3. a) Given $f(x)$,

$f(x) = 0$, if $x \leq 0$

$f(x) = 5 - x$, if $0 < x < 4$

$f(x) = \frac{1}{5-x}$, if $x \geq 4$

Left hand derivative at $x = 4$:

The left hand derivative is defined as:

$$f'_-(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

For $x < 4$, $f(x) = 5 - x$. Therefore:

$$f_-(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$$

Substituting $f(4+h) = 5 - (4+h) = 1 - h$, and $f(4) = \frac{1}{5-4} = 1$:

$$f'_-(4) = \lim_{h \to 0} \frac{(1-h) - 1}{h} = \lim_{h \to 0} \frac{-h}{h} = \lim_{h \to 0} -1 = -1$$

So, $f'_-(4) = -1$.

Right-hand derivative at $x=4$:
The right hand derivative is defined as:

$$f'_+(4) = \lim_{h \to 0^+} \frac{f(4+h)+f(4)}{h}.$$

For $x \geq 4$, $f(x) = \frac{1}{5-x}$,

$$\therefore f'_+(4) = \lim_{h \to 0} \frac{f(4+h)-f(4)}{h}$$

Substituting,

$$f'_+(4) = \lim_{h \to 0} \frac{\frac{1}{1-h} - 1}{h}$$

Simplifying the numerator,

$$\frac{1}{1-h} - 1 = \frac{1-(1-h)}{1-h} = \frac{h}{1-h}$$

So, $f'(+)\,4 = \lim_{h \to 0^+} \frac{\frac{h}{1-h}}{h} = \lim_{h \to 0^+} \frac{1}{1-h}$

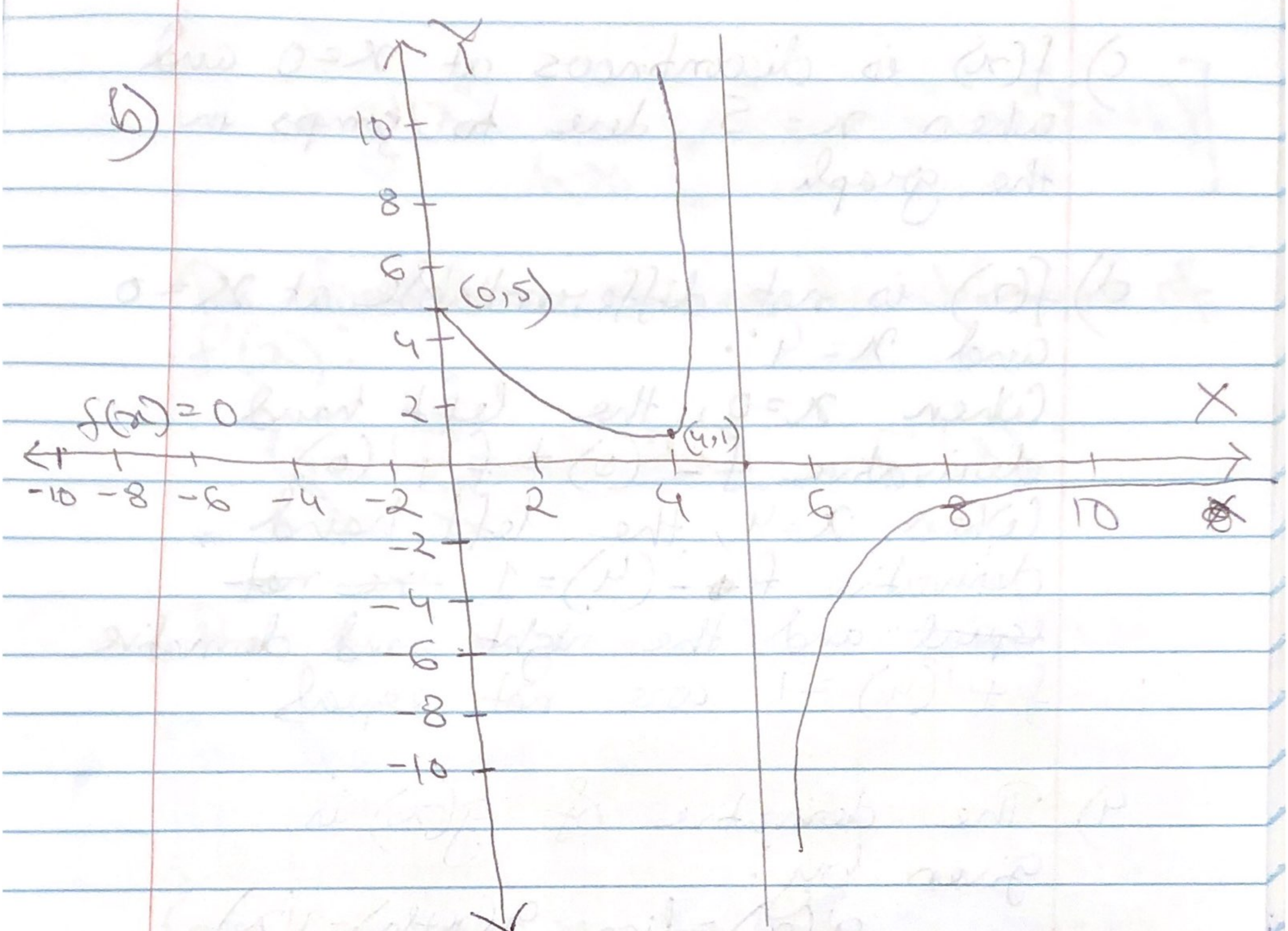
As $h \to 0^+$, $1-h \to 1$:

$$f'_+(4) = \frac{1}{1} = 1.$$

So, $f'_+(4) = 1$

Since, $f'_-(4) \neq f'_+(4)$, the derivative $f'(4)$ does not exist.

b)



For $x \leq 0$, $f(x) = 0$, a horizontal line at $y = 0$

For, $0 < x < 4$, $f(x) = 5 - x$, a straight line with slope -1

For $x \geq 4$, $f(x) = \dfrac{1}{5-x}$, a curve with a vertical asymptote at $x = 5$.

c) $f(x)$ is discontinous at $x=0$ and when $x=5$ due to jumps in the graph.

d) $f(x)$ is not differentiable at $x=0$ and $x=4$.
When $x=0$, the left hand derivative $f_-'(0) \neq f_+'(0)$
When $x=4$, the left hand derivative $f_-'(4)=1$ ~~are not equal~~ and the right hand derivative $f_+'(4)=1$ are not equal.

4) The derivative of $g(x)$ is given by:

$$g'(x) = \lim_{h \to 0} \frac{g(x+h)-g(x)}{h}$$

Since, $g(x) = xf(x)$,

$$g'(x) = \lim_{h \to 0} \frac{(x+h)f(x+h) - xf(x)}{h}$$

$$= \lim_{h \to 0} \frac{x(f(x+h)-f(x)) + hf(x+}{h}$$

$$= g'(x) = \lim_{h\to 0}\left[x\frac{f(x+h)-f(x)}{h} + f(x+h)\right]$$

The first term contains the definition of $f'(x)$:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x)$$

Thus, the first term becomes $xf'(x)$.

For the first term, $\lim_{h\to 0} f(x+h) = f(x)$, since $f(x)$ is continous (all differentiable functions are continous).
Combining the results of the two limits:

$$g'(x) = xf'(x) + f(x)$$

5. a) Boyle's law states that $P \cdot V = K$, where K is a constant. Given,

$V = 0.106\ m^3$

$P = 50\ KPa$

$\therefore K = P \cdot V$

$= 50\ KPa \times 0.106\ m^3$

$= 5.3\ KPa \cdot m^3$

$$\therefore V(P) = \frac{K}{P} = \frac{5.3}{P}$$

b) The derivative of $V(P)$ with respect to P is:

$$\frac{dV}{dP} = \frac{d}{dP}\left(\frac{5.3}{P}\right) = -\frac{5.3}{P^2}$$

At $P = 50\ KPa$:

$$\frac{dV}{dP} = -\frac{5.3}{(50)^2} = \frac{-5.3}{2500}$$

$= -0.00212\ m^3/KPa$

The derivative $\frac{dV}{dP}$ represents the rate of change of the volume V with respect to the pressure P.

The units of $\frac{dV}{dP}$ are m^3/KPa.

