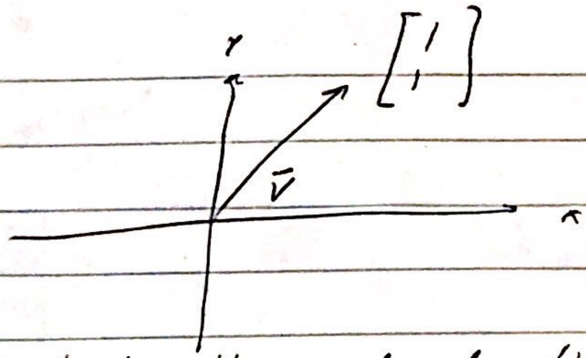
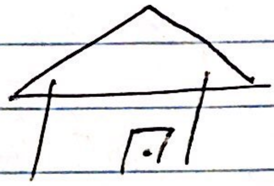
0. Linear Algebra

0.1 Vector



physics: arrow w/ length and direction.
CS: "tuple" list of numbers.

ex/



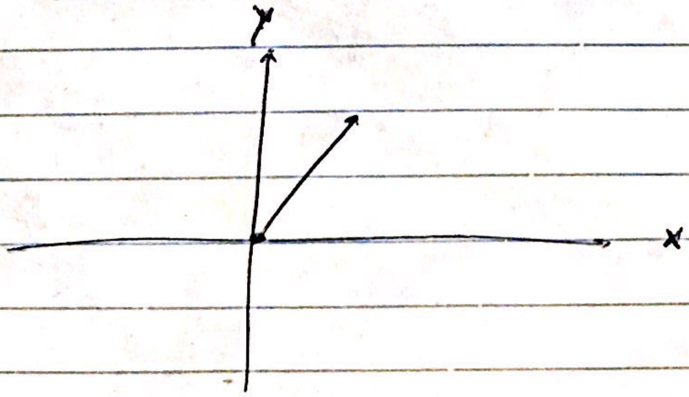
How much is this house? Price?

$$\begin{bmatrix} 26{,}000\ ft^2 \\ \$300{,}000 \end{bmatrix}$$

Math: 1. $\vec{v} + \vec{w}$
2. $2\vec{v}$

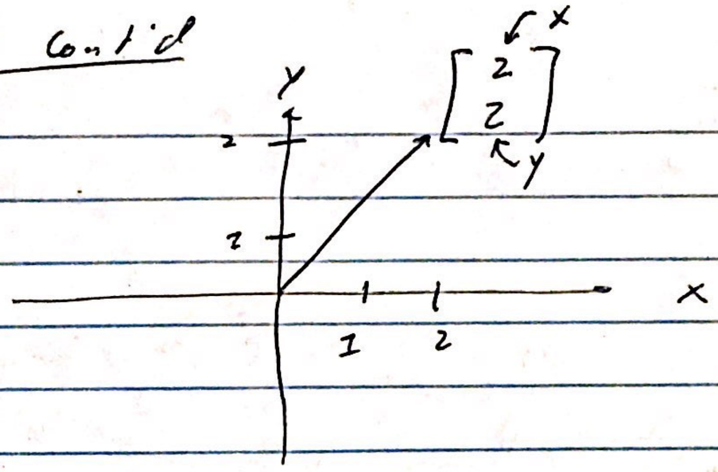
anything you can add and multiply by a number

0.1.1 Arrows.

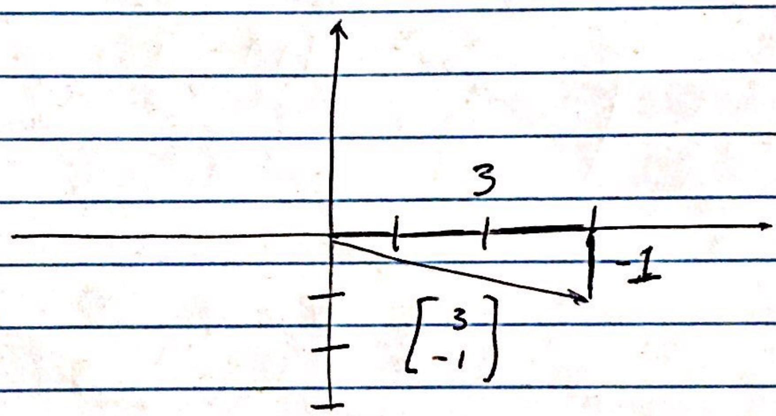


Arrow w/ Tail at the anchor.

