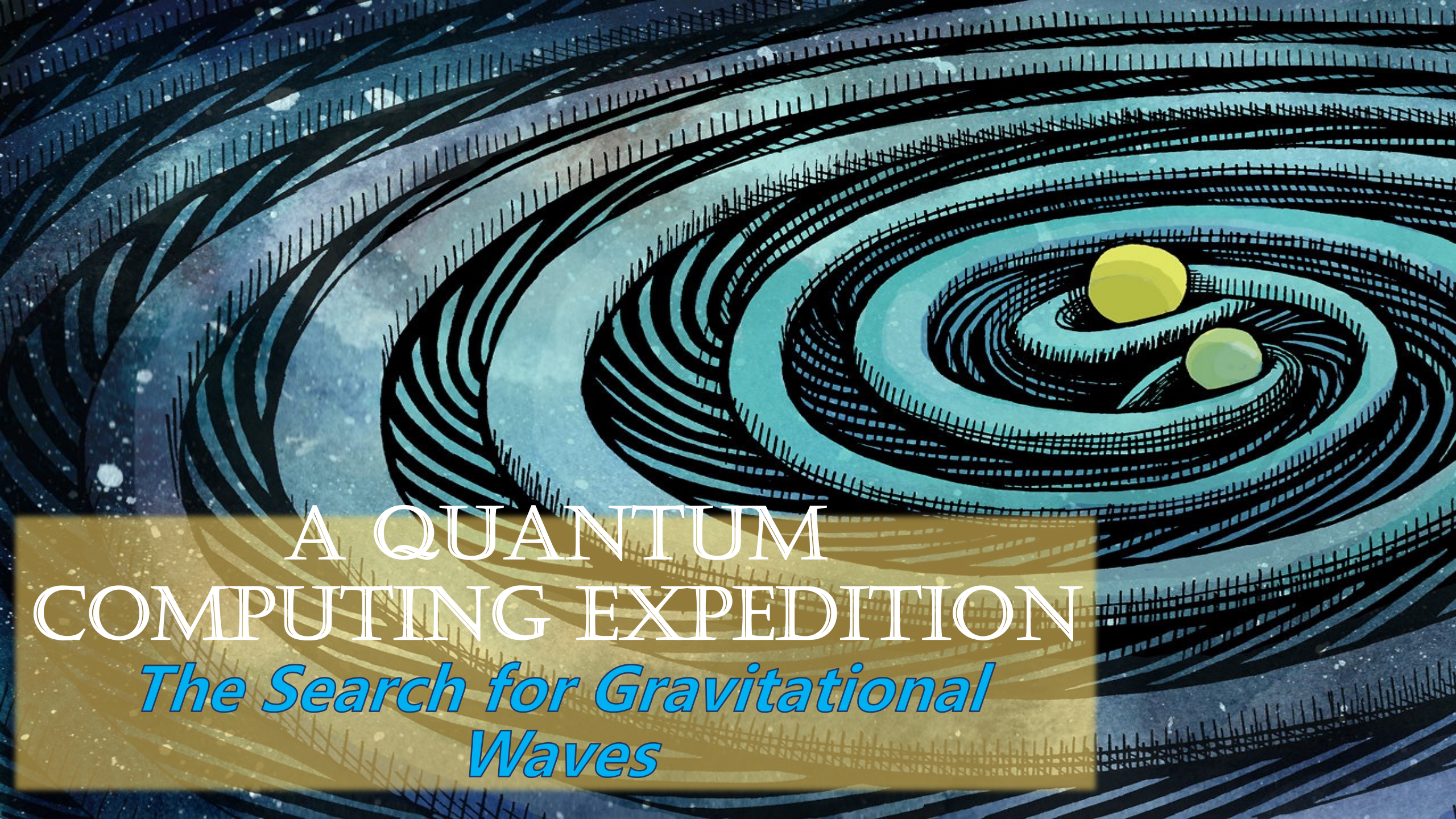
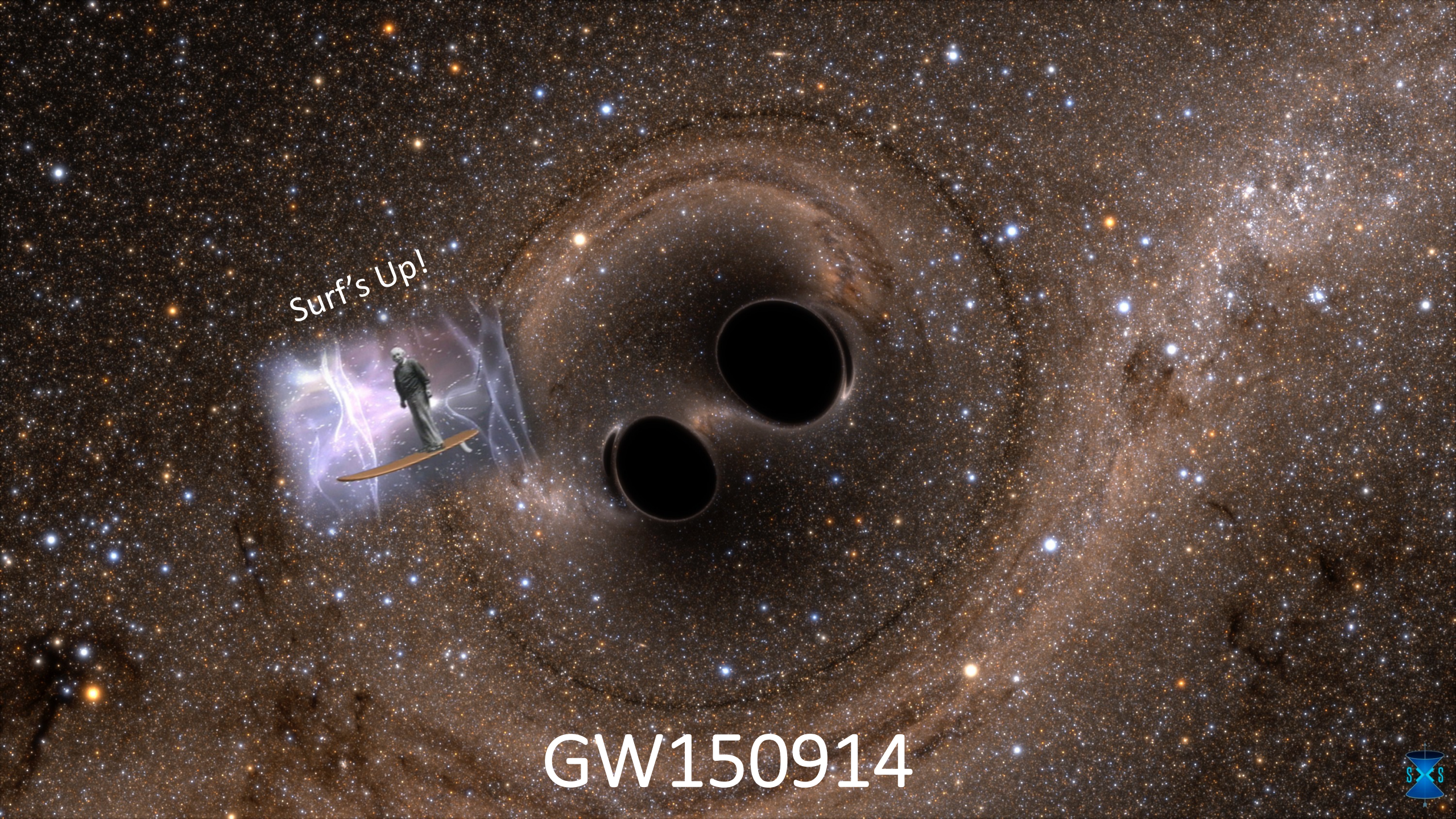
