Quantum Error Correction

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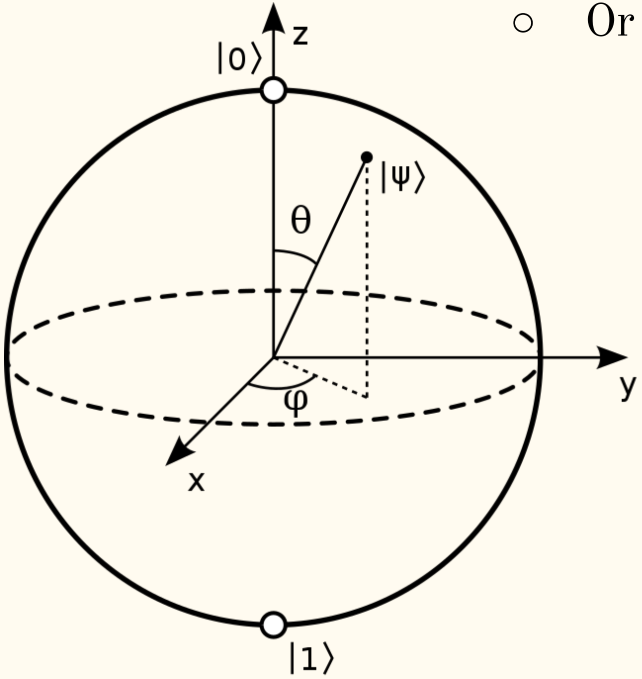
Why Quantum Error Correction?

- To solve more complex algorithms
  - Ex. Shor’s factor finder
- Big future - $106B potential quantum technology market size by 2040

Classical vs. Quantum Bits

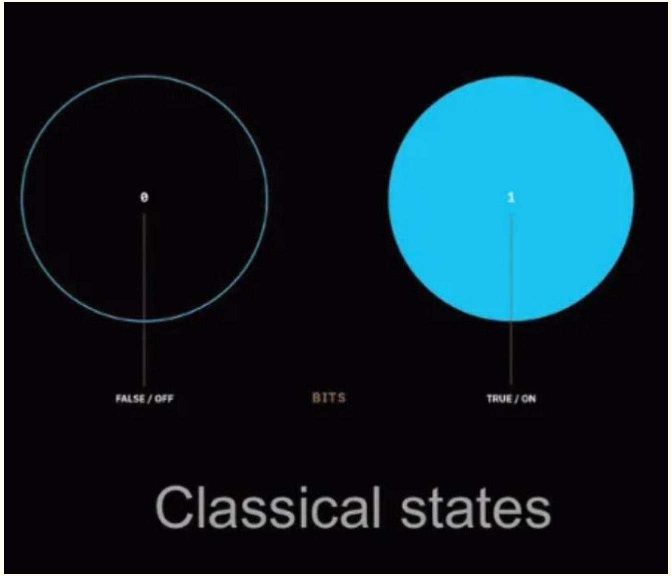
- Classical: 0 or 1
- Quantum: 0, 1
    - Or a superposition of 0 and 1



