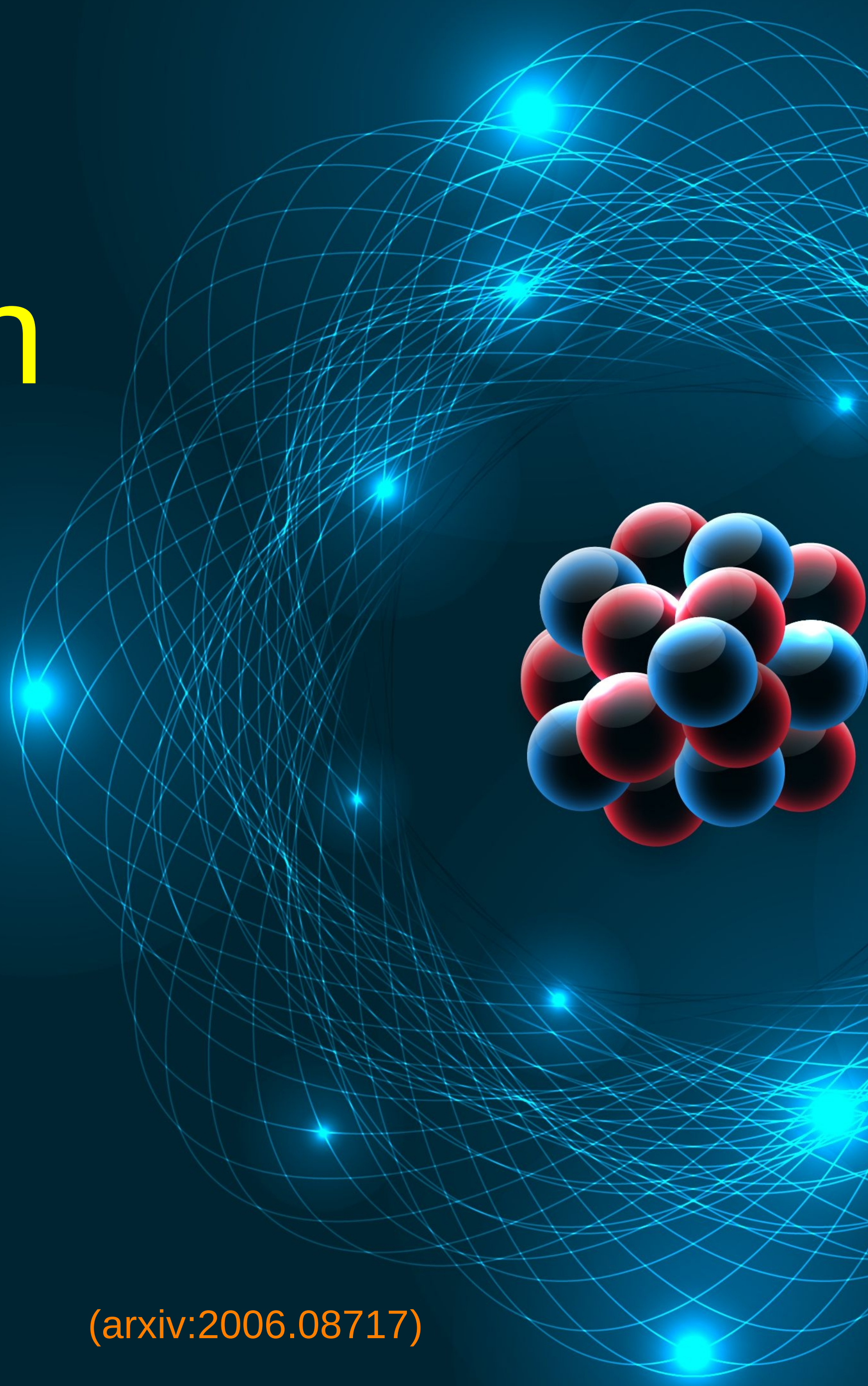
Anomalous exciton Hall effect



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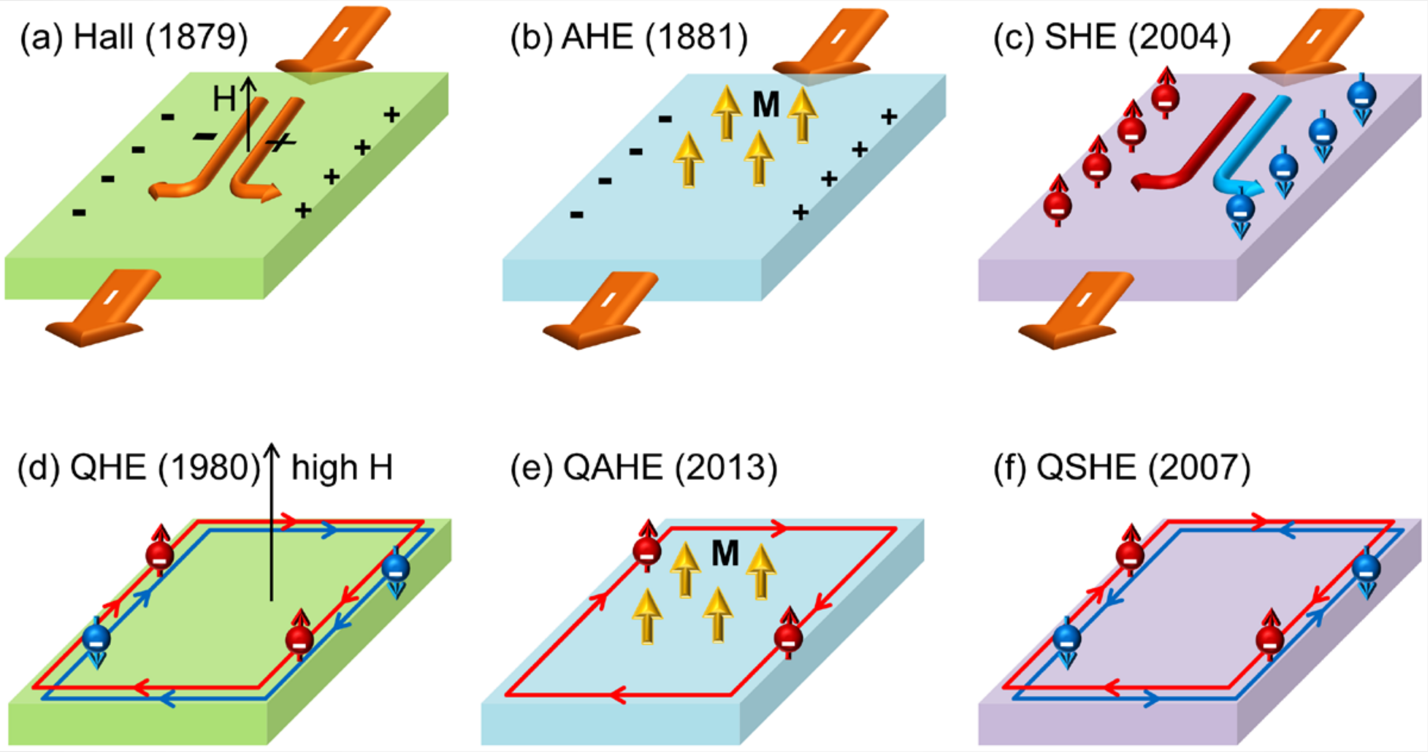
(arxiv:2006.08717)