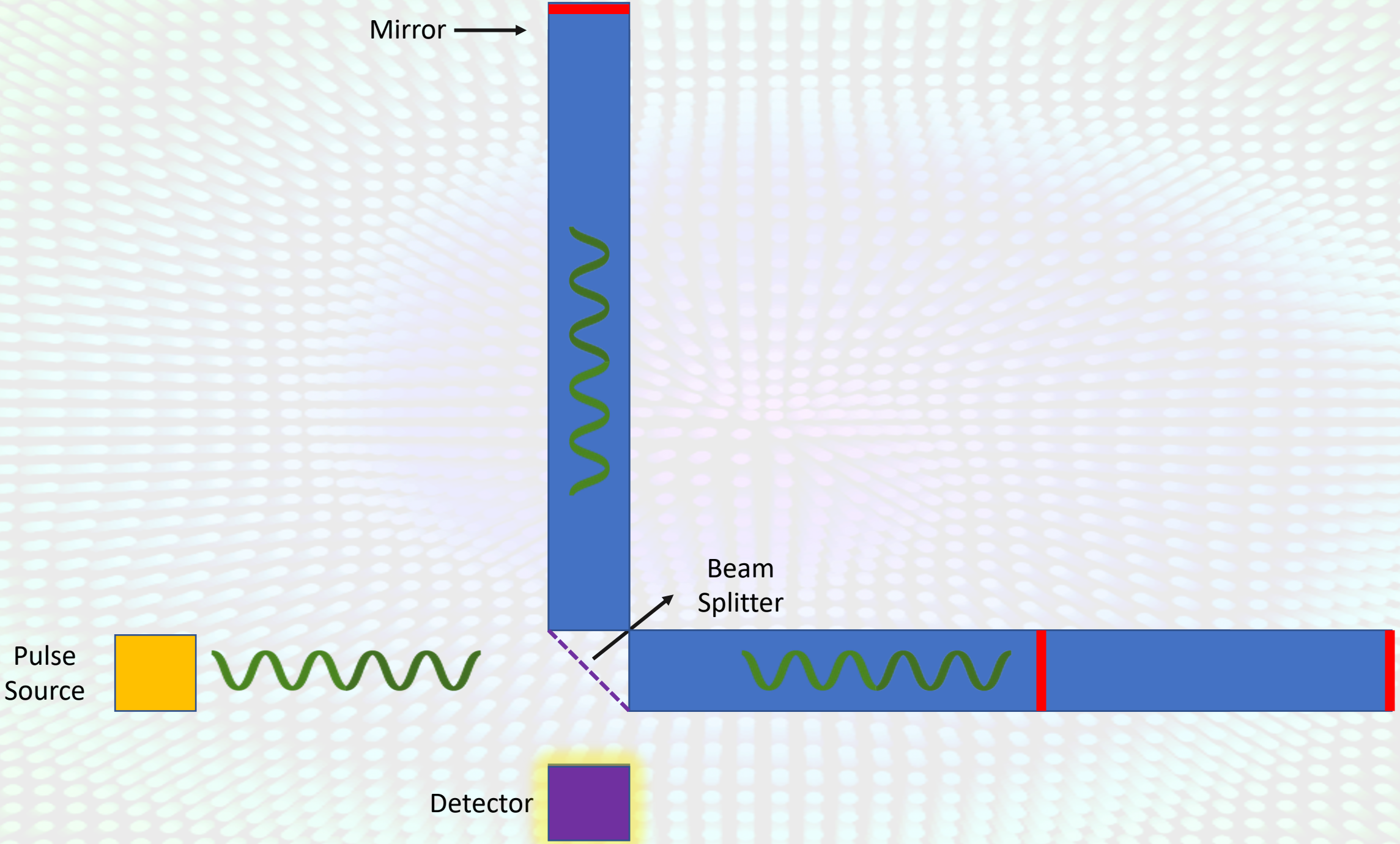
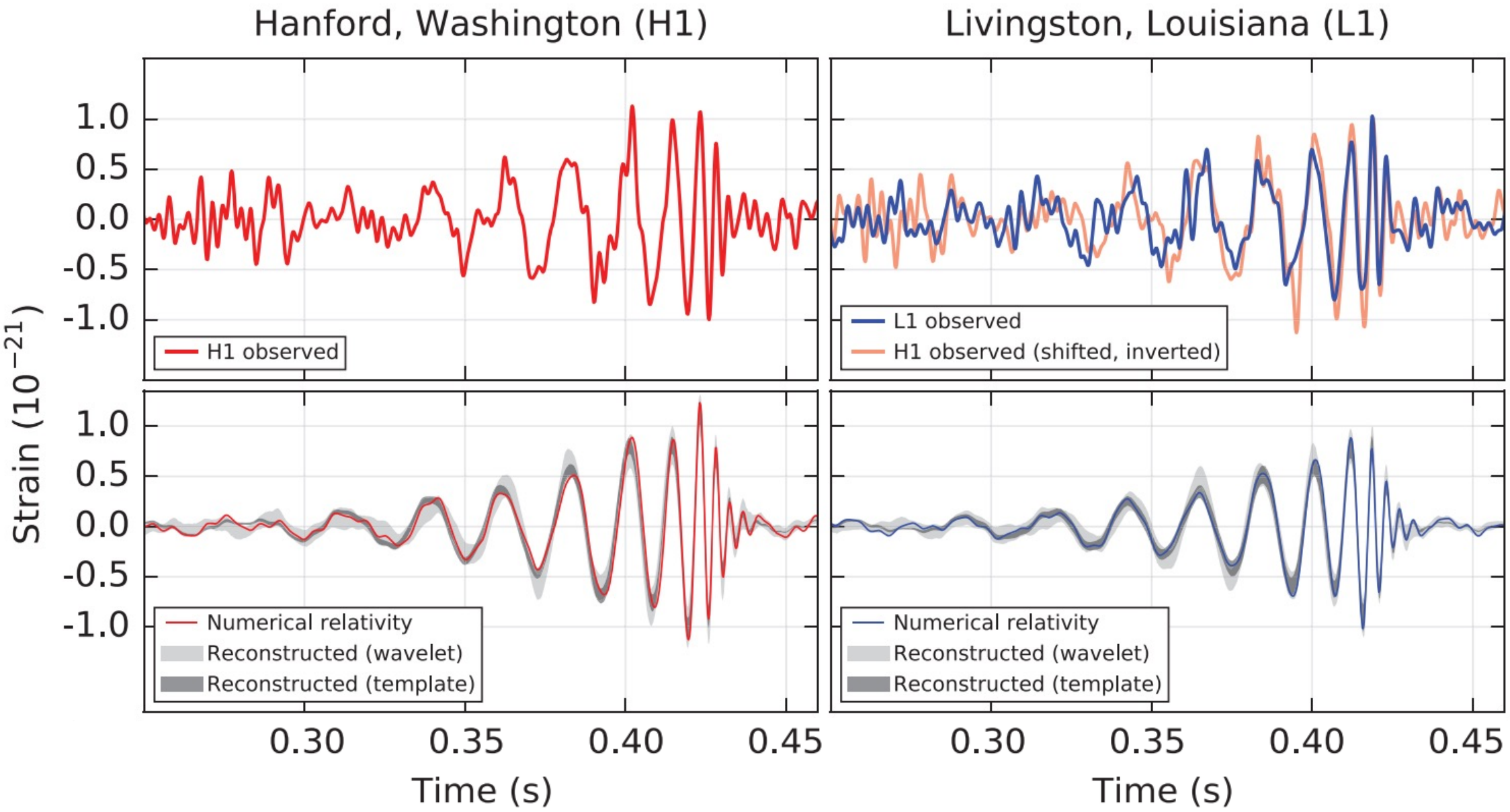