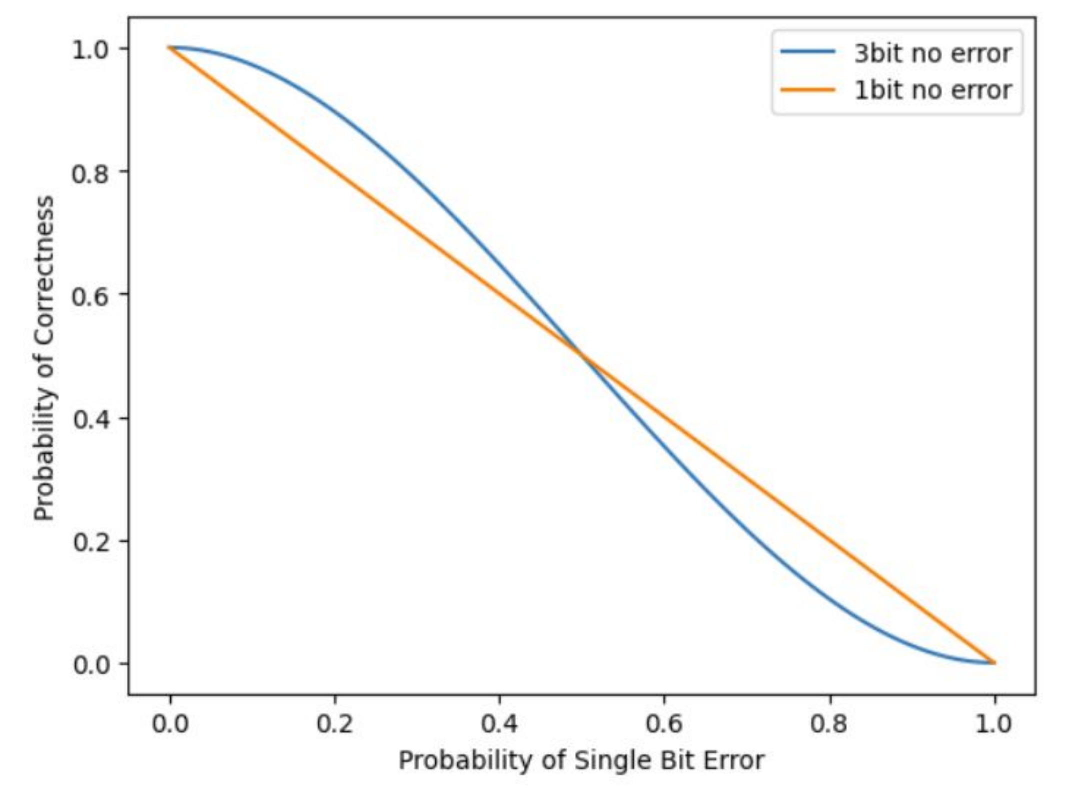
Classical Error Correction

Classical Error Correction

- Corrects errors during transmission and storage of information
- To send a single bit over a binary symmetric channel, then we can encode the bit, by repeating it three times
  - $0 \rightarrow 000$
  - $1 \rightarrow 111$
- Error:
  - $0 \rightarrow 001$



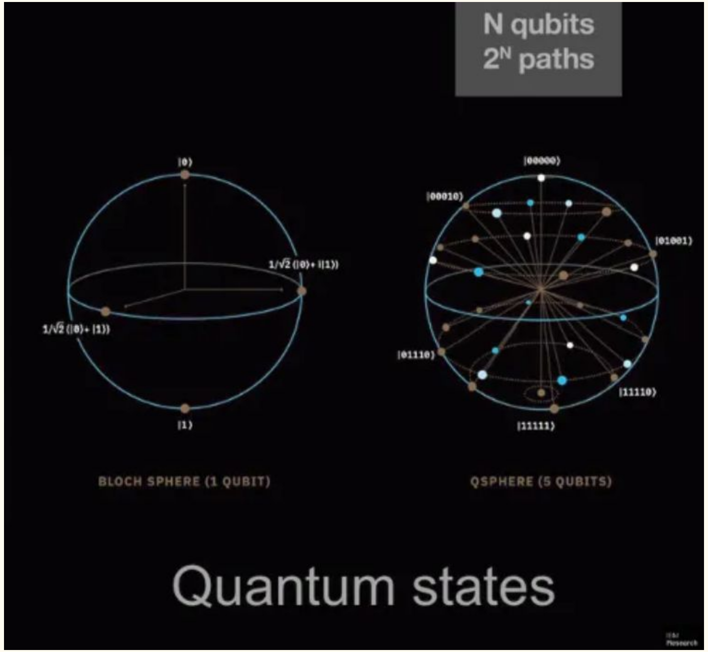
Potential Drawbacks of Repetition



Quantum Error Correction

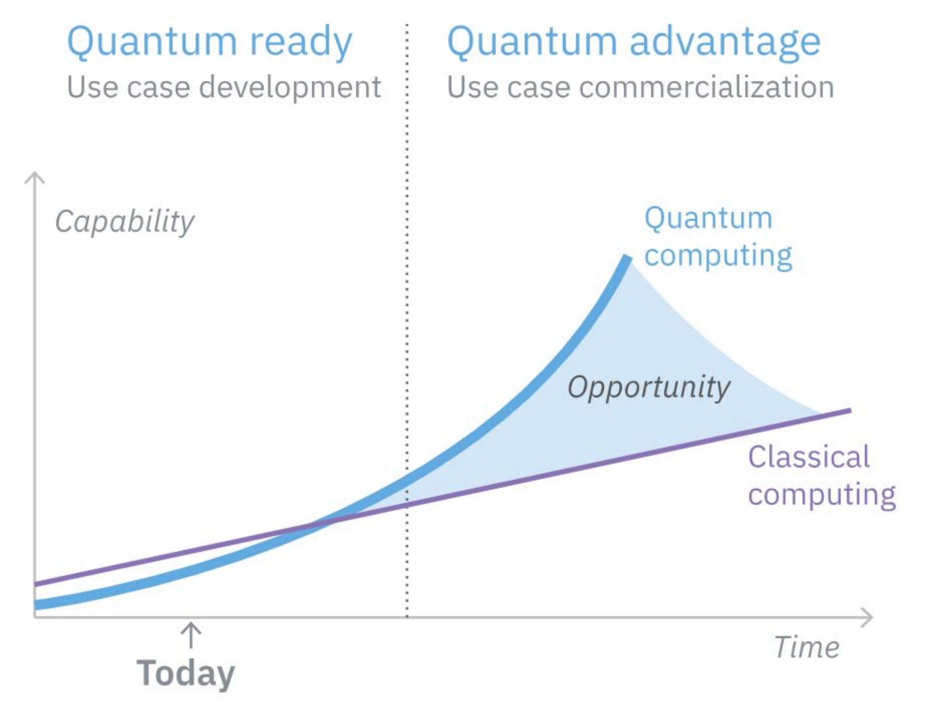
Preserve Quantum Information

- Quantum error correction ensures that delicate quantum information remains accurate and usable despite environmental disturbances and errors
- Very important for large scale calculations