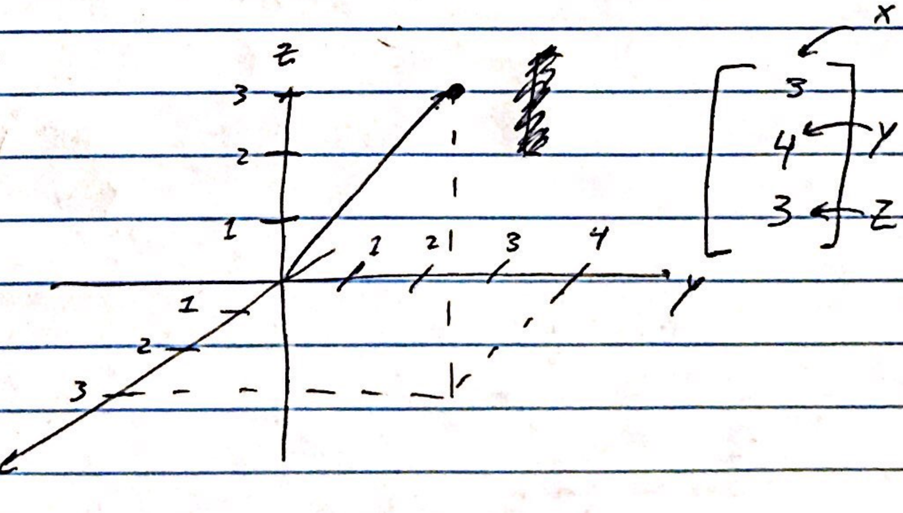
0.1.1 Cont'd



$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} \Rightarrow$$ From 0, walk 2 steps up and 2 steps right.

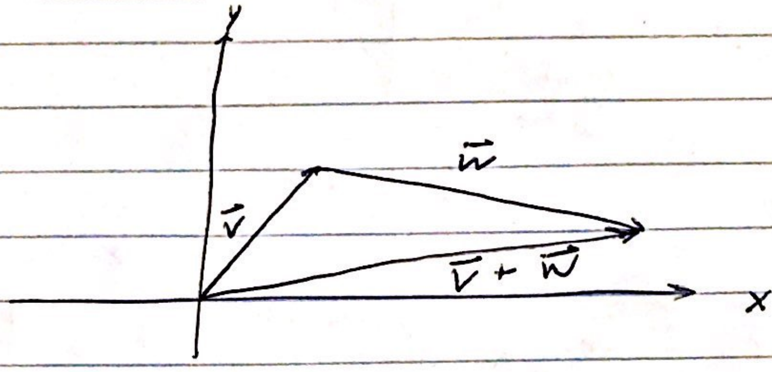


3 D?



0.1.2 Operations

0.1.2.1 Adding

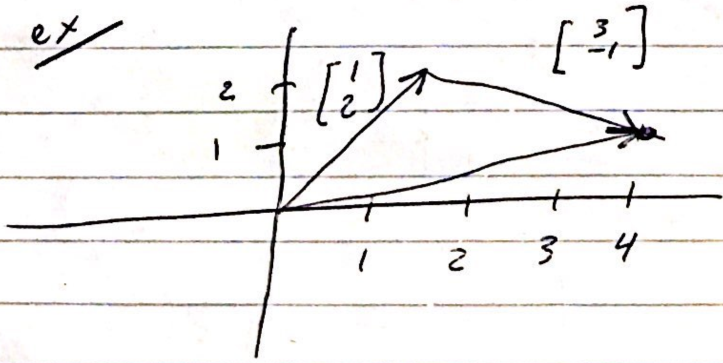


How to add two vectors:

↳ Put $\vec{w}$ tail at the tip of $\vec{v}$. The vector from tail of $\vec{v}$ to tip of $\vec{w}$ is $\vec{v}+\vec{w}$.

