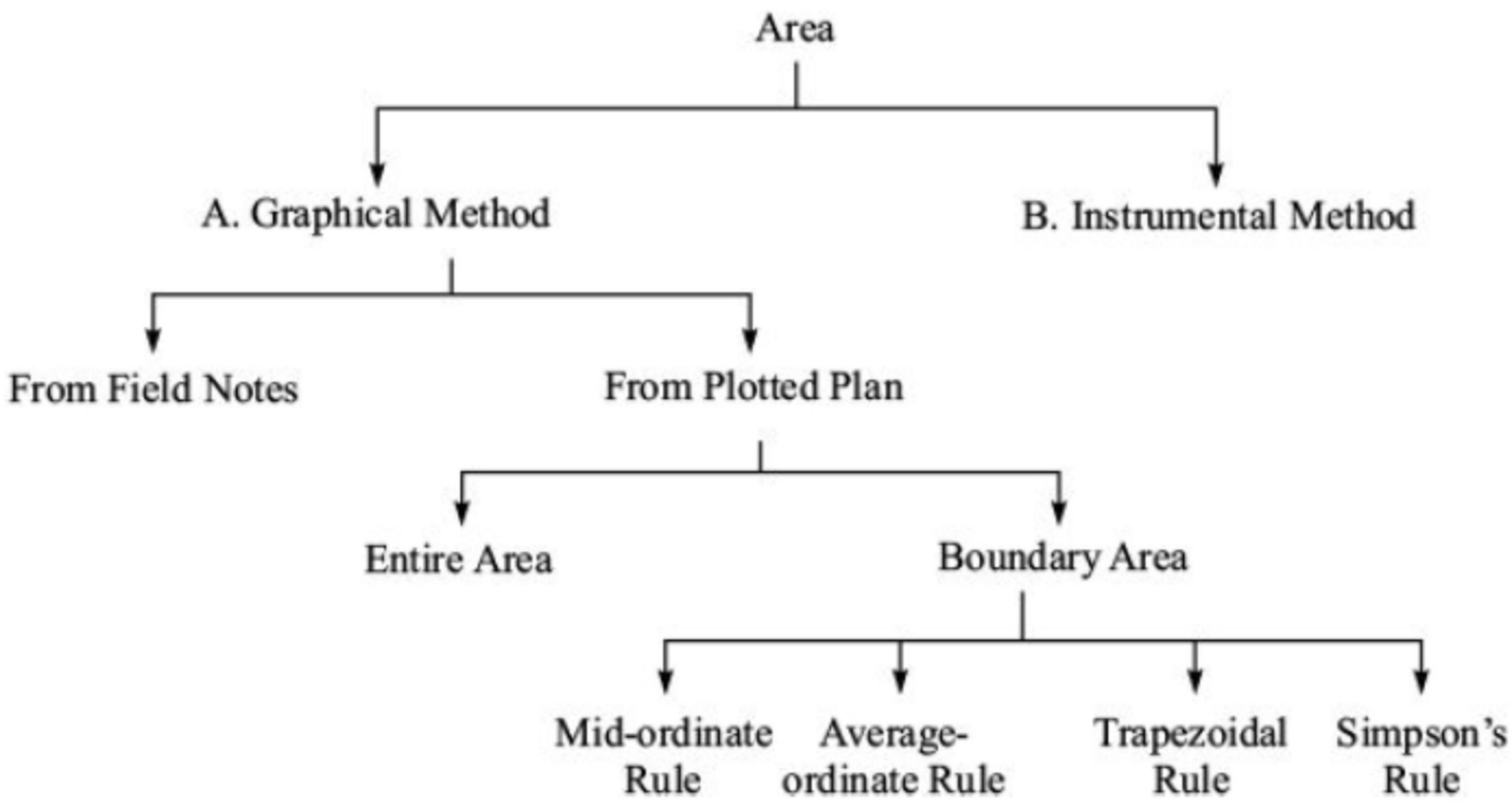
Calculations of Area and Volume

The term ‘area’ in the context of surveying refers to the area of a tract of land projected upon the horizontal plane, and not to the actual area of the land surface.

Area may be expressed in the following units:

1. Square metres
2. Hectares (1 hectare = 10,000 m^2)
3. Square feet
4. Acres (1 acre = 4840 sq. yd. = 43.560 sq. ft.)

The following is a hierarchical representation of the various methods of computation of area.



COMPUTATION OF AREA FROM FIELD NOTES

This is done in two steps.

Step 1

- In cross-staff survey, the area of field can be directly calculated from field notes. During survey work the whole area is divided into some geometrical figures, such as triangles, rectangles, squares and trapeziums.

1. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where a, b and c are the sides,

and s = $\frac{a+b+c}{2}$

or Area of triangle = $1/2 \times b \times h$

where b = base

and h = altitude

2. Area of rectangle = $a \times b$

where a and b are the sides.

3. Area of square = a^2

where a is the side of the square.

4. Area of trapezium = $1/2\ (a + b) \times d$

where a and b are the parallel sides, and d is the perpendicular distance between them.

