Surface Codes

Logical Qubits for Error Correction

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Austin G. Fowler, Matteo Mariantoni, John M. Martinis, Andrew N. Cleland, Surface codes: Towards practical large-scale quantum computation

- Error Correction
- Error Detection
- Surface Code
- Logical Operators

Qubit Gates / Pauli Operators

Qubit: 2 eigenstates |g>, |e>

Gates are Hermitian: $AA^H = I$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{X}^2 = -\hat{Y}^2 = \hat{Z}^2 = \hat{I}$$

$$\hat{X}\hat{Z} = -\hat{Z}\hat{X}$$

$$[\hat{X}, \hat{Y}] \equiv \hat{X}\hat{Y} - \hat{Y}\hat{X} = -2\hat{Z}$$

Qubit Errors

- X bit-flip
- Z phase-flip
- Y = ZX combined
- measurement
- ...

Gates are Hermitian: $AA^H = I$

Error Detection

Detect errors by repeatedly measuring each qubit

Measurement operators must commute!!

$$[X, Z] \neq 0$$

$$[X_a X_b, Z_a Z_b] = 0$$

$\hat{Z}_a \hat{Z}_b$	$\hat{X}_a \hat{X}_b$	$\|\psi\rangle$
$+1$	$+1$	$(\|gg\rangle + \|ee\rangle)/\sqrt{2}$
$+1$	-1	$(\|gg\rangle - \|ee\rangle)/\sqrt{2}$
-1	$+1$	$(\|ge\rangle + \|eg\rangle)/\sqrt{2}$
-1	-1	$(\|ge\rangle - \|eg\rangle)/\sqrt{2}$

Error Detection

Errors cannot be uniquely identified

Initial State: $(|gg\rangle + |ee\rangle)/\sqrt{2}$

X_a or X_b error: $(|ge\rangle + |eg\rangle)/\sqrt{2}$

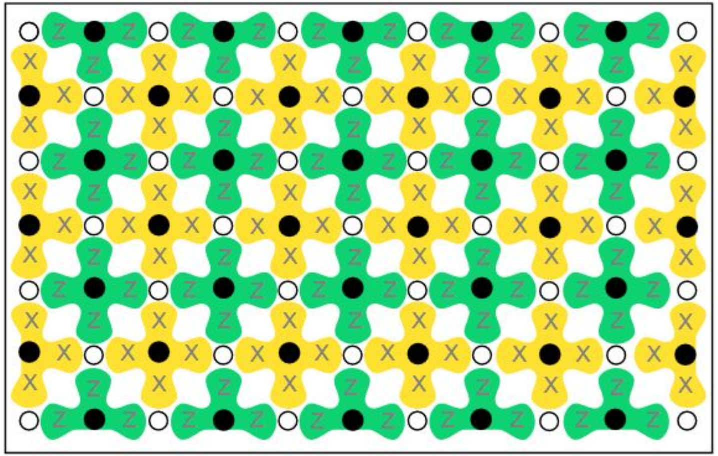
$\hat{Z}_a\hat{Z}_b$	$\hat{X}_a\hat{X}_b$	$\vert\psi\rangle$
$+1$	$+1$	$(\vert gg\rangle + \vert ee\rangle)/\sqrt{2}$
$+1$	-1	$(\vert gg\rangle - \vert ee\rangle)/\sqrt{2}$
-1	$+1$	$(\vert ge\rangle + \vert eg\rangle)/\sqrt{2}$
-1	-1	$(\vert ge\rangle - \vert eg\rangle)/\sqrt{2}$