A QUANTUM
COMPUTING EXPEDITION
The Search for Gravitational Waves


Surf's Up!
GW150914


Mirror
Beam
Splitter
Pulse
Source
Detector



Abbott, B. P., et al. 2016. **"Observation of Gravitational Waves from a Binary Black Hole Merger."** Physical Review Letters. American Physical Society (APS).

Matched Filtering

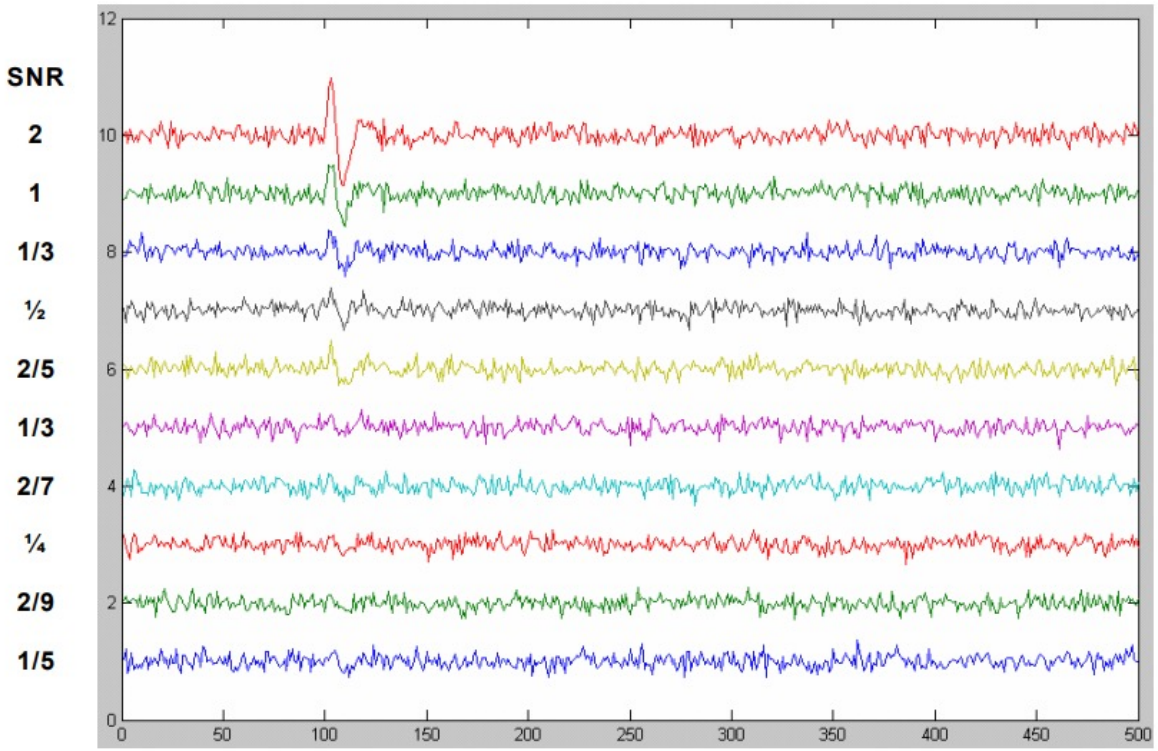


FIG. 3: Ten noisy traces and wavelets with a SNR that varies from 2.0 to 0.2.

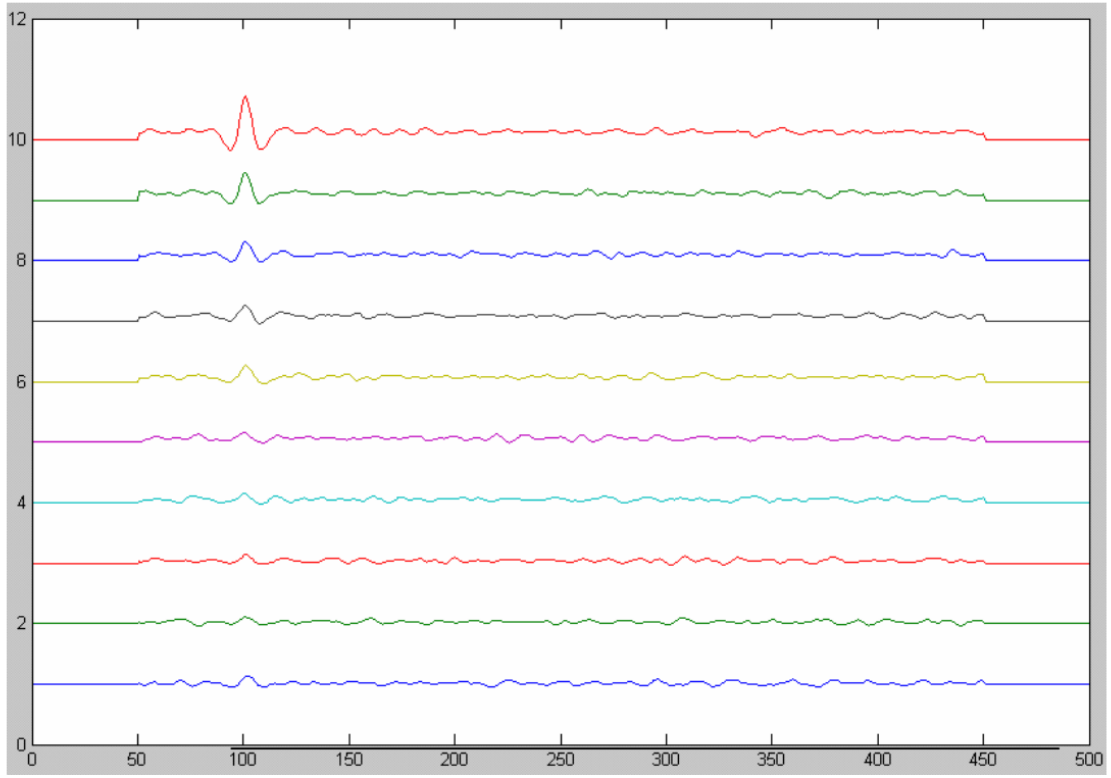


FIG. 4: Results of a matched filter from cross-correlating the wavelet in Figure 2 with the noisy signals in Figure 3,

J. C. Bancroft, Introduction to matched filters - Crewes, 2002

A quantum algorithm for gravitational wave matched filtering

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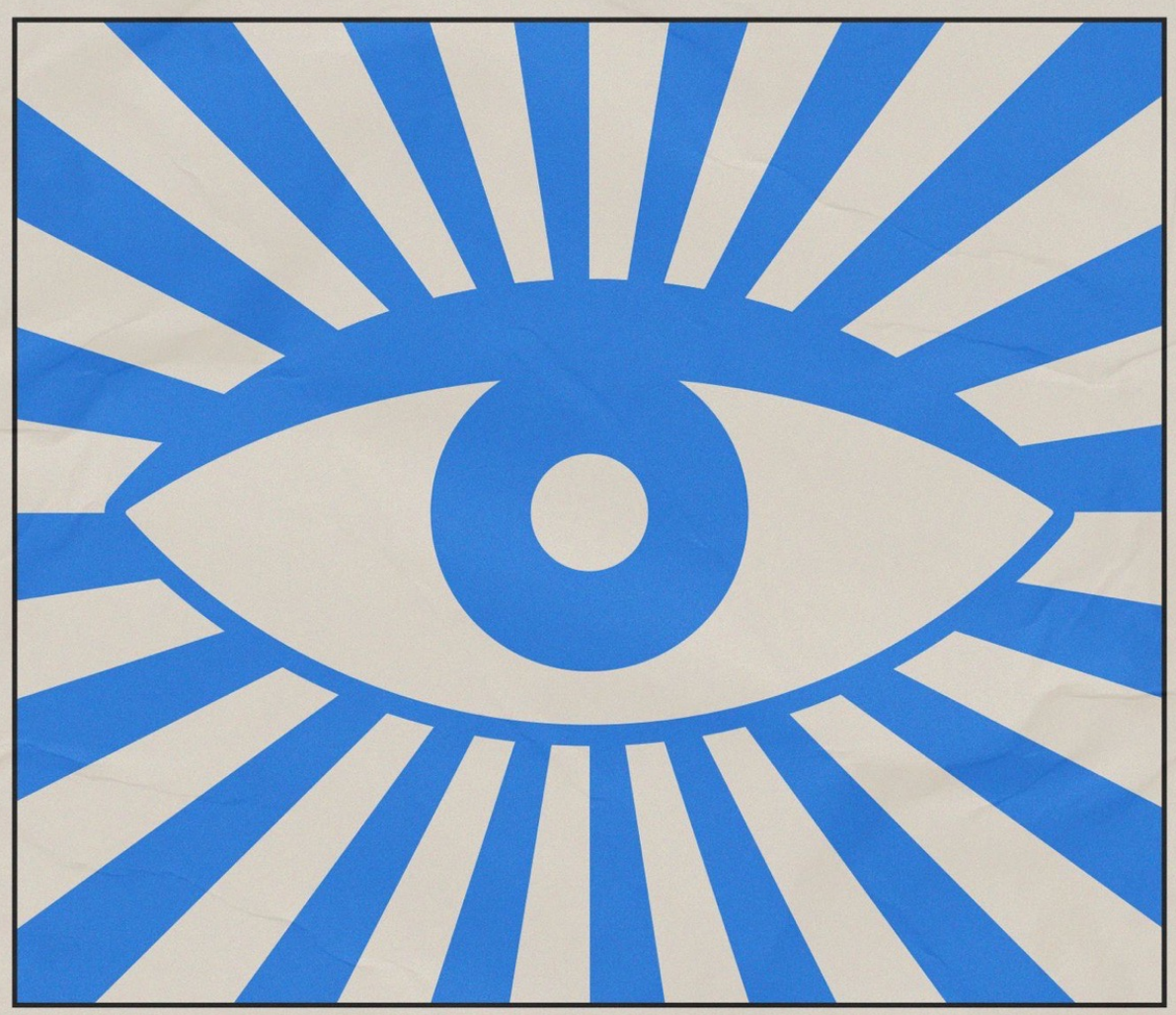
[1] *SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom*

(Dated: September 6, 2021)

Quantum computational devices, currently under development, have the potential to accelerate data analysis techniques beyond the ability of any classical algorithm. We propose the application of a quantum algorithm for the detection of unknown signals in noisy data. We apply Grover's algorithm to matched-filtering, a signal processing technique that compares data to a number of candidate signal templates. In comparison to the classical method, this provides a speed-up proportional to the square-root of the number of templates, which would make possible otherwise intractable searches. We demonstrate both a proof-of-principle quantum circuit implementation, and a simulation of the algorithm's application to the detection of the first gravitational wave signal GW150914. We discuss the time complexity and space requirements of our algorithm as well as its implications for the currently computationally-limited searches for continuous gravitational waves.

