6. a) Given,

$$\lim_{x \to 0} \frac{x^3 - 1}{\sqrt{x} - 1}$$

$$\Rightarrow \lim_{x \to 0} \frac{(x-1)(x^2+x+1)}{\sqrt{x} - 1}$$

$$\Rightarrow \lim_{x \to 0} \frac{(x-1)(x^2+x+1)}{\frac{x-1}{\sqrt{x}+1}}$$

$$\Rightarrow \lim_{x \to 0} (x^2+x+1)(\sqrt{x}+1)$$

Substituting $x=1$, limit $= 6$

So, the limit is 6.

Also, after calculating x and $f(x)$ values in excel, we observe that as x gets closer to 1, $f(x)$ approaches 6. This numerical verification will confirm the limit.