Surface Codes

- Error Detection
- Logical Qubits

Surface Code

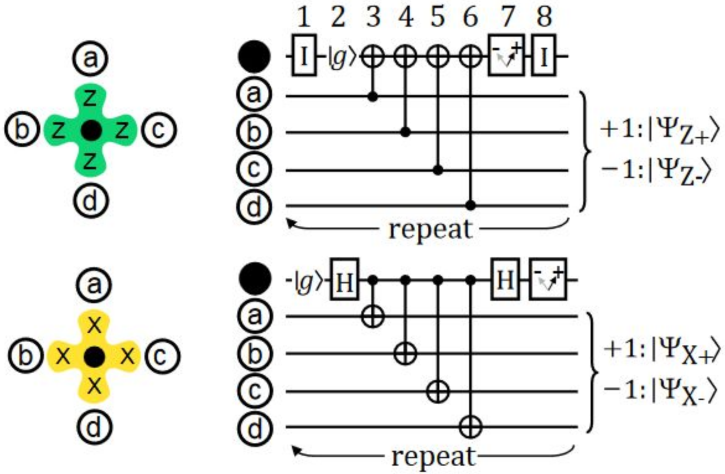
- Data qubits
- Measurement qubits



$$Z_aZ_bZ_cZ_d$$

Stabilizers

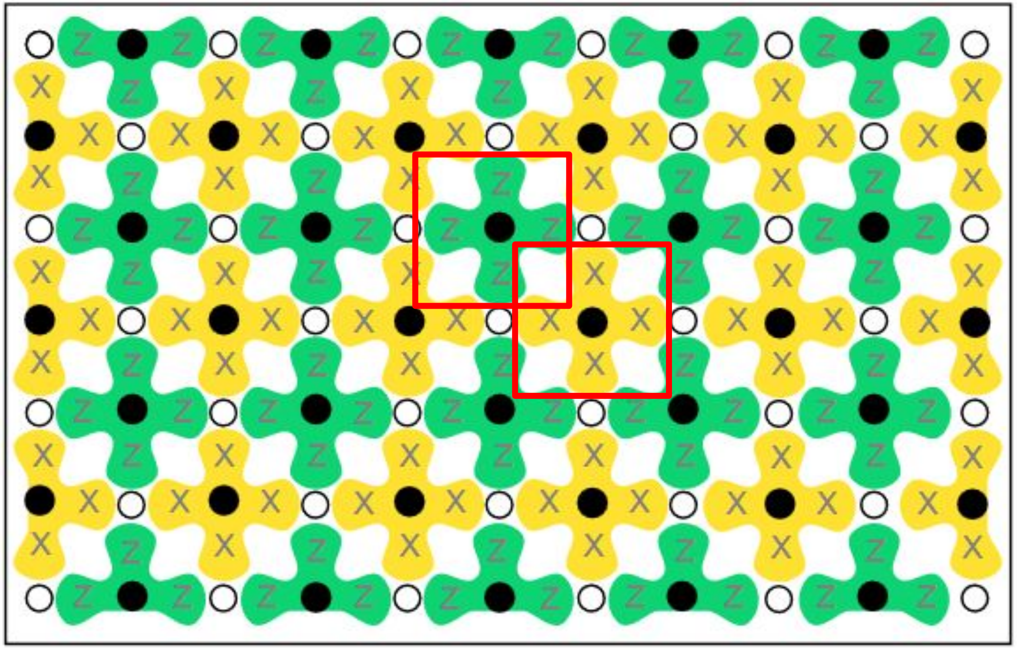
$$X_aX_bX_cX_d$$



Error Detection

Stabilizers are commute

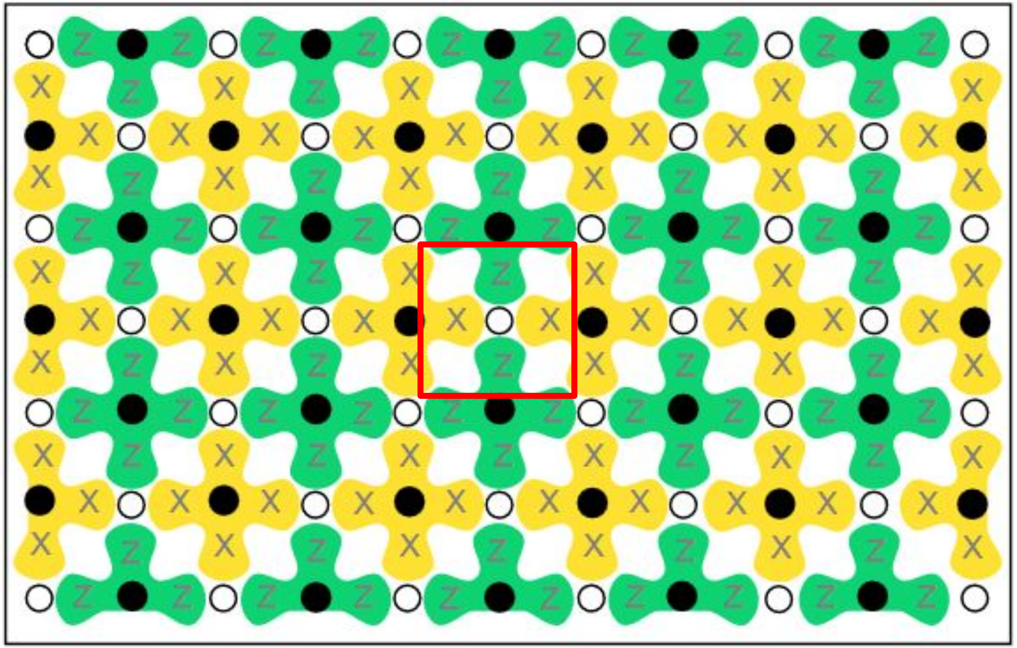
$$
\begin{aligned}
\left[\hat{X}_a\hat{X}_b\hat{X}_c\hat{X}_d\ ,\hat{Z}_a\hat{Z}_b\hat{Z}_e\hat{Z}_f\right]
&= \left(\hat{X}_a\hat{Z}_a\right)\left(\hat{X}_b\hat{Z}_b\right)\hat{X}_c\hat{X}_d\hat{Z}_e\hat{Z}_f \\
&- \left(\hat{Z}_a\hat{X}_a\right)\left(\hat{Z}_b\hat{X}_b\right)\hat{X}_c\hat{X}_d\hat{Z}_e\hat{Z}_f \\
&= 0
\end{aligned}
$$