↳ "If you take a step along the first vector, then take a step along the second vector, the overall effect is the same as moving along the vector $\vec{v}+\vec{w}$.

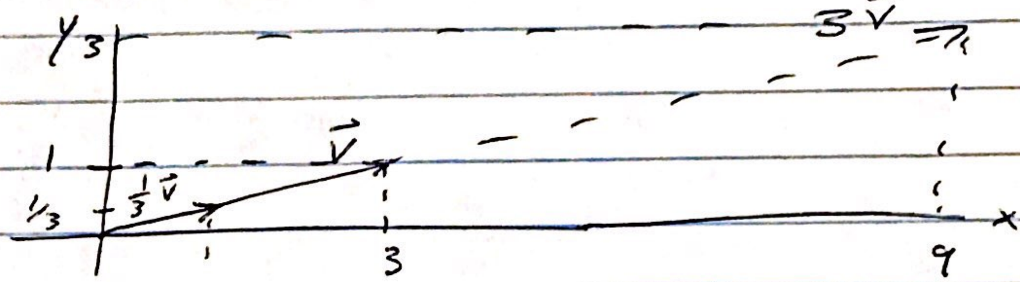
ex/



$$\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}3\\-1\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$$

"move 1+3 right, then 2-1 up."

0.1.2.2 Multiplying



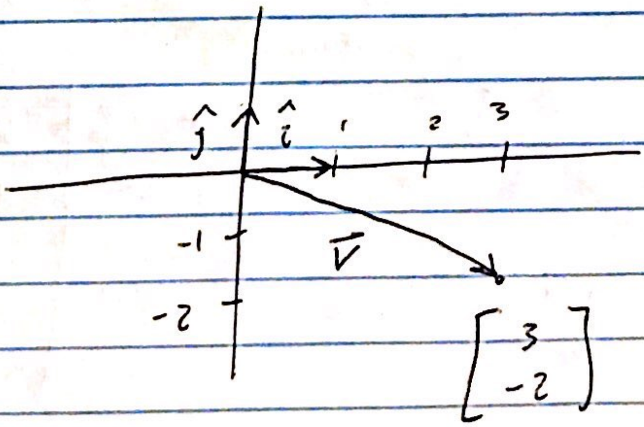
$$\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\frac{1}{3}\vec{v} = \begin{bmatrix} 1 \\ 1/3 \end{bmatrix}$$

$$3\vec{v} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

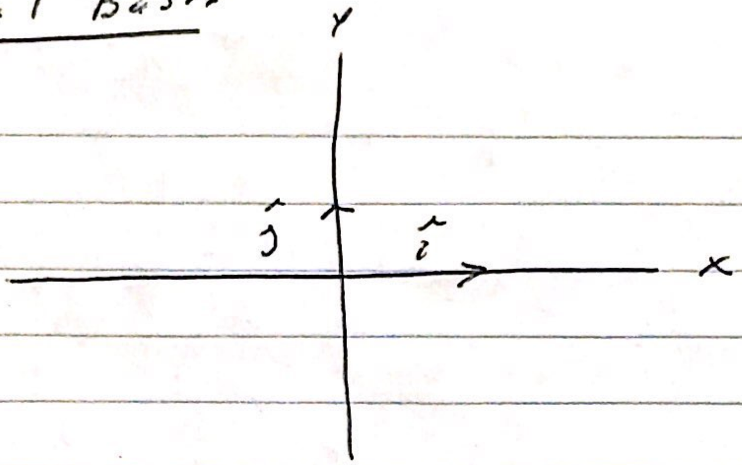
3 "scales" the vector, so we call it a "scalar"

