Quantum Approximate Optimization Algorithm for the Shortest Vector Problem

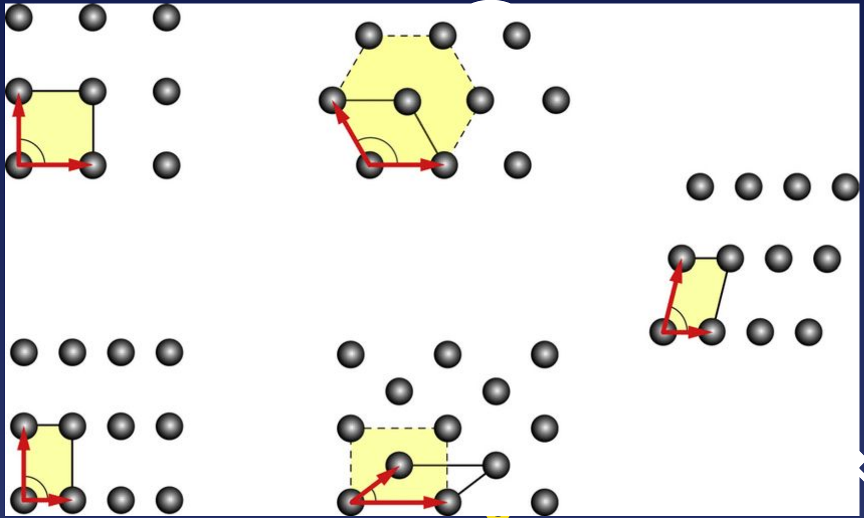
Miriam Kim
High School Apprentice through AEOP

01

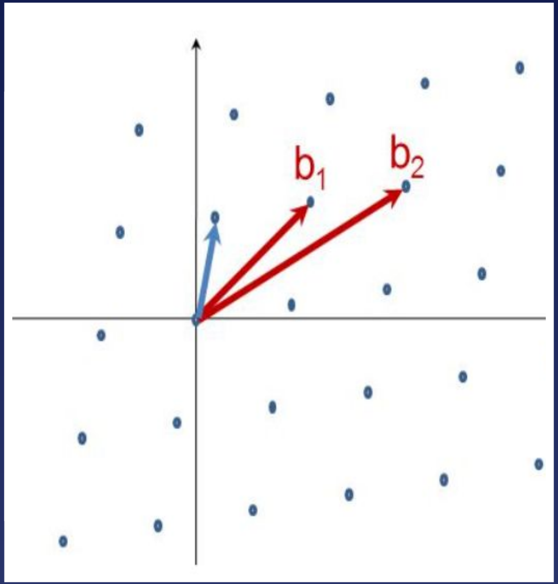
Intro and Background

Important Concepts

- Lattices
    - Lattice based cryptography
- Optimization Problems
    - How are they solved?
    - Applications
- Hamiltonian

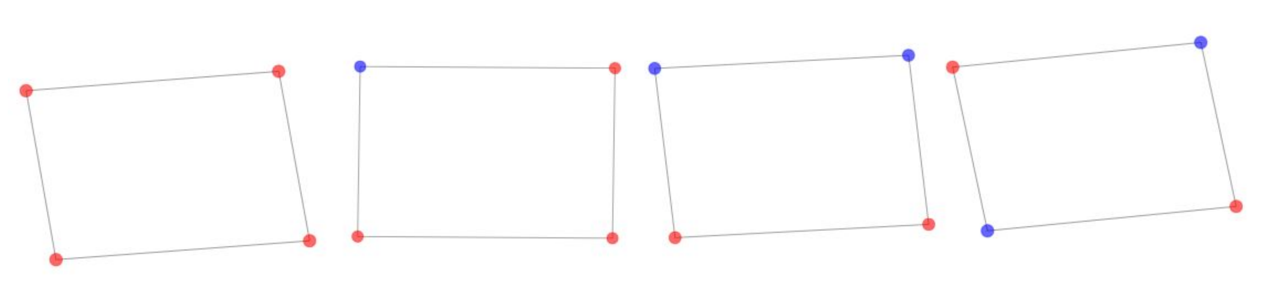


Shortest Vector Problem



- A **Lattice Problem**
- Given a **basis** of vector space V, a norm N, for lattice L, **find the shortest non-zero vector in L**
- Relevance