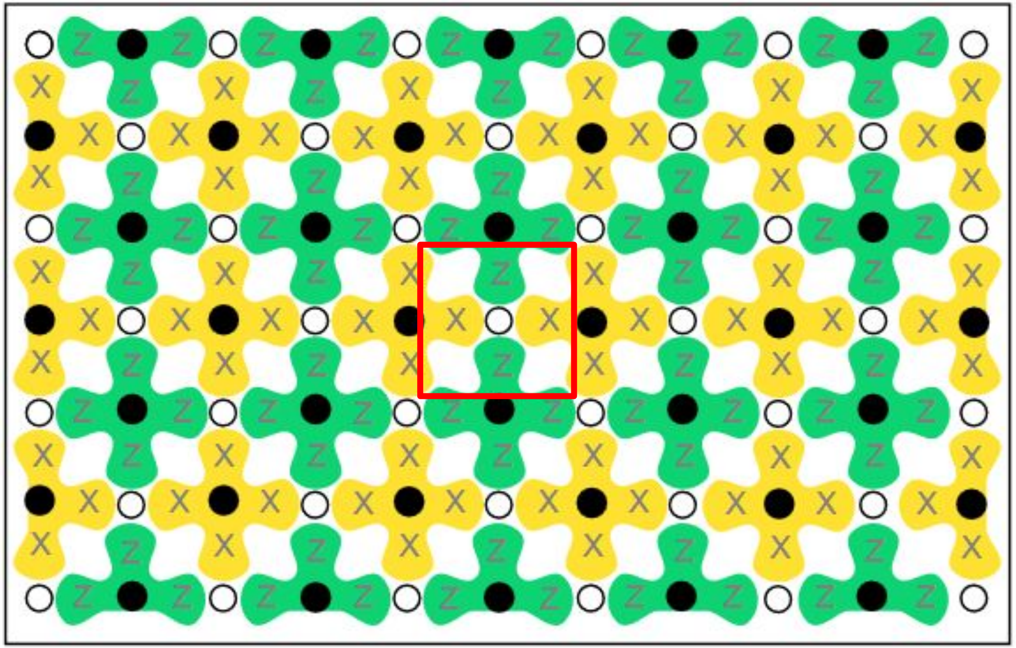
Error Detection

Z error

$$\hat{X}_a\hat{X}_b\hat{X}_c\hat{X}_d\left(\hat{Z}_a|\psi\rangle\right) = -\hat{Z}_a\left(\hat{X}_a\hat{X}_b\hat{X}_c\hat{X}_d|\psi\rangle\right)$$
$$= -X_{abcd}\left(\hat{Z}_a|\psi\rangle\right)$$
$$\hat{Z}_a\hat{Z}_b\hat{Z}_c\hat{Z}_d\left(\hat{Z}_a|\psi\rangle\right) = \hat{Z}_a\left(\hat{Z}_a\hat{Z}_b\hat{Z}_c\hat{Z}_d|\psi\rangle\right)$$
$$= Z_{abcd}\left(\hat{Z}_a|\psi\rangle\right),$$