0.2 Space



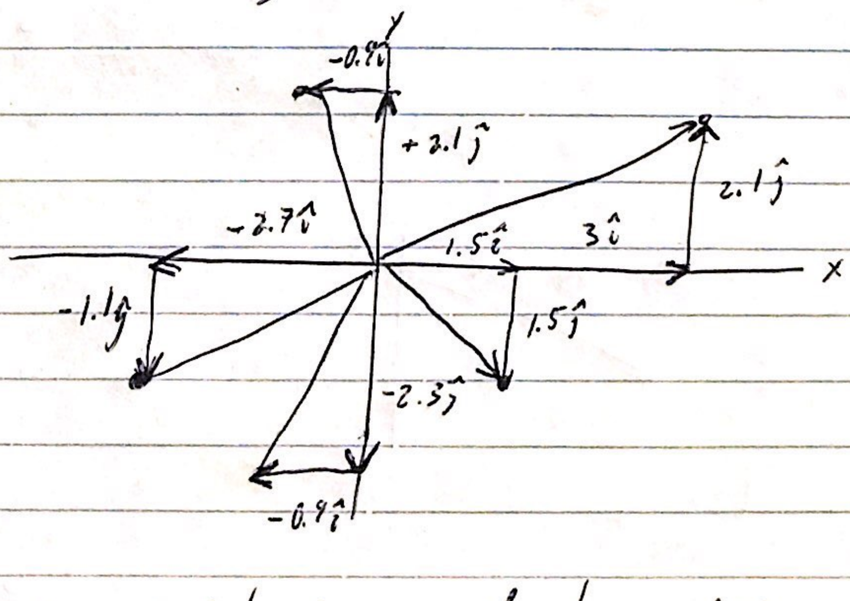
3 and -2 are scalars. that multiply $\hat{i}$ and $\hat{j}$. $\vec{v} = 3\hat{i} + -2\hat{j}$

0.2.1 Basis



$$\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

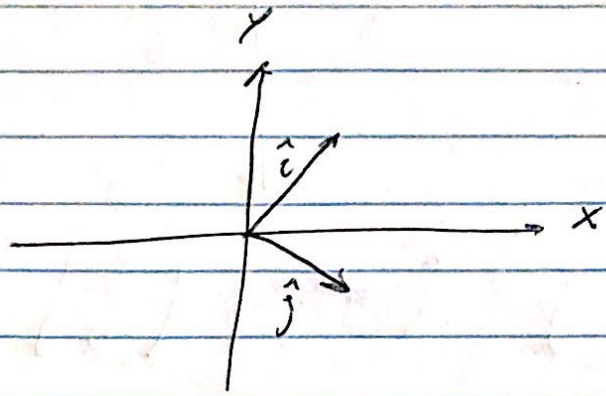
" $\hat{i}$ and $\hat{j}$ are the basis vectors of 2-D space."



Every point is covered by adding and scaling $\hat{i}$ and $\hat{j}$

0.2.1.1 Aside

What about $\hat{\imath} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ $\hat{\jmath} = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$



↳ still good, and you can relate these to the old basis.

0.2.2. Linear Combination

→ Any addition of two scaled vectors.

$$a\vec{v} + b\vec{w}$$

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} \Longleftrightarrow 3\hat{\imath} + 1\hat{\jmath}$$

is a linear combination.

0.2.3 Span

→ All vectors you get when adding and scaling the two vectors.