Quantum Advantage

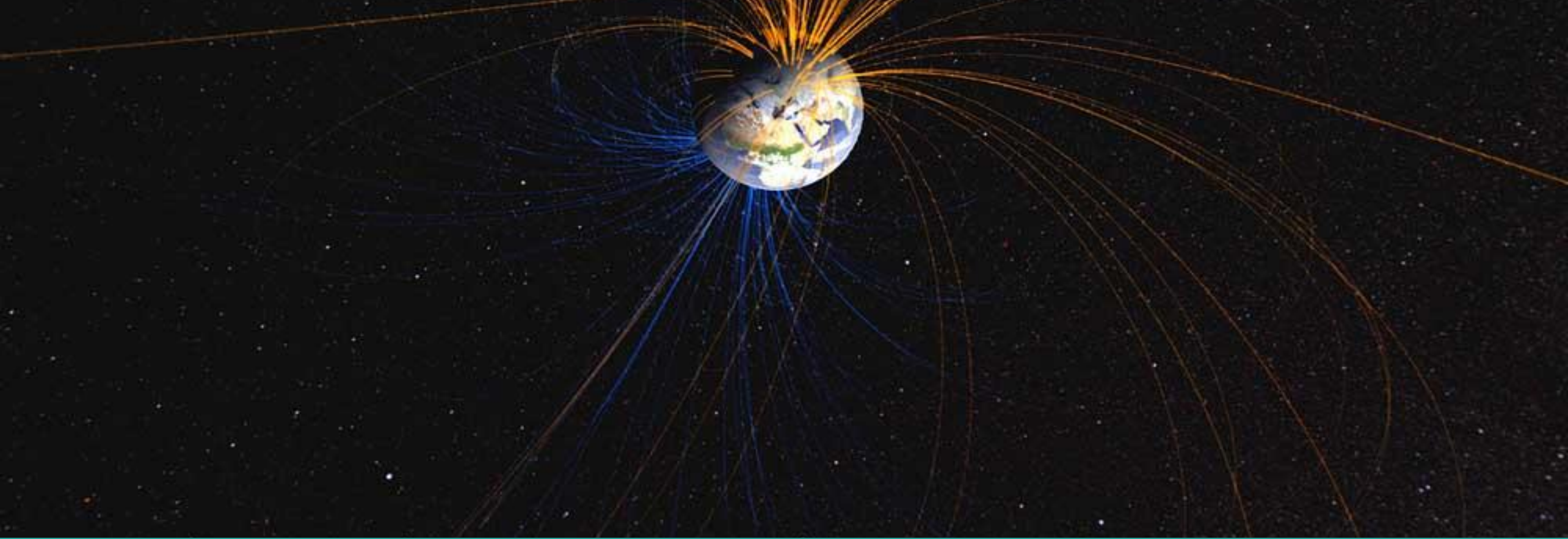
- Quantum algorithms promise to outperform classical algorithms for specific tasks
- Quantum error correction helps maintain the advantage of quantum algorithms by mitigating the impact of errors and noise



Causes of Error

External Forces

- Any external force can lead to inaccuracies in quantum computations and, in certain quantum computer architectures, complete loss of memory state