Outline

- Introduction (historical reference)
- Formulation of the problem
- Phenomenological model
- Microscopic theory
- Scattering rate
- Conclusion

Introduction - Various Hall effects



Members of the Hall family

(from: Cui-Zu Chang and Mingda Li - Quantum anomalous Hall effect in time-reversal-symmetry breaking topological insulators)

Introduction – Some results about excitons

- Thomas and Hopfield – excitons, propagating in the presence of an external magnetic field orthogonal to their velocity, acquire stationary dipole polarisation perpendicular to both the magnetic field and their propagation direction (due to the Lorentz force).
- Imamoglu – exciton, placed in crossed electric and magnetic fields, starts moving as a whole in the direction perpendicular to the directions of both fields.
- Onga – the experimental observation of an exciton Hall effect in atomically thin layers of MoS_2 in the presence of a magnetic field. The effect is caused by the strong spin-valley locking.

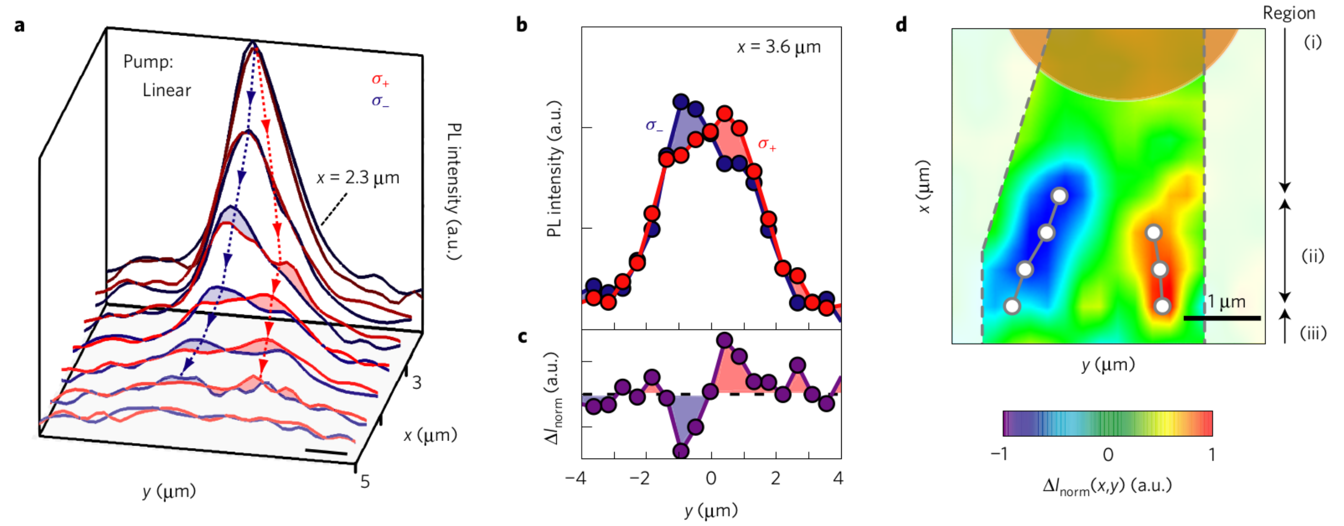


Figure 3 | The exciton Hall effect. a, Polarization-resolved PL mapping under linearly polarized excitation. The red (blue) lines show the σ_+ (σ_-) components of the luminescence. The space between two pairs of lines along the *x*-axis is around 0.46 μm, and the scale bar along the *y*-axis is 1 μm. **b**, Extracted cross-sectional profile of **a** at $x = 3.6$ μm. **c**, ΔI_{norm} versus *y*. The sign reversal at the centre of the flake represents the occurrence of the valley-contrasting EHE. **d**, Colour plot of ΔI_{norm}, which exhibits a real-space picture of the EHE. The white circles shows the peak positions of ΔI_{norm}. The edges of the flake and the laser spot are shown as grey dashed lines and an orange circle (the diameter is twice as large as the standard deviation of the laser profile).

