2. Measurement

The prefactors tell us about the probabilities to find our state in different states $|0\rangle$ and $|1\rangle$

e.g.

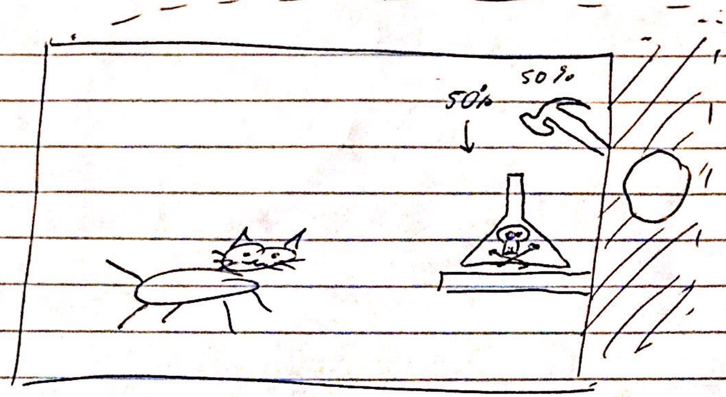
$$|q_0\rangle = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$P(|0\rangle) = |\langle 0|q_0\rangle|^2 = \underline{\frac{1}{2}}$$

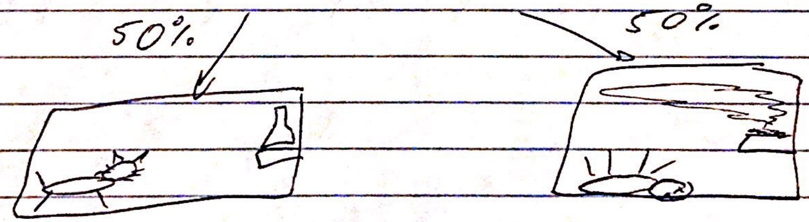
$$P(|1\rangle) = |\langle 1|q_0\rangle|^2 = \underline{\frac{1}{2}}$$

$\rightarrow$ Measure the state of $|q_0\rangle$, find it in ~~state~~ $|0\rangle$ 50% of the time, and $|1\rangle$ 50% of the time.

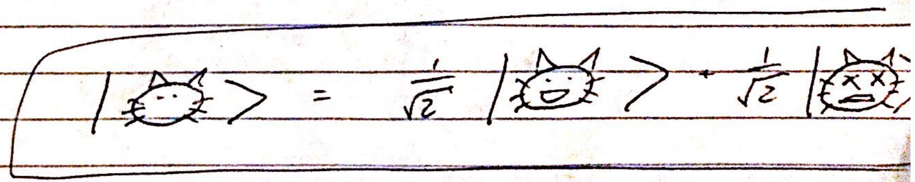
2.0 Schrödinger's Cat



Let's say there is a cat in a box that also has a vile of poisonous gas. There's a hammer that has a 50% chance of smashing the vile.



The cat is in a "superposition of both dead and alive"



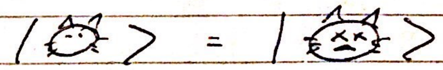
2.1 Measurement Problem

- Until you measure your quantum state, you have no idea what state it's actually in. So the state is "in both states simultaneously"

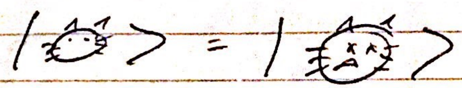
  ↳ The cat is both dead and alive!?

- After measuring the cat to be dead, there is 100% probability that if you measure the cat again, it will be dead.

$\Downarrow$ "Is the cat dead or alive"



$\Downarrow$ "Let's check again"



2.1 Measurement Problem Cont'd

Similarly ...

$$|q_0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\Downarrow \text{ "Is } |q_0\rangle \text{ in state } |0\rangle \text{ or } |1\rangle\text{?"}$$

$$|q_0\rangle = |0\rangle$$

$$\Downarrow \text{ "Let's check again"}$$

$$|q_0\rangle = |0\rangle$$

Phenomenon is known as "collapse of the wave function"

$|q_0\rangle$ ↳ wave function collapses to a single state. $|0\rangle$

Probability before measurement?