PACS numbers: 03.67.Ac; 04.30.-w; 07.05.Kf
Keywords: Quantum algorithm, matched filtering, Grover's algorithm, gravitational waves, continuous waves, data analysis



The Oracle

The Set Up

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle,$$

$|s\rangle$ = The superposition of all theoretical data templates in a template bank.
$|i\rangle$ = The i-th template in your template bank
N = The number of templates in your template bank

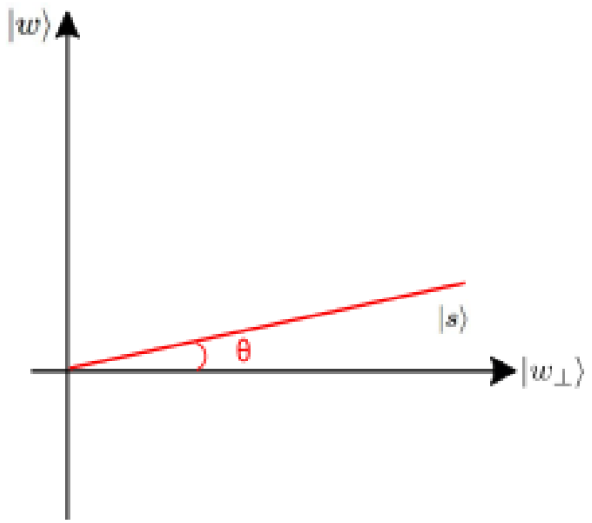
$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle, \longrightarrow |s\rangle = \sqrt{\frac{r}{N}}|w\rangle + \sqrt{\frac{N-r}{N}}|w_\perp\rangle.$$

$|w\rangle$ = The superposition of all the templates that would be **marked** as matching templates.

$|w_\perp\rangle$ = The superposition of all the other templates in our bank.

r = Number of matching templates

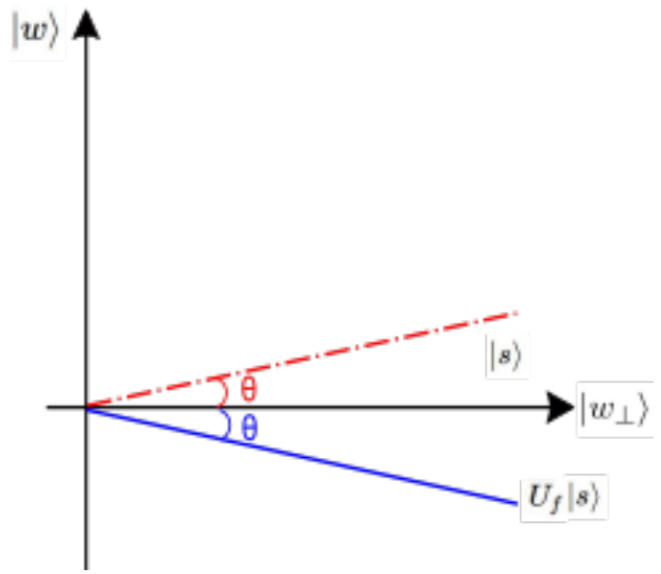
$$\theta = \arcsin\left(\langle w|s\rangle\right) = \arcsin\left(\sqrt{\frac{r}{N}}\right).$$



(a) The input state, represented by the red line.

We then apply an oracle U_f that is constructed to mark a solution with a negative sign.

$$U_f|s\rangle = -\sqrt{\frac{r}{N}}|w\rangle + \sqrt{\frac{N-r}{N}}|w_\perp\rangle,$$



(b) The state after the oracle is applied, represented by the blue line.