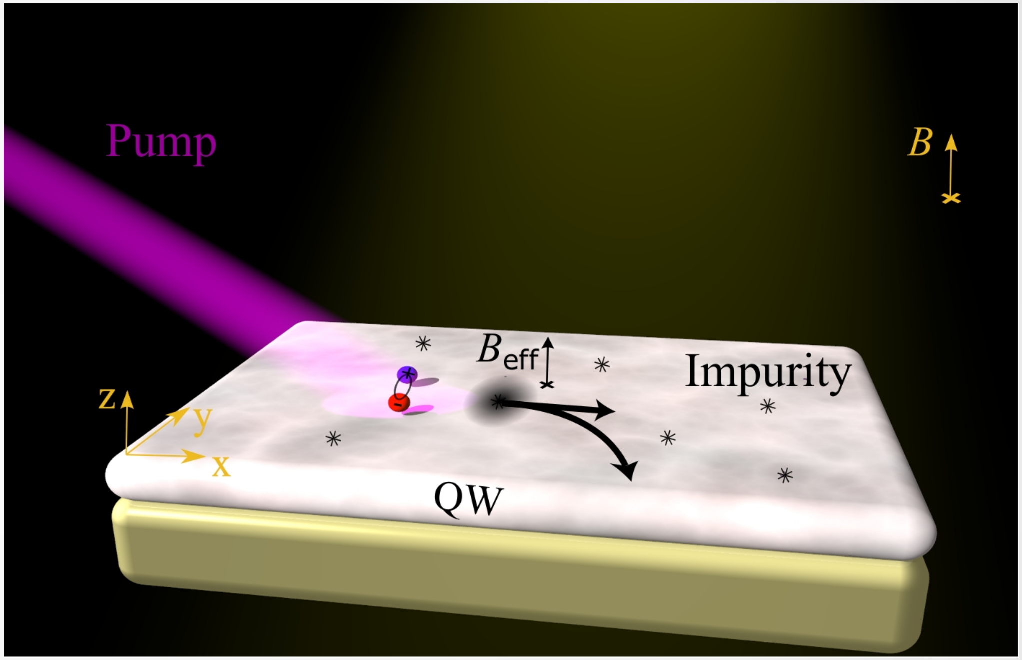
(from: Masaru Onga et al. - Exciton Hall effect in monolayer MoS2)

Formulation of the problem

We need:

- conventional GaAs (2D)
- charged impurities
- external magnetic field
- pump, to produce excitons

We showed that in conventional GaAs quantum wells containing charged impurities an exciton flow may be reoriented in the real space due to the combined effect of the local electrostatic potential created by charged impurities and the magnetic field applied in normal to the plane direction.



Conceptually, this effect is similar to the cross-field effect proposed by Imamoglu and to the exciton Hall effect studied by Onga *et al*. However, in comparison with their effects, some initial conditions are not required in our work:

- an external electric field
- spin-valley locking

Phenomenological model

1) Hamiltonian:

$$\hat{H}^{\text{dip}} = \frac{\hat{\mathbf{p}}^2}{2M} + \hat{V}^{\text{dip}} = \frac{\hat{\mathbf{p}}^2}{2M} + U_{sc}(\mathbf{R}) - \frac{1}{2}(\hat{\mathbf{d}} \cdot \mathbf{E} + \mathbf{E} \cdot \hat{\mathbf{d}})$$

- $U_{sc}(\mathbf{R})$ — scattering potential, created by impurities and fluctuations of the doped quantum well width.
- $\mathbf{R}$ — the exciton center-of-mass.
- $\mathbf{E}(\mathbf{R})$ — the electric field, generated by impurities.
- $\hat{d} = f(B)\,[\mathbf{e}_z \times \mathbf{k}]$ — the electric dipole moment of the exciton, where $f(\mathbf{B})$ is a function of the magnetic field, which depends linearly on B at weak fields, but becomes inversely proportional to B at the large fields in the magneto-exciton regime.

The last statement can be derived in case of weak magnetic field by passing to the center-of-mass reference frame:

$$\mathbf{E}' = \frac{\hbar}{M}[\mathbf{k}\times\mathbf{B}] \Rightarrow \langle\hat{d}\rangle = \alpha\mathbf{E}' = \frac{\alpha\hbar}{M}[\mathbf{k}\times\mathbf{B}],\ \text{where } M \text{ — exciton mass, } \alpha = \frac{21 \cdot 4\pi a_B^3 \varepsilon_0 \varepsilon}{128}$$

For strong magnetic fields:

$$\langle\hat{d}\rangle = -e[\mathbf{e}_z \times \mathbf{k}]l_B^2,\ \text{where } l_B = \sqrt{\frac{\hbar}{eB}} \text{ — magnetic length}$$

Phenomenological model

2) We transform our Hamiltonian to the form:

$$\hat{H}^{\rm dip} = \frac{1}{2M}(\hat{p} - e\mathbf{A}_{\rm eff}(\mathbf{R}))^2 + U_{\rm eff}(\mathbf{R})$$

where:

- $\mathbf{A}_{\rm eff}(\mathbf{R}) = \frac{Mf(B)}{\hbar e}(\mathbf{e}_x E_y(\mathbf{R}) - \mathbf{e}_y E_x(\mathbf{R}))$
- $\mathbf{B}_{\rm eff}(\mathbf{R}) = \frac{Mf(B)}{\hbar e}\,{\rm div}\,\mathbf{E}(\mathbf{R})$
- $\mathbf{U}_{\rm eff}(\mathbf{R}) = U_{\rm sc}(\mathbf{R}) - \frac{Mf^2(B)}{2\hbar^2}\mathbf{E}^2(\mathbf{R})$

