4.5 Cont'd

Last Time

$$CNOT\,|-+\rangle = CNOT \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}}\Big[|00\rangle - |01\rangle - |10\rangle + |11\rangle\Big]$$

$$= \frac{1}{\sqrt{2}}\Big[|0\rangle \otimes (|0\rangle - |1\rangle) - |1\rangle \otimes (|0\rangle - |1\rangle)\Big]$$

$$= \frac{1}{\sqrt{2}}\Big((|0\rangle - |1\rangle) \otimes (|0\rangle - |1\rangle)\Big)$$

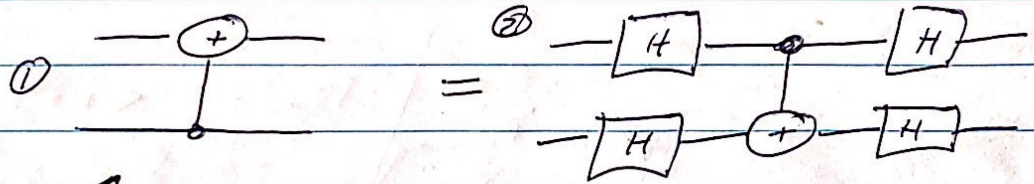
$$= |-\rangle \otimes |-\rangle = \underline{|--\rangle}$$

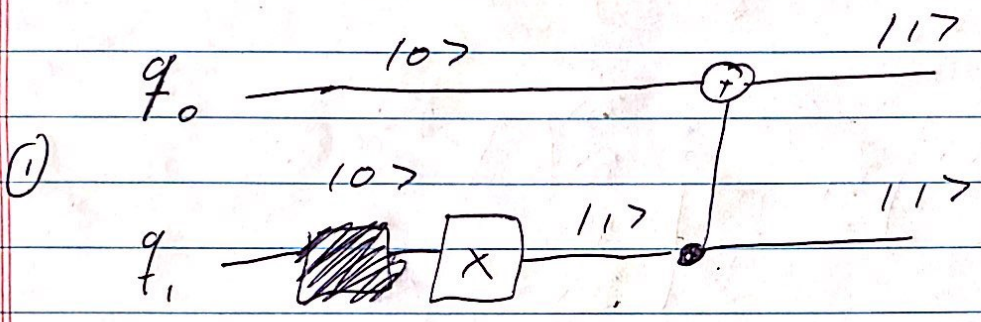
$$\boxed{CNOT\,|-+\rangle = |--\rangle}$$

Flips control qubit!

4.5 cont'd

Claim

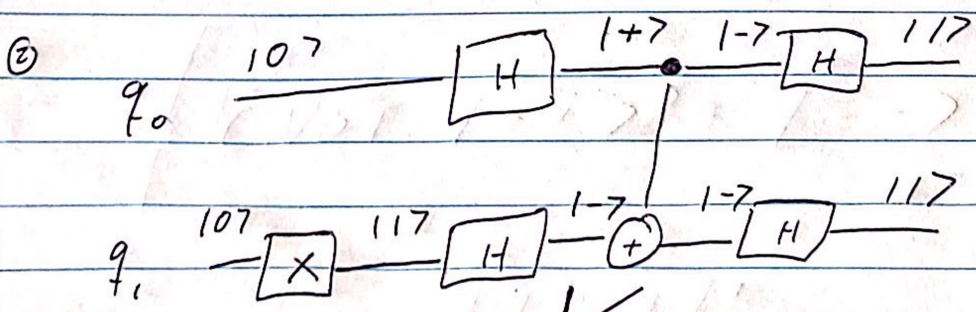




or

CNOT $|10\rangle$ = $\underline{|11\rangle}$ ← flip target

control target



CNOT $|-+\rangle$ = $|--\rangle$

target control

$\boxed{\text{flip control}}$

4.5 Cont'd

We get the same thing out in the two circuits!

q_1 control
q_0 ~~[illegible]~~ target

Hadamard → CNOT → Hadamard

||

CNOT
q_1 target
q_0 control.

