Bloch Sphere

Before ...

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

or generally

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where α and β can be complex

We know

$$|\alpha|^2 + |\beta|^2 = 1$$

Instead of α and β, let's use a and $be^{i\phi}$, for a and b Real.

$$|\psi\rangle = a|0\rangle + e^{i\phi}b|1\rangle$$

Now

$$a^2 + b^2 = 1$$

Bloch Sphere Cont'd

Use

$$\sin^2 x + \cos^2 x = 1$$

and set $a = \cos\frac{\theta}{2}$

$b = \sin\frac{\theta}{2}$

Then

$$\boxed{|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle}$$

for two angles θ and ϕ.

Visually representing a qubit

Plot $|\psi\rangle$ along $|0\rangle, |1\rangle$

and Real, Imaginary.

↳ angles θ and ϕ tell you

which direction the vector

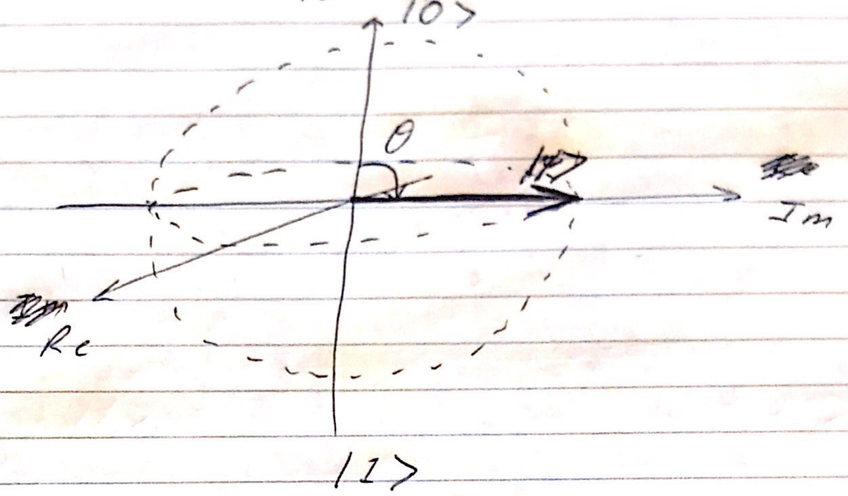
$|\psi\rangle$ points along in the unit

sphere.

Visually representing qubits [illegible]

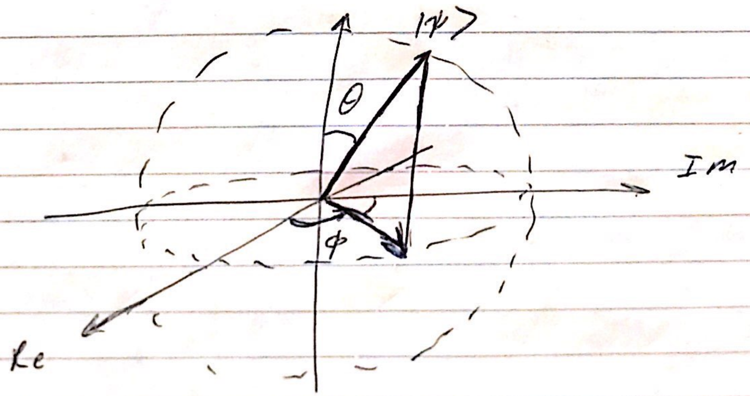
e.g.

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$



generally

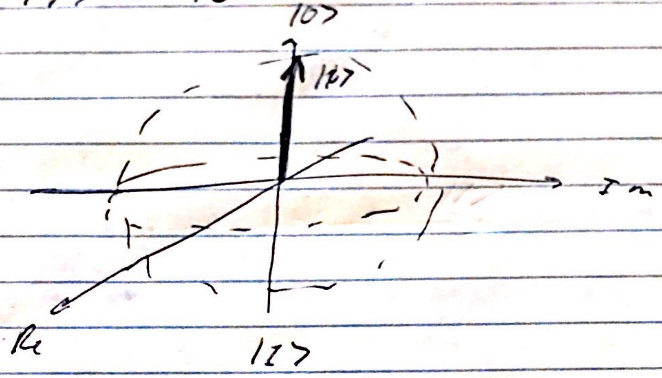
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$