$$P(|0\rangle) = |\langle 0|q_0\rangle|^2$$

$$= \left|\langle 0|\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\right|^2$$

$$= \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

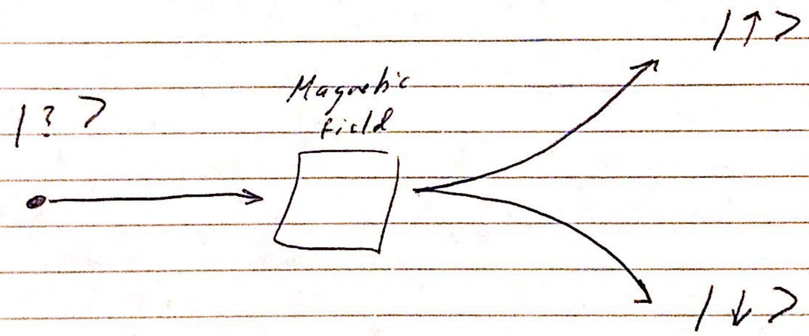
2.1 Cont'd

Probability after measurement?

$$P(|0\rangle) = |\langle 0 | \psi_0 \rangle|^2$$
$$= |\langle 0 | 0 \rangle|^2$$
$$= \underline{1}$$

↳ 50% before, 100% after.

2.1.1 Stern Gerlach Experiment.



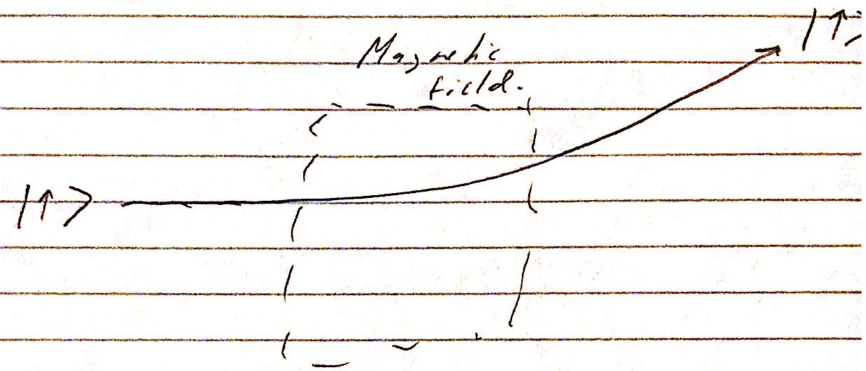
→ Measure the "spin" of a particle.

↳ strong magnetic field.

Up spins go one way, down spins go another.

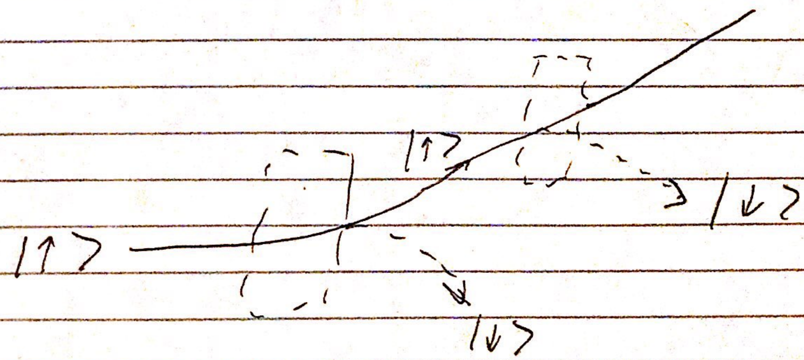
2.7.1 cont'd

Basis $\{|\uparrow\rangle, |\downarrow\rangle\}$



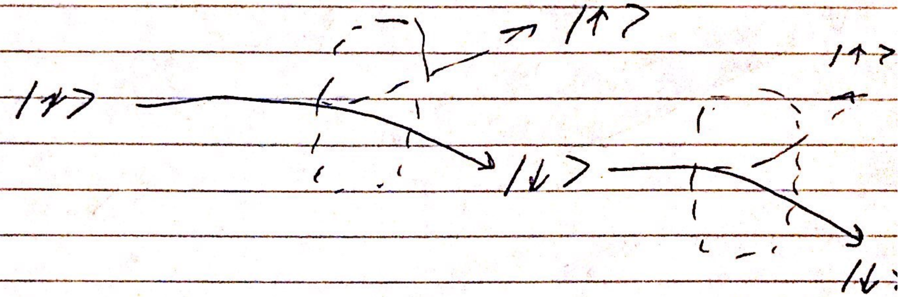
↳ Measure the spin of a particle to be in the up state.

→ Do a second measurement.



2.1.1 Cont'd

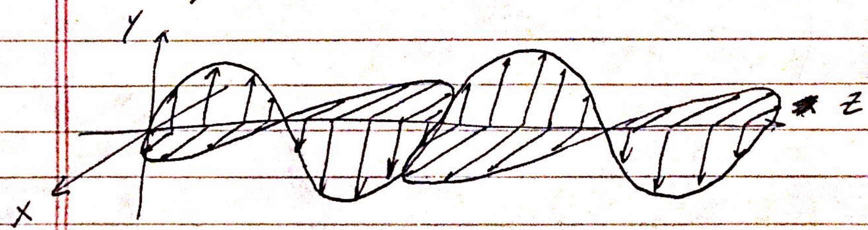
Now starting state is $|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$



After measuring $|\rightarrow\rangle$ in the $|\uparrow\rangle$ state, it stays in the $|\uparrow\rangle$ state and vice versa.