Example of Phase kick back

Remember ~~[illegible]~~ =

$$X|-\rangle = -|-\rangle$$

$$\uparrow$$

$$-1$$

Use X-gate in representing CNOT

$$CNOT|-0\rangle = |-\rangle \otimes |0\rangle$$

$$= |-0\rangle$$

$$\text{CNOT} \, |-1\rangle = X|-\rangle \otimes |1\rangle$$

control in $|1\rangle$, apply a "flip" to target.

$$= -|-\rangle \otimes |1\rangle$$

$$= \underline{-|-1\rangle}$$

→ "phase" or minus sign does not matter for measurement. but does matter in some cases.

$$\text{CNOT} \, |-+\rangle = \cancel{}$$

$$\text{CNOT}\left(|-\rangle \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left[\text{CNOT}\left(|-0\rangle + |-1\rangle\right)\right]$$

$$\text{CNOT}|-0\rangle = |-0\rangle$$

$$\text{CNOT}|-1\rangle$$

$$= X|-\rangle \otimes |1\rangle$$

$$= -|-1\rangle$$

$$\frac{1}{\sqrt{2}}\left[CNOT\,|{-}0\rangle + CNOT\,|{-}1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\,|{-}0\rangle - |{-}1\rangle$$

$$= |-\rangle \otimes \left(\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\right)$$

$$= |-\rangle \otimes |-\rangle = \boxed{|--\rangle}$$

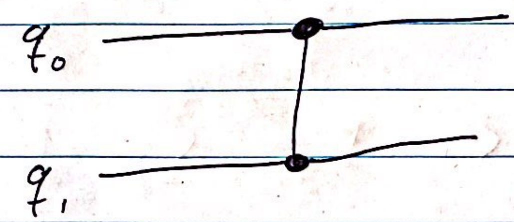
* writing states in $\{|-\rangle, |+\rangle\}$, superposition of $\{|0\rangle, |1\rangle\}$, you see effects of phase kickback.

4.5.1 Other examples of kickback

Recall:
$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$

* rotation by $\frac{\pi}{4}$ *

Circuit Diagram for $U_1(\pi/4)$



Apply a T-gate to qubit in $|1\rangle$ state.

$$T|1\rangle = e^{i\pi/4}|1\rangle$$

Add another qubit?

$$|1+\rangle = |1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
$$= \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$$

$\Rightarrow$ Controlled T-gate: Apply T to qubit 0

$$\text{Controlled } T|1+\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + e^{i\pi/4}|11\rangle\right)$$

4.5.1 Cont'd

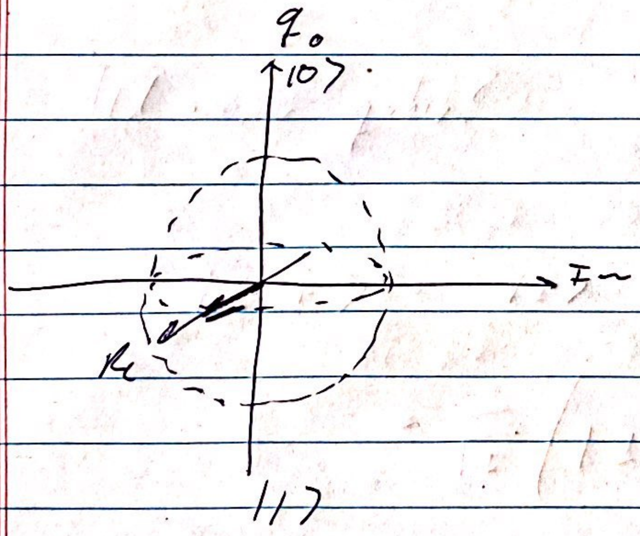
$$\text{Controlled } T\,|1+\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + e^{i\pi/4}\,|11\rangle\right)$$

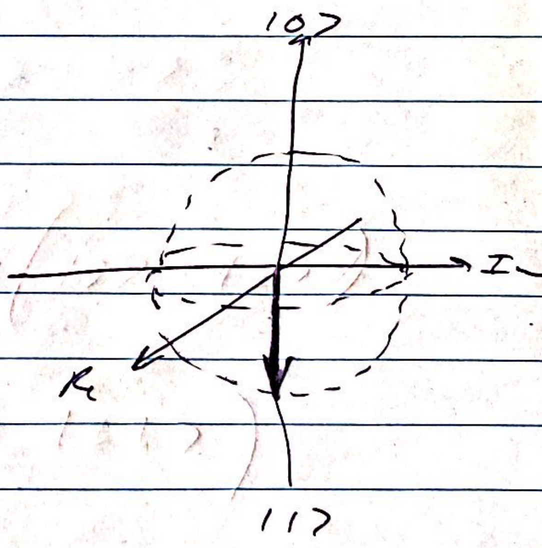
$$= \frac{1}{\sqrt{2}}\left(|1\rangle \otimes |0\rangle + e^{i\pi/4}\,|1\rangle\right)$$

rotates the phase of qubit 0 by $\frac{\pi}{4}$.