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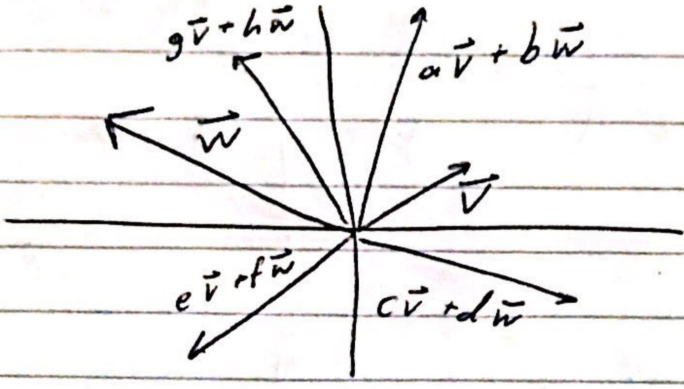
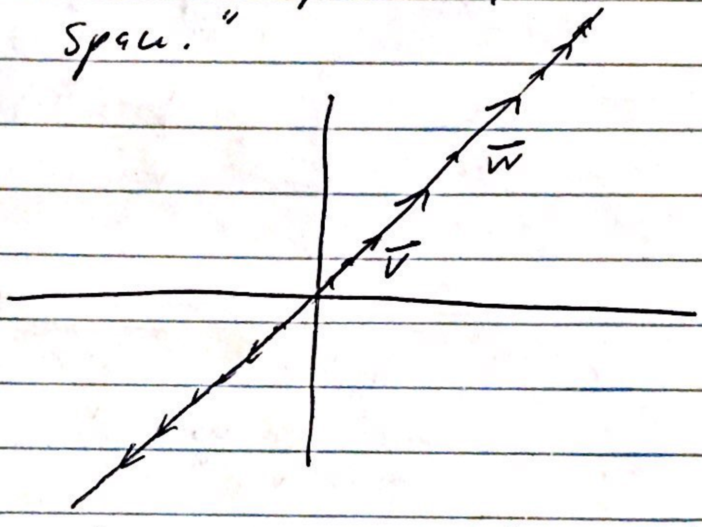
0.2.3 Cont'd



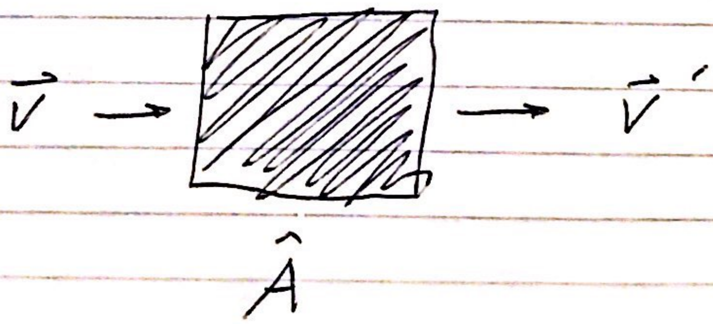
"$\vec{v}$ and $\vec{w}$ span all of 2-D space."



"$\vec{v}$ and $\vec{w}$ span a 1-D line"

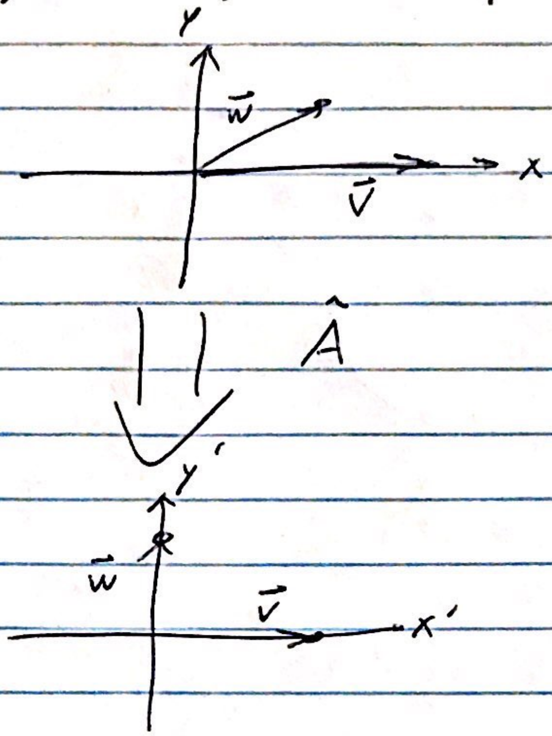
0.3 Matrices as Transformations

Linear Transformation



0.3 Cont'd

Given transformation $\hat{A}$ and some space spanned by $\vec{v}$ $\vec{w}$



Linear Transformations:

1) Keeps gridlines straight

2) Keeps origin in the same place.