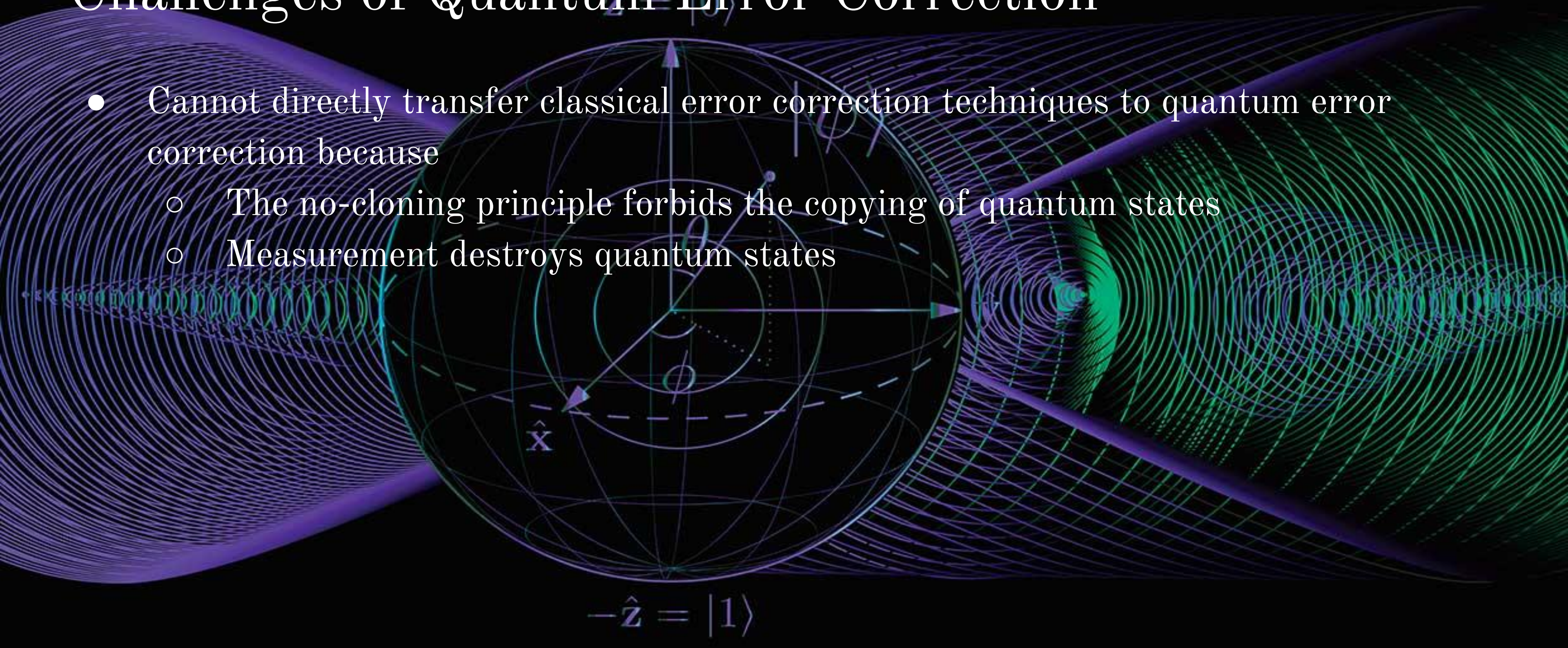


Gate Errors

- Imperfections in quantum gates cause errors
  - Noise and imperfections - these imperfections can arise due to limitations in hardware, fluctuations in control parameters, and external disturbances

Challenges of Quantum Error Correction

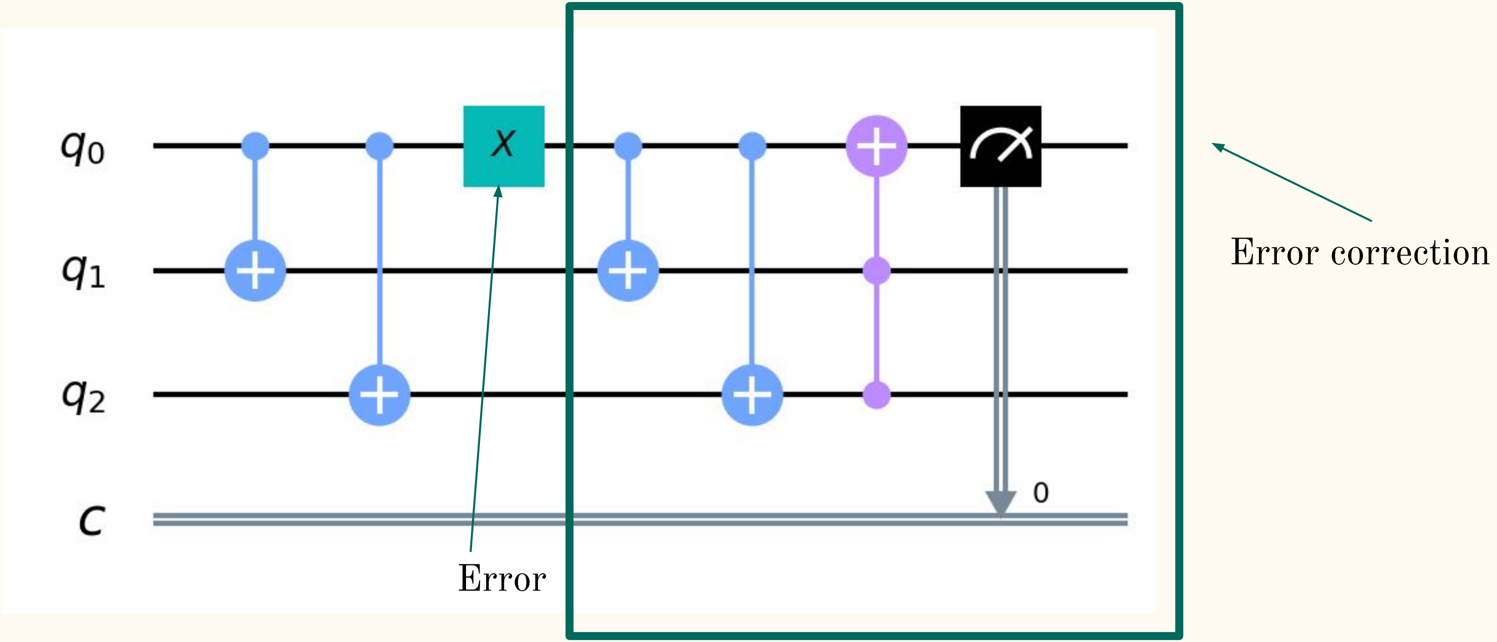
- Cannot directly transfer classical error correction techniques to quantum error correction because
  - The no-cloning principle forbids the copying of quantum states
  - Measurement destroys quantum states