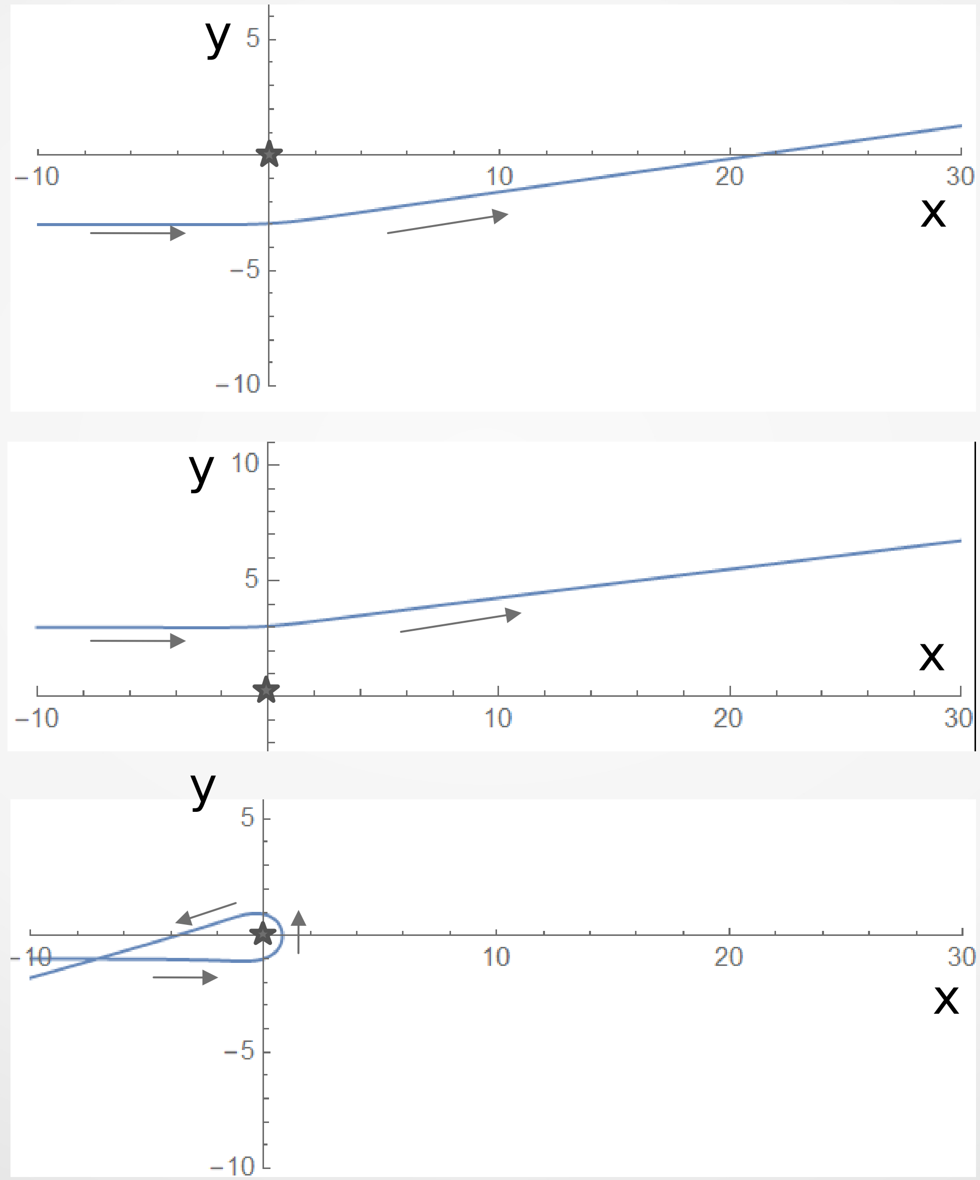
3) We consider the elastic scattering of an exciton by a single radially symmetrical impurity with $U_{sc}(\mathbf{R}) = U_{sc}(R)$ and $E(\mathbf{R}) = E(R)\mathbf{R}/R$. Scattering matrix elements between the exciton states with the center-of-mass momenta $\mathbf{k}$ and $\mathbf{k}'$:

$$V^{\rm dip}_{\mathbf{k},\mathbf{k}'} = U_{\rm sc}(\Delta\mathbf{k}) + \frac{i\hbar f\left[\mathbf{k}'\times\mathbf{k}\right]_z}{4\pi|\Delta\mathbf{k}|}\int_0^\infty {\rm d}R^2 E J_1(|\Delta\mathbf{k}|R)$$

Where: $U_{\rm sc}(\Delta\mathbf{k}) \equiv (2\pi)^{-1}\int_0^\infty R{\rm d}R U_{sc}(R) J_0(|\Delta\mathbf{k}|R)$

The most important result is: $V^{dip}_{\mathbf{k},\mathbf{k}'} \neq V^{dip}_{-\mathbf{k}',-\mathbf{k}}$

Phenomenological model – simple model



Microscopic theory

1) Hamiltonian:

$$\hat{H} = \frac{1}{2m_e}\left(-i\hbar\boldsymbol{\nabla}_e + e\mathbf{A}_e\right)^2 + \frac{1}{2m_h}\left(-i\hbar\boldsymbol{\nabla}_h - e\mathbf{A}_h\right)^2 - \frac{e^2}{4\pi\varepsilon_0\varepsilon|\mathbf{r}_e - \mathbf{r}_h|}$$