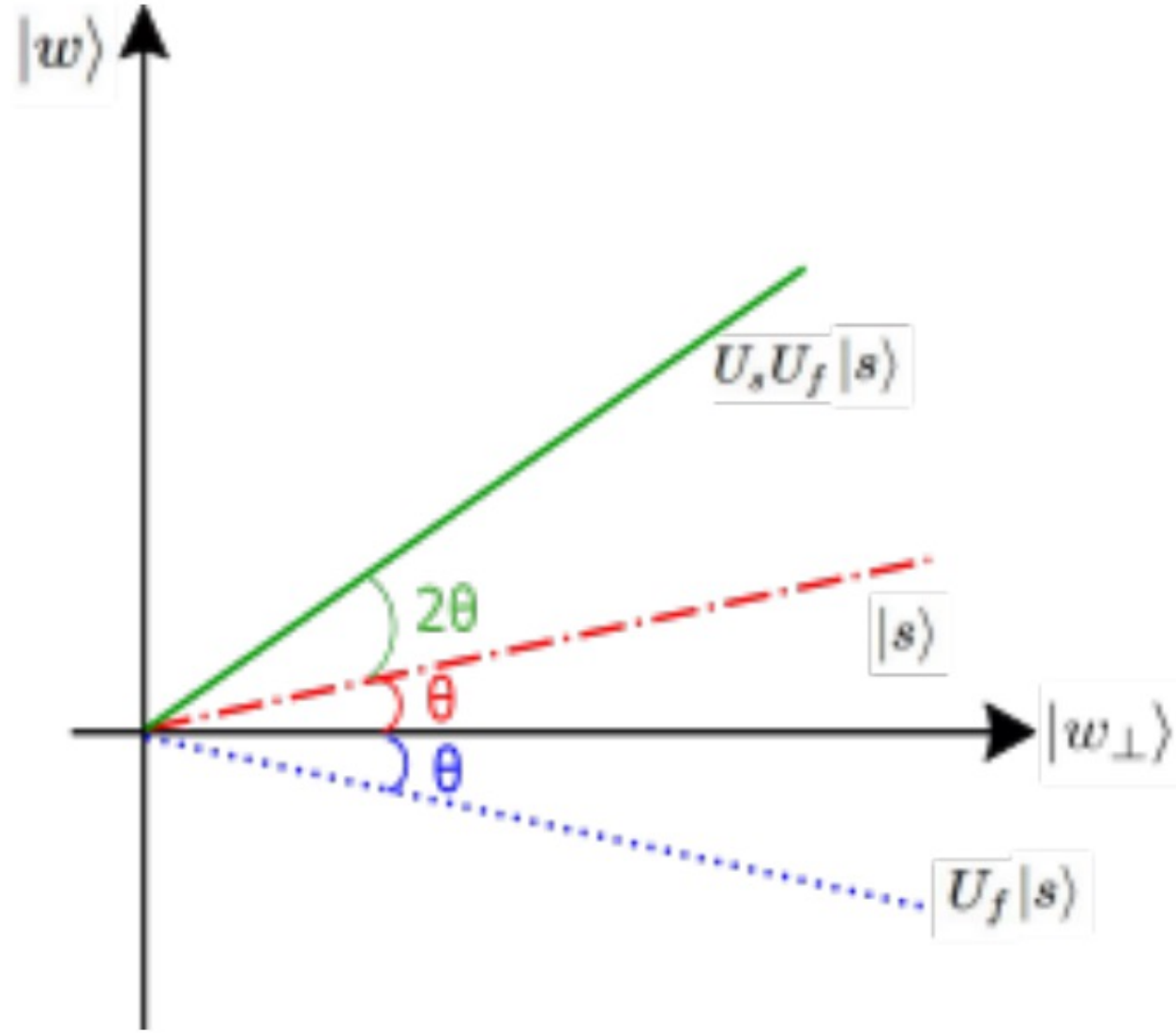
We finally apply the Diffusion Operator

$$U_s = 2|s\rangle\langle s| - \hat{\mathrm{I}},$$

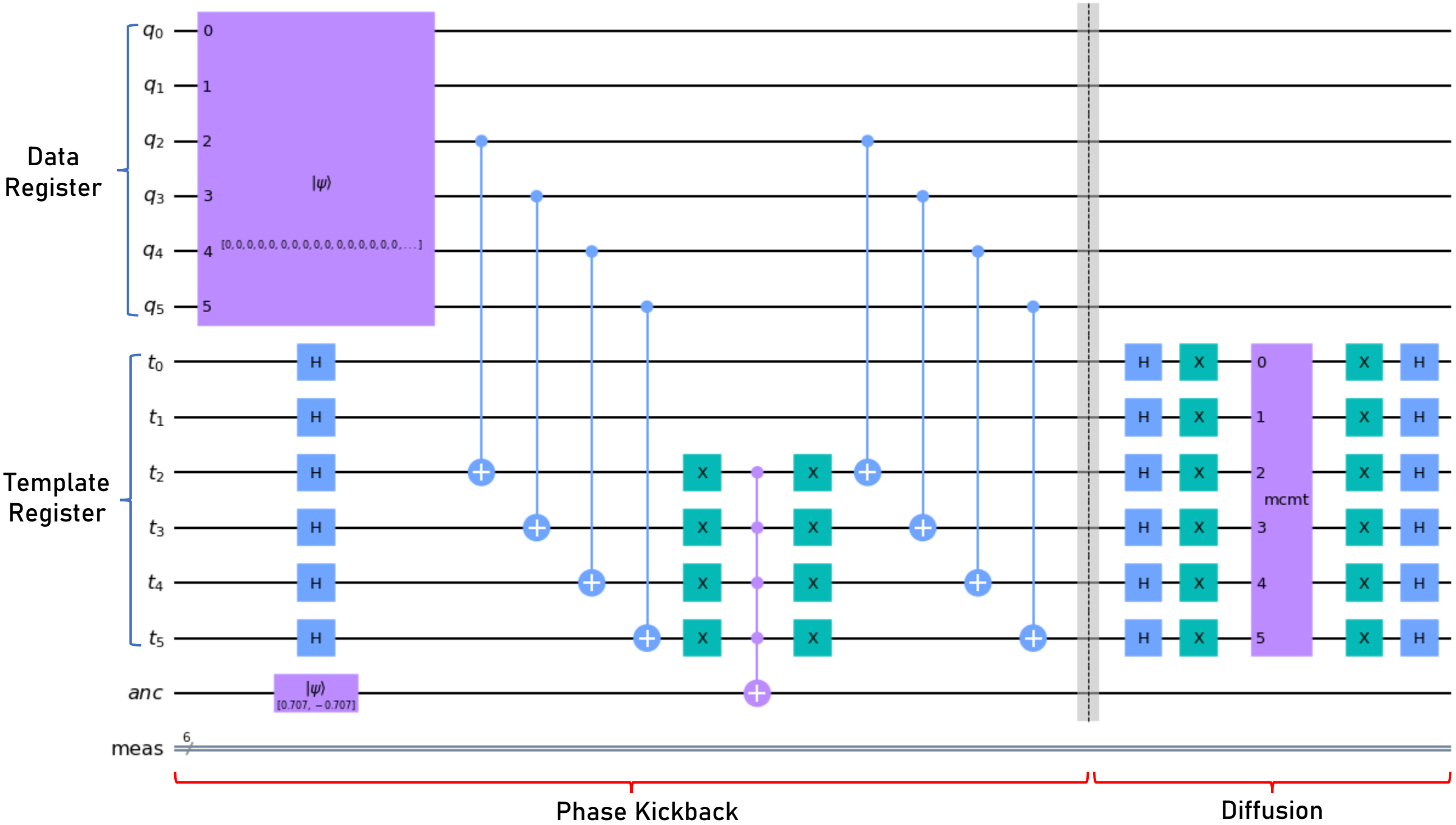
$$\mathrm{U}_s \left[-\sqrt{\tfrac{r}{N}}|w\rangle + \sqrt{\tfrac{N-r}{N}}|w_\perp\rangle \right] = (2|s\rangle\langle s| - I) \left[-\sqrt{\tfrac{r}{N}}|w\rangle + \sqrt{\tfrac{N-r}{N}}|w_\perp\rangle \right]$$

$$\left(3 - \tfrac{4r}{N}\right) \sqrt{\tfrac{r}{N}}|w\rangle + \left(1 - \tfrac{4r}{N}\right) \sqrt{\tfrac{N-r}{N}}|w_\perp\rangle$$

Matching Templates Superposition Amplified!

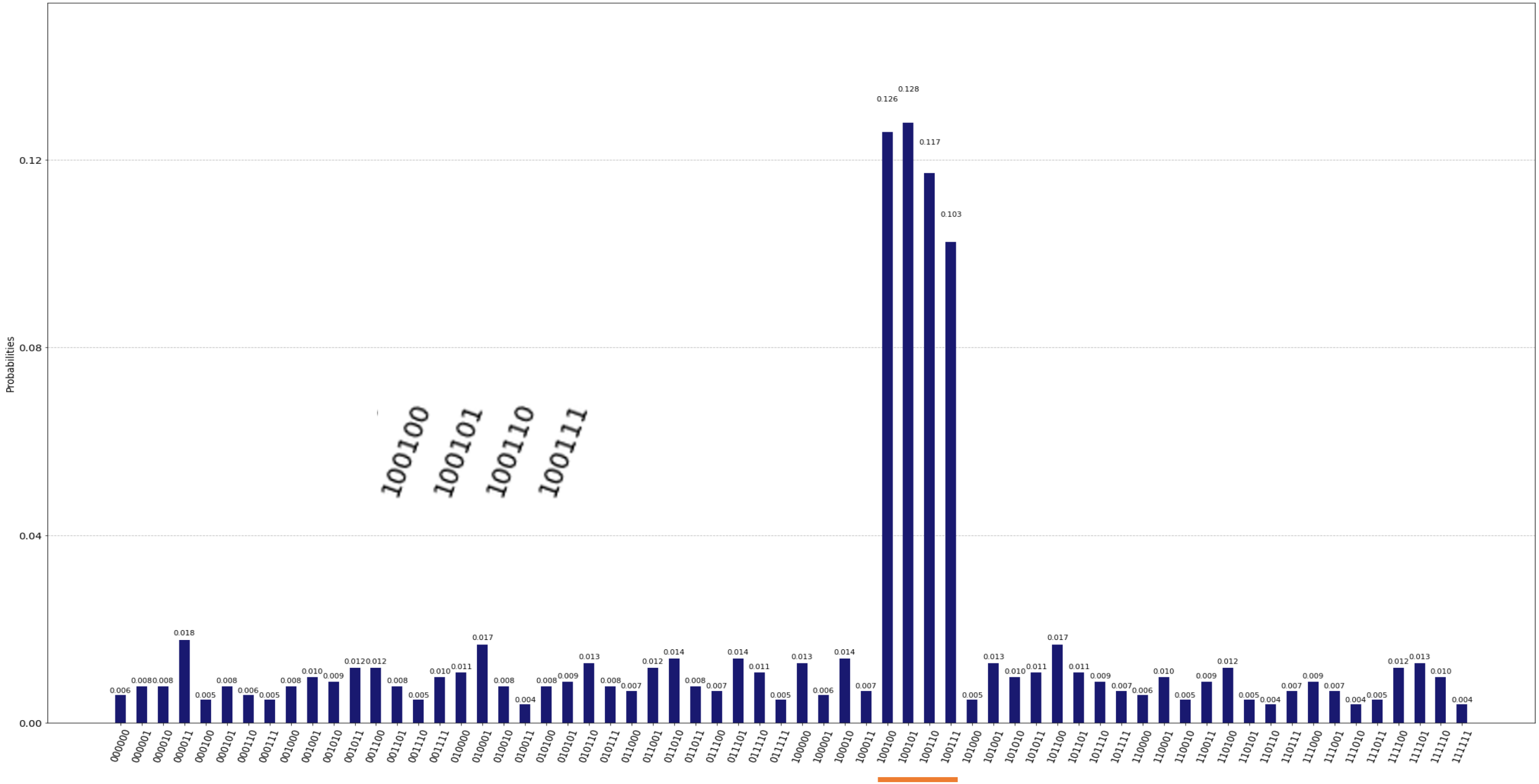


(c) The state after the diffusion operator, represented by the green line.