The area along the boundaries is calculated as follows) - step 2

o_1, o_2 = ordinates

x_1, x_2 = chainages

Area of shaded portion = $\frac{o_1 + o_2}{2} \times (x_2 - x_1)$

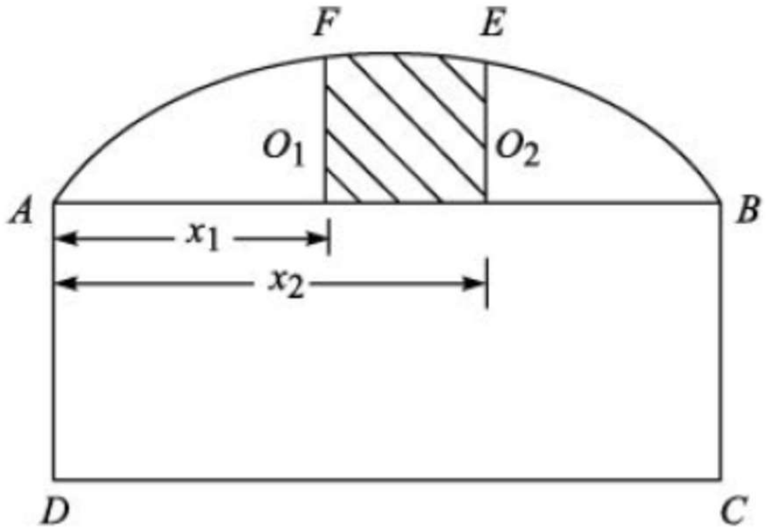


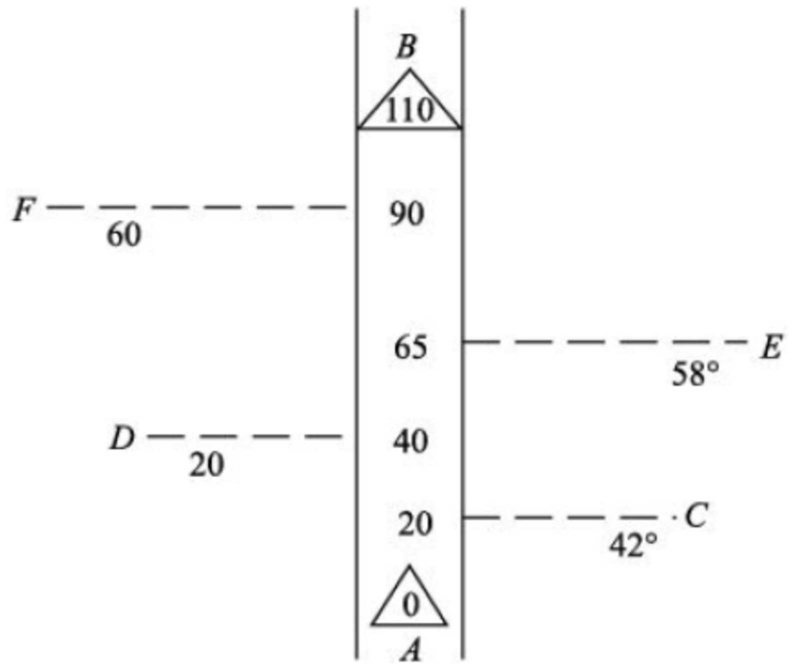
Fig. 7.1 Area Calculation

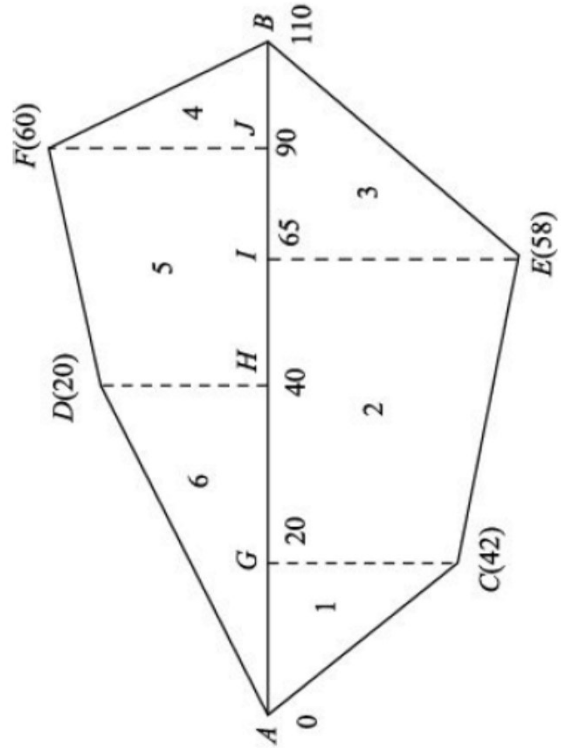
Similarly, the areas between all pairs of ordinates are calculated and added to obtain the total boundary area.

Hence, Total area of the field = Area of geometrical figure + Boundary areas

PROBLEMS ON COMPUTING AREA FROM FIELD NOTES

A page of the field book of a cross-staff survey is given in. Plot the required figure and calculate the relevant area





The result is given in the following table:

Sl No.	Figure	Chainage (m)	Base (m)	Offset (m)	Mean offset (m)	Area (m^2) +ve	Area (m^2) –ve	Remark
1	2	3	4	5	6	7	8	9
1	ΔACG	0 and 20	20	0 and 42	21	420	—	Area = col. 4 × col. 6
2	Trap. GCEI	20 and 65	45	42 and 58	50	2,250	—	
3	ΔIEB	65 and 110	45	58 and 0	29	1,305	—	
4	ΔBFJ	90 and 110	20	0 and 60	30	600	—	
5	Trap. FJHD	40 and 90	50	60 and 20	40	2,000	—	
6	ΔDHA	0 and 40	40	20 and 0	10	400	—	

6,975

Area of field = 6975 sq. m

COMPUTATION OF AREA FROM PLOTTED PLAN

The area may be calculated in the two following ways.