If we look for the wave function if the form: $\Psi_{\mathbf{k}}(\mathbf{R},\mathbf{r}) = \exp\left\{i\frac{\mathbf{R}}{\hbar}[\hbar\mathbf{k} + e\mathbf{A}(\mathbf{r})\right\}\Phi_{\mathbf{k}}(\mathbf{r})$, then Hamiltonian for $\Phi_{\mathbf{k}}(\mathbf{r})$ of the relative motion of an e – h pair will take the form:

$$\hat{H}_{\mathrm{rel}} = -\frac{\hbar^2}{2\mu}\nabla_{\mathrm{r}}^2 - \frac{i\hbar eB}{2}\left(\frac{1}{m_e} - \frac{1}{m_h}\right)[\mathbf{r}\times\nabla_{\mathbf{r}}]_z + \frac{e^2B^2}{8\mu}r^2 + \frac{e\hbar}{M}\mathbf{B}\cdot[\mathbf{r}\times\mathbf{k}] - \frac{e^2}{4\pi\varepsilon_0\varepsilon|\mathbf{r}|} + \frac{\hbar^2\mathbf{k}^2}{2M}$$

Our goal is to find scattering matrix elements: $V_{\mathbf{k},\mathbf{k}'} = \langle\Psi_{\mathbf{k}}|\hat{V}|\Psi_{\mathbf{k}'}\rangle$, where:
$\hat{V} = V_e(\mathbf{r}_e) + V_h(\mathbf{r}_h)$ — potential of impurity

Microscopic theory – Weak magnetic field

2) If one takes into account the terms up to the second order in B (weak field) and the lowest order in $|\mathbf{k}|a_\mathrm{B}$ (exciton is always in the ground state), perturbation theory yields:

$$V_{\mathbf{k},\mathbf{k}'} = \tilde{V}_e(\Delta\mathbf{k})\mathcal{F}_e(\Delta\mathbf{k}) + \tilde{V}_h(\Delta\mathbf{k})\mathcal{F}_h(\Delta\mathbf{k}) + i\,[\mathbf{k}' \times \mathbf{k}]_z\, a_\mathrm{B}^2 \left(\frac{a_\mathrm{B}}{l_B}\right)^2 \left(\tilde{V}_e(\Delta\mathbf{k})\alpha_e - \tilde{V}_h(\Delta\mathbf{k})\alpha_h\right)$$

Here:

- $\tilde{V}_j(\mathbf{k})$ — two-dimensional Fourier transform of the potentials $V_j(\mathbf{r})$
- $\mathcal{F}_{e(h)}(\Delta\mathbf{k}) = \left[1 - 3a_\mathrm{B}^2 m_{h(e)}^2 (\Delta\mathbf{k})^2 / \left(32M^2\right) + \beta_{e(h)}(\Delta\mathbf{k})^2 a_\mathrm{B}^2 \left(a_\mathrm{B}/\ell_B\right)^4\right]$
- $\Delta\mathbf{k} = \mathbf{k}' - \mathbf{k}$ — transferred momentum.
- $\beta_{e(h)} = 4^{-6}M^{-2}\left(105m_{h(e)}^2 - 159\mu^2/2\right)$
- $\alpha_{e(h)} = -21\mu/\left(16^2 M\right)$
- $a_B = 4\pi\varepsilon_0\varepsilon\hbar^2/(\mu e^2)$

One can see anisotropy in $V_{\mathbf{k},\mathbf{k}'}^{dip} \neq V_{-\mathbf{k}',-\mathbf{k}}^{dip}$

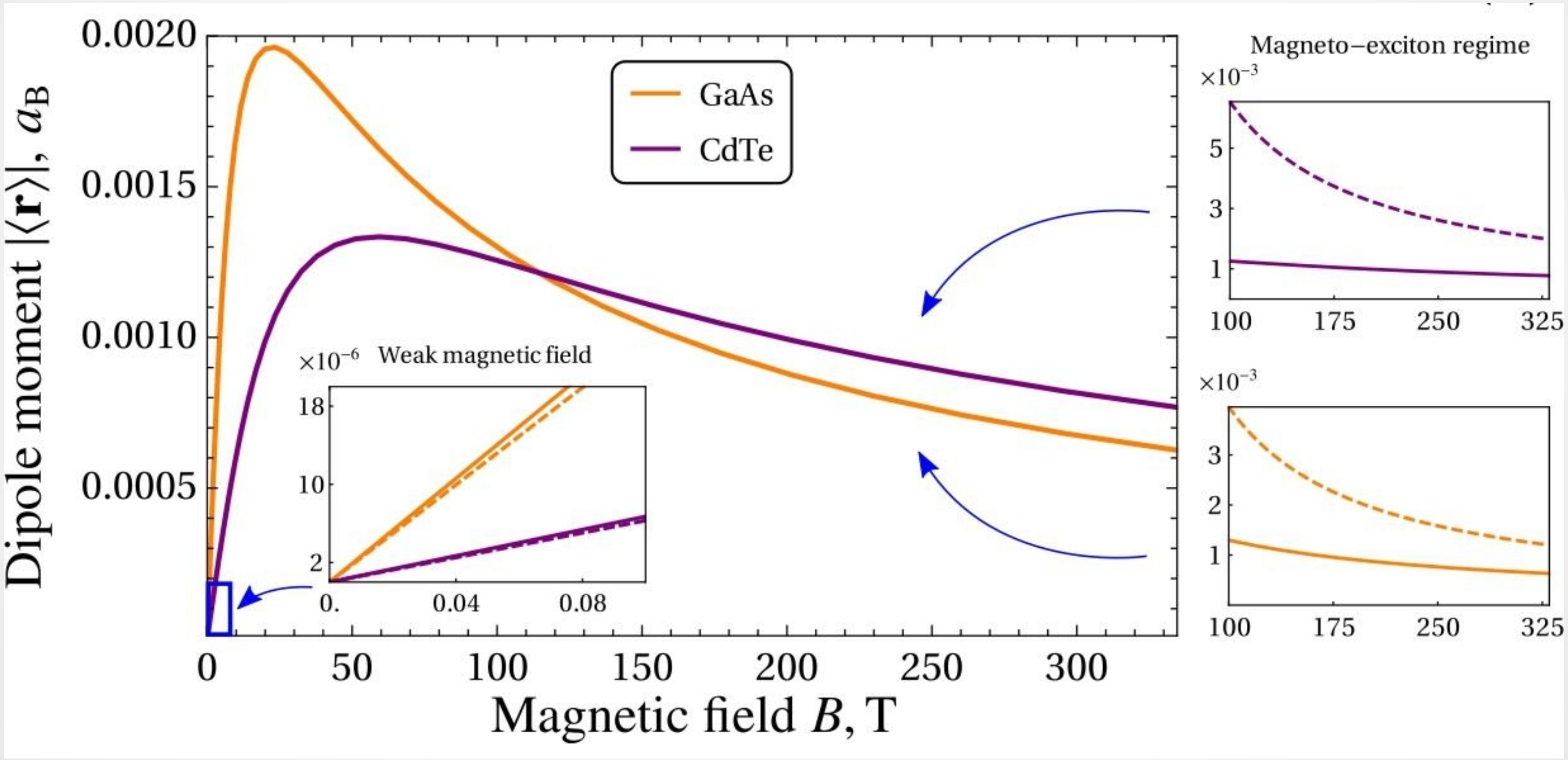
Microscopic theory – Magneto-exciton regime