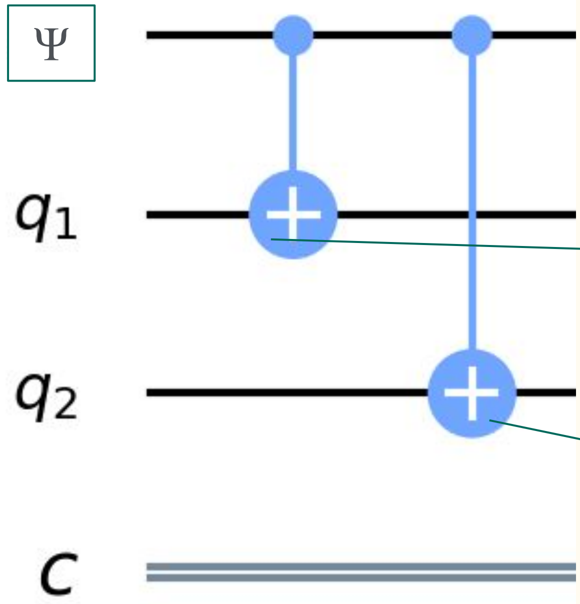
Fixing the Error

Bit-Flip Code

- Only the first qubit would ever be measured unlike classical code repetition



$$\Psi = a|0\rangle + b|1\rangle$$

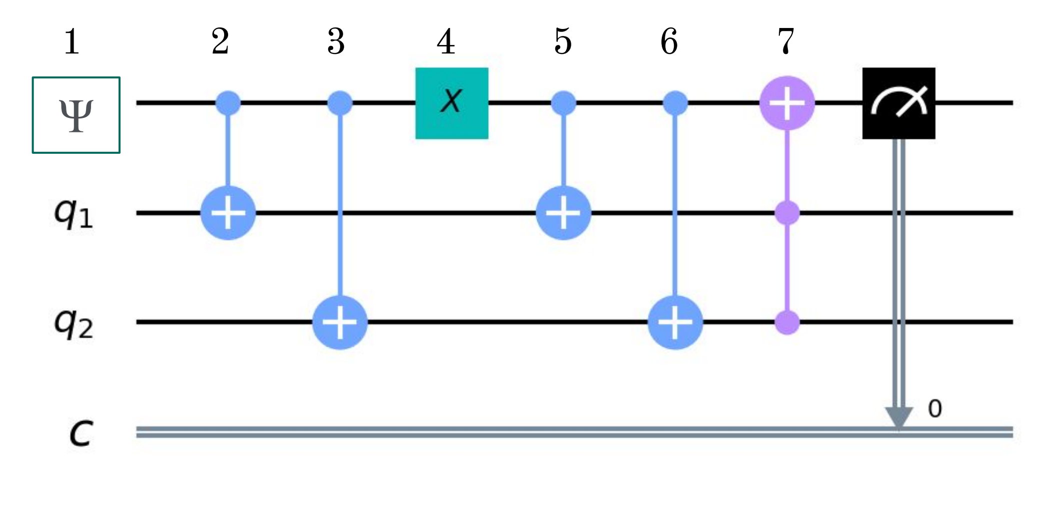


Initial State

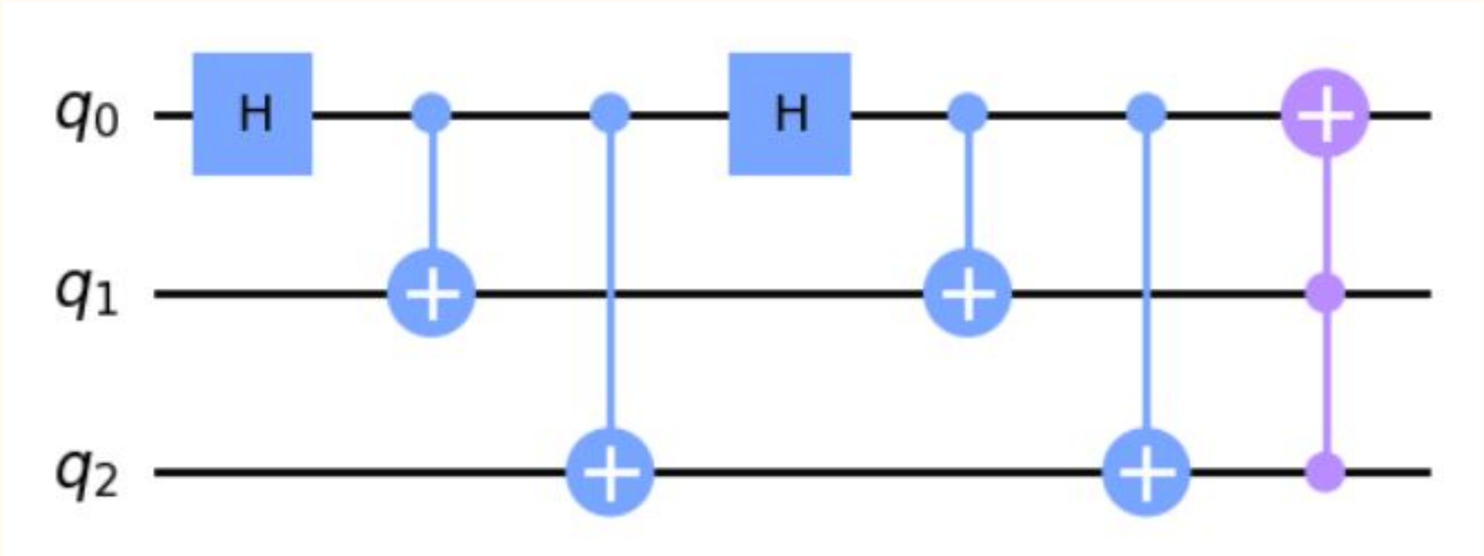
$$a|000\rangle + b|100\rangle$$

$$a|000\rangle + b|110\rangle$$

$$a|000\rangle + b|111\rangle$$