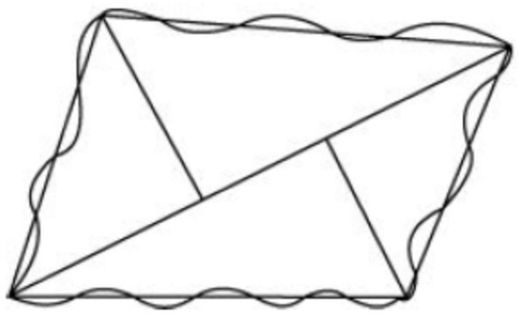
Case I—Considering the Entire Area

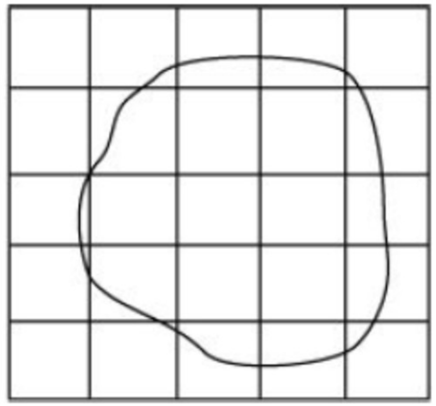
The entire area is divided into regions of a convenient shape, and calculated as follows:

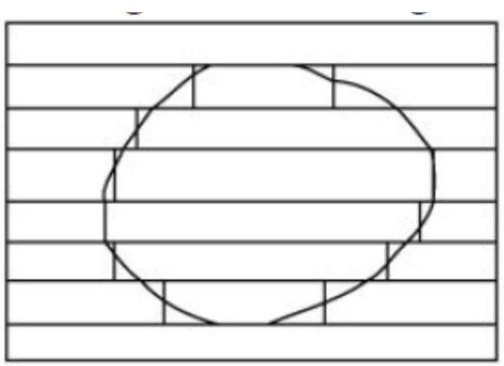
By Dividing the Area into Triangles

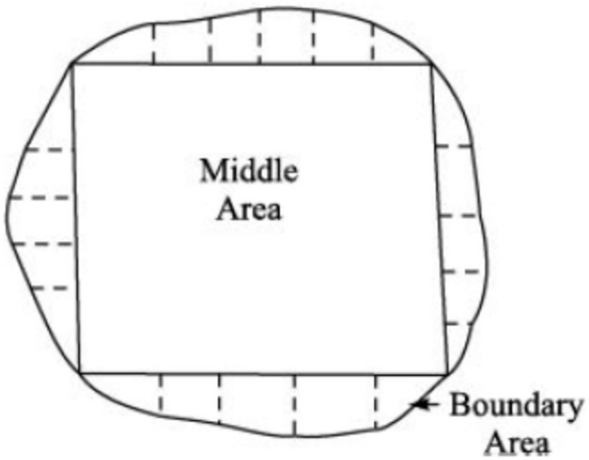
By Dividing the Area into Squares

By Drawing Parallel Lines and Converting them to Rectangles








Middle
Area
Boundary
Area

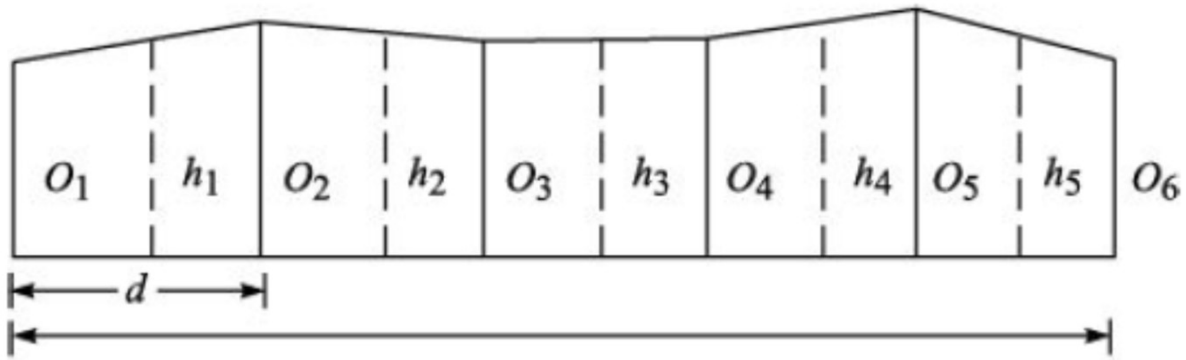
Boundary area

In this method, a large square or rectangle is formed within the area in the plan. Then ordinates are drawn at regular intervals from the side of the square to the curved boundary.

