Week 1

1. Section: Intro

Q: Which is a bit?

↳ A blade used by carpenter?

↳ Smallest unit of information (1 or 0)?

↳ Something you put in horses mouth?

A: All of them.

1.2 Bits

Represent any information by 2 numbers 0 or 1.

eg $\underline{61} = 6 \times 10^1 + 1 \times 10^0$

or $\underline{61} = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3$
$+ 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 111101$

in binary

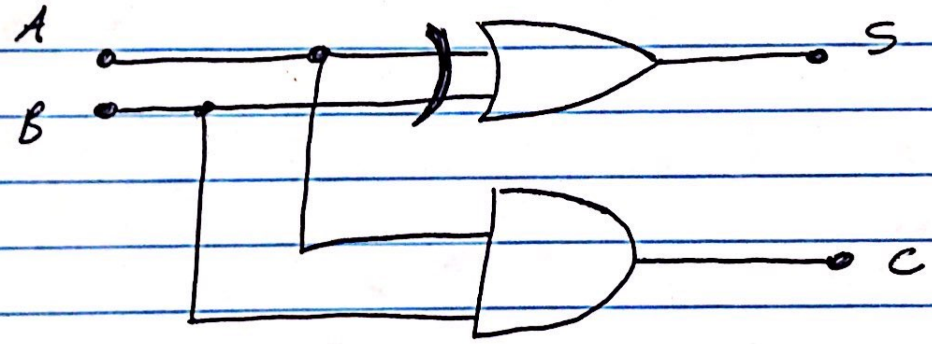
1.1 Cont'd

These strings of bits can be used to represent letters, numbers, colors ...

1.2 Diagrams

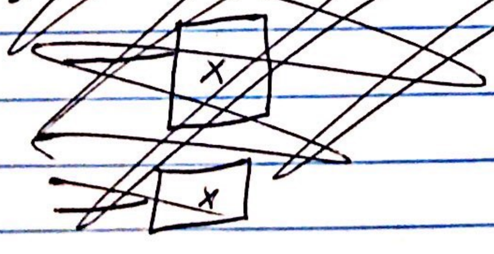
Manipulate ~~Use~~ bits and qubits to go from inputs to outputs.

Use circuit diagram



Important part is that you put in inputs, manipulate them, and get out outputs.

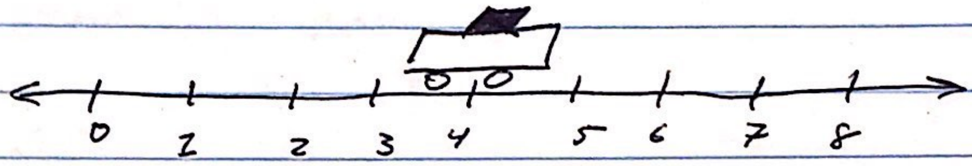
~~Some thing in a quantum circuit.~~



1.3 Classical vs. Quantum bits

State vectors:

ex/ Describe position of a car on a track.



→ use number, x.

$$x = 4$$

→ use the probability of finding the car in a certain place.

$$x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

probability of finding the car at ~~pos.~~ pos. 1 is 0%

prob. of finding car at pos. 4 is 100%.

1.3 cont'd

Qubit Notation:

for classical bits, can only assign a 0 or 1.

ex: $c = 0$

for qubits, we can have something more complex.

↳ prepare qubit to give a 0.

Call the state that gives the val. 0 $|0\rangle$. similarly, val. 1 gets $|1\rangle$

→ Use 2 orthogonal vectors.

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The notation of $|i\rangle$ is Bra-ket notation,

↳ remind us we are talking about states which give the value 0 or 1.

1.3 Cont'd

$|0\rangle$ and $|1\rangle$ form an orthonormal basis, so any 2D vector is represented as a linear combination of $|0\rangle$ and $|1\rangle$

$$\hookrightarrow \quad |q_0\rangle = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}|0\rangle + -\frac{1}{\sqrt{2}}|1\rangle$$

$|q_0\rangle$ describes the state of the qubit.

↳ not entirely in state $|0\rangle$ and not entirely in state $|1\rangle$.

↳ call this a "superposition"

1.3 Cont'd

Given state

$$|q_0\rangle = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

Let's measure it. (whether it's in $|0\rangle$ or $|1\rangle$)

↳ we said that $\begin{bmatrix} x \\ y \end{bmatrix}$ represents

a probability $\underline{x}$ to be in state

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and prob. $\underline{y}$ to be in state

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

In math terms.

$$\boxed{p(|\psi\rangle) = |\langle q_0 | \psi\rangle|^2}$$

Scanned with CamScanner

1.3 Cont'd

$\langle \psi | q_0 \rangle$ is the inner product of two state vectors.

ex

$$|\psi\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$|q_0\rangle = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$\langle \psi | q_0 \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$= 1 \times \frac{1}{\sqrt{2}} + 0 \times \left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}$$

$$\boxed{p(|\psi\rangle)} = p(|0\rangle)$$

$$= |\langle \psi | q_0 \rangle|^2$$

$$= \left|\frac{1}{\sqrt{2}}\right|^2 = \boxed{\frac{1}{2}}$$

Just saw that the state

$|q_0\rangle = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$ has $\frac{1}{2}$

probability to be in state

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

↳ Orthogonal states have the property that

① $\langle 0|0\rangle \neq 0$

② $\langle 0|1\rangle = 0$

① $\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \underline{1}\times 1 + 0\times 0 = \underline{1}$

② $\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1\times 0 + 0\times 1 = \underline{0}$

1.3 Cont'd

In this way, we can keep things in terms of bras and kets.

↳ Inner product.

$$\langle \psi | q_0 \rangle = \langle 0 | q_0 \rangle$$

$$= \langle 0 | \left[\frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \right]$$

$$= \frac{1}{\sqrt{2}} \underbrace{\langle 0|0\rangle}_{1} - \frac{1}{\sqrt{2}} \underbrace{\cancel{\langle 0|1\rangle}}_{0}$$

$$= \underline{\frac{1}{\sqrt{2}}}$$ and $p(|0\rangle) = \left|\frac{1}{\sqrt{2}}\right|^2$

$$= \underline{\frac{1}{2}}$$