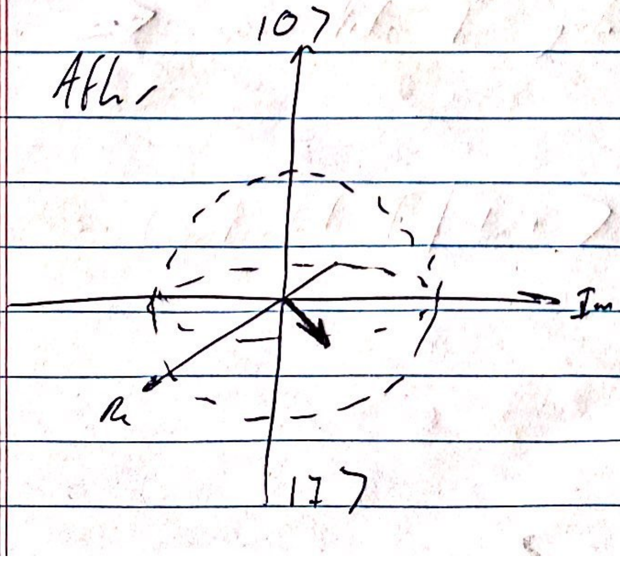
Bloch Sphere:

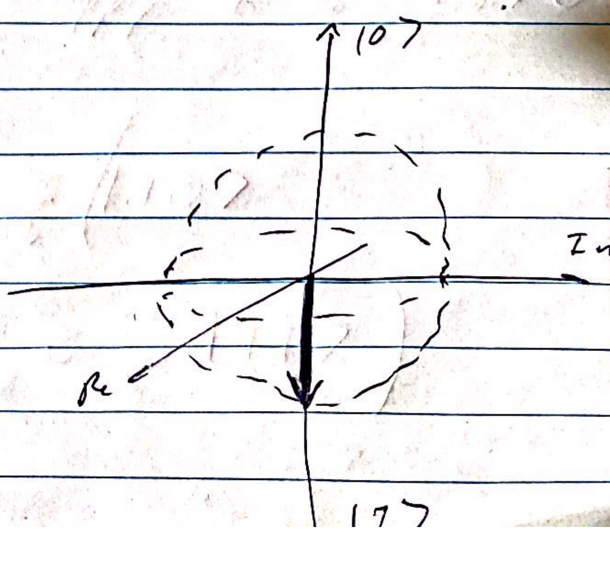
	qubit 0	qubit 1
Initially	~~$\vert +\rangle$~~	$\vert 1\rangle$
	$\vert +\rangle$	$\vert q_1\rangle$



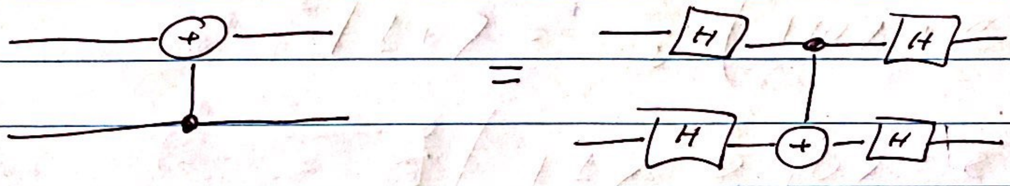


After





4.6 Circuit Identities



Is an example of a circuit identity.

Useful to transform some combination of gates into another gate.

↳ Also remember

$$X = HZH$$

4.6.1 Controlled Z from a CNOT.

Circuit Diagram for controlled Z.

↳

Apply Z-gate if control is in state $|1\rangle$.

How to transform CNOT to controlled-Z.

↳ Use single qubit circuit Identities.

H changes $|0\rangle \leftrightarrow |+\rangle$
$|1\rangle \leftrightarrow |-\rangle$

and Z changes $|+\rangle \leftrightarrow |-\rangle$

$\therefore \quad HXH = Z$ ①

and $HZH = X$ ②

① $|0\rangle$?