The middle area is calculated in the usual way. The boundary area is calculated according to one of the following rules:

1. The mid-ordinate rule
2. The average-ordinate rule
3. The trapezoidal rule
4. Simpson’s rule

The mid-ordinate rule



Let O_1, O_2, O_3, …, O_n = ordinates at equal intervals

l = length of base line

d = common distance between ordinates

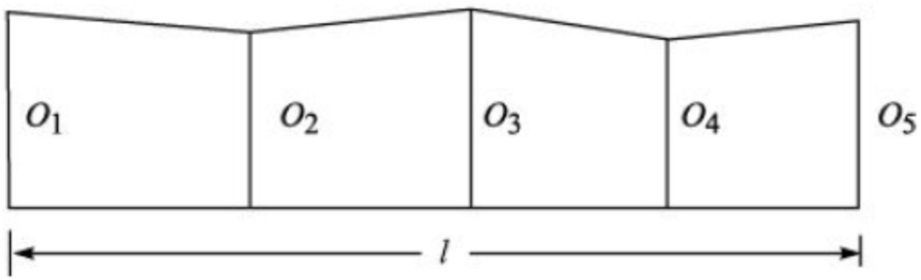
h_1 h_2, …, h_n = mid-ordinates

Area of plot = $h_1 \times d + h_2 \times d + \ldots + h_n \times d$

$= d(h_1 + h_2 + \ldots + h_n)$

i.e. Area = Common distance × sum of mid-ordinates

Average ordinate rule



Let $O_1\ O_2, \ldots, O_n$ = ordinates or offsets at regular intervals

l = length of base line

n = number of divisions

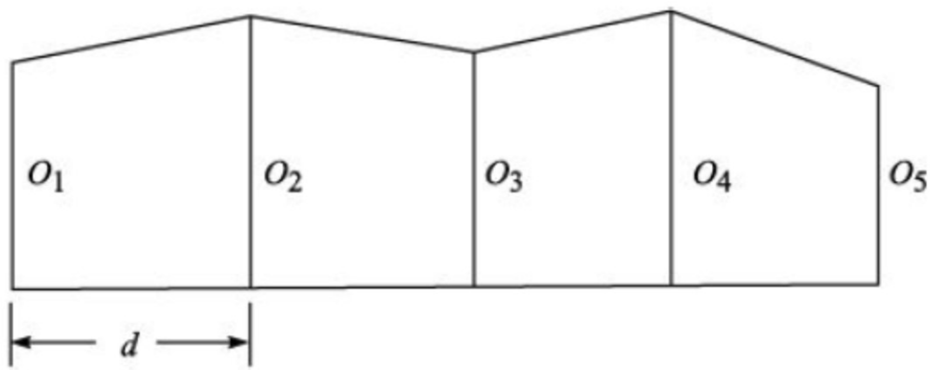
$n + 1$ = number of ordinates

$$\text{Area} = \frac{O_1 + O_2 + \cdots + O_n}{O_{n+1}} \times l \quad (7.2)$$

$$\text{i.e. Area} = \frac{\text{sum of ordinates}}{\text{no. of ordinates}} \times \text{length of base line}$$

Trapezoidal Rule

- While applying the trapezoidal rule, boundaries between the ends of ordinates are assumed to be straight. Thus the areas enclosed between the base line and the irregular boundary line are considered as trapezoids.



Let O_1 O_2, …, O_n = ordinates at equal intervals

d = common distance

First area = $\frac{O_1 + O_2}{2} \times d$

Second area = $\frac{O_2 + O_3}{2} \times d$

Third area = $\frac{Q_3 + Q_4}{2} \times d$

…………………………..

Last area = $\frac{Q_{n-1} + Q_n}{2} \times d$

Total area = $\frac{d}{2}$ $\{O_1 + 2O_2 + \ldots + 2O_{n-1} + O_n\}$ (7.3)

$= \frac{\text{common distance}}{2} \{(\text{1st ordinate} + \text{last ordinate} + 2\,(\text{sum of other ordinate})\}$

Thus, the trapezoidal rule may be stated as follows:

To the sum of the first and the last ordinate, twice the sum of intermediate ordinates is added. This total sum is multiplied by the common distance. Half of this product is the required area.

Simpson’s Rule

- In this rule, the boundaries between the ends of ordinates are assumed to form an arc of a parabola. Hence, Simpson’s rule is sometimes called the *parabolic rule*.

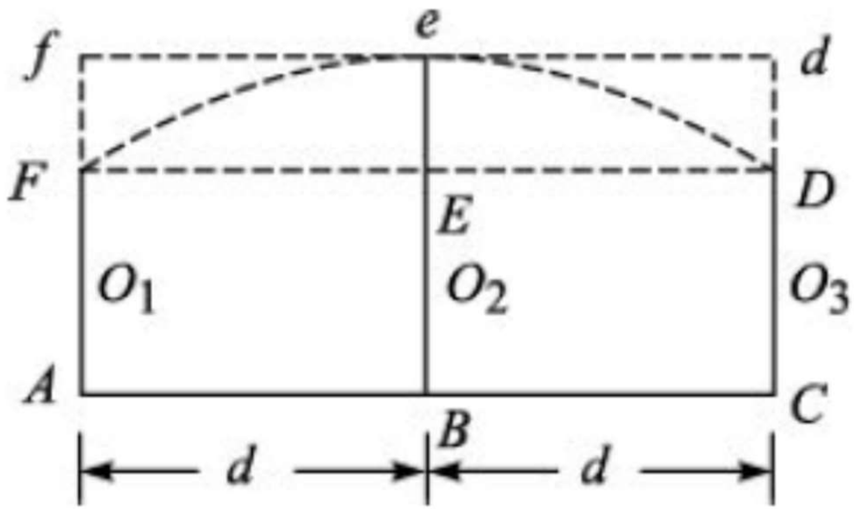


Fig. 7.9 Simpson’s Rule

Let

O_1 O_2, O_3 = three consecutive ordinates

d = common distance between the ordinates

Area *AFeDC* = area of trapezium *AFDC*

+ area of segment *FeDEF*

Here,

Area of trapezium = $\frac{O_1 + O_3}{2} \times 2d$

Area of segment = $\frac{2}{3}$ area of parallelogram *FfdD*

$= \frac{2}{3} \times Ee \times 2d = \frac{2}{3} \times \left\{ O_2 - \frac{O_1 + O_3}{2} \right\} \times 2d$

