Quantum Money

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Defining The Problem

What Is Quantum Money?

Protocol to create and validate banknotes impossible to forge

Two Sectors:

- Private-key
- Public-key

Stephen Wiesner (1942-2021)

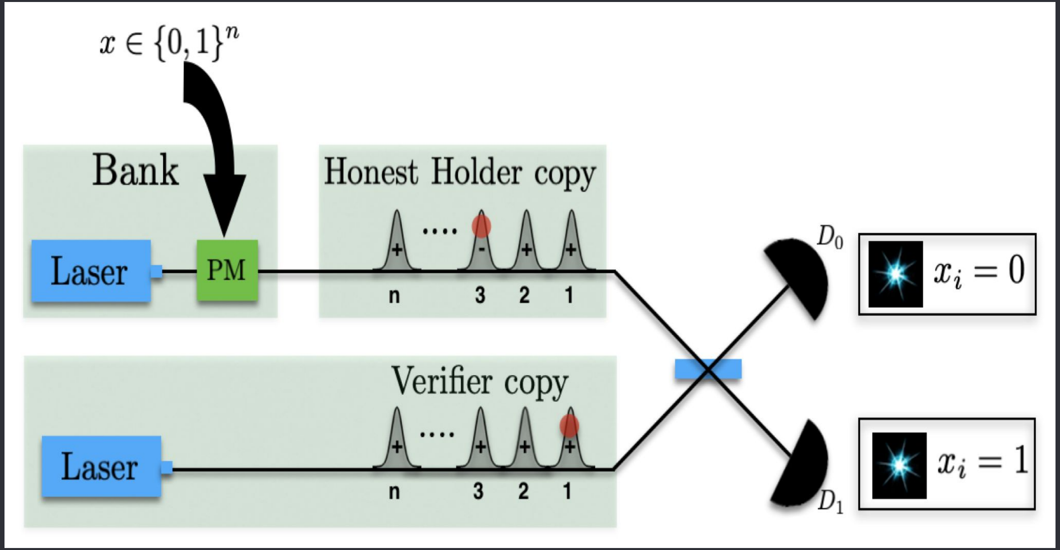
Wiesner's Quantum Money Scheme

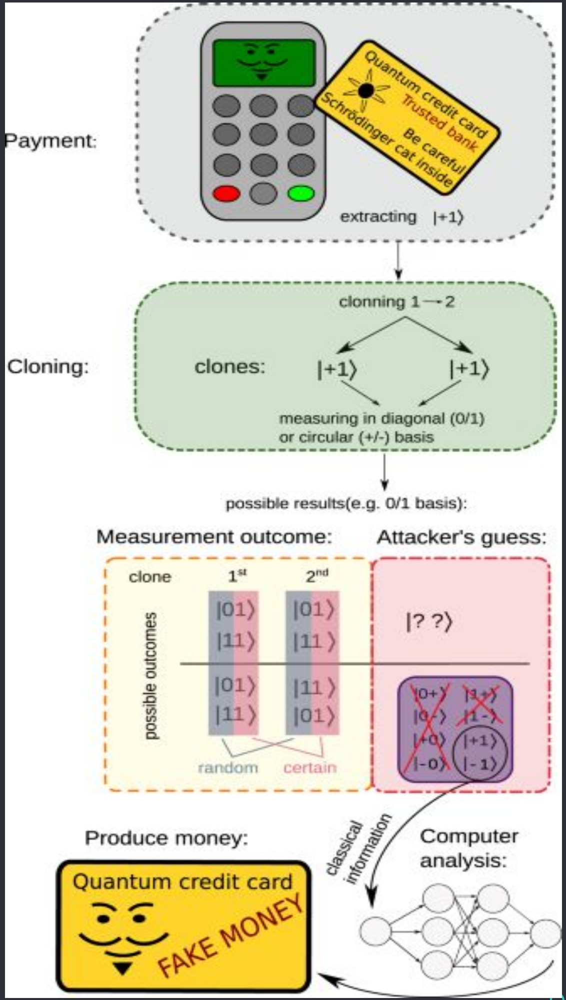
- ➢ Central bank prints "quantum bills"
  - » $|0\rangle$,
    » $|1\rangle$,
    » $|+\rangle = (|0\rangle+|1\rangle)/\sqrt{2}$, or
    » $|-\rangle = (|0\rangle-|1\rangle)/\sqrt{2}$.
- ➢ Measures each qubit in the correct basis
- ➢ Counterfeiter would fail with a a probability of $(3/4)^N$ with n qubits