6. a) Given,

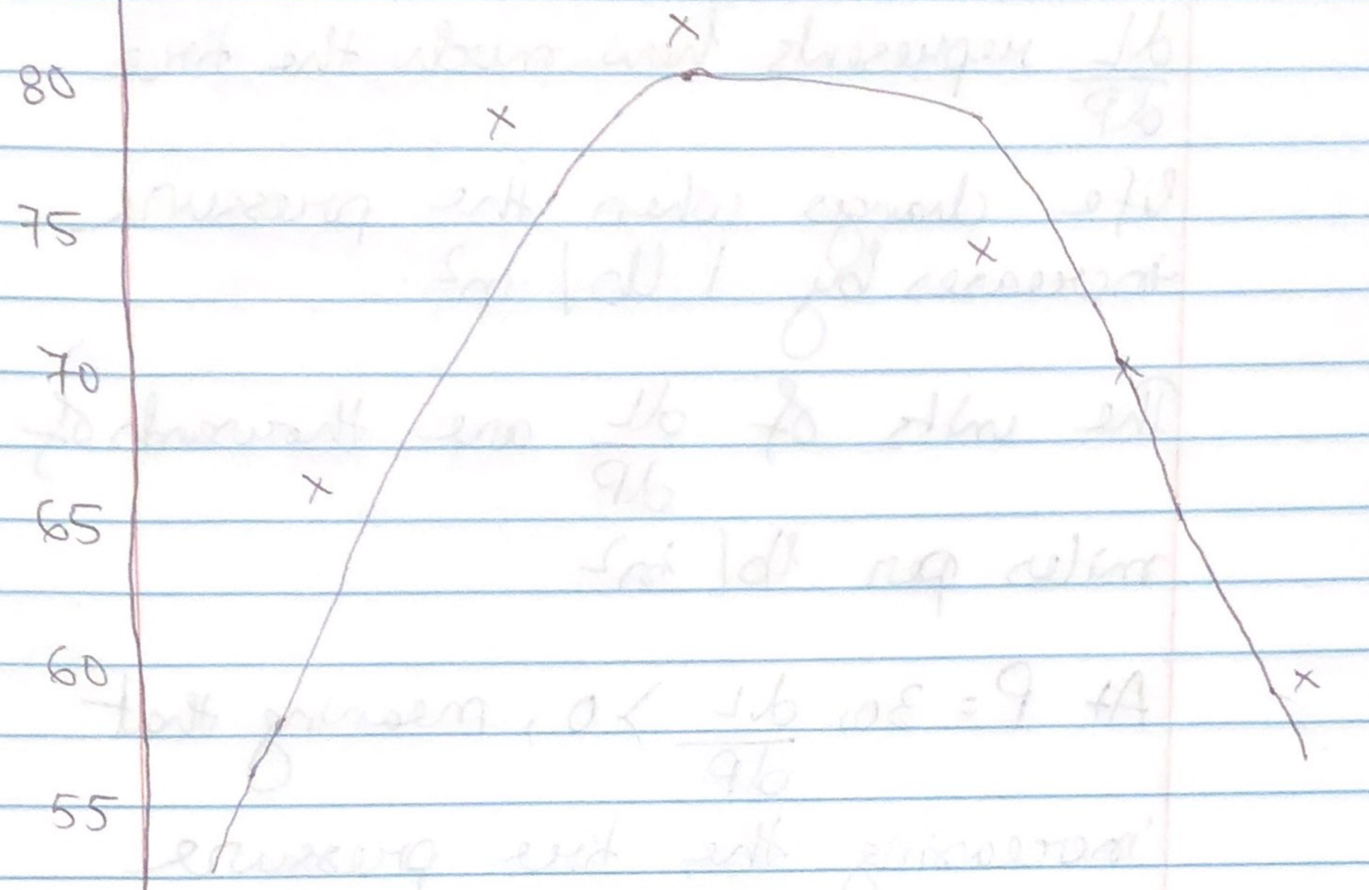
$P = [26, 28, 31, 35, 38, 42, 45]$

$L = [50, 66, 78, 81, 74, 70, 59]$

The quadratic function that models tire life as a function of pressure is:

$$L(P) = -0.275P^2 + 19.75P - 273.55$$

The graph shows both the data points and the fitted quadratic curve.



b) The rate of change of tire life with respect to pressure $\left(\frac{dL}{dp}\right)$ is:

At $P = 30$: $\frac{dL}{dp} = 3.22$ thousand miles per lb/in^2

At $P = 40$: $\frac{dL}{dp} = -2.29$ "

$\frac{dL}{dP}$ represents how much the tire life changes when the pressure increases by 1 lb/ in^2.

The units of $\frac{dL}{dP}$ are thousands of miles per lb/ in^2

At $P = 30$, $\frac{dL}{dP} > 0$, meaning that increasing the tire pressure slightly increases tire life.

At $P = 40$, $\frac{dL}{dP} < 0$, meaning that increasing the tire pressure reduces tire life, ~~suggesting~~ suggesting overinflating leads to premature wear.