



Time: 03 hours

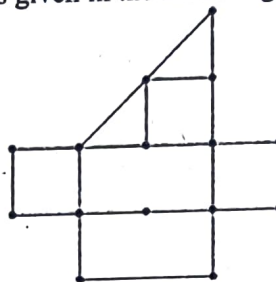
Total Marks: 90

[N.B. The figure in the right margin indicates the marks allocated for respective question.
 Split answer of any question is not allowed.]

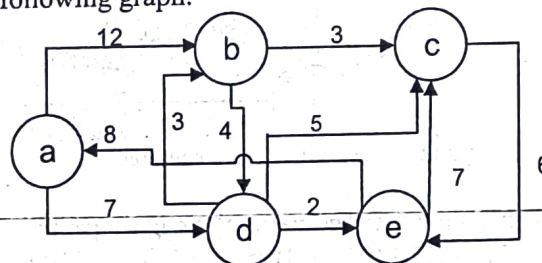
Section-A

(Answer any 03(three) from the following questions)

1. a) Define the terms: simple graph, complete graph, sub-graph, regular graph and star graph. 5
- b) Write the application of graph theory. 4
- c) A possible floor plan for a museum is given in the following graph. 2+2



- i) Can you find an Euler tour? If not then correct the floor plan for this purpose.
- ii) Can you find a Hamiltonian cycle? If not then correct the floor plan for this purpose. 1+1
- d) What does it mean for two graphs G_1 and G_2 to be isomorphic? Show an example. 1+1
2. a) What is connected component and k -edge connected? Draw a graph with 3 connected components. 2+2
- b) Using Dijkstra's algorithm, find the length of shortest path from the source vertex 'a' to each of the other vertices of the following graph. 9



- c) Differentiate between walk and path. 2
 3. a) What is complete bipartite graph? Give an example. 2+1
 - b) Define tree and spanning tree. Write the properties of a tree. 2+2
 - c) What is duality? Write the characteristic of dual graph. 2+3
 - d) Proof the theory, "A graph G is planar if and only neither K_5 nor $K_{3,3}$ is a minor of G ". 3
 4. a) Define planar graph, tournament graph and face with example. 6
 - b) Draw a planar graph with five faces. 3
 - c) Prove the following theory with a suitable example. 6
- "If a connected graph has planar embedding then $v - e + f = 2$ "
 Where v is the number of vertices, e is the number of edges and f is the number of faces.

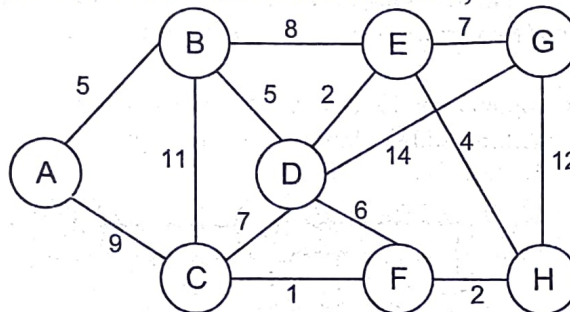
Section-B

(Answer any 03(three) from the following questions)

1. a) Let $G = (V, E)$ be a graph. Define any five from the following terms with example. 5
 - i. Trail
 - ii. Cycle
 - iii. Euler trail
 - iv. Euler tour
 - v. Hamiltonian cycle
 - vi. Hamiltonian path
- b) Give an example of a graph which is 6
 - i) Eulerian trail but not Eulerian tour.
 - ii) Hamiltonian cycle but not Eulerian tour.
 - iii) Eulerian tour but not Hamiltonian cycle.
- c) True or false: If a graph is bipartite then it is 2-colorable. Justify your answer. 2
- d) Draw the graph having following matrix as its adjacency matrix. 2

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

2. a) Define dual graph with an example. 3
- b) What is a Minimum Spanning Tree (MST)? Find the MST of the following graph using Prim's or Kruskal's algorithm. {Take vertex A as initial vertex if needed.} 2+8



- c) True or false: MST is always unique. Justify your answer. 1+1
3. Consider the network flow problem with the following edge capacities, $c(u, v)$ for edge (u, v) :
 $c(s, 2)=2, c(s, 3)=13, c(2, 5)=12, c(2, 4)=10, c(3, 4)=5, c(3, 7)=6, c(4, 5)=1, c(4, 6)=1, c(6, 5)=2, c(6, 7)=3, c(5, t)=6, c(7, t)=2$
 - a) Draw the network. 3
 - b) Find the maximum flow by using the Ford-Fulkerson algorithm also show each residual graph. 9
 - c) Show the minimum cut. 3
4. a) Define chromatic number. 2
- b) Prove the theory, "A graph with maximum degree at most k is $(k+1)$ colorable." 5
- c) A group of eight CSE students plans to take part in special repeated exams in the courses C_1, C_2, C_3, C_4, C_5 and C_6 as follows:

Student1: C_1, C_3	Student3: C_2, C_6	Student5: C_3, C_4	Student7: C_5, C_6
Student2: C_1, C_4, C_5	Student4: C_2, C_3, C_6	Student6: C_3, C_5	Student8: C_1, C_5

 - i) Now, schedule the exam by using coloring graph theory and consider that, a student can take only one test during a particular time slot. 5
 - ii) Determine the chromatic number of this problem. 1
 - iii) What does this number tell us in this case? 2