So, the area between the first two divisions,

$$\Delta_1 = \frac{O_1 + O_3}{2} \times 2d + \frac{2}{3} \left\{ O_2 - \frac{O_1 + O_3}{2} \right\} \times 2d$$

$$= \frac{d}{3}(O_1 + 4O_2 + O_3)$$

Similarly, the area between next two divisions,

$\Delta_2 = \frac{d}{3}(O_3 + 4O_4 + O_5)$, and so on.

$\therefore$ Total area $= \frac{d}{3}(O_1 + 4O_2 + 2O_3 + 4O_4 + \cdots + O_n)$

$$= \frac{d}{3}\{O_1 + O_n + 4(O_2 + O_4 + \ldots) + 2(O_3 + O_5 + \ldots)\} \quad (7.4)$$

$= \frac{\text{common distance}}{3}$ {first ordinate + last ordinate)

+ 4 (sum of even ordinates)

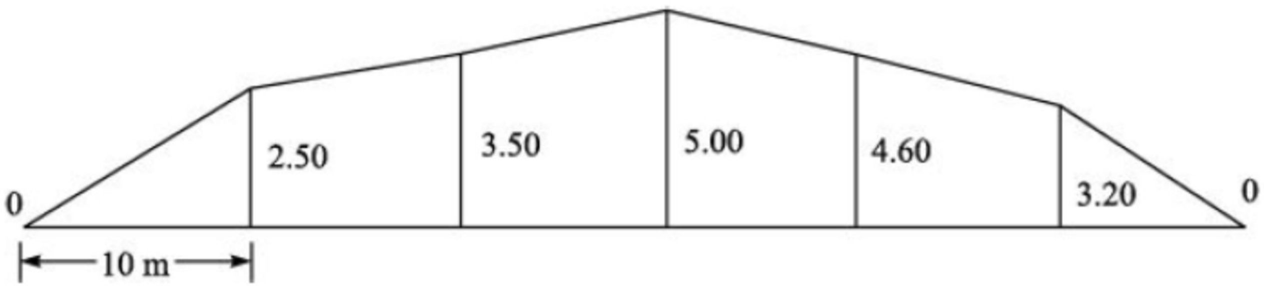
+ 2 (sum of remaining odd ordinates)}

Limitation This rule is applicable only when the number divisions is even, i.e. the number of ordinates is odd.

Trapezoidal rule	Simpson's rule
1. The boundary between the ordinates is considered straight.	1. The boundary between the ordinates is considered an arc of a parabola.
2. There is no limitation. It can be applied for any number of ordinates.	2. To apply this rule, the number of ordinates must be odd. That is, the number of divisions must be even.
3. It gives an approximate result.	3. It gives a more accurate result.

Example…The following offsets were taken from a chain line to an irregular boundary line at an interval of 10 m:
0, 2.50, 3.50, 5.00, 4.60, 3.20, 0 m compute the area between the chain line, the irregular boundary line and the end offsets by

(a) The mid-ordinate rule
(b) The average-ordinate rule
(c) The trapezoidal rule
(d) Simpson's rule



Calculation of volume

For computation of the volume of earth work, the sectional areas of the cross section which are taken transverse to the longitudinal section during profile levelling are first calculated.

the cross-sections may be of different types,

(i) level

(ii) two-level,

(iii) three-level,

(iv) side-hill two-level,

and

(v) multi-level.

After calculation of cross-sectional areas, the volume of earth work is calculated by

(i) the trapezoidal (or average end area) rule, and

(ii) the prismoidal rule.

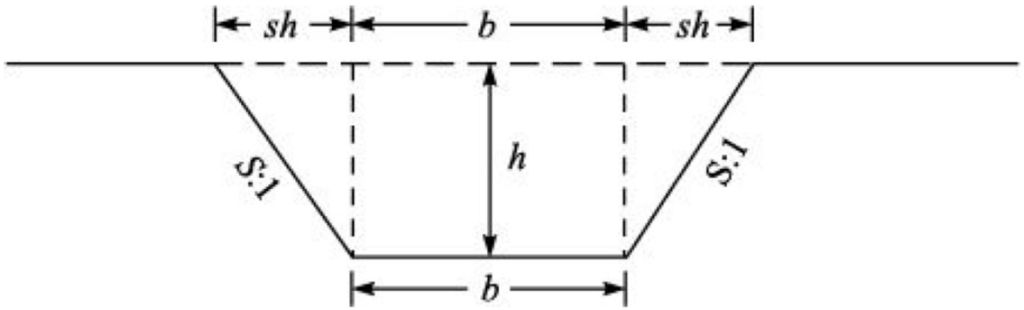
NOTES

1. The prismoidal rule gives the correct volume directly.

2. The trapezoidal rule does not give the correct volume. Prismoidal correction should be applied for this purpose. This correction is always subtractive.