3) In case of strong magnetic field scattering matrix elements are in the form:

$$V_{\mathbf{k},\mathbf{k}'} = \tilde{V}_e(\Delta\mathbf{k})\exp\left(-\frac{i}{2}\left[\mathbf{k}'\times\mathbf{k}\right]_z l_B^2 - \frac{\Delta\mathbf{k}^2 l_B^2}{4}\right) + \tilde{V}_h(\Delta\mathbf{k})\exp\left(+\frac{i}{2}\left[\mathbf{k}'\times\mathbf{k}\right]_z l_B^2 - \frac{\Delta\mathbf{k}^2 l_B^2}{4}\right)$$

(from: A. B. Dzyubenko, G. E. W. Bauer - Low-temperature transport of excitons in type-II GaAs/AlAs quantum wells)

Microscopic theory – Dipole moment



Numerical results for dipole moments of exciton for GaAs and CdTe.

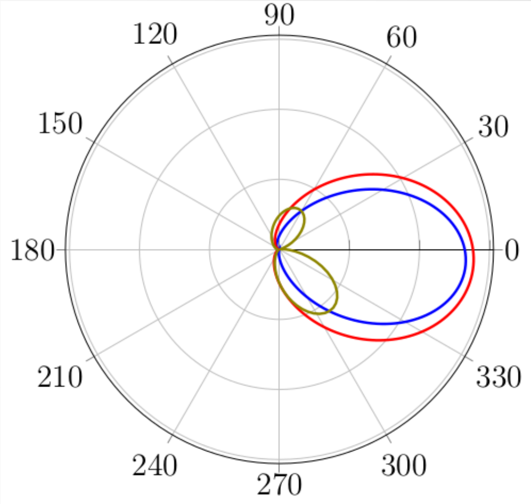
Scattering rate – Few words about T-matrix

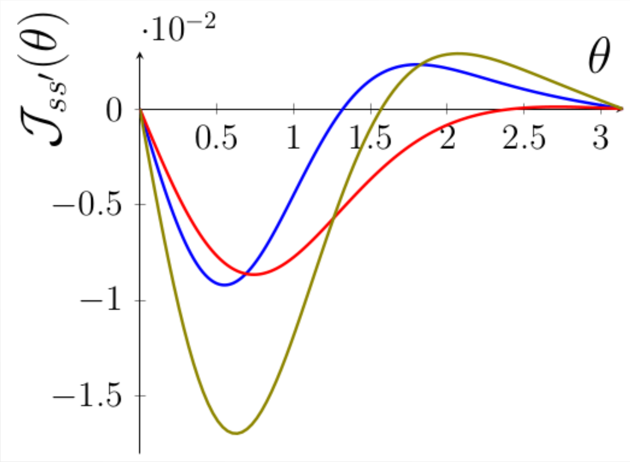
1) Scattering T-matrix:

$$T_{\mathbf{k},\mathbf{k}'} = \left\langle \Psi_{\mathbf{k}} | \hat{V} | \tilde{\Psi}_{\mathbf{k}'} \right\rangle$$

Where:

- $\hat{V} = V_e\left(\mathbf{r}_e\right) + V_h\left(\mathbf{r}_h\right)$ — the impurity potential operator
- $\left|\Psi_{\mathbf{k}}\right\rangle$ — the eigenstate of the free Hamiltonian $\hat{H}_0$
- $\left|\tilde{\Psi}_{\mathbf{k}'}\right\rangle$ — the eigenstate of the full Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}$





Symmetric and asymmetric dimensionless functions:

- $\nu_0^2 |T_{\mathbf{k},\mathbf{k}'}|^2 = \mathcal{G}_{\mathbf{k},\mathbf{k}'} + \mathcal{J}_{\mathbf{k},\mathbf{k}'}$
- $\mathcal{G}_{\mathbf{k},\mathbf{k}'} \equiv \mathcal{G}(\theta) = \mathcal{G}(-\theta)$
- $\mathcal{J}_{\mathbf{k},\mathbf{k}'} \equiv \mathcal{J}(\theta) = -\mathcal{J}(-\theta)$

(from: K.S. Denisov - Theory of an electron asymmetric scattering on skyrmion textures in two-dimensional systems)

Scattering rate – T-matrix

2) The asymmetric part $\mathcal{J}_{\mathbf{k},\mathbf{k}'}$ of exciton scattering by impurities gives rise to the Hall current. Our final goal is to calculate the Hall angle numerically.