Max-Cut Problem



- Given a graph, partition nodes into two sets such that the edges between the sets is **maximum**
- Problem Hamiltonian up to a constant:

$$H_P = \frac{1}{2}(Z_0 \otimes Z_1 \otimes I_2 \otimes I_3) + \frac{1}{2}(I_0 \otimes Z_1 \otimes Z_2 \otimes I_3) + \frac{1}{2}(Z_0 \otimes I_1 \otimes I_2 \otimes Z_3) + \frac{1}{2}(I_0 \otimes I_1 \otimes Z_2 \otimes Z_3)$$

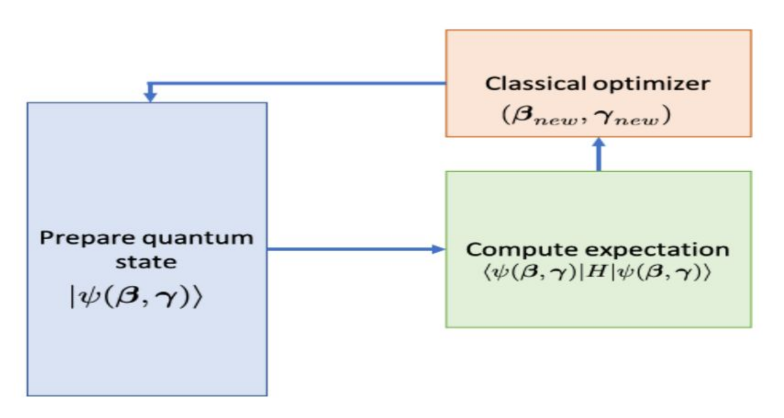
About QAOA

Farhi et al.

Quantum Approximate Optimization Algorithm

A quantum algorithm that attempts to solve combinatorial optimization problems

- Applies Hamiltonians to find optimal parameters (β, γ) to find the minimum eigenvalue of H_p
- $U(\beta, \gamma) \rightarrow |\psi(\beta, \gamma)>$
- Classical bit string expected to have good approximation ratio



QAOA

01

Initial State

- Apply Hadamard gate to each qubit

02

Construct Unitaries from Hamiltonians

- $U(H_B) = e^{-i\beta H\_b}$
- $U(H_P) = e^{-i\gamma H\_p}$

03

Pick a *p* and initialize β, γ

04

Apply Unitaries for *p* times to form state

$$|\psi(\boldsymbol{\beta},\boldsymbol{\gamma})\rangle = \underbrace{U(\beta)U(\gamma)\cdots U(\beta)U(\gamma)}_{p \text{ times}}|\psi_0\rangle$$

05

Measure $|\beta, \gamma>$

06

Evaluate expectation value classically

Classical and Quantum

Classical Solution

- Gram-Smith Orthogonalization
- LLL Algorithm

Quantum Solution

- Superposition states → function can be applied to many possible inputs simultaneously

02

Implementation

Construction

$$H_P = \sum_{i,j}^{N} \hat{Q}^{(i)} \hat{Q}^{(j)} \mathbf{G}_{ij},$$

Problem Hamiltonian

$$H_{\mathrm{M}} = \sum_{j=1}^{n} X_j,$$

Mixing Hamiltonian

$$|\psi_0\rangle = \left(\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \right)^{\otimes n}$$

Initial State

Problem Hamiltonian

$$H_P = \sum_{i,j}^{N} \hat{Q}^{(i)}\hat{Q}^{(j)}\mathbf{G}_{ij},$$

(Joseph et al.)

$$H_p = \sum_{i,j}^{N=2} \hat{Q}^i \hat{Q}^j G_{i,j}$$

$$= \hat{Q}^1 \hat{Q}^1 G_{11}$$
$$+ \hat{Q}^1 \hat{Q}^2 G_{12}$$
$$+ \hat{Q}^2 \hat{Q}^2 G_{22}$$
$$+ \hat{Q}^2 \hat{Q}^1 G_{21}$$