Further Study In Weisner's Model

- Chance of getting "stolen" by an adversary
- Tokunada (2003) → prevent mint from tracking each bill
- Mosca and Stebila (2007) → *quantum coins*

Classical Variation





Qiskit Implementation

```python
def create_money(bits, bases):
    message = []
    for i in range(3):
        qc = QuantumCircuit(1,1)
        if bases[i] == 0: # Prepare qubit in Z-basis
            if bits[i] == 0:
                pass
            else:
                qc.x(0)
        else: # Prepare qubit in X-basis
            if bits[i] == 0:
                qc.h(0)
            else:
                qc.x(0)
                qc.h(0)
        qc.barrier()
        message.append(qc)
    return message
```

```python
bank_fixednote = np.array([1,0,0])
bank_fixedbase = np.array([0,1,1])

banknote = create_money(bank_fixednote, bank_fixedbase)

bank_bases = randint(2, size=3)
```

```python
def measure_message(message, bases):
    backend = Aer.get_backend('aer_simulator')
    measurements = []
    for q in range(3):
        if bases[q] == 0: # measuring in Z-basis
            message[q].measure(0,0)
        if bases[q] == 1: # measuring in X-basis
            message[q].h(0)
            message[q].measure(0,0)
        aer_sim = Aer.get_backend('aer_simulator')
        qobj = assemble(message[q], shots=1, memory=True)
        result = aer_sim.run(qobj).result()
        measured_bit = int(result.get_memory()[0])
        measurements.append(measured_bit)
    return measurements
```

```python
prob = 0
shots = 100

for i in range(100):
    forge_bits = randint(2, size=3)
    forge_bases = randint(2, size=3)
    forge_note = create_money(forge_bits, forge_bases)
    bank_measure = measure_message(forge_note, bank_bases)
    bank_fixed = measure_message(banknote, bank_fixedbase)
    if(bank_fixed==bank_measure):
        print("Bank Serial Number: " + str(bank_fixed))
        print("Verfied Bill: " + str(bank_measure))
    else:
        print("Bank Serial Number: " + str(bank_fixed))
        print("Forged Bill: " + str(bank_measure))
        prob = prob+1
    print("")

print("Probability of bills being forged:" + str(prob/shots))
```

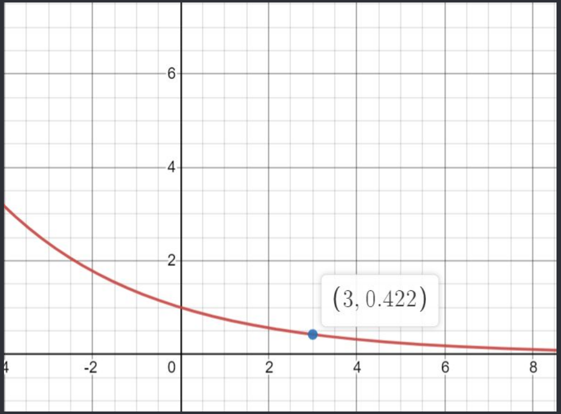
Example Result:

```
Bank Serial Number: [1, 1, 1]
Verfied Bill: [1, 1, 1]
Bank Serial Number: [1, 1, 1]
Forged Bill: [0, 1, 0]
Bank Serial Number: [1, 1, 0]
Forged Bill: [0, 1, 1]

Bank Serial Number: [1, 0, 1]
Forged Bill: [0, 1, 0]
Probability of bills being forged:0.42
```

Proof:

(¾)^3 = 0.421875

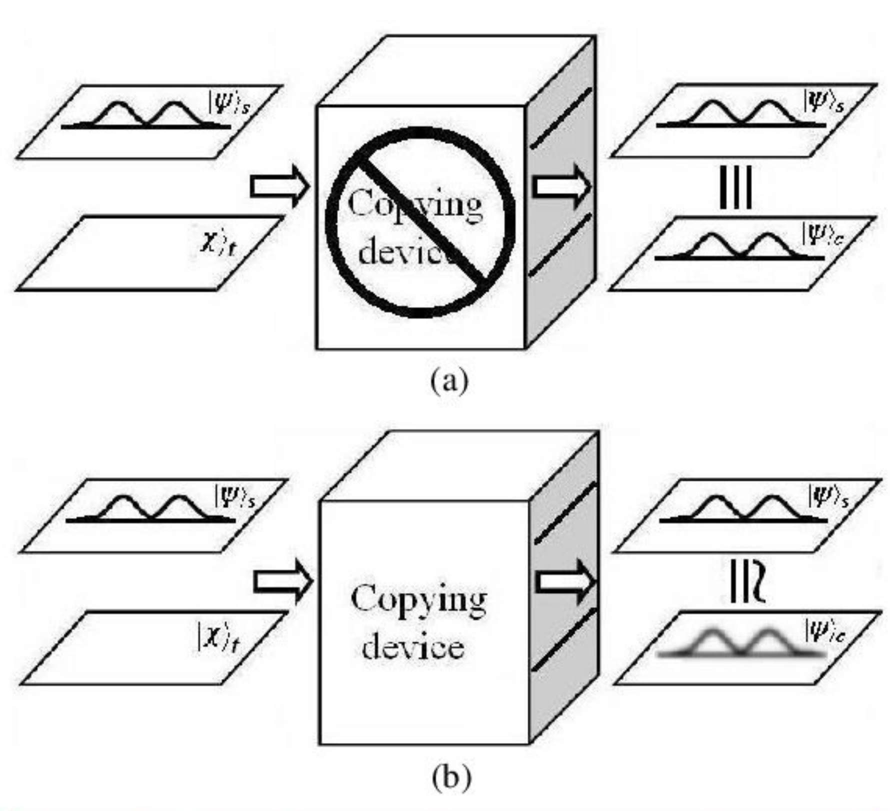


Advantages of this Scheme

- Coins become exponentially harder to counterfeit as attempts increase
- Bank's database can be static
- Does not require 3-party authentication
- Dependence between the number of verifications and the number of qubits it contains is optimal

No-Cloning Theorom

Cannot create a copy of an arbitrary quantum state

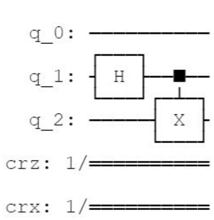


Qiskit Implementation

```python
bill = QuantumRegister(3, name="q")
crz = ClassicalRegister(1, name="crz")
crx = ClassicalRegister(1, name="crx")
bank_circuit = QuantumCircuit(bill, crz, crx)
```

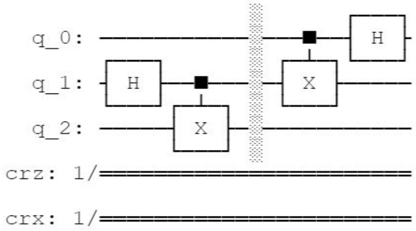
```python
def create_bell_pair(qc, a, b):
    qc.h(a)
    qc.cx(a,b)
```

```python
create_bell_pair(bank_circuit, 1, 2)
bank_circuit.draw()
```



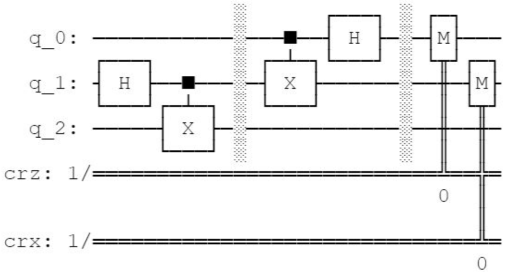

```python
def alice_bill(qc, psi, a):
    qc.cx(psi, a)
    qc.h(psi)
```

```python
bank_circuit.barrier()
alice_bill(bank_circuit, 0, 1)
bank_circuit.draw()
```