3) The T-matrix satisfies the Lippmann-Schwinger equation:

$$T_{\mathbf{k},\mathbf{k}'} = V_{\mathbf{k},\mathbf{k}'} + \int \frac{d^2\mathbf{g}}{(2\pi)^2} \frac{V_{\mathbf{k},\mathbf{g}} T_{\mathbf{g},\mathbf{k}'}}{E - \epsilon(\mathbf{g}) + i0}$$

We assume: $V_e(r) = -V_h(r) = -eq_{\mathrm{imp}}/(4\pi\varepsilon_0\varepsilon r)$. In that case the equation above can not be solved using perturbation theory. Thus, we calculated T-matrix using numerical calculation.

Scattering rate – Derivation of the Hall angle

4) We used the semiclassical Boltzmann equation (classical transport regime $|k|l \gg 1$, where l is the exciton mean free path) to calculate the occupation numbers of exciton states $n_{\mathbf{k}}$ with wave vectors $\mathbf{k}$:

$$\frac{dn_{\mathbf{k}}}{dt} = P_{\mathbf{k}} - \Gamma n_{\mathbf{k}} + \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \left(W_{\mathbf{k},\mathbf{k}'} n_{\mathbf{k}'} - W_{\mathbf{k}',\mathbf{k}} n_{\mathbf{k}} \right)$$

Here $P_{\mathbf{k}}$ is the coherent pump term, $W_{\mathbf{k},\mathbf{k}'}$ — the elastic scattering rate and $\Gamma = 1/\tau_0$ is the particle decay rate.

5) We used Fermi golden rule to calculate elastic scattering rate:

$$W_{\mathbf{k},\mathbf{k}'} = 2\pi\hbar^{-1} n_{\mathrm{imp}} \left| T_{\mathbf{k},\mathbf{k}'} \right|^2 \delta \left(\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}'} \right)$$

Here: n_{imp} — the surface density of impurities.

Scattering rate – Derivation of the Hall angle

6) In the stationary regime, performing the integration over the absolute value of $\mathbf{k}'$:

$$0 = P_{\mathbf{k}} - \Gamma n_{\mathbf{k}} + \int d\varphi' \left(w_{\mathbf{k},\mathbf{k}'} n_{\mathbf{k}'} - w_{\mathbf{k}',\mathbf{k}} n_{\mathbf{k}} \right),$$

where $w_{\mathbf{k},\mathbf{k}'} = n_{\mathrm{imp}} \nu_0 / (\hbar \nu(\mathbf{k})^2) \left(\mathcal{G}_{\mathbf{k},\mathbf{k}'} + \mathcal{J}_{\mathbf{k},\mathbf{k}'} \right)$ and the exciton density of states $\nu(\mathbf{k}) = \left| \partial \epsilon(\mathbf{k}) / \partial k \right|^{-1} k/(2\pi)$.

Scattering rate – Derivation of the Hall angle

7) We represent $n_{\mathbf{k}} = n_0(k) + \delta n(\mathbf{k})$, where $\delta n(\mathbf{k}) = n_+(k)\cos\varphi + n_-(k)\sin\varphi$, $n_0(k)$ is the isotropic part of the distribution function which depends only on energy. Also we consider that $P_{\mathbf{k}} = P_0\delta(\mathbf{k}-\mathbf{k}_0) = (P_0/k)\delta(k-k_0)\delta(\varphi)$. Finally, we get:

$$0 = \frac{P_0}{k}\delta(k-k_0)\delta(\varphi) + \cos\varphi\left(\Omega(k)n_-(k) - \frac{n_+(k)}{\tau(k)}\right)$$
$$- \sin\varphi\left(\Omega(k)n_+(k) + \frac{n_-(k)}{\tau(k)}\right) - \frac{n_{\mathbf{k}}}{\tau_0},$$

where:

- $\theta = \varphi - \varphi'$ is the scattering angle.
- $\Omega(k) = -n_{\mathrm{imp}}\nu_0/\left(\hbar\nu(\mathbf{k})^2\right)\int_0^{2\pi}\mathcal{J}_{\mathbf{k},\mathbf{k}'}\sin\theta d\theta$
- $\tau(k)^{-1} = n_{\mathrm{imp}}\nu_0/(\hbar\nu(\mathbf{k})^2)\int_0^{2\pi}\mathcal{G}_{\mathbf{k},\mathbf{k}'}(1-\cos\theta)d\theta$

Scattering rate – Derivation of the Hall angle

8) Using the orthogonality of $\sin\varphi$ and $\cos\varphi$, we solved this equation and found $n_+(k)$ and $n_-(k)$, which we substitute into the definition of the Hall current j_y and the longitudinal current j_x:

$$j_{x,y} = \int \hbar k_{x,y} \delta n(\mathbf{k}) d^2\mathbf{k}/(2\pi)^2$$

And, eventually, obtain the Hall angle:

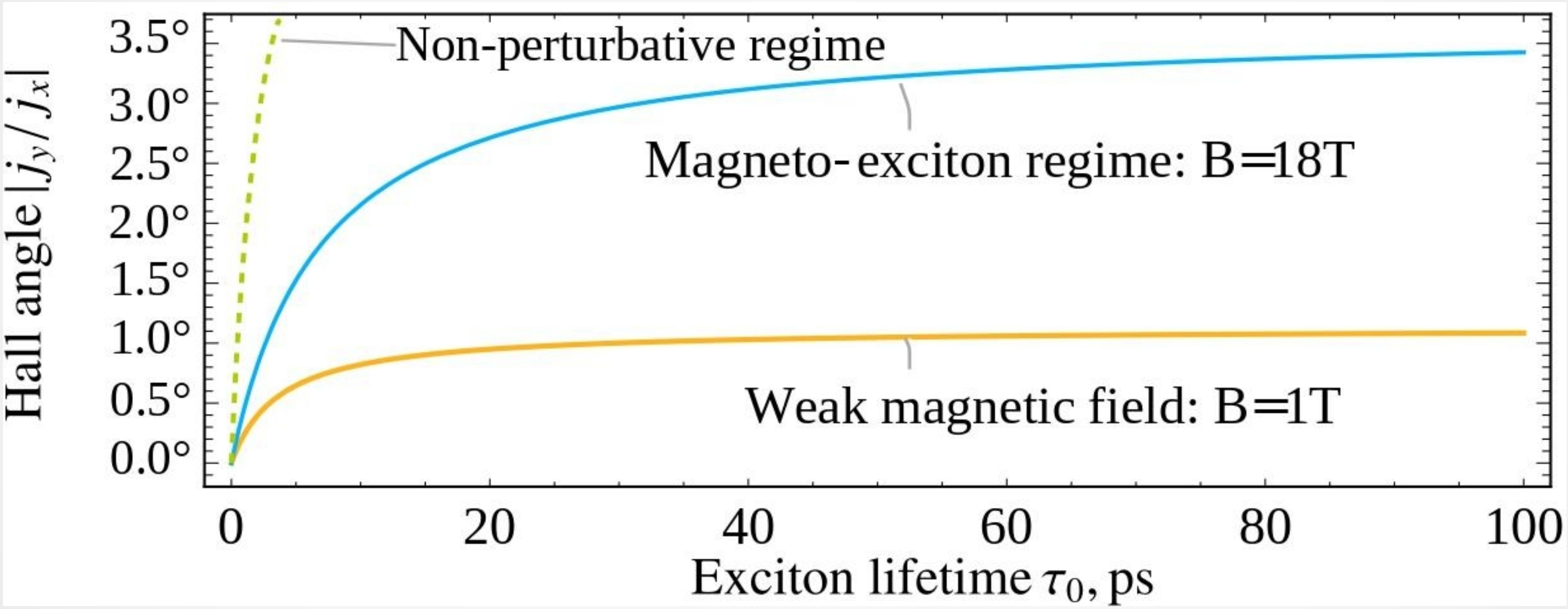
$$j_y/j_x = -\Omega(k_0)\tau_{\mathrm{tot}}(k_0),$$

where: $\tau_{\mathrm{tot}}(k) = (\tau_0^{-1} + \tau(k)^{-1})^{-1}$ — total relaxation time.

9) Numerical result for GaAs:

- Weak magnetic field: $j_y/j_x \approx 0.8\%$
- Strong magnetic field: $j_y/j_x \approx 1.8\%$

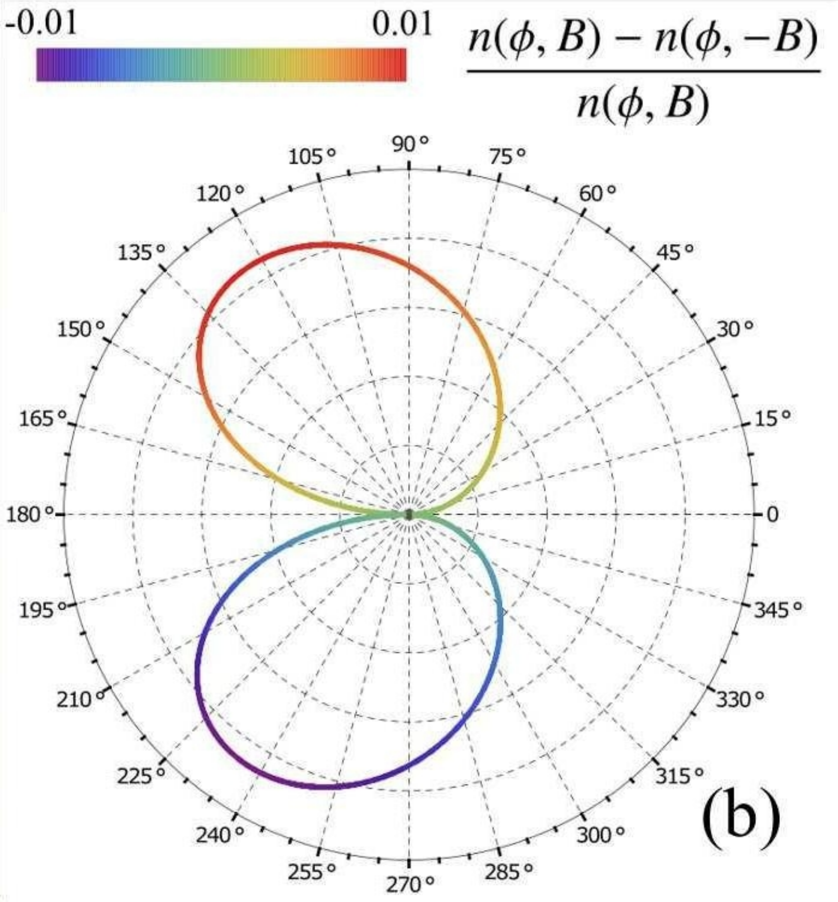
Scattering rate – The Hall angle



Numerical calculation of the Hall angle for excitons in GaAs

Scattering rate – How to measure anisotropy

The angular distribution of the exciton emission: $n(\phi) = \int\limits_0^\infty n_{\mathbf{k}} \frac{kdk}{(2\pi)^2}$,
where $n_{\mathbf{k}}$ — occupation numbers of exciton states having wave vectors $\mathbf{k}$.



Conclusion

The effect may have a significant magnitude in fluids of optically inactive, dark excitons due to their long lifetimes, and it is much weaker for shortliving bright excitons in conventional GaAs-based quantum wells. This makes the anomalous exciton Hall effect a powerful tool for spatial separation of dark and bright excitons.

Thank you for your attention!