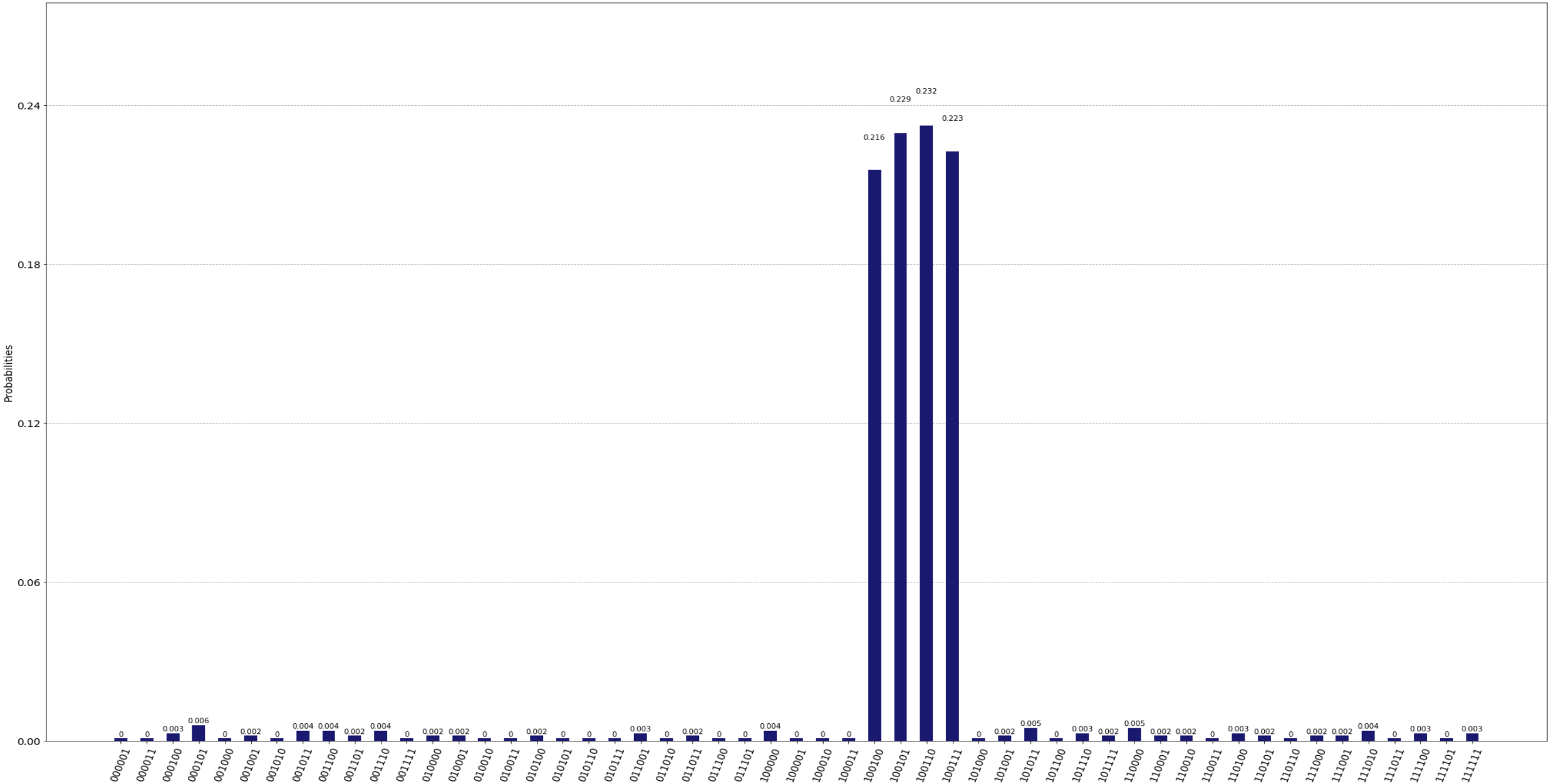

Data Register
Template Register
q_0
q_1
q_2
q_3
q_4
q_5
t_0
t_1
t_2
t_3
t_4
t_5
anc
meas
$|\psi\rangle$
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...]
$|\psi\rangle$
[0.707, −0.707]
H
X
mcmt
6
Phase Kickback
Diffusion