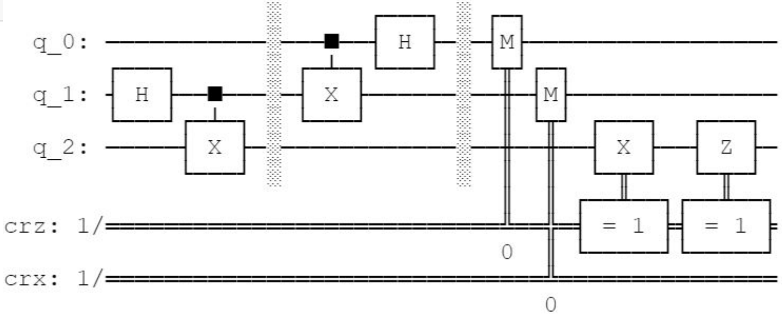

```python
def measure_and_send(qc, a, b):
    """Measures qubits a & b and 'sends' the results to Bob"""
    qc.barrier()
    qc.measure(a,0)
    qc.measure(b,1)
```

```python
measure_and_send(bank_circuit, 0 ,1)
bank_circuit.draw()
```




```python
def bank_apply(qc, qubit, crz, crx):
    qc.x(qubit).c_if(crx, 1)
    qc.z(qubit).c_if(crz, 1)
```

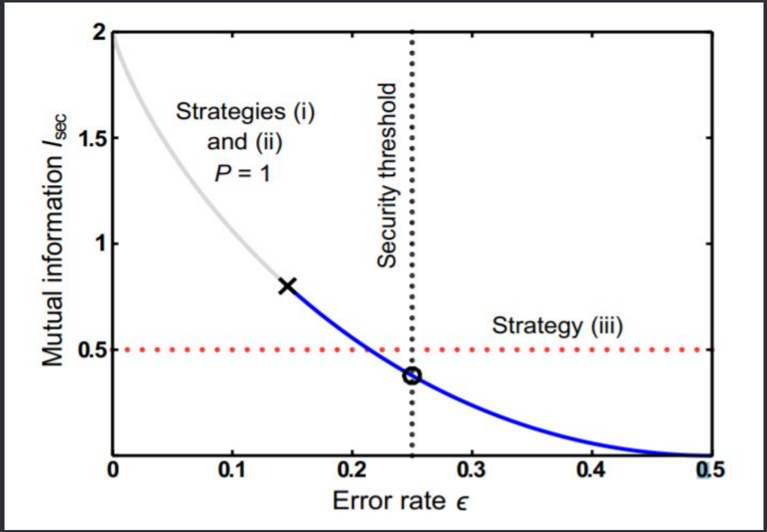
```python
bank_apply(bank_circuit, 2, crz, crz)
bank_circuit.draw()
```

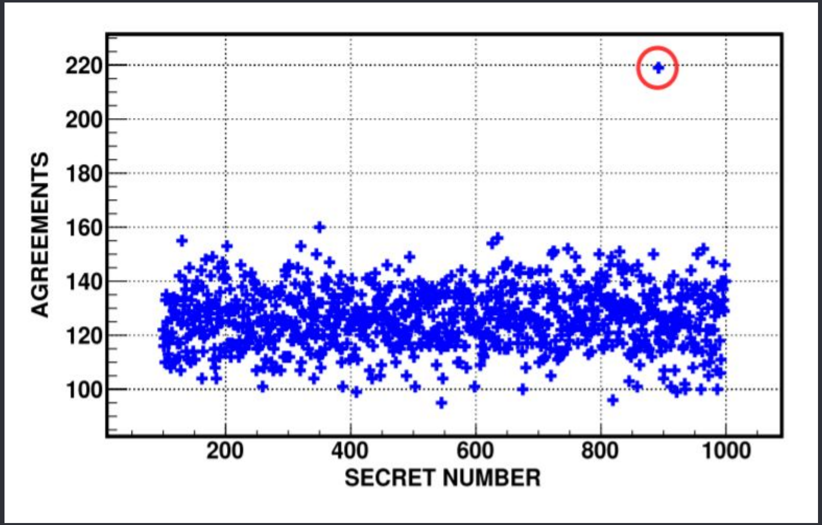


Benefits of this Model

- ➢ Obtain exponential security
- ➢ Produce notes that are publicly verifiable
- ➢ Cannot copy even with access to a verifier

Possible Attacks and Security Measures





(Jiráková, K. and Lemr, K.)

Future Outlook on Quantum Money

- ➢ Implementations of Quantum Key Distribution
- ➢ Practical implementation not proved yet
- ➢ Government and commercial sectors

References

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Thank You!

Questions?