↳ for any vector, output is not curved, and its tail ends up in the same place.

0.3 Cont'd

"We know where any vector will land as long as we keep a record of where $\hat{i}$ and $\hat{j}$ land"

$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} = x\hat{i} + y\hat{j} = x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Downarrow \hat{A}$$

$$\vec{v}' = \begin{bmatrix} x' \\ y' \end{bmatrix} = x\hat{i}' + y\hat{j}'$$

$$= x\begin{bmatrix} 1 \\ -2 \end{bmatrix} + y\begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1x + 3y \\ -2x + 0y \end{bmatrix}$$

$\Rightarrow$ represent this as a set of 4 numbers.

where $\hat{j}$ goes.

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$$

where $\hat{i}$ goes

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0.3 cont'd

If you know what A does to

$\hat{i}$ and $\hat{j}$, you know what A does to

any vector

ex

$$A = \begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\vec{v}' = 5 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + 7 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

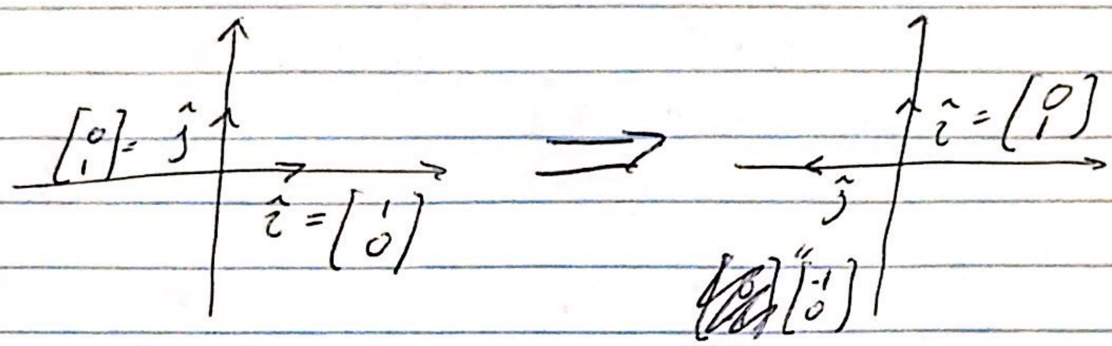
$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\vec{v}' = x \begin{bmatrix} a \\ c \end{bmatrix} + y \begin{bmatrix} b \\ d \end{bmatrix}$$

$$= \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

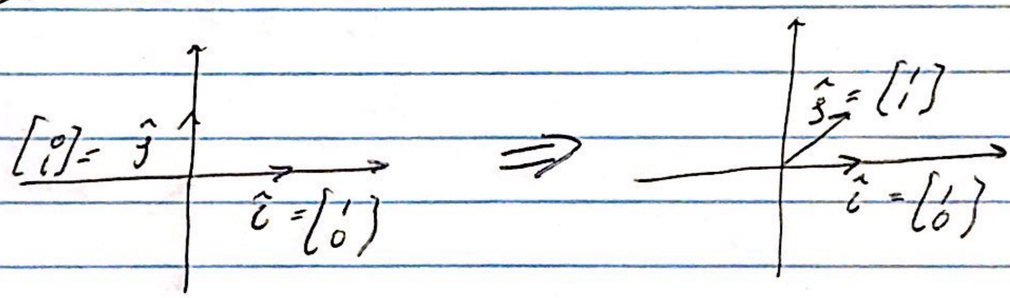
0.3 Cont'd

practice : 1. 90° rotation



$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

2. Shear



$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$