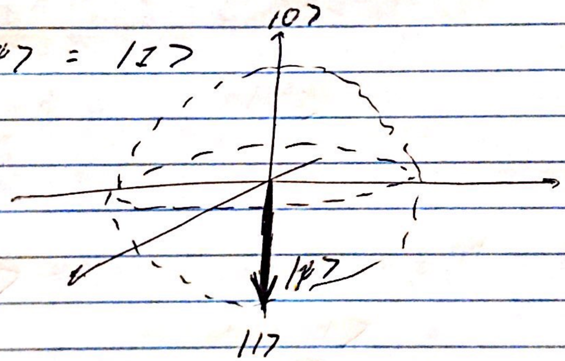
Visually Representing Qubits cont'd

ex

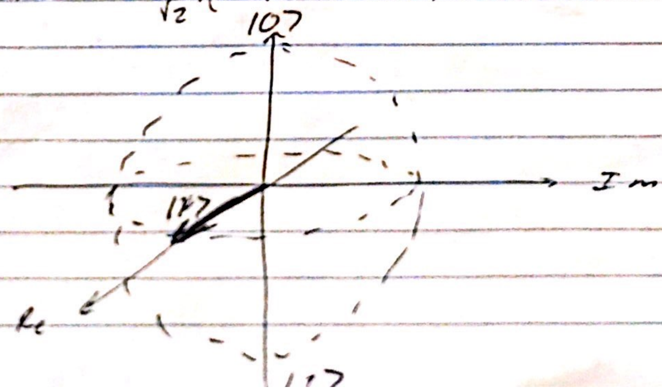
$|\psi\rangle = |0\rangle$



$|\psi\rangle = |1\rangle$



$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$



3.1.1 Cont'd Last time.

Pauli Gates

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

---

ex

$$\underline{X|0\rangle} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{|1\rangle}$$

$$\underline{Y|0\rangle} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{pmatrix} 0 \\ i \end{pmatrix} = \underline{i|1\rangle}$$

3.1.1 Cont'd

ex Cont'd

$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= |0\rangle$$

3.1.2 X-Y-Z Basis

Eigen vectors of X-gate

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

are $|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

and $|-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\Rightarrow |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Eigen values are $+1$ for $|+\rangle$ and -1 for $|-\rangle$

3.1.2 Cont'd

Eigen vectors of Y-gate are

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$|\circlearrowright\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} ; \quad |\circlearrowleft\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

w/ eigen values $+1$ for $|\circlearrowright\rangle$ and
-1 for $|\circlearrowleft\rangle$

Eigen vectors of Z-gate are

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

w/ eigen values $+1$ for $|0\rangle$ and
-1 for $|1\rangle$

3.1.3 Changing Bases

Work in basis of X, Y, and Z gates.

$$Z \rightarrow \{ |0\rangle, |1\rangle \}$$

$$Y \rightarrow \{ |\circlearrowright\rangle, |\circlearrowleft\rangle \}$$

$$X \rightarrow \{ |+\rangle, |-\rangle \}$$

Must express eigenvectors of X and Y in terms of eigenvectors of Z.

↳ Also can do the opposite. Express eigenvectors of Z in terms of eigenvectors of X and Y.

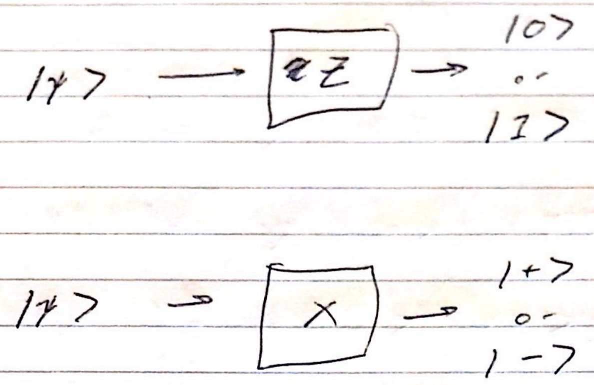
$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$

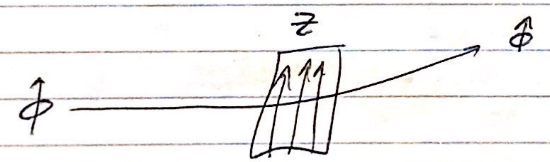
3.1.4 Stern Gerlach Exp. Analog.