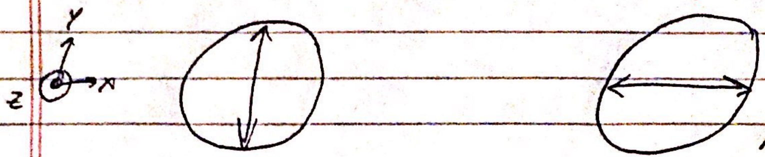
2.1.2 Photon Polarization

Light Waves.



2.1.2 Cont'd

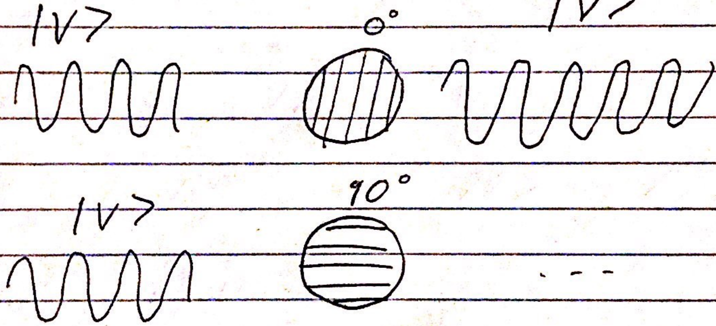
Or ... can be seen as



$|V\rangle$ – "vertically" polarized

$|H\rangle$ – "horizontally" polarized.

Polarizer



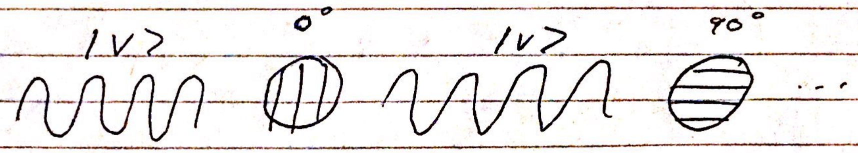
The amplitude of the outgoing wave represents the **probability** of finding the wave in that state.

↳ 0° polarizer measures if the wave is vertically polarized.

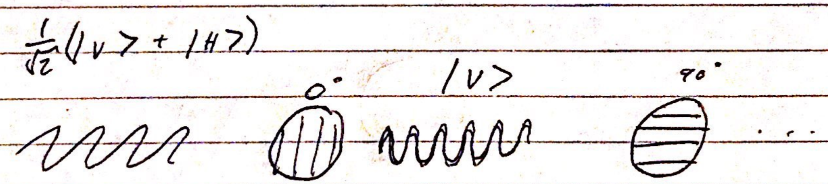
2.1.2 Cont'd

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|V\rangle + |H\rangle) \; ?$$

Imagine two polarizers.



First polarizer collapses the wave to the vertical state, second polarizer further collapses the wave ...



Again, collapse to $|V\rangle$ or measure the probability of finding $|\psi\rangle$ in $|V\rangle$. Then collapse to $|H\rangle$ or measure probability of finding $|V\rangle$ in $|H\rangle$ (0%)

3. Operators

3.1 Single Qubit Gates.

3.1.1 Pauli Gates.

Given ~~[illegible]~~ basis states

$$\{ |0\rangle, |1\rangle \}$$

How can we transform our states?

1) Swap or Exchange, or X-gate.

↳ flip the state $|1\rangle$ to $|0\rangle$
and $|0\rangle$ to $|1\rangle$.

if $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and

$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

then $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3.1.1 cont'd

eg/

$$X|0\rangle = |1\rangle$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \underline{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

$\therefore$ X gate transforms $|0\rangle$ to $|1\rangle$.

2) Y Gate.

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

↳ does the same thing as the X gate, but adjusts with a complex factor.

eg/ $$Y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ +i \end{bmatrix}$$

$$= \underline{+i|1\rangle}$$

3.1.1 cont'd

ex
$$Y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -i \\ 0 \end{bmatrix} = \underline{-i|0\rangle}$$

3) Z - Gate.

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

↳ gives you a +1 if you are in the $|0\rangle$ state and a ~~minus~~ -1 if you're in the $|1\rangle$ state.

ex
$$Z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \underline{|0\rangle}$$

3.1.16.1.v

ex

$$Z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = + \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{-|1\rangle}$$

Z gate also similar to the

Stern - Gerlach Experiment...