Error Detection



X error: flip the 2 neighboring Z-measurement qubits

Y error: flip all 4 neighboring qubits

measurement error: flip 1 measurement qubits

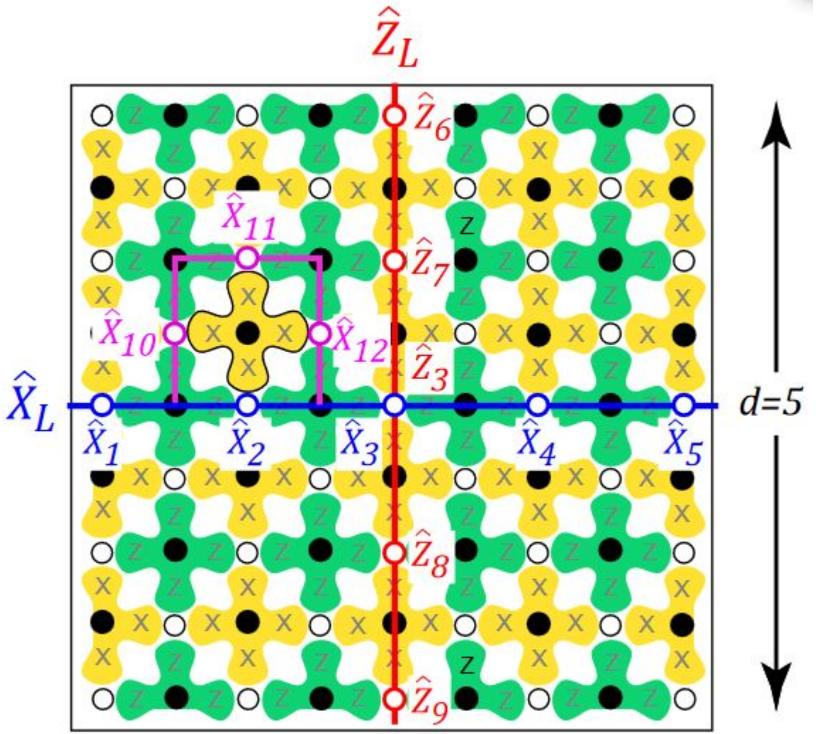
From Physical Qubits to Logical Qubits

additional degrees of freedom

– logical operators

$$\hat{X}_L = \hat{X}_1\hat{X}_2\hat{X}_3\hat{X}_4\hat{X}_5$$

$$\hat{Z}_L = \hat{Z}_6\hat{Z}_7\hat{Z}_3\hat{Z}_8\hat{Z}_9$$

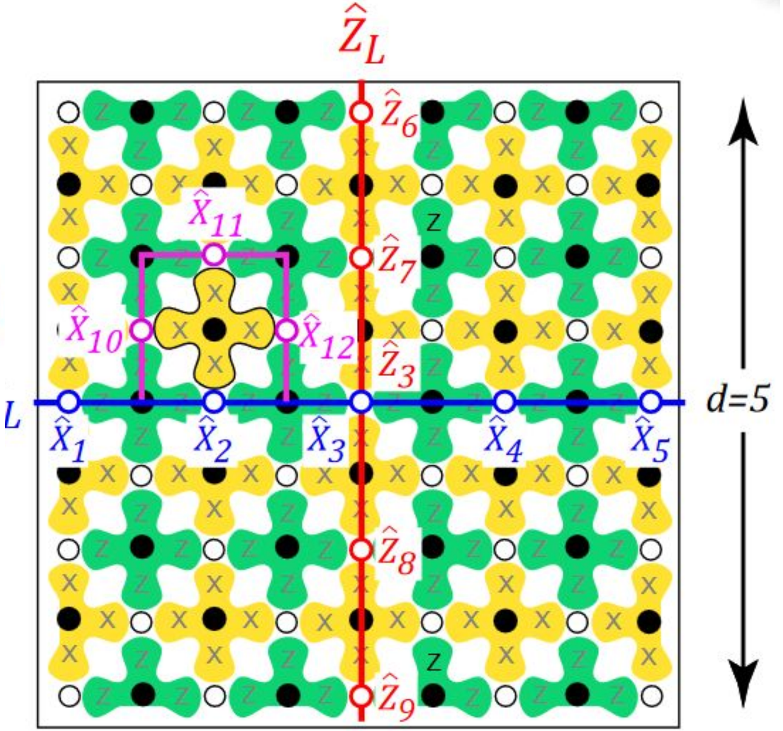


Logical Qubits

More operators?

$$
\begin{aligned}
\hat{X}'_L &= \hat{X}_1\hat{X}_{10}\hat{X}_{11}\hat{X}_{12}\hat{X}_3\hat{X}_4\hat{X}_5 \\
&= \left(\hat{X}_2\hat{X}_{10}\hat{X}_{11}\hat{X}_{12}\right)\left(\hat{X}_1\hat{X}_2\hat{X}_3\hat{X}_4\hat{X}_5\right) \\
&= \boxed{\left(\hat{X}_2\hat{X}_{10}\hat{X}_{11}\hat{X}_{12}\right)}\hat{X}_L,