$|\psi\rangle \rightarrow \boxed{Z} \rightarrow |0\rangle$ or $|1\rangle$

$|\psi\rangle \rightarrow \boxed{X} \rightarrow |+\rangle$ or $|-\rangle$

Z gate and X gate can be thought of as magnetic fields oriented in different directions.



3.1.4 Gates

Effect of Measurement:

Wave function collapse.

$$|\psi\rangle \longrightarrow \boxed{Z} \longrightarrow \begin{matrix} |0\rangle \\ \text{or} \\ |1\rangle \end{matrix}$$

Measuring eigenvalue of Z-gate means you collapse to an eigenstate of Z. Same with X.

Imagine simultaneous measurements of $|\psi\rangle$

$$|\psi\rangle \longrightarrow \boxed{Z} \rightarrow |0\rangle \rightarrow \boxed{X} \longrightarrow ???$$

if $|0\rangle \rightarrow \boxed{X}$, we know the wavefunction collapses to an eigenstate of X. $\{|+\rangle, |-\rangle\}$

Scanned with CamScanner

3.1.4 Cont'd

Writing eigenstates of Z in terms of

$\{|+\rangle, |-\rangle\}$

$$|0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$$|0\rangle \longrightarrow \boxed{X} \begin{array}{l} \xrightarrow{50\%} |+\rangle \\ \xrightarrow{50\%} |-\rangle \end{array}$$

wave function will collapse to either $|+\rangle$ or $|-\rangle$ w/ 50% probability for each. We now measure Z.

$$|+\rangle \longrightarrow \boxed{Z} \longrightarrow ???$$

Writing $|+\rangle$ in terms of $|0\rangle$ and $|1\rangle$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

measuring Z collapses wavefunction to $|0\rangle$ or $|1\rangle$

$$|+\rangle \to \boxed{Z} \xrightarrow{50\%} |1\rangle$$
$$\searrow_{50\%} |0\rangle$$

Conclusion: Measuring Z then X can change the state of the qubit to an eigenstate in respective bases.

ex1

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|+\rangle \to \boxed{Z} \to |1\rangle \to \boxed{X} \text{---}$$
$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$$

$$\to |+\rangle \to \boxed{Z} \to \boxed{|0\rangle}$$
$$|+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$$

ex2

$$|+\rangle \to \boxed{Z} \to |0\rangle \to \boxed{X} \text{---}$$
$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$$\to |+\rangle \to \boxed{Z} \to \underline{|0\rangle}$$
$$|+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$$

3.1.4 Grid

Also can put in an eigen state into its respective "box" and get the same thing back.

ex

$$|0\rangle \to \boxed{Z} \to \underline{|0\rangle}$$

$$|+\rangle \to \boxed{X} \to |+\rangle \to \boxed{X} \to \underline{|+\rangle}$$

$$|+\rangle \to \boxed{Z} \to |0\rangle \to \boxed{Z} \to \underline{|0\rangle}$$

3.1.5 Table

Gate	eigen basis	eigen value	output of X-gate	output of Z
X	$\lvert+\rangle$	1	$\lvert+\rangle$	$\lvert0\rangle$ or $\lvert1\rangle$
	$\lvert-\rangle$	-1	$\lvert-\rangle$	$\lvert0\rangle$ or $\lvert1\rangle$
Z	$\lvert0\rangle$	1	$\lvert+\rangle$ or $\lvert-\rangle$	$\lvert0\rangle$
	$\lvert1\rangle$	-1	$\lvert+\rangle$ or $\lvert-\rangle$	$\lvert1\rangle$