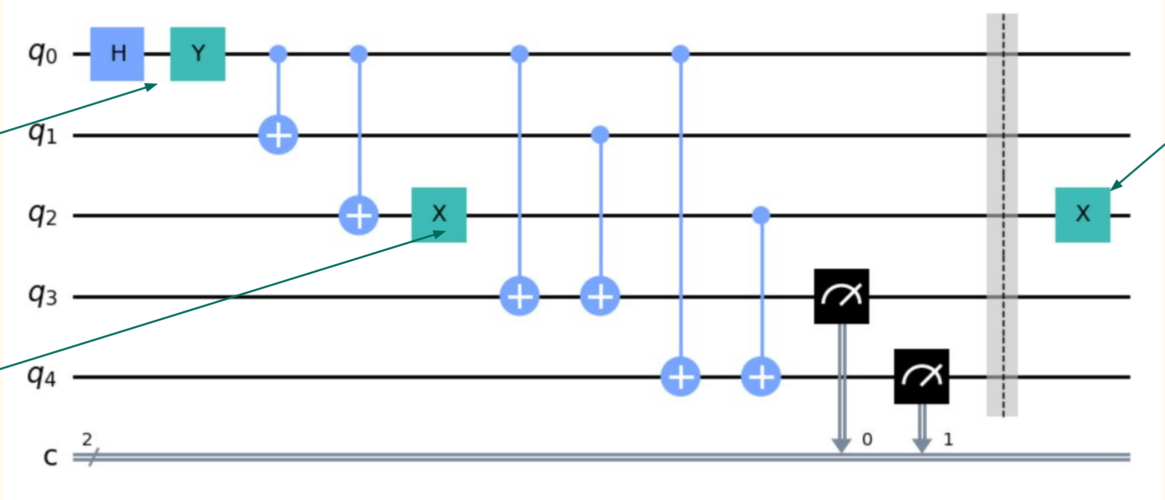
1. $a|000\rangle + b|100\rangle$
2. $a|000\rangle + b|110\rangle$
3. $a|000\rangle + b|111\rangle$
4. Error:
   $a|100\rangle + b|011\rangle$
5. $a|110\rangle + b|011\rangle$
6. $a|111\rangle + b|011\rangle$
7. $a|011\rangle + b|111\rangle$



$$\frac{1}{2}|000\rangle + \frac{1}{2}|011\rangle - \frac{1}{2}|100\rangle + \frac{1}{2}|111\rangle$$