$H|0\rangle = |+\rangle$

~~$X|+\rangle = |+\rangle$~~

$H|+\rangle = |0\rangle$ ✓

$Z|0\rangle = |0\rangle$ ✓

$|1\rangle$? $\quad H|1\rangle = |-\rangle$

$X|-\rangle = -1|-\rangle$

$H(-|-\rangle) = \underline{-|1\rangle}$ ✓

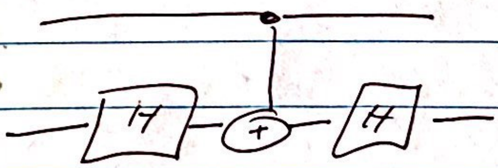
4.6.1 Contd

$Z|1\rangle = -|1\rangle$ ✓

→ Use the same trick to turn CNOT into controlled-Z.

↳ CNOT is an X if control is in $|1\rangle$.

so if control is in $|1\rangle$



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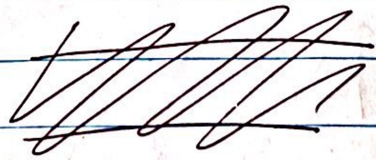

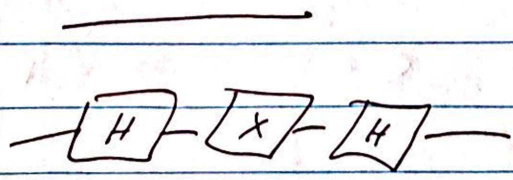

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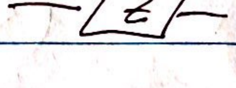


✓

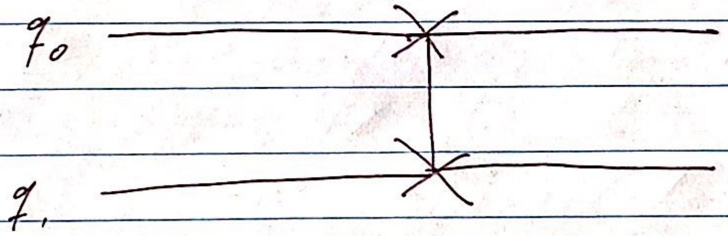
4.6.1 Cont'd

Since $HXH = Z$

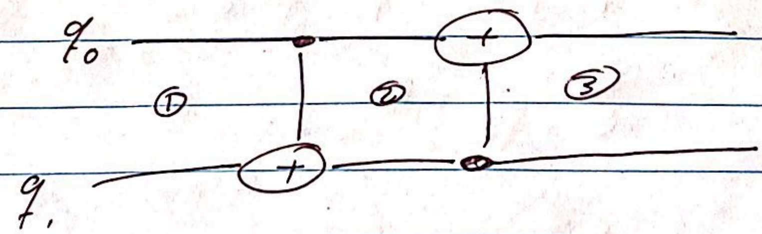
we have $H\, CNOT\, H$ = Controlled Z.

4.6.2 Swapping Qubits

SWAP Gate:



or



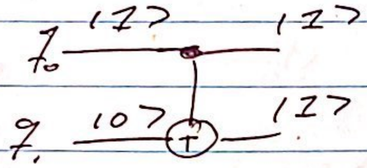
$|q_0\rangle = |1\rangle$

$|q_1\rangle = |0\rangle$

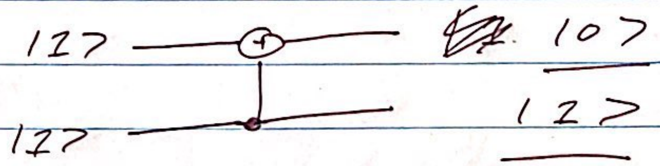
[illegible]

4.6.1 Cont'd

Step 1:

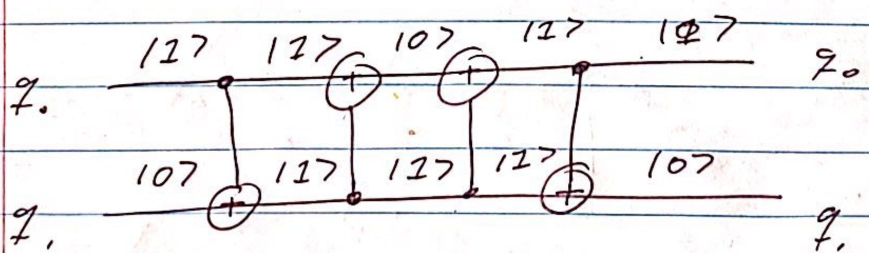


Step 2:



Before	After
$\lvert q_0\rangle = \lvert 1\rangle$	$\lvert q_0\rangle = \lvert 0\rangle$
$\lvert q_1\rangle = \lvert 0\rangle$	$\lvert q_1\rangle = \lvert 1\rangle$

Two swaps?



Get the same thing out!