3. Cutting is denoted by a positive sign and filling by a negative sign.

A. Level Section

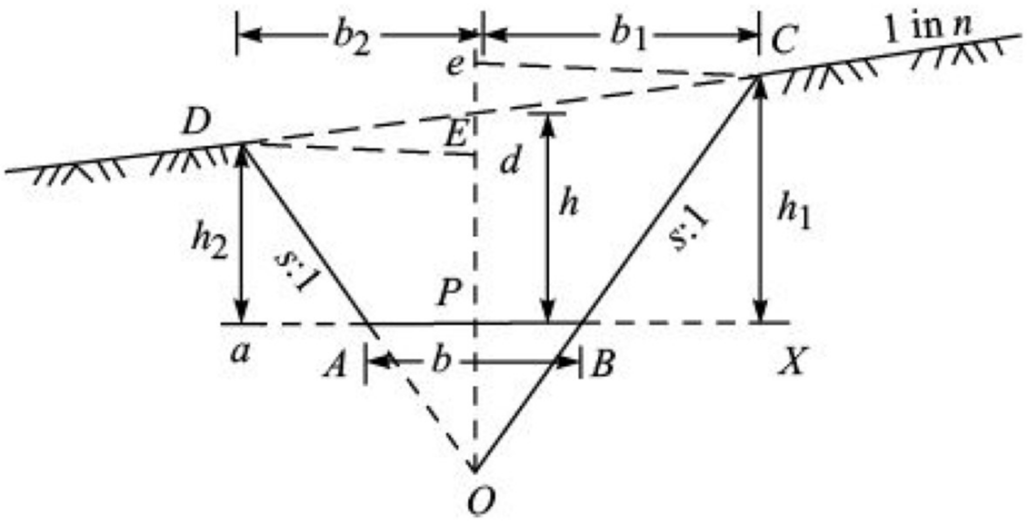


$$\text{Cross-sectional area} = \frac{b + b + 2sh}{2} \times h$$

$$= (b + sh)h \quad (8.1)$$

Two-Level Section

When the ground surface has a transverse slope



$$\text{Area } ABCED = \Delta DOE + \Delta COE - \Delta AOB$$

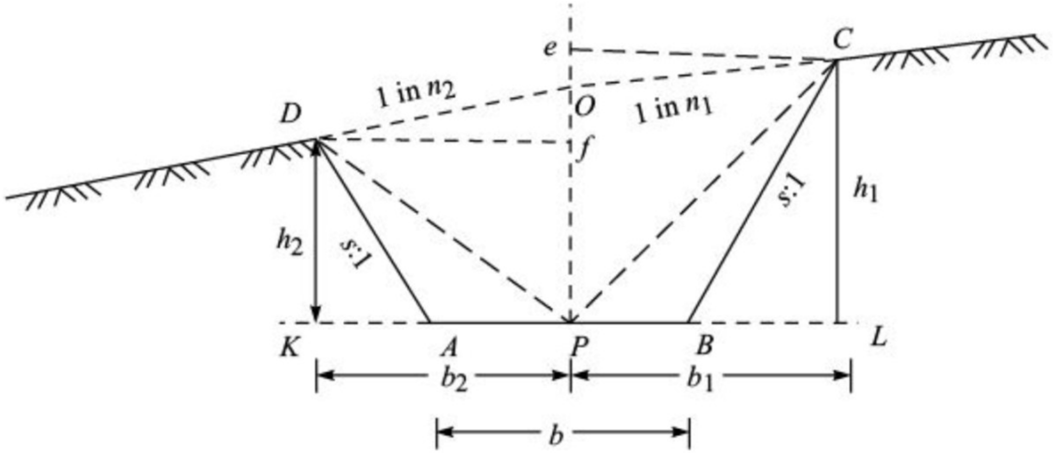
$$= \frac{1}{2} OE \times Dd + \frac{1}{2} OE \times Ce - \frac{1}{2} AB \times OP$$

$$\text{Area} = \frac{1}{2}\left\{\left(\frac{b}{2s}+h\right)b_2 + \frac{1}{2}\left(\frac{b}{2s}+h\right)b_1 - \frac{1}{2}b \times \frac{b}{2s}\right\}$$

$$= \frac{1}{2}\left\{\left(\frac{b}{2s}+h\right)(b_1+b_2) - \frac{b^2}{2s}\right\} \quad (8.6)$$

Three-Level Section

When the transverse slope is not uniform:



Area $ABCOD = \Delta DOP + \Delta COP + \Delta DAP + \Delta BCP$

$$= \frac{1}{2} \times h \times b_2 + \frac{1}{2}\, h \times b_1 + \frac{1}{2} \times \frac{b}{2}\, h_2 + \frac{1}{2} \times \frac{b}{2} \times h_1$$