Syndrome Measurement

- Method for measuring the states of qubits indirectly

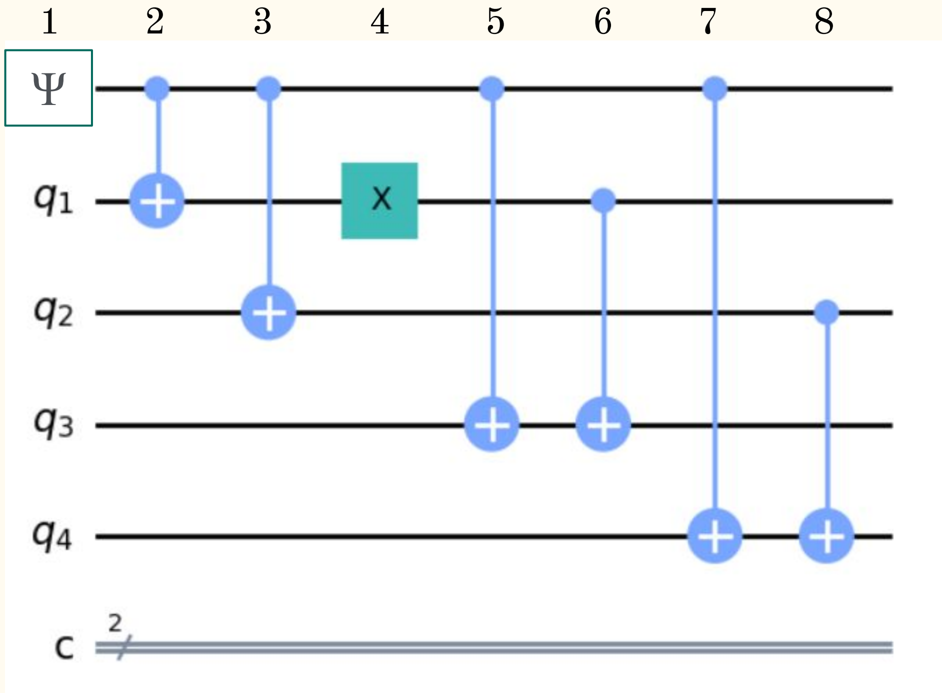


Initial State

Error

Error Correction

$$\Psi = a|0\rangle + b|1\rangle$$



1. $a|000\rangle + b|100\rangle$

2. $a|000\rangle + b|110\rangle$

3. $a|000\rangle + b|111\rangle$

4. Error:
   $a|010\rangle + b|101\rangle$

   $a|010\rangle \otimes |00\rangle + b|101\rangle \otimes |00\rangle$

5. $a|010\rangle \otimes |00\rangle + b|101\rangle \otimes |10\rangle$

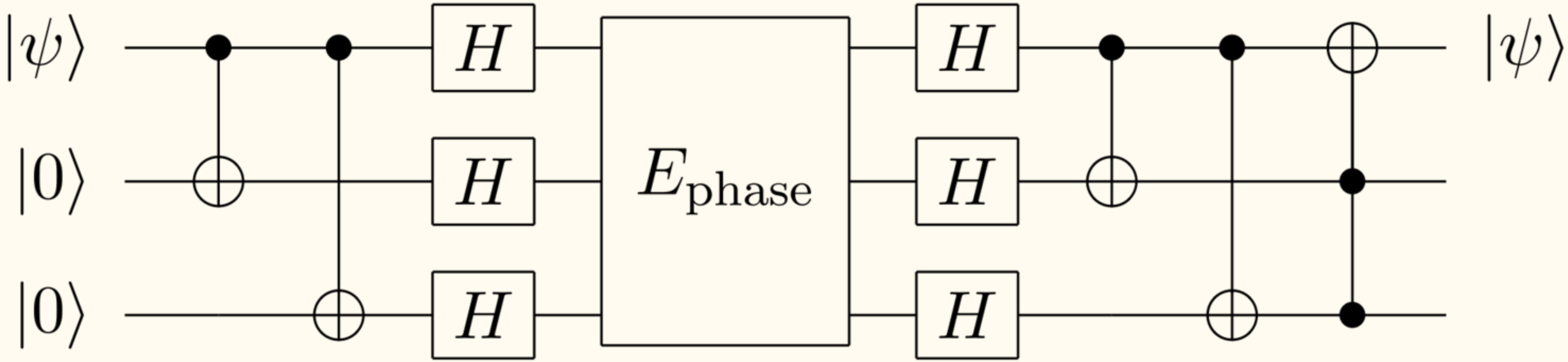
6. $a|010\rangle \otimes |10\rangle + b|101\rangle \otimes |10\rangle$