$$G_{11} = \vec{b}_1 \cdot \vec{b}_1 = a^2 + b^2$$

$$G_{12} = \vec{b}_1 \cdot \vec{b}_2 = ac + bd$$

$$G_{22} = \vec{b}_2 \cdot \vec{b}_2 = c^2 + d^2$$

$$G_{21} = \vec{b}_2 \cdot \vec{b}_1 = ca + db$$

$$Q_1 = (z_i^0 + z_i^1 \; z_i^2 \ldots z_i^7)$$

$$Q_2 = (z_j^0 + z_j^1 + z_j^2 \ldots z_j^7)$$

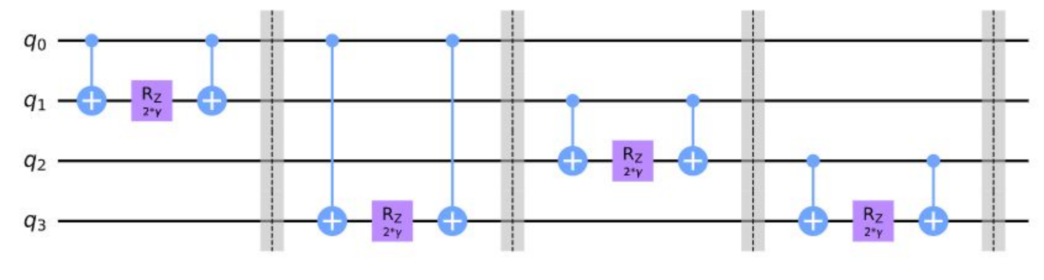
$$U(\gamma) = e^{-i\gamma H_p}$$

$$U(\gamma) = e^{-i\gamma H_P}$$

$$U(H_B) = e^{-i\beta H_B}$$

```
gamma = Parameter("$\\gamma$")
qc_p = QuantumCircuit(nqubits)
for pair in list(G.edges()):  # pairs of nodes
    qc_p.rzz(2 * gamma, pair[0], pair[1])
    qc_p.barrier()

qc_p.decompose().draw()
```



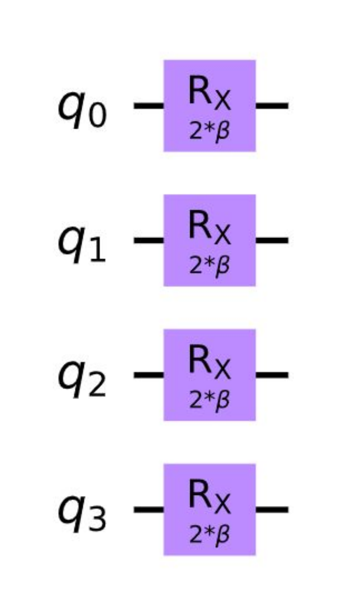
Problem Unitary

Mixing Unitary

```python
nqubits = 4

beta = Parameter("$\\beta$")
qc_mix = QuantumCircuit(nqubits)
for i in range(0, nqubits):
    qc_mix.rx(2 * beta, i)

qc_mix.draw()
```



Classical Optimization

```
from scipy.optimize import minimize


expectation = get_expectation(G, p=1)

res = minimize(expectation,
                      [1.0, 1.0],
                      method='COBYLA')
res
```

After the initial state is prepared apply $U(\beta, \gamma)$ and use classical techniques to optimize the parameters

- Prepare $|\psi(\beta,\gamma)\rangle$
- Measure the state
- Compute $\langle\psi(\beta,\gamma)|H_p|\psi(\beta,\gamma)\rangle$
- Find the new set of parameters
- Set the current parameters equal to the new and repeat

Analyzing Results

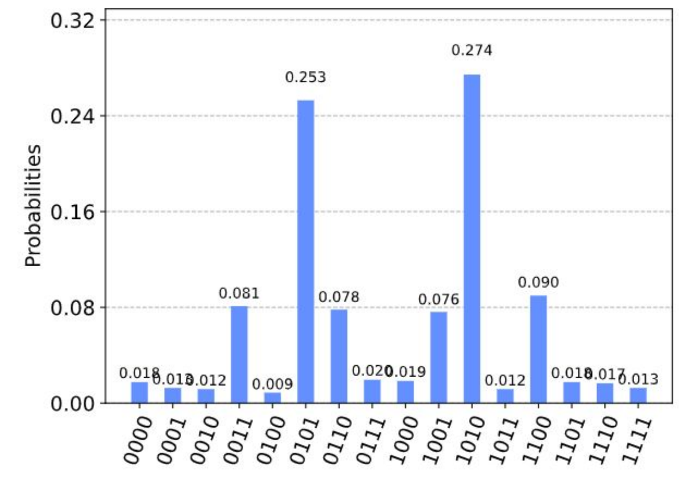
```
from qiskit.visualization import plot_histogram

backend = Aer.get_backend('aer_simulator')
backend.shots = 512

qc_res = create_qaoa_circ(G, res.x)

counts = backend.run(qc_res, seed_simulator=10).result().get_counts()

plot_histogram(counts)
```



THANKS!

CREDITS: This presentation template was created by Slidesgo, including icons by Flaticon, and infographics & images by Freepik