\end{aligned}
$$

Stabilizer



Logical Operators

$$
\begin{aligned}
\hat{X}_L\hat{Z}_L &= \left(\hat{X}_1\hat{X}_2\hat{X}_3\hat{X}_4\hat{X}_5\right)\left(\hat{Z}_9\hat{Z}_{10}\hat{Z}_3\hat{Z}_{11}\hat{Z}_{12}\right) \\
&= \hat{X}_3\hat{Z}_3\left(\hat{X}_1\hat{X}_2\hat{X}_4\hat{X}_5\right)\left(\hat{Z}_9\hat{Z}_{10}\hat{Z}_{11}\hat{Z}_{12}\right) \\
&= -\hat{Z}_3\hat{X}_3\left(\hat{Z}_9\hat{Z}_{10}\hat{Z}_{11}\hat{Z}_{12}\right)\left(\hat{X}_1\hat{X}_2\hat{X}_4\hat{X}_5\right) \\
&= -\hat{Z}_L\hat{X}_L,
\end{aligned}
$$

$$\hat{Y}_L = \hat{Z}_L\hat{X}_L$$

Recall:

$$\hat{X}^2 = -\hat{Y}^2 = \hat{Z}^2 = \hat{I}$$

$$\hat{X}\hat{Z} = -\hat{Z}\hat{X}$$

$$[\hat{X},\hat{Y}] \equiv \hat{X}\hat{Y} - \hat{Y}\hat{X} = -2\hat{Z}$$

Questions?