Input Signal (Solution): 100101

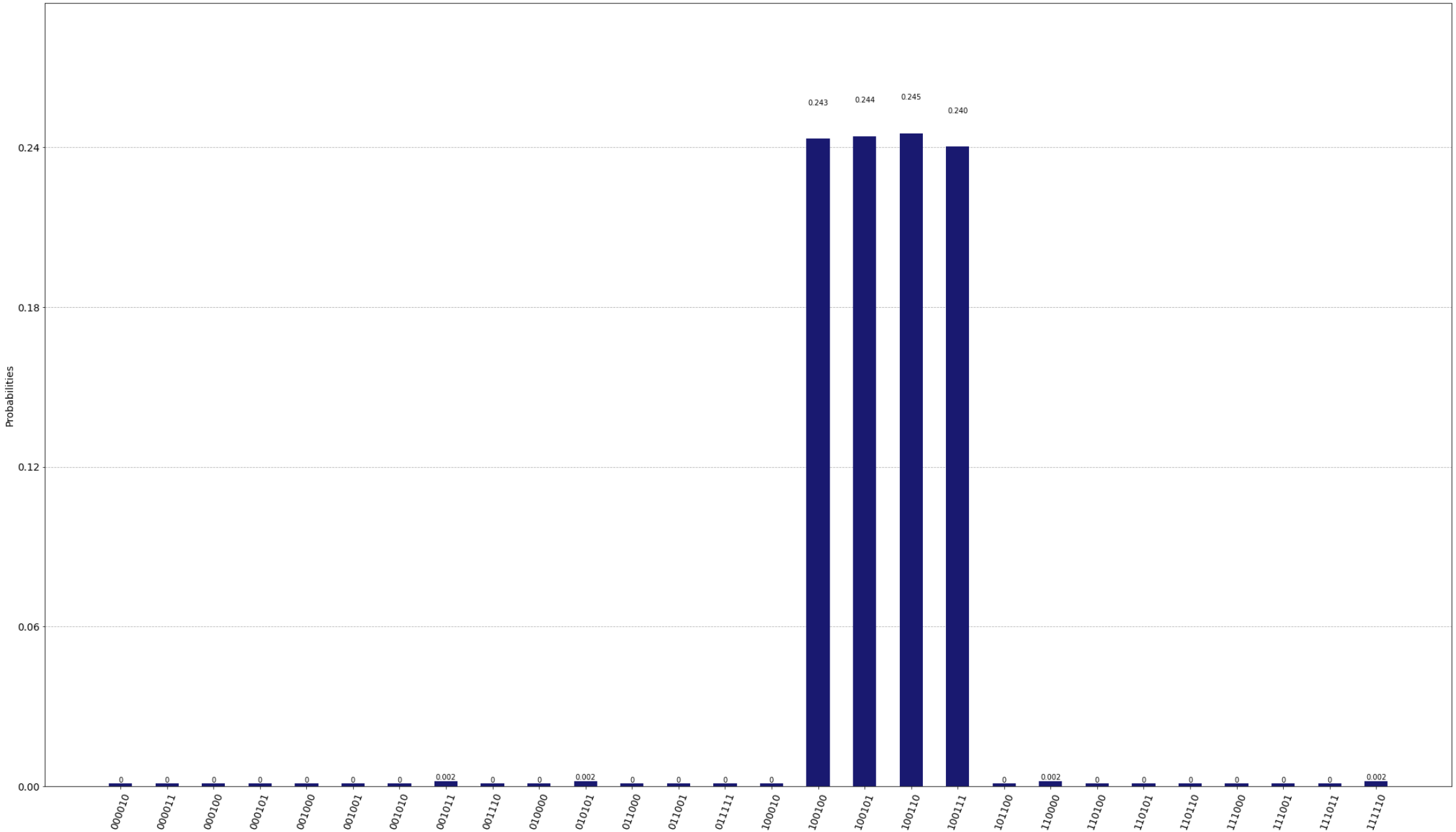
1 Grover Iteration



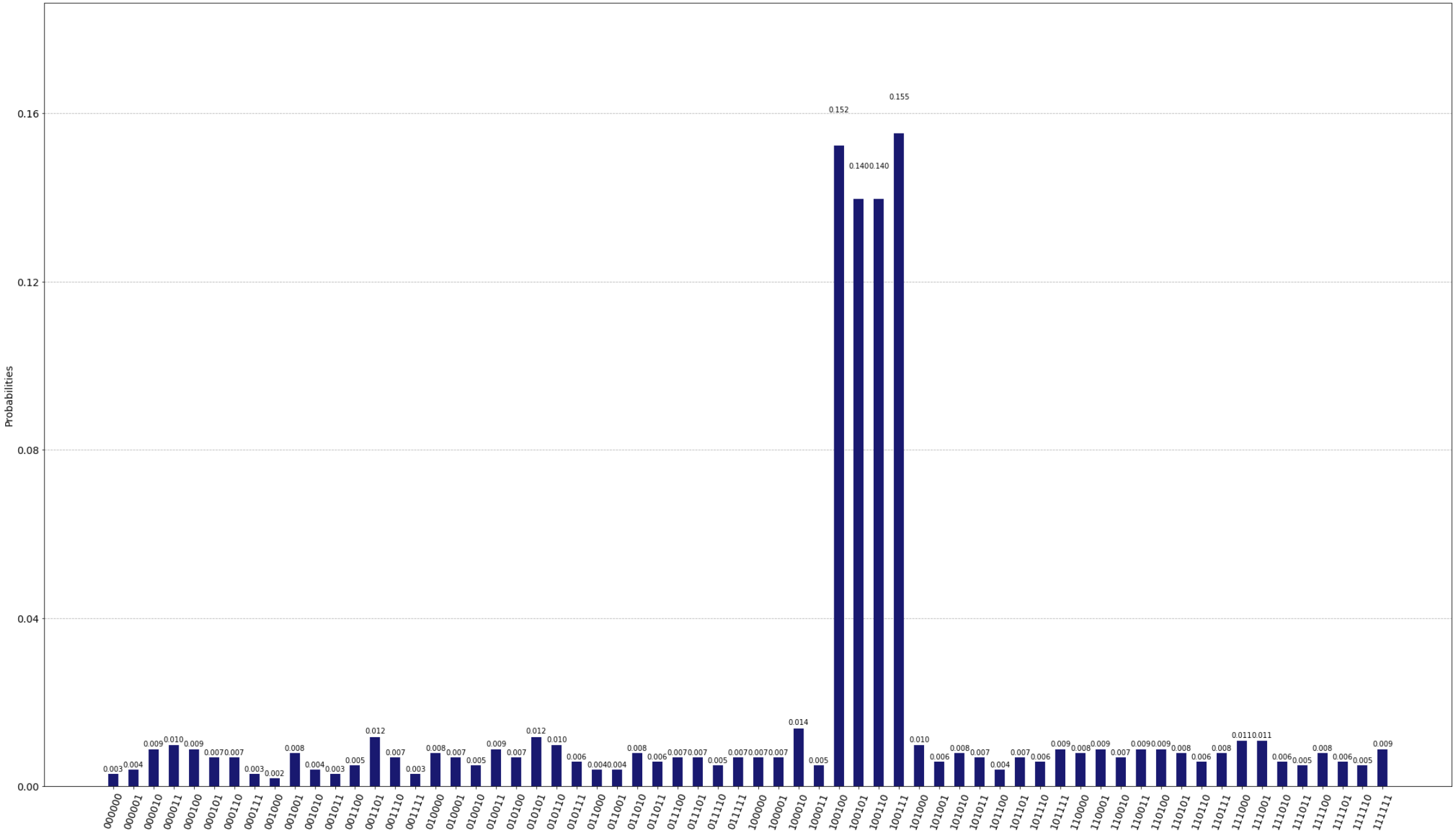
2 Grover Iterations

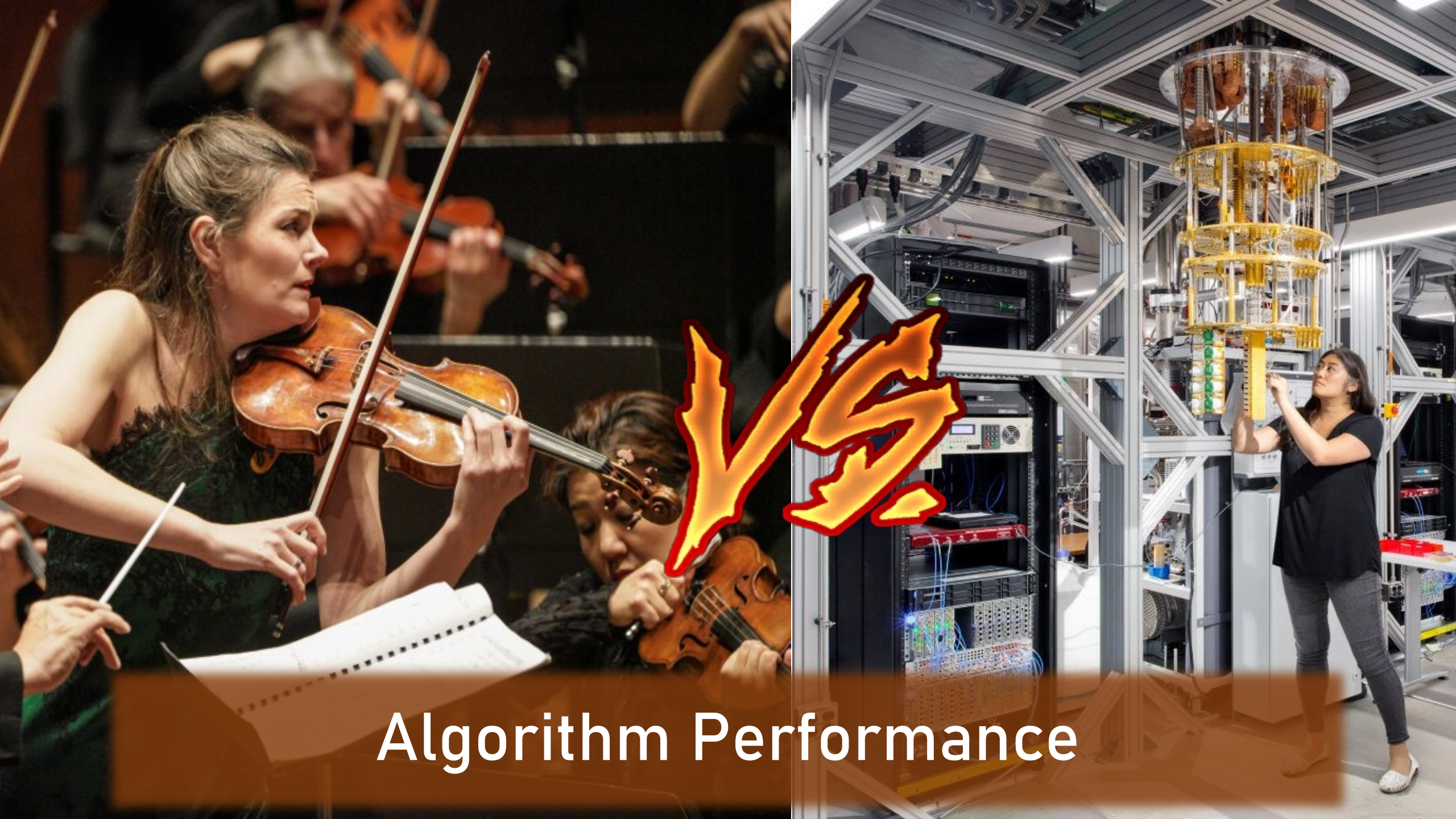


3 Grover Iterations

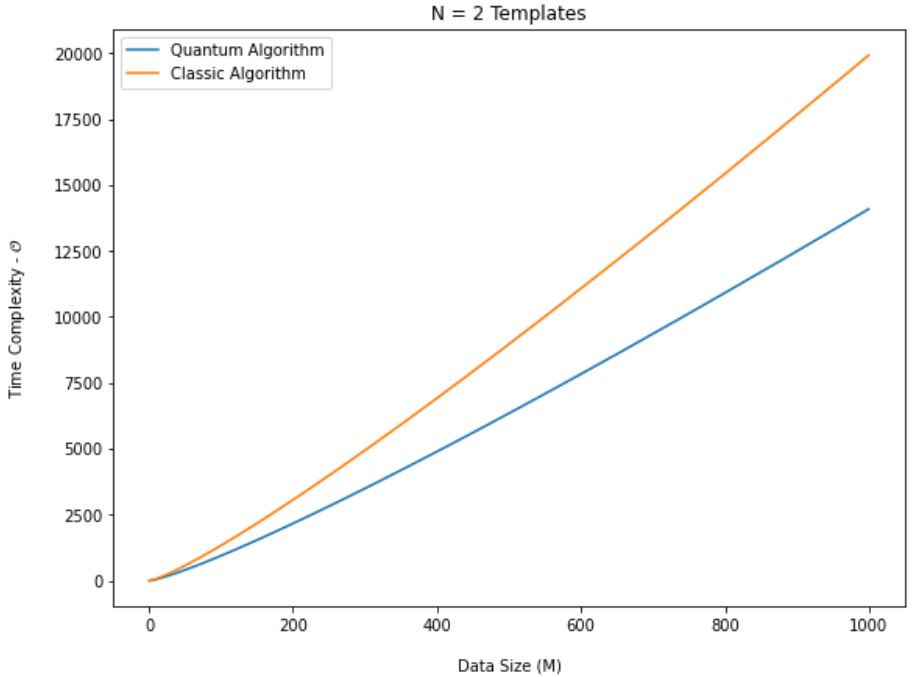


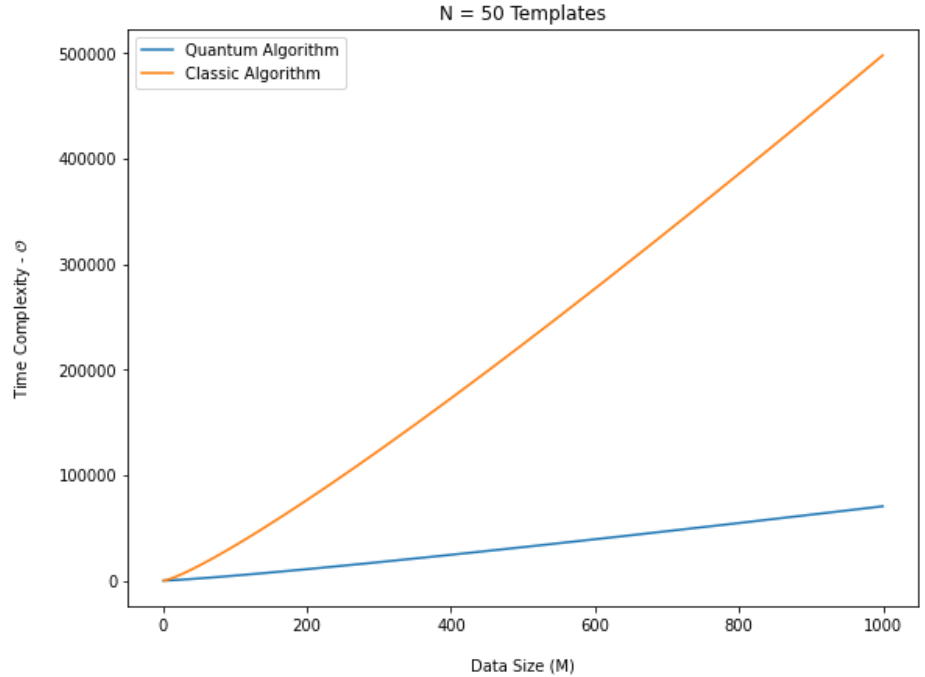
4 Grover Iterations

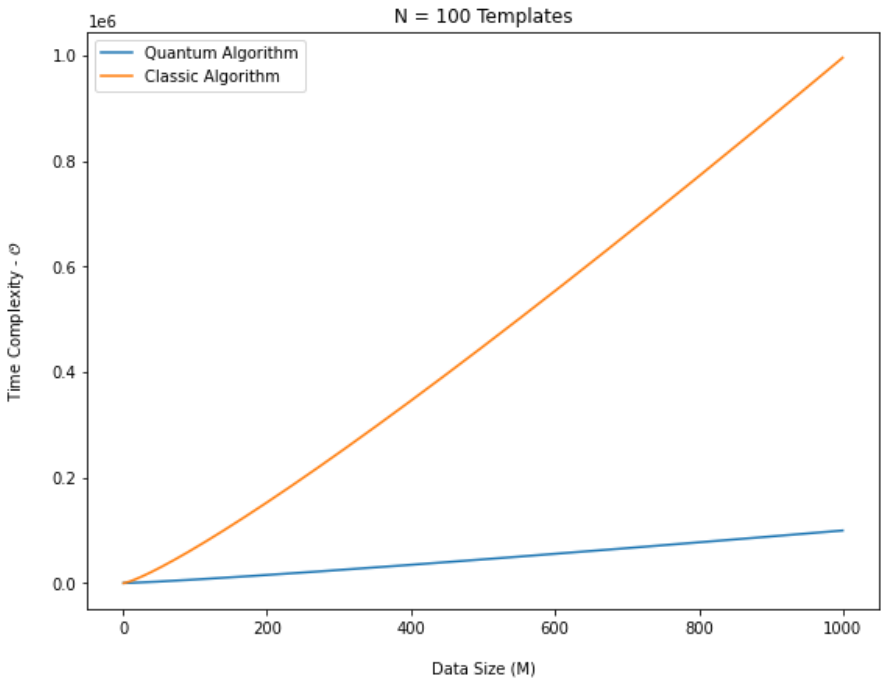


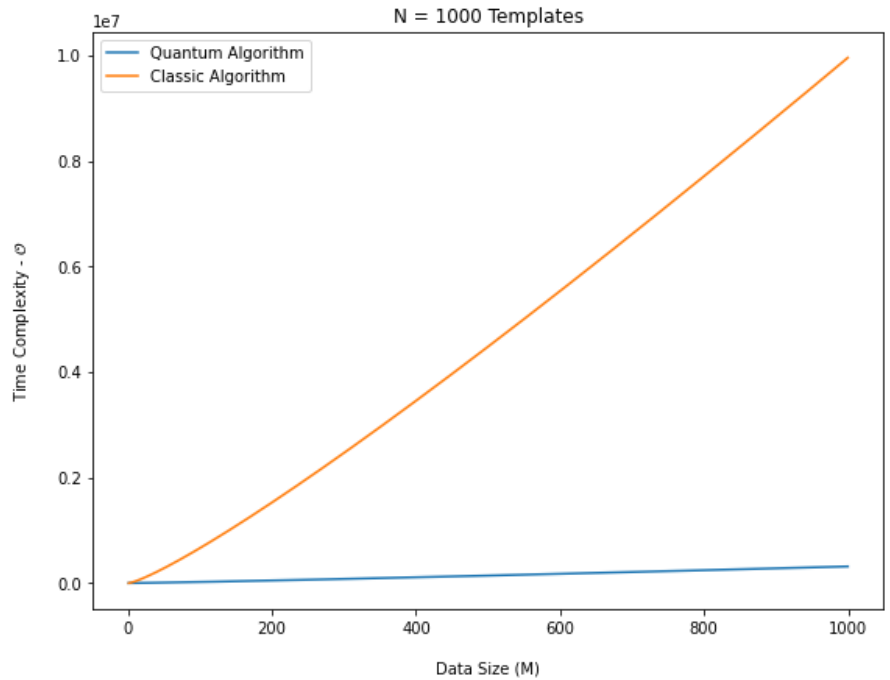


Algorithm Performance


N = 2 Templates
Quantum Algorithm
Classic Algorithm
Time Complexity - $\mathcal{O}$
Data Size (M)
0
2500
5000
7500
10000
12500
15000
17500
20000
0
200
400
600
800
1000


N = 50 Templates
Quantum Algorithm
Classic Algorithm
Time Complexity - $\mathcal{O}$
Data Size (M)
0
100000
200000
300000
400000
500000
0
200
400
600
800
1000


N = 100 Templates
1e6
Quantum Algorithm
Classic Algorithm
Time Complexity - $\mathcal{O}$
Data Size (M)
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000


N = 1000 Templates
1e7
Quantum Algorithm
Classic Algorithm
Time Complexity - $\mathcal{O}$
Data Size (M)
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000


YOUR ATTENTION
I THANK YOU FOR