7. $a|010\rangle \otimes |10\rangle + b|101\rangle \otimes |11\rangle$

8. $a|010\rangle \otimes |10\rangle + b|101\rangle \otimes |10\rangle$

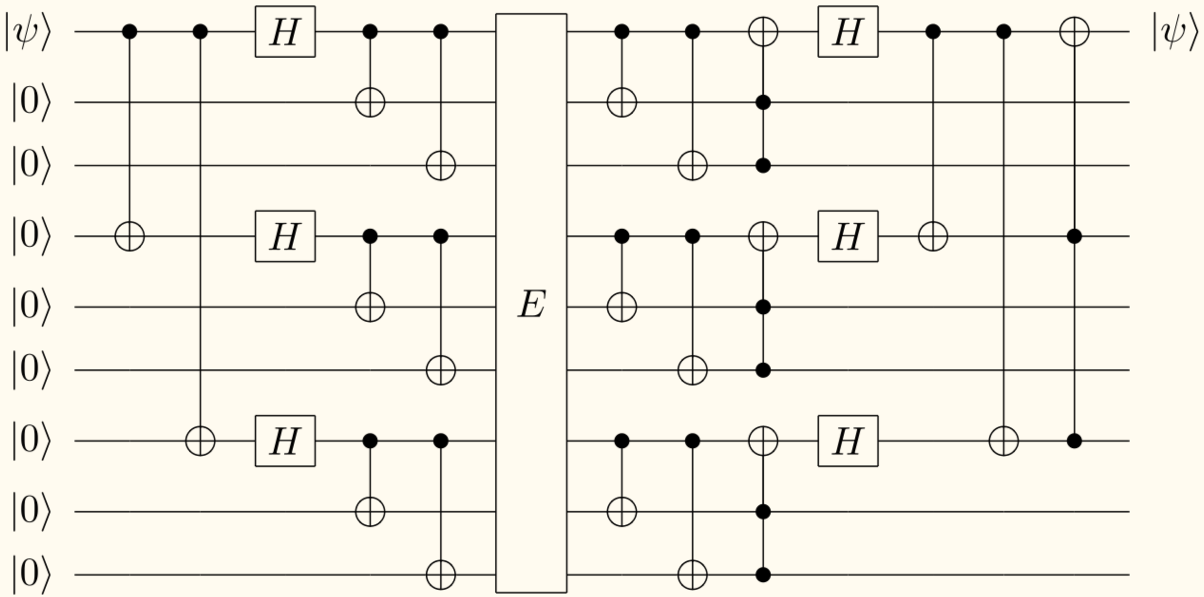
Sign Flip Error Correction

- Hadamard gates rotate the qubit around the x and y axes to allow for error correction along them since the bit flip code only corrects errors along the z-axis



Shor's Code

- It combines the techniques used to correct for bit flips and sign flips in order to be effective against any type of error



Quantum Error Correction (Code)

```
from qiskit import Aer, QuantumCircuit, execute
from qiskit.circuit.library import RGQFTMultiplier
qc = QuantumCircuit(12, 6)

qc.x(0)
qc.x(2) #sets 5

qc.x(4)
qc.x(5) #sets 6

circuit = RGQFTMultiplier(num_state_qubits=3, num_result_qubits=6)
qc = qc.compose(circuit)

qc.measure(6, 0)
qc.measure(7, 1)
qc.measure(8, 2)
qc.measure(9, 3)
qc.measure(10,4)
qc.measure(11,5)

backend = Aer.get_backend('qasm_simulator')
job = execute(qc, backend, shots=100)
counts = job.result().get_counts()
print(counts)
```

```
{'011110': 100}
```

```python
from qiskit import Aer, QuantumCircuit, execute
from qiskit.circuit.library import RGQFTMultiplier
qc = QuantumCircuit(12, 6)

qc.x(0)
qc.x(2) #sets 5

qc.x(4)
qc.x(5) #sets 6

#Error
qc.y(0)
qc.h(1)

circuit = RGQFTMultiplier(num_state_qubits=3, num_result_qubits=6)
qc = qc.compose(circuit)

qc.measure(6, 0)
qc.measure(7, 1)
qc.measure(8, 2)
qc.measure(9, 3)
qc.measure(10,4)
qc.measure(11,5)

backend = Aer.get_backend('qasm_simulator')
job = execute(qc, backend, shots=100)
counts = job.result().get_counts()
print(counts)
```

```
{'011000': 46, '100100': 54}
```

```python
from qiskit import Aer, QuantumCircuit, execute
from qiskit.circuit.library import RGQFTMultiplier
import numpy as np
qc = QuantumCircuit(20, 6)

qc.x(0)
qc.x(2) #sets 5

qc.x(4)
qc.x(5) #sets 6

#Shor's Algorithm Part 1
qc.cx(0,12)
qc.cx(0,13)
qc.h(0)
qc.h(12)
qc.h(13)
q = [0, 12, 13]
for i in range(6):
  qc.cx(q[i % 3], 19-i)

#Error
qc.y(0)
qc.h(0)
qc.x(12)
```

```python
#Shor's Algorithm Part 2
for i in range(6):
  qc.cx(q[i % 3], 19-i)
for i in range(3):
  qc.ccx(13+i, 16+i, q[i])
qc.h(0)
qc.h(12)
qc.h(13)
qc.cx(0,12)
qc.cx(0,13)
qc.ccx(12, 13, 0)

circuit = RGQFTMultiplier(num_state_qubits=3, num_result_qubits=6)
qc = qc.compose(circuit)

qc.measure(6, 0)
qc.measure(7, 1)
qc.measure(8, 2)
qc.measure(9, 3)
qc.measure(10,4)
qc.measure(11,5)

backend = Aer.get_backend('qasm_simulator')
job = execute(qc, backend, shots=100)
counts = job.result().get_counts()
print(counts)
```

```
{'011110': 100}
```

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