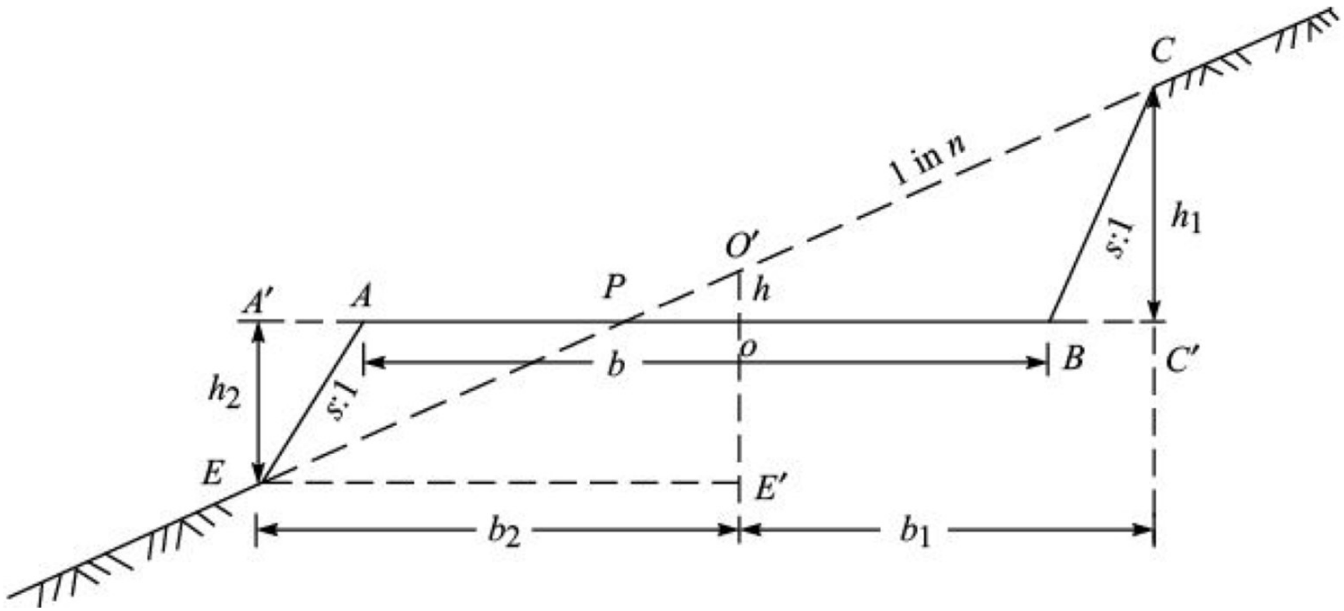
i.e. Area $= \left\{\frac{h}{2}(b_1 + b_2) + \frac{b}{4}(h_1 + h_2)\right\}$ (8.7)

Here $h_1 = OP + Oe = h + \frac{b_1}{n_1}$ (8.8)

$h_2 = OP - ef = h - \frac{b_2}{n_2}$ (8.9)

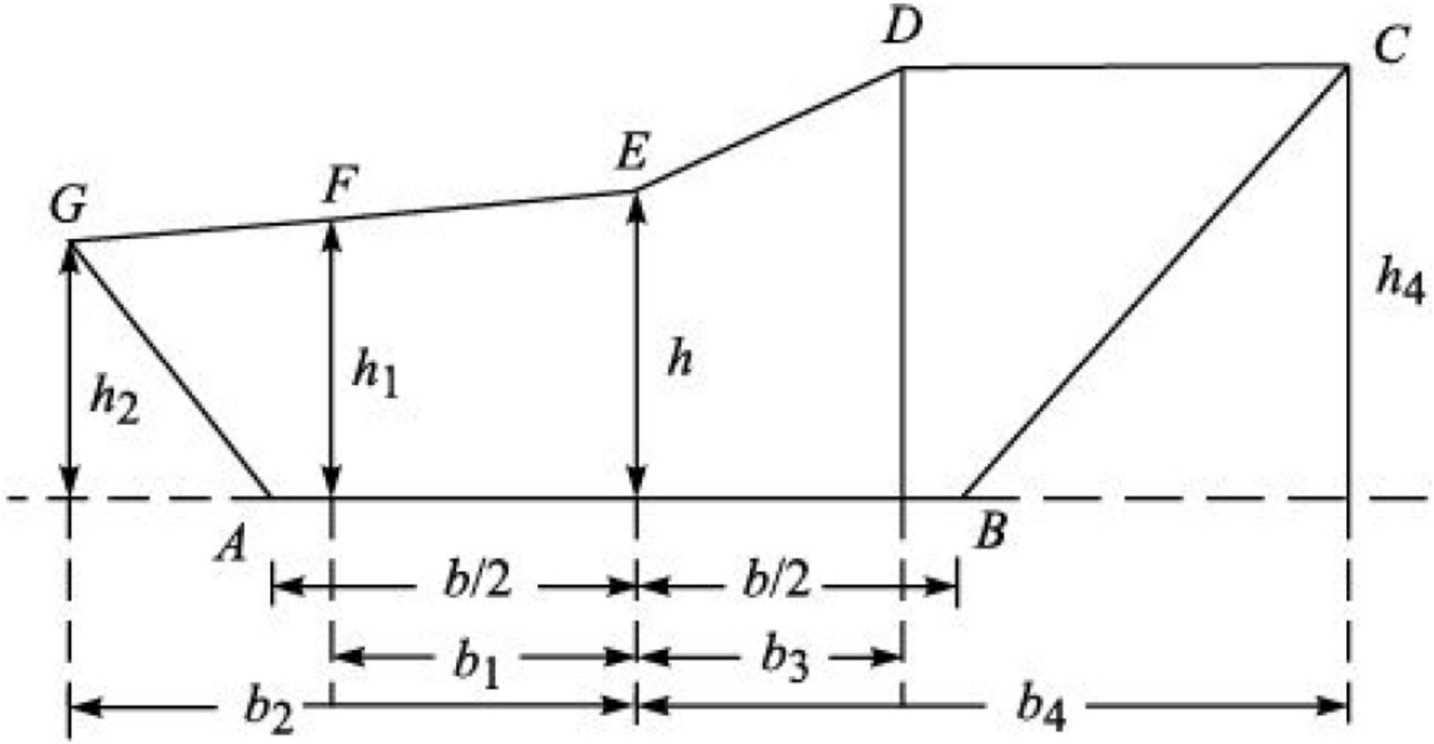
Side-Hill Two-Level Section

- When the ground surface has a transverse slope, but the slope of the ground cuts the formation level partly in cutting and partly in filling, the following method is adopted.



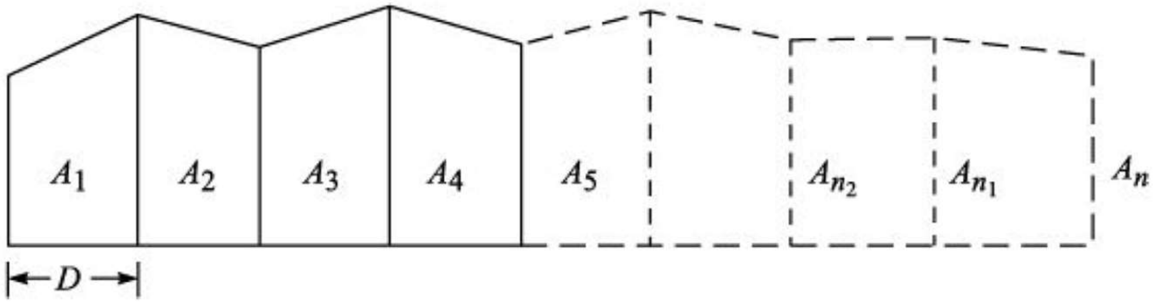
Multi-Level Section

The cross-sectional data pertaining to an irregular section are noted in the following form:



FORMULA FOR CALCULATION OF VOLUME

D = common distance between sections



A. Trapezoidal Rule (Average-End-Area Rule)

Volume (cutting or filling), $V = \frac{D}{2}\{A_1 + A_n + 2(A_2 + A_3 + \ldots + A_{n-1})\}$

i.e. volume $= \frac{\text{common distance}}{2}$ {area of first section + area of last section + 2 (sum of area of other sections)}

B. Prismoidal Formula

$$\text{Volume (cutting or filling), } V = \frac{D}{3}\{A_1 + A_n + 4(A_2 + A_4 + A_{n-1}) + 2(A_3 + A_5 + \ldots + A_{n-2})\}$$

$$\text{i.e., } V = \frac{\text{common distance}}{2}\{\text{area of first section} + \text{area of last section} + 4\,(\text{sum of areas of even sections}) + 2\,(\text{sum of areas of odd sections})\}$$

Note: The prismoidal formula is applicable when there are an odd number of sections. If the number of sections is even, the end strip is treated separately and the area is calculated according to the trapezoidal rule. The volume of the remaining strips is calculated in the usual manner by the prismoidal formula. Then both the results are added to obtain the total volume.

Example 2

An embankment of 10 m width and side slopes 1½ : 1 is required to be made on a ground which is level in a direction transverse to the centre

line. The central heights at 40 m intervals are as follows:

0.90, 1.25, 2.15, 2.50, 1.85, 1.35, and 0.85

Calculate the volume of earth work according to (a) the trapezoidal

formula, and (b) the prismoidal formula

The cross-sectional areas are calculated:

Area, $\Delta = (b + sh) \times h$

$\Delta 1 = (10 + 1.5 \times 0.90) \times 0.90 = 10.22$ m2

$\Delta 2 = (10 + 1.5 \times 1.25) \times 1.25 = 14.84$ m2

$\Delta 3 = (10 + 1.5 \times 2.15) \times 2.15 = 28.43$ m2

$\Delta 4 = (10 + 1.5 \times 2.50) \times 2.50 = 34.38$ m2

$\Delta 5 = (10 + 1.5 \times 1.85) \times 1.85 = 23.63$ m2

$\Delta 6 = (10 + 1.5 \times 1.35) \times 1.35 = 16.23$ m2

$\Delta 7 = (10 + 1.5 \times 0.85) \times 0.85 = 9.58$ m2

(a) Volume according to trapezoidal formula:

$$V = \frac{40}{2}\{10.22 + 9.58 + 2(14.84 + 28.43 + 34.38 + 23.63 + 16.23)\}$$

$$= 20\{19.80 + 235.02\} = 5{,}096.4 \text{ m}^3$$

(b) Volume calculated in prismoidal formula:

$$V = \frac{40}{3}\{10.22 + 9.58 + 4(14.84 + 34.38 + 16.23) + 2(28.43 + 23.63)\}$$

$$= \frac{40}{3}(19.80 + 261.80 + 104.12) = 5{,}142.9 \text{ m}^3$$

Example 3

An excavation is to be made for a 40 m reservoir long and 30 m wide at the bottom. The side slope of the excavation has to be 2 : 1.

Calculate the volume of earth work if the depth of excavation is 5 m.

Assume level ground at the site.

Example 4

An excavation is to be made for a 40 m reservoir long and 30 m wide at the bottom. The side slope of the excavation has to be 2 : 1.

Calculate the volume of earth work if the depth of excavation is 5 m.

Assume level ground at the site.

Example 4

The formation width of a certain cutting is 8 m and the side slope is 1 : 1. The surface of the ground has a uniform slope of 1 in 10. If the depths of cutting at the centres of three sections 40 m apart are 2, 3 and 4 m